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Incentive-compatible mechanisms for online resource allocation in Mobility-as-a-Service systems

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Abstract

In the context of Mobility-as-a-Service (MaaS), the transportation sector has been evolving towards user-centric business models, which put the user experience and tailored mobility solutions at the center of the offer. The emerging concept of MaaS emphasizes that users value experience-relevant factors, e.g., service time, inconvenience cost, and travel delay, over segmented travel modes choices. This study proposes an auction-based mechanism and tractable optimization models for the demand-side management of MaaS systems wherein users' trip requests are represented as mode-agnostic mobility resources. Users' requests arrive dynamically in the MaaS system and users compete for mobility resources by bidding for mobility services based on their willingness to pay and experience-relevant preferences. We take the perspective of a MaaS platform regulator who aims to maximize social welfare by optimally allocating mobility resources to users in real-time. The MaaS regulator first decides whether to offer each user a MaaS bundle and identifies the optimal allocation of mobility resources for the selected users. Users have the possibility to accept or reject offered MaaS bundles by comparing the associated utility obtained from MaaS with a reserve utility obtained from other travel options. We introduce mixed-integer programming formulations for this online mobility resource allocation problem. We show that the proposed MaaS mechanism is incentive-compatible, individually rational, budget balanced, and computationally efficient. We propose a polynomial-time online algorithm and derive its competitive ratio relative to an offline algorithm. We also explore rolling horizon configurations with varying look-ahead policies to implement the proposed mechanism. Extensive numerical simulations conducted on large-scale instances generated from realistic mobility data highlight the benefits of the proposed mechanism.

Keywords: Auctions; Incentive-compatibility; Online resource allocation; Mobility-as-a-Service

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1. Introduction

The concept of Mobility-as-a-Service (MaaS) is emerging and expected to shift future mobility trends. MaaS aims to convert users' mobility behavior from the possession of vehicles to the usage of mobility services. Hensher (2017) hypothesized that MaaS would shift public transportation contracts from output-based form (i.e., delivering kilometers on defined modes) to mode-agnostic, outcome-based contracts (i.e., delivering accessibility using any mode). This study explores a user-centric MaaS paradigm wherein users submit trip requests in real-time based on their preferences, such as service delay time and travel inconvenience. Although the concept of MaaS is attractive, Karlsson et al. (2020) have shown that most users are reluctant to change their travel habits and purchase mobility resources via a dedicated MaaS platform. The authors concluded that the perception of users towards the ability of the MaaS platform to meet their mobility requirements is a decisive factor in users' adoption of MaaS.

The goal of a MaaS platform is to provide a diverse menu of mobility options across multiple travel modes, including public transport, ride-, car- or bike-sharing, and taxi; hereby referred to as a *MaaS bundle*. To provide personalized MaaS bundles, the MaaS platform regulator must elicit private information from users, such as their preferences and willingness to pay (WTP) for mobility services. This motivates us to explore the potential of MaaS solutions through the lens of incentive-compatible (IC) auction-based mechanism design, which aims to identify the economic incentives to achieve targeted objectives (Haeringer, 2018). Furthermore, we focus on the design of personalized MaaS bundles in an online fashion, i.e., via a Pay-as-You-Go (PAYG) model where users submit trip requests in real time.

In this study, we take the perspective of a MaaS platform regulator that coordinates users' trip requests. To link users' trip requests to the supply of mode-specific transportation service providers (TSP), we introduce the concept of *mobility resources*. We represent users' trip requests as mode-agnostic mobility resources that are used to generate personalized MaaS bundles. We consider that users' requests arrive in batches and, at each time period, the MaaS regulator decides whether to offer users a MaaS bundle or not by solving an online mobility resource allocation problem. If a user is offered a MaaS bundle, then the user decides whether to accept this MaaS bundle or not by comparing the utility obtained from using MaaS to her reserve utility obtained from using other travel options such as private transportation. We propose a time-varying pricing framework wherein the unit price of mobility resources varies based on the capacity of the MaaS system. The resulting auction-based mechanism is used to determine optimal MaaS bundles based on users' trip requests. We show that this mechanism is IC, individually rational, and budget balanced. We also develop an efficient primal-dual online algorithm and derive its competitive ratio relative to an offline optimization formulation. Numerical experiments are conducted to examine the behavior of the MaaS mechanism and the performance of the solution methods developed.

We next review the literature on MaaS systems (Section 1.1), auctions and incentive compatibility (IC) in mobility services (Section 1.2) and online resource allocation problems (Section 1.3); before outlining the contributions of this study (Section 1.4).

1.1. Overview of MaaS

MaaS is a framework for delivering a portfolio of multimodal mobility services that place the user experience at the center of the offer. The design of MaaS bundles has been a hot topic for transportation researchers and practitioners due to their centrality in business plans. Most academic research thus far has investigated the problem of how to design *MaaS bundles* based on

Table 1: Main features of MaaS systems

Features	Study
Bundle different public and private transportation modes	Caiati et al. (2020); Reck et al. (2020)
Customize a service based on a user's preference and WTP	Ho et al. (2021a); Kim et al. (2021)
Users can request or book a service from a digital third platform	Matyas and Kamargianni (2019)
Offer on-demand services and tariff	Shaheen and Cohen (2020)
Integration of transport, information, payment and ticketing	Hensher and Mulley (2020); MA (2020)

stated choice surveys to elicit consumer preferences, e.g., for example, Ho et al. (2018), Reck et al. (2020), Caiati et al. (2020), Guidon et al. (2020), Ho et al. (2021b). A central element of MaaS is to provide multimodal mobility services via a single payment (Hensher and Mulley, 2020), thus facilitating user adoption and delivering fully integrated mobility ecosystems. In MaaS systems, two tariff options are typically available: Pay-as-You-Go (PAYG) and subscription plans. PAYG charges users for the immediate use of mobility services. In contrast, subscription plans allow users to subscribe to mobility packages over a longer period, such as a week or a month package (Ho et al., 2018). Aapaoja et al. (2017) suggested that PAYG may be the more suitable and acceptable for users given the variation in customer demand and service provision, and thus we focus on PAYG in this study. Ho et al. (2020) explored the potential demand for MaaS under different business models, such as monthly subscription and PAYG, based on the stated choice experiments. Ho et al. (2021a) evaluated the users' interest in various MaaS subscription bundles and identified key drivers of users' choices using Sydney trial data. Wong et al. (2020) explored the role of the government in an emerging MaaS system and identified two scenarios: government-contracted and economic deregulation. In the former, the government is assumed to be in charge of coordinating the allocation of mobility resources to users. In the latter, third-party service providers are assumed to compete for market demand. Xi et al. (2022a) proposed a single-leader multi-follower games (SLMFG) framework to investigate hierarchical configurations between a MaaS regulator, TSPs, and users in a two-sided MaaS market. van den Berg et al. (2022) analyzed three archetypical models in which MaaS could be operationalized: integrator, platform, and intermediary, and then tested prices, profits, consumer surplus, and welfare under these different models. In this study, we adopt the government-contracted model wherein TSPs are assumed to cooperate with the MaaS regulator which aims to maximize social welfare. The main features of MaaS systems discussed in the literature can be categorized into five categories as summarized in Table 1.

1.2. Auctions and incentive-compatibility in mobility services

To provide customized mobility services, the regulator must elicit private information from users, such as their preferences and WTP. Auction theory and mechanism design have thus received increasing attention in transportation for the purposes of allocating mobility resources to users. In particular, several incentive-compatible mechanisms which promote truthful user bidding behavior have been developed to elicit private economic and mobility-driven information from users due to the fact that mobility services often involve private information games. We next review recent efforts in different types of mobility services.

In the context of transportation service procurement, Xu and Huang (2014) proposed efficient auction-based mechanisms for the distributed transportation procurement problem to induce truthful bidding from carriers. In the context of public transit regulation, Sun et al. (2020) studied the regulation of a monopolistic public transit operator whose marginal cost is unknown to the regulator by presenting an IC regulatory policy that captures interactions between the regulator and the transit operator under asymmetric information provision. In the content of the parking reservation problem, Shao et al. (2020) considered an auction-based parking reservation problem where a parking management platform is the auctioneer and drivers are bidders, and then proposed an IC multi-stage Vickrey-Clarke-Groves (VCG) auction mechanism. In the context of traffic intersection management, Rey et al. (2021) presented online mechanisms for traffic intersection auctions in which users bid for priority service, which are shown to be IC in the dynamic sense. In the context of a ridesharing market, Tafreshian and Masoud (2022) proposed an IC auction-based mechanism to implement a subsidy scheme for a ridesharing platform. In this study, we aim to propose a MaaS resource allocation mechanism where users bid for mobility services represented as MaaS bundles. In contrast to classical VCG-like approaches, the proposed mechanism only requires users to report their preferences for their trip requests and thus obviates the need for users to report their preferences over other mobility services.

1.3. Online resource allocation problems

The implementation of MaaS systems is based on real-time data acquisition and processing. In particular, MaaS systems aim to provide users the possibility to purchase mobility resources on-the-fly to meet their travel needs. This requires that mobility resources be allocated in a dynamic fashion when user requests are submitted to the MaaS platform. Online resource allocation problems have received considerable attention in the Operations Research, Management Science, and Computer Science literature. Online resource allocation problems have been extensively studied in the context of computing. Buyya (2002) proposed and developed a distributed computational economy-based framework for resource allocation to regulate supply and demand, which incentivizes users to trade off deadline, budget, and quality of service. Zhang et al. (2013), Shi et al. (2015), and Zhou et al. (2016) proposed a truthful online cloud auction framework for cloud computing and developed online algorithms which guarantee certain competitive ratios. Xiao et al. (2018) studied two truthful double-auction mechanisms for a shared parking problem. They consider a parking platform with flexible schedules that aims to promote the typical daily 'go out early and come back at dusk' pattern. Cohen et al. (2019) introduced a model that quantifies the value of over-commitment in cloud computing and developed competitive online algorithms for solving their problem. Several online resource allocation problems can be cast as knapsack problems (Marchetti-Spaccamela and Vercellis, 1995). Bapna et al. (2005) explored a joint preference-elicitation, pricing, and capacity allocation problem faced by a firm offering one-time digital products in the form of data streams and proposed a computationally efficient, revenue-maximizing knapsack formulation. Zhou et al. (2008) and Chakrabarty et al. (2008) designed online algorithms for the knapsack problem which can achieve provably optimal competitive ratios. Buchbinder et al. (2007) designed online algorithms for fractional versions of the online knapsack problem with a guaranteed competitive ratio. Wang and Truong (2018) proposed an online algorithm that can minimize the total cost due to waiting, cancellations, and overtime capacity usage and proved that the algorithm could reach the best possible competitive ratio for this class of problems. Recently, Asadpour et al. (2020) studied a class of online resource-allocation problems over the different demand classes with limited flexibility and showed the effectiveness in mitigating supply-demand mismatch under a myopic online allocation policy. Stein et al. (2020) studied an online resource allocation problem with heterogeneous customers having specific preferences for each resource and introduced online algorithms with bounded competitive ratios. In this study, we build on the existing literature for online resource allocation problems and develop a customized online algorithm for allocating mobility resources in the proposed MaaS system.

1.4. Our contributions

The main contributions of this study relative to the existing literature are next summarized.

- We propose an auction-based MaaS mechanism where users compete with others and can bid for mobility services by submitting mode-agnostic trip requests represented in terms of mobility resources. Each user's bid consists of trip-specific preferences (service time and bidding price) and user-specific preferences (inconvenience tolerance and travel delay budget). We show that the proposed MaaS mechanism is IC, individually rational, and budget balanced.

- We cast this MaaS mechanism as an online mobility resource allocation problem and propose mixed integer linear programming (MILP) formulations to optimize the allocation of mobility resources to users in terms of MaaS bundles. We take the perspective of a MaaS platform regulator to decide which users will be provided a MaaS offer and allocated mobility resources to the selected users, maximizing social welfare. Users are modeled as utility-maximizing agents who decide whether to accept a MaaS offer provided by the platform by comparing their utilities obtained from MaaS with reserve utilities obtained from other travel options.

- We develop a customized polynomial-time, primal-dual online algorithm to solve this online mobility resource allocation problem and derive its competitive ratio relative to an optimal offline formulation. We also explore rolling horizon configurations with varying look-ahead policies to implement the proposed mechanism and compare social welfare and computational performance under different configurations.

- We conduct numerical experiments to test the proposed mechanism and illustrate its performance on a series of realistic scenarios. The results show that the proposed methods can solve large-scale instances, i.e., involving thousands of users in a competitive time, based on the simulations conducted with real travel data in Sydney, Australia. This highlights the potential of the proposed approach to supporting the deployment of auction-based mechanisms for resource allocation in MaaS systems.

The rest of this paper is organized as follows. Section 2 introduces the proposed auction-based mechanism for MaaS systems. Section 3 presents online mobility resource allocation formulations as well as a customized primal-dual algorithm to solve this online problem. The properties of the proposed MaaS resource allocation mechanism are discussed, and a competitive analysis is conducted to measure the performance of the proposed primal-dual algorithm. Section 4 introduces a rolling horizon setting to implement the proposed mechanism. Section 5 summarizes the numerical experiments and results. Section 6 concludes this study and discusses future research directions.

2. Auction-based mechanism for MaaS systems

In this section, we outline the motivation for designing such a mechanism in MaaS systems and problem statement (Section 2.1), introduce the auction setting (Section 2.2), time-varying pricing and user payment (Section 2.3), the objective function of the MaaS regulator (Section 2.4), user utility (Section 2.5), auction process (Section 2.6), and give an example to illustrate the process of the proposed MaaS mechanism (Section 2.7).

2.1. Problem statement

The emerging concept of MaaS emphasizes users' mobility behavior changes from the possession of vehicles to the usage of mobility services. Thus users value experience-relevant factors, such as service time (including in-vehicle travel time, waiting time, transfer time, and operational stop time, etc.), service delay time brought by transferring between different travel modes, and inconvenience cost brought by sharing with other riders, over the choices of travel modes.

Hensher and Xi (2022) noted that "effort" and "seamlessness" are two key factors to MaaS uptake, therefore we propose a MaaS framework that can quantify these two factors in terms of inconvenience cost and travel delay. Each travel mode can be assigned an inconvenience cost per unit of time, and users indicate preferences on their maximum acceptable inconvenience cost, hereby referred to as user *inconvenience tolerance*, which denotes the maximum inconvenient cost that a user can tolerate brought by transferring between different travel modes during a multi-modal trip. Intuitively, the inconvenience cost of travel modes aims to capture discomfort in shared and public transportation modes (Bian and Liu, 2019). The inconvenience tolerance is user-specific, while the inconvenience cost per unit of time is assumed to be mode-specific and based on vehicle occupancy (the number of shared riders on a vehicle). We consider travel delay to quantify the "seamlessness" of a multi-modal trip, and users indicate their preferences on the maximum acceptable travel delay, hereby referred to as user *travel delay budget*. In this study, inconvenience tolerance and travel delay budget are proposed to reflect the thresholds of inconvenient cost and travel delay that can be withstood by a user and are relevant to the user's socio-demographic characteristics, thus are modeled as users' constraints.

This study is based on the premise that mobility resources can be regarded as mode-agnostic, continuous quantities. For instance, distance, service time, and price can be regarded as continuous features of a mobility service. We propose a modeling framework that aims to provide a versatile MaaS system wherein users' trip requests are represented in terms of mode-agnostic mobility resources. In the proposed MaaS system, the average speed of travel modes is assumed to be known and representative of the *commercial speed* of each mode, defined as the average speed that incorporates service delays, e.g., waiting time, transfer time, and operational stop time into the average driving speed (Cortés et al., 2011). Commercial speed is a key factor in the operation of public transport systems since it represents a direct measure of the quality of service and also considerably affects system costs. In this study, we use the commercial speed of travel modes to determine users' service time in the MaaS system. On the demand side, users' mobility service requests can be represented as mode-agnostic mobility resources which can be matched to various travel modes. On the supply side, we assume that TSPs have limited service capacities which can be expressed in terms of mobility resources. To link users' heterogeneous travel requests to the supply of service providers, we measure the quantity of mobility resources required for a user's trip from both spatial and temporal metrics of a mobility service, i.e., the trip distance and the average commercial speed of a mobility service, and is expressed in speed-weighted travel distance units.

Users have preferences towards mobility services, including service time, WTP, inconvenience tolerance, and travel delay budget. To capture the preferences of users towards mobility services and elicit users' private information, such as users' WTP, we propose an IC auction-based MaaS mechanism where users submit heterogeneous bids to request for mode-agnostic mobility services on their arrival. A user bid consists of the requested service time and the bidding price for a specific trip. We assume that users also provide user-specific preferences on service quality during a trip, i.e., travel delay budget and inconvenience tolerance while registering for the MaaS platform. The

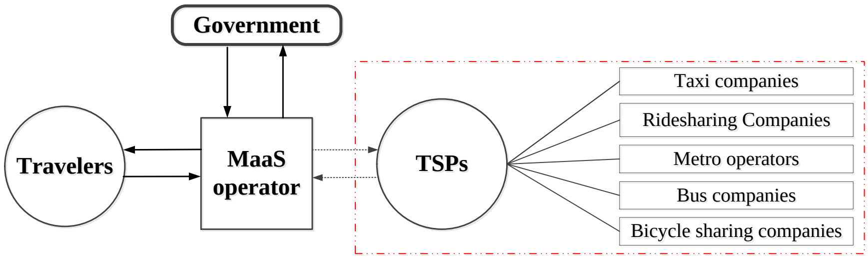


Figure 1: Illustration of a MaaS system with a regulator under government contracting

MaaS regulator aims to allocate mobility resources to meet user requests in terms of MaaS bundles consisting of different travel modes in real-time.

In this paper, a *MaaS bundle*, consisting of the service time of different travel modes during a multi-modal trip, is a result of resource allocation across travel modes that meets a user's trip request. The problem addressed in this study is to determine the optimal allocation of mobility resources to users in terms of the customized MaaS bundles while accounting for users' trip requests and capacity constraints in an online fashion. Since users' mode-agnostic requests arrive dynamically in the MaaS system under PAYG tariff, we adopt an online optimization approach to allocate mobility resources to heterogeneous mobility requests in terms of the MaaS bundle timely.

This study takes the perspective of a MaaS platform regulator who integrates mobility resources from various transportation service providers (TSPs) that provide on-demand mobility services without fixed schedules and stops. We assume that the MaaS regulator is under contract with the government and thus aims to maximize social welfare. In this context, the MaaS regulator and the TSPs are under reselling contracts, in which TSPs are paid by the MaaS regulator to satisfy their reservation utility regardless of whether the provided mobility resources are utilized.[1] The interaction among stakeholders in the MaaS system is illustrated in Fig.1.

2.2. Auction setting

In the proposed MaaS system, the auctioneer is the MaaS regulator, and the bidders are users who seek mobility services to fulfill their trip requests. We propose a sealed-bid online auction. We consider a discrete-time process and denote Ω, the set of periods in the auction. At each time period, users bid for mobility services, and the MaaS regulator determines which bids are served by allocating mobility resources subject to resource availability constraints. We denote $\mathcal{I}(t)$ the set of users bidding at time period $t \in \Omega$. A user bid consists of a pair of service time and WTP, also hereby referred to as *bidding price*. For each user $i \in \mathcal{I}(t)$, we denote T_i the service time and b_i the bidding price, respectively, corresponding to user i's bid.

Let D_i be the travel distance of user i's trip request, which is fixed and representative of the shortest path distance for this trip. We denote O_i the requested departure time of user i. We assume that time O_i occurs in the near future, e.g., within minutes or hours. Further, we assume that along with her bid, user i submits her *travel delay budget* denoted Φ_i and her *inconvenience tolerance* denoted Γ_i to the MaaS system. A user's trip request is formally defined as follows.

Definition 1. *User i's trip request is defined as a tuple* $\mathcal{B}_i \triangleq \{O_i, D_i, \Phi_i, \Gamma_i, (T_i, b_i)\}$.

[1]The reselling model has already been discussed by many existing literatures for shared mobility, e.g., Zhang et al. (2020); Liu et al. (2022). One may refer to the literature for a more detailed discussion regarding the reselling model.

We next define the quantity of *mobility resources* corresponding to a user bid.

Definition 2. *For each user $i \in \mathcal{I}(t)$, the quantity of mobility resources corresponding to the user's bid is defined as $Q_i \triangleq \frac{D_i}{T_i} D_i$, which is expressed in speed-weighted travel distance units.*

The quantity of mobility resources Q_i as defined in Definition 2 represents the quantity of resources needed to travel distance D_i at speed $\frac{D_i}{T_i}$. The quantity of mobility resources Q_i increases quadratically with the distance D_i and decreases with the trip service time T_i. We assume that the capacity of the MaaS system is expressed in terms of mobility resources, i.e., in speed-weighted travel distance units. The quantity of mobility resources in the MaaS system measures how much mobility resources in terms of distance and service time a user consumes for a multi-modal trip. For example, a user allocated with a MaaS bundle consisting of '10 min-taxi' consumes more mobility resources than that allocated with a MaaS bundle consisting of '30 min-taxi', given the same travel distance. Quantifying mobility resources connects users' mode-agnostic trip requests with mode-specific TSPs' supply.

To design user payment functions, it will be convenient to determine the ratio of bidding price to mobility resources, i.e., b_i/Q_i, for each user $i \in \mathcal{I}(t)$. We hereby refer to this ratio as the unit bidding price of user $i \in \mathcal{I}(t)$. We next introduce the payment mechanism.

2.3. Time-varying pricing and user payment

User payments are based on the available mobility resources in the MaaS system and users' bids. To determine user payments, we propose a second-best pricing mechanism where the payment of the highest bidder is determined as a function of the second-highest user bid. In turn, the payment of non-highest bidders is determined as a function of the highest user bid. Let $\boldsymbol{b} = [b_i]_{i \in \mathcal{I}(t)}$ be the vector of all user bids. Without any loss of generality, we assume that the user set $\mathcal{I}(t)$ is sorted by increasing per-unit bids, that is: $\frac{b_1}{Q_1} < \frac{b_2}{Q_2} < \ldots < \frac{b_k}{Q_k} < \ldots < \frac{b_{n_t-1}}{Q_{n_t-1}} < \frac{b_{n_t}}{Q_{n_t}}$ where $n_t = |\mathcal{I}(t)|$ is the number of bidders at time period t and k is the critical index satisfying $\sum_{i=k+1}^{n_t} Q_i \leq A_t < \sum_{i=k}^{n_t} Q_i$. Let $\bar{b}_i^{\max}(t, \boldsymbol{b})$ be the following function:

$$\bar{b}_i^{\max}(t, \boldsymbol{b}) = \begin{cases} \frac{b_{n_t-1}}{Q_{n_t-1}} & \text{if } i = n_t, \\ \frac{b_{n_t}}{Q_{n_t}} & \text{otherwise.} \end{cases} \tag{1}$$

$\bar{b}_i^{\max}(t)$ represents the maximum per-unit bidding price relative to user $i \in \mathcal{I}(t)$, i.e., it is the second-highest per-unit bid if i is the highest bidder at time period t; and it is the highest per-unit bid otherwise. Let p_{res} denote the unit reserve price of the MaaS regulator which is assumed exogenous. Note that if there is only one bidder i at time period t, $\bar{b}_i^{\max}(t, \boldsymbol{b})$ is set as p_{res}.

We consider three user payment functions to determine the unit price of mobility resources: linear, quadratic, and exponential functions, which are commonly discussed in the literature (Cropper et al., 1988; Pels and Rietveld, 2007). Let C be the total capacity of the MaaS system, which represents the total supply of mobility resources by TSPs. Let $A_t \leq C$ be the available mobility resources at time period $t \in \Omega$. The quantity $z_t = C - A_t$ represents the quantity of mobility resources unavailable at time period t. These unavailable resources correspond to mobility resources allocated at previous time periods. At each time period $t \in \Omega$, the MaaS regulator determines the unit price of mobility resources as a function of z_{t-1}. Recall that p_{res} denotes the unit reserve price of the MaaS regulator which is assumed exogenous and let $\frac{b_{k+1}(t)}{Q_{k+1}}$ be the unit price of the $k+1$th user. The lower bound on the unit payment function $p_{\min}(t)$ is defined in Eq.(2):

$$p_{\min}(t, \boldsymbol{b}) = \begin{cases} p_{\text{res}} & \text{if } \sum_{i=1}^{n_t} Q_i \leq A_t, \\ \frac{b_{k+1}(t)}{Q_{k+1}} & \text{otherwise.} \end{cases} \tag{2}$$

Let $\alpha_t \geq 0$ be a time-varying parameter. Given a user $i \in \mathcal{I}(t)$, the three linear, quadratic and exponential unit payment functions, denoted p_i^{lin}, p_i^{quad} and p_i^{exp}, respectively, are:

$$p_i^{\mathsf{lin}}(t, \boldsymbol{b}) = \frac{\bar{b}_i^{\max}(t, \boldsymbol{b})}{C} z_{t-1} + p_{\min}(t, \boldsymbol{b}), \tag{3}$$

$$p_i^{\mathsf{quad}}(t, \boldsymbol{b}) = \frac{1}{C^2}(z_{t-1})^2 + \frac{\bar{b}_i^{\max}(t, \boldsymbol{b})}{C} z_{t-1} + p_{\min}(t, \boldsymbol{b}), \tag{4}$$

$$p_i^{\mathsf{exp}}(t, \boldsymbol{b}) = \frac{\bar{b}_i^{\max}(t, \boldsymbol{b})}{\alpha_t - 1} \left(\alpha_t^{\frac{z_{t-1}}{C}} - 1 \right) + p_{\min}(t, \boldsymbol{b}). \tag{5}$$

The parameter α_t in Eq. (5) is defined as $\alpha_t = (1 + \overline{R}_t)^{\frac{1}{\overline{R}_t}}$ wherein $\overline{R}_t$ denotes the ratio of the maximum requested resources among travellers (Q_i) to the available mobility resources at time period t (A_t), i.e., $\overline{R}_t = \max_{i \in \mathcal{I}(t)} \left\{ \frac{Q_i}{A_t} \right\}$. Let $p_i^{\mathsf{pay}}(t, \boldsymbol{b})$ be one of the three unit payment functions Eq. (3)-Eq. (5), observe that $p_{\min}(t, \boldsymbol{b}) \leq p_i^{\mathsf{pay}}(t, \boldsymbol{b}) \leq \bar{b}_i^{\max}(t) + p_{\min}(t, \boldsymbol{b})$.

A possibility to design MaaS systems is to consider the cost of various travel modes and design trip-based fares that integrates various mode-specific costs. In this study, we adopt an alternative stance with mode-agnostic trip fares: we consider that travel modes are characterized by their commercial speed. We use the commercial speed of travel modes to quantify how much mobility resources that a multimodal trip consumes (Definition 2), and then determine trip service fares accordingly by combining with the unit payment function. Thus the payment rule of user i is:

$$p_i(t, \boldsymbol{b}) = Q_i p_i^{\mathsf{pay}}(t, \boldsymbol{b}), \quad \forall i \in \mathcal{I}(t). \tag{6}$$

This payment rule is used in the proposed MaaS resource allocation mechanism to determine user payments for users who are allocated mobility services.

2.4. Objective function of the MaaS regulator

The goal of the MaaS regulator is to maximize social welfare. We define social welfare at time period t as the sum of consumer surplus and the revenue generated from user payments which are assumed to be transferred to TSPs. Let $\mathcal{I}^\star(t)$ be the set of accepted users. Consumer surplus is defined as the difference between users' bids and payments, while the revenue generated is the sum of users' payments. Thus, social welfare at time period t is the sum of accepted user bids:

$$\max \underbrace{\sum_{i \in \mathcal{I}^\star(t)} b_i}_{\text{social welfare}} = \underbrace{\sum_{i \in \mathcal{I}^\star(t)} (b_i - p_i)}_{\text{consumer surplus}} + \underbrace{\sum_{i \in \mathcal{I}^\star(t)} p_i}_{\text{TSPs' total revenue}} \tag{7}$$

2.5. User utility

Users are assumed to be utility-maximizing and to have private WTP and reserve utility features. The reserve utility of a user represents the user's utility when using other travel options outside of the MaaS platform (e.g., private vehicles). A user accepts an offered MaaS bundle only if the associated utility exceeds her reserve utility. User i's true valuation for a mobility service, i.e., WTP, is denoted ν_i and is assumed to be private data. If user i is truthful then $b_i = \nu_i$ holds. Let $r_i \geq 0$ denote user i's reserve utility which represents user i's utility obtained from using other travel options outside of the MaaS platform and is also assumed to be the private data. Let x_i be a binary variable denoting whether user i's bid is accepted (1) or not (0). This motivates the following definition of user utility.

Definition 3. *The utility of user $i \in \mathcal{I}(t)$ is defined as:*

$$u_i \triangleq \max\{x_i(\nu_i - p_i(t, \boldsymbol{b})), r_i\}. \tag{8}$$

Note that if user i's bid is accepted by the MaaS regulator, i.e., $x_i = 1$, then the utility of this user for using MaaS is $\nu_i - p_i(t, \boldsymbol{b})$, which depends on the payment rule. To determine if a user will proceed with using MaaS, this utility is compared with the user's reserve utility, i.e., $u_i = \max\{\nu_i - p_i(t, \boldsymbol{b}), r_i\}$. In turn, if user i's bid is rejected by the MaaS regulator, i.e., $x_i = 0$, user i does not have the option to use MaaS, she derives zero utility from choosing MaaS, and $u_i = r_i$. The user utility defined in Eq. (8) aims to capture the choice of users, i.e., each user maximizes her utility by deciding whether to accept or reject the offered MaaS bundle with price $p_i(t, \boldsymbol{b}), \forall i \in \mathcal{I}$. Since user payments as defined in Eq. (6) are function users' bids $\boldsymbol{b} = [b_i]_{i \in \mathcal{I}(t)}$, it is critical to develop a resource allocation mechanism which is IC to prevent users from gaming the MaaS system.

2.6. Auction process

At each time period $t \in \Omega$, the auction consists of the following five main steps. i) **Trip requests**: users arriving at time period t submit their trip requests $\mathcal{B}_i, \forall i \in \mathcal{I}(t)$; 2) **Auction setup**: the MaaS regulator determines the mobility resources allocated by time $t-1$, i.e., z_{t-1}, and determines for each user $i \in \mathcal{I}(t)$ the unit price at the current time period $p_i^{\mathsf{pay}}(t, \boldsymbol{b})$ using one of the three payment functions Eq.(3), (4) or (5). User i's payment is determined by Eq.(6). iii) **Mobility resource allocation**: the MaaS regulator solves an online resource allocation problem to identify the optimal allocation of mobility resources to users subject to resource availability constraints. This step will be discussed in Section 3. iv) **User payments**: users decide whether or not to accept the offered MaaS bundle by comparing the utility obtained from using MaaS with her reserve utility. If a user decides to accept her MaaS bundle, she is charged the corresponding payment based on Eq. (6). v) **Resource availability update**: Mobility resources are allocated to users uniformly over their service period. For user $i \in \mathcal{I}(t)$, we denote O_i the departure time of user i, and we denote L_i the total service time of user i's MaaS bundle, which is determined based on the output of Step 3. Further details of this step are provided in Section 3.

2.7. MaaS mechanism illustration

We illustrate the auction-based mechanism with a MaaS system consisting of five travel modes: taxi, ridesharing, metro, bus, and bicycle-sharing. Fig. 2 illustrates how MaaS bundles satisfying users' requests could be designed for each of these three bids by allocating mobility resources across multiple travel modes, wherein the customized MaaS bundles A, B, and C correspond to the bids of user 1, 2 and 3, respectively. The inconvenience cost per unit of time is assumed to be mode-specific and based on vehicle occupancy as shown in Fig. 2. The unit inconvenience cost of a taxi ($0/min)

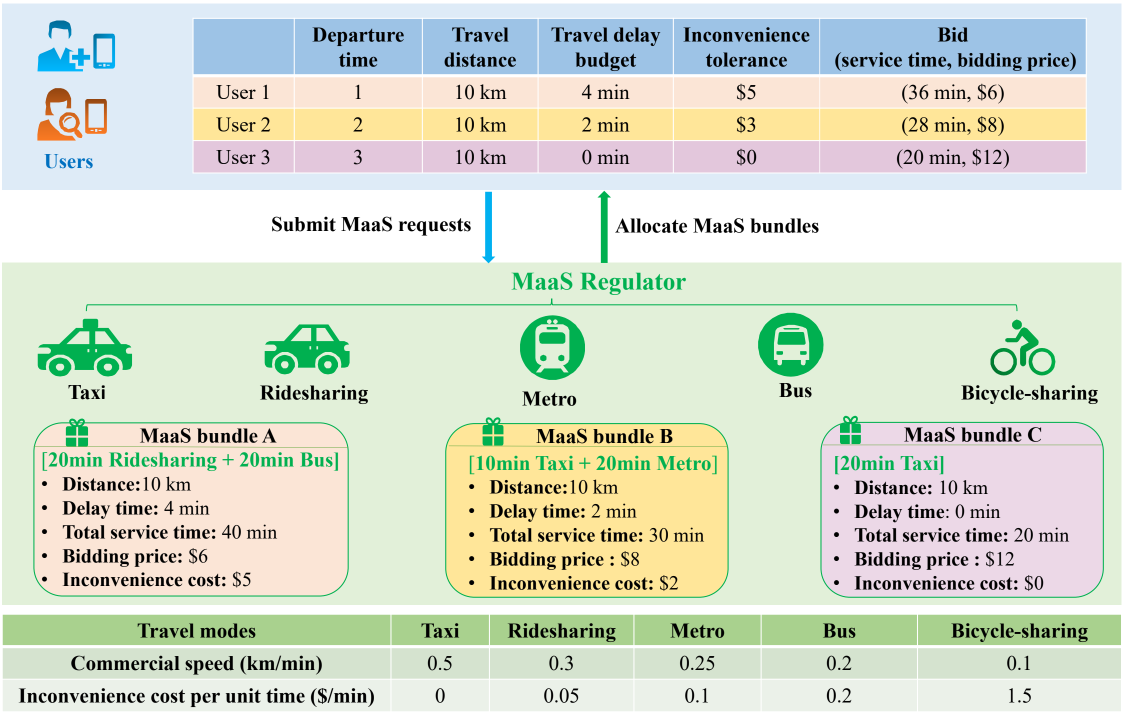


Figure 2: Illustration of the mechanism: user trip requests and MaaS bundles

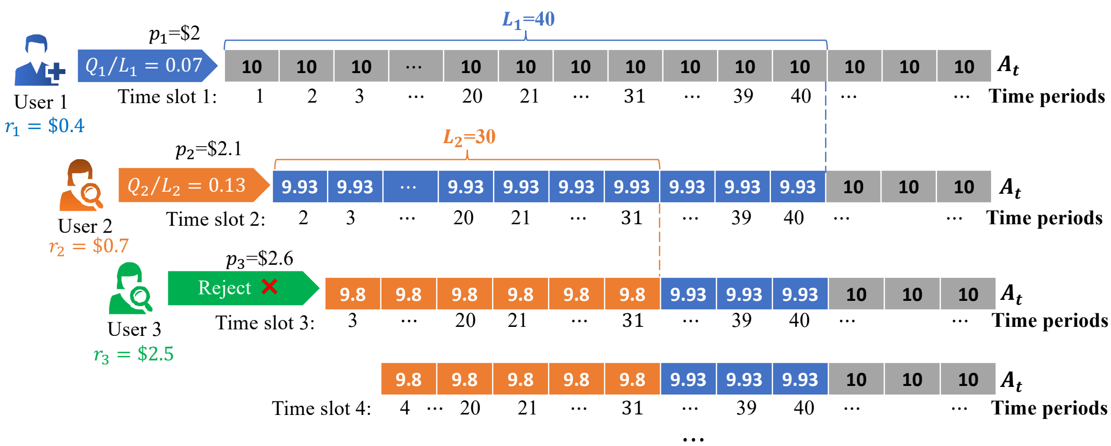


Figure 3: Illustration of the auction process

is lower than rideshare with two travelers ($0.05/min) or public transit ($0.2/min). Observe that all MaaS bundles are feasible with regard to users' trip requests. For example, considering the 'MaaS bundle B' allocated to user 2's trip request, the delay time (2 min) and inconvenience cost ($2) of 'MaaS bundle B' is smaller than travel delay budget and inconvenience tolerance of user 2, and the total service time (30 min) of 'MaaS bundle B' does not exceed the requested service time (28 min) and travel delay budget (2 min) of user 2.

To illustrate the auction process described in Section 2.6, we assume that user 1, user 2, and user 3 submit a trip request for a trip of 10 km to the MaaS platform and arrive sequentially in three time periods. We assume that users bid truthfully, i.e., $b_i = \nu_i$ for all $i \in \mathcal{I}(t)$. The update of the available mobility resources and the unit price in each time period is shown in Fig. 3. We use the linear user payment function Eq. (3); the system capacity (C) is set to 10, and the unit price of the MaaS regulator (p_{res}) is assumed to be \$2. Note that there is only one user and mobility resources are available in each time period, we have $\bar{b}_i^{max}(t, \boldsymbol{b}) = p_{min}(t, \boldsymbol{b}) = 1$, and $p_i^{\text{lin}}(t, \boldsymbol{b}) = 0.2z_{t-1} + 2$.

► **At time period 1:**

Step 1: User 1 with a reserve utility of \$0.4 submits a bid $\mathcal{B}_1$ to request mobility services:

$$\mathcal{B}_1 = \{O_1 = 1, D_1 = 10 \text{ km}, \Phi_1 = \$4, \Gamma_1 = \$5, (T_1 = 36 \text{ min}, b_1 = \$6)\},$$

and the corresponding quantity of mobility resource is: $Q_1 = D_1 \frac{D_1}{T_1} = 10 \times \frac{10}{36} = 2.78$,

Step 2: Assume that all mobility resources are available at time period 1, according to Eq. (3), the unit price at time period 1 is $p_1^{\text{lin}}(1) = \$2$. User 1's payment is $p_1(1) = p_1^{\text{lin}}(1)Q_1 = \$5.56$.

Step 3: The MaaS regulator solves an optimization problem based on user 1's preferences (see Model 1 in Section 3) to determine 'MaaS bundle A' with a total service time of 40 min (20 min ridesharing and 20 min bus).

Step 4: User 1 compares the utility obtained from MaaS with her reserve utility ($r_1 = \$0.4$), i.e., $u_1 = \max\{b_1 - p_1(1), r_1\} = \$0.46$, and accepts the MaaS offer. User 1's payment is $p_1(1) = p_1^{\text{lin}}(1)Q_1 = \$5.56$. User 1 is charged with a payment of \$5.56.

Step 5: The mobility resources corresponding to 'MaaS bundle A' are uniformly allocated over the service period $[O_1, \ldots, O_1 + L_1 - 1]$, i.e., the quantity of $\frac{Q_1}{L_1}(0.07)$ is deduced from the system capacity $C(10)$ for all time periods $1 \ldots 40$. The quantity of available mobility resources within time periods $2 \sim 40$ is updated to 9.93 (marked as blue in Fig. 3).

►**At time period 2:**

Step 1: User 2 with a reserve utility of \$0.7 submits a bid $\mathcal{B}_2$ to request mobility services:

$$\mathcal{B}_2 = \{O_2 = 2, D_2 = 10 \text{ km}, \Phi_2 = \$2, \Gamma_2 = \$3, (T_2 = 28 \text{ min}, b_2 = \$8)\},$$

and the corresponding mobility resources are $Q_2 = D_2 \frac{D_2}{T_2} = 10 \times \frac{10}{28} = 3.57$.

Step 2: The allocated mobility resources by time period 2 is 0.07 (see Fig. 3), according to Eq. (3), the unit price at time period 2 is $p_1^{\text{lin}}(2) = \$2.01$. User 2's payment is $p_1(2) = p_1^{\text{lin}}(2)Q_2 = \$7.19$.

Step 3: The MaaS regulator allocates 'MaaS bundle B' to user 2 with a total service time of 30 min (10 min taxi and 20 min metro).

Step 4: User 2 compares the utility obtained from MaaS with her reserve utility ($r_2 = \$0.7$), i.e., $u_2 = \max\{b_2 - p_1(2), r_2\} = \$0.81$, and accepts the MaaS offer, and is charged \$7.19.

Step 5: The mobility resources corresponding to 'MaaS bundle B' are allocated over the service period $[O_2, \ldots, O_2 + L_2 - 1]$, and $\frac{Q_2}{L_2}(0.13)$ is deduced from the available resources for all time periods $2 \ldots 31$. The quantity of available mobility resources within time periods $3 \sim 31$ is updated to 9.8 (marked as orange in Fig. 3).

►**At time period 3:**

Step 1: User 3 with a reserve utility of \$2.5 submits a bid $\mathcal{B}_3$ to request mobility services:

$$\mathcal{B}_3 = \{O_3 = 3, D_3 = 10 \text{ km}, \Phi_3 = \$0, \Gamma_3 = \$0, (T_3 = 20 \text{ min}, b_3 = \$12)\},$$

and the corresponding mobility resources are $Q_3 = D_3 \frac{D_3}{T_3} = 10 \times \frac{10}{20} = 5$.

Step 2: The allocated mobility resources by time period 3 is 0.2, according to Eq. (3), the unit price at time period 3 is $p_1^{\text{lin}}(3) = \$2.04$. Hence, user 3's payment is $p_1(3) = p_1^{\text{lin}}(3)Q_3 = \$10.2$.

Step 3: The MaaS regulator allocates 'MaaS bundle C' to user 3 with a total service time of 20 min.

Step 4: User 3 compares the utility obtained from MaaS with her reserve utility ($r_3 = \$2.5$), i.e., $u_3 = \max\{b_3 - p_1(3), r_3\} = \$2.5$, and rejects the MaaS offer since the reserve utility is higher than the utility obtained from MaaS ($b_3 - p_1(3) = \$1.8$). Thus no payment is charged.

Step 5: Since user 3 rejects the MaaS offer, no mobility resources are consumed in time period 3.

This example illustrates how users with identical trip distance but different preferences can be allocated customized MaaS bundles. We next present optimization methods to solve the online mobility resource allocation problems which arise in Step 3 of the auction process.

3. Online mobility resource allocation

This section presents formulations and algorithms for online mobility resource allocation in MaaS systems. We first introduce mathematical programming formulations for the online resource allocation problems considered in Section 3.1. We then develop an online primal-dual algorithm in Section 3.3 and conduct a competitive analysis of this online algorithm in Section 3.4.

3.1. Mathematical programming formulations

For each user $i \in \mathcal{I}(t)$, let x_i be a binary variable denoting whether user i's bid is allocated mobility resources (1) or not (0). Recall that Q_i represents the quantity of mobility resources requested for user i's bid and that A_t is the available mobility resources at time period t. The resource availability constraint ensures that the total quantity of allocated mobility resources does not exceed the system capacity at time t:

$$\sum_{i \in \mathcal{I}(t)} Q_i x_i \leq A_t. \tag{9}$$

For each user $i \in \mathcal{I}(t)$, user i's bid can be accepted if her bidding price b_i is greater or equal to the user payment determined by the payment function $p_i(t, \boldsymbol{b})$ and will be rejected otherwise. For conciseness, we drop the arguments of this function and denote p_i the payment of user i. Specifically, we require:

$$x_i(b_i - p_i) \geq 0, \quad \forall i \in \mathcal{I}(t). \tag{10}$$

Thus, if $b_i < p_i$ then $x_i = 0$, otherwise $x_i \geq 0$. Let $\mathcal{M}$ be the set of travel modes available in the MaaS system. To map allocation decisions to mobility resources provided by different travel modes, we introduce a real positive variable l_i^m which represents the service time allocated to mode

$m \in \mathcal{M}$ in a MaaS bundle if user i's bid is accepted. Let v_m be the *commercial speed* of mode $m \in \mathcal{M}$. Allocation decisions x_i are linked to variable l_i^m via constraint (11):

$$\sum_{m \in \mathcal{M}} v_m l_i^m = D_i x_i, \quad \forall i \in \mathcal{I}(t). \tag{11}$$

If $x_i = 1$, then the requested travel distance D_i must be distributed across speed-weighted, mode-based service times; otherwise, the vector of variables $\boldsymbol{l}_i = [l_i^m]_{m \in \mathcal{M}}$ is null. We consider that users have a travel delay budget, denoted Φ_i, which represents the upper bound on the excess service time relative to the requested service times T_i, for each user $i \in \mathcal{I}(t)$. The delay time between the total service time of a MaaS bundle ($\sum_{m \in \mathcal{M}} l_i^m$) and user i's requested service time must not exceed this upper bound, which yields:

$$0 \leq \sum_{m \in \mathcal{M}} l_i^m - T_i x_i \leq \Phi_i, \quad \forall i \in \mathcal{I}(t). \tag{12}$$

Further, we assume that users perceive inconvenience cost per time using different travel modes differently, which is a common assumption in the literature (Janjevic et al., 2020). We use σ_m to denote the inconvenience cost of mode $m \in \mathcal{M}$ per unit of time and use Γ_i to denote the *inconvenience tolerance* (the upper bound of inconvenience cost) of user $i \in \mathcal{I}(t)$. We require:

$$\sum_{m \in \mathcal{M}} \sigma_m l_i^m \leq \Gamma_i, \quad \forall i \in \mathcal{I}(t). \tag{13}$$

As introduced in Section 2.4, we assume that the goal of the MaaS regulator is to maximize social welfare, which is defined as the sum of consumer surplus and the revenue of TSPs, i.e., (15b),

$$\max \sum_{i \in \mathcal{I}(t)} b_i x_i = \sum_{i \in \mathcal{I}(t)} (b_i - p_i) x_i + \sum_{i \in \mathcal{I}(t)} p_i x_i. \tag{14}$$

The resulting mixed-integer linear programming (MILP) formulation for the online mobility resource allocation problem at time period $t \in \Omega$ is summarized in Model 1.

Model 1 (Online mobility resource allocation at time period t)**.**

$$\max \quad \sum_{i \in \mathcal{I}(t)} b_i x_i, \tag{15a}$$

$$\text{s.t.:} \quad \text{Eqs. (9)–(13)},$$

$$l_i^m \geq 0, \quad \forall i \in \mathcal{I}(t), m \in \mathcal{M}, \tag{15b}$$

$$x_i \in \{0, 1\}, \quad \forall i \in \mathcal{I}(t). \tag{15c}$$

Solving Model 1 corresponds to Step 3 of the auction process outlined in Section 2.6. After solving the online resource allocation problem at time period t, the available mobility resources at subsequent time periods $t' \geq t+1$ are updated (Step 4). Specifically, for each user $i \in \mathcal{I}(t)$ such that $x_i = 1$, we compare r_i and $\nu_i - p_i(t, \boldsymbol{b})$ to determine user i's choice. If the latter is greater than the former, the number of time periods affected by user i's allocation is $L_i = \lceil \sum_{m \in \mathcal{M}} l_i^m x_i \rceil$ and for each $t' \in [O_i, O_i + L_i]$, the quantity of available mobility resources $A_{t'}$ is decreased by $\frac{1}{L_i} Q_i x_i$.

To study the proposed online mobility resource allocation formulation, we start by showing that it can be reformulated as a multidimensional knapsack problem (MKP). For this, we introduce the concept of user-based feasible MaaS bundle set.

Definition 4. *For any user $i \in \mathcal{I}(t)$, let $\boldsymbol{l}_i = [l_i^m]_{m \in \mathcal{M}}$. Let $\mathcal{S}_i$ be the set defined as:*

$$\mathcal{S}_i \triangleq \left\{ \boldsymbol{l}_i \in \mathbb{R}^{|\mathcal{M}|} : (9) - (11), b_i \geq p_i, x_i = 1 \right\}. \tag{16}$$

We say that $\mathcal{S}_i$ is the set of feasible MaaS bundles for user i.

Note that in the definition of $\mathcal{S}_i$ we implicitly assume that the constraints (9)-(11) are restricted to those corresponding to user $i \in \mathcal{I}(t)$. For any MaaS bundle $s \in \mathcal{S}_i$, the corresponding mobility resources are denoted $Q_{i,s} = Q_i$ and the corresponding user bid is denoted $b_{i,s} = b_i$. We can reformulate Model 1 using the concept of feasible MaaS bundles to obtain a compact formulation.

Lemma 1. *For each user $i \in \mathcal{I}(t)$ and for each MaaS bundle $s \in \mathcal{S}_i$, let $\chi_{i,s}$ be a binary variable indicating whether MaaS bundle s is allocated to user i or not. Consider the following compact integer program (IP):*

$$\max \quad \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} b_{i,s} \chi_{i,s}, \tag{17a}$$

$$\text{s.t.:} \quad \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_{i,s} \chi_{i,s} \leq A_t, \tag{17b}$$

$$\sum_{s \in \mathcal{S}_i} \chi_{i,s} \leq 1, \qquad \forall i \in \mathcal{I}(t), \tag{17c}$$

$$\chi_{i,s} \in \{0, 1\}, \qquad \forall i \in \mathcal{I}(t), s \in \mathcal{S}_i. \tag{17d}$$

The compact IP (17) is equivalent to Model 1. Further, the LP-relaxation of (17) is equivalent to the LP-relaxation of Model 1.

The proof of Lemma 1 is provided in Appendix B.1.

3.2. Properties of the mechanism

We next show that the proposed auction-based MaaS resource allocation mechanism is i) *incentive compatible* (IC), i.e., bidding truthfully is a weakly dominant strategy, ii) *individually rationality* (IR), i.e., each user has a nonnegative utility from participating in the auction, and iii) *budget balanced* (BB), i.e., the MaaS regulator (auctioneer) has a nonnegative total payoff.

Proposition 1. *The proposed MaaS resource allocation mechanism is IC, i.e., bidding truthfully is a weakly dominant strategy.*

The proof of Proposition 1 is provided in Appendix B.2.

Remarks. (**Truthfulness in travel delay budget and inconvenience tolerance**). Travel delay budget and inconvenience tolerance are users' personal characteristics. The inconvenience tolerance and delay budget have no impact on a user's utility (see Eq. (8)), but act as constraints in the online mobility resource allocation problem. Note that if a user reports a travel delay budget or inconvenience cost higher than her true valuations for the features, she may experience a higher inconvenience cost or travel delay time, thus there is no incentive to do so. Conversely, reporting lower values for these features may result in the user not being allocated a MaaS bundle due to the reduced feasible region of the users' bundle feasible set, e.g., $\mathcal{S}_i$ (Eq. (16)). Consider the example given in Fig. 2; if user 1 misreports her travel delay budget as 0 min and inconvenience tolerance as \$0, MaaS bundles A and C are no longer feasible, and user 1 can be only allocated MaaS bundle B which does not maximize her utility.

Proposition 2. *The proposed MaaS resource allocation mechanism is individually rational and budget balanced.*

The proof of Proposition 2 is provided in Appendix B.4.

3.3. Primal-dual online algorithm

The compact integer program (IP) for online mobility resource allocation given in Lemma 1 falls within the literature on the MKP (Bertsimas and Demir, 2002) which is NP-hard if the decision variable is binary, and thus motivates us to design a heuristic algorithm. In this section, we propose a customized primal-dual algorithm for the online mobility resource allocation problem at hand (Borodin and El-Yaniv, 2005; Buchbinder and Naor, 2009). The compact IP (17) can be converted into a compact LP by relaxing the binary variable $\chi_{i,s} \in \{0,1\}$ to the continuous variable $\chi_{i,s} \geq 0$. The relaxed compact LP $OPT_1(t)$ and its dual problem $OPT_2(t)$ are summarized below.

$$
\begin{aligned}
OPT_1(t) : &\max \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} b_{i,s}\chi_{i,s}, \\
&\text{s.t.:} \quad \sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} Q_{i,s}\chi_{i,s} \leq A_t, \sum_{s \in \mathcal{S}_i} \chi_{i,s} \leq 1, \forall i \in \mathcal{I}(t), \chi_{i,s} \geq 0, \forall i \in \mathcal{I}(t), s \in \mathcal{S}_i. \\
OPT_2(t) : &\min A_t q_t + \sum_{i \in \mathcal{I}(t)} y_i, \\
&\text{s.t.:} \quad y_i \geq b_{i,s} - Q_{i,s}q_t, \forall i \in \mathcal{I}(t), s \in \mathcal{S}_i, q_t \geq 0, y_i \geq 0, \forall i \in \mathcal{I}(t).
\end{aligned}
$$

The primal variable $\chi_{i,s}$ of $OPT_1(t)$ corresponds to the dual constraint of $OPT_2(t)$; and q_t and y_i are the dual variables of $OPT_2(t)$ corresponding to the two constraints of the primal problem $OPT_1(t)$. According to primal-dual complementary slackness, if $y_i > 0$, constraint (17c) is binding, $\sum_{s\in\mathcal{S}_i} \chi_{i,s} = 1, \forall i \in \mathcal{I}(t)$, namely, user i is allocated with one of the MaaS bundle $s \in \mathcal{S}_i$. Further, if $\chi_{i,s} > 0$, constraint (17b) is binding, $y_i = b_{i,s} - Q_{i,s}q_t$. Since $y_i \geq 0$, primal-dual optimality conditions imply that the optimal value of y_i denoted y_i^* is:

$$
y_i^* = \max\left\{0, \max_{s\in\mathcal{S}_i}\{b_{i,s} - Q_{i,s}q_t^*\}\right\}, \forall i \in \mathcal{I}(t). \tag{18}
$$

where q_t^* denote the optimal value of q_t. If we interpret q_t as the unit price at time period t, then $Q_{i,s}q_t$ can be interpreted as user i's payment for MaaS bundle s and y_i can be interpreted as user i's utility. According to Eq. (18), if $y_i^* > 0$, $y_i^* = \max_{s\in\mathcal{S}_i}\{b_{i,s} - Q_{i,s}q_t^*\} = b_i - Q_i q_t^*$. We use this

Algorithm 1: PAYG online mobility resource allocation algorithm

Input: $\mathcal{B}_i = \{O_i, \Phi_i, \Gamma_i, (T_i, b_i)\}, \forall i \in \mathcal{I}(t)$
Output: $\boldsymbol{x}$, $\boldsymbol{q}$ and $\boldsymbol{l}$
1 $\boldsymbol{A}[1, 2, \cdots, |\Omega|] \leftarrow C$ ▷ initialize available mobility resources
2 **for** $t \in \Omega$ **do**
3 $\quad \bar{\mathcal{I}}(t) \leftarrow$ sort $\mathcal{I}(t)$ by decreasing unit bidding price b_i/Q_i
4 $\quad$ select the critical index k satisfying $\sum_{i=1}^{k-1} Q_i \leq A_t < \sum_{i=1}^{k} Q_i$
5 $\quad \bar{\mathcal{I}}_k(t) \leftarrow [1, 2, \cdots, k-1]$ ▷ select participants for the auction
6 $\quad \overline{R}_t \leftarrow \max_{i \in \mathcal{I}(t)} \left\{ \frac{Q_i}{A_t} \right\}$
7 $\quad \overline{\alpha}_t \leftarrow (1 + \overline{R}_t)^{\frac{1}{\overline{R}_t}}$
8 $\quad A_t \leftarrow \boldsymbol{A}[t]$ ▷ update available mobility resource at t
9 $\quad q_t \leftarrow 0$
10 $\quad \boldsymbol{x} \leftarrow \boldsymbol{0}$
11 $\quad$ **for** $i \in \bar{\mathcal{I}}_k(t)$ **do**
12 $\quad\quad$ **if** $q_t \leq \frac{b_i}{Q_i}$ *and* $\mathcal{S}_i \neq \emptyset$ **then**
13 $\quad\quad\quad q_t \leftarrow q_t(1 + \frac{Q_i}{A_t}) + \frac{b_i}{(\overline{\alpha}_t - 1)A_t}$
14 $\quad\quad\quad x_i \leftarrow 1$ ▷ accept user i's bid
15 $\quad\quad\quad [l_{i^\star}^m]_{m \in \mathcal{M}} \leftarrow$ solve $\mathcal{S}_i$ ▷ solve a linear feasibility problem
16 $\quad\quad\quad L_i \leftarrow \lceil \sum_{m \in \mathcal{M}} l_{i^\star}^m \rceil$ ▷ determine allocated service time
17 $\quad\quad\quad \boldsymbol{A}[O_i : O_i + L_i - 1] \leftarrow \boldsymbol{A}[O_i : O_i + L_i - 1] - \frac{Q_i}{L_i}$
18 $\quad\quad$ **else**
19 $\quad\quad\quad x_i \leftarrow 0$ ▷ reject user i's bid

property to design the proposed primal-dual algorithm.

In Algorithm 1, let $|\Omega|$ denote the last time period, Line 1 initializes the quantity of available resources at each time period as C, Line 3 $\sim$ Line 5 show critical index selection in the auction

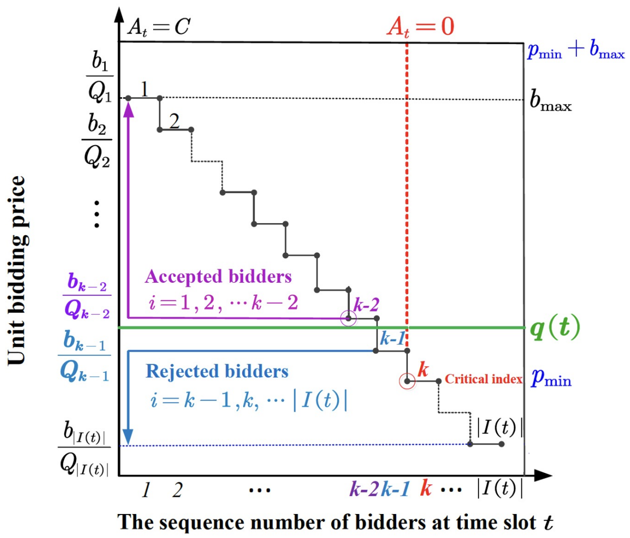


Figure 4: Critical index selection and acceptance/rejection determination

process, which are illustrated in Fig. 4. Line 3 sorts users' maximum unit bidding price in non-increasing order; Line 4 determines the critical index k based on the available mobility resources in time period t, and Line 5 selects users $1, 2, \ldots, k-1$ as participants for the auction at time t. Line 6 and Line 7 define the parameters $\overline{R}_t$ and $\overline{\alpha}_t$. Line 9 $\sim$ Line 14 show the iterations of primal and dual variables in a time loop. The iteration rule of q_t (Line 13) is determined by the exponential unit price function in Eq. (5), Line 12 and Line 18 indicate the acceptance determination in the auction process, which is illustrated in Fig. 4. Line 15 solves a feasibility problem $\mathcal{S}_i$ (an LP with 4 constraints, $|\mathcal{M}|$ variables and $|\mathcal{M}|$ bound constraints) to obtain the vector $[l_{i^\star}^m]_{m\in\mathcal{M}}$, which denotes a feasible MaaS bundle corresponding to user i's bid. Line 16 counts the total number of time periods arranged for each user in a MaaS bundle. Line 17 updates the available mobility resources based on the resources allocated to user i, i.e., the available mobility resources A_t are reduced by $\frac{1}{L_i}Q_i x_i$ within her service period $t \in [O_i, O_i + L_i]$, where O_i is user i's departure time period, and L_i is the total service time allocated to user i.

Lemma 2. *The worst-case time complexity of Algorithm 1 is $\mathcal{O}(|\Omega||\mathcal{I}(t)|LP)$, where LP denotes the worst-case time complexity of algorithms for solving linear programming problems.*

The proof of Lemma 2 is provided in Appendix B.5.

Lemma 3. *In each time iteration of Algorithm 1, $\Delta\mathcal{P}(i) = (1 - \frac{1}{\overline{\alpha}_t})\Delta\mathcal{D}(i)$ holds, where $\Delta\mathcal{D}(i)$ and $\Delta\mathcal{P}(i)$ denotes the change value in the objective functions of dual and primal problems, respectively.*

The proof of Lemma 3 is provided in Appendix B.6.

Lemma 4. *Algorithm 1 constructs feasible solutions for both the primal ($OPT_1(t)$) and dual ($OPT_2(t)$) online problems.*

The proof of Lemma 4 is provided in Appendix B.7.

We next analyze the performance of the proposed online algorithm in terms of solution quality.

3.4. Competitive ratio analysis

To evaluate the performance of the proposed online algorithm (Algorithm 1), we formulate an offline resource allocation problem that takes as input the complete travel demand data over the entire optimization period. For this analysis, we restrict our attention to linear user payment function, i.e., Eq. (3). The offline mobility resource allocation problem is summarized in Model 2.

Model 2 (Offline mobility resource allocation)**.**

$$\max \quad \sum_{i\in\mathcal{I}} \sum_{t\in\Omega: t\le O_i} b_i x_i^t, \tag{19a}$$

$$\text{s.t.:} \quad \sum_{i\in\mathcal{I}} Q_i x_i^t \le A_t, \quad \forall t \in \Omega, \tag{19b}$$

$$\sum_{t\in\Omega: t\le O_i} x_i^t \le 1, \quad \forall i \in \mathcal{I}, \tag{19c}$$

$$\sum_{m\in\mathcal{M}} v_m l_i^{mt} = D_i x_i^t, \quad \forall i \in \mathcal{I}, t \in \Omega : t \le O_i, \tag{19d}$$

$$0 \leq \sum_{m \in \mathcal{M}} l_i^{mt} - T_i x_i^t \leq \Phi_i, \qquad \forall i \in \mathcal{I}, t \in \Omega : t \leq O_i, \tag{19e}$$

$$\sum_{m \in \mathcal{M}} \sigma_m l_i^{mt} \leq \Gamma_i, \qquad \forall i \in \mathcal{I}, t \in \Omega : t \leq O_i, \tag{19f}$$

$$x_i^t (b_i - Q_i p_t) \geq 0, \qquad \forall i \in \mathcal{I}, t \in \Omega : t \leq O_i, \tag{19g}$$

$$p_t = \frac{\bar{b}_i^{\max}}{C} \left(\sum_{i \in \mathcal{I}} Q_i x_i^t \right) + p_{\min}, \qquad \forall i \in \mathcal{I}, t \in \Omega, \tag{19h}$$

$$l_i^{mt} \geq 0, \qquad \forall i \in \mathcal{I}, m \in \mathcal{M}, t \in \Omega : t \leq O_i, \tag{19i}$$

$$p_{\min} \leq p_t \leq p_{\min} + b_{\max}, \qquad \forall t \in \Omega, \tag{19j}$$

$$x_i^t \in \{0, 1\}, \qquad \forall i \in \mathcal{I}, t \in \Omega : t \leq O_i. \tag{19k}$$

Model 2 can be viewed as a time-extended version of Model 1, where variable x_i^t indicates whether user i's bid will be allocated at time period t or not, variable l_i^{mt} represents the number of time periods served by mode m in the MaaS bundle customized for user i's bid, p_t denotes the unit price at time period t, and p_i^t denotes user i's actual payment at time period t. The objective function (19a) aims to maximize social welfare overall all time periods. Constraints (19d), (19e), and (19f) guarantee that the MaaS bundle can satisfy the user's requested distance requirement, service time requirement, and inconvenience tolerance, respectively. Constraint (19g) illustrates that the actual payment at time period t is determined by the unit price at time period t and the requested weighted quantity of mobility resources, and when the bidding price is smaller than the actual payment, the bid will be rejected. Constraint (19h) adjust the unit price based on the linear user payment function and are divided into two separate constraints based on the definition of $\bar{b}_i^{\max}$ given in Eq. (1). Constraint (19b) restricts the weighted quantity of requested mobility resources at time period t can not exceed the weighted quantity of available mobility resources at time period t. Constraint (19c) guarantees that user i's is accepted in at most one time period. Observe that if $|\Omega| = 1$, Model 2 is equivalent to Model 1.

Definition 5. *For any user $i \in \mathcal{I}(t)$, let $\boldsymbol{l}_i^t = [l_i^{mt}]_{m \in \mathcal{M}}$. Let $\mathcal{S}_i^t$ be the set defined as:*

$$\mathcal{S}_i^t \triangleq \left\{ \boldsymbol{l}_i^t \in \mathbb{R}^{|\mathcal{M}|} : (19d) - (19h), b_i \geq p_i^t, x_i^t = 1 \right\}. \tag{20}$$

We say that $\mathcal{S}_i^t$ is the set of feasible MaaS bundles user i at time period $t \in \Omega$.

Lemma 5. *For each user $i \in \mathcal{I}(t)$ and for each MaaS bundle $s \in \mathcal{S}_i^t, \forall t \in \Omega$, let $\chi_{i,s}^t$ be a binary variable representing the allocation of s to i at time period $t \in \Omega$. Consider the compact IP:*

$$\max \quad \sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}_i^t} \sum_{t \in \Omega} b_{i,s} \chi_{i,s}^t, \tag{21a}$$

$$\text{s.t.:} \quad \sum_{i \in \mathcal{I}} \sum_{s \in \mathcal{S}_i^t} Q_{i,s} \chi_{i,s}^t \leq A_t, \qquad \forall t \in \Omega, \tag{21b}$$

$$\sum_{s \in \mathcal{S}_i^t} \sum_{t \in \Omega : t \leq O_i} \chi_{i,s}^t \leq 1, \qquad \forall i \in \mathcal{I}, \tag{21c}$$

$$\chi_{i,s}^t \in \{0, 1\}, \qquad \forall i \in \mathcal{I}, s \in \mathcal{S}_i^t, t \in \Omega : t \leq O_i. \tag{21d}$$

The compact IP (21) is equivalent to Model 2.

Proof. The proof follows from that of Lemma 1. □

The LP-relaxation of IP (21) is summarized in OPT_3 and its dual in OPT_4.

$$OPT_3 : \max \sum_{i\in\mathcal{I}} \sum_{t\in\Omega:t\leq O_i} \sum_{s\in\mathcal{S}_i^t} b_{i,s}\chi_{i,s}^t,$$

$$\text{s.t.: } \sum_{i\in\mathcal{I}} \sum_{s\in\mathcal{S}_i^t} Q_{i,s}\chi_{i,s}^t \leq A_t, \forall t\in\Omega, \sum_{t\in\Omega:t\leq O_i} \sum_{s\in\mathcal{S}_i^t} \chi_{i,s}^t \leq 1, \forall i\in\mathcal{I}, \chi_{i,s}^t \geq 0, \forall i\in\mathcal{I}, s\in\mathcal{S}_i^t, t\in\Omega : t\leq O_i.$$

$$OPT_4 : \min \sum_{t\in\Omega:t\leq O_i} A_t q_t + \sum_{i\in\mathcal{I}} y_i,$$

$$\text{s.t.: } Q_{i,s}q_t + y_i \geq b_{i,s}, \forall i\in\mathcal{I}, s\in\mathcal{S}_i^t, t\in\Omega : t\leq O_i, q_t\geq 0, \forall t\in\Omega, y_i\geq 0, \forall i\in\mathcal{I}.$$

For any input sequence $\boldsymbol{\tau}$, let $\mathcal{Z}^*(\boldsymbol{\tau})$ denote the maximum value of the offline problem (OPT_3), if Algorithm 1 outputs a solution which is at least $\Theta\cdot\mathcal{Z}^*(\boldsymbol{\tau})$, then we say that its competitive ratio is Θ (Borodin and El-Yaniv, 2005).

Proposition 3. *The competitive ratio of Algorithm 1 is* $\Theta = (1-\mathbb{R}_{max})\left(1-\frac{1}{\overline{\alpha}}\right)$*, where* $\overline{\alpha} = \min_{t\in\boldsymbol{\tau}} \overline{\alpha}_t$*,* $\mathbb{R}_{max} = \max_{t\in\boldsymbol{\tau}} \overline{R}_t$ *,* $\overline{\alpha}_t = (1+\overline{R}_t)^{\frac{1}{\overline{R}_t}}$*,* $\overline{R}_t = \max_{i\in\mathcal{I}(t)}\left\{\frac{Q_i}{A_t}\right\}$*,* $\forall t\in\Omega$*.*

The proof of Proposition 3 is provided in Appendix B.8.

Corollary 1. *When the available mobility resources at each time period approach infinity, i.e.,* $A_t\to+\infty, t\in\Omega$*, we have* $\Theta = (1-\mathbb{R}_{max})\left(1-\frac{1}{\overline{\alpha}}\right) = 1-\frac{1}{e}\approx 0.632$*, where* e *is Euler's number.*

This result shows that the ratio of the social welfare obtained by Algorithm 1 to the social welfare obtained by the offline model is always bounded by $1-\frac{1}{e}\approx 0.632$. Hence, this implies that expanding the capacity of the MaaS system will not reduce the efficiency of the online algorithm (Algorithm 1) in terms of social welfare.

4. Rolling horizon configurations

Travel demand data are input into the model as a stream, and the mechanism is executed periodically based on the available data without knowledge of future demand. We use a rolling horizon algorithm (RHA) framework to implement the proposed MaaS resource allocation mechanism where Δt denotes the time step at which the optimization problems are solved, and $\mathcal{T}$ denotes the length of the time horizon considered at the current iteration. Hence, at time period t, only users with a requested departure time in the time window $[t, t+\mathcal{T}]$ are considered. Fig. 5 illustrates the proposed rolling horizon algorithm. The optimization horizon $\mathcal{T}$ rolls forward per Δt time periods and solves the corresponding mobility resource allocation problem. Let ω denote the last time period, and the number of rolling processes (n) is written as $\omega/\Delta t$. The detailed procedure of the rolling horizon configuration is introduced in Algorithm 2 in Appendix C, where we consider three algorithm configurations with different time steps (Δt), time horizon lengths ($\mathcal{T}$) and optimization methods:

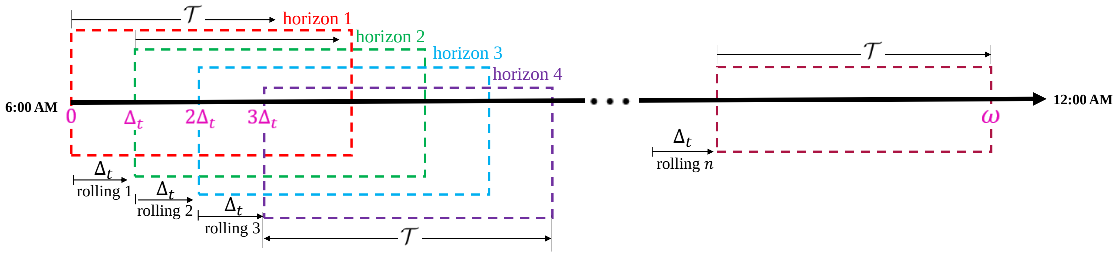


Figure 5: Rolling horizon algorithm (RHA) configurations

1) RHA ($\Delta t = 1, \mathcal{T} = 1$): rolls forward per unit time period, and users placing an order at time period t can request a service departing at time period t ($O_i = t$).
2) RHA ($\Delta t = 1, 1 < \mathcal{T} \leq \omega$): rolls forward per unit time period, and users placing an order at time period t can book a service departing within the time window $O_i \in [t, t + \mathcal{T}]$.
3) SHA ($\Delta t = \omega, \mathcal{T} = \omega$): this is a single horizon configuration aiming to provide a benchmark to evaluate alternative configurations. In this configuration, the offline problem is solved with full knowledge of the future travel demand overall time periods.

The first two RHA configurations run in an online manner with a time step of $\Delta t = 1$, and the corresponding online mobility resources allocation problem is solved either exactly using Model 1 or heuristically using Algorithm 1. The third configuration is referred to as SHA, and the mobility resource allocation problem is solved offline using Model 2, which is used as an oracle to benchmark the performance of the online RHA configurations.

5. Numerical experiments

We conduct a series of numerical experiments to evaluate the performance of the proposed MaaS resource allocation mechanism. Specifically, we discuss the impacts of several parameters, i.e., the maximum bidding price, travel demand in a time period, speed factor, and capacity. We also examine the impact of different types of user payment functions on social welfare, and numerically validate the derived competitive ratio of the online algorithm. All numerical experiments are conducted using Python and CPLEX Python API All numerical experiments are conducted using Python and CPLEX Python API on an Apple MacBook Pro M1 machine with a 10-Core CPU and 32GB unified Memory.

5.1. Input data and parameter settings

Real-time trip request information is an input to the proposed MaaS resource allocation mechanism. To obtain users' request data in the proposed MaaS system, we generate artificial instance data in which the parameters are set as follows. We consider five types of transport modes with

Table 2: The commercial speed and inconvenience cost of different travel modes

Modes	$m = 1$ Taxi	$m = 2$ Ride sharing	$m = 3$ Metro	$m = 4$ Bus	$m = 5$ Bicycle-sharing
Commercial speed (km/min)	0.5	0.3	0.25	0.2	0.1
Inconvenience cost (\$)/min	0	0.05	0.1	0.2	1.5

Table 3: The travel demand of each time period (λ_t) in the simulations

	non-peak hours	peak hours	non-peak hours	peak hours	non-peak hours
Time	$6:00-7:00$	$8:00-9:00$	$10:00-17:00$	$18:00-19:00$	$20:00-01:00$
time period	$[1,2\cdots,120]$	$[121,122,\cdots 240]$	$[241,182\cdots,720]$	$[721,722\cdots,840]$	$[841,842,\cdots,1200]$
λ_t	$\lambda_t \sim \mathcal{N}\left(2,1^2\right)$	$\lambda_t \sim \mathcal{N}\left(8,2^2\right)$	$\lambda_t \sim \mathcal{N}\left(2,1^2\right)$	$\lambda_t \sim \mathcal{N}\left(8,2^2\right)$	$\lambda_t \sim \mathcal{N}\left(2,1^2\right)$

different commercial speeds (km/min) and inconvenience costs per unit of time ($/min) given in Table 2. Recall that user i's trip request is $\mathcal{B}_i = \{D_i, O_i, \Phi_i, \Gamma_i, T_i, b_i\}$. User trip distance $[D_i]$ for each trip is randomly generated within the range [1km,18km]. User i's requested departure time $[O_i]$ is generated in different ways under different rolling horizon configurations as discussed in Section 4. Both user travel delay budget $[\Phi_i]$ and inconvenience tolerance $[\Gamma_i]$ are assumed to have a reverse relationship with user bidding price: $[\Phi_i]$ is randomly generated within $\left[0, \frac{100}{b_i}\right]$, and $[\Gamma_i]$ is randomly generated within $\left[0, \frac{100 D_i}{b_i}\right]$. To simulate a realistic MaaS system, the requested service time $[T_i]$ for each trip is set within the range of service time taken by the fastest mode (taxi) and the slowest mode (bicycle-sharing), namely, the requested service time of user i's bid $[T_i]$ is randomly generated within $\left[\frac{D_i}{v_5}, \frac{D_i}{v_1}\right]$. User bidding price $[b_i]$ is generated based on user trip distance and based on the tariff of the mobility system in Sydney, Australia. At each time period, the minimum unit price ($p_{\min}$) is set based on the price of public transit in Sydney[2], and the maximum unit bidding price ($b_{\max}$) is set based on the price of UberX in Sydney, which varies over the time during one day[3]. Accordingly, user i's bidding price (b_i) is randomly generated within $[p_{\min} Q_i, b_{\max} Q_i]$, and user i's reserve utility (r_i) is randomly generated within $[\gamma_t^{\min} p_{\min}, \gamma_t^{\max} b_{\max}]$, where parameters $\gamma_t^{\min}$ and $\gamma_t^{\max}$ vary during different time periods.

The operation time of the MaaS system is set to 20 hours every day (6:00 am-01:00 am). We consider time periods of 1 min; hence there are 1200 time periods per day, and the system capacity (C) is set to 500. The number of users placing an order at time period t, denoted λ_t is assumed to be normally distributed, where the mean value and standard deviation are set to different values between peak-hours and non-peak hours as indicated in Table 3.

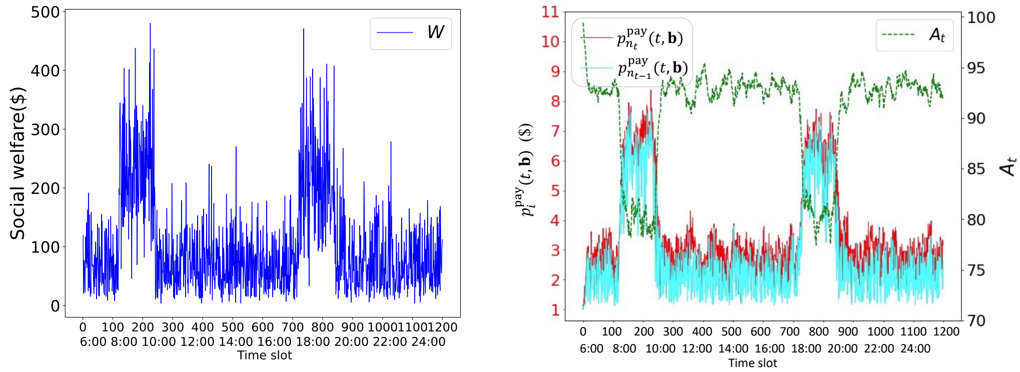
5.2. Numerical results

We conduct sensitivity analyses on a series of parameters in Section 5.2.1. Empirical validation of the derived competitive ratio of the primal-dual algorithm is provided in Section 5.2.2, and we analyse the impact of flexible booking in Section 5.2.3. We benchmark the proposed solution methods under different RHA configurations in Section 5.2.4. The obtained managerial insights are summarized in Observations 1–4.

5.2.1. Sensitivity analysis of the MaaS mechanism

We execute Algorithm 1 and examine the variation of social welfare, unit price, and available mobility resources over time periods. Fig. 6a depicts the variation of social welfare over 1200 time periods (representative of one day). We find that social welfare during peak hours (time periods $120 \sim 240$ and $720 \sim 840$) is higher than that during off-peak hours. Fig. 6b reports the variation of the unit price $p_i^{\text{pay}}(t, \boldsymbol{b})$ and the available mobility resources A_t over time. Let $p_{n_t-1}^{\text{pay}}(t, \boldsymbol{b})$ denote

[2] Opal (2020) Trip Planner can be used to estimate the fare of different public transport modes in NSW, Australia.
[3] Uber (2020)'s Real-time estimator provides real-time fare estimates on each trip.



(a) $t - W$

(b) $t - p_t$ and $t - A_t$

Figure 6: Social welfare (W), unit price ($p_i^{\text{pay}}(t, \boldsymbol{b})$) and available mobility resources (A_t).

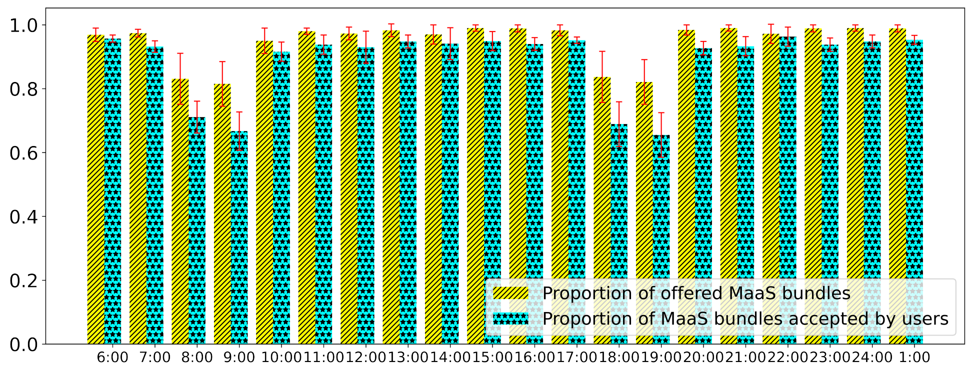


Figure 7: Proportion of offered MaaS bundles and proportion of MaaS bundles accepted by users.

the unit price of the highest bidder, and $p_{n_t}^{\text{pay}}(t, \boldsymbol{b})$ denote the unit price of the non-highest bidders. These simulations reveal that the unit price $p_i^{\text{pay}}(t, \boldsymbol{b})$ exhibits a reverse relationship with the available mobility resource A_t, and that $p_{n_t}^{\text{pay}}(t, \boldsymbol{b})$ is always higher than $p_{n_t-1}^{\text{pay}}(t, \boldsymbol{b})$. During one day, when the available mobility resources A_t varies within $0 \sim 100$, the unit price p_t varies within \$1$\sim$ \$11. During peak hours, the value of A_t is smaller, and the value of unit price $p_i^{\text{pay}}(t, \boldsymbol{b})$ is larger; during off-peak hours, the value of A_t is larger, and the value of $p_i^{\text{pay}}(t, \boldsymbol{b})$ is smaller.

We also compare the hourly average proportion of offered MaaS bundles with the proportion of MaaS bundles accepted by users in Fig. 7 and summarize the obtained insights as follows.

Observation 1. *Social welfare decreases if users' maximum bidding price increases, since a larger maximum bidding price will induce a larger unit price, reducing of the number of accepted users.*

Observation 2. *The hourly proportion of offered MaaS bundles is higher than the proportion accepted by users during one day, and is significantly higher during the peak hours, e.g., 8:00-9:00, and 18:00-19:00. The reason why more users incline to reject MaaS offers during peak hours is that the price of using MaaS system is higher during peak hours than during non-peak hours.*

Additional sensitivity analyses on the maximum bidding price, the type of user payment function, the influence of the system capacity, and the influence of the commercial speed of available

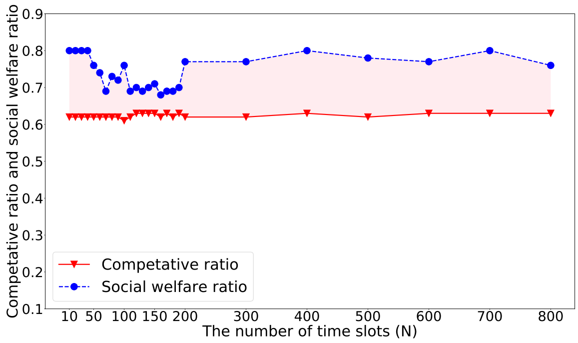


Figure 8: Competitive ratio (Θ) and social welfare ratio ($\mathcal{R}$)

travel modes are provided in Appendix D. Notably, we find that social welfare increases with capacity and that the speed factor has a limited influence on social welfare.

5.2.2. Competitive ratio analysis

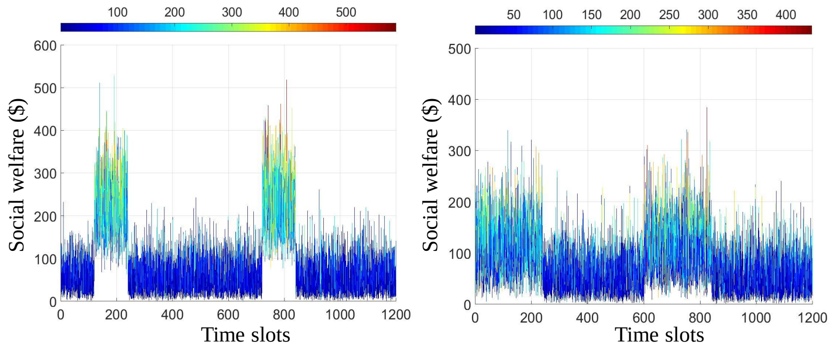
We analyze the competitive ratio (Θ) given in Proposition 3 by comparing this value with the social welfare ratio ($\mathcal{R}$) defined as the ratio of social welfare obtained by Algorithm 1 to that obtained by Model 2. Fig. 8 shows the relationship between the value of the competitive ratio (Θ) as well as the social welfare ratio ($\mathcal{R}$) and the number of time periods (N), respectively. Fig. 8 shows that the value of Θ is within $0.6114 \sim 0.6297$, and the value of $\mathcal{R}$ is within $0.7572 \sim 0.8014$. Detailed values of Θ and $\mathcal{R}$ are given in Table E.7 in Appendix E. Fig. 8 shows that the competitive ratio (Θ) given in Proposition 3 is a lower bound of the social welfare ratio ($\mathcal{R}$). The empirical gap between $\mathcal{R}$ and Θ is within $0.1275 \sim 0.1717$. This insight can be summarized as follows.

Observation 3. *The social welfare ratio ($\mathcal{R}$) is bounded by below by the competitive ratio (Θ) given in Proposition 3.*

5.2.3. Impact of booking flexibility

We investigate the impact of booking flexibility in the MaaS system. We consider the case where users can request trips in advance with a later departure time while charging the price at the current time period. We compare two types of RHA configurations with the same time step (Δt) and different time horizon lengths ($\mathcal{T}$): RHA without booking flexibility ($\Delta t = 1$, $\mathcal{T} = 1$) and RHA with booking flexibility ($\Delta t = 1$, $\mathcal{T} = 240$). We simulate users' booking behavior and generate users' requested departure times, given the same total demand for both configurations. The relationship between social welfare and time slots under RHA ($\Delta t = 1$, $\mathcal{T} = 1$) and RHA ($\Delta t = 1$, $\mathcal{T} = 240$) are given in Fig. 9a and Fig. 9b, respectively. Compared with Fig. 9a, Fig. 9b show that the social welfare during non-peak hours (e.g., time periods 0∼240 and $600 \sim 720$) starts to increase in RHA ($\Delta t = 1$, $\mathcal{T} = 240$) configuration. Since the total travel demand remains unchanged, social welfare increases during non-peak hours and decreases during peak hours.

Observation 4. *Using a RHA configuration with a longer time horizon can improve users' booking flexibility and balance social welfare between peak hours and non-peak hours.*



(a) $t - W$ with RHA ($\mathcal{T} = 1$)

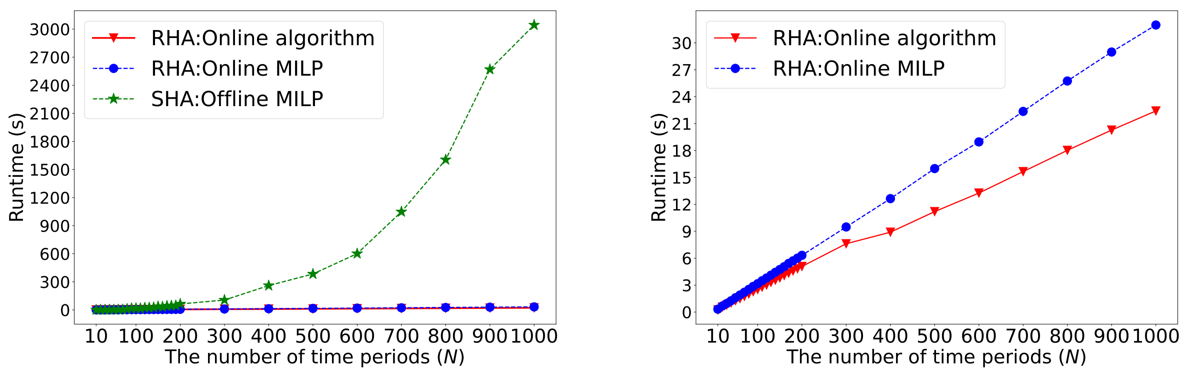
(b) $t - W$ with RHA ($\mathcal{T} = 240$)

Figure 9: RHA (Δt=1,$\mathcal{T}$=1) and RHA (Δt=1,$\mathcal{T}$=240)

5.2.4. Comparison of rolling horizon algorithm configurations

We conduct numerical experiments to compare the average value of computation time and social welfare obtained by the proposed online algorithm, online formulation, and offline formulation under the three types of RHA/SHA configurations introduced in Section 4. To compare the computational performance and solution quality under different RHA configurations, we ignore the influence of booking flexibility and set the time horizon ($\mathcal{T}$) to be the same as the time step (Δt), which is fixed in the first three configurations and increases from 10 $\sim$ 600 in SHA. In addition, since the first three configurations run in an online manner, they are referred to as online configurations. Instead, SHA is referred to as the offline configuration and is used as an oracle to benchmark the performance of the online RHA configurations.

The first RHA configuration uses Algorithm 1 with RHA parameters $\Delta t = 1$ and $\mathcal{T} = 1$. The second RHA configuration solves Model 1 as a MILP with RHA parameters $\Delta t = 1$ and $\mathcal{T} = 240$. The SHA configuration solves the offline MILP Model 2. The variation of the social welfare and the algorithms' computational performance under the three configurations tested are respectively



(a) Computation time

(b) A closer look at online configurations

Figure 10: Runtime under three RHA/SHA configurations

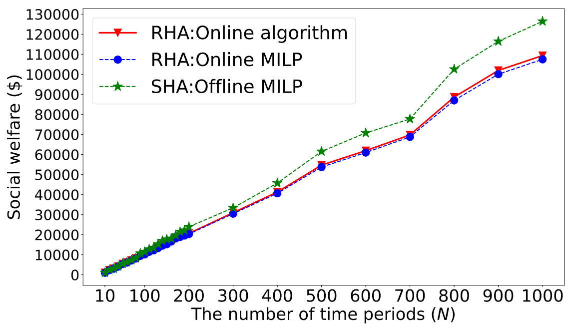


Figure 11: Social welfare under three RHA/SHA configurations

displayed in Fig. 10 and Fig. 11, where the number of time periods (N) is increased from 10 $\sim$ 1000 in each group, and the average over 20 instances of each group are reported. Hence, a total of 560 instances are solved using each configuration. We observe that the social welfare obtained by Algorithm 1 yields higher social welfare than online MILP Model 1. We find that Algorithm 1 is faster compared to solving Model 1 using CPLEX. Detailed average values of runtime and social welfare using each configuration are reported in Table E.7 in Appendix E.

6. Conclusion and remarks

We summarize the main contributions of this research and discuss future research directions.

6.1. Conclusion

This study makes four broad methodological contributions. The first is to propose an auction-based MaaS resource allocation mechanism, which allows users to bid for mode-agnostic mobility resources expressed in speed-weighted travel distance units. We show that the proposed mechanism is incentive-compatible (IC), individually rational (IR), and budget-balanced (BB). These theoretical results ensure that both end-users and the regulator benefit from the MaaS platform and support the adoption of the proposed resource allocation mechanism in MaaS systems. The second is to propose novel mathematical programming formulations for the online and offline mobility resource allocation problems that provide a new modeling framework to design MaaS bundles based on users' preferences and WTP. After introducing the concept of a user-based feasible MaaS bundle set, we propose compact reformulations of these mobility resource allocation problems as multidimensional knapsack problems. Third, we customize a polynomial-time, primal-dual online algorithm for the proposed mechanism, which is computationally efficient and can achieve a competitive ratio of 0.6321 when available mobility resources approach infinity. Fourth, we propose different RHA configurations to show how the proposed framework can be tuned to improve user booking flexibility and the tradeoff between improving solution quality and computational efficiency. Our numerical results uncover managerial insights into how the proposed MaaS resource allocation mechanism behaves in relation to different parameters, such as the maximum bidding price and the capacity of the MaaS system, highlight the performance of the online algorithm, as well as the benefits obtained through the RHA configurations. We show that the competitive ratio of the online algorithm can provide a lower bound on the social welfare ratio of the mechanism. Further, we find that RHA configurations with larger time horizons can improve users' booking flexibility. We also find that the proportion of MaaS bundles offered by the MaaS regulator is higher than the proportion accepted by users per day and is significantly higher during peak hours.

The global economic transition and information technologies are driving the transformation of the transport sector from an infrastructure/manufacturing-focused industry to a service/experience-focused industry. This renders the proposed models and algorithms essential tools to evaluate and operate futuristic MaaS systems. In line with the transition from a focus on "products" to "service" to "user experience", we propose a MaaS paradigm emphasizing the nature of service and user experience, where users submit experience-relevant preferences and WTP on different mobility services, instead of selecting segmented travel modes. The proposed MaaS paradigm quantifies the "effort" and "seamlessness" to advocate users' MaaS uptake and seeks out new perspectives to operate the MaaS systems. In sum, this study proposes a unified framework and tractable optimization models for the innovative MaaS paradigm, exploits the potentialities of designing MaaS bundles by resorting to the mobility resource allocation problems, proposes computationally efficient algorithms which can be implemented in real markets with large-scale data sets and provide meaningful managerial insights for the regulation of MaaS systems.

6.2. Remarks and future research

Sochor et al. (2018) proposed a four-level taxonomy to evaluate different MaaS schemes, and most of the current MaaS schemes, such as UbiGo have not reached Level 3 (integration of the service offer). In comparison, the proposed MaaS paradigm aims to reach the highest level (integration of societal goals). Although the proposed MaaS paradigm may provide several advantages as discussed above, several practical issues remain to address before such MaaS systems can be deployed in practice. At the micro level, users' habits and attitudes are recognized as essential factors. In the proposed MaaS paradigm, users are expected to quantify their travel preferences (e.g., inconvenience tolerance and travel delay budget) and report them to the MaaS regulator, which might be difficult to identify in practice. Karlsson et al. (2020) showed that it is difficult to change people's travel behavior due to the established habits and users' perceived "action space"; thus future research will investigate user adoption and attempt to quantify to which degree users are willing to change their travel habits. In addition, the proposed mechanism is not robust to collusion effects, and user coalitions may affect user payments. Given the expected scale of MaaS systems in which thousands of users may compete and the near real-time resolution of on-demand mobility services, collusion effects are unlikely to prevail, but further research may investigate the design of mechanisms that are IC at the group level. At the macro level, Merkert et al. (2020) identified the importance of integration and the elimination of the influence of boundary effects on different travel modes in a MaaS system and indicated that the combined operation of MaaS systems across public and private sectors might increase the pressure upon TSPs to provide multimodal and seamless services. Thus, the potential collaboration of different stakeholders in the MaaS system needs to be further investigated, and supply-side incentives might be required to promote cooperation across TSPs.

This study made a first step towards designing a mechanism for the demand side of MaaS systems by emphasizing the experience-relevant preferences of heterogeneous users in the markets under government contracting. However, supply-side interaction and market dynamics in MaaS systems are not accounted for in this study. This includes the integration of the transportation network structure into the resource allocation formulation and the modeling of detailed supply-side fleets and operations. Extensions of towards greater MaaS ecosystems also hold the potential to improve the sustainability of urban mobility and logistics systems. For example, Xi et al. (2022b) considered independent origin-destination pairs and derived a two-class bundle choice user equilibrium for mobility and delivery users in MaaS ecosystems to improve the sustainability of urban transportation systems. Future research can be extended to consider the interaction between OD pairs to model the oversaturated condition where congestion effects are taken into account for users' utility and system performance. The consideration of user incentives such as discounts, fare credits, and possibly subsidies leading to benefits that users cannot obtain through uni-modal trips also warrants further investigation.

Acknowledgments

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Appendix A. Mathematical notations

Variables	
l_i^m	Real variable denoting the service time served by mode m in the MaaS bundle for user i's bid
p_t	Parameter/real variable denoting the unit price at time period t in the online/offline resource allocation problem
p_i^t	Real variable denoting the actual payment of user i's bid at time period t
x_i	Binary variable denoting whether user i's bid is accepted
x_i^t	Binary variable denoting whether user i's bid is accepted at time period t
$\chi_{i,s}$	Binary variable denoting whether MaaS bundle s is allocated to user i
$\chi_{i,s}^t$	Binary variable denoting whether MaaS bundle s is allocated to user i at time period t
Parameters	
A_t	The quantity of available mobility resources at time period t
b_i	Bidding price of user i's bid
$b_{i,s}$	Bidding price of user i's MaaS bundle s
C	The capacity of the mobility resources in each time period
D_i	user i's shortest traveling distance arranged by MaaS regulator based on Origin destination information
L_i	The total service time periods allocated to user i's MaaS bundle
O_i	user i's requested departure time in MaaS online mechanism
p_i	The actual payment of user i's bid, which is a constant in each time period
Q_i	speed-weighted travel distance: distance weighted by user i's requested speed
$Q_{i,s}$	speed-weighted travel distance of user i's MaaS bundle s
T_i	The requested service time of user i
v_m	Average commercial speed of transport mode m
Γ_i	The maximum inconvenience degrees that user i can tolerate during a service
σ_m	inconvenience cost per unit of time for travel mode m
Φ_i	user i's *travel delay budget*: maximum delay that user i can accept during a service

Appendix B. Proof of Lemmas and Propositions

Appendix B.1. Proof of Lemma 1

Proof. Observe that all MaaS bundles $s \in \mathcal{S}_i$ may be allocated to user i since by definition $b_i \geq p_i$. Constraint (17c) imposes that at most one MaaS bundle is allocated to i. If $\chi_{i,s} = 0$ for all $s \in \mathcal{S}_i$, then no mobility resources are allocated to i which is equivalent to $x_i = 0$. Otherwise, if $\chi_{i,s} = 1$, then $s \in \mathcal{S}_i$. Since, by definition, all MaaS bundles $s \in \mathcal{S}_i$ verify constraints $(9) - (11)$, any solution $[\chi_{i,s}]_{i \in \mathcal{I}(t), s \in \mathcal{S}_i}$ of the compact IP (17) corresponds to a solution $[(x_i, \boldsymbol{l}_i)]_{i \in \mathcal{I}(t)}$ of Model 1 and vice-versa. Further, since the compact IP (17) is equivalent to Model 1, the LP-relaxation of (17) is equivalent to the LP-relaxation of Model 1. □

Appendix B.2. Proof of Proposition 1

Proof. Observe that there are two ways a user can manipulate the user payments: i) by bidding lower/higher than her WTP without changing the user ranking of the user set $\mathcal{I}(t)$; and ii) by bidding lower/higher than her WTP to change the user ranking of set $\mathcal{I}(t)$. We treat each type of manipulation separately.

For manipulations of type i), we assume that the user ranking of the set $\mathcal{I}(t)$ does not change as a result of user i's untruthful behavior. In this case, observe that the critical index does not change as a result of non-truthful behavior and thus the lower bound on the unit payment function $p_{\min}(t, \boldsymbol{b})$ is identical in truthful and non-truthful user behaviors.

For manipulations of type ii), we assume that the user i's declared bid may affect the user ranking of the set $\mathcal{I}(t)$. For this type of manipulations, observe that changes in the ranking may correspond to changes in the highest bidder but also changes in the critical index.

Note that user i's payment function could be impacted by the change of order in terms of $b_i^{\max}$ and $p_{\min}$, let $p_i(t, \tilde{\boldsymbol{b}})$ denote the user payment corresponding to user i's untruthful bidding, we consider the changes in payments in the following manipulations:

- manipulations of type i): $p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}})$ because $p_{\min}(t, \boldsymbol{b}) = p_{\min}(t, \tilde{\boldsymbol{b}})$ since user ranking is assumed to not change as a result of user i non-truthful behavior.

- manipulations of type ii):
  when resources are sufficient for each user, $p_i(t, \boldsymbol{b})$ is only impacted by $\bar{b}_i^{\max}(t)$
  when are not sufficient for each user, $p_i(t, \boldsymbol{b})$ are impacted by $\bar{b}_i^{\max}(t)$ and $p_{\min}(t, \boldsymbol{b}) = \frac{b_{k+1}}{Q_{k+1}}$

We first consider each sub-type of manipulation in terms of $\bar{b}_i^{\max}(t)$. For changes in the highest bidder, only two sub-cases are of interest: ii-a) user i is moving from highest (truthful) to non-highest (non-truthful) rank, and ii-b) user i is moving from non-highest (truthful) to highest (non-truthful) rank. For sub-case ii-a), we assume that: $\frac{b_1}{Q_1} < \frac{b_2}{Q_2} < \ldots < \frac{b_{n_t-1}}{Q_{n_t-1}} < \frac{\nu_{n_t}}{Q_{n_t}} = \frac{\nu_i}{Q_i}$, and that if user i bids non-truthfully then $\frac{b_i}{Q_i} < \frac{b_{n_t-1}}{Q_{n_t-1}}$. Observe that if user i is truthful, then Eq. (1) gives $\bar{b}_i^{\max}(t) = \frac{b_{n_t-1}}{Q_{n_t-1}}$. Conversely, if user i is non-truthful, then the highest bidder is user $n_t - 1$ and $\bar{b}_i^{\max}(t) = \frac{b_{n_t-1}}{Q_{n_t-1}}$. Hence, in this case, regardless of whether user i is truthful or not her payment is determined $\bar{b}_i^{\max}(t) = \frac{b_{n_t-1}}{Q_{n_t-1}}$. For sub-case ii-b), we assume that: $\frac{b_1}{Q_1} < \frac{b_2}{Q_2} < \ldots < \frac{\nu_i}{Q_i} < \frac{\nu_{b_t}}{Q_{n_t}}$, and that if user i bids non-truthfully then $\frac{b_i}{Q_i} > \frac{b_{n_t-1}}{Q_{n_t-1}}$. Observe that if user i is truthful, then Eq. (1) gives $\bar{b}_i^{\max}(t) = \frac{b_{n_t}}{Q_{n_t}}$. Conversely, if user i is non-truthful, then the second-highest bidder is user n_t and $\bar{b}_i^{\max}(t) = \frac{b_{n_t}}{Q_{n_t}}$. Hence, in this case, regardless of whether user i is truthful or not her payment is determined $\bar{b}_i^{\max}(t) = \frac{b_{n_t}}{Q_{n_t}}$. Observe that moving of more than one rank from highest to non-highest (or vice-versa) yields the same changes in terms of $\bar{b}_i^{\max}(t)$ as moving to the second highest rank (or vice-versa).

We have proved that $\bar{b}_i^{\max}(t)$ will not be impacted by the changes in order for manipulations of type i) and ii). We now consider changes in the ranking of set $\mathcal{I}(t)$ that affect the critical index and therefore the lower bound on the unit payment function $p_{\min}(t, \boldsymbol{b})$. Observe that if there is sufficient mobility resources to serve all users, then since $p_{\min} = p_{\text{res}}$ is a constant, then the payments of users are not affected by changes in the ranking.

When mobility resources are deficient in time period t, users' payments might be affected by changes in the ranking since $p_{\min} = \frac{b_{k+1}}{Q_{k+1}}$ where b_{k+1} denotes the bidding price of the last user that could be allocated with the resource in time period t. Since $b_{k+1} \leq p_i(t, \boldsymbol{b})$ always holds as shown in Eq.(5), if $p_i(t, \boldsymbol{b}) < b_i$ (Cases 1, 3, 6), then $b_{k+1} \leq b_i$ always holds, indicating that when the mobility resources are limited in time period t, mobility resources can be guaranteed for a user whose bidding price is higher than the payment.

To prove that the mechanism is IC, we enumerate all possible cases to compare the utility of user i with regard to truthful and non-truthful bidding behaviors. We add a '$\sim$' above the letter to denote the value of variables in non-truthful scenarios. Truthful bidding corresponds to user i reporting a bidding price of ν_i, whereas non-truthful bidding corresponds to user i reporting a bidding price of b_i. Recall that Constraint (12) indicates that if $b_i \geq p_i(t, \boldsymbol{b})$ (or $\nu_i \geq p_i(t, \boldsymbol{b})$ if bidding truthfully), then x_i is free; and if $b_i < p_i(t, \boldsymbol{b})$ (or $\nu_i < p_i(t, \boldsymbol{b})$ if bidding truthfully), then $x_i = 0$.

To analyse manipulation of type i) and ii), we enumerate all possible cases in Table B.5, if user i bids higher than her true valuation, i.e., $b_i > \nu_i$, her utility obtained from non-truthful bidding is equal to that obtained from truthful bidding (Cases 1, 2 and 3). If user i bids lower than her true valuation, i.e., $b_i < \nu_i$, her utility obtained from non-truthful bidding is no greater than that of bidding truthfully (Cases 4, 5 and 6). Note that $b_{k+1} \leq p_i(t, \boldsymbol{b})$ always holds as shown in Eq.(5), and that if $\nu_i < b_{k+1} < b_i <$, then $p_{\min}$ will be increased and $p_i(t, \boldsymbol{b}) < p_i(t, \tilde{\boldsymbol{b}})$.

In Case 1.1, since $b_{k+1} < \nu_i < b_i$, the payment corresponding to the untruthful bidding $p_i(t, \tilde{\boldsymbol{b}})$ and the payment corresponding to the truthful bidding $p_i(t, \boldsymbol{b})$ are same since $p_{\min}$ is not changed. In 1.2, since $\nu_i < b_{k+1} < b_i$, the payment corresponding to the untruthful bidding $p_i(t, \tilde{\boldsymbol{b}})$ will be higher than $p_i(t, \boldsymbol{b})$ since $p_{\min}$ will be higher. If i is non-truthful, then $\nu_i - p_i(t, \tilde{\boldsymbol{b}}) < 0$ in Cases 1.1 and 1.2, therefore $\tilde{u}_i = \max\left\{\nu_i - p_i(t, \tilde{\boldsymbol{b}}), r_i\right\} = r_i$, and since $b_i < p_i(t, \boldsymbol{b})$ in case 1.3, user i will be rejected and $\tilde{u}_i$=r˙i; if i bids truthfully $u_i = r_i$, it holds that $\tilde{u}_i = u_i$ in case 1.

In Cases 2.1, 2.2 and 2.3, since $b_i < p_i(t, \tilde{\boldsymbol{b}})$ always holds if she bids untruthfully and $\nu_i < p_i(t, \boldsymbol{b})$ always holds if she bids truthfully, user i will be rejected if she bids truthfully or not ($x_i = \tilde{x}_i = 0$), therefore $\tilde{u}_i = u_i = r_i$.

In Cases 3 and 6, since b_i and ν_i are always greater than b_{k+1}, the payments under untruthful and truthful bidding are same, i.e., $p_i(t, \tilde{\boldsymbol{b}}) = p_i(t, \boldsymbol{b})$, user i may be accepted or rejected, and her utility from MaaS if accepted is $\nu_i - p_i(t, \boldsymbol{b}) > 0$. If $x_i = 1$, then $u_i = \max\{\nu_i - p_i(t, \boldsymbol{b}), r_i\} \geq \tilde{u}_i = \max\{\tilde{x}_i(\nu_i - p_i(t, \boldsymbol{b})), r_i\}$. We now show that it is sub-optimal that $x_i = 0$ if there exists a feasible MaaS bundle for user i. Note that if no feasible MaaS bundle exist, then $x_i = \tilde{x}_i = 0$ and $u_i = \tilde{u}_i = r_i$. Now assume there exists a feasible MaaS bundle for user i, if there is sufficient resources to serve all users $\sum_{i=1}^{n_t} Q_i \leq A_t$, then $x_i = 0$ is sub-optimal since accepting i only increases the social welfare as defined in Eq. (7). If there is not enough resources to serve all users, since by assumption $p_i(t, \boldsymbol{b})$ is lesser than ν_i and b_i, then the critical index k is such that $k+1 \leq i$. Observe that $x_j = 0$, for any $j \leq k$ because $p_j \geq Q_j p_{\min}(t, \boldsymbol{b}) = Q_j \frac{b_{k+1}}{Q_{k+1}}$, therefore:

$$b_j - p_j \leq b_j - Q_j \frac{b_{k+1}}{Q_{k+1}} < 0$$

since $\frac{b_j}{Q_j} < \frac{b_{k+1}}{Q_{k+1}}$ and therefore Eq. (10) imposes $x_j = 0$. This implies that it is guaranteed that there is enough mobility resources for users $k+1, \ldots, n_t$ and thus $x_i = 0$ is sub-optimal if there exists a feasible MaaS bundle for user i.

In case 4.1, since $b_{k+1} < b_i < \nu_i$ and $p_{\min}$ will not be changed, the payments under untruthful and truthful bidding are same, i.e., $p_i(t, \tilde{\boldsymbol{b}}) = p_i(t, \boldsymbol{b})$, since $b_i < p_i(t, \tilde{\boldsymbol{b}})$ always holds if she bids untruthfully, user i will be rejected ($\tilde{x}_i = 0$), therefore $\tilde{u}_i = r_i$. In case 4.2 and 4.3, since $b_i < b_{k+1}$, user i will be directly rejected due to the insufficient mobility resource, and her utility is $\tilde{u}_i = r_i$. If she bids truthfully, she will be rejected since $\nu_i < p_i(t, \boldsymbol{b})$ always holds, and $u_i = r_i$. Thus we have $\tilde{u}_i = u_i = r_i$.

In Case 5, if user i bids untruthfully, she will be rejected since $b_i < p_i(t, \tilde{\boldsymbol{b}})$ in case 5.1 and be rejected since mobility resources are deficient for user i, i.e., $b_i < b_{k+1}$, and thus $\tilde{u}_i = r_i$; if user i bids truthfully, we have $u_i = \max\{r_i, \nu_i - p_i(t, \boldsymbol{b})\} \geq r_i$, therefore $u_i \geq \tilde{u}_i$.

Table B.5: Utility comparison under truthful and non-truthful bidding.

No.	Cases	Non-truthful bidding $\tilde{u}_i$		Truthful bidding u_i		Utility comparison
		$\tilde{x}_i = 0$	$\tilde{x}_i = 1$	$x_i = 0$	$x_i = 1$	
1	$\nu_i < p_i(t, \boldsymbol{b}) < b_i$					
1.1	$b_{k+1} < \nu_i < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}}) < b_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	$\tilde{u}_i = u_i$
1.2	$\nu_i < b_{k+1} < p_i(t, \boldsymbol{b}) < p_i(t, \tilde{\boldsymbol{b}}) < b_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	$\tilde{u}_i = u_i$
1.3	$\nu_i < b_{k+1} < p_i(t, \boldsymbol{b}) < b_i < p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
2	$\nu_i < b_i < p_i(t, \boldsymbol{b})$					
2.1	$b_{k+1} < \nu_i < b_i < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
2.2	$\nu_i < b_{k+1} < b_i < p_i(t, \boldsymbol{b}) < p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
2.3	$\nu_i < b_i < b_{k+1} < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
3	$p_i(t, \boldsymbol{b}) < \nu_i < b_i$					
3.1	$b_{k+1} < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}}) < \nu_i < b_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i = u_i$
4	$b_i < \nu_i < p_i(t, \boldsymbol{b})$					
4.1	$b_{k+1} < b_i < \nu_i < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
4.2	$b_i < b_{k+1} < \nu_i < p_i(t, \boldsymbol{b})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
4.3	$b_i < \nu_i < b_{k+1} < p_i(t, \boldsymbol{b})$	r_i	$-$	r_i	$-$	$\tilde{u}_i = u_i$
5	$b_i < p_i(t, \boldsymbol{b}) < \nu_i$					
5.1	$b_{k+1} < b_i < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}}) < \nu_i$	r_i	$-$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i \leq u_i$
5.2	$b_i < b_{k+1} < p_i(t, \boldsymbol{b}) < \nu_i$	r_i	$-$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i \leq u_i$
6	$p_i(t, \boldsymbol{b}) < b_i < \nu_i$					
6.1	$b_{k+1} < p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}}) < b_i < \nu_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i = u_i$

Overall, since user i's utility obtained by bidding truthfully is no less than her utility obtained by bidding non-truthfully in both types of manipulation i) and manipulation ii), i.e., $u_i \geq \tilde{u}_i$, IC always holds.

This analysis shows that bidding truthfully is the weakly dominant strategy for both types of manipulations. Hence the proposed MaaS resource allocation mechanism is IC. □

Appendix B.3. Proof of IC

There are two non-truthful user behavior: overbidding ($b_i > \nu_i$) and underbidding ($b_i < \nu_i$). Non-truthful behavior may affect the ranking of users in set $\mathcal{I}(t)$ which may itself affect the payment of users. User payment is also function of the availability of mobility resources. Our proof logic is based on comparing the payment of user i when she is truthful or not and determining the associated utilities.

We denote $p_i(t, \boldsymbol{b})$ the payment of user i if i is truthful, i.e. $b_i = \nu_i$ and $p_i(t, \tilde{\boldsymbol{b}})$ the payment of i if i is non-truthful, i.e. $b_i \neq \nu_i$. Observe that users' payment function depends on two terms that may be affected by users' bids: $\bar{b}_i^{\max}(t)$ and $p_{\min}(t, \boldsymbol{b})$. According to Eq. (2), the lower bound $p_{\min}(t, \boldsymbol{b})$ is fixed to p_{res} unless the amount of mobility resources is not sufficient to serve all users. Further, if there is only a single bidder, then either the user is served and its payment is p_{res} or if there is not sufficient mobility resources the user is rejected. Therefore in the remaining, we assume that there is at least two users in the auction mechanism.

By definition, the term $\bar{b}_i^{\max}(t)$ as defined in Eq. (1) is not affected by the bid of user i. Specifically, if user i is the highest bidder whether she is truthful or not, then $\bar{b}_i^{\max}(t)$ does not change. If user i is not the highest bidder when truthful, i.e. $i < n_t$, then Eq. (1) gives $\bar{b}_i^{\max}(t) = \frac{b_{n_t}}{Q_{n_t}}$. If i bids non-truthfully, either she becomes the highest-bidder and user n_t becomes the second-highest bidder, in which case $\bar{b}_i^{\max}(t) = \frac{b_{n_t}}{Q_{n_t}}$; or $\bar{b}_i^{\max}(t)$ does not change. Hence, regardless of whether user i is truthful or not $\bar{b}_i^{\max}(t)$ is not affected by the bid of user i. Hence, the non-truthful behavior of user i can only affect the payment function $p_i(t, \boldsymbol{b})$ by affecting its lower bound $p_{\min}(t, \boldsymbol{b})$.

We state the following lemmas to characterize the relationship between non-truthful bidding and the user payment function, and the behavior of the mechanism in the case of deficient mobility resources.

Lemma 6. *If user i overbids, i.e. $b_i > \nu_i$, then its payment cannot decrease, i.e. $p_i(t, \tilde{\boldsymbol{b}}) \geq p_i(t, \boldsymbol{b})$. Reciprocally, if user i underbids, i.e. $b_i < \nu_i$, then its payment cannot increase, i.e. $p_i(t, \tilde{\boldsymbol{b}}) \leq p_i(t, \boldsymbol{b})$.*

Proof. As specified in Eq. (2), this lower bound is fixed if $\sum_{i=1}^{n_t} Q_i \leq A_t$. Further, if the non-truthful behavior of i does not affect user ranking, this lower bound does not change. Thus the lemma holds in these cases. Assume now that there is not enough resources to serve all users and that the non-truthful behavior of i affects user ranking. In this case, $p_{\min}(t, \boldsymbol{b}) = b_{k+1}/Q_{k+1}$ where $k < i$ is the critical index if i is truthful. If when truthful $i = k+1$ and if i overbids, then the lower bound increases to $b_{k+2}/Q_{k+2} > b_{k+1}/Q_{k+1}$, thus $p_i(t, \tilde{\boldsymbol{b}}) \geq p_i(t, \boldsymbol{b})$. If when truthful $i > k+1$ and if i underbids such that $b_i/Q_i < b_{k+1}/Q_{k+1}$, then the lower bound decreases to b_i/Q_i, thus $p_i(t, \tilde{\boldsymbol{b}}) \leq p_i(t, \boldsymbol{b})$. In all other cases, the lower bound does not change as a result of i's non-truthful behavior and thus $p_i(t, \tilde{\boldsymbol{b}}) = p_i(t, \boldsymbol{b})$. □

Lemma 7. *If mobility resources are deficient, i.e. there exists a critical index k such that $\sum_{i=k+1}^{n_t} Q_i \leq A_t < \sum_{i=k}^{n_t} Q_i$, then:*

- *if $i \leq k$ then user i is rejected, i.e. $x_i = 0$;*
- *if $i \geq k+1$ then, if there exists a feasible MaaS bundle for user i, it is sub-optimal to reject user i.*

Proof. Observe that if $i \leq k$, then $p_i(t, \boldsymbol{b}) \geq Q_i p_{\min}(t, \boldsymbol{b}) = Q_i \frac{b_{k+1}}{Q_{k+1}}$. Since by assumption $\frac{b_i}{Q_i} < \frac{b_{k+1}}{Q_{k+1}}$, we have:

$$b_i - p_i(t, \boldsymbol{b}) \leq b_i - Q_i \frac{b_{k+1}}{Q_{k+1}} < 0,$$

and Eq. (10) imposes $x_i = 0$. This implies that it is guaranteed that there is enough mobility resources for users $k+1, \ldots, n_t$ and thus it is sub-optimal to reject a user $i \geq k+1$ if there exists a feasible MaaS bundle for user i. □

Using Lemma 6, we enumerate all feasible permutations of ν_i, b_i, $p_i(t, \boldsymbol{b})$ and $p_i(t, \tilde{\boldsymbol{b}})$; and compare the utility of user i in truthful (u_i) and non-truthful ($\tilde{u}_i$) bidding scenarios. This analysis is summarized in Table B.6. Cases 1-6 correspond to overbidding and Cases 7-12 correspond to underbidding. Recall that the mechanism rejects users whose bids are lower than their payment (see Constraint (10)), thus if $b_i = \nu_i$ (truthful) and $b_i < p_i(t, \boldsymbol{b})$, then $x_i = 0$; and analogously if

Table B.6: Utility comparison under truthful and non-truthful bidding.

No.	Cases	Non-truthful		Truthful		Utility comparison
		$\tilde{x}_i = 0$	$\tilde{x}_i = 1$	$x_i = 0$	$x_i = 1$	
1	$\nu_i < b_i < p_i(t, \boldsymbol{b}) \leq p_i(t, \tilde{\boldsymbol{b}})$	r_i	–	r_i	–	$\tilde{u}_i = u_i$
2	$\nu_i < p_i(t, \boldsymbol{b}) \leq b_i < p_i(t, \tilde{\boldsymbol{b}})$	r_i	–	r_i	–	$\tilde{u}_i = u_i$
3	$\nu_i < p_i(t, \boldsymbol{b}) \leq p_i(t, \tilde{\boldsymbol{b}}) \leq b_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	–	$\tilde{u}_i = u_i$
4	$p_i(t, \boldsymbol{b}) \leq \nu_i < b_i < p_i(t, \tilde{\boldsymbol{b}})$	r_i	–	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i \leq u_i$
5	$p_i(t, \boldsymbol{b}) \leq \nu_i < p_i(t, \tilde{\boldsymbol{b}}) \leq b_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i \leq u_i$
6	$p_i(t, \boldsymbol{b}) \leq p_i(t, \tilde{\boldsymbol{b}}) \leq \nu_i < b_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i \leq u_i$
7	$b_i < \nu_i < p_i(t, \tilde{\boldsymbol{b}}) \leq p_i(t, \boldsymbol{b})$	r_i	–	r_i	–	$\tilde{u}_i = u_i$
8	$b_i < p_i(t, \tilde{\boldsymbol{b}}) \leq \nu_i < p_i(t, \boldsymbol{b})$	r_i	–	r_i	–	$\tilde{u}_i = u_i$
9	$b_i < p_i(t, \tilde{\boldsymbol{b}}) \leq p_i(t, \boldsymbol{b}) \leq \nu_i$	r_i	–	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i \leq u_i$
10	$p_i(t, \tilde{\boldsymbol{b}}) \leq b_i < \nu_i < p_i(t, \boldsymbol{b})$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	–	$\tilde{u}_i ?\, u_i$
11	$p_i(t, \tilde{\boldsymbol{b}}) \leq b_i < p_i(t, \boldsymbol{b}) \leq \nu_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i ?\, u_i$
12	$p_i(t, \tilde{\boldsymbol{b}}) \leq p_i(t, \boldsymbol{b}) \leq b_i < \nu_i$	r_i	$\nu_i - p_i(t, \tilde{\boldsymbol{b}})$	r_i	$\nu_i - p_i(t, \boldsymbol{b})$	$\tilde{u}_i ?\, u_i$

$b_i \neq \nu_i$ (non-truthful) $b_i < p_i(t, \tilde{\boldsymbol{b}})$, then $\tilde{x}_i = 0$. This scenario occurs in Cases 1, 2, 7 and 8 where both truthful and non-truthful bids are less than their corresponding payments, thus $x_i = \tilde{x}_i = 0$ and $u_i = \tilde{u}_i = r_i$.

In Case 3, user i is rejected if truthful thus $u_i = r_i$. If i is non-truthful, $\tilde{x}_i$ is free but $\nu_i - p_i(t, \tilde{\boldsymbol{b}}) < 0$, therefore and $\tilde{u}_i = \max\{\tilde{x}_i(\nu_i - p_i(t, \tilde{\boldsymbol{b}})), r_i\} = r_i = u_i$. In Case 4, user i is rejected if non-truthful thus $\tilde{u}_i = r_i$. If i is truthful, her utility from MaaS may be larger than r_i since $\nu_i - p_i(t, \boldsymbol{b}) \geq 0$, therefore $u_i \geq r_i = \tilde{u}_i$. In Case 5, both truthful and non-truthful bids are greater or equal to their corresponding payments, but $\nu_i - p_i(t, \tilde{\boldsymbol{b}}) < 0$ and $\nu_i - p_i(t, \boldsymbol{b}) \geq 0$. Therefore $\tilde{u}_i = r_i \leq \max\{x_i(\nu_i - p_i(t, \boldsymbol{b}))\} = u_i$. In Case 6, user i may derive positive utility from MaaS whether truthful or not but since $p_i(t, \boldsymbol{b}) \leq p_i(t, \tilde{\boldsymbol{b}})$, $\nu_i - p_i(t, \boldsymbol{b}) \geq \nu_i - p_i(t, \tilde{\boldsymbol{b}})$ and $\tilde{u}_i \leq u_i$.

In Case 9, user i is rejected if non-truthful thus $\tilde{u}_i = r_i$. If i is truthful, her utility from MaaS may be larger than r_i since $\nu_i - p_i(t, \boldsymbol{b}) \geq 0$, therefore $u_i \geq r_i = \tilde{u}_i$. For Cases 10, 11 and 12:

- If there are sufficient resources to serve all users, i.e. $\sum_{i=1}^{n_t} Q_i \leq A_t$, then $p_{\min}(t, \boldsymbol{b}) = p_{\min}(t, \tilde{\boldsymbol{b}})$ and $p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}})$, thus Cases 10 and 11 are infeasible. In Case 12, user i derives the same utility from MaaS whether she bids truthfully or not. Furthermore, it is optimal to serve i whether she is truthful or not, i.e. $x_i = \tilde{x}_i = 1$ and $u_i = \tilde{u}_i = \max\{\nu_i - p_i(t, \boldsymbol{b}), r_i\}$.
- If resources are deficient but the user ranking does not change due to i underbidding, then $p_i(t, \boldsymbol{b}) = p_i(t, \tilde{\boldsymbol{b}})$, and the reasoning of the previous setting (sufficient resources) holds for Cases 10, 11 and 12.
- If the user ranking changes due to user i underbidding and if resources are deficient,
    - Case 10: if i is truthful then i is rejected and $u_i = r_i$. enumerate cases: $i \leq k, i = k+1, i > k+1$........

Appendix B.4. Proof of Proposition 2

Proof. Individual rationality. According to the bidding rule of the MaaS mechanism, a user is only accepted if her biding price is larger than or equal to the user payment, i.e., $b_i \geq p_i(t, \boldsymbol{b})$; otherwise, her bid is rejected. If a user bids truthfully, i.e., $\nu_i = b_i$ and her bid is accepted, the user utility for using MaaS is $\nu_i - p_i$, which is nonnegative. If a user's bid is rejected, her utility is the reserve utility r_i, which is nonnegative. Therefore, it follows that each user obtains a nonnegative utility from bidding truthfully. Thus, the proposed MaaS mechanism is individually rational.

Budget balance. According to the proposed time-varying pricing strategy, since $z_t \in [0, C]$, $p_i^{\mathsf{pay}}(t, \boldsymbol{b}) \in [p_{\min}, \bar{b}_i^{\max}(t) + p_{\min}]$, where $p_{\min}$ denotes the reserve price of the MaaS regulator and $\bar{b}_i^{\max}$ is the maximum unit bidding price relative to user i. Since the payoff of the MaaS regulator, i.e., $p_i^{\mathsf{pay}}(t, \boldsymbol{b})Q_i - p_{\min}Q_i$, is always nonnegative, the proposed MaaS mechanism is budget balanced. □

Appendix B.5. Proof of Lemma 2

Proof. The operations in Lines 3 to 10 have a worst-case time complexity of $\mathcal{O}(|\mathcal{I}(t)|\log|\mathcal{I}(t)|)$ corresponding to the sort operation in Line 3. This is dominated by the **for** loop starting from Line 11, which requires $\mathcal{O}(|\mathcal{I}(t)|LP)$ time since in the worst-case one linear feasibility problem must be solved for each user in $\mathcal{I}(t)$. □

Appendix B.6. Proof of Lemma 3

Proof. In each iteration of Algorithm 1, Δq_t is obtained from Line 18 in Algorithm 1, and then let y_i be $b_i - Q_i q_t$, $\Delta\mathcal{D}(i)$ can be written as Eq. (B.1):

$$\begin{aligned}
\Delta\mathcal{D}(i) = A_t\Delta q_t + u_i &= A_t\left[q_t\frac{Q_i}{A_t} + \frac{b_i}{(\overline{\alpha}_t - 1)A_t}\right] + u_i, \\
&= Q_i q_t + \frac{b_i}{\overline{\alpha}_t - 1} + b_i - Q_i q_t, \\
&= b_i\left(\frac{1}{\overline{\alpha}_t - 1} + 1\right).
\end{aligned} \tag{B.1}$$

Since $x_j = 1$ (Line 17), the change value in the objective function of the primal problem is $\Delta\mathcal{P}(i) = b_i$. Thus the relationship between $\Delta\mathcal{D}(i)$ and $\Delta\mathcal{P}(i)$ is:

$$\Delta\mathcal{D}(i) = \left(\frac{1}{\overline{\alpha}_t - 1} + 1\right)\Delta\mathcal{P}(i). \tag{B.2}$$

□

Appendix B.7. Proof of Lemma 4

Proof. We first prove that Algorithm 1 yields dual feasible solutions of $OPT_2(t)$. Let q_t denote the value of the dual variable at the end of each iteration in a time loop (Line 2). If $q_t \geq \frac{b_i}{Q_i}$, then the dual constraint always holds; else if $q_t \leq \frac{b_i}{Q_i}$, the dual variables will be increased until the dual constraints are satisfied. Let the dual variable u_i be $b_i - Q_i q_t$, the subsequent increase of q_t is always feasible due to the acceptance determination rules: if user i's unit bidding price is no less than the unit price at time period t, i.e $q_t \leq \frac{b_i}{Q_i}$, user i will be accepted; otherwise, if $q_t \geq \frac{b_i}{Q_i}$, user i will be rejected, and the dual variables will not be updated.

We next show that Algorithm 1 yields primal feasible solutions of $OPT_1(t)$. The iteration rule of q_t (Line 14) ensures that q_t is bounded by the sum of a geometric sequence with the common ratio $(1 + \frac{Q_i}{A_t})$. Consider a geometric sequence produced by the iterations of q_t for user m: the first item is $\frac{b_m}{(\overline{\alpha}_t-1)A_t}$, in which the value of b_m is fixed in each iteration (Line 11), and the common ratio is $1+\frac{Q_m}{A_t}$. Summing the terms of this geometric sequence gives:

$$q_t \geq \frac{b_m}{Q_m} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left[\left(1+\frac{Q_m}{A_t}\right)^{\sum\limits_{i\in\mathcal{I}(t)} x_i} - 1 \right], \tag{B.3}$$

where the number of iterations in each time loop is larger than $\sum_{i\in\mathcal{I}(t)} x_i$. Eq.(B.3) is rewritten as:

$$q_t \geq \frac{b_m}{Q_m} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left[\left(1+\frac{Q_m}{A_t}\right)^{\frac{A_t}{Q_m}\cdot\frac{\sum\limits_{i\in\mathcal{I}(t)} Q_m x_i}{A_t}} - 1 \right]. \tag{B.4}$$

Let $\overline{R}_t$ denote the maximum ratio of a user's requested quantity of mobility resources to the available resources at time period t, $\overline{R}_t = \max_{i\in\mathcal{I}(t)} \left\{\frac{Q_i}{A_t}\right\}$. Since $0 \leq \frac{Q_i}{A_t} \leq \frac{Q_m}{A_t} \leq 1$, we have:

$$q_t \geq \frac{b_m}{Q_m} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left[\left(1+\frac{Q_m}{A_t}\right)^{\frac{A_t}{Q_m}\cdot\frac{\sum\limits_{i\in\mathcal{I}(t)} Q_i x_i}{A_t}} - 1 \right]. \tag{B.5}$$

Since $0 \leq \frac{Q_i}{A_t} \leq \overline{R}_t \leq 1$, we have $\frac{\ln(1+\frac{Q_i}{A_t})}{\frac{Q_i}{A_t}} \geq \frac{\ln(1+\overline{R}_t)}{\overline{R}_t}$, thus, $(1+\frac{Q_i}{A_t})^{\overline{R}_t} \geq (1+\overline{R}_t)^{\frac{Q_i}{A_t}}$, and:

$$1+\frac{Q_i}{A_t} \geq (1+\overline{R}_t)^{\frac{1}{\overline{R}_t}\cdot\frac{Q_i}{A_t}}. \tag{B.6}$$

Since $\alpha_t = \left(1+\overline{R}_t\right)^{\frac{1}{\overline{R}_t}}$, substituting Eq.(B.6) into Eq.(B.5) yields:

$$q_t \geq \frac{b_m}{Q_m} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left(\overline{\alpha}_t^{\frac{\sum\limits_{i\in\mathcal{I}(t)} Q_i x_i}{A_t}} - 1 \right). \tag{B.7}$$

According to Eq. (B.7), if $\sum_{i\in\mathcal{I}(t)} Q_i x_i \geq A_t$, then $q_t \geq \frac{b_m}{Q_m}$. Since Algorithm 1 does not update the primal solution if $q_t \geq \frac{b_m}{Q_m}$, the primal solutions will only be updated once $\sum_{i\in\mathcal{I}(t)} Q_i x_i \leq A_t$ or $\sum_{i\in\mathcal{I}(t)} \sum_{s\in\mathcal{S}_i} Q_{i,s}\chi_{i,s} \leq A_t$. Thus, Algorithm 1 yields dual feasible solutions of $OPT_2(t)$ and primal solutions of $OPT_1(t)$. □

Appendix B.8. Proof of Proposition 3

Proof. Let k be the critical index determined at Line 4 of Algorithm 1. Let q_t^{end} and q_t^{start} denote the value of q_t before and after each iteration in the loop of $i = k$ in Algorithm 1, respectively.

Substituting q_t^{end} and q_t^{start} into Line 14 of Algorithm 1, yields Eq. (B.8):

$$q_t^{\text{end}} = q_t^{\text{start}} \left(1 + \frac{Q_k}{A_t}\right) + \frac{b_k}{(\overline{\alpha}_t - 1)A_t}. \tag{B.8}$$

According to Eq.(B.7), before examining user k's bids, the value of q_t^{start} is bounded as:

$$q_t^{\text{start}} \geq \frac{b_k}{Q_k} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left(\overline{\alpha}_t^{\frac{\sum\limits_{i \in \mathcal{I}(t)\setminus\{k\}} Q_i x_i}{A_t}} - 1 \right). \tag{B.9}$$

Substituting Eq.(B.9) into Eq.(B.8), and simplifying yields:

$$q_t^{\text{end}} \geq \frac{b_k}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left(\overline{\alpha}_t^{\frac{\sum\limits_{i \in \mathcal{I}(t)\setminus\{k\}} Q_i x_i}{A_t}} - 1 \right) \cdot \left(1 + \frac{Q_k}{A_t}\right) + \frac{b_k}{(\overline{\alpha}_t - 1) \cdot A_t}, \tag{B.10}$$

$$\geq \frac{b_k}{Q_k} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left[\overline{\alpha}_t^{\frac{\sum\limits_{i \in \mathcal{I}(t)\setminus\{k\}} Q_i x_i}{A_t}} \cdot \left(1 + \frac{Q_k}{A_t}\right) - 1 \right]. \tag{B.11}$$

According to Eq.(B.6), we have $1 + \frac{Q_k}{A_t} \geq (1 + \overline{R}_t)^{\frac{1}{\overline{R}_t} \cdot \frac{Q_k}{A_t}}$, thus Eq.(B.11) can be rewritten as:

$$q_t^{\text{end}} \geq \frac{b_k}{Q_k} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left[\overline{\alpha}_t^{\frac{\sum\limits_{i \in \mathcal{I}(t)\setminus\{k\}} Q_i x_i}{A_t}} \cdot (1 + \overline{R}_t)^{\frac{1}{\overline{R}_t} \cdot \frac{Q_k}{A_t}} - 1 \right]. \tag{B.12}$$

Assume that user k is accepted at time period t, then $\sum_{i \in \mathcal{I}(t)\setminus\{k\}} Q_i x_i + Q_k x_k = \sum_{i \in \mathcal{I}(t)} Q_i x_i$, and since $\overline{\alpha}_t = (1 + \overline{R}_t)^{\frac{1}{\overline{R}_t}}$ Eq.(B.12) can be written as:

$$q_t^{\text{end}} \geq \frac{b_k}{Q_k} \cdot \frac{1}{\overline{\alpha}_t - 1} \cdot \left(\overline{\alpha}_t^{\frac{\sum\limits_{i \in \mathcal{I}(t)\setminus\{k\}} Q_i x_i}{A_t}} \cdot \overline{\alpha}_t^{\frac{Q_k}{A_t}} - 1 \right), \tag{B.13}$$

$$= \frac{b_k}{Q_k} \cdot \frac{1}{\alpha_t - 1} \cdot \left(\overline{\alpha}_t^{\frac{\sum\limits_{i \in \mathcal{I}(t)} Q_i x_i}{A_t}} - 1 \right). \tag{B.14}$$

Eq. (B.14) illustrates that if the requested quantity of mobility resources exceeds the available mobility resources at time period t, i.e., $\sum_{i \in \mathcal{I}(t)} Q_i x_i > A_t$, then user k will not be allocated with any resources due to the fact that $q^{\text{end}} \geq \frac{b_k}{Q_k}$; thus $\sum_{i \in \mathcal{I}(t)} Q_i x_i \geq A_t - \max_{i \in \mathcal{I}(t)}\{Q_i\}$, $\forall t \in \Omega$.

Since $\overline{R}_t = \max_{i \in \mathcal{I}(t)} \left\{ \frac{Q_i}{A_t} \right\}, \forall t \in \Omega$, the social welfare obtained by Algorithm 1 at time period t is at least:

$$\sum_{i \in \mathcal{I}(t)} b_i x_i \cdot \frac{A_t - \max_{i \in \mathcal{I}(t)} \{Q_i\}}{A_t} = \sum_{i \in \mathcal{I}(t)} b_i x_i \cdot \left(1 - \overline{R}_t\right). \tag{B.15}$$

Based on Lemma1, we have $\sum_{i \in \mathcal{I}(t)} \sum_{s \in \mathcal{S}_i} b_{i,s} \chi_{i,s} = \sum_{i \in \mathcal{I}(t)} b_i x_i$. According to Lemma 3, $\Delta\mathcal{D}(i)$ and $\Delta\mathcal{P}(i)$ denote the change value in the objective functions of dual problem $OPT_2(t)$ and primal problem $OPT_1(t)$, respectively, at each iteration of Algorithm 1. Let $\mathcal{P}(t)$ and $\mathcal{D}(t)$ denote the objective value of the dual and primal problems obtained by Algorithm 1. In time loop t, we have $\mathcal{D}(t) = \sum_{i \in \mathcal{I}(t)} \Delta\mathcal{D}(i)$ and $\mathcal{P}(t) = \sum_{i \in \mathcal{I}(t)} \Delta\mathcal{P}(i)$. Based on Lemma 3, the relationship between $\mathcal{D}(t)$ and $\mathcal{P}(t)$ is:

$$\mathcal{P}(t) = \left(1 - \frac{1}{\alpha_t}\right) \mathcal{D}(t). \tag{B.16}$$

Given the input time sequence $\boldsymbol{\tau} = [1, 2, \cdots, t-1, t]$, let $\mathcal{P}(\boldsymbol{\tau})$ and $\mathcal{D}(\boldsymbol{\tau})$ denote the objective values of OPT_3 and OPT_4, respectively, obtained by Algorithm 1. Since $\alpha = \min_{t \in \boldsymbol{\tau}} \alpha_t$, Eq. (B.16) implies:

$$\mathcal{P}(\boldsymbol{\tau}) \geq \left(1 - \frac{1}{\alpha}\right) \mathcal{D}(\boldsymbol{\tau}). \tag{B.17}$$

Let $W_{\text{Alg1}}(\boldsymbol{\tau})$ denote the social welfare obtained by Algorithm 1 corresponding to the sequence $\boldsymbol{\tau}$. Since $\mathbb{R}_{max} = \max_{t \in \boldsymbol{\tau}} \overline{R}_t$, according to Eq. (B.15), we have $W_{\text{Alg1}}(\boldsymbol{\tau}) \geq (1 - \mathbb{R}_{max}) \mathcal{P}(\boldsymbol{\tau})$ and:

$$W_{\text{Alg1}}(\boldsymbol{\tau}) \geq (1 - \mathbb{R}_{max}) \left(1 - \frac{1}{\alpha_t}\right) \mathcal{D}(\boldsymbol{\tau}). \tag{B.18}$$

Given the input sequence $\boldsymbol{\tau}$, let $\mathcal{Z}^{IP}(\boldsymbol{\tau})$ denote the optimal value of the offline compact IP (21), let $\mathcal{Z}^*(\boldsymbol{\tau})$ and $\mathcal{D}(\boldsymbol{\tau})$ denote the optimal value of the offline LP-relaxation OPT_3 and its dual problem OPT_4, respectively. According to weak duality, we have $\mathcal{D}(\boldsymbol{\tau}) \geq \mathcal{Z}^*(\boldsymbol{\tau})$, and Eq. (B.18) can be rewritten as follows:

$$W_{\text{Alg1}}(\boldsymbol{\tau}) \geq (1 - \mathbb{R}_{max}) \left(1 - \frac{1}{\alpha_t}\right) \mathcal{Z}^*(\boldsymbol{\tau}), \tag{B.19}$$

$$\geq (1 - \mathbb{R}_{max}) \left(1 - \frac{1}{\alpha}\right) \mathcal{Z}^{IP}(\boldsymbol{\tau}). \tag{B.20}$$

Hence the competitive ratio of Algorithm 1 is $\Theta = (1 - \mathbb{R}_{max}) \left(1 - \frac{1}{\alpha}\right)$. □

Appendix C. Procedure of the rolling horizon configuration

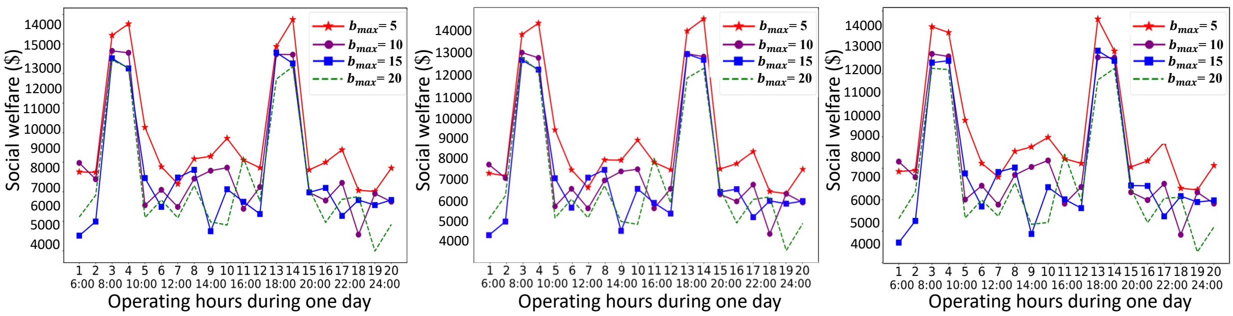
Algorithm 2: Rolling horizon configurations

Input: $\mathcal{B}_i = \{O_i, D_i, \Phi_i, \Gamma_i, T_i, b_i\}, \forall i \in \mathcal{I}(t)$
Output: $\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{l}$

```
initialize time step Δt and time horizon lengths 𝒯
A_t = A[t]
if Δt = 1 then
    [x_i, q_t, l_i^m] ← Algorithm 1(B_i, A_t) or Model 1 (B_i, A_t)
    p_i ← q_t Q_i x_i
    L_i ← ⌈Σ_{m∈M} l_i^m⌉
    A[O_i : O_i + L_i − 1] ← A[O_i : O_i + L_i − 1] − Q_i x_i / L_i
if 1 < Δt ≤ ω then
    [x_i^t, p_t, p_i^t, l_i^{mt}] ← Model 2 (B_i, Ω_n, A_t)
return x, p, l
```

Appendix D. Detailed sensitivity analysis of the MaaS resource allocation mechanism

We evaluate the performance of the MaaS resource allocation mechanism by conducting a sensitivity analysis on the maximum bidding price and the type of user payment function. We report the variation of hourly average social welfare under different maximum bidding prices ($b_{\max}$) using different types of unit price functions in Fig. D.12. If $b_{\max} = 5$, the value of social welfare is higher than its counterparts under all three types of user payment functions. Furthermore, the hourly average social welfare exhibits a similar pattern using all three user payment functions.

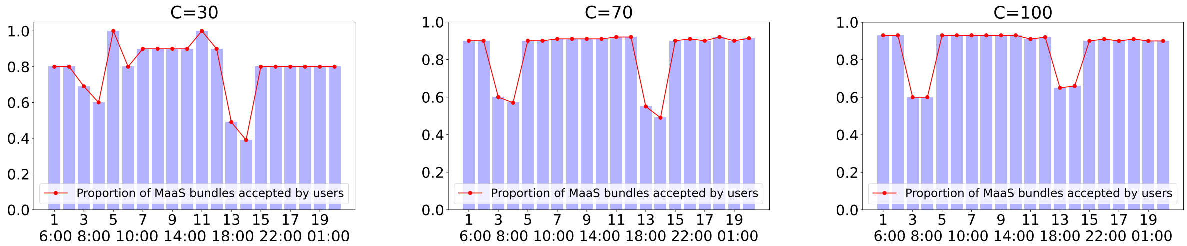


(a) Linear user payment function (b) Quadratic user payment function (c) Exponential user payment function

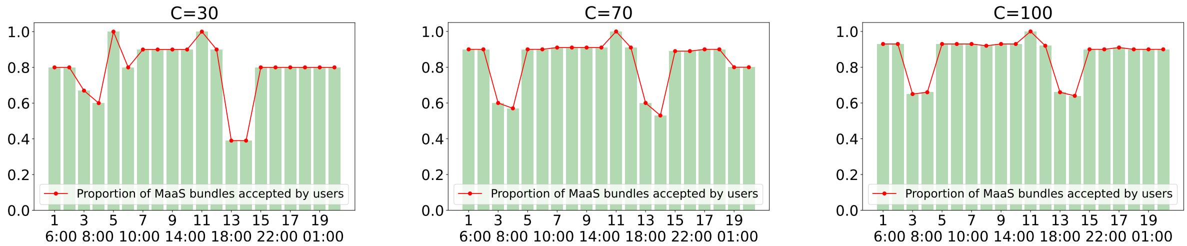
Figure D.12: Sensitivity analysis on different types of pricing functions under different $b_{\max}$.

To further explore the behavior of the proposed mechanism, we analyze the proportion of MaaS bundles accepted by users, which is defined as the ratio of the number of users who are offered MaaS bundles and also willing to accept MaaS bundles to the total number of users participating in the auction per time period. For clarity, we only show the hourly average proportion, i.e., the average proportion of MaaS bundles accepted by users for each bin of 60 time periods. Fig. D.13a, Fig. D.13b and Fig. D.13c show the acceptance ratio under linear (Eq. (3)), quadratic (Eq. (4)) and exponential (Eq. (5)) user payment functions for varying system capacities, respectively. Fig. D.13

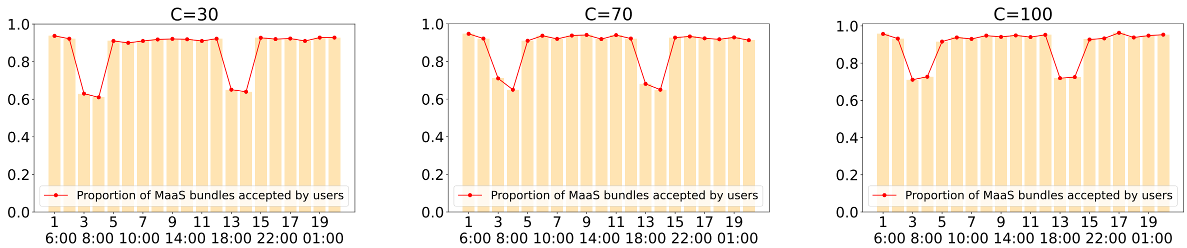
shows that the acceptance ratio under the exponential user payment function is higher than that under linear and quadratic user payment functions, thus motivating our choice to use the exponential user payment function as the iteration rule in Line 13 of Algorithm 1.



(a) Linear user payment function



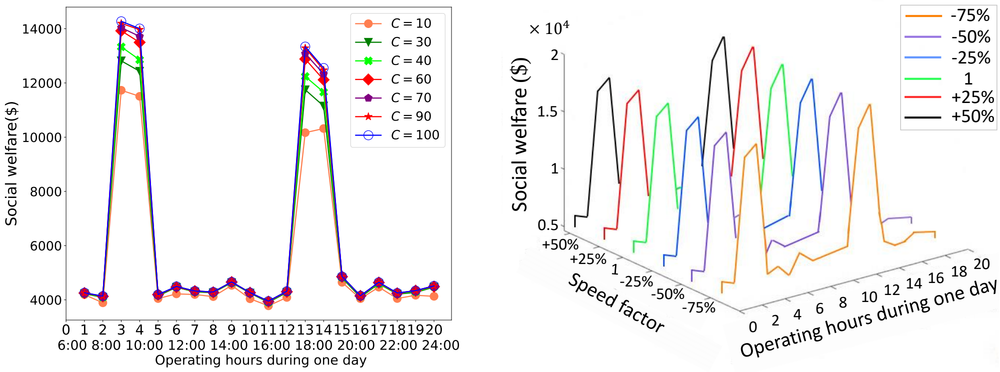
(b) Quadratic user payment function



(c) Exponential user payment function

Figure D.13: Proportion of MaaS bundles accepted by users under different capacity and user payment functions.

To observe the influence of capacity C and commercial speed of different travel modes on social welfare, we set the value of $b_{\max}$ as 5 and conduct a sensitivity analysis on these parameters. The results are reported in Fig. D.14. Fig. D.14a show that social welfare increases with the increase of capacity over all time periods and that social welfare remain unchanged beyond a capacity of 100. The impact of the commercial speed on social welfare is examined by applying speed factors of varying percentages (-75%,-50%,-25%,0%,$+25\%$,$+50\%$) on the commercial speed of each travel mode v_m. The results of this experiment are reported in Fig. D.14b. We find that the speed factor has a very limited influence on the pattern of social welfare over time.



(a) Sensitivity analysis on capacity (C) (b) Sensitivity analysis on speed factor (Δv)

Figure D.14: Sensitivity analysis on capacity (C) and speed factor (Δv)

Appendix E. Analysis of the computational performance and solution quality under different RHA and SHA configurations and the competitive ratio

Table E.7 provides the varying tradeoffs between runtime and solution quality of online and offline configurations. On the one hand, as the number of time periods (N) increases, the offline configuration ('SHA: Offline MILP') runs in exponential time and takes significantly more time than online configurations ('RHA: Online MILP' and 'RHA: Online algorithm'). On the other hand, the social welfare obtained by the offline configuration is significantly larger than that obtained by the online configurations. This highlights the tradeoff between improving solution quality using offline configuration and computational efficiency using online configurations.

Table E.7: Computational performance and solution quality under different RHA/SHA configurations, and competitive ratio analysis

	RHA: Online MILP (Model 1)			RHA: Online algorithm (Algorithm 1)				SHA: Offline MILP (Model 2)				
N	k	W($)	T(s)	k	W($)	T(s)	*Gap* (%)	k	W($)	T(s)	$\mathcal{R}$	Θ
10	10	1016.08	0.32	10	1032.02	**0.30**	1.54%	1	1046.58	0.06	0.99	0.62
20	20	2350.16	0.67	20	2397.90	**0.56**	1.99%	1	2443.42	0.27	0.98	0.62
30	30	3048.25	0.95	30	3096.08	**0.90**	1.54%	1	3139.72	0.52	0.99	0.62
40	40	4064.34	1.26	40	4128.11	**1.07**	1.54%	1	4186.33	1.18	0.99	0.62
50	50	5373.27	1.60	50	5467.58	**1.16**	1.72%	1	5553.86	1.23	0.98	0.62
60	60	6096.50	1.90	60	6192.16	**1.21**	1.54%	1	6279.19	3.01	0.99	0.62
70	70	7122.89	2.21	70	7234.84	**1.79**	1.55%	1	7337.65	3.55	0.99	0.62
80	80	8128.67	2.53	80	8256.21	**1.97**	1.54%	1	8354.24	9.41	0.99	0.62
90	90	9424.93	2.86	90	9582.26	**2.20**	1.64%	1	10727.75	15.45	0.89	0.62
100	100	10160.84	3.16	100	10320.26	**2.68**	1.54%	1	11465.82	17.01	0.90	0.61
110	110	11454.85	3.49	110	11643.53	**2.96**	1.62%	1	12818.14	15.70	0.91	0.62
120	120	12193.01	3.79	120	12384.32	**3.41**	1.54%	1	13558.99	22.64	0.91	0.63
130	130	13304.91	4.11	130	13514.47	**3.89**	1.55%	1	15285.69	24.18	0.88	0.63
140	140	14521.69	4.43	140	14757.54	**4.18**	1.60%	1	16975.62	26.00	0.87	0.63
150	150	15241.26	4.74	150	15480.40	**4.52**	1.54%	1	17698.75	35.86	0.87	0.63
160	160	16596.22	5.06	160	16865.76	**4.82**	1.60%	1	18113.44	32.35	0.93	0.63
170	170	18141.84	5.42	170	18455.75	**5.14**	1.70%	1	19744.42	41.85	0.93	0.63
180	180	18849.86	5.71	180	19164.52	**5.36**	1.64%	1	21455.50	44.10	0.89	0.63
190	190	19573.70	6.01	190	19888.04	**5.69**	1.58%	1	22179.43	46.96	0.90	0.63
200	200	20321.68	6.32	200	20640.53	**5.96**	1.54%	1	23931.67	63.39	0.86	0.63
300	300	30482.52	9.48	300	30960.79	**6.33**	1.54%	1	33397.48	105.11	0.93	0.62
400	400	40643.36	12.64	400	41281.06	**6.71**	1.54%	1	45733.00	259.69	0.90	0.63
500	500	53732.68	15.99	500	54675.78	**7.59**	1.72%	1	61538.50	383.57	0.89	0.63
600	600	60964.99	18.96	600	61921.59	**8.07**	1.54%	1	70735.19	601.18	0.88	0.63
700	700	68781.23	22.36	700	69756.57	**12.50**	1.40%	1	77723.20	1048.10	0.90	0.63
800	800	86991.55	25.75	800	88645.89	**15.13**	1.87%	1	102561.43	1604.53	0.86	0.63
900	900	100058.62	28.98	900	101934.83	**18.75**	1.84%	1	116443.72	2568.86	0.88	0.63
1000	1000	107465.36	31.99	1000	109351.56	**20.53**	1.72%	1	126432.61	3043.65	0.86	0.63

Notes. N: the number of total time periods in an instance; k: the number of online models (algorithms) or offline models solved in an instance; W: the average social welfare value; T: the average value on the summarized runtime of N online models in each group under 'RHA: Online MILP', the average value on the summarized runtime of N online algorithms in each group under 'RHA: Online algorithm', and the average value on the runtime of the offline model in each group under 'SHA: Offline MILP'; *Gap*: the average optimality gaps of social welfare between Algorithm 1 and online MILP Model 1 (solved by CPLEX); $\mathcal{R}$: the average social welfare ratio; Θ: the average competitive ratio; Numbers in bold denotes the smallest CPU runtime in each group (line).