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Game-Theoretic Multi-Energy Trading Framework for Strategic Biogas-Solar Renewable Energy Provider with Heterogeneous Consumers

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Abstract

This paper proposes a game-theoretic multi-energy trading framework for a biogas-solar renewable energy provider with heterogeneous consumers to promote the efficient utilization of local renewable energy resources. Within the proposed framework, the multi-energy provider utilizes biogas-solar complementarities to enhance the operational flexibility for electricity, biogas, and heat energy supplies, and consumers are enabled to actively participate in demand response under different multi-energy prices from the provider and utility companies. A multi-energy pricing model is then proposed based on the Stackelberg game to interactively and dynamically determine the internal trading prices for optimal multi-energy trading between the provider and consumers. Furthermore, a bi-level optimization method is formulated to solve the Stackelberg game-based multi-energy trading problem to maximize the provider's profit at the upper level and the welfare of each consumer at the lower level. Case studies show that the provider's profit is improved by 33.69% using the proposed scheme compared with the benchmark scheme, and meanwhile the average welfare of consumers for consuming biogas and electricity are approximately 3.0 and 1.4 times greater than those using the benchmark scheme.

Highlights

Proposal of an interactive framework for multi-energy trading at the community level.
Development of a biogas-solar energy hub for flexible multi-carrier energy supplies.
Formulation of the internal multi-energy pricing problem into a Stackelberg game.
Utilization of bi-level optimization method to coordinate multi-user demand response.
Performances of enhancing economic benefits for both the provider and consumers.

Keywords: Energy hub, multi-energy trading, optimal pricing, demand response, renewable energy, bi-level optimization.

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1 Introduction

1.1 Motivation

Biogas is a renewable energy source produced by anaerobic digestion from widely available biomass such as crop residues, animal manure, and organic wastes. It can be used as a fuel or as a starting material for the production of biomethane, hydrogen, and syngas with high energy densities, and can then easily be captured in a gas storage tank [1]. Biogas is the low-cost fuel commonly used to replace traditional firewood and kerosene fuels in remote areas, especially in developing countries. It is reported in Ref. [2] that Global biogas usage has increased from an estimated 14.5 gigawatts (GW) in 2012 to 29.5 GW in 2022. Unlike wind and solar energies with strong intermittency and periodic variability, biogas energy is independent of climatic influences and can provide flexible energy generation with high predictability and controllability [3]. Instead of burning biomass materials, anaerobic digestion of biodegradable waste can effectively reduce the garbage treatment cost and pollutant emissions [4]. However, the metabolic rate of biogas fermentation decreases dramatically under low-temperature conditions as anaerobic microorganisms slow their metabolism [5]. To improve biogas production in cold climate regions, most anaerobic digesters are heated by thermal energy recovered from the combined heat and power (CHP) system, or else from a furnace when electricity is not generated. In addition, solar energy is progressively applied to heat anaerobic digesters as it is clean and abundantly available [6-9].

Biogas-based energy harvesting, storage, and management technologies can make sustainable electrification affordable and promote the development of electrification system applications in developing countries [3]. As a sustainable substitute for natural gas, biogas has enormous potential to be combined with CHP plants and photovoltaic thermal (PVT) systems to form multi-energy systems, especially in mountainous and remote regions [7,8]. The biogas-solar hybrid renewable energy system can provide increased system efficiency and greater energy supply capability, and is regarded as a multi-energy provider of diversified energy services to local heterogeneous consumers. In general, the provider strategically offers multi-energy prices to consumers for profit maximization considering operating costs, while consumers modify multi-energy consumption behaviours in response to varying energy prices for welfare maximization. Consumers will be attracted to trade energy with the provider once it offers lower energy prices; otherwise, they will spontaneously trade with the competitor of the provider. Therefore, it faces a challenge when dealing with interactions between the provider and heterogeneous consumers, where each participant strives to maximize its benefit. To analyse the situation in which the decision variables of participants are independently controlled, the game model is gradually introduced and applied in multi-energy systems. The entire system can be modelled as a one-leader multi-follower Stackelberg game, where the provider and consumers act as a leader and followers, respectively [10,11].

The multi-energy trading between the provider and consumers based on the Stackelberg game is a hierarchical non-cooperative decision problem that can be formulated as a bi-level optimization model [11,12]. At the upper level, the provider should offer reasonable energy prices to attract consumers to purchase energy during the competition with the utility companies, and solve the optimal operation problem through coordination of various energy conversion devices for profit maximization. At the lower level, consumers aim to purchase the optimal amounts of energy from the provider for maximum welfare in responding to the provider's pricing strategies. Consequently, consumers' behaviours in turn influence the provider's decision. In addition, unlike multi-energy systems with fossil fuel or natural gas as a primary energy source, the multi-energy system managed by the provider in this paper relies entirely on solar and biogas. Thus, it is a 100% renewable energy system and is termed the biogas-solar renewable energy provider (BSREP). Multi-energy operation without considering the interactions between the provider and consumers will result in inefficient utilization of biogas-solar renewable energy and unoptimized demand scheduling. Therefore, developing a multi-energy trading framework and trading mechanism is imperative for BSREP and heterogeneous consumers to fully utilize demand flexibility to promote the local consumption of biogas-solar renewable energy and the sustainable development of the multi-energy system. On the other hand, since the interest of the provider is conflicted with the self-interested energy consumption behaviours of consumers, the strategic behaviours of consumers should be considered during the energy pricing of the provider. Thus, the game-theoretic approach should be used to further investigate the multi-energy interactions between the provider and consumers to achieve a balanced split of benefits among participants.

1.2 Relevant Background

The energy hub (EH) concept was introduced in Ref. [13] to investigate the optimal power flows of multi-energy carriers and the coordination of various energy conversion devices. An EH model represents an interface between energy conversion infrastructures and various energy system carriers [14]. Much effort has been focused on the hub layout and topology [14-18], coupling matrix modelling [19], multi-energy flow analysis [18,19], and optimal energy management [16,17,20]. Furthermore, the multi-energy demand response has been integrated into EH to increase the reliability, resilience, and flexibility of energy supply [21,22]. With EH as technical support, the multi-energy system can integrate two or more local renewable resources at remote sites, such as wind-solar [19,23], geothermal-solar-wind [24] and wind-solar- hydro

[25]. Furthermore, the biogas-dominated energy hub is increasingly being applied in multi-energy systems to meet the various energy demands of hospitals, schools, and residential buildings in rural areas [23,26].

Much effort has been devoted to the optimal operation of the energy system with biogas technology in pursuit of economic and environmental benefits [27-29]. However, the lack of synergies between biogas and other types of energy results in a low efficiency of energy utilization. To solve this problem, recent research works [8,9,30,31] have studied the multi-energy complementarities between biogas and other energy sources. In particular, on the basis of digesting thermodynamic effects proposed in Ref. [9], the biogas-solar-wind energy complementarities were fully exploited to facilitate the mitigation of renewable intermittency, and renewable energy can be converted to biogas in a gas storage tank. Also, the authors in Ref. [9] adjusted biogas production by changing the fermentation temperature using heat recovered from PVT systems and CHP units. Coordination between energy providers and consumers can enhance renewable energy utilization and further improve system efficiency. However, ignoring the impact of multi-energy consumption behaviours of consumers on the system operation, the present biogas-solar energy system inevitably results in energy wastage and further makes it difficult to sustain the long-term economic and reliable operation.

It has been widely recognized that both energy providers and consumers can reciprocally benefit from direct energy trading [21]. In multi-energy trading markets, energy providers can design appropriate pricing mechanisms to influence consumer behaviours to cooperate with their energy production. A real-time price-based demand-response algorithm was developed in Ref. [32] for smart grids to achieve optimal load control of devices in a facility. A decision-making model for heat-power EH was proposed in Ref. [33] to maximize its profits by selling electricity and heat energy. Natural gas has been taken into consideration in Ref. [10] to provide energy supplies by the proposed energy pricing strategy, and the equilibrium of the coupled natural gas and electricity markets was studied considering the strategic offering behaviours of providers. The energy-sharing provider was proposed in Ref. [11] to facilitate the energy sharing of multiple PV prosumers, and a Stackelberg game formulated as a bi-level programming was adopted to design the dynamical pricing. However, all the aforementioned studies mainly focus on designing pricing mechanisms for a single type of energy and thus may not be applicable to the multi-energy trading framework for BSREP and heterogeneous consumers.

1.3 Contributions and Paper Organization

In this paper, a novel multi-energy trading framework is proposed for BSREP and heterogeneous consumers aiming at improving the efficient utilization of biogas and solar renewable energy and the benefits of all participants. The contributions of this paper are threefold as listed below:

(1) A multi-energy trading framework is proposed for BSREP and consumers. Within the proposed framework, the provider is able to optimize its internal operation and external transactions jointly. Solar and biomass energy can be converted into electricity and biogas for storage or selling to nearby consumers and utility companies. The biogas-solar complementarities are fully exploited on the basis of biogas digesting thermodynamics effects for the interactive coordination among multi-energy carriers including electricity and biogas.

(2) A multi-energy trading mechanism that allows heterogeneous consumers to actively participate in energy trading with the provider by introducing the welfare function is proposed, which promotes renewable energy consumption. In addition, to further model the strategic behaviours of the provider and consumers, a decision-making model based on a Stackelberg game is proposed to maximize the profit of the provider and the welfare of each consumer by using time-of-use energy prices.

(3) An optimal multi-energy pricing strategy is developed by constructing the energy trading mechanism as a bi-level optimization model. This multi-energy optimal pricing strategy can achieve the mutually beneficial outcome for the provider and consumers, and further accommodates renewable energy and improves energy utilization efficiency. The upper-level problem aims to maximize the provider's profit, while the lower-level problem represents the welfare maximization of each consumer. The bi-level optimization model is reformulated as a single-level one by replacing the lower-level problem with its Karush-Kuhn-Tucker (KKT) conditions.

The remainder of this paper is organized as follows: In Section 2, the market trading mechanism between BSREP and consumers is described. In Section 3, the model for the provider's optimal decision problem is presented. The proposed methodology is evaluated through test cases in Section 4. The main conclusions and contributions of the paper are discussed in Section 5.

2 Multi-Energy Trading Problem Formulation

2.1 Multi-Energy Trading Framework

The multi-energy trading framework among BSREP, utility companies (such as power grid and natural gas companies), and heterogeneous consumers is depicted in Fig. 1. For the provider, solar and biomass energy are imported into the EH and converted into electricity, heat and biogas to provide flexible and desirable multi-energy carriers for consumers and utility companies. In addition, biogas could be further purified into pipeline-quality natural gas, injected into the natural gas network, and sold to the natural gas company. On the demand side, consumers prefer to purchase heat, electricity and biogas energy from the provider due to its attractive energy prices and would trade energy with utility companies once the energy prices offered by the provider are higher than those offered by utility companies. Moreover, the provider will trade surplus energy with utility companies when the total available energy supply surpasses the total demand of consumers.

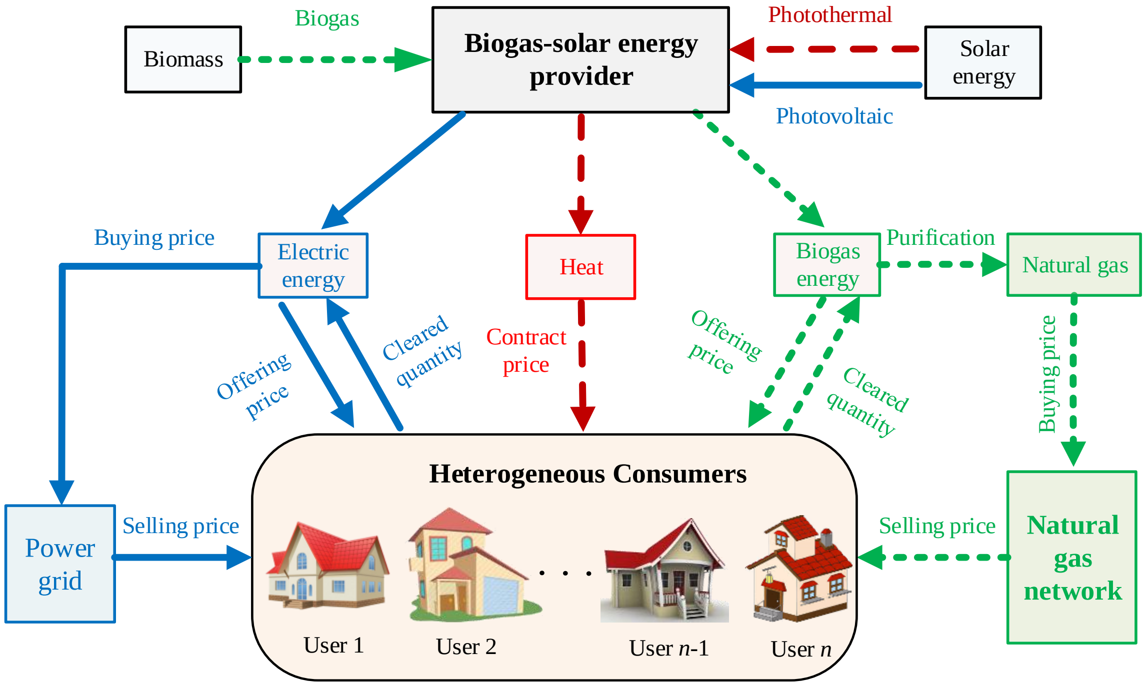

Fig. 1 Multi-energy trading framework.

As the provider and consumers are normally all self-interested, each participant strives to maximize its benefit during multi-energy trading. The provider must offer attractive energy prices to consumers to compete with utility companies. Given the energy prices, each consumer can exert influence on energy prices by manipulating their energy demand. For example, consumers reduce the energy purchased from the provider to make the provider passively lower the energy prices. The profit-seeking behaviours of the provider and consumers form a game relationship. The provider strategically offers multi-energy prices to consumers for profit maximization, while consumers influence the provider by optimizing their energy consumption for welfare maximization. Hence, a Stackelberg game model is proposed to analyse the situation in which the strategies of the provider and consumers are independently decided. For simplicity, the provider trades heat energy directly with consumers at a fixed price.

Throughout the paper, let $\mathcal{K}$ denote the set of indices of time periods, where $\mathcal{K} \triangleq \{1,2,\ldots,K\}$, and let $\mathcal{N}$ denote the set of indices of consumers, where $\mathcal{N} \triangleq \{1,2,\ldots,N\}$. The energy prices offered by the provider to consumers are denoted by

$$\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptc}} \triangleq \{\lambda_{\mathrm{e},1}^{\mathrm{ptc}}, \lambda_{\mathrm{e},2}^{\mathrm{ptc}}, \ldots, \lambda_{\mathrm{e},K}^{\mathrm{ptc}}\} \tag{1}$$

$$\boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptc}} \triangleq \{\lambda_{\mathrm{g},1}^{\mathrm{ptc}}, \lambda_{\mathrm{g},2}^{\mathrm{ptc}}, \ldots, \lambda_{\mathrm{g},K}^{\mathrm{ptc}}\} \tag{2}$$

$$\boldsymbol{\lambda}_{\mathrm{h}}^{\mathrm{ptc}} \triangleq \{\lambda_{\mathrm{h},1}^{\mathrm{ptc}}, \lambda_{\mathrm{h},2}^{\mathrm{ptc}}, \ldots, \lambda_{\mathrm{e},K}^{\mathrm{ptc}}\} \tag{3}$$

where $\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptc}}$, $\boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptc}}$, and $\boldsymbol{\lambda}_{\mathrm{h}}^{\mathrm{ptc}}$ are the set of the electricity, biogas, and heat prices offered by the provider to consumers, respectively.

Assuming that energy prices between the provider and utility companies are predetermined by the utility companies and are known to all entities. The prices are denoted by

$$\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptu}} \triangleq \{\lambda_{\mathrm{e},1}^{\mathrm{ptu}}, \lambda_{\mathrm{e},2}^{\mathrm{ptu}}, \ldots, \lambda_{\mathrm{e},K}^{\mathrm{ptu}}\} \tag{4}$$

$$\boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptu}} \triangleq \{\lambda_{\mathrm{g},1}^{\mathrm{ptu}}, \lambda_{\mathrm{g},2}^{\mathrm{ptu}}, \ldots, \lambda_{\mathrm{g},K}^{\mathrm{ptu}}\} \tag{5}$$

where $\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptu}}$ and $\boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptu}}$ are the set of electricity and biogas prices offered by the provider to utility companies, respectively.

Utility companies can also sell energy to consumers at a predetermined price once the provider cannot satisfy consumers' demands. The energy prices offered by utility companies to consumers are denoted by

$$\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{utc}} \triangleq \{\lambda_{\mathrm{e},1}^{\mathrm{utc}}, \lambda_{\mathrm{e},2}^{\mathrm{utc}}, \ldots, \lambda_{\mathrm{e},K}^{\mathrm{utc}}\} \tag{6}$$

$$\boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{utc}} \triangleq \{\lambda_{\mathrm{g},1}^{\mathrm{utc}}, \lambda_{\mathrm{g},2}^{\mathrm{utc}}, \ldots, \lambda_{\mathrm{g},K}^{\mathrm{utc}}\} \tag{7}$$

where $\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{utc}}$ and $\boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{utc}}$ are the set of electricity and biogas prices offered by the power grid and the natural gas company to consumers, respectively.

For guaranteeing the profitability, the provider must apply incentive measures to attract consumers to trade energy with the provider. Additionally, biogas should first be purified and then sold to the natural gas company. In this case, the constraints of energy prices of the provider are

$$\begin{cases} \lambda_{\mathrm{e},k}^{\mathrm{ptu}} \le \lambda_{\mathrm{e},k}^{\mathrm{ptc}} \le \rho_{\mathrm{e}} \lambda_{\mathrm{e},k}^{\mathrm{utc}}, \ \forall k \in \mathrm{K} \\ \lambda_{\mathrm{g},k}^{\mathrm{ptu}} \le \lambda_{\mathrm{g},k}^{\mathrm{ptc}} \le \rho_{\mathrm{g}} \varepsilon_{\mathrm{g}} \lambda_{\mathrm{g},k}^{\mathrm{utc}}, \ \forall k \in \mathrm{K} \end{cases} \tag{8}$$

where ρ_{e} and ρ_g are the coefficients for attracting consumers to trade directly with the provider at a lower energy price compare to utility companies, $0 < \rho_{\mathrm{e}} < 1$ and $0 < \rho_{\mathrm{g}} < 1$; ε_{g} is the ratio of the heat value of natural gas and biogas.

2.2 Multi-Energy Modelling of BSREP

A framework of the biogas-solar EH considered as BSREP is depicted in Fig. 2. The proposed biogas-solar EH is supplied by solar and biomass energy, which can be converted and conditioned through a PVT system and digester into different energy carriers, e.g., electricity, heat, and biogas. Diverse energy conversion and storage devices in the EH, such as CHP units, battery, biogas tanks, etc., are used to convert and control these energy carriers into the desired quality and quantity to supply the demand for electricity, heat, and biogas from consumers at the output ports. The biomass energy is converted directly into biogas through a digester, and then be transformed into electricity and heat by CHP units and furnace. The PVT system generates heat and electricity simultaneously from solar radiation, and the electricity generated can be further converted into heat by an electric boiler. Both the battery and the biogas tank can offer large storage capacities for the available electricity and biogas for later use once solar and biomass energy is insufficient.

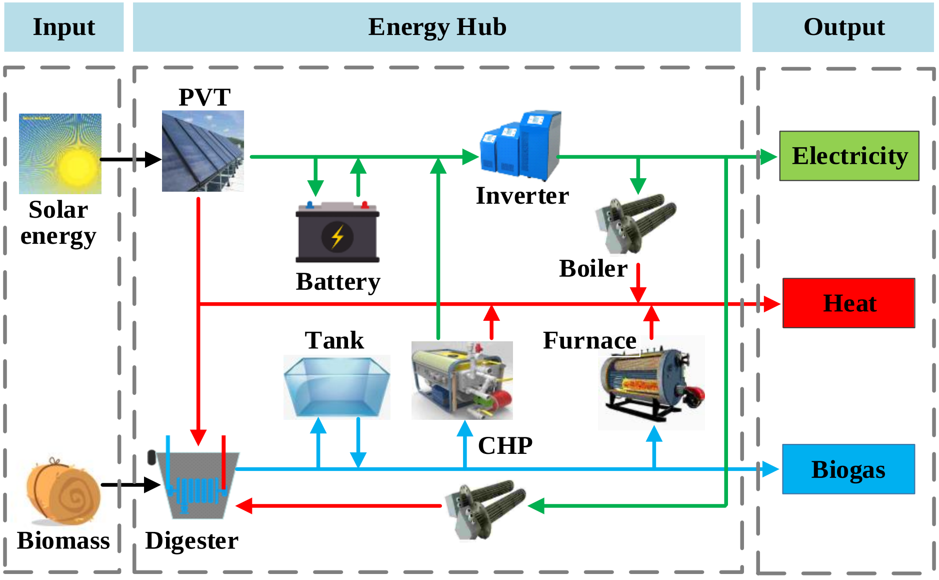

Fig. 2 Biogas-solar EH framework.

As the positive effect of temperature on the efficiency of anaerobic digestion, the available electricity and heat energy from solar energy can be recovered to heat the digester to accelerate the microbial fermentation for increasing biomethane yields. Thus, the provider can take advantage of the biogas-solar complementarities based on biogas digesting thermodynamics effects to improve system energy efficiency for achieving a high profit and reliable multi-energy supply [9]. To analyse the conversion relationship among the devices in the EH, some formulas are proposed to model the conversion of different energy sources, as shown in (9)-(11).

$$f_{\mathrm{e,PVT}} G_{\mathrm{PVT},k} + P_{\mathrm{BES},k} - \frac{S_{\mathrm{B},k}}{\eta_{\mathrm{B}}} + S_{\mathrm{CHP},k} - S_{\mathrm{eg},k} = P_k^{\mathrm{ptc}} + P_k^{\mathrm{ptu}} \tag{9}$$

$$f_{\mathrm{h,PVT}} G_{\mathrm{PVT},k} + S_{\mathrm{B},k} + \frac{\eta_{\mathrm{h,CHP}} S_{\mathrm{CHP},k}}{\eta_{\mathrm{e,CHP}}} + S_{\mathrm{F},k} - S_{\mathrm{hg},k} = H_k^{\mathrm{ptc}} \tag{10}$$

$$E_{\mathrm{bio},k} + V_{\mathrm{GS},k} - \frac{S_{\mathrm{CHP},k}}{Q_{\mathrm{bio}} \eta_{\mathrm{e,CHP}}} - \frac{S_{\mathrm{F},k}}{Q_{\mathrm{bio}} \eta_{\mathrm{F}}} = G_k^{\mathrm{ptc}} + G_k^{\mathrm{ptu}} \tag{11}$$

where P_k^{ptc}, H_k^{ptc} and G_k^{ptc} are total electricity, heat, and biogas purchased by consumers from the provider at period *k*, respectively; $P_k^{\mathrm{ptc}} = \sum_{i=1}^{N} P_{i,k}^{\mathrm{ptc}}$, $H_k^{\mathrm{ptc}} = \sum_{i=1}^{N} H_{i,k}^{\mathrm{ptc}}$ and $G_k^{\mathrm{ptc}} = \sum_{i=1}^{N} G_{i,k}^{\mathrm{ptc}}$, where $P_{i,k}^{\mathrm{ptc}}$, $H_{i,k}^{\mathrm{ptc}}$ and $G_{i,k}^{\mathrm{ptc}}$ are the electricity, heat and biogas purchased by consumer *i* at period *k*, respectively; P_k^{ptu} and G_k^{ptu} are the electricity and biogas sold from the provider to utility companies at period *k*, respectively; $G_{\mathrm{PVT},k}$

and $E_{\mathrm{bio},k}$ are solar energy and biogas inputs at period *k*, respectively; $P_{\mathrm{BES},k}$ and $V_{\mathrm{GS},k}$ are the net outputs of electricity and biogas for battery and biogas tank at period *k*, respectively; $f_{\mathrm{e,PVT}}$ and $f_{\mathrm{h,PVT}}$ are electrical and thermal efficiency of the PVT system, respectively; $\eta_{\mathrm{e,CHP}}$ and $\eta_{\mathrm{h,CHP}}$ are the gas-electric and gas-thermal efficiencies of CHP unit, respectively; η_{B} and η_{F} are conversion efficiencies of electric boiler and furnace, respectively; $S_{\mathrm{B},k}$ and $S_{\mathrm{F},k}$ are the thermal output of electric boiler and furnace at period *k*, respectively; $S_{\mathrm{CHP},k}$ is the electricity output of CHP unit at period *k*; $S_{\mathrm{eg},k}$ and $S_{\mathrm{hg},k}$ are the electricity and thermal energy for heating digester at period *k*, respectively; Q_{bio} is the heat value of biogas.

Although biogas production increases as the internal temperature of the digester, the over-temperature environment will inactivate microorganisms. In this paper, fresh swine manure is used as the feedstock for anaerobic digestion and the biogas production is a function of digestion temperature according to the data in Ref. [34], as follows,

$$E_{\mathrm{bio},k} = \mathrm{m}_1 (T_{\mathrm{d},k} - T_{\mathrm{o}})^2 + \mathrm{m}_2 \tag{12}$$

where $T_{\mathrm{d},k}$ is the actual digestion temperature at period *k*; T_{o} is the optimal temperature for most of mesophilic organisms, set to 35 °C; m_1 and m_2 are the coefficients obtained from the data fitting.

A complex thermal interaction model based on RC thermal network is adopted to investigate the biogas digesting thermodynamic effects under external energy injections, and can be further transformed into a linearized state-space realization model. Its concise form is shown in (13). The interested reader can refer to a previous work Ref. [9] of the authors for more information on it.

$$\begin{cases} x_{k+1}(T_{\mathrm{d}}) = Ax_k(T_{\mathrm{d}}) + Bu_k(\eta_{\mathrm{B}} S_{\mathrm{eg}}, S_{\mathrm{hg}}) + Dd_k(T_{\mathrm{out}}) \\ y_k(T_{\mathrm{d}}) = Yx_k(T_{\mathrm{d}}) \end{cases} \tag{13}$$

where x_k is the state vector denoting the temperatures of the nodes in the thermal network at period *k*; y_k is the output vector denoting the temperature of the digesting zone at period *k*; u_k represents the input vector of controllable electricity and thermal energy for digester heating at period *k*; d_k stores the uncontrollable inputs and system disturbances at period *k*, e.g., ambient temperature T_{out}; *A*, *B*, *D*, and *Y* are the state matrices corresponding to the vectors, respectively.

Formulas (9)-(11) can be expressed as a matrix to indicate the coupling between the operation status of different devices and output/input variables, as follows,

$$\begin{bmatrix} P_k^{\text{ptc}} + P_k^{\text{ptu}} \\ H_k^{\text{ptc}} \\ G_k^{\text{ptc}} + G_k^{\text{ptu}} \end{bmatrix} = \begin{bmatrix} f_{\text{e,PVT}} & 0 & 1 & 0 & -1/\eta_{\text{B}} & 1 & 1 & -1 & 0 \\ f_{\text{h,PVT}} & 0 & 0 & 0 & 1 & \eta_{\text{h,CHP}}/\eta_{\text{e,CHP}} & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1/(Q_{\text{bio}}\eta_{\text{e,CHP}}) & -1/(Q_{\text{bio}}\eta_{\text{F}}) & 0 & 0 \end{bmatrix} \begin{bmatrix} G_{\text{PVT},k} \\ E_{\text{bio},k} \\ P_{\text{BES},k} \\ V_{\text{GS},k} \\ S_{\text{B},k} \\ S_{\text{CHP},k} \\ S_{\text{F},k} \\ S_{\text{eg},k} \\ S_{\text{hg},k} \end{bmatrix} \tag{14}$$

There are multiple constraints during the operating time of the provider, as shown below.

1) Battery Capacity Constraints

$$E_{\text{BES},k} = E_{\text{BES},k-1} + P_{\text{BES},k}\Delta k \tag{15}$$

$$P_{\text{BES},k} = \eta_{\text{ch}} P_{\text{ch},k} - P_{\text{dis},k}/\eta_{\text{dis}} \tag{16}$$

$$E_{\text{BES,min}} \le E_{\text{BES},k} \le E_{\text{BES,max}} \tag{17}$$

$$E_{\text{BES},K} = E_{\text{BES},0} \tag{18}$$

where $P_{\text{ch},k}$ and $P_{\text{dis},k}$ are the charging and discharging power at period k, respectively; η_{ch} and η_{dis} are the charging and discharging efficiency, respectively; $E_{\text{BES},k}$ represents the battery capacity at period k; $E_{\text{BES,min}}$ and $E_{\text{BES,max}}$ are the lower and upper bounds of battery capacity, respectively; $E_{\text{BES},0}$ is the initial capacity of the battery.

2) Battery Charging/Discharging Rate Constraints

$$0 \le P_{\text{ch},k} \le P_{\text{ch,max}} \tag{19}$$

$$0 \le P_{\text{dis},k} \le P_{\text{dis,max}} \tag{20}$$

where $P_{\text{ch,max}}$ and $P_{\text{dis,max}}$ are the allowable maximum charging and discharging power, respectively. Generally, there are binary variables or complementary constraint $P_{\text{ch},k}P_{\text{dis},k} = 0$ to control the state of battery energy flow (charging or discharging). However, numerous studies such as Ref. [42] have clarify that the binary variables and this complementary constraint can be omitted, and there will always be the case that only one of the variables $P_{\text{ch},k}$ and $P_{\text{dis},k}$ can be non-zero at a time in an optimal solution of the optimization model of the provider. Therefore, the battery model represented by (15)-(20) is still valid without introducing additional non-convexity.

3) CHP Unit, Boiler, and Furnace Constraints

$$S_{l,\min} \le S_{l,k} \le S_{l,\max} \quad \forall l \in \{\text{CHP, B, F}\} \tag{21}$$

$$-\Delta S_{\text{CHP,ramp}} \le \frac{1}{\Delta t}\left(S_{\text{CHP},k} - S_{\text{CHP},k-1}\right) \le \Delta S_{\text{CHP,ramp}} \tag{22}$$

where $S_{l,\min}$ and $S_{l,\max}$ are the lower and upper bounds of outputs of device l, respectively; $\Delta S_{\text{CHP,ramp}}$ is the ramp rate of CHP unit.

4) Biogas Tank Constraints

$$E_{\text{gas},k} = E_{\text{gas},k-1} - V_{\text{GS},k-1}\Delta k \tag{23}$$

$$E_{\text{gas,min}} \le E_{\text{gas},k} \le E_{\text{gas,max}} \tag{24}$$

$$V_{\text{GS,min}} \le V_{\text{GS},k} \le V_{\text{GS,max}} \tag{25}$$

$$E_{\text{gas},K} = E_{\text{gas},0} \tag{26}$$

where $E_{\text{gas},k}$ is the volume of biogas in the biogas tank at period *k*; $E_{\text{gas,min}}$ and $E_{\text{gas,max}}$ are the lower and upper bounds of volume of biogas tank , respectively; $V_{\text{GS,min}}$ and $V_{\text{GS,max}}$ are the lower and upper bounds of biogas tank output, respectively; $E_{\text{gas},0}$ is the initial capacity of the biogas tank.

5) Digesting Temperature Constraint

The digestion temperature should be constrained to ensure mesophilic conditions and the survival of anaerobic organisms, as follows,

$$T_{\text{d,min}} \le T_{\text{d},k} \le T_{\text{d,max}} \tag{27}$$

Where $T_{\text{d,min}}$ and $T_{\text{d,max}}$ are the lower and upper bounds of the digestion temperature, respectively.

2.3 Multi-Energy Demand Response of Consumers

Different consumers would purchase different amounts of energy according to their heterogeneous consumption preferences. The consumer's utility is defined as the satisfaction gained from consuming electricity and biogas energy, which are related to the amounts of energy purchased from the provider and utility companies [35]. In this paper, a widely used quadratic function is adopted to measure the consumer's utility [36], which is strictly concave and continuously differentiable, as follows,

$$U_{\text{e},i,k} = b_{i,k}P_{i,k} - a_{i,k}P_{i,k}^2 \tag{28}$$

$$U_{\text{g},i,k} = d_{i,k}G_{i,k} - c_{i,k}G_{i,k}^2 \tag{29}$$

where $U_{\text{e},i,k}$ and $U_{\text{g},i,k}$ are the consumer *i*'s utility function for consuming electricity and biogas energy at period *k*, respectively; $P_{i,k}$ and $G_{i,k}$ are the electricity and biogas demand of the consumer *i*, respectively; $a_{i,k}$, $b_{i,k}$, $c_{i,k}$ and $d_{i,k}$ are the weight coefficients of the quadratic utility function. Variation of weight coefficients at different time slots of the utility function can capture the dynamics of consumers' demand.

When the energy prices offered by the provider to consumers are higher than those offered by utility companies, consumers would purchase energy from utility companies. Therefore, the electricity demand $P_{i,k}$ and the biogas demand $G_{i,k}$ of the consumer *i* are met by energy purchased from both the provider and utility companies, as follows,

$$P_{i,k} = P_{i,k}^{\text{ptc}} + P_{i,k}^{\text{utc}} \tag{30}$$

$$\begin{cases} P_{i,k}^{\text{utc}} \geq 0 \quad \text{if } \lambda_{\text{e},k}^{\text{ptc}} \leq \lambda_{\text{e},k}^{\text{utc}} \\ P_{i,k}^{\text{ptc}} = 0 \quad \text{if } \lambda_{\text{e},k}^{\text{ptc}} > \lambda_{\text{e},k}^{\text{utc}} \end{cases} \tag{31}$$

$$G_{i,k} = G_{i,k}^{\text{ptc}} + G_{i,k}^{\text{utc}} \tag{32}$$

$$\begin{cases} G_{i,k}^{\text{utc}} \geq 0 \quad \text{if } \lambda_{\text{g},k}^{\text{ptc}} \leq \lambda_{\text{g},k}^{\text{utc}} \\ G_{i,k}^{\text{ptc}} = 0 \quad \text{if } \lambda_{\text{g},k}^{\text{ptc}} > \lambda_{\text{g},k}^{\text{utc}} \end{cases} \tag{33}$$

where $P_{i,k}^{\text{utc}}$ and $G_{i,k}^{\text{utc}}$ are the electricity and biogas purchased by the consumer *i* from the power grid and the natural gas company, respectively.

Then, the welfare of the consumer *i* is defined as the difference between its utility and its energy usage cost, as follows,

$$S_{i,k} = S_{\text{e},i,k} + S_{\text{g},i,k} \tag{34}$$

$$S_{\text{e},i,k} = \begin{cases} U_{\text{e},i,k} - (\lambda_{\text{e},k}^{\text{ptc}} P_{i,k}^{\text{ptc}} + \lambda_{\text{e},k}^{\text{utc}} P_{i,k}^{\text{utc}}) & \text{if } \lambda_{\text{e},k}^{\text{ptc}} \leq \lambda_{\text{e},k}^{\text{utc}} \\ U_{\text{e},i,k} - \lambda_{\text{e},k}^{\text{utc}} P_{i,k}^{\text{utc}} & \text{if } \lambda_{\text{e},k}^{\text{ptc}} > \lambda_{\text{e},k}^{\text{utc}} \end{cases} \tag{35}$$

$$S_{\text{g},i,k} = \begin{cases} U_{\text{g},i,k} - (\lambda_{\text{g},k}^{\text{ptc}} G_{i,k}^{\text{ptc}} + \lambda_{\text{g},k}^{\text{utc}} G_{i,k}^{\text{utc}}) & \text{if } \lambda_{\text{g},k}^{\text{ptc}} \leq \lambda_{\text{g},k}^{\text{utc}} \\ U_{\text{g},i,k} - \lambda_{\text{g},k}^{\text{utc}} G_{i,k}^{\text{utc}} & \text{if } \lambda_{\text{g},k}^{\text{ptc}} > \lambda_{\text{g},k}^{\text{utc}} \end{cases} \tag{36}$$

where $S_{i,k}$ is the welfare of the consumer *i* at period *k*; $S_{\text{e},i,k}$ and $S_{\text{g},i,k}$ are the welfare from consuming electricity and biogas energy, respectively.

Hence, an optimization problem based on the consumer's welfare is formulated to achieve the demand response, that is

$$\max_{P_{i,k}, G_{i,k}} S_{i,k} \tag{37}$$

Given the constraint (8), the provider offers lower energy prices to consumers compared to the utility companies. On the other hand, when the energy produced by the provider is sufficient to meet the demand of consumers, consumers would purchase energy entirely from the provider, thus $P_{i,k}^{\text{utc}} = 0$. In this case, $S_{\text{e},i,k}$ and $S_{\text{g},i,k}$ can be formulated as follows,

$$\begin{cases} S_{\text{e},i,k} = U_{\text{e},i,k} - \lambda_{\text{e},k}^{\text{ptc}} P_{i,k} \\ S_{\text{g},i,k} = U_{\text{g},i,k} - \lambda_{\text{g},k}^{\text{ptc}} G_{i,k} \end{cases} \tag{38}$$

As the consumer's demand is elastic, the consumer can always attain optimal welfare by adjusting its demand to a point. When the marginal utility equals the energy price, each consumer attains the maximum welfare according to the optimality condition $\nabla S_{i,k} = 0$:

$$P_{i,k}^{*} = \frac{b_{i,k} - \lambda_{\text{e},k}^{\text{ptc}}}{2a_{i,k}} \tag{39}$$

$$G_{i,k}^{*} = \frac{d_{i,k} - \lambda_{\text{g},k}^{\text{ptc}}}{2c_{i,k}} \tag{40}$$

where $P_{i,k}^{*}$ and $G_{i,k}^{*}$ are the optimal consumption of electricity and biogas of the consumer *i* at period *k*, respectively, and they correspond to the maximum welfare of the consumer *i*.

Without loss of generality, the energy consumption of each consumer must be within a bounded range $[P_{i,k}^{\min}, P_{i,k}^{\max}]$ and $[G_{i,k}^{\min}, G_{i,k}^{\max}]$, where $P_{i,k}^{\min}$ (resp. $G_{i,k}^{\min}$) and $P_{i,k}^{\max}$ (resp. $G_{i,k}^{\max}$) represent the lower and upper bounds of the electricity (resp. biogas) required by the consumer *i* at period *k*, respectively.

3 Optimal Decision of BSREP on the Stackelberg Game

3.1 Profit of BSREP

As the energy input of the provider comes entirely from solar and biomass energy without energy production cost, the proposed optimal scheduling scheme aims to control various devices within the EH for optimal coordination among multi-carrier energy conversions, complementary cooperation between bio-mass and solar energy, and optimal charging and discharging behaviours of the battery. Thus, the operating costs of BSREP include the costs incurred in the production, conversion, and storage, specifically, the op-erating and maintenance costs due to the energy conversions of various converters C_{L} and the battery degradation cost C_{B}, as follows,

$$C_{\mathrm{L}} = \sum_{k=1}^{K} (\mu_{\mathrm{B}} S_{\mathrm{B},k} + \mu_{\mathrm{B}} \eta_{\mathrm{B}} S_{\mathrm{eg},k} + \mu_{\mathrm{CHP}} S_{\mathrm{CHP},k} + \mu_{\mathrm{F}} S_{\mathrm{F},k}) \Delta k \tag{41}$$

$$C_{\mathrm{B}} = \sum_{k=1}^{K} \frac{q}{\delta \eta_{\mathrm{sr}}} (P_{\mathrm{ch},k} + P_{\mathrm{dis},k}) \Delta k \tag{42}$$

where μ_{CHP}, μ_{B} and μ_{F} are the unit costs of the operating and maintenance of the CHP unit, electric boiler, and biogas furnace, respectively; q and δ denote the replacement cost and lifetime throughput of battery; η_{sr} is the square root of the roundtrip efficiency of the battery.

Thus, the provider's profit R_{P} consists of revenue from energy sales and operating costs, which can be formulated as

$$R_{\mathrm{P}} = \sum_{k=1}^{K} \left[(\lambda_{\mathrm{e},k}^{\mathrm{ptc}} P_k^{\mathrm{ptc}} + \lambda_{\mathrm{g},k}^{\mathrm{ptc}} G_k^{\mathrm{ptc}} + \lambda_{\mathrm{h},k}^{\mathrm{ptc}} H_k^{\mathrm{ptc}}) + (\lambda_{\mathrm{e},k}^{\mathrm{ptu}} P_k^{\mathrm{ptu}} + \lambda_{\mathrm{g},k}^{\mathrm{ptu}} G_k^{\mathrm{ptu}}) \right] \Delta k - (C_{\mathrm{L}} + C_{\mathrm{B}}) \tag{43}$$

3.2 Optimal Pricing Strategy of BSREP

The multi-energy trading problem between BSREP and consumers can be described as a Stackelberg game with BSREP as the leader and consumers as followers. This game is formally defined by its strategic form [11], as follows,

$$G = \left\{ (\mathrm{N} \cup \mathrm{L}); \{\mathbf{P}_k, \mathbf{G}_k\}, \{\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptc}}, \boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptc}}\}; \{\boldsymbol{S}_k\}, \{R_{\mathrm{P}}\} \right\} \tag{44}$$

where Λ denotes the set of leaders, with $\Lambda := \{\mathrm{BSREP}\}$ since BSREP is the only one leader in the game

G; $\{\boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptc}}, \boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptc}}\}$ is the strategy set of BSREP; $\{\mathbf{P}_k, \mathbf{G}_k\}$ is the strategy set of all consumers; R_{p} is the profit of BSREP determined by (43); $\boldsymbol{S}_k$ is the welfare of consumers determined by (34).

A set of strategies ($\lambda_{\mathrm{e},k}^{\mathrm{ptc}*}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}*}, \mathbf{P}_{i,k}^{*}, \mathbf{G}_{i,k}^{*}$) forms a Stackelberg equilibrium (SE) of this game, if and only if the following series of inequalities are satisfied [11]:

$$S_{\mathrm{e},i,\tau}(\mathbf{P}_{i,k}^{*}, \lambda_{\mathrm{e},k}^{\mathrm{ptc}*}) \geq S_{\mathrm{e},i,k}(P_{i,k}, \mathbf{P}_{-i,k}^{*}, \lambda_{\mathrm{e},k}^{\mathrm{ptc}*}), \quad \forall P_{i,k} \in \mathbf{P}_{i,k} \tag{45}$$

$$S_{\mathrm{g},i,k}(\mathbf{G}_{i,k}^{*}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}*}) \geq S_{\mathrm{g},i,k}(G_{i,k}, \mathbf{G}_{-i,k}^{*}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}*}), \quad \forall G_{i,k} \in \mathbf{G}_{i,k} \tag{46}$$

$$R_{\mathrm{p}}(\mathbf{P}_{i,k}^{*}, \mathbf{G}_{i,k}^{*}, \lambda_{\mathrm{e},k}^{\mathrm{ptc}*}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}*}) \geq R_{\mathrm{p}}(\mathbf{P}_{i,k}^{*}, \mathbf{G}_{i,k}^{*}, \lambda_{\mathrm{e},k}^{\mathrm{ptc}}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}}), \ \forall \lambda_{\mathrm{e},k}^{\mathrm{ptc}} \in \boldsymbol{\lambda}_{\mathrm{e}}^{\mathrm{ptc}}, \forall \lambda_{\mathrm{g},k}^{\mathrm{ptc}} \in \boldsymbol{\lambda}_{\mathrm{g}}^{\mathrm{ptc}} \tag{47}$$

where $\mathbf{P}_{-i,k}^{*} \triangleq [P_{1,k}^{*}, P_{2,k}^{*}, ..., P_{i-1,k}^{*}, P_{i+1,k}^{*}, ..., P_{N,k}^{*}]$ and $\mathbf{G}_{-i,k}^{*} \triangleq [G_{1,k}^{*}, G_{2,k}^{*}, ..., G_{i-1,k}^{*}, G_{i+1,k}^{*}, ..., G_{N,k}^{*}]$.

When all the participants in game G are at an SE, BSREP cannot improve its profit by adjusting energy prices from the SE prices ($\lambda_{\mathrm{e},k}^{\mathrm{ptc}*}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}*}$), and similarly, no consumer can increase its welfare by adjusting its energy consumption ($P_{i,k}, G_{i,k}$).

The constraint (12) is nonconvex, which not only increases the computational complexity of the optimization problem but also is an obstacle to obtaining the SE of the Stackelberg game model in this paper. According to Ref. [37], constraint (12) can be approximated by a quadratic function with negative quadratic coefficients and a linear function, as shown in Fig. 3. Therefore, constraint (12) can be replaced by

$$E_{\mathrm{bio},k} \leq \mathrm{m}_1(T_{\mathrm{d},k} - T_{\mathrm{o}})^2 + \mathrm{m}_2 \tag{48}$$

$$E_{\mathrm{bio},k} \geq \mathrm{m}_1(T_{\mathrm{d,max}} + T_{\mathrm{d,min}} - 2T_{\mathrm{o}})T_{d,k} + \mathrm{m}_1 T_{\mathrm{o}}^2 + \mathrm{m}_2 - \mathrm{m}_1 T_{\mathrm{d,max}} T_{\mathrm{d,min}} \tag{49}$$

where (48) is a second-order cone constraint and (49) is a linear one, and both are convex.

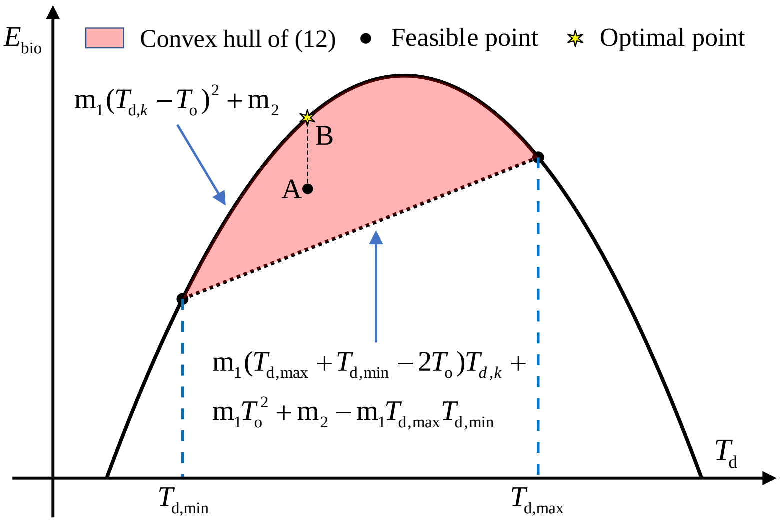

Fig. 3 Convex hull relaxation schematic of constraint (12).

Inequality constraints (48) and (49) define a convex feasible region (i.e., convex hull) denoted by Ξ. Assume that there is a solution $(T_{\mathrm{d}}^{0}, E_{\mathrm{bio}}^{0})$ denoted by A that lies in the relative interior of Ξ. Also, there always exists a point $(T_{\mathrm{d}}^{0}, E_{\mathrm{bio}}^{*}) \in \Xi$ denoted by B with $E_{\mathrm{bio}}^{*} > E_{\mathrm{bio}}^{0}$, which is directly above A and located on the quadratic curve. Since the objective function (43) of the provider is linear, the more biogas yield E_{bio}, the greater the profit of the provider. When the digestion temperature reaches T_{d}^{0}, the provider will try to find the greater E_{bio} in Ξ to increase the profit. Therefore, *B* is the best choice, not *A*, which means that the optimal solution will be found on the quadratic curve defined by $E_{\mathrm{bio},k} = \mathrm{m}_1(T_{\mathrm{d},k} - T_{\mathrm{o}})^2 + \mathrm{m}_2$. In conclusion, the introduced constraints (48) and (49) are equivalent to (12) and sufficiently exact for the proposed optimization model to represent the reality well.

Proposition 1: a unique Stackelberg equilibrium (SE) of the proposed game *G* exists if the following conditions are satisfied simultaneously [32].

1) The strategy set of each participant is nonempty, compact, and convex.

2) Each consumer has a unique optimal response strategy informed of the provider's strategy.

3) The provider develops a unique optimal strategy given the identified best strategies of all consumers.

Before proceeding to prove *Proposition 1*, the topological concepts of convex set and compact set are given, as follows:

1) A set *C* is convex if the line segment between any two points in *C* lies in *C*, i.e., if for any $x_1, x_2 \in C$ and any θ with $0 \le \theta \le 1$, $\theta x_1 + (1-\theta)x_2 \in C$ is satisfied.

2) A subset *S* of a topological space *X* is compact if for every open cover of *S* there exists a finite subcover of *S*.

Proof of Proposition 1: Since the approximation by (48) and (49), the constraints of both the provider and consumers are convex, and apparently nonempty and compact. For the demand side, the welfare function is strictly concave as $\partial^2 S_{i,k} / \partial(P_{i,k})^2 < 0$ and $\partial^2 S_{i,k} / \partial(G_{i,k})^2 < 0$, and problem (37) is a standard convex maximization problem. As a result, the response strategy of each consumer is guaranteed to be optimal and unique. For the provider side, the profit function is also strictly concave and the provider's determined optimal strategy is guaranteed to be unique due to the strict negative definiteness of the Hessian matrix, which are proven in Appendix A. Hence, conditions 1)–3) are satisfied and the *Proposition 1* holds.

The provider has priority in determining energy prices and imposes its strategies on consumers, then consumers observe the strategies of the provider and adapt their strategies accordingly. As the sequence of actions of the provider and consumers, the leader-follower structure in game *G* can be reformulated as a bi-level optimization model to iteratively solve the game *G* for profit maximization of the provider at the upper level and the welfare optimization of consumers at the lower level, as shown below.

Upper level: find the optimal energy prices ($\lambda_{\mathrm{e},k}^{\mathrm{ptc}*}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}*}$)

$$\max \ R_{\mathrm{P}}(\lambda_{\mathrm{e},k}^{\mathrm{ptc}}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}}) \tag{50}$$

$$s.t.\begin{cases}\lambda_{\mathrm{e},k}^{\mathrm{ptu}} \le \lambda_{\mathrm{e},k}^{\mathrm{ptc}} \le \rho_{\mathrm{e}}\lambda_{\mathrm{e},k}^{\mathrm{utc}},\ \forall k \in \mathrm{K}\\ \lambda_{\mathrm{g},k}^{\mathrm{ptu}} \le \lambda_{\mathrm{g},k}^{\mathrm{ptc}} \le \rho_{\mathrm{g}}\varepsilon_{\mathrm{g}}\lambda_{\mathrm{g},k}^{\mathrm{utc}},\ \forall k \in \mathrm{K}\\ (9)-(11),\ (13)-(27),\ (48),\ \text{and}\ (49),\quad \forall k \in \mathrm{K}\end{cases} \tag{51}$$

Lower level: get the optimal energy consumptions $(\mathbf{P}_k^{*}, \mathbf{G}_k^{*})$

$$\max\ S_{i,k}(P_{i,k}, G_{i,k}) \tag{52}$$

$$s.t.\begin{cases}P_{i,k}^{\min} \le P_{i,k} \le P_{i,k}^{\max},\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\\ G_{i,k}^{\min} \le G_{i,k} \le G_{i,k}^{\max},\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\end{cases} \tag{53}$$

3.3 Solution of the Stackelberg Game

The bi-level optimization problem cannot be solved directly by commercial solvers available as its hierarchical structure and nonlinear objective function (52). One of the common and exact solution methods for this problem is to replace lower-level problems with their KKT conditions [10]. In this bi-level optimization problem, the optimal pricing strategies of BSREP consist of the price variables $(\lambda_{\mathrm{e},k}^{\mathrm{ptc}}, \lambda_{\mathrm{g},k}^{\mathrm{ptc}})$, which are known parameters for the lower-level problem. Furthermore, the energy prices set by the provider will always be lower than those set by the utility company given the constraint (8). Therefore, this bi-level optimization model is transformed into a single-level one through KKT conditions, as follows,

$$\max \sum_{k=1}^{K}\left[(\lambda_{\mathrm{e},k}^{\mathrm{ptc}}P_k^{\mathrm{ptc}} + \lambda_{\mathrm{g},k}^{\mathrm{ptc}}G_k^{\mathrm{ptc}} + \lambda_{\mathrm{h},k}^{\mathrm{ptc}}H_k^{\mathrm{ptc}}) + (\lambda_{\mathrm{e},k}^{\mathrm{ptu}}P_k^{\mathrm{ptu}} + \lambda_{\mathrm{g},k}^{\mathrm{ptu}}G_k^{\mathrm{ptu}})\right]\Delta k - (C_{\mathrm{L}} + C_{\mathrm{B}}) \tag{54}$$

$$s.t.\begin{cases}\lambda_{\mathrm{e},k}^{\mathrm{ptc}} - b_{i,k} + 2a_{i,k}P_{i,k} + \mu_{i,k}^{\max} - \mu_{i,k}^{\min} = 0,\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\\ \lambda_{\mathrm{g},k}^{\mathrm{ptc}} - d_{i,k} + 2c_{i,k}G_{i,k} + \phi_{i,k}^{\max} - \phi_{i,k}^{\min} = 0,\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\\ 0 \le (P_{i,k} - P_{i,k}^{\min}) \perp \mu_{i,k}^{\min} \ge 0,\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\\ 0 \le (P_{i,k}^{\max} - P_{i,k}) \perp \mu_{i,k}^{\max} \ge 0,\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\\ 0 \le (G_{i,k} - G_{i,k}^{\min}) \perp \phi_{i,k}^{\min} \ge 0,\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\\ 0 \le (G_{i,k}^{\max} - G_{i,k}) \perp \phi_{i,k}^{\max} \ge 0,\quad \forall i \in \mathrm{N}, \forall k \in \mathrm{K}\end{cases} \tag{55}$$

where (55) is the KKT conditions of the lower-level problem; $\mu_{i,k}^{\min}$, $\mu_{i,k}^{\max}$, $\phi_{i,k}^{\min}$ and $\phi_{i,k}^{\max}$ are Lagrange multipliers corresponding to the inequality constraints (53); the notation $0 \le x \perp y \ge 0$ represents the complementary slackness condition: $x \ge 0$, $y \ge 0$, and $xy = 0$.

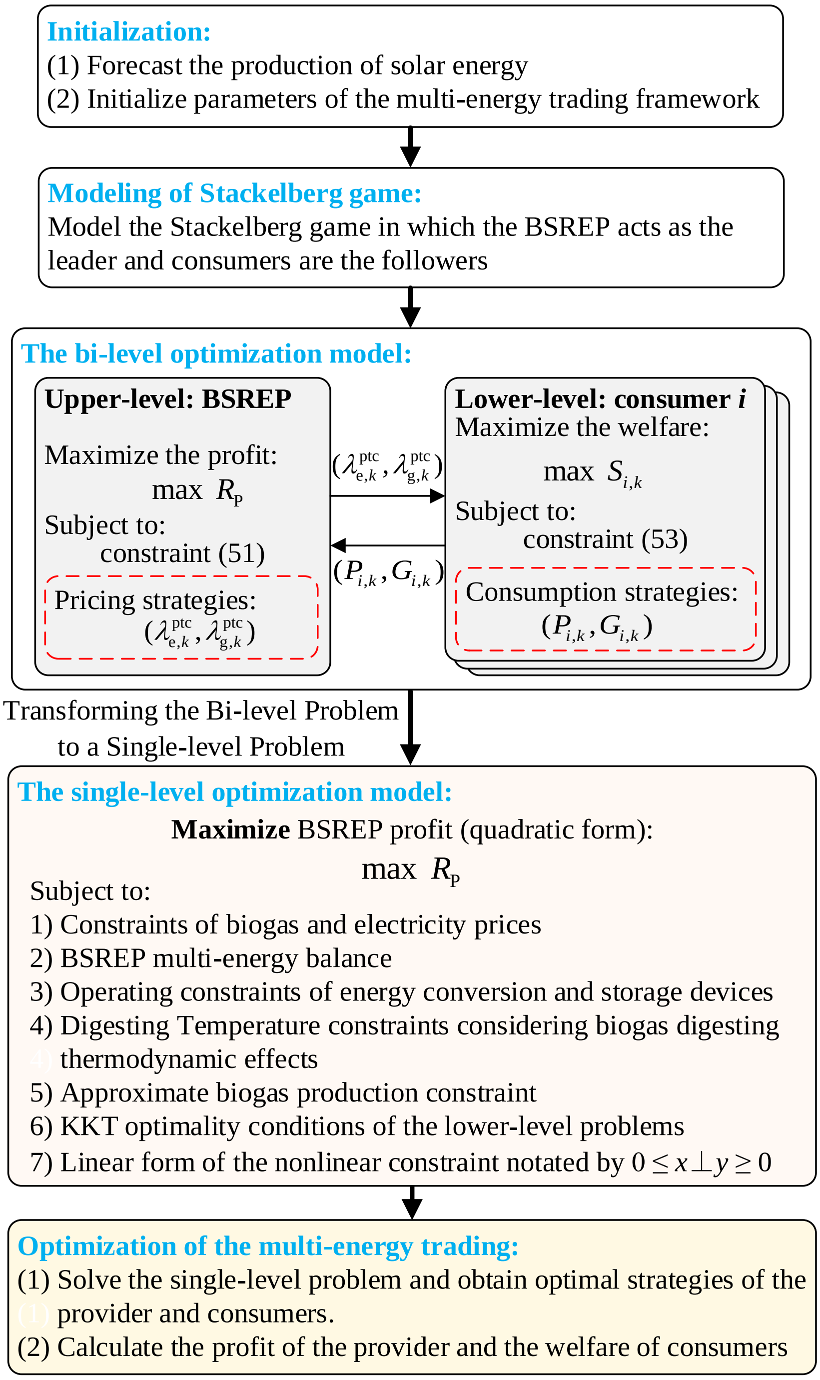

Fig. 4 Multi-energy trading optimization problem for participation of the provider and consumers.

The nonlinear expressions $\lambda_{\mathrm{e},k}^{\mathrm{ptc}} P_k^{\mathrm{ptc}}$ and $\lambda_{\mathrm{g},k}^{\mathrm{ptc}} G_{\mathrm{g},k}^{\mathrm{ptc}}$ can be transformed into quadratic ones by strong duality theory and KKT conditions. The procedure is described in Appendix B. The transformed objective function is

$$\max \left[\begin{array}{l} \sum_{k=1}^{K} \sum_{i=1}^{N} \left(\begin{array}{l} b_{i,k} P_{i,k} - 2a_{i,k} P_{i,k}^2 + d_{i,k} G_{i,k} - 2c_{i,k} G_{i,k}^2 + \lambda_{\mathrm{h},k}^{\mathrm{ptc}} H_{i,k}^{\mathrm{ptc}} \\ + \mu_{i,k}^{\min} P_{i,k}^{\min} - \mu_{i,k}^{\max} P_{i,k}^{\max} + \phi_{i,k}^{\min} G_{i,k}^{\min} - \phi_{i,k}^{\max} G_{i,k}^{\max} \end{array} \right) \Delta k + \\ \sum_{k=1}^{K} (\lambda_{\mathrm{e},k}^{\mathrm{ptu}} P_k^{\mathrm{ptu}} + \lambda_{\mathrm{g},k}^{\mathrm{ptu}} G_k^{\mathrm{ptu}}) \Delta k - (C_{\mathrm{L}} + C_{\mathrm{B}}) \end{array} \right] \tag{56}$$

To simplify the mathematical model, the nonlinear constraint notated by $0 \le x \perp y \ge 0$ can be rewritten as equivalent linear constraints, as follows,

$$\begin{cases} 0 \le x \le \mathrm{M}u \\ 0 \le y \le \mathrm{M}(1-u) \end{cases} \tag{57}$$

where u is a binary variable and M is a sufficiently large positive number.

Hence, the bi-level optimization model is equivalently converted to a single-level one, which can be solved easily using various available commercial solvers. Fig. 4 shows the flow chart of the multi-energy trading optimization problem based on the Stackelberg game.

4 Case Studies

4.1 Basic Configuration

In this paper, a biogas-solar energy system, denoted as BSREP, in a rural stand-alone biogas-solar Microgrid project, Hunan Province, China, is taken as the study object. In the system, a 600 m^3 in-ground tubular digester was built for anaerobic digestion, and a 100 m^3 biogas tank is equipped for biogas storage. In rural areas, livestock and organic waste, such as swine manure, are abundantly available and can be used as feedstock for anaerobic digestion under mesophilic conditions. N=5 consumers with different consumption preferences are selected to participate in the multi-energy trading and Fig. 5 shows the weight coefficients in the utility function of each consumer. As the methane content in biogas produced is about 60%, the heat value of biogas Q_{bio} is set to 6.11kWh/m^3. The technical specifications of BSREP in the case studies are summarized and listed in Table I. Fig. 6 shows the predicted solar energy and the external temperature of the digester within 24 hours. The structure and parameters of the R-C thermal network of the digester are obtained from Ref. [9].

Based on the feed-in tariff of China, the curves of energy prices $\lambda_{\mathrm{e},k}^{\mathrm{ptu}}$ and $\lambda_{\mathrm{e},k}^{\mathrm{utc}}$ are shown in Fig. 7 [38]. $\lambda_{\mathrm{g},k}^{\mathrm{ptu}}$ and $\lambda_{\mathrm{g},k}^{\mathrm{utc}}$ are kept at 0.186 \$/m^3 and 0.480 \$/m^3, respectively. Heat energy is only traded directly at the fixed price $\lambda_{\mathrm{h},k}^{\mathrm{ptc}}$, set to 0.031 \$/kWh. Other parameters ρ_{e}, ρ_{g}, ε_{g}, and M are set to 0.95, 0.95, 0.625, and 1 E4, respectively. In the case studies, hourly-based pricing is adopted by dividing a day into K=24 equal time periods. The transformed single-level optimization problem is a mixed-integer quadratic programming (MIQP) problem and is implemented with the freely available YALMIP toolbox [39] in MATLAB software and solved using the GUROBI 9.5.1. GUROBI solver is one of the fastest and most powerful mathematical programming solver available for MIQP problem. It is able to find both feasible and proven optimal solutions fast through the most advanced algorithms such as deterministic parallel, nontraditional search and symmetry breaking [40].

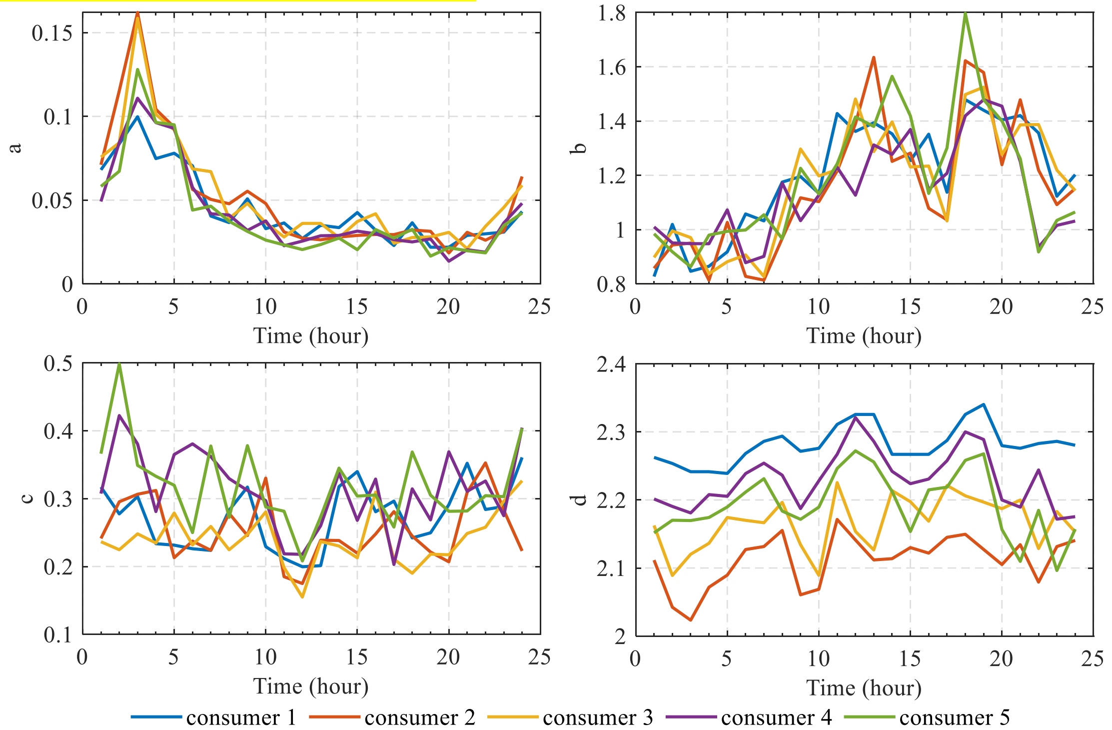

Fig. 5 The curves of weight coefficients in the utility function of consumers.

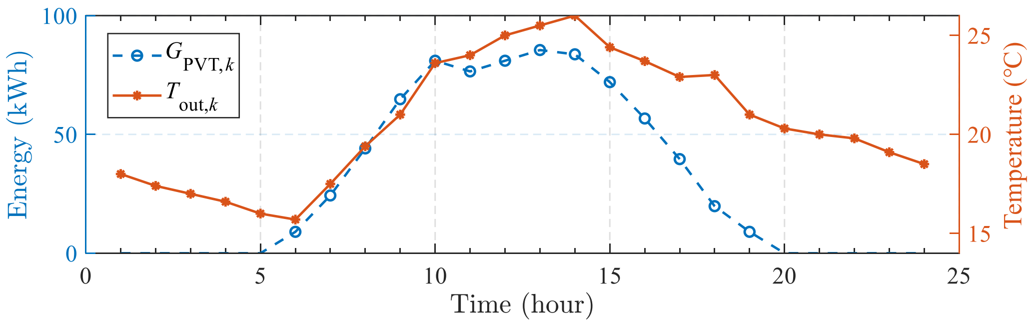

Fig. 6 The curves of daily predicted solar energy and the external temperature of digester.

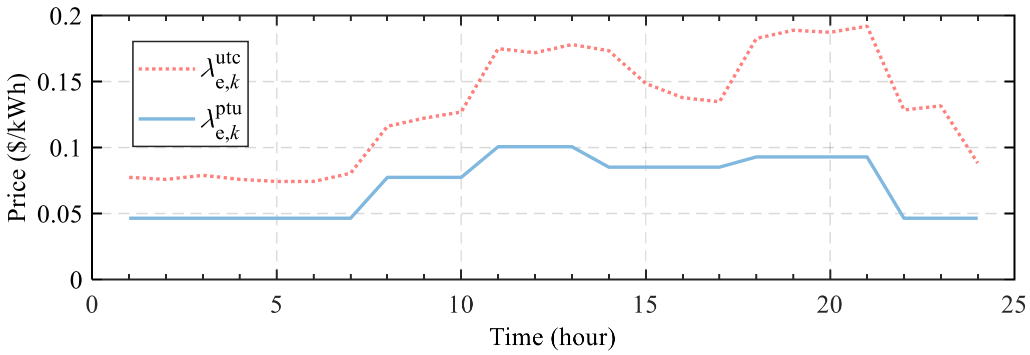

Fig. 7 The curves of daily energy prices $\lambda_{\mathrm{e},k}^{\mathrm{utc}}$ and $\lambda_{\mathrm{e},k}^{\mathrm{ptu}}$.

TABLE I

Main technical specifications of BSREP

Boiler	$S_{\mathrm{B,max}} = 30\mathrm{kW}$	$\eta_{\mathrm{B}} = 0.8$
Furnace	$S_{\mathrm{F,max}} = 30\mathrm{kW}$	$\eta_{\mathrm{F}} = 0.8$
CHP	$\Delta S_{\mathrm{CHP,ramp}} = 25\mathrm{kW/h}$	
	$\eta_{\mathrm{h,CHP}} = 0.45$	$\eta_{\mathrm{e,CHP}} = 0.36$
	$S_{\mathrm{CHP,min}} = 10\mathrm{kW}$	$S_{\mathrm{CHP,max}} = 50\mathrm{kW}$
Battery	$\eta_{\mathrm{sr}} = 0.89$	$\delta = 7.5\times10^{4}\mathrm{kWh}$
	$q = 12384\$$	$\eta_{\mathrm{ch}} = \eta_{\mathrm{dis}} = 0.93$
	$P_{\mathrm{dis,max}} = 20\mathrm{kW}$	$P_{\mathrm{ch,max}} = 20\mathrm{kW}$
	$E_{\mathrm{BES,min}} = 10\mathrm{kWh}$	$E_{\mathrm{BES,max}} = 50\mathrm{kWh}$
Biogas tank	$V_{\mathrm{GS,min}} = -50\ \mathrm{m^3/h}$	$V_{\mathrm{GS,max}} = 50\ \mathrm{m^3/h}$
	$E_{\mathrm{gas,min}} = 10\mathrm{m^3}$	$E_{\mathrm{gas,max}} = 100\mathrm{m^3}$
Digester	$\mathrm{m_1} = -0.0625$	$\mathrm{m_2} = 25$
	$Q_{\mathrm{bio}} = 6.11\ \mathrm{kWh/m^3}$	$T_{\mathrm{o}} = 35℃$
	$T_{\mathrm{d,min}} = 30℃$	$T_{\mathrm{d,max}} = 37℃$
Unit cost	$\mu_{\mathrm{CHP}} = 9.288\times10^{-3}\ \$/\mathrm{kWh}$	
	$\mu_{\mathrm{F}} = 3.870\times10^{-3}\ \$/\mathrm{kWh}$	$\mu_{\mathrm{B}} = 4.644\times10^{-3}\ \$/\mathrm{kWh}$

4.2 Comparative Studies and Analysis

For in-depth investigations on the effectiveness and superiority of the proposed multi-energy trading framework, three schemes are considered for comparative analysis and discussions: 1) Scheme 1 adopts the proposed multi-energy trading framework illustrated in Sections 2 and 3; 2) Scheme 2 implements the model of BSREP without considering biogas-solar complementarities, i.e. the heat and electricity from the solar energy are not recovered to heat the digester; 3) Scheme 3 performs the multi-energy trading model in the previous work Ref. [16] in which the provider and consumers only trade energy with utility companies at fixed prices. The lower bounds of consumer’s energy demand are their basic demand.

Fig. 8 illustrates the energy injected into the digester for heating as well as the digestion temperature and biogas production with schemes 1-3. In scheme 2, the thermal energy recovered from the CHP unit and furnace is insufficient and simply maintains the digestion temperature at the lower bound. Therefore, a deficiency of thermal energy to heat the digester causes biogas production to decrease significantly. In schemes 1 and 3, as solar energy is not available during hours 1-6 and 20-24, the digestion temperature and biogas production decrease. However, since solar energy is continuously injected into the digester through the PVT system during hours 10-17, the digester temperature rises to 33.6 °C at hour 15, and biogas production increases accordingly. Therefore, solar energy can effectively increase biogas production by heating the digester through PVT systems, which benefits the multi-energy supply capability of the provider.

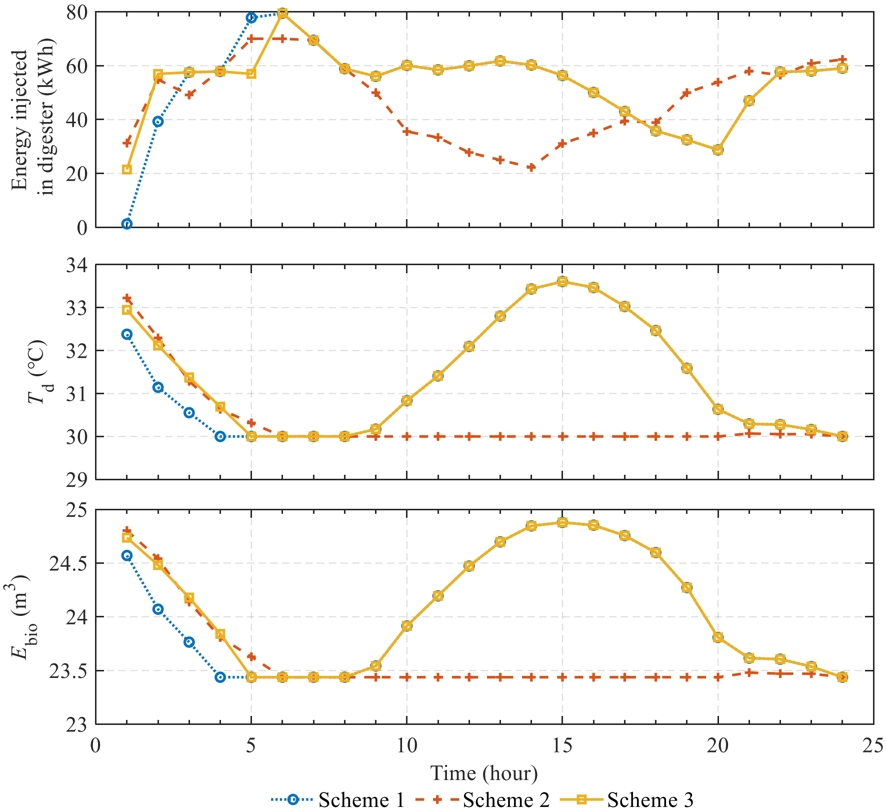

Fig. 8 Comparison of injected energy profile, the digestion temperature, and the biogas production between the proposed scheme and two benchmark schemes.

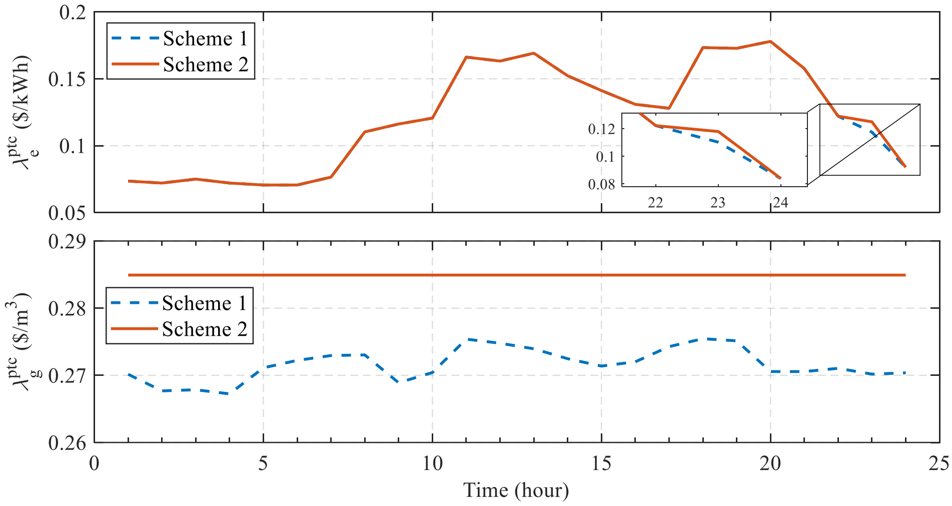

Fig. 9 Comparison of energy prices offered by the provider to consumers between schemes 1 and 2.

Fig. 9 illustrates the electricity and biogas prices offered by the provider to consumers with schemes 1-2. The provider in scheme 1 has sufficient energy to meet the multi-energy demand of consumers and thus sets optimal energy prices. In scheme 2, the provider would still strive to achieve the highest profit with the limited amount of biogas although the biogas production is insufficient. It is more beneficial for the provider to sell the electricity converted from the biogas rather than directly selling the biogas since the electricity price is relatively higher than the biogas prices taking into account the energy conversion efficiency. Therefore, the provider in scheme 2 sets the electricity price to be the same as that in scheme 1 to improve the revenue from selling electricity and raises the biogas price to the upper bound to indirectly reduce consumers' biogas demands.

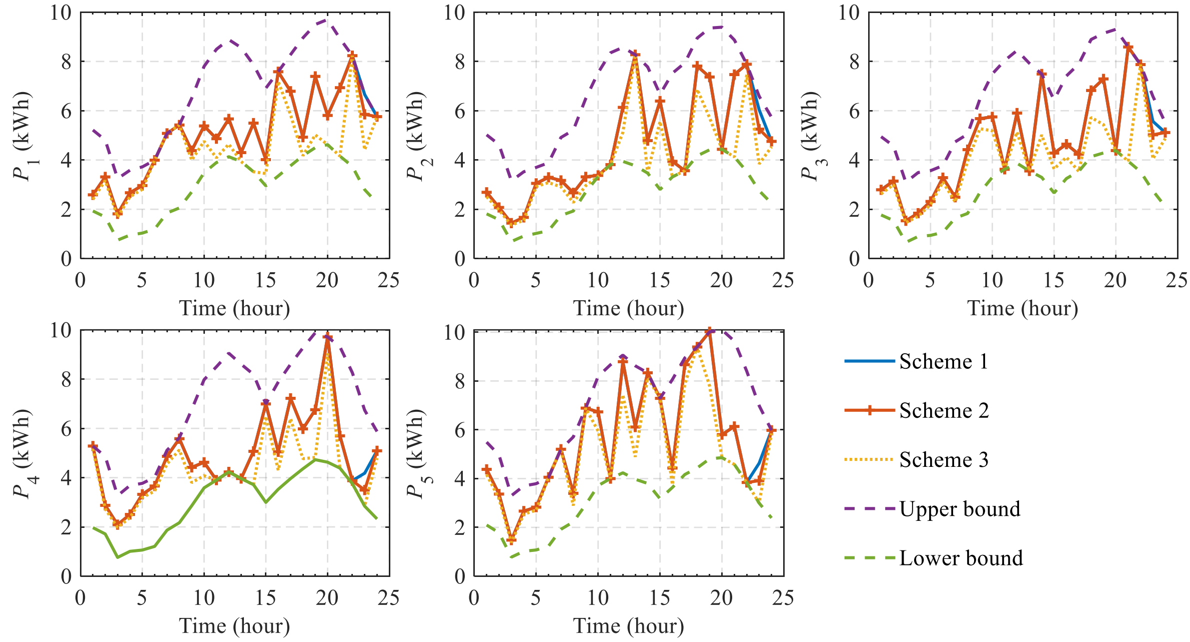

Fig. 10 Comparison of electricity demand of each consumer between the proposed scheme and two benchmark schemes.

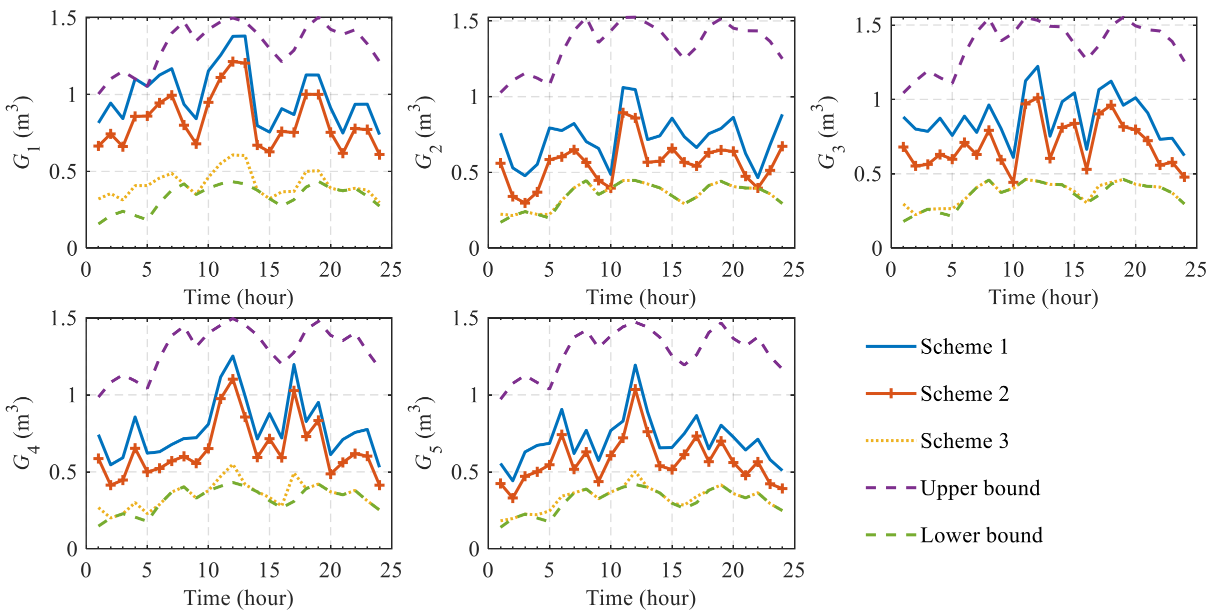

Fig. 11 Comparison of biogas demand of each consumer between the proposed scheme and two benchmark schemes.

From the perspective of consumers, the energy quantity ordered can be adjusted to achieve their desired welfare according to multi-energy prices. Comparison of the electricity and biogas demands between the proposed scheme and two benchmark schemes is shown in Fig. 10 and Fig. 11, respectively. Since consumers in scheme 3 purchase energy directly from utility companies at the highest price, their energy demands are the lowest. Because the electricity price offered by the provider to consumers in scheme 1 is approximately the same as that in scheme 2, the difference in electricity demand between the two schemes is small. However, the biogas demand in scheme 2 is consistently lower than that in scheme 1 during hours 1-24 due to the higher biogas price in scheme 2. Thus, the proposed multi-energy trading framework can effectively promote energy trading between consumers and BSREP, which facilitates local consumption of renewable energy.

Taking the sum of the maximum welfare for consuming electricity and biogas of each consumer in scheme 1 as the base values, Fig. 12 illustrates the normalized welfare of consumers for consuming electricity and biogas in each time period with scheme 1-3. It can be found that scheme 1 outperforms schemes 2 and 3 in terms of improving the welfare of consumers. Since the energy prices from utility companies in scheme 3 are not attractive for consumers, accordingly consumers reduce their energy consumption, and thus their welfare declines. Similarly, the high biogas price in scheme 2 leads to the reduction of consumers' welfare. As the provider in scheme 1 offers the lowest energy prices and sufficient biogas to consumers, consumers' welfare is greatest compared to that in schemes 2-3.

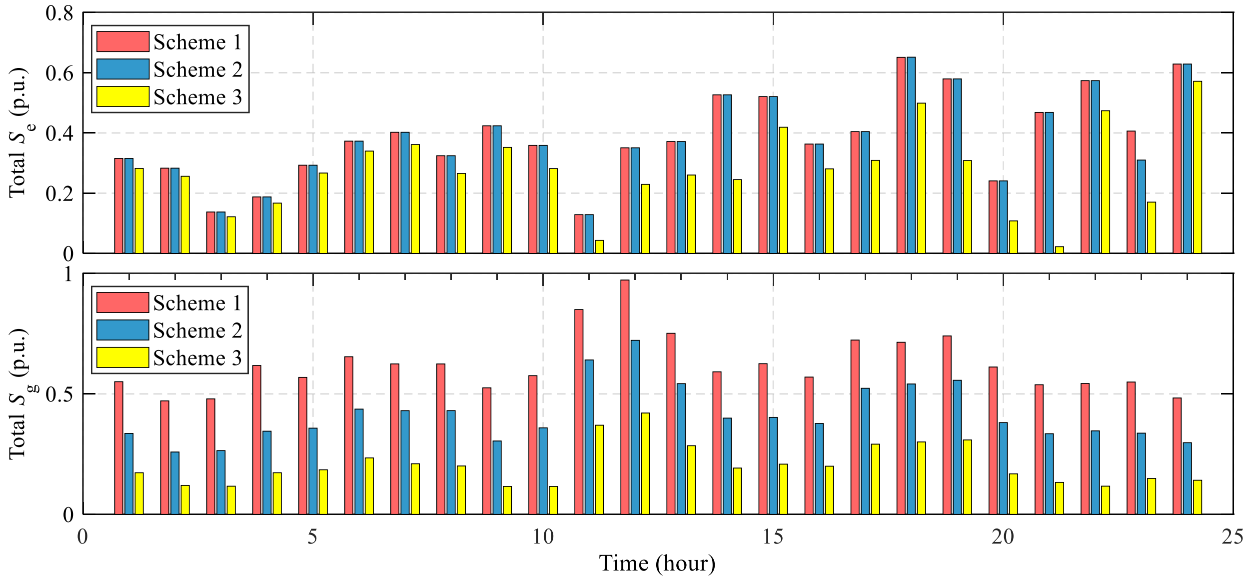

Fig. 12 Comparison of normalized welfare for consuming electricity and biogas of consumers at each time slot among schemes 1-3.

With the maximum daily welfare of consumers in scheme 1 as the base value, Table II summarizes the normalized welfare of each consumer with schemes 1-3. Since the electricity prices in schemes 1 and 2 are approximately the same, the average welfare for consuming electricity of consumers in scheme 1 is nearly the same as that in scheme 2, and is approximately 1.4 times greater than that in scheme 3. In scheme 1, as the biogas price is lowest and the biogas demand is highest, the average welfare for consuming biogas is the largest and approximately 3.0 times greater than that in scheme 3. As a result, the proposed trading framework can promote consumers' energy consumption as well as improve their welfare, and thus motivate them to actively participate in multi-energy trading to achieve efficient utilization of renewable energy.

Table II

Comparison of normalized welfare for consuming electricity and biogas of each consumer among schemes 1-3

Consumer	Welfare for consuming electricity (p. u.)			Welfare for consuming biogas (p. u.)		
	Scheme 1	Scheme 2	Scheme 3	Scheme 1	Scheme 2	Scheme 3
1	0.8794	0.8692	0.6532	1.0000	0.7068	0.4311
2	0.6964	0.6873	0.4784	0.4995	0.2952	0.0671
3	0.7308	0.7222	0.5047	0.6945	0.4422	0.1998
4	0.6663	0.6602	0.4549	0.7221	0.4939	0.2705
5	1.0000	0.9931	0.7393	0.6008	0.3963	0.1911
Average	0.7946	0.7864	0.5661	0.7034	0.4669	0.2319

Table III

Comparative performance results of schemes 1-3

Number of consumers	5			10			15			20		
Scheme	1	2	3	1	2	3	1	2	3	1	2	3
Revenue from selling energy to consumers ($)	**115.7**	111.5	11.3	**171.7**	170.4	11.3	**220.6**	219.5	22.6	**264.1**	263.6	37.6
Revenue from selling energy to utilities ($)	59.0	45.8	122.7	50.0	13.3	158.4	86.2	23.2	219.6	89.4	18.7	243.9
System operating cost ($)	12.0	12.1	12.3	10.8	10.2	13.7	12.4	12.5	16.7	14.0	14.2	19.7
Profit of provider ($)	**162.7**	145.2	121.7	**210.9**	173.4	156.0	**294.3**	230.1	225.4	**339.5**	268.0	261.8

To demonstrate the superior performance of the proposed multi-energy trading framework, the neighbourhood systems with the number of consumers from 5 to 20 are used for performance comparisons. Table III compares the system performance in terms of revenue of energy sales, system operating cost, and profit of provider with schemes 1-3. Since energy prices in scheme 1 are optimized by the provider aiming at increasing its profit, and energy prices in scheme 3 are fixed without optimization, the profit of the provider in scheme 1 with 5 consumers is increased by 33.69% compared to that in scheme 3. With the expansion of the transaction size, the proposed multi-energy trading framework is still outperformed by other schemes on increasing the provider's profit. To sum up, these comparisons indicate that, compared to the comparative cases, the proposed multi-energy trading framework enables BSREP to make trade-offs in energy trading between consumers and utility companies while ensuring sufficient biogas production, and can maximize the profit of the provider as well as the welfare of consumers.

5 Conclusions

In this paper, a novel multi-energy trading framework is proposed for biogas-solar renewable energy provider and heterogeneous consumers. The proposed framework can provide lots of economic and environmental benefits and is practically applicable. Firstly, the renewable utilization and biogas production can be enhanced by the provider with a significant improvement in its profit, and the optimal pricing strategy can be determined by the provider in the electricity and biogas energy market; secondly, consumers in this framework can promote their energy consumption as well as improve their welfare. Finally, market regulators can use this framework to investigate the market equilibrium between multi-energy providers and consumers. The proposed framework and the biogas-solar energy hub are beneficial to the harvesting, storage, and management of renewable energy resources, which have great potential to be extended in multiple fields, such as in rural and intercity railway electrification systems.

Appendix A

Considering the limits on energy consumption of consumers, the amounts of energy traded with consumers by BSREP can be formulated as:

$$P_k^{\mathrm{ptc}} = \sum_{i \in \boldsymbol{P}_{\mathrm{all}} - \boldsymbol{P}_{\mathrm{L}} - \boldsymbol{P}_{\mathrm{U}}} \frac{(b_{i,k} - \lambda_{\mathrm{e},k}^{\mathrm{ptc}})}{2a_{i,k}} + \sum_{i \in \boldsymbol{P}_{\mathrm{L}}} P_{i,k}^{\min} + \sum_{i \in \boldsymbol{P}_{\mathrm{U}}} P_{i,k}^{\max} \tag{A.1}$$

$$G_k^{\mathrm{ptc}} = \sum_{i \in \boldsymbol{G}_{\mathrm{all}} - \boldsymbol{G}_{\mathrm{L}} - \boldsymbol{G}_{\mathrm{U}}} \frac{(d_{i,k} - \lambda_{\mathrm{g},k}^{\mathrm{ptc}})}{2c_{i,k}} + \sum_{i \in \boldsymbol{G}_{\mathrm{L}}} G_{i,k}^{\min} + \sum_{i \in \boldsymbol{G}_{\mathrm{U}}} \mathrm{G}_{i,k}^{\max} \tag{A.2}$$

where $\boldsymbol{P}_{\mathrm{all}}$ and $\boldsymbol{G}_{\mathrm{all}}$ are the sets of the electricity and biogas demand of consumers, respectively; $\boldsymbol{P}_{\mathrm{L}}$, $\boldsymbol{G}_{\mathrm{L}}$, $\boldsymbol{P}_{\mathrm{U}}$, and $\boldsymbol{G}_{\mathrm{U}}$ are the set of the electricity and biogas demand of consumers whose energy consumptions have already reached their lower or upper limits.

Therefore, the profit function of BSREP shown in (A.3) is obtained by substituting (9)-(11), (A.1), and (A.2) in (43).

$$R_{\mathrm{P}}=\sum_{k=1}^{K}[(\sum_{i\in \boldsymbol{P}_{\mathrm{all}}-\boldsymbol{P}_{\mathrm{L}}-\boldsymbol{P}_{\mathrm{U}}}\frac{\lambda_{\mathrm{e},k}^{\mathrm{ptc}}b_{i,k}-\left(\lambda_{\mathrm{e},k}^{\mathrm{ptc}}\right)^{2}}{2a_{i,k}}+\lambda_{\mathrm{e},k}^{\mathrm{ptc}}\sum_{i\in \boldsymbol{P}_{\mathrm{L}}}P_{i,k}^{\min}+\lambda_{\mathrm{e},k}^{\mathrm{ptc}}\sum_{i\in \boldsymbol{P}_{\mathrm{U}}}P_{i,k}^{\max})+$$
$$\lambda_{\mathrm{e},k}^{\mathrm{ptu}}(f_{\mathrm{e,PVT}}G_{\mathrm{PVT},k}+P_{\mathrm{BES},k}-\frac{S_{\mathrm{B},k}}{\eta_{\mathrm{B}}}+S_{\mathrm{CHP},k}-S_{\mathrm{eg},k}-\sum_{i\in \boldsymbol{P}_{\mathrm{all}}-\boldsymbol{P}_{\mathrm{L}}-\boldsymbol{P}_{\mathrm{U}}}\frac{(b_{i,k}-\lambda_{\mathrm{e},k}^{\mathrm{ptc}})}{2a_{i,k}}-\sum_{i\in \boldsymbol{P}_{\mathrm{L}}}P_{i,k}^{\min}-\sum_{i\in \boldsymbol{P}_{\mathrm{U}}}P_{i,k}^{\max})+$$
$$(\sum_{i\in \boldsymbol{G}_{\mathrm{all}}-\boldsymbol{G}_{\mathrm{L}}-\boldsymbol{G}_{\mathrm{U}}}\frac{\lambda_{\mathrm{g},k}^{\mathrm{ptc}}d_{i,k}-\left(\lambda_{\mathrm{g},k}^{\mathrm{ptc}}\right)^{2}}{2c_{i,k}}+\lambda_{\mathrm{g},k}^{\mathrm{ptc}}\sum_{i\in \boldsymbol{G}_{\mathrm{L}}}G_{i,k}^{\min}+\lambda_{\mathrm{g},k}^{\mathrm{ptc}}\sum_{i\in \boldsymbol{G}_{\mathrm{U}}}G_{i,k}^{\max})+ \tag{A.3}$$
$$\lambda_{\mathrm{g},k}^{\mathrm{ptu}}(f_{\mathrm{D}}E_{\mathrm{bio},k}+V_{\mathrm{GS},k}-\frac{S_{\mathrm{CHP},k}}{Q_{\mathrm{bio}}\eta_{\mathrm{e,CHP}}}-\frac{S_{\mathrm{F},k}}{Q_{\mathrm{bio}}\eta_{\mathrm{F}}}-\sum_{i\in G-\boldsymbol{G}_{\mathrm{L}}-\boldsymbol{G}_{\mathrm{U}}}\frac{(d_{i,k}-\lambda_{\mathrm{g},k}^{\mathrm{ptc}})}{2c_{i,k}}-$$
$$\sum_{i\in \boldsymbol{G}_{\mathrm{L}}}G_{i,k}^{\min}-\sum_{i\in \boldsymbol{G}_{\mathrm{U}}}G_{i,k}^{\max})+\lambda_{\mathrm{h},k}^{\mathrm{ptc}}H_{i,k}^{\mathrm{ptc}})]\Delta k-(C_{\mathrm{L}}+C_{\mathrm{B}})$$

The Hessian matrix of the R_{P} based on $\lambda_{\mathrm{e},k}^{\mathrm{ptc}}$ and $\lambda_{\mathrm{g},k}^{\mathrm{ptc}}$ is given by:

$$\mathbf{H}=\begin{bmatrix}-\sum_{k=1}^{K}\sum_{i\in \boldsymbol{P}_{\mathrm{all}}-\boldsymbol{P}_{\mathrm{L}}-\boldsymbol{P}_{\mathrm{U}}}\frac{1}{a_{i,k}} & 0\\ 0 & -\sum_{k=1}^{K}\sum_{i\in \boldsymbol{G}_{\mathrm{all}}-\boldsymbol{G}_{\mathrm{L}}-\boldsymbol{G}_{\mathrm{U}}}\frac{1}{c_{i,k}}\end{bmatrix} \tag{A.4}$$

The Hessian matrix **H** is negative definite since $a_{i,k}>0$ and $c_{i,k}>0$, thus the profit function of BSREP is strictly concave.

Appendix B

The objective function of the dual problem of the lower-level problem can be rewritten by substituting the KKT conditions of the lower-level problem into (54) as follows,

$$\min_{\mu_{i,k},\phi_{i,k}}\max_{P_{i,k},G_{i,k}}\sum_{i,k}\left[S_{\mathrm{e},i,k}+S_{\mathrm{g},i,k}+\mu_{i,k}^{\min}(P_{i,k}^{\min}-P_{i,k})+\mu_{i,k}^{\max}(P_{i,k}-P_{i,k}^{\max})+\phi_{i,k}^{\min}(G_{i,k}^{\min}-G_{i,k})+\phi_{i,k}^{\max}(G_{i,k}-G_{i,k}^{\max})\right]$$
$$=\min_{\mu_{i,k},\phi_{i,k}}\max_{P_{i,k},G_{i,k}}\sum_{i,k}\begin{bmatrix}(b_{i,k}P_{i,k}-a_{i,k}P_{i,k}^{2}-\lambda_{\mathrm{e},k}^{\mathrm{ptc}}P_{i,k})+(d_{i,k}G_{i,k}-c_{i,k}G_{i,k}^{2}-\lambda_{\mathrm{g},k}^{\mathrm{ptc}}G_{i,k})+\\ \mu_{i,k}^{\min}(P_{i,k}^{\min}-P_{i,k})+\mu_{i,k}^{\max}(P_{i,k}-P_{i,k}^{\max})+\phi_{i,k}^{\min}(G_{i,k}^{\min}-G_{i,k})+\phi_{i,k}^{\max}(G_{i,k}-G_{i,k}^{\max})\end{bmatrix}$$
$$=\min_{\mu_{i,k},\phi_{i,k}}\max_{P_{i,k},G_{i,k}}\sum_{i,k}\begin{bmatrix}b_{i,k}P_{i,k}-a_{i,k}P_{i,k}^{2}+P_{i,k}(b_{i,k}-2a_{i,k}P_{i,k}-\mu_{i,k}^{\max}+\mu_{i,k}^{\min})+\\ d_{i,k}G_{i,k}-c_{i,k}G_{i,k}^{2}+G_{i,k}(d_{i,k}-2c_{i,k}G_{i,k}-\phi_{i,k}^{\max}+\phi_{i,k}^{\min})+\\ \mu_{i,k}^{\min}(P_{i,k}^{\min}-P_{i,k})+\mu_{i,k}^{\max}(P_{i,k}-P_{i,k}^{\max})+\phi_{i,k}^{\min}(G_{i,k}^{\min}-G_{i,k})+\phi_{i,k}^{\max}(G_{i,k}-G_{i,k}^{\max})\end{bmatrix}$$
$$=\min_{\mu_{i,k},\phi_{i,k}}\max_{P_{i,k},G_{i,k}}\sum_{i,k}(a_{i,k}P_{i,k}^{2}+c_{i,k}G_{i,k}^{2}-\mu_{i,k}^{\min}P_{i,k}^{\min}+\mu_{i,k}^{\max}P_{i,k}^{\max}-\phi_{i,k}^{\min}G_{i,k}^{\min}+\phi_{i,k}^{\max}G_{i,k}^{\max})$$
$$\tag{B.1}$$

According to the strong duality theory, the objective function of the lower-level problem is equal to its dual problem at the optimum. Combining (38)-(40), the following equation can be obtained.

$$
\begin{aligned}
&\sum_{i,k}(S_{\mathrm{e},i,k}+S_{\mathrm{g},i,k}) \\
&= \sum_{i,k}(b_{i,k}P_{i,k}-a_{i,k}P_{i,k}^2-\lambda_{\mathrm{e},k}^{\mathrm{ptc}}P_{i,k}^{\mathrm{ptc}}+d_{i,k}G_{i,k}-c_{i,k}G_{i,k}^2-\lambda_{\mathrm{g},k}^{\mathrm{ptc}}G_{i,k}^{\mathrm{ptc}}) \\
&= \sum_{i,k}(a_{i,k}P_{i,k}^2+c_{i,k}G_{i,k}^2-\mu_{i,k}^{\min}P_{i,k}^{\min}+\mu_{i,k}^{\max}P_{i,k}^{\max}-\phi_{i,k}^{\min}G_{i,k}^{\min}+\phi_{i,k}^{\max}G_{i,k}^{\max})
\end{aligned} \tag{B.2}
$$

Finally, the equivalent expression for $\lambda_{\mathrm{e},k}^{\mathrm{ptc}}P_{i,k}^{\mathrm{ptc}}$ and $\lambda_{\mathrm{g},k}^{\mathrm{ptc}}G_{i,k}^{\mathrm{ptc}}$ is obtained, as follows,

$$
\begin{aligned}
&\sum_{i,k}(\lambda_{\mathrm{e},k}^{\mathrm{ptc}}P_{i,k}^{\mathrm{ptc}}+\lambda_{\mathrm{g},k}^{\mathrm{ptc}}G_{i,k}^{\mathrm{ptc}}) \\
&= \sum_{i,k}(b_{i,k}P_{i,k}-2a_{i,k}P_{i,k}^2+d_{i,k}G_{i,k}-2c_{i,k}G_{i,k}^2+\mu_{i,k}^{\min}P_{i,k}^{\min}-\mu_{i,k}^{\max}P_{i,k}^{\max}+\phi_{i,k}^{\min}G_{i,k}^{\min}-\phi_{i,k}^{\max}G_{i,k}^{\max})
\end{aligned} \tag{B.3}
$$

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