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On ride-sourcing services of electric vehicles considering cruising for charging and parking

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Abstract

Cruising of electric ride-sourcing vehicles (ERVs) when waiting for trip orders can create additional vehicle miles, which increase congestion and waste electricity. Reducing cruising is an important issue. This study investigates the strategy of allocating a portion of road space as parking for ERVs. Considering ERVs cruising for parking/charging, we analytically examine the trade-off between road capacity reduction due to reserving road space as parking and less cruising. We evaluate the effects of parking provision on reducing congestion and charging demand. We also investigate the optimal fare and fleet size of ERV services to achieve profit or social welfare maximization. Numerical studies indicate that vehicles cruising for charging might be reduced significantly with a mild increase of charging pile supply, where cruising can increase sharply after charging pile occupancy rate is at critical levels. By providing parking to ERVs, ride-sourcing demand increases, charging demand reduces, profit and social welfare increase.

Keywords: Electric ride-sourcing vehicles; Cruising for charging; Parking; Fare; Fleet size

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1. Introduction

Ride-sourcing services (also known as Transportation Network Companies which is abbreviated as TNCs) such as Uber, Lyft and Didi Chuxing, become popular with the rapid development of mobile communication devices and applications. For ride-sourcing services, customers request the rides by inputting the information on the e-hailing platform, and then the platform matches the orders to the affiliated drivers who drive their own vehicles to provide on-demand transportation services (Rayle et al., 2016). Compared to the traditional street-hailing taxi services, the on-demand and e-hailing shared transportation platform reduces the searching and meeting frictions between the customers and drivers, and the ride-sourcing services have become popular in many cities (Cramer and Krueger, 2016; Nie, 2017).

With the rapid growth of the ride-sourcing services, there are many vacant vehicles cruising on the road while waiting for matching with the next customers, which created additional empty vehicle miles traveled (VMT) and even led to severe traffic congestion (Wenzel et al., 2019; Wei et al., 2020; Beojone and Geroliminis, 2021). Therefore, it is important to reduce cruising of ride-sourcing vehicles, especially in congested city regions. At the same time, there is a growing trend to adopt electric vehicles to reduce greenhouse gas emissions (Sheng et al., 2021, 2022). The provision of charging piles is necessary to support the operation of ride services by electric ride-sourcing vehicles (ERVs) with limited driving ranges. It is also worthwhile to investigate the impact of reducing cruising traffic on saving of battery electricity.

Considering the impacts of cruising traffic on traffic congestion and energy consumption of ERVs, we investigate a parking strategy by reserving a portion of the road space as parking spaces for the ERVs cruising on street. The parking provision could reduce the traffic due to reduced cruising vehicles. Besides, as these vacant ERVs can park rather than cruise on streets, energy consumption can be reduced, and less charging demand will be generated. In city regions where it is difficult to find an unoccupied charging pile, the saving of battery energy can be particularly beneficial for operating ERVs. However, the parking provision will reduce the utilizable road space for these running vehicles. Therefore, it is critical to appropriately balance the trade-off among reduction of cruising traffic, saving of energy consumption and the loss of road capacity for running vehicles. Xu et al. (2017b) has proposed a parking strategy for reducing cruising ride-sourcing traffic to improve travel speed. This paper further investigates the parking strategy under the ERV environment where a portion of road space is reserved as the parking spaces for the cruising ERVs. Specifically, by allocating a portion of the road space as parking spaces, the empty ERVs after completing the order of the last request will cruise for parking as well as waiting for the next matching with customers. With limited parking spaces, some empty vehicles can park and others may keep cruising while waiting for the matching. Once the vacant ERVs are successfully matched, the matched ERVs will pick up the customers and take them to their destinations. After dropping off the customer, the driver can check the battery status. If the remaining battery electricity is low, the driver can cruise for the charging pile. Otherwise, the driver will wait for the next matching while cruising for parking again. We model the process of empty ERVs cruising for parking and the ERV cruising for charging when the level of remaining battery electricity of the ERV is low. We construct and analyze the market equilibrium of the ride-sourcing services with consideration of parking and charging at an aggregate level. We then study the optimal pricing and fleet sizing strategies for the ERVs.

The major contributions of the paper are three-fold. (i) This study investigates the effects of parking strategy by reserving a portion of the road space as parking spaces for empty ERVs on reducing cruising traffic as well as saving battery electricity. The complex interactions of the ride-sourcing services considering ERVs cruising for charging and parking are modelled with the market equilibrium model including a system of nonlinear equations, and the existence of the equilibrium solution is investigated. (ii) This study analytically investigates the impacts of the numbers of parking spaces and charging piles on customer demand and ERV service level in terms of customer average waiting time. In particular, this study analytically determines the number of ERVs in the network to service the maximum customer demand under the constraint on the number of charging piles. (iii) This study investigates the optimal fleet size of ERVs and fare to achieve either profit maximization or social welfare maximization based on the market equilibrium model. We also analyze the impacts of the number of charging piles on the profit and social welfare.

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 introduces the model formulation of ride-sourcing services and then investigates the equilibrium properties of the ride-sourcing services with cruising for charging and parking. Section 4 examines the optimal fare and fleet size of ERVs to achieve either profit maximization or social welfare maximization. Section 5 conducts numerical experiments to illustrate the findings of the ride-sourcing services with cruising for charging and parking, and the effects of parking provision on system performance. Section 6 concludes the paper.

2. Literature review

There is a large body of studies on ride-sourcing services and e-hailing taxis in recent years. One can refer to Wang and Yang (2019) for a recent comprehensive review. The rapid increase of ride-sourcing services largely depends on the high efficiency of matching between customers and ride-sourcing drivers (or vehicles)(Yang et al., 2020; Xu et al., 2018; Wei et al., 2022), flexible labor supply and surge pricing to coordinate the supply and demand (Cachon et al., 2017; Nourinejad and Ramezani, 2020; Chen et al., 2020). The driver-rider matching is an important research question. Yang et al. (2020) examined the impacts of the matching time interval and matching radius on the system performance and then optimize the two variables in different levels of supply and demand. Wang et al. (2020) investigated the customer confirmed-order cancellation behaviour considering a coupled market of ride-sourcing and taxi services. Castillo et al. (2022) investigated a matching failure (called “wild goose chases”), where drivers are sent far away to pick up riders and thus the earning and the welfare are low. In terms of the impacts of introduction of ride-sourcing services on transportation system, Rayle et al. (2016) conducted a survey in San Francisco to compare taxis, transit and ride-sourcing services, and found that there were differences in user properties, wait times and trips served between taxis and ride-sourcing services, and at least half of ride-sourcing trips replaced non-taxi trips. Nie (2017) examined the impacts of the ride-sourcing services on taxi markets based on taxi data collected in Shenzhen, China, and found that taxi industry can compete with ride-sourcing services effectively in peak periods and in the areas with high population density. Su and Wang (2019) analyzed the impact of ride-sourcing services on the morning commute considering the parking supply constraints. Many studies have adopted data-driven approaches to predict the ride-hailing demand (Ke et al., 2017; Chen et al., 2022). From a network equilibrium perspective, He and Shen (2015) modeled the taxi services with both the emerging e-hailing applications and traditional street hailing by constructing the spatial equilibrium model. Ban et al. (2019) constructed a general network equilibrium model with the e-hailing transportation services and examined the impacts on congestion. Xu et al. (2021) further explicitly modeled the cruising trips for customers and deadheading trips to pick up customers in the transportation network with ride-sourcing services. Focusing on ride-sourcing markets, Zha et al. (2016) modeled the ride-sourcing services using an aggregate model and examined and compared different ride-sourcing market scenarios. Wang et al. (2016) investigated the prices for the taxi hailing platform paid by both the customers and drivers who used e-hailing applications, and following studies further considered the customer order cancellation behavior and the corresponding penalty strategy with the taxi e-hailing application (He et al., 2018; Wang et al., 2020; Xu et al., 2022). Li et al. (2019) investigated the impacts of three regulation strategies on the ride-sourcing system: (i) the minimum wage for ride-sourcing drivers, (ii) the cap on the number of ride-sourcing drivers, and (iii) congestion surcharge. Yu et al. (2020) investigated the impacts of regulation policies for ride-sourcing services on the welfare of stakeholders including customers, taxi drivers, ride-sourcing company, and independent drivers for ride-sourcing services.

Besides, with the increasing concern about carbon emissions and the advancement of the battery technology, the usage of electric vehicles is growing. One can refer to Shen et al. (2019) for a comprehensive review for operation management of electric vehicles such as planning of charging infrastructures, charging operations, public policies and business models. Specifically, in terms of planning charging stations, Tu et al. (2016) proposed a spatial-temporal demand coverage approach to optimize locations of charging stations for electric taxis. Yang et al. (2017) introduced a data-driven optimization-based approach to planning charging stations for electric taxis. Yan et al. (2018) investigated an wireless charger deployment scheme to maximize the drivers’ opportunities to pick up customers at the chargers and support the taxicabs' continuous operability with enough energy to drive. Brandstätter et al. (2017) constructed the optimization model to determine the charging station locations for the electric car-sharing system under stochastic demand. Liu et al. (2018) studied the planning of fast-charging stations in a battery electric bus system with consideration of energy consumption uncertainty of buses. Hu et al. (2022) constructed a joint optimization model of charger locating and electric bus en-route charging schedule considering uncertain passenger demands and travel times. Liu et al. (2021, 2022a) optimized the charging strategies for electric buses under resource constraints or energy consumption uncertainty. Sun et al. (2015, 2016) investigated the charge timing choice behavior and fast-charging station choice behavior. From a network equilibrium perspective, Xu et al. (2017a) modeled the user equilibrium for mixed battery electric vehicles and gasoline vehicles in transportation network with battery swapping station. The effects of road gradient on electricity consumption rate is further investigated by Liu et al. (2017). Liu and Song (2018) proposed the network equilibrium model for battery electric vehicles considering flow-dependent electricity consumption, and studied the congestion pricing problem. With the increasing trips of ride-sourcing services, there is a trend to electrifying the ride-sourcing vehicles (Jenn, 2020), while one of the biggest barriers is the vehicle charging problem. Ke et al. (2019) constructed a time-expanded network to model working schedules for both electric vehicle drivers and gasoline vehicle drivers in a ride-sourcing market, and the recharging schedules for electric vehicle drivers. Bauer et al. (2019) conducted agent-based simulations to analyze the charging infrastructure needed for ride-sourcing electrification and found that coordinated use of charging infrastructure can improve robustness of fleet performance. Zhan et al. (2022) proposed a simulation-optimization framework considering dynamic matching of ride-hailing sharing (multiple passengers can share a vehicle) and dynamic ride-hailing electric vehicle charging. Li et al. (2021) used an improved genetic algorithm to optimize the locations of the public charging stations for electric ride-hailing vehicles. Ma and Xie (2021) investigated both the charging station planning and charging operation for the electric ridesharing system. Mo et al. (2020) investigated subsidies on electric vehicle purchase and charging stations under the competition of heterogeneous ride-sourcing platforms, where one platform hired drivers who own gasoline or electric vehicles while the other platform hired drivers to drive the electric vehicles owned by the platform. Li et al. (2022) analyzed optimal fare and fleet size of taxi/ride-sourcing markets with gasoline/electric vehicles, where congestion effects and emission are considered. Liu et al. (2022b) investigated the regulatory policies to electrify ride-sourcing systems such as annual permit fees, different trip-based fees or differential commission caps. While these studies highlight the importance of charging facility planning and operation for ride-sourcing vehicles, existing studies have not explicitly modeled the effect of reducing cruising traffic of ERVs for customers on the saving of battery electricity consumption.

Considering the impacts of the empty and cruising vehicles on traffic congestion, some studies have investigated the parking strategies to reduce the cruising traffic of shared mobility services. Xu et al.

(2017b) proposed the strategy of allocating a portion of road space as parking spaces for the cruising ride-sourcing vehicles, and found that travel speed could be improved. Beojone and Geroliminis (2021) conducted agent-based simulation experiments to investigate the impacts of ride-sourcing services on traffic congestion and adopted the parking strategy to reduce cruising traffic. Kondor et al. (2020) applied a data-driven methodology to investigate the minimum parking infrastructure needed for the given on-demand mobility demand. Zhang and Guhathakurta (2017) investigated the parking demand in the age of Shared Autonomous Vehicles and explored the difference of parking demand and parking land under free and charged parking scenarios. Ruch et al. (2021) further investigated different parking operation policies in terms of how to assign idle vehicles to the parking spaces considering different parking space distributions for a mobility-on-demand system. Kontou et al. (2020) investigated the effect of future travel information on reduction of empty ride-sourcing vehicle travel. To make more efficient use of limited parking resources, Jian et al. (2020) proposed an integrated carsharing and parking sharing services, where the carsharing operator can provide more parking spaces to carsharing users via collaborating with the parking sharing platform. The parking sharing service is also introduced for ride-sourcing services to reduce the empty and cruising vehicles on the road (Gao et al., 2022).

However, the effects of parking strategy on reducing both the traffic congestion caused by cruising traffic of ERVs for customers and battery electricity consumption have not been explicitly modeled and analyzed. In particular, in some city central regions, where traffic congestion is severe and there is lack of enough charging piles to satisfy the charging demand, the negative impacts of cruising traffic can be particularly significant. Therefore, how to quantify and manage the cruising traffic of ERVs in city regions with limited charging piles is an important question to be addressed.

3. Model formulation

In this section, we introduce the travel, parking and charging process of ride-sourcing services considering ERVs' cruising for parking and charging in Section 3.1, which includes matching between vacant ERVs and customers, vacant ERVs cruising for parking, the elastic customer demand, the charging demand, and network average speed in Sections 3.1.1-3.1.5, respectively. The user-vehicle matching, serving trip requests, parking and charging process is formulated as a system of nonlinear equations, which describes the ride-sourcing market equilibrium. Some properties of the equilibrium are then investigated in Section 3.2. We also consider the special case with no parking provision and a constant speed to generate additional understanding on the impacts of limited charging piles on ride-sourcing services in Section 3.3. The main notations used in this paper are listed in Appendix A.

3.1. The travel, parking and charging process of ride-sourcing services

Considering an isotropic (spatially homogeneous) road network (Arnott and Inci, 2010), the user-vehicle matching, serving trip requests, parking and charging process of ERVs considering cruising for parking and charging is shown in Fig. 1. We first introduce matching between vacant ERVs and customers. These vacant vehicles, including cruising vehicles on streets and parked vehicles, join the user-vehicle matching in the ride-sourcing platform. Once the vehicle is successfully matched with one customer, the driver will pick up the customer, and drive the customer to the destination as requested. After dropping off the customer, the driver will check whether the remaining battery power is higher than a threshold (e.g., enough to take/serve the next customer). If the remaining battery power is sufficient, the driver directly rejoins the matching process for the next customer. If the remaining battery power is too low, the driver will cruise for unoccupied charging piles. When there are fewer unoccupied charging piles, ERV drivers have to spend more time on average to find an unoccupied charging pile. After finding an unoccupied charging pile, the driver spends a certain length of time for vehicle recharging, and then enters the ride-sourcing market again. For those drivers of vacant ERVs waiting for matching, they can cruise on roads if there is no parking space. The cruising vehicles not only increase traffic and cause traffic congestion but also consume electricity. In this paper, we consider that a part of road area can be allocated as the parking spaces, which are considered to be evenly distributed on the road network. A cruising vehicle could park while waiting to be matched if it finds an unoccupied curbside parking space before being successfully matched. The remaining vacant vehicles still cruise on roads to search for parking spaces while waiting to be matched. We will formulate the above in the following subsections.

3.1.1. Matching between the unmatched ERVs and customers

We first introduce the matching rate between the unmatched ERVs and customers, which is defined as the number of successful matches between the waiting ERVs and customers in one time period such as one hour. Although the e-hailing applications could conveniently get the locations of these waiting drivers and riders, there are still matching frictions due to temporal and spatial gaps between them in a road network. For example, the rider sends the order to the platform, which may not be matched immediately due to no vacant ERV in the matching radius (Yang et al., 2020). To capture matching friction and competition among these unmatched drivers and riders, an aggregate matching function introduced by Yang et al. (2010) is adopted as follows:

$$m^{c-t} = M^c(N_{vt}, N_c), \tag{1}$$

where m^{c-t}, $M^c(\cdot)$, N_{vt}, and N_c respectively denote the matching rate, matching function, the number of unmatched drivers for matching, and the number of unmatched customers. Intuitively, matching rate increases with the numbers of waiting drivers and customers, i.e., $\frac{\partial M^c}{\partial N_{vt}} > 0$ and $\frac{\partial M^c}{\partial N_c} > 0$. The number of unmatched ERVs including these vacant vehicles cruising on road while waiting for matching, and the others parking while waiting for matching. Note that as the studied region is considered to be isotropic, there are not hotpots with higher customer demand, and thus it is considered that the cruising ERVs

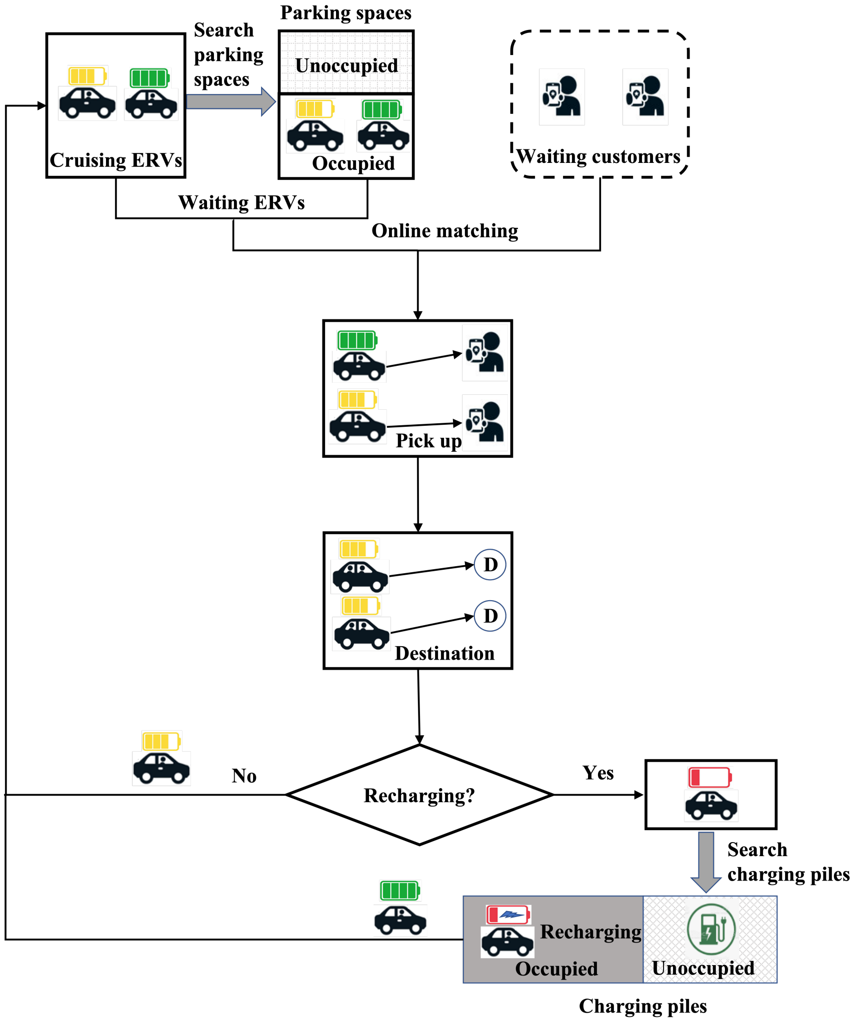


Fig. 1. The process of the ride-sourcing services considering parking and charging

have the same matching probability as the parked ERVs.

In this paper, we consider that the system is at a stationary state (to allow tractability), where the market equilibrium is reached, i.e., the customer demand is always satisfied by ERV supply. Thus, the customer demand, denoted as Q, is equal to the arrival rate (supply) of vacant ERVs, denoted as $\mathit{\Gamma}$, and is also equal to the matching rate m^{c-t}:

$$Q = \mathit{\Gamma} = m^{c-t}. \tag{2}$$

With a number of customers N_c and a number of ERVs N_{vt} waiting for matching at any time, the matching rate m^{c-t} is equal to Q in Eq. (2), and thus we can give the customer average waiting time to be matched, denoted as w_c, and ERV average waiting time to be matched, denoted as w_t, as follows:

$$w_c = \frac{N_c}{Q}, \tag{3}$$

$$w_t = \frac{N_{vt}}{Q}. \tag{4}$$

After being matched successfully, the matched driver will pick up the matched customer. The average pick-up time for the matched driver and customer to meet each other is denoted as w_r. We assume that w_r is constant for simplicity.

We further adopt a Cobb-Douglas type production function (Varian, 1992; Yang et al., 2010), which specifies the aggregate matching function in Eq. (1) as follows:

$$m^{c-t} = M^c(N_{vt}, N_c) = A(N_{vt})^{\alpha_1}(N_c)^{\alpha_2}, \tag{5}$$

where A is the positive model parameter of matching rate which is related to the spatial properties of the ride-sourcing markets and matching technology, and α_1 and α_2 denote the elasticities of the matching rate with respect to the number of waiting ERVs N_{vt} and the number of waiting customers N_c, respectively $\left(\alpha_1 = \frac{\partial M^c}{\partial N_{vt}}\frac{N_{vt}}{M^c}, \alpha_2 = \frac{\partial M^c}{\partial N_c}\frac{N_c}{M^c}\right)$. It is considered that $0 < \alpha_1, \alpha_2 \leqslant 1$. The matching function is homogeneous of degree $\alpha_1 + \alpha_2$, and is increasing, constant and decreasing return to scale when $\alpha_1 + \alpha_2 > 1$, $\alpha_1 + \alpha_2 = 1$ and $\alpha_1 + \alpha_2 < 1$, respectively.

From Eqs. (2)-(5), we can get

$$Q = A(Qw_t)^{\alpha_1}(Qw_c)^{\alpha_2}, \tag{6}$$

and we can derive the average customer waiting time to be matched based on Eq. (6) as follows:

$$w_c = W_c(Q, w_t) = A^{-\frac{1}{\alpha_2}} Q^{\frac{1-\alpha_1-\alpha_2}{\alpha_2}} w_t^{-\frac{\alpha_1}{\alpha_2}}, \tag{7}$$

where $W_c(\cdot)$ denotes the average customer waiting time function of customer demand Q and average ERV waiting time w_t. In practice, the matching efficiency for different areas or regions might be approximated by different matching functions and the function parameters should also be estiamted based on local data. One can refer to Wei et al. (2022) for detailed discussions.

3.1.2. Cruising of the ERVs for parking spaces

Next, we introduce the cruising process of the ERVs for parking. The unmatched ERVs can cruise for parking while waiting for matching. The parking vacancy rate P_p is defined as the ratio of the number of vacant parking spaces to the total number of of parking spaces as follows:

$$P_p = \frac{N_p - N_p^o}{N_p}, \tag{8}$$

where N_p and N_p^o denote the total number of parking spaces in the region and the occupied parking spaces by the ERVs, respectively.

We consider the cruising of the unmatched ERV for an unoccupied parking space as a stochastic process (Arnott and Rowse, 1999; Anderson and De Palma, 2004; Liu and Geroliminis, 2016), and the expected searching time for a vacant parking space, denoted as w_p, can be given as follows:

$$w_p = \frac{L_a}{N_p P_p v} = \frac{L_a}{(N_p - N_p^o)v}, \tag{9}$$

where L_a, N_p, N_p^o and v are respectively denote the total length of the road system in the region, the total number of parking spaces, the number of occupied parking spaces, and network average speed. And $\frac{L_a}{N_p}$ denotes the average distance between two adjacent parking spaces.

An unmatched ERV will end cruising for a parking space if it successfully gets matched with a customer or it finds a vacant parking space, where matching with customers and finding a parking space are independent. We consider that both the matching process in Section 3.1.1 and cruising process for parking are stochastic, and the time periods of successful matching t_m and finding a parking space t_p both follow exponential distribution. And the corresponding probability density functions, denoted as $g_m(t)$ and $g_p(t)$, respectively, are given as follows:

$$g_m(t_m) = \frac{1}{w_t} e^{-\frac{t_m}{w_t}}, \quad t_m \geqslant 0, \tag{10}$$

$$g_p(t_p) = \frac{1}{w_p} e^{-\frac{t_p}{w_p}}, \quad t_p \geqslant 0. \tag{11}$$

The cumulative distribution function for $g_m(t)$, denoted as $G_m(t)$, can be given as follows:

$$G_m(t_m) = 1 - e^{-\frac{t_m}{w_t}}. \tag{12}$$

The probability of an unmatched ERV finding a parking space before being matched, denoted as P_p^m, is given as follows:

$$P_p^m = \int_0^{+\infty} \frac{1}{w_p} e^{-\frac{t}{w_p}} (1 - G_m(t))\, dt = \frac{w_t}{w_t + w_p}, \tag{13}$$

and we can get the parking rate m^{t-p}, defined as the number of ERVs finding parking spaces in one time period, as follows:

$$m^{t-p} = Q P_p^m = Q \frac{w_t}{w_t + w_p}. \tag{14}$$

As we consider that the matching probabilities are the same for the ERVs cruising on the road and parked in Section 3.1.1, we can get

$$\frac{m^{t-p}}{m^{c-t}} = \frac{m^{t-p}w_t}{m^{c-t}w_t} = \frac{N_p^o}{N_{vt}}. \tag{15}$$

where the number of unmatched ERVs N_{vt} is equal to the product of matching rate and average ERV waiting time to be matched ($m^{c-t}w_t$) based on Eqs. (2) and (4), and the number of occupied parked spaces (equal to the number of ERVs finding parking spaces) is equal to the product of the parking rate and the average ERV waiting time to be matched ($m^{t-p}w_t$).

With Eqs. (2), (4) and (15), we get the number of occupied parking spaces as follows:

$$N_p^o = \frac{Qw_t^2}{w_p + w_t}. \tag{16}$$

3.1.3. The travel cost and customer demand

After picking up the customer, the ERV will drive the customer to his/her destination. The average in-vehicle travel time is given as follows:

$$T = \frac{L_r}{v} \tag{17}$$

where T and L_r denote the average in-vehicle travel time and average travel distance for each ride trip, respectively.

The travel cost μ for each customer is given as follows:

$$\mu = F + \beta(w_c + w_r) + \gamma T \tag{18}$$

where F denotes the fare for each trip, β and γ denote the out-of-vehicle (waiting for being matched and picked up) and in-vehicle value of time, respectively.

The customer travel demand for ride-sourcing services, denoted as Q, is assumed to be a strictly monotonically decreasing function of the travel cost, as follows:

$$Q = f(\mu) \tag{19}$$

where $f(\cdot)$ is a continuous, positive and bounded function with $f(\mu) > 0$ and $\lim_{\mu \to +\infty} f(\mu) = 0$, and $f(\cdot)$ is differentiable and strictly monotonically decreasing, i.e., $f' = \frac{df}{d\mu} < 0$.

3.1.4. The demand of charging

As we consider that all the ride-sourcing vehicles are electrically driven, it is necessary to provide charging piles for these ERVs. With the total number of ERVs N given, the total charging demand, denoted as q_e, is given as follows:

$$q_e = \frac{(N - N_p^o - N_e^o)v}{L_e}, \tag{20}$$

where L_e and N_e^o respectively denote the maximum driving range of each ERV fully charged, and the number of occupied charging piles. Specifically, except these ERVs parking to wait for being matched and charging on the charging piles, the remaining vehicles are moving on the road network, the number of which is $N - N_p^o - N_e^o$. The total driving mileage of these moving vehicles per unit time is $(N - N_p^o - N_e^o)v$. Under the stationary state, the completion rate of charging is equal to the charging demand, which means that the supply of these ERVs fully charged per unit time is equal to the charging demand q_e. And the total mileage supplied by these ERVs fully charged per unit time is $q_e L_e$ as each fully charged vehicle can serve a mileage of L_e. As a result, the total supply of mileage is equal to the consumed mileage of these moving vehicles, i.e., $q_e L_e = (N - N_p^o - N_e^o)v$. Therefore, the charging demand q_e equals the total mileage consumed by the moving vehicles per unit time divided by the maximum driving mileage of one ERV fully charged. Note that we only consider the charging need of the ERVs and the charging demand of other background vehicles is not considered for simplicity.

The number of occupied charging piles, denoted as N_e^o, is

$$N_e^o = q_e t_e, \tag{21}$$

where t_e is the average charging time for an ERV to be fully charged.

We define the occupancy rate of the charging piles as the ratio of the number of occupied charging piles to the total number of charging piles, denoted as O_e, as follows:

$$O_e = \frac{N_e^o}{N_e}, \tag{22}$$

where N_e is the total number of charging piles.

Similar to the expected searching time of finding a vacant parking space in Eq. (9), the expected time of finding an unoccupied charging pile t_s is

$$t_s = \frac{L_a}{(N_e - N_e^o)v}. \tag{23}$$

The number of ERVs cruising for charging piles N_e^s is

$$N_e^s = q_e t_s. \tag{24}$$

The conservation condition for the ERV fleet size under the stationary state is

$$N = Q(w_t + w_r + T) + q_e(t_s + t_e), \tag{25}$$

where the number of ERVs N is equal to the sum of the number of vehicles waiting to be matched Qw_t, the number of vehicles on the roads to pick up the customers Qw_r, the number of occupied vehicles with customers heading for customers' destinations QT, the number of vehicles cruising for charging $q_e t_s$, and the number of vehicles being charging $q_e t_e$.

We define the utilization efficiency of ERVs U as the ratio of the occupied ERVs with customers to the total number of ERVs in the markets as follows:

$$U = \frac{QT}{N}, \tag{26}$$

where a higher U indicates that more vehicles carry customers to destinations under the given number of ERVs.

3.1.5. Network average speed

We model the network average speed as a function of the traffic density (or traffic accumulation) in the isotropic area (Mahmassani et al., 1987; Geroliminis and Daganzo, 2008):

$$v = V\left(\frac{k_a}{k_j}\right) \tag{27}$$

where k_a and k_j respectively denote the network average density of the running vehicles and the jam density in the network, and $\frac{k_a}{k_j}$ can be regarded as the network density ratio, which is defined as the ratio of network average density to the jam density. $V(\cdot)$ is the network average speed function. $V(\cdot)$ is differentiable and monotonically decreasing, i.e., $V' < 0$, and $V(\cdot)$ is a positive and bounded function with $V(0) = v_f$ and $\lim_{\frac{k_a}{k_j}\to 1} V\left(\frac{k_a}{K_j}\right) = 0$, where v_f denotes the free-flow travel speed.

We can get the network average density k_a as follows:

$$k_a = \frac{N_R}{A_R}, \tag{28a}$$

$$N_R = N - N_p^o - N_e^o + N_b, \tag{28b}$$

$$A_R = A_u - A_p - A_e, \tag{28c}$$

where N_R and N_b denote the total number of running vehicles and the given number of background vehicles, respectively, and A_u, A_R, A_p, and A_e denote the total road area, the utilizable road area for running vehicles, the road area allocated as parking spaces, and the road area allocated for charging piles, respectively. To allow analytical tractability, the potential impacts of ERVs on background traffic (e.g., traffic redistribution) are not considered in the current study, where future studies may further examine this. The first equation in Eq. (28) indicates that the network average density k_a equals the total number of running vehicles N_R divided by the the utilizable road area for running vehicles A_R. The second equation indicates that the total number of running vehicles on the roads N_R is the sum of the number of running ERVs (equal to the difference between the total number of ERVs and the number of these parking and charging vehicles, i.e., $N - N_p^o - N_e^o$) and the given number of background vehicles N_b. The third equation indicates that the utilizable road area for running vehicles A_R is equal to the difference between the total road area A_u and the areas allocated for parking spaces and charging piles $(A_p + A_e)$. Note that the speed of these vehicles cruising for parking and charging and the speed of running vehicles to pick up customers or carrying customers to the destinations are assumed to be identical. Heterogeneous speeds for different traffic and the impacts of cruising traffic can be further investigated in a future study.

We further consider that both average occupied space of one parking space and one charging pile is same as the average occupied space of one vehicle under the jam state. The total number of parking spaces N_p is equal to the product of allocated area for parking and the jam density, i.e, $N_p = A_p k_j$. And similarly, the total number of charging piles N_e is equal to the product of allocated area for charging piles and the jam density, i.e., $N_e = A_e k_j$. With $N_p = A_p k_j$, $N_e = A_e k_j$, and Eqs. (27)-(28), the network average speed can be rewritten as

$$v = V\left(\frac{N - N_p^o - N_e^o + N_b}{A_u k_j - N_p - N_e}\right), \tag{29}$$

where it is considered that there should be remaining spaces for running vehicles after allocating the area for parking spaces and charging piles, i.e., the sum of the numbers of the parking spaces and parking piles $(N_p + N_e)$ is smaller than the maximum number of vehicles that the total road space can carry $(A_u k_j)$, i.e., $A_u k_j > N_p + N_e$. Besides, we consider that the sum of the numbers of ERVs and background vehicles $(N + N_b)$ is smaller than the maximum number of vehicles that the utilizable road area for running vehicles can carry $(A_u k_j - N_p - N_e)$ to avoid traffic jam, i.e., $N + N_b < A_u k_j - N_p - N_e$.

3.2. Summary and analysis of the general equilibrium under the stationary state

Section 3.2.1 summarizes and analyzes the general equilibrium about the ride-sourcing services described in Section 3.1. Section 3.2.2 further discusses the special case when the speed is constant (i.e., ERVs create minor congestion and road condition is dominated by background traffic) to provide further understanding.

3.2.1. Summary and analysis of the equilibrium

Summarizing all the relationships derived in in Section 3.1 yields a system of nonlinear equations for the ride-sourcing market equilibrium with cruising for parking and charging as follows:

$$w_c = A^{-\frac{1}{\alpha_2}} Q^{\frac{1-\alpha_1-\alpha_2}{\alpha_2}} w_t^{-\frac{\alpha_1}{\alpha_2}}, \tag{30a}$$

$$Q = f\left(F + \beta(w_c + w_r) + \gamma \frac{L_r}{v}\right), \tag{30b}$$

$$N = Q\left(w_t + w_r + \frac{L_r}{v}\right) + q_e(t_s + t_e), \tag{30c}$$

$$N_p^o = \frac{Q w_t^2}{w_p + w_t}, \tag{30d}$$

$$w_p = \frac{L_a}{(N_p - N_p^o)v}, \tag{30e}$$

$$q_e = \frac{(N - N_p^o - q_e t_e)v}{L_e}, \tag{30f}$$

$$t_s = \frac{L_a}{(N_e - q_e t_e)v}, \tag{30g}$$

$$v = V\left(\frac{N - N_p^o - q_e t_e + N_b}{A_u k_j - N_p - N_e}\right), \tag{30h}$$

where Eq. (30a) is from Eq. (7), which introduces customer waiting time related to the matching; Eq. (30b) is the elastic demand function and is obtained based on Eqs. (17)-(19); Eq. (30c) is the conservation condition of the ERV fleet size, which is obtained by combining Eq. (17) and Eq. (25); Eq. (30d) is from Eq. (16) to state the number of parking spaces being occupied; Eq. (30e) is from Eq. (9), which is the expected cruising time for finding a vacant parking space; Eq. (30f) is the charging demand, which is obtained by combining Eqs. (20)-(21); Eq. (30g) is the expected cruising time for finding a vacant charging piles, which is obtained by combining Eq. (21) and Eq. (23); Eq. (30h) represents the network average speed, which is obtained by combining Eq. (21) and Eq. (29).

Note that we consider that ERVs are re-charged during different times (i.e., their charging time is distributed over the day), and the above model focuses the steady state for a given modeling duration. In case there is a non-working duration (e.g., night times) for ERVs that can be used for charging, the current model can be readily extended to incorporate that, i.e., some ERV charging demand disappears as they will only be charged during non-working hours, and the impacts of provision of parking and charging piles will be smaller and the numbers of parking and charging piles needed will also be smaller.

There are four supply related variables in Eq. (30), i.e., number of parking spaces N_p, the number of charging piles N_e, the trip fare F, the number of ERVs N. Given the four supply variables (N_p, N_e, F, N), there are eight market equilibrium-dependent endogenous variables (we also have eight equations in Eq. (30)), i.e., the customer average waiting time to be matched w_c, the ERV average waiting time to be matched w_t, the customer demand Q, the charging demand q_e, the expected cruising time for finding an unoccupied charging pile t_s, the number of occupied parking spaces N_p^o, the expected cruising time for finding an unoccupied charging pile w_p, and the network average speed v.

All the above exogenous and endogenous variables are considered to be greater than zero and the fare F is considered to be no less than zero. Based on the system of nonlinear equations in Eq. (30), we investigate the existence of the solution to the system (i.e., the ride-sourcing market equilibrium).

Proposition 1. *The solution to the system of nonlinear equations in Eq. (30) exists, when the system satisfies the following conditions:*

(i) the number of charging piles is larger than the number of the ERVs being charging, i.e., $N_e > q_e t_e$*; the total number of the ERVs cruising for charging and being charging is smaller than the total number of ERVs, i.e.,* $q_e(t_s + t_e) < N$*;*

(ii) the total number of vehicles on the roads is smaller than the maximum number of vehicles that the utilizable road space for running vehicles can carry, i.e., $N + N_b < A_u k_j - N_p - N_e$.

The proof is provided in Appendix B. The two conditions in Proposition 1(i) requires that the supply of charging piles is sufficient. The number of charging piles should be able to service the charging demand, otherwise, the operation cannot be maintained due to a lack of battery electricity ($N_e > q_e t_e$). In addition, there should be remaining ERVs serving customers besides those vehicles cruising for charging piles and being charging ($q_e(t_s + t_e) < N$).

Proposition 1(ii) requires that the total number of vehicles on the road including ERVs and background traffic is smaller than the maximum capacity of the road network, i.e., traffic gridlock is avoided ($N + N_b < A_u k_j - N_p - N_e$).

While the existence of solutions to the system of nonlinear equations in Eq. (30) is ensured, the uniqueness of the equilibrium solutions can not be guaranteed due to the complex nonlinear relations in the modeled ride-sourcing transportation system considering cruising for charging and parking.

We now further discuss the impacts of parking provision on ride-sourcing transportation system with cruising for charging and parking. We will compare two scenarios where parking provision for ride-sourcing electric vehicles is greater than zero and equal to zero, respectively (the equilibrium outcomes are summarized in Table 1).

By providing parking spaces to ERVs waiting to be matched, we expect to save the battery electricity of the ERVs as well as to reduce traffic congestion. Based on Eq. (30f), we can rewrite the charging demand q_e as

$$q_e = \frac{N - N_p^o}{\frac{L_e}{v} + t_e}. \tag{31}$$

The charging demand is positively related to both the total number of the ERVs except those parked ERVs ($N - N_p^o$) and network average speed v. The more vehicles to park rather than cruising on the road expects to help reduce the charging demand q_e.

Table 1
The impacts of the provision of parking spaces

	No parking provision	Parking provision
Charging demand q_e	$\frac{N}{\frac{L_e}{v}+t_e}$	$\frac{N-N_p^o}{\frac{L_e}{v}+t_e}$
The number of unoccupied charging piles $N_e - N_e^o$	$N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}$	$N_e - \frac{(N-N_p^o)t_e}{\frac{L_e}{v}+t_e}$
Average cruising time for charging t_s	$\frac{L_a}{\left(N_e-\frac{Nt_e}{\frac{L_e}{v}+t_e}\right)v}$	$\frac{L_a}{\left(N_e-\frac{(N-N_p^o)t_e}{\frac{L_e}{v}+t_e}\right)v}$
The number of ERVs cruising for charging and being charging $q_e(t_e+t_s)$	$\frac{N}{\frac{L_e}{v}+t_e}\left(t_e+\frac{L_a}{\left(N_e-\frac{Nt_e}{\frac{L_e}{v}+t_e}\right)v}\right)$	$\frac{N-N_p^o}{\frac{L_e}{v}+t_e}\left(t_e+\frac{L_a}{\left(N_e-\frac{(N-N_p^o)t_e}{\frac{L_e}{v}+t_e}\right)v}\right)$
Average cruising time for parking w_p	N/A	$\frac{L_a}{(N_p-N_p^o)v}$
The number of parked ERVs N_p^o	N/A	$\frac{N_{vt}w_t}{w_t+w_p}$
Network density ratio $\frac{k_a}{k_j}$	$\frac{N-q_et_e+N_b}{A_uk_j-N_e}$	$\frac{N-N_p^o-q_et_e+N_b}{A_uk_j-N_p-N_e}$

The change of charging demand due to the parking provision directly has an effect on the cruising or searching time for finding an unoccupied charging pile. If there is less charging demand q_e and fewer occupied parking spaces (q_et_e), there are thus more unoccupied charging piles $(N_e - q_et_e)$. Based on Eq. (30g), the average cruising time for finding an unoccupied charging pile t_s is negatively related to both the unoccupied charging piles $(N_e - q_et_e)$ and network average speed v. When there are more unoccupied charging piles, t_s may be reduced. Particularly, when there are fewer unoccupied charging piles and drivers have to spend significant time to find an unoccupied charging pile, the effect of reduction of charging demand due to the parking provision can be substantial. Based on Eq. (30c), with fewer vehicles cruising for charging piles (q_et_s) and being charging (q_et_e), there are more utilizable vehicles to serve customers, which is denoted as N_u and the number is $N_u = (N - q_e(t_s + t_e))$. This highlights the effect of parking provision on improving the number of utilizable ERVs to provide services.

We then further discuss the impacts of parking on network average speed. Allocating a portion of road space as on-street parking spaces for these ERVs waiting for matching with customers helps reduce the cruising traffic of unmatched ERVs, while it also reduces the road area for running vehicles. Specifically, when we do not consider parking provision strategy to ERVs, the network density ratio, which is defined as the ratio of the network average density k_a to the jam density k_j in Section 3.1.5, is $\frac{N-q_et_e+N_b}{A_uk_j-N_e}$ based on Eqs. (28)-(29). After allocating a portion of road area as parking spaces, the network average density ratio is $\frac{N-N_p^o-q_et_e+N_b}{A_uk_j-N_p-N_e}$ based on Eqs. (28)-(29). One then can verify that if the ratio of the number of occupied parking spaces to the total number of parking spaces $\left(\frac{N_p^o}{N_p}\right)$ is relatively large after the parking provision strategy for ERVs, the network average speed can be improved by the parking provision strategy.

Next, we analyze the marginal impact of parking provision on the equilibrium. Based on Eqs. (30a)-

(30c), we can derive the partial derivatives with respect to N_p as follows:

$$\frac{\partial Q}{\partial N_p} = \frac{-f'\beta W'_{c,w_t}\frac{\partial T}{\partial N_p} - f'\beta\frac{1}{Q}W'_{c,w_t}\frac{\partial(q_et_s+q_et_e)}{\partial N_p} + f'\gamma\frac{\partial T}{\partial N_p}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{32}$$

$$\frac{\partial w_c}{\partial N_p} = \frac{1}{f'\beta}\frac{\partial Q}{\partial N_p} - \frac{\gamma}{\beta}\frac{\partial T}{\partial N_p}, \tag{33}$$

$$\frac{\partial(q_et_e+q_et_s)}{\partial N_p} = \frac{-1}{\frac{L_e}{v}+t_e}\left(t_e+t_s+\frac{q_et_s^2vt_e}{L_a}\right)\frac{\partial N_p^o}{\partial N_p}$$
$$+\left(\frac{q_eL_e(t_e+t_s)}{v(L_e+t_ev)} + \frac{q_e^2t_s^2L_et_e}{L_a(L_e+t_ev)} - \frac{q_et_s}{v}\right)\frac{\partial v}{\partial N_p}, \tag{34}$$

$$\frac{\partial T}{\partial N_p} = -\frac{L_r}{v^2}\frac{\partial v}{\partial N_p}, \tag{35}$$

where $W'_{c,Q} = \frac{\partial w_c}{\partial Q} = \frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q}$ and $W'_{c,w_t} = \frac{\partial w_c}{\partial w_t} = -\frac{\alpha_1}{\alpha_2}\frac{w_c}{w_t}$ are obtained based on Eq. (7), and the average in-vehicle travel time T is equal to $\frac{L_r}{v}$ in Eq. (17). The detailed derivation about $\frac{\partial N_p^o}{\partial N_p}$ and $\frac{\partial v}{\partial N_p}$ is provided in Appendix C.

We analyze the impacts of increasing parking spaces on customer demand. In Eq. (30b), the customer demand is negatively related to both the customer waiting time w_c for matching and the in-vehicle travel time T as $f' < 0$ (introduced in Section 3.1.3). There are three terms in the numerator on the right-hand side of Eq. (32), which reflect the impacts of marginal increase of parking spaces on the change of customer demand. The first two terms are the effects of the change of customer waiting time for matching on the change of customer demand, where the change of customer waiting time is related to the change of in-vehicle trip time $\left(\frac{\partial T}{\partial N_p}\right)$ and the change of the total number of ERVs cruising for charging and being charging $\left(\frac{\partial(q_et_s+q_et_e)}{\partial N_p}\right)$. And the third term is the impact of the change of in-vehicle trip time on the customer demand change. If the number of parking spaces increases, the travel speed improves and thus the in-vehicle trip time decreases $\left(\frac{\partial T}{\partial N_p} < 0\right)$, and the total number of ERVs cruising for charging and being charging reduces $\left(\frac{\partial(q_et_s+q_et_e)}{\partial N_p} < 0\right)$ and thus more utilizable vehicles to provide services for customers. The demand will increase $\left(\frac{\partial Q}{\partial N_p} > 0\right)$ based on Eq. (32). $\frac{\partial Q}{\partial N_p} > 0$ can be shown by noting the below: we have $f' < 0$ introduced in Section 3.1.3 and $W'_{c,w_t} < 0$; and we also have

$$1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q} = 1 - f'\beta\frac{1-\alpha_2}{\alpha_2}\frac{w_c}{Q} - f'\beta\frac{\alpha_1}{\alpha_2}\frac{w_c}{Q}\left(\frac{w_r+T}{w_t}\right) > 0. \tag{36}$$

Similarly, we investigate the marginal impacts of the fare F, the total number of ERVs N, and the number of installed charging piles N_e on the stationary equilibrium, where we have

$$\frac{\partial Q}{\partial F} = \frac{f' - f'\beta W'_{c,w_t}\frac{\partial T}{\partial F} + f'\gamma\frac{\partial T}{\partial F} - f'\beta\frac{1}{Q}W'_{c,w_t}\frac{\partial(q_et_s+q_et_e)}{\partial F}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{37}$$

$$\frac{\partial w_c}{\partial F} = \frac{1}{f'\beta}\frac{\partial Q}{\partial F} - \frac{1}{\beta} - \frac{\gamma}{\beta}\frac{\partial T}{\partial F}, \tag{38}$$

$$\frac{\partial Q}{\partial N} = \frac{f'\beta\frac{1}{Q}W'_{c,w_t} - f'\beta W'_{c,w_t}\frac{\partial T}{\partial N} + f'\gamma\frac{\partial T}{\partial N} - f'\beta\frac{1}{Q}W'_{c,w_t}\frac{\partial(q_e t_s + q_e t_e)}{\partial N}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q}}, \tag{39}$$

$$\frac{\partial w_c}{\partial N} = \frac{1}{f'\beta}\frac{\partial Q}{\partial N} - \frac{\gamma}{\beta}\frac{\partial T}{\partial N}, \tag{40}$$

$$\frac{\partial Q}{\partial N_e} = \frac{-f'\beta W'_{c,w_t}\frac{\partial T}{\partial N_e} + f'\gamma\frac{\partial T}{\partial N_e} - f'\beta\frac{1}{Q}W'_{c,w_t}\frac{\partial(q_e t_s + q_e t_e)}{\partial N_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q}}, \tag{41}$$

$$\frac{\partial w_c}{\partial N_e} = \frac{1}{f'\beta}\frac{\partial Q}{\partial N_e} - \frac{\gamma}{\beta}\frac{\partial T}{\partial N_e}, \tag{42}$$

The above derivatives are obtained based on Eqs. (7), (17), (30a)-(30c) and more detailed derivation is summarized in Appendix C. Due to complex interactions in the system considering charging, parking, and endogenous speed, whether increasing trip fare F will increase or reduce customer demand and whether increasing the number of ERVs N will increase or reduce customer demand is indeterminate. This is different from existing literature on ride-sourcing with no consideration of the charging issue (Yang et al., 2010; Zha et al., 2016). When the unoccupied charging piles are fewer, more time is needed to find an unoccupied charging pile and more vehicles are cruising for charging rather than providing services for customers, which can reduce customer demand. Increasing the number of ERVs N may cause fierce competition for finding an unoccupied charging pile and may reduce network average speed, and the customer demand will decrease with the increase of the number of ERVs, which will be illustrated by the numerical study in Section 5.2. Besides, increasing the number of charging piles N_e can help reduce the cruising time and the number of vehicles cruising for charging, and the customer demand may increase. However, excessive charging piles can be inefficient (a waste of resource) due to low occupation rates as tested numerically in Section 5.2.

This study mainly investigates the effect of parking space provision on reducing traffic speed and charging demand, but did not fully examine parking and congestion pricing strategies. In Appendix E, we further discuss how incorporating parking pricing will affect the drivers' cost formulation, and thus will further impact drivers' choices of parking and charging and the system equilibrium.

3.2.2. Analysis of the equilibrium when speed is constant

We now further focus on how parking provision to save the battery electricity can affect system performance of ride-sourcing services. We assume the network average speed v in Eq. (30h) as a constant (e.g., background vehicles account for relatively large part of the traffic and governs the network speed v). We investigate the effects of increasing parking spaces on customer demand and ERV service level in terms of customer average waiting time. Given a constant network average speed, based on Eqs. (7), (17), (30a)-(30g), we can readily get

$$\frac{\partial Q}{\partial N_p} = \frac{-f'\beta\frac{1}{Q}W'_{c,w_t}\frac{\partial(q_e t_s + q_e t_e)}{\partial N_p}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q}}, \tag{43}$$

$$\frac{\partial w_c}{\partial N_p} = \frac{1}{f'\beta}\frac{\partial Q}{\partial N_p}, \tag{44}$$

$$\frac{\partial (q_e t_e + q_e t_s)}{\partial N_p} = -\frac{1}{\frac{L_e}{v} + t_e}\left(t_e + t_s + \frac{q_e t_s^2 t_e v}{L_a}\right)\frac{\partial N_p^o}{\partial N_p}, \tag{45}$$

$$\frac{\partial N_p^o}{\partial N_p} = \frac{1}{\psi}\frac{Q^2 w_t^2 w_p^2 v}{L_a (w_t + w_p)^2}\left(1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q}\right), \tag{46}$$

where ψ denotes

$$\psi = \left(1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q}\right)\left(Q + \frac{Q^2 w_t^2 w_p^2 v}{L_a (w_t + w_p)^2}\right) - \frac{Q w_t}{w_t + w_p}\frac{1}{\frac{L_e}{v} + t_e}\left(t_e + t_s + \frac{q_e t_s^2 t_e v}{L_a}\right)\left((1 - f'\beta W'_{c,Q})\frac{w_t + 2w_p}{w_t + w_p} + f'\beta W'_{c,w_t}\frac{w_t}{Q}\right), \tag{47}$$

If $\psi > 0$, one can derive that $\frac{\partial N_p^o}{\partial N_p} > 0$ based on Eq. (46) and $1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q} > 0$ introduced in Eq. (36). We also note that Eqs. (43)-(45) are the special case of Eqs. (32)-(34) with $\frac{\partial v}{\partial N_p} = 0$ (the network average speed v being constant).

Observation 1. *Considering a constant network average speed v, if the number of ERVs that choose to park and wait for matching increases with the number of parking spaces, i.e., $\frac{\partial N_p^o}{\partial N_p} > 0$, the customer demand increases and the average customer waiting time for matching decreases, i.e., $\frac{\partial Q}{\partial N_p} > 0$, $\frac{\partial w_c}{\partial N_p} < 0$.*

When $\frac{\partial N_p^o}{\partial N_p} > 0$, one can verify that $\frac{\partial (q_e t_e + q_e t_s)}{\partial N_p} < 0$ based on Eq. (45). It means that if there are more parked ERVs with the increase of parking spaces, the total number of ERVs cruising for charging and being charging decreases, and thus there are more utilizable ERVs to provide services for customers. When $\frac{\partial N_p^o}{\partial N_p} > 0$, one can also derive that $\frac{\partial Q}{\partial N_e} > 0$ based on Eq. (43), and $\frac{\partial w_c}{\partial N_p} > 0$ based on Eq. (44).

3.3. Analysis of equilibrium under no parking provision ($N_p = 0$) and a constant speed

In this section, we focus on the impacts of the number of charging piles on ride-sourcing transportation system with electric vehicles. In particular, we explore the impacts of cruising for charging on the number of utilizable ERVs to provide services for customers. To ease the analysis, we now consider that parking provision to ERVs is zero (i.e., $N_p = 0$) and the speed is constant.

Without consideration of parking provision to ERVs ($N_p = 0$), the equilibrium model in Eq. (30) reduces to the system of equations as $w_c = A^{-\frac{1}{\alpha_2}} Q^{\frac{1-\alpha_1-\alpha_2}{\alpha_2}} w_t^{-\frac{\alpha_1}{\alpha_2}}$, $Q = f\left(F + \beta(w_c + w_r) + \gamma\frac{L_r}{v}\right)$, $N = Q\left(w_t + w_r + \frac{L_r}{v}\right) + q_e(t_s + t_e)$, $q_e = \frac{(N - q_e t_e)v}{L_e}$, and $t_s = \frac{L_a}{(N_e - q_e t_e)v}$, which follow Eqs. (30a)-(30c), Eq. (30f), and Eq. (30g), given that $N_p^o = 0$. Similar to the proof in Appendix B for Proposition 1, we can establish the existence of equilibrium solutions. Moreover, following Lemma 1 in Appendix B, given $N_p^o = 0$ and a constant speed v, it can be shown that the solution (q_e, t_s, Q, w_c, w_t) can be uniquely determined under mild conditions, which include that the number of charging piles is sufficient to satisfy charging demand ($N_e > q_e t_e$), and there are still remaining ERVs to provide services except these vehicles cruising for charging and being charging ($q_e(t_s + t_e) < N$).

Based on Eq. (30f) without consideration of parking, i.e., $N_p^o = 0$, we can get the charging demand as follows:

$$q_e = \frac{N}{\frac{L_e}{v} + t_e}. \tag{48}$$

With the increase of the number of ERVs, there will be more ERVs that compete for charging, where the number of occupied charging piles increases, the number of remaining unoccupied charging piles decreases, and thus the average cruising time for finding an unoccupied charging pile t_s in Eq. (30g) increases. Considering that these ERVs cruising for charging (N_e^s) and being charging (N_e^o) can not provide services for customers, the number of utilizable ERVs N_u to provide services can be given as follows:

$$N_u = N - N_e^o - N_e^s = N - q_e(t_e + t_s) = N - \frac{N}{\frac{L_e}{v} + t_e}\left(t_e + \frac{L_a}{\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)v}\right). \tag{49}$$

Proposition 2. *Considering a constant network average speed v and no parking provision to ERVs, with the given number of charging piles N_e,*

(i) *if $0 < N \leqslant \frac{\left(t_e+\frac{L_e}{v}\right)\left(L_eN_e-(L_aL_eN_e)^{0.5}\right)}{L_eN_e}$, with the increase of the total number of ERVs, the number of ERVs being charging, the number of ERVs cruising for charging piles, and the number of utilizable ERVs all increase, i.e., $\frac{\partial N_e^o}{\partial N} > 0$, $\frac{\partial N_e^s}{\partial N} > 0$, and $\frac{\partial N_u}{\partial N} \geqslant 0$;*

(ii) *if $\frac{\left(t_e+\frac{L_e}{v}\right)\left(L_eN_e-(L_aL_eN_e)^{0.5}\right)}{L_eN_e} < N < \frac{N_eL_e^2-L_aL_e-L_at_ev+L_eN_et_ev}{L_et_ev}$, with the increase of the total number of ERVs, both the number of ERVs being charging and the number of ERVs cruising for charging piles increase, while the number of utilizable ERVs N_u decreases, i.e., $\frac{\partial N_e^o}{\partial N} > 0$, $\frac{\partial N_e^s}{\partial N} > 0$, and $\frac{\partial N_u}{\partial N} < 0$;*

Proof. We can verify Proposition 2 by the following derivatives, which can be obtained by taking partial derivatives with respect to the equilibrium model defined by Eqs. (30a)-(30c), Eq. (30f), and Eq. (30g) with $N_p^o = 0$, i.e.,

$$\frac{\partial N_e^o}{\partial N} = \frac{t_e}{\frac{L_e}{v} + t_e}, \tag{50}$$

$$\frac{\partial N_e^s}{\partial N} = \frac{L_aN_e}{(L_e + t_ev)\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2}, \tag{51}$$

$$\frac{\partial N_u}{\partial N} = 1 - \frac{\partial N_e^o}{\partial N} - \frac{\partial N_e^s}{\partial N} = 1 - \frac{t_e}{\frac{L_e}{v} + t_e} - \frac{L_aN_e}{(L_e + t_ev)\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2}. \tag{52}$$

When $\frac{\partial N_u}{\partial N} = 0$ in Eq. (52), N is equal to $\frac{\left(t_e+\frac{L_e}{v}\right)\left(L_eN_e-(L_aL_eN_e)^{0.5}\right)}{L_eN_e}$. And when $N_u = 0$, N is equal to zero or $\frac{N_eL_e^2-L_aL_e-L_at_ev+L_eN_et_ev}{L_et_ev}$. It can be readily verified that if $0 < N \leqslant \frac{\left(t_e+\frac{L_e}{v}\right)\left(L_eN_e-(L_aL_eN_e)^{0.5}\right)}{L_eN_e}$, there is $\frac{\partial N_u}{\partial N} \geqslant 0$, and if $\frac{\left(t_e+\frac{L_e}{v}\right)\left(L_eN_e-(L_aL_eN_e)^{0.5}\right)}{L_eN_e} < N < \frac{N_eL_e^2-L_aL_e-L_at_ev+L_eN_et_ev}{L_et_ev}$, there is $\frac{\partial N_u}{\partial N} < 0$. And we have $\frac{\partial N_e^o}{\partial N} > 0$ in Eq. (50) and $\frac{\partial N_e^s}{\partial N} > 0$ in Eq. (51) for $0 < N < \frac{N_eL_e^2-L_aL_e-L_at_ev+L_eN_et_ev}{L_et_ev}$. This completes the proof. □

Proposition 2 is illustrated by Fig. 2. With the increase of the total number of ERVs N in the market, the number of ERVs being charging N_e^o increases linearly as shown by the dash-dotted line and thus the number of unoccupied charging piles reduces linearly. The number of ERVs cruising for charging N_e^s represented by the dotted line increases slowly with N as there are enough charging piles. However, when there are fewer unoccupied charging piles due to more ERVs and larger charging demand, the number of ERVs cruising for charging increases exponentially since the cruising time t_s will increase exponentially for finding an unoccupied charging piles. The number of utilizable ERVs N_u represented by the solid line reaches the peak at point A in Fig. 2. This is the largest number of utilizable ERVs to provide services for customers under the given number of charging piles N_e. However, further increase of N will reduce the number of utilizable ERVs N_u, and when N is close to point B in Fig. 2, the number of utilizable ERVs approaches zero. If the number of ERVs N is greater than the right-hand-side term of the inequality of Proposition 2(ii), the sum of the number of ERVs being charging N_e^o and the number of ERVs cruising for charging piles N_e^s is greater than N, which means that the number of utilizable ERVs N_u is negative as $N_u = N - N_e^o - N_e^s$ in Eq. (49). This case is not meaningful, and thus is not considered.

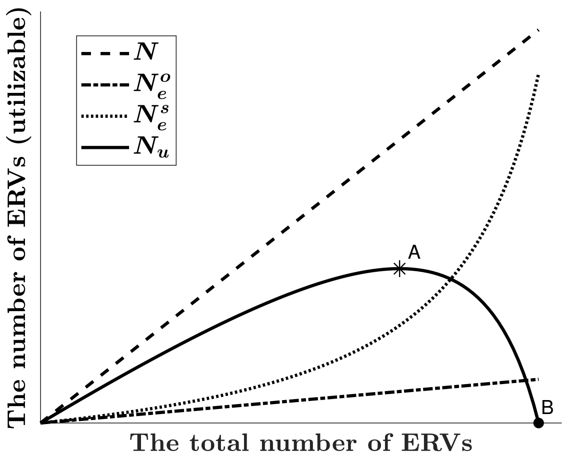


Fig. 2. The utilizable ERVs under the given number of charging piles

We then investigate the effects of increasing the number of ERVs N in the network on customer demand Q, customer waiting time for matching w_c, driver waiting time for matching w_t and the number of vacant ERVs waiting for matching N_{vt}. Based on the equilibrium defined by Eqs. (30a)-(30c), Eq. (30f), Eq. (30g) and Eq. (49) with $N_p^o = 0$, by taking partial derivatives, we can derive that

$$\frac{\partial Q}{\partial N} = \frac{f'\beta \frac{1}{Q} W'_{c,w_t} \frac{\partial N_u}{\partial N}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t} \frac{w_r + w_t + T}{Q}}, \tag{53}$$

$$\frac{\partial w_c}{\partial N} = \frac{\frac{1}{Q} W'_{c,w_t} \frac{\partial N_u}{\partial N}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t} \frac{w_r + w_t + T}{Q}}, \tag{54}$$

$$\frac{\partial w_t}{\partial N} = \frac{\frac{1}{Q}\left(1 - f'\beta \frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{w_c}{Q}\right) \frac{\partial N_u}{\partial N}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t} \frac{w_r + w_t + T}{Q}}, \tag{55}$$

$$\frac{\partial N_{vt}}{\partial N} = \frac{\left(1 - f'\beta \frac{1-\alpha_2}{\alpha_2} \frac{w_c}{Q}\right) \frac{\partial N_u}{\partial N}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t} \frac{w_r + w_t + T}{Q}}, \tag{56}$$

where $\frac{\partial N_u}{\partial N}$ is from Eq. (52).

Observation 2. *Considering the constant network average speed v and no parking provision, with the given number of charging piles N_e,*

(i) *if $0 < N \leqslant \frac{\left(t_e + \frac{L_e}{v}\right)\left(L_e N_e - (L_a L_e N_e)^{0.5}\right)}{L_e N_e}$, with the increase of the total number of ERVs N, the customer demand Q increases, the customer waiting time for matching w_c decreases, and the number of vacant ERVs waiting for matching N_{vt} increases, i.e., $\frac{\partial Q}{\partial N} \geqslant 0$, $\frac{\partial w_c}{\partial N} \leqslant 0$, $\frac{\partial N_{vt}}{\partial N} \geqslant 0$;*

(ii) *if $\frac{\left(t_e + \frac{L_e}{v}\right)\left(L_e N_e - (L_a L_e N_e)^{0.5}\right)}{L_e N_e} < N < \frac{N_e L_e^2 - L_a L_e - L_a t_e v + L_e N_e t_e v}{L_e t_e v}$, with the increase of the total number of ERVs N, the customer demand Q decreases, the customer waiting time for matching w_c increases, and the number of vacant ERVs waiting for matching N_{vt} decreases, i.e., $\frac{\partial Q}{\partial N} < 0$, $\frac{\partial w_c}{\partial N} > 0$, $\frac{\partial N_{vt}}{\partial N} < 0$.*

We can readily verify the conclusions in Observation 2 based on Proposition 2. When $0 < N \leqslant \frac{\left(t_e + \frac{L_e}{v}\right)\left(L_e N_e - (L_a L_e N_e)^{0.5}\right)}{L_e N_e}$, there are more utilizable ERVs with the increase of the total number of ERVs in the market, i.e., $\frac{\partial N_u}{\partial N} \geqslant 0$ in Proposition 2. When there are more utilizable ERVs, the customer demand will increase and the customer waiting time will decrease, and there are more vacant ERVs waiting for the matching. In particular, if $(\alpha_1 + \alpha_2) \leqslant 1$, we have $\frac{\partial w_t}{\partial N} > 0$; if $(\alpha_1 + \alpha_2) > 1$, the sign of $\frac{\partial w_t}{\partial N}$ depends on the values of α_1, α_2, f', β, w_c and Q, where if $f'\beta \frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{w_c}{Q} \leqslant 1$ (> 1), we have $\frac{\partial w_t}{\partial N} \geqslant 0$ (< 0). In contrast, when $\frac{\left(t_e + \frac{L_e}{v}\right)\left(L_e N_e - (L_a L_e N_e)^{0.5}\right)}{L_e N_e} < N < \frac{N_e L_e^2 - L_a L_e - L_a t_e v + L_e N_e t_e v}{L_e t_e v}$, there are fewer utilizable ERVs with the increase of the total number of ERVs, and the customer demand will decrease, the customer waiting time will increase, and there are fewer vacant ERVs waiting for matching with customers. This implies that although there are more ERVs in the market, the ride-sourcing customer demand decreases and the customer waiting time is longer since more ERVs are cruising for charging due to fierce competition for charging rather than providing services.

We further examine the impacts of fare F, the number of charging piles N_e, average charging time

t_e and maximum driving range L_e on the ride-sourcing transportation system equilibrium as follows:

$$\frac{\partial N_e^o}{\partial F}=0,\ \frac{\partial N_e^s}{\partial F}=0,\ \frac{\partial N_u}{\partial F}=0,\ \frac{\partial Q}{\partial F}<0,\ \frac{\partial w_c}{\partial F}<0,\ \frac{\partial w_t}{\partial F}>0,\ \frac{\partial N_{vt}}{\partial F}>0, \tag{57a}$$

$$\frac{\partial N_e^o}{\partial N_e}=0,\ \frac{\partial N_e^s}{\partial N_e}<0,\ \frac{\partial N_u}{\partial N_e}>0, \tag{57b}$$

$$\frac{\partial Q}{\partial N_e}>0,\ \frac{\partial w_c}{\partial N_e}<0,\ \frac{\partial w_t}{\partial N_e}>0\ (if\ \alpha_1+\alpha_2\leqslant 1),\ \frac{\partial N_{vt}}{\partial N_e}>0, \tag{57c}$$

$$\frac{\partial N_e^o}{\partial L_e}<0,\ \frac{\partial N_e^s}{\partial L_e}<0,\ \frac{\partial N_u}{\partial L_e}>0, \tag{57d}$$

$$\frac{\partial Q}{\partial L_e}>0,\ \frac{\partial w_c}{\partial L_e}<0,\ \frac{\partial w_t}{\partial L_e}>0\ (if\ \alpha_1+\alpha_2\leqslant 1),\ \frac{\partial N_{vt}}{\partial L_e}>0, \tag{57e}$$

$$\frac{\partial N_e^o}{\partial t_e}>0,\ \frac{\partial N_e^s}{\partial t_e}>0\ (if\ N>N_e),\ \frac{\partial N_e^s}{\partial t_e}\leqslant 0\ (if\ N\leqslant N_e),\ \frac{\partial N_u}{\partial t_e}<0, \tag{57f}$$

$$\frac{\partial Q}{\partial t_e}<0,\ \frac{\partial w_c}{\partial t_e}>0,\ \frac{\partial w_t}{\partial t_e}<0\ (if\ \alpha_1+\alpha_2\leqslant 1),\ \frac{\partial N_{vt}}{\partial t_e}<0, \tag{57g}$$

The detailed derivation is provided in Appendix D.

With the increase of the fare F, the number of ERVs being charging N_e^o, the number of ERVs cruising for charging N_e^s and the number of utilizable ERVs N_u are constant. However, with the increase of the fare F, customer demand Q reduces, and there are more vacant ERVs waiting for matching ($\frac{\partial N_{vt}}{\partial N_e}>0$) and longer waiting time for ERVs w_t but shorter waiting time for customers w_c to be successfully matched.

With the increase of the number of charging piles, the number of ERVs cruising for charging N_e^s decreases, and thus there are more utilizable ERVs to provide the services for the customers. It follows that the customer demand is higher and the customer waiting time for matching is shorter. However, there are more vacant ERVs waiting to be matched. In particular, when the matching rate with the decreasing/constant return to scale ($\alpha_1+\alpha_2\leqslant 1$), which is less efficient, the waiting time of ERVs for being matched w_t is larger $\left(\frac{\partial w_t}{\partial N_e}>0\right)$. When $\alpha_1+\alpha_2>1$, if $f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\leqslant 1\ (>1)$, we have $\frac{\partial w_t}{\partial N_e}\geqslant 0$ (<0), which are derived in Appendix D.

If the maximum driving range increases with the development of battery technologies, fewer ERVs need to charge $\left(\frac{\partial N_e^o}{\partial L_e}<0\right)$, and there are more vacant charging piles and fewer ERVs cruising for charging. It follows that there are more utilizable ERVs, and there are more unmatched ERVs waiting for matching with customers, higher customer demand, shorter waiting time of customer for matching.

When the charging time is longer, there are more ERVs being charging but lower charging demand $\left(\frac{\partial q_e}{\partial t_e}<0\right)$, and more vehicles cruising for charging if the total number of ERVs is larger than the number of charging piles ($N>N_e$) as derived in Appendix D. With the increase of the charging time t_e, the number of utilizable ERVs reduces, and the customer demand decreases and customer waiting time for matching increases.

4. Profit maximization and social welfare maximization

In this section, we investigate the optimal trip fare and fleet size to maximize the profit of ride-sourcing operator and social welfare respectively in Sections 4.1-4.2.

4.1. Profit maximization

Section 4.1.1 investigates the optimal trip fare and fleet size to maximize the profit subject to the ride-sourcing system equilibrium considering parking provision, as defined in Section 3.2.1. Section 4.1.2 conducts the same analysis for the special case where there is no parking provision ($N_p = 0$) and and the speed is constant, as introduced in Section 3.3.

4.1.1. Profit maximization subject to the general equilibrium

We first consider the ride-sourcing operator to maximize the profit $\mathit{\Pi}$:

$$\max_{(F \geqslant 0, N > 0)} \mathit{\Pi} = FQ - CN - C_e q_e, \tag{58}$$

subject to the general equilibrium defined in Section 3.2.1. The first term on the right-hand side of Eq. (58) is the total fare revenue, the second term is the cost of operating these ERVs with C denoting the cost of operating one vehicle per unit time, and the third term is the total charging toll with C_e denoting the charging toll each time.

Assuming interior optimal solutions, the first-order optimality conditions with respect to the fare F and the fleet size N can be derived as follows:

$$\frac{\partial \mathit{\Pi}}{\partial F} = 0 \Rightarrow F\frac{\partial Q}{\partial F} + Q - C_e\frac{\partial q_e}{\partial F} = 0, \tag{59}$$

$$\frac{\partial \mathit{\Pi}}{\partial N} = 0 \Rightarrow F\frac{\partial Q}{\partial N} - C - C_e\frac{\partial q_e}{\partial N} = 0. \tag{60}$$

Eq. (59) implies that at profit maximization, the marginal change of revenue from the collected fare $\left(F\frac{\partial Q}{\partial F} + Q\right)$ due to marginal increase of the fare should exactly offset the marginal change of the charging toll $\left(C_e\frac{\partial q_e}{\partial F}\right)$ each other. Eq. (60) states that marginal change of collected fare $\left(F\frac{\partial Q}{\partial N}\right)$ due to marginal increase of the fleet size should be equal to the sum of the additional operation cost and the marginal change of charging toll $\left(C + C_e\frac{\partial q_e}{\partial N}\right)$.

We can further write Eqs. (59) and (60) as follows:

$$Q\delta_F^Q + Q - \frac{C_e q_e}{F}\delta_F^{q_e} = 0, \tag{61}$$

$$\frac{FQ}{N}\delta_N^Q - C - \frac{C_e q_e}{N}\delta_N^{q_e} = 0, \tag{62}$$

where δ_x^y represents the elasticity of y with respect to x, i.e., $\delta_x^y = \frac{x}{y}\frac{\partial y}{\partial x}$. We then can get

$$F = \frac{C_e q_e \delta_F^{q_e}}{Q(1+\delta_F^Q)}, \tag{63}$$

$$N = \frac{C_e q_e}{C}\left(\frac{\delta_F^{q_e}\delta_N^Q}{1+\delta_F^Q} - \delta_N^{q_e}\right), \tag{64}$$

where we consider $1+\delta_F^Q$ is not equal to zero. If $\delta_F^{q_e} = \frac{F}{q_e}\frac{\partial q_e}{\partial F} < 0$, which indicates the charging demand decreases with the increase of fare $\left(\frac{\partial q_e}{\partial F} < 0\right)$, we can get $\delta_F^Q < -1$, which means that profit-maximizing ride-sourcing operation is on the elastic portion of the customer demand curve for the ride-sourcing services.

4.1.2. Profit maximization: no parking provision ($N_p = 0$) and constant speed

To provide further understanding of the impacts of cruising for charging on the operation strategy for the ERVs, in this section, we simplify the analysis by assuming $N_p = 0$ and a constant speed as introduced in Section 3.3. We investigate the optimal fare and fleet size to achieve ride-sourcing operator's profit maximization.

The formulation of the ride-sourcing operator to maximize the profit Π follows that in Eq. (58), which is now subject to the equilibrium under no parking provision ($N_p = 0$) and a constant speed as defined in Section 3.3. We consider the interior optimal solution and the first-order optimality conditions with respect to F and N are similar to those in Eqs. (59) and (60) while we now have $C_e \frac{\partial q_e}{\partial F} = 0$ in Eqs. (59) based on Eq. (57a) $\left(\frac{\partial N_e^o}{\partial F} = \frac{t_e \partial q_e}{\partial F} = 0\right)$. By substituting the derivatives $\frac{\partial Q}{\partial F}$ in Eq. (D.2) and $\frac{\partial Q}{\partial N}$ in Eq. (53) into Eqs. (59)-(60), respectively, we can obtain

$$F = -\frac{Q}{f'} + \frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}}\left(w_t + w_r + T + \frac{1-\alpha_1-\alpha_2}{\alpha_1} w_t\right), \tag{65}$$

$$N_{vt}\frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}} = N_c \beta \frac{\alpha_1}{\alpha_2}, \tag{66}$$

where the unit combined cost considering both the charging toll and the number of utilizable ERVs as cruising for charging is derived, i.e., $\frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}}$ (different from the related literature when there is no cruising for charging, e.g., Yang et al., 2010; Zha et al., 2016). At profit maximization, the fare in Eq. (65) includes the monopoly markup $\left(-\frac{Q}{f'}\right)$ (Lerner, 1934), the combined cost of an ERV to serve a customer with the total time of $\frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}}(w_t + w_r + T)$ and the additional cost related to the matching, i.e., $\frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}} \frac{1-\alpha_1-\alpha_2}{\alpha_1} w_t$. Eq. (66) states that at profit maximization, the total combined cost of operating vacant ERVs waiting for matching equals the total waiting time cost of these waiting customer for matching multiplied by a coefficient of $\frac{\alpha_1}{\alpha_2}$.

Proposition 3. *Given the optimal fare F and the fleet size N in Eq. (65) and Eq. (66) for profit maximization, (i) the maximized profit of the ride-sourcing operator Π increases with the number of* *charging piles N_e, i.e., $\frac{d\Pi}{dN_e} = F\frac{\partial Q}{\partial N_e} > 0$; (ii)*

The proof of Proposition 3 is provided in Appendix F. Proposition 3 highlights the positive effect of providing charging piles on the profit of ride-sourcing operator.

4.2. Social welfare maximization

Section 4.2.1 investigates the optimal trip fare and fleet size to maximize the social welfare under the general equilibrium defined in Section 3.2.1, and Section 4.2.2 conducts the same analysis subject to the special case with no parking provision ($N_p = 0$) and a constant speed as introduced in Section 3.3.

4.2.1. Social welfare maximization subject to the general equilibrium

We now investigate the social welfare maximization, where the fare and the fleet size of ERVs are chosen to maximize total social welfare S as follows:

$$\max_{(F\geqslant 0, N>0)} S = \int_0^Q f^{-1}(z)dz - Q(\beta(w_c + w_r) + \gamma T) - CN - q_e C_e - C_h N_e - \gamma N_b \left(1 - \frac{v}{v_f}\right), \quad (67)$$

subject to the equilibrium defined in Section 3.2.1, where the first four terms on the right-hand side of Eq. (67) ($\int_0^Q f^{-1}(z)dz - Q(\beta(w_c + w_r) + \gamma T) - CN - q_e C_e$) represent the sum of consumer surplus for the customers and the profit of the ride-sourcing operator. We note that the money transfer (the total collected fare FQ) within the system is excluded in the social welfare. C_h denotes the average installation, operation and maintenance cost of one charging pile and $C_h N_e$ is the total cost of constructing and maintaining the charging piles with the number of N_e. And the last term reflects the benefit loss of background traffic due to the decrease of speed. Specifically, we consider that the value of in-vehicle time of the travelers of background traffic is same as that of the ride-sourcing customers and the average trip length of the background traffic is denoted as L_b. The benefit loss of background traffic is $\gamma\frac{N_b v}{L_b}\left(\frac{L_b}{v_f} - \frac{L_b}{v}\right) = -\gamma N_b\left(1 - \frac{v}{v_f}\right)$, i.e., the loss is measured in relation to the difference between the experienced speed and the free-flow speed. Note that we analytically examine the trade-off between road capacity reduction due to reserving road space as parking spaces and less cruising traffic, and the cost of providing the on-street parking spaces is not considered (this might overestimate the benefit of providing parking spaces since the costs are underestimated).

Considering the interior optimal solutions, the first-order optimality conditions of S with respect to the fare F and the fleet size of the ERVs N are as follows:

$$\frac{\partial S}{\partial F} = 0 \Rightarrow -\beta Q\frac{\partial w_c}{\partial F} - \gamma Q\frac{\partial T}{\partial F} + F\frac{\partial Q}{\partial F} - C_e\frac{\partial q_e}{\partial F} + \gamma\frac{N_b}{v_f}\frac{\partial v}{\partial F} = 0, \quad (68)$$

$$\frac{\partial S}{\partial N} = 0 \Rightarrow -\beta Q\frac{\partial w_c}{\partial N} - \gamma Q\frac{\partial T}{\partial N} + F\frac{\partial Q}{\partial N} - C - C_e\frac{\partial q_e}{\partial N} + \gamma\frac{N_b}{v_f}\frac{\partial v}{\partial N} = 0. \quad (69)$$

Eq. (68) describes that at social welfare maximization, with the marginal increase of the fare, the sum of the marginal changes of the consumer surplus, the profit of the ride-sourcing operator, and the benefit loss of the background traffic is equal to zero. Specifically, with the increase of the fare, the change of consumer surplus includes additional fare loss for all customers due to fare increase ($-Q$), the marginal change of the cost for the customer waiting time $\left(-\beta Q \frac{\partial w_c}{\partial F}\right)$, and the marginal change of the cost for customer in-vehicles travel time $\left(-\gamma Q \frac{\partial T}{\partial F}\right)$. The marginal change of the profit of the ride-sourcing operator includes the marginal fare revenue $\left(F \frac{\partial Q}{\partial F} + Q\right)$, the marginal charging toll $\left(-C_e \frac{\partial q_e}{\partial N}\right)$. Note that the fare loss due to fare increase ($-Q$) in the change of consumer surplus and the additional fare revenue for all customers due to fare increase (Q) in the change of profit of the ride-sourcing operator offset each other and are not included in Eq. (68). Similarly, Eq. (69) states that at social welfare maximization, with the marginal increase of the fleet size, the sum of the marginal changes of the consumer surplus, the profit of the ride-sourcing operator and the benefit loss of background traffic are equal to zero. Specifically, with the increase of the fleet size, the marginal change of consumer surplus includes the marginal costs of customer waiting time and in-vehicle travel time $\left(-\beta Q \frac{\partial w_c}{\partial N} - \gamma Q \frac{\partial T}{\partial N}\right)$. The marginal change of the profit includes marginal fare revenue $\left(F \frac{\partial Q}{\partial N}\right)$, the additional operation cost of one another ERV ($-C$) and marginal charging toll $\left(-C_e \frac{\partial q_e}{\partial N}\right)$.

With $\frac{\partial w_c}{\partial F}$ in Eq. (38) and $\frac{\partial w_c}{\partial N}$ in Eq. (40) being substituted into Eqs. (68) and (69), respectively, we further get

$$\left(F - \frac{Q}{f'}\right) \frac{\partial Q}{\partial F} + Q - C_e \frac{\partial q_e}{\partial F} + \gamma \frac{N_b}{v_f} \frac{\partial v}{\partial F} = 0, \tag{70}$$

$$\left(F - \frac{Q}{f'}\right) \frac{\partial Q}{\partial N} - C - C_e \frac{\partial q_e}{\partial N} + \gamma \frac{N_b}{v_f} \frac{\partial v}{\partial N} = 0. \tag{71}$$

By comparing the first-order optimality conditions with respect to the fare F and the fleet size N under profit maximization in Eqs. (59)-(60) and those under social welfare maximization in Eqs. (70)-(71), it is observed that under social welfare maximization, the monopoly markup $-\frac{Q}{f'}$ is excluded from the fare, i.e., we have $\left(F - \frac{Q}{f'}\right)$, while terms related to the benefit loss of the background traffic appears in the optimality conditions.

4.2.2. Social welfare maximization: no parking provision ($N_p = 0$) and constant speed

Similarly, we further investigate social welfare maximization under the equilibrium in a special case with no parking provision ($N_p = 0$) and a constant speed, as defined in Section 3.3. The formulation to maximize the social welfare S follows that in Eq. (67) while the last term related to background traffic is constant, and thus is dropped. Again, we consider the interior optimal solution and the first-order optimality conditions with respect to F and N are similar to those in Eqs. (68)-(69) while we note that $\gamma Q \frac{\partial T}{\partial F} = 0$, $C_e \frac{\partial q_e}{\partial F} = 0$ and $\gamma \frac{N_b}{v_f} \frac{\partial v}{\partial F} = 0$ in Eq. (68), and $\gamma Q \frac{\partial T}{\partial N} = 0$ and $\gamma \frac{N_b}{v_f} \frac{\partial v}{\partial N} = 0$ in Eq. (69).

With $\frac{\partial Q}{\partial F}$ in Eq. (D.2), $\frac{\partial w_c}{\partial F}$ in Eq. (D.3), $\frac{\partial Q}{\partial N}$ in Eq. (53) and $\frac{\partial w_c}{\partial N}$ in Eq. (54) being substituted into Eqs. (68)-(69), we can get

$$F = \frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}} \left(w_t + w_r + T + \frac{1 - \alpha_1 - \alpha_2}{\alpha_1} w_t \right), \tag{72}$$

$$N_{vt} \frac{C + C_e \frac{\partial q_e}{\partial N}}{\frac{\partial N_u}{\partial N}} = N_c \beta \frac{\alpha_1}{\alpha_2}, \tag{73}$$

where the optimal fare F at social welfare maximization in Eq. (72) is similar to that under profit maximization in Eq. (65), and the monopoly markup $\left(-\frac{Q}{f'}\right)$ is not included in the fare at social optimum in Eq. (72). The formulation governing the optimal feet size in Eq. (73) at social welfare maximization are the same as that in Eq. (66) for profit maximization, reflecting the same trade-off involved in relation to feet size under social welfare maximization and profit maximization.

Proposition 4. *Given the optimal fare F and fleet size of ERVs N for social welfare maximization, the social welfare S increases with the number of charging piles N_e, i.e., $\frac{dS}{dN_e} > 0$, if and only if $F \frac{\partial Q}{\partial N_e} - \beta Q \frac{\partial w_c}{\partial N_e} - C_h > 0$.*

The proof is given in Appendix G. We have $\frac{\partial Q}{\partial N_e} > 0$ and $\frac{\partial w_c}{\partial N_e} < 0$ in Eq. (57c) and thus we have $F \frac{\partial Q}{\partial N_e} - \beta Q \frac{\partial w_c}{\partial N_e} > 0$, where we focus on the interior optimal solutions with $F > 0$ at social welfare maximization. Proposition 4 indicates that when the sum of the marginal increase of the fare revenue from demand increase $\left(F \frac{\partial Q}{\partial N_e}\right)$ and the marginal decrease of the cost loss of the customer waiting time $\left(-\beta Q \frac{\partial w_c}{\partial N_e}\right)$ is larger than the cost of constructing and maintaining a charging pile C_h, the total social welfare increases with the number of charging piles.

At social welfare maximization, based on Eqs. (72)-(73), the profit of the ride-sourcing operator Π based on Eq. (58) can be derived as follows:

$$\Pi = \beta N_c \left(\frac{\alpha_1}{\alpha_2} \frac{w_r + T}{w_t} + \frac{1 - \alpha_2}{\alpha_2} \right) - CN - C_e q_e. \tag{74}$$

5. Numerical studies

In this section, we conduct numerical studies to illustrate the equilibrium of the ERV services with cruising for parking and charging and the effects of the parking provision on system performance. Firstly, we introduce numerical settings in Section 5.1, followed by comparative statics to investigate the impacts of the number of charging piles, the total number of ERVs, and the number of parking spaces on the equilibrium in Section 5.2. Then we investigate profit maximization and social welfare maximization in Section 5.3.

5.1. Numerical settings

The demand function introduced in Section 3.1.3 is specified as the following exponential function form:

$$Q = Q_0 \exp\{-k(F + \beta(w_c + w_r) + \gamma T)\}, \tag{75}$$

where Q_0 is the potential customer demand, and k is the demand sensitivity parameter.

The network average speed function introduced in Section 3.1.5 is specified as follows:

$$v = v_f \cdot \left(1 - \frac{N - N_p^o - N_e^o + N_b}{A_u k_j - N_p - N_e}\right). \tag{76}$$

The parameters related to the road network are given as follows: the free-flow speed $v_f = 50$ (km/h), the total length of the road system $L_a = 300$ (km), the total utilizable road space $A_u = 800$ (km·lane) and the jam density $k_j = 160$ (veh/(km·lane)).

The parameters about the vehicle charging are set as follows: the average charging time $t_e = 0.5$ (h) and the maximum driving range of an ERV with fully charged $L_e = 200$ (km).

For travelers, we set the average travel length of ride-sourcing trips as $L_r = 10$ (km), the average pick-up time $w_r = 0.05$ (h), the value of waiting time $\beta = 50$ (AUD/h) and the value of in-vehicle time $\gamma = 30$ (AUD/h). In the demand function, we set $k = 0.1$, the potential customer demand $Q_0 = 3 \times 10^6$ (trip/h) and the number of background traffic $N_b = 25000$. Regarding the matching efficiency, we let $A = 5 \times 10^{-3}$, $\alpha_1 = 1$ and $\alpha_2 = 1$.

In terms of cost, we set the hourly cost of operating one ERV $C = 30$ (AUD), the cost of charging each time $C_e = 15$ (AUD), and the average hourly cost for establishing and operating one charging pile $C_h = 30$ (AUD). And we set the fare $F = 30$ (AUD) in Section 5.2.

5.2. Comparative statics

In this section, we conduct comparative statics about the number of charging piles, the total number of ERVs and the number of parking spaces, respectively. Fig. 3 shows the impacts of the number of charging piles on equilibrium, where we set the fare $F = 30$ (AUD) and the number of ERVs $N = 20000$. We focus on the impacts of charging pile supply and consider no parking provision here with $N_p = 0$, and the effects of parking provision will be illustrated later.

As shown in Fig. 3(a), the number of charging piles N_e increases from 1520 to 1540, while the number of ERVs cruising for charging piles N_e^s reduces from about 7179 to 1156. And as shown in Fig. 3(d), the occupancy rate of the charging piles O_e is nearly 100% when there are 1520 charging piles. Therefore, many ERVs are cruising for charging rather than providing services and the customer demand Q is the lowest as shown in Fig. 3(b). With a mild increase of the charging piles, there are more unoccupied

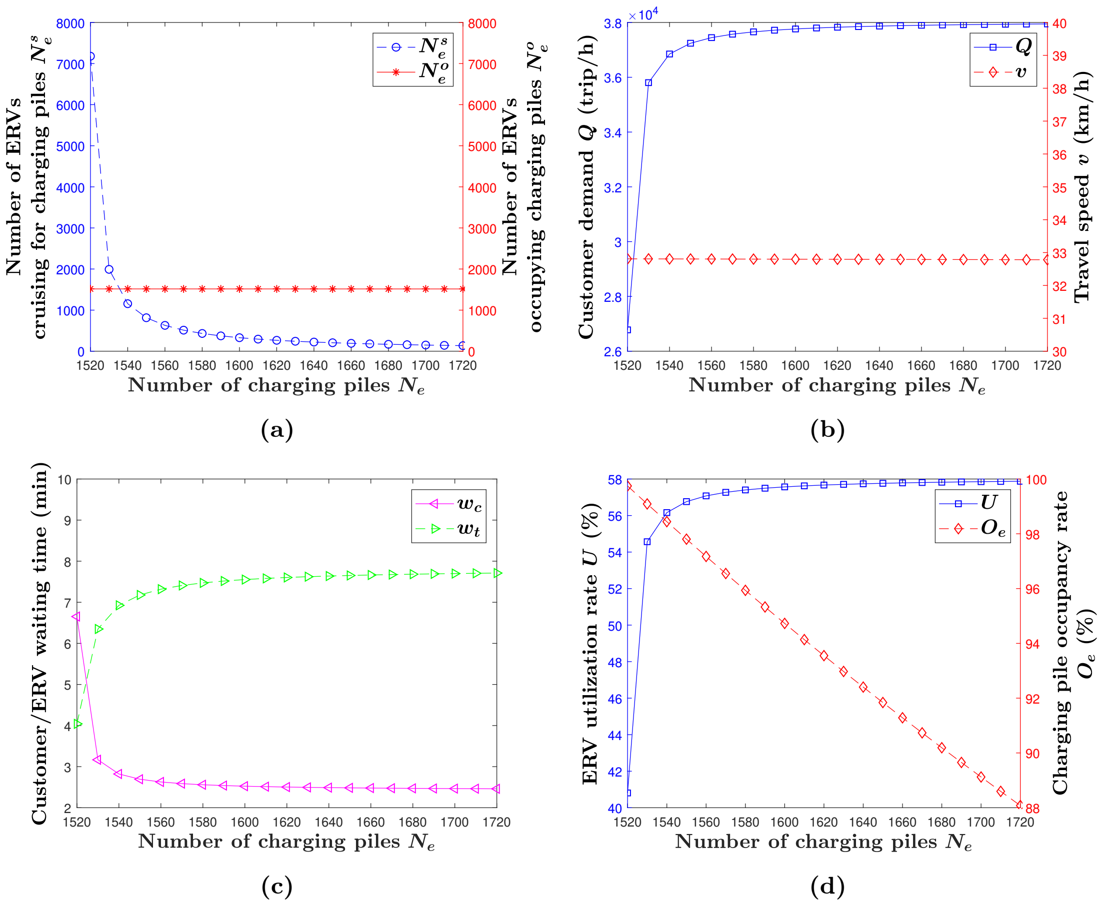


Fig. 3. The impacts of the number of charging piles N_e

charging piles (lower occupancy rate of charging pile shown in Fig. 3(d)) and the number of ERVs cruising for charging reduces significantly. This is because, when charging pile occupancy rate is above critical level, cruising increases very sharply. A mild increase of charging pile supply helps to reduce the charging pile occupancy rate to a level under the critical value, cruising then decreases very sharply. It follows that the customer demand increases and the ERV utilization rate U increases as shown in Fig. 3(d). As shown in Fig. 3(c), the customer waiting time w_c also reduces by about four minutes correspondingly when the number of charging piles N_e increases from 1520 to 1540. Additional charging piles beyond 1600 are less cost-effective as charging piles will be less utilized as shown in Fig. 3(d) and the customer demand increase is limited as shown in Fig. 3(b). The numerical results in Fig. 3 highlight the importance of charging pile supply on ride-sourcing system with electric vehicles.

We then investigate that whether increasing the number of ERVs could improve customer demand, and the number of charging piles is set as 2100. Fig. 4 shows the impacts of number of ERVs on equilibrium. As shown in Fig. 4(a), with the increase of the number of ERVs from 1.5×10^4, the number of ERVs being charging N_e^o increases nearly linearly, while the number of ERVs cruising for charging N_e^s increases slowly at first but then increases nearly exponentially. As shown in Fig. 4(b), the customer demand Q reduces with the increase of number of ERVs from about 2.4×10^4. This indicates that increasing the fleet

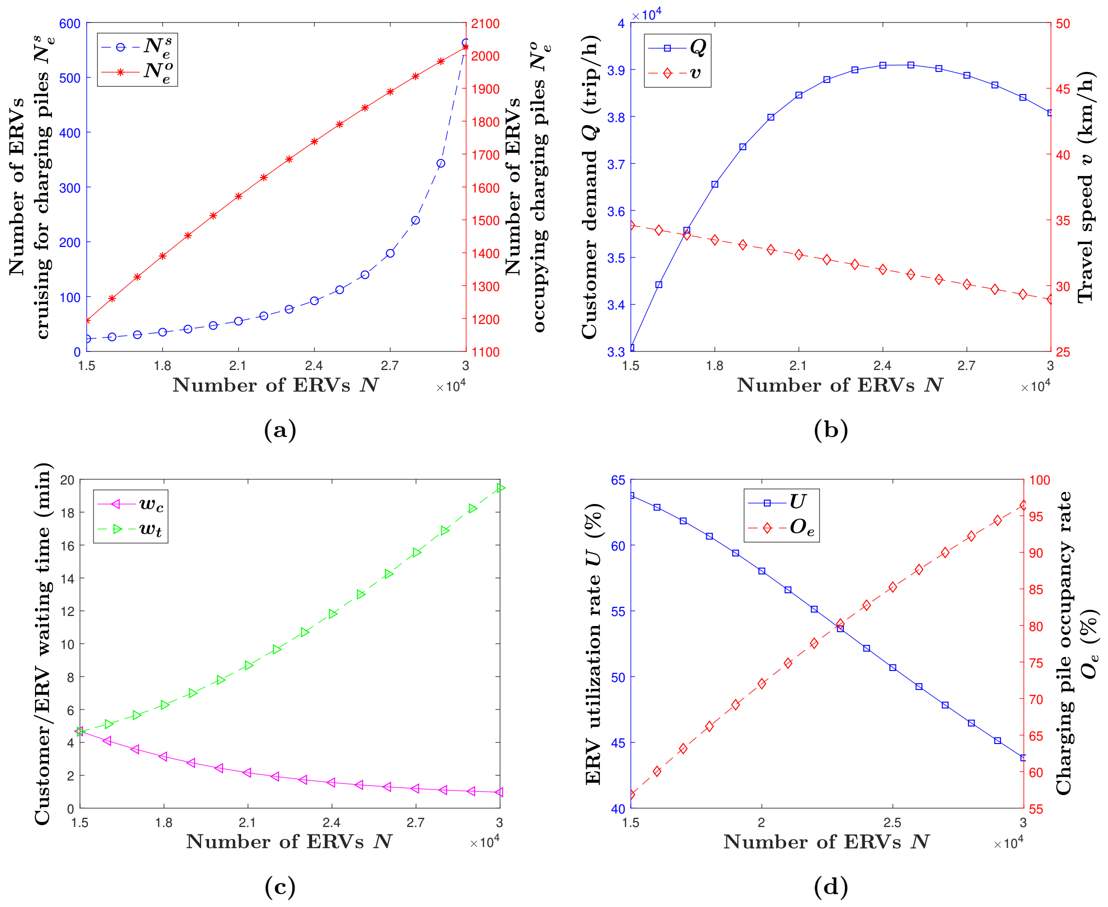


Fig. 4. The impacts of the number of ERVs N

size N cannot necessarily improve customer demand, which is different from the related studies due to the vehicle charging issue (Yang et al., 2010; Zha et al., 2016). Also, the average network travel speed v reduces with the increase of the fleet size shown in Fig. 4(b). As shown in Fig. 4(c), the customer waiting time w_c reduces with the increase of the fleet size while the ERV waiting time w_t increases significantly. And the utilization rate of ERVs drops as shown in Fig. 4(d) with the increase of the fleet size. These results indicate that control of ERVs fleet size in the transportation network is important.

We investigate the effects of the provision of parking spaces for the cruising ERVs shown in Fig. 5, where N_e is given as 1550 and N is set as 20000. As shown in Fig. 5(a), N_e^s drops from 815 to 45 and so does N_e^o from about 1516 to 1090 with the increase of the number of parking spaces N_p from 0 to 2×10^4. Fig. 5(b) shows that both the customer demand Q and speed v can reach the peak when N_p is about 5700. However, when there are more parking spaces, both Q and v drop. Fig. 5(c) shows that w_t increases while w_c reduces. And Fig. 5(d) indicates that the occupation rate of parking spaces N_p^o/N_p drops and the rate between the parked ERVs and the total ERVs (N_p^o/N) increases with the increase of parking spaces. Note that in Fig. 5(d), when $N_p = 0$ we set the N_p^o/N_p equal to 100% and N_p^o/N equal to 0.

We now compare the results under no parking provision ($N_p = 0$) and under a parking provision of

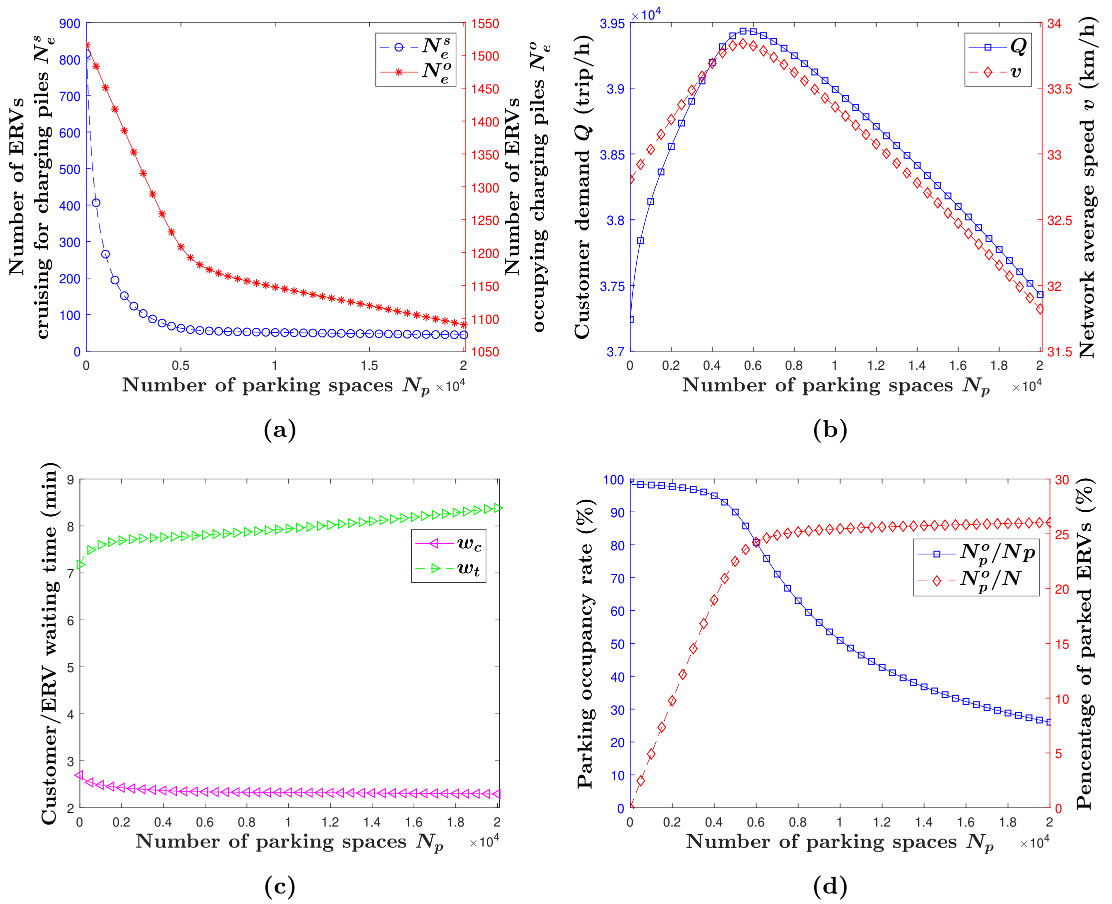


Fig. 5. The impacts of the number of parking spaces N_p

Table 2
No parking provision vs. Parking provision

Metrics	no parking provision ($N_p = 0$)	parking provision ($N_p = 5700$)
w_c (min)	2.69	2.34
w_t (min)	7.18	7.80
Q (trip/h)	37242.00	39436.45
v (km/h)	32.81	33.84
q_e (trip/h)	3031.92	2374.81
N_e^s	815	58
N_e^o	1516	1187
N_{vt}	4455	5127
t_s (min)	16.12	1.47
T (min)	18.29	17.73
w_p (min)	–	0.58
N_p^o/N_p	–	83.78%

$N_p = 5700$ in Table 2. The customer waiting time w_c drops from 2.69 min under no parking provision to 2.34 min under a parking provision of $N_p = 5700$, and the customer demand Q increases from 37242 to 39436. The speed v increases from 32.81 km/h to 33.83 km/h. The charging demand q_e drops from 3032 to 2375. The average cruising time for finding an unoccupied charging pile t_s is significantly reduced from 16.12 min to 1.47 min. These results imply the effectiveness of providing the parking spaces on improving customer demand and ride-sourcing service level in terms of customer waiting time, and reducing both charging demand and average cruising time for finding an unoccupied charging pile.

5.3. *Profit maximization and social welfare maximization*

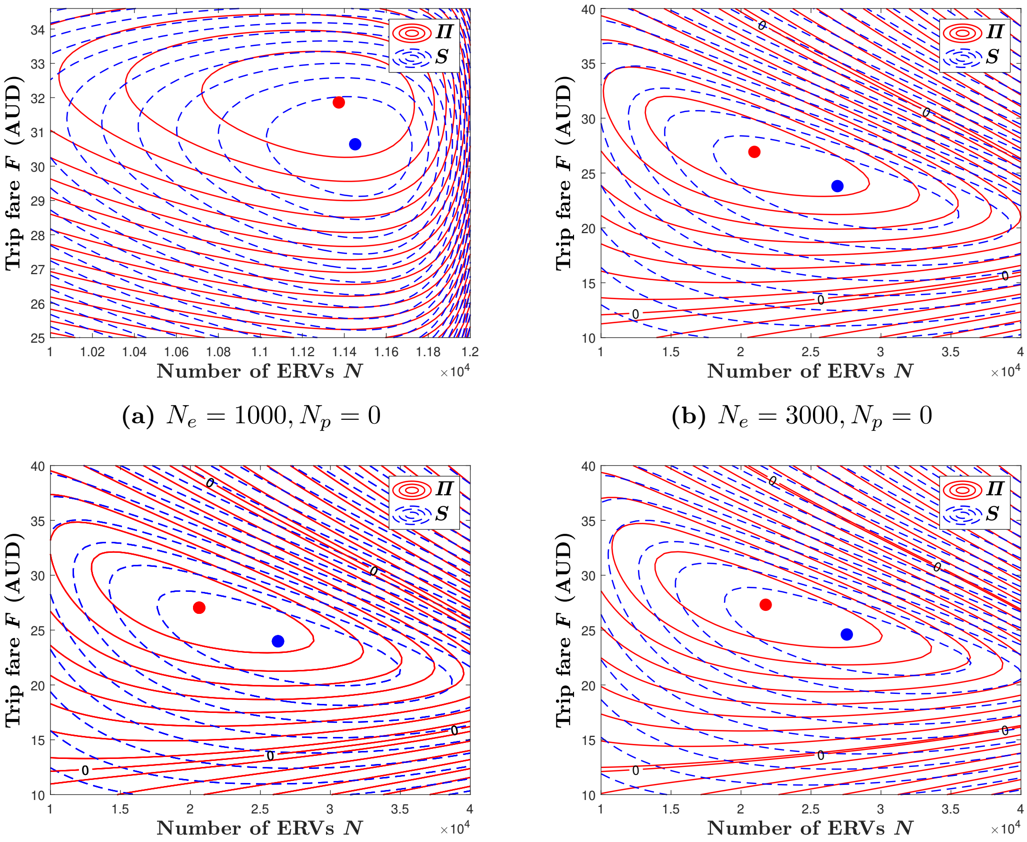
We now conduct numerical studies to investigate the optimal fare and the fleet size to achieve profit maximization of the ride-sourcing operator or social welfare maximization. We investigate four different combinations of different numbers of charging piles and parking spaces, i.e, ($N_e = 1000, N_p = 0$), ($N_e = 3000, N_p = 0$), ($N_e = 5000, N_p = 0$), and ($N_e = 3000, N_p = 5000$), where $N_p = 0$ means that the parking provision is not considered. Fig. 6 shows the contours of the profit Π (the red solid line) and the social welfare S (the blue dash line). Table 3 summarizes the optimal profit and total social welfare. Fig. 6(a) shows that when the number of charging piles is 1000 and the parking provision is not considered, the trip fare is the highest and the number of ERVs is the lowest both under the profit optimum and social optimum compared to the other three cases. This emphasizes the impacts of the limited number of charging piles. However, when the number of charging piles is 5000 and $N_p = 0$, based on Table 3, the profit Π is 532855 and the total social profit S is 600412, which are both lower than that under $N_e = 3000, N_p = 0$. And we note that the occupancy rate of charging piles N_e^o/N_e is only 30.63% under profit maximization with $N_e = 5000, N_p = 0$. In contrast to the results under $N_e = 3000, N_p = 0$, with parking spaces set with $N_e = 3000, N_p = 5000$, both the company profit and total social welfare are increased. These results indicate that by allocating parking spaces for the vacant ERVs, both the company profit and social welfare can be improved.

Table 3
The profit maximization (PM) and social welfare maximization (SWM) (currency: AUD, time unit: min)

Variables	$N_e = 1000, N_p = 0$		$N_e = 3000, N_p = 0$		$N_e = 5000, N_p = 0$		$N_e = 3000, N_p = 5000$	
	PM	SWM	PM	SWM	PM	SWM	PM	SWM
Π	431140	426993	541038	499360	532855	494352	574919	539449
S	443193	448098	636503	681345	556932	600412	494352	729963
F	31.86	30.64	26.93	23.81	27.04	23.98	27.31	24.60
N	11374	11452	20956	26891	20630	26261	21764	27559
Q	25131	26079	45182	57208	44297	55749	46467	57604
w_c	6.17	7.19	3.87	3.94	3.92	4.01	3.18	3.04
w_t	4.64	3.84	4.12	3.20	4.15	3.22	4.87	4.12
v	36.05	36.02	32.24	29.99	32.07	29.90	32.55	30.30
N_e^o/N_e (%)	94.03	94.61	52.11	62.53	30.63	36.54	45.79	56.26
N_p^o/N_p (%)	–	–	–	–	–	–	70.17	71.73

6. Conclusion

This paper investigates ride services by ERVs considering the provision of parking spaces and charging piles for ERVs. Since the vacant ERVs cruising on streets for customers will not only increase traffic but also waste battery electricity, we consider to allocate a portion of road area as parking spaces for these cruising ERVs. While the cruising traffic will be reduced and battery electricity will be saved by



(a) $N_e = 1000, N_p = 0$ **(b)** $N_e = 3000, N_p = 0$

(c) $N_e = 5000, N_p = 0$ **(d)** $N_e = 3000, N_p = 5000$

Fig. 6. Contours of the profit and social welfare in the domain of (F, N)

parking provision, the road space allocation to parking can reduce the utilizable road area for running vehicles. To investigate this trade off, we model the ride-sourcing services with cruising for charging and parking. We analytically investigate the impacts of the number of charging piles and parking spaces on ride-sourcing system. In particular, we analytically investigate the impacts of the cruising process for finding an unoccupied charging pile on ride-sourcing services. We analytically determine the number of ERVs in the network to achieve the maximum utilizable ERVs to provide services under limited charging piles. We also investigate the optimal fare and fleet size to achieve profit maximization and social welfare maximization.

The numerical results indicate that when the number of charging piles is insufficient, a substantial number of ERVs will be cruising to find an unoccupied charging pile, and the number of utilizable ERVs to service customers will be small. By an appropriate increase of the number of charging piles, the number of ERVs cruising for charging drops significantly, especially when the charging pile occupancy rate is at a critical level. By allocating some parking spaces for the vacant vehicles, the number of ERVs cruising for charging and being charging is reduced and the customer demand increases. The network average speed can also be improved. The profit of ride-sourcing operator and social welfare can also be improved with parking provision. These results indicate the effectiveness of parking space provision for the vacant ERVs.

This paper adopts stylized models to generate strategic-level analytical understanding on the operation, charging pile and parking provision for ERVs, which does have many limitations. Firstly, the aggregate model in this paper ignores the spatial dimension (e.g., no spatial demand/supply heterogeneity). Future extensions may consider more network details (such as a multi-zone model or a detailed link-node network). Secondly, this study conducts system equilibrium analysis and ignores time-dependent variations and system dynamics. A future study may look into how demand-supply interactions over time in the ride-sourcing system with EVs. Thirdly, this study considers "average" driver and customer, where individual attributes are not modeled. A future study may further develop operational level models to account individual level heterogeneity, e.g., heterogeneous trip lengths and the specific electricity consumption of each vehicle, heterogeneous pick-up time, value of waiting time and value of in-vehicle time. Last but not least, due to limited access to relevant real-world data, we mainly conducted numerical studies based on assumed parameters to illustrate the model. In the future, it is of our interest to carry out empirical case study when more sensible data becomes available.

7. Acknowledgement*

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Appendix A. Notations

Table 4
List of notations

Notation	Interpretation
A	The positive model parameter of matching rate which is related to the spatial properties of the ride-sourcing markets and matching technology
A_e	The road area allocated for charging piles
A_p	The road area allocated for parking spaces
A_R	The road area for running vehicles
A_u	Total utilizable road area for running vehicles, parking spaces and charging piles, which is measure as total length of all the lanes
C	The hourly cost of operation one ERV
C_a	The expected cost of drivers choosing to park while waiting for the matching
C_b	The expected cost of drivers not considering parking while waiting for the matching
C_e	The cost for charging each time

Notation	Interpretation
C_h	The average cost for installing, operating and maintaining one charging pile
$f(\cdot)$	The customer demand function related to total ride-sourcing travel cost
F	The trip fare of the ride-sourcing service
k_a	Average Network traffic density
k_j	Jam density
L_a	Total length of the road system
L_e	The maximum driving range of an ERV fully charged
L_r	Average travel distance of ride-sourcing trips
m^{c-t}	The matching rate between the unmatched customers and ERVs under the ride-sourcing platform
m^{t-p}	The parking rate of the cruising ERVs
$M(\cdot)$	The function for matching rate between the unmatched customers and ERVs under the ride-sourcing platform
$M^p(\cdot)$	The function for meeting rate between the cruising ERVs and the parking spaces
N	Total number of ERVs
N_b	Number of background vehicles
N_c	Number of unmatched customers
N_e	Number of charging piles
N_e^o	Number of ERVs being charging, or the number of occupied charging piles
N_e^s	Number of ERVs cruising for unoccupied charging piles
N_p	Number of parking spaces
N_p^o	Number of unmatched vehicles which are parked on the roadside, or the number of occupied parking spaces
N_p^u	Number of unoccupied parking spaces
N_R	Number of running vehicles
N_u	Number of ERVs to serve customers except for the ERVs cruising for charging and being charging
N_{vt}	Number of unmatched vehicles
N_{vt}^s	Number of unmatched vehicles cruising on the roads
O_e	Charging pile occupancy rate
p_a	The parking toll each time
P_p	The parking vacancy rate
P_p^m	The probability of an unmatched ERV finding an unoccupied parking space before been matched

Notation	Interpretation
q_e	The traffic flow of ERVs cruising for charging
Q	Customer demand for ride-sourcing services
S	Social welfare
t_s	Expected searching time for an unoccupied charging pile
t_e	Charging time for the ERV to get fully charged
T	Average in-vehicle travel time of ride-sourcing trips
U	The ratio of the occupied ERVs with customers to the total number of ERVs
v	Network average speed for car traffic
v_f	free flow travel speed
$V(\cdot)$	Network average speed function for car traffic
w_c	Expected customer waiting time to be matched with one ERV (or driver) by an online platform
w_t	Expected ERV (or driver) waiting time to be matched with a customer by an online platform
w_r	The average meeting time between the customers and ERVs after successfully matched
$W_c(\cdot)$	The function for customer waiting time to be matched
α_1	The elasticity of the matching rate for the number of waiting customers
α_2	The elasticity of the matching rate for the number of unmatched ERVs
β	Value of waiting time
γ	Value of in-vehicle time
$\mathit{\Gamma}$	Arrival rate or supply of vacant ERV
μ	Total travel cost
π	Average profit of each ERV
$\mathit{\Pi}$	Ride-sourcing operator's profit

Appendix B. The existence of solutions to the system of nonlinear equations

Under the given values of the exogenous variables F, N, N_e and N_p, we need to prove the existence of the solution of the eight endogenous variables, i.e., w_c, w_t, Q, q_e, t_s, N_p^o, w_p, and v, in the eight equations in Eq. (30).

A nonempty, compact and convex set Θ is constructed for the two variables (N_p^o, v), i.e., $\Theta = \{N_p^o, v | 0 \leqslant N_p^o \leqslant N, v_s \leqslant v \leqslant v_f\}$, where $v_s = V\left(\frac{N+N_b}{A_u k_j - N_p - N_e}\right)$. For each one $(\tilde{N}_p^o, \tilde{v}) \in \Theta$, we try to solve five dependent variables (q_e, t_s, Q, w_c, w_t) according to the equations as follows:

$$q_e = \frac{N - \tilde{N}_p^o}{\frac{L_e}{\tilde{v}} + t_e}, \tag{B.1a}$$

$$t_s = \frac{L_a}{(N_e - q_e t_e)\tilde{v}}, \tag{B.1b}$$

$$\frac{N - q_e(t_s + t_e)}{w_t + w_r + \frac{L_r}{\tilde{v}}} = f\left(F + \beta(L_1(w_t) + w_r) + \gamma\frac{L_r}{\tilde{v}}\right), \tag{B.1c}$$

$$Q = \frac{N - q_e(t_s + t_e)}{w_t + w_r + \frac{L_r}{\tilde{v}}}, \tag{B.1d}$$

$$w_c = A^{-\frac{1}{\alpha_2}} Q^{\frac{1-\alpha_1-\alpha_2}{\alpha_2}} w_t^{-\frac{\alpha_1}{\alpha_2}}, \tag{B.1e}$$

where if $0 < \alpha_2 < 1$, $L_1(w_t) = A^{\frac{-1}{\alpha_2}} \left(\frac{N-q_e(t_s+t_e)}{w_t+w_r+\frac{L_r}{\tilde{v}}}\right)^{\frac{1-\alpha_2}{\alpha_2}} \left(\frac{(N-q_e(t_s+t_e))w_t}{w_t+w_r+\frac{L_r}{\tilde{v}}}\right)^{\frac{-\alpha_1}{\alpha_2}}$; if $\alpha_2 = 1$, $L_1(w_t) = A^{\frac{-1}{\alpha_2}} \left(\frac{(N-q_e(t_s+t_e))w_t}{w_t+w_r+\frac{L_r}{\tilde{v}}}\right)^{\frac{-\alpha_1}{\alpha_2}}$. $L_1(w_t)$ represents the customer waiting time for matching which is derived based on Eqs. (30a) and (30c). Eqs. (B.1a)-(B.1b) respectively represents charging demand and cruising time for charging, which are given based on Eqs. (30f)-(30g), respectively. Eq. (B.1c) is derived based on Eqs. (30a)-(30c), which indicates that the supply of unmatched ERVs is equal to customer demand. Eq. (B.1d) is supply of unmatched ERVs derived from Eq. (30c). Eq. (B.1e) indicates customer waiting time, which is directly given based on Eq. (30a).

We now prove that the solution (q_e, t_s, Q, w_c, w_t) is uniquely determined by $(\tilde{N}_p^o, \tilde{v}) \in \Theta$ and changes continuously as well. And the remaining equations in the system of nonlinear equations in Eq. (30) are:

$$\bar{N}_p^o \left(1 + \frac{L_a}{(N_p - \tilde{N}_p^o)\tilde{v} w_t}\right) = Q w_t, \tag{B.2a}$$

$$\bar{v} = V\left(\frac{N - \tilde{N}_p^o - q_e t_e + N_b}{A_u k_j - N_p - N_e}\right), \tag{B.2b}$$

where Eq. (B.2a) integrates Eqs. (30d) and (30e) about the cruising time for parking w_p, and Eq. (B.2b) is from Eq. (30h).

We then verify that Eqs. (B.1)-(B.2) constructs a continuous mapping from $(\tilde{N}_p^o, \tilde{v}) \in \Theta$ to the variable space of $(\bar{N}_p^o, \bar{v})$, and prove that $(\bar{N}_p^o, \bar{v})$ is in the set Θ. The existence of the equilibrium or the solution to the systems Eqs. (B.1) and (B.2) can then be proved according to Brouwer's fixed-point theorem (De la Fuente, 2000).

Lemma 1. *For the given $(\tilde{N}_p^o, \tilde{v}) \in \Theta$, and considering $N_e > q_e t_e$ and $q_e(t_s + t_e) < N$, the solution (q_e, t_s, Q, w_c, w_t) is uniquely determined based on Eq. (B.1), q_e is no less than zero, and t_s, Q, w_c, w_t are greater than zero. (q_e, t_s, Q, w_c, w_t) are continuous with $(\tilde{N}_p^o, \tilde{v})$.*

Proof. Based on Eq. (B.1a), q_e is uniquely determined with the given $(\tilde{N}_p^o, \tilde{v}) \in \Theta$, and we have $q_e \geqslant 0$. Obviously, q_e changes continuously with $(\tilde{N}_p^o, \tilde{v})$. Similarly, based on Eqs. (B.1a)-(B.1b), t_s is uniquely determined and changes continuously with $(\tilde{N}_p^o, \tilde{v})$. Note that $N_e > q_e t_e$ is considered so that the number of charging piles is sufficient to satisfy the charging demand, and we have $t_s > 0$.

Based on Eq. (B.1c), we let $L_2(w_t)$ denote as:

$$L_2(w_t) = \frac{N - q_e(t_s + t_e)}{w_t + w_r + \frac{L_r}{\tilde{v}}} - f\left(F + \beta(L_1(w_t) + w_r) + \gamma \frac{L_r}{\tilde{v}}\right), \tag{B.3}$$

We first consider $0 < \alpha_2 < 1$, and $L_1(w_t) = A^{\frac{-1}{\alpha_2}} \left(\frac{N - q_e(t_s+t_e)}{w_t + w_r + \frac{L_r}{\tilde{v}}}\right)^{\frac{1-\alpha_2}{\alpha_2}} \left(\frac{(N - q_e(t_s+t_e))w_t}{w_t + w_r + \frac{L_r}{\tilde{v}}}\right)^{\frac{-\alpha_1}{\alpha_2}}$. When $w_t \to 0$, we have $L_1(w_t) \to +\infty$, and $\lim_{w_t \to 0} f = f(+\infty) = 0$ as discussed in Section 3.1.3. Therefore, we have $\lim_{w_t \to 0} L_2(w_t) = \frac{N - q_e(t_s+t_e)}{w_r + \frac{L_r}{\tilde{v}}} > 0$. Note that we consider $N > q_e(t_s + t_e)$, so that there are remaining ERVs to provide service rather than all the vehicles being cruising for charging piles and charging. When $w_t \to +\infty$, we have $L_1(w_t) \to 0$, and $\lim_{w_t \to +\infty} f = f\left(F + \beta w_r + \gamma \frac{L_r}{\tilde{v}}\right) > 0$ as discussed in Section 3.1.3. Therefore, we have $\lim_{w_t \to +\infty} L_2(w_t) < 0$. $L_2(w_t)$ is continuous and strictly decreasing with w_t $\left(\frac{\partial L_2(w_t)}{\partial w_t} < 0\right)$ and thus there is unique positive w_t to solve $L_2(w_t) = 0$. The first-order partial derivatives $\frac{\partial L_2(w_t)}{\partial \tilde{N}_p^o}$ and $\frac{\partial L_2(w_t)}{\partial \tilde{v}}$ exist and are continuous. Especially, we have $\frac{\partial L_2(w_t)}{\partial w_t} \neq 0$. According to implicit function theorem, w_t is continuous function of $(\tilde{N}_p^o, \tilde{v})$. Therefore, Q is also continuous function of $(\tilde{N}_p^o, \tilde{v})$ in Eq. (B.1d). And w_t is continuous function of $(\tilde{N}_p^o, \tilde{v})$ in Eq. (B.1e). Both Q and w_c are greater than zero.

We then consider $\alpha_2 = 1$ and $L_1(w_t) = A^{\frac{-1}{\alpha_2}} \left(\frac{(N - q_e(t_s+t_e))w_t}{w_t + w_r + \frac{L_r}{\tilde{v}}}\right)^{\frac{-\alpha_1}{\alpha_2}}$. Similarly, when $w_t \to 0$, we have $\lim_{w_t \to 0} L_2(w_t) > 0$, and when $w_t \to +\infty$, we have $\lim_{w_t \to +\infty} L_1(w_t) = A^{\frac{-1}{\alpha_2}} (N - q_e(t_s + t_e))^{\frac{-\alpha_1}{\alpha_2}} > 0$ and thus $\lim_{w_t \to +\infty} f = f\left(F + \beta(A^{\frac{-1}{\alpha_2}} (N - q_e(t_s + t_e))^{\frac{-\alpha_1}{\alpha_2}} + w_r) + \gamma \frac{L_r}{\tilde{v}}\right) > 0$. Therefore, we have $\lim_{w_t \to +\infty} L_2(w_t) < 0$. There is unique positive w_t to solve $L_2(w_t) = 0$ as $L_2(w_t)$ is continuous and strictly decreasing with w_t $\left(\frac{\partial L_2(w_t)}{\partial w_t} < 0\right)$. Besides, obviously, the first-order partial derivatives $\frac{\partial L_2(w_t)}{\partial \tilde{N}_p^o}$ and $\frac{\partial L_2(w_t)}{\partial \tilde{v}}$ exist and continuous. Therefore, when $\alpha_2 = 1$, w_t is also continuous function of $(\tilde{N}_p^o, \tilde{v})$ according to implicit function theorem. And both Q and w_c in Eqs. (B.1d)-(B.1e) are also continuous functions of $(\tilde{N}_p^o, \tilde{v})$ and are greater than zero.

This completes the proof. □

Lemma 2. *Both $\bar{N}_p^o$ and $\bar{v}$ are continuous functions of $(\tilde{N}_p^o, \tilde{v})$ and the domains are $0 < \bar{N}_p^o < N$ and $v_s < \bar{v} < v_f$.*

Proof. Based on Eq. (B.2a), we let $L_3(\bar{N}_p^o)$ denote

$$L_3(\bar{N}_p^o) = \bar{N}_p^o \left(1 + \frac{L_a}{(N_p - \bar{N}_p^o)\tilde{v} w_t}\right) - Q w_t, \tag{B.4}$$

Under the given $(\tilde{N}_p^o, \tilde{v}) \in \Theta$, we have $Qw_t > 0$ based on Lemma 1, and based on Eq. (B.1d), we also have $Qw_t = N - q_e(t_e + t_s) - Q(w_r + \frac{L_r}{\tilde{v}}) < N$. Therefore, we have $0 < Qw_t < N$ when $(\tilde{N}_p^o, \tilde{v}) \in \Theta$. When $\bar{N}_p^o \to 0$, we have $\lim_{\bar{N}_p^o \to 0} L_3(\bar{N}_p^o) < 0$. If $N_p < N$, we have $\lim_{\bar{N}_p^o \to N_p} L_3(\bar{N}_p^o) > 0$. If $N_p \geqslant N$, we have $\lim_{\bar{N}_p^o \to N} L_3(\bar{N}_p^o) > 0$. In particular, if $N_p = N$, we have $\lim_{\bar{N}_p^o \to N_p} L_3(\bar{N}_p^o) = +\infty$. $L_3(\bar{N}_p^o)$ is continuous function and strictly increasing with $\bar{N}_p^o$ $\left(\frac{\partial L_3(\bar{N}_p^o)}{\partial \bar{N}_p^o} > 0\right)$. Therefore, for the given $(\tilde{N}_p^o, \tilde{v})$, $\bar{N}_p^o$ $(0 < \bar{N}_p^o < N)$ is uniquely determined, where if $N_p < N$ we have $0 < \bar{N}_p^o < N_p < N$, and if $N_p \geqslant N$, we have $\bar{N}_p^o < N$. Obviously, the first-order partial derivatives $\frac{\partial L_3(\bar{N}_p^o)}{\partial \bar{N}_p^o}$ and $\frac{\partial L_3(\bar{N}_p^o)}{\partial \tilde{v}}$ exist and are continuous. Besides, we have $\left(\frac{\partial L_3(\bar{N}_p^o)}{\partial \bar{N}_p^o} \neq 0\right)$. Therefore, $\bar{N}_p^o$ $(0 < \bar{N}_p^o < N)$ is a continuous function of $(\tilde{N}_p^o, \tilde{v}) \in \Theta$ according to the implicit function theorem.

By combining q_e in Eq. (B.1a) and v in Eq. (B.2b), we have

$$\bar{v} = V\left(\frac{(N - \tilde{N}_p^o)\frac{L_e}{\tilde{v}}\frac{1}{\frac{L_e}{\tilde{v}} + t_e} + N_b}{A_u k_j - N_p - N_e}\right), \tag{B.5}$$

Since $V(\cdot)$ is strictly decreasing function as $V' < 0$ introduced in Section 3.1.5, We have $\bar{v} > v_s$, where $v_s = V\left(\frac{N + N_b}{A_u k_j - N_p - N_e}\right)$, and $\bar{v} < v_f$ $(v_f = V(0))$. Therefore, we have $v_s < \bar{v} < v_f$ and $\bar{v}$ is a continuous function of $(\tilde{N}_p^o, \tilde{v}) \in \Theta$.

This completes the proof. □

Based on Lemma 1 and Lemma 2, Eqs. (B.1)-(B.2) construct a continuous mapping from $(\tilde{N}_p^o, \tilde{v}) \in \Theta$ to the subset $(0 < \bar{N}_p^o < N, v_s < \bar{v} < v_f)$, where $\Theta = \{N_p^o, v | 0 \leqslant N_p^o \leqslant N, v_s \leqslant v \leqslant v_f\}$ is a nonempty, compact and convex set. Therefore, the solution $(0 < N_p^o < N, v_s < v < v_f)$ to the system Eq. (B.2) exists according to Brouwer's fixed-point theorem. We then have solution w_p based on Eq. (30e). And the solution (q_e, t_s, Q, w_c, w_t) is determined based on Eq. (B.1) and Lemma 1. Therefore, the solution to the system of nonlinear equations Eq. (30) exists.

Note that in the above proof process, the following conditions are assumed: (i) the number of charging piles is sufficient to satisfy the charging demand $(N_e > q_e t_e)$; (ii) there are still remaining ERVs to provide services to customers besides those vehicles cruising for charging and being charging $(N > q_e(t_s + t_e))$; (iii) the total number of vehicles is smaller than the utilizable road space to avoid gridlock $((N + N_b) < A_u k_j - N_p - N_e)$.

Appendix C. Comparative statics for the ride-sourcing transportation system

With w_c in Eqs. (7) and (30a), w_p in Eq. (30e), $q_e = \frac{N - N_p^o}{\frac{L_e}{v} + t_e}$ obtained in Eq. (30f), and t_s in Eq. (30g) being substituted into Eqs. (30b)-(30d) and Eq. (30h), we can get

$$Q = f\left(F + \beta(W_c(Q, w_t) + w_r) + \gamma\frac{L_r}{v}\right), \tag{C.1}$$

$$N = Q\left(w_t + w_r + \frac{L_r}{v}\right) + \frac{N - N_p^o}{\frac{L_e}{v} + t_e}\left(\frac{L_a}{\left(N_e - \frac{N - N_p^o}{\frac{L_e}{v} + t_e}t_e\right)v} + t_e\right), \tag{C.2}$$

$$N_p^o = \frac{Q w_t^2}{\frac{L_a}{(N_p - N_p^o)v} + w_t}, \tag{C.3}$$

$$v = V\left(\frac{N - N_p^o - \frac{N-N_p^o}{\frac{L_e}{v}+t_e}t_e + N_b}{A_u k_j - N_p - N_e}\right). \tag{C.4}$$

Comparative statics with respect to N_p. By taking partial derivatives with respect to the number of parking spaces N_p on both sides of the system of Eqs. (C.1)-(C.4), we can get

$$(f'\beta W'_{c,Q} - 1)\frac{\partial Q}{\partial N_p} + f'\beta W'_{c,wt}\frac{\partial w_t}{\partial N_p} - f'\gamma\frac{L_r}{v^2}\frac{\partial v}{\partial N_p} = 0, \tag{C.5}$$

$$\left(w_t + w_r + \frac{L_r}{v}\right)\frac{\partial Q}{\partial N_p} + Q\frac{\partial w_t}{\partial N_p} - \frac{1}{\frac{L_e}{v}+t_e}\left(t_e + t_s + \frac{q_e t_s^2 v t_e}{L_a}\right)\frac{\partial N_p^o}{\partial N_p}$$
$$+\left(\frac{q_e L_e(t_e+t_s)}{v(L_e+t_e v)} + \frac{q_e^2 t_s^2 L_e t_e}{L_a(L_e+t_e v)} - \frac{q_e t_s}{v} - \frac{Q L_r}{v^2}\right)\frac{\partial v}{\partial N_p} = 0, \tag{C.6}$$

$$\frac{w_t^2}{w_t+w_p}\frac{\partial Q}{\partial N_p} + \frac{Qw_t^2 + 2Qw_t w_p}{(w_t+w_p)^2}\frac{\partial w_t}{\partial N_p}$$
$$-\left(1 + \frac{Qw_t^2 w_p^2 v}{L_a(w_t+w_p)^2}\right)\frac{\partial N_p^o}{\partial N_p} + \frac{Qw_t^2 w_p^2(N_p - N_p^o)}{L_a(w_t+w_p)^2}\frac{\partial v}{\partial N_p} = \frac{-Qw_t^2 w_p^2 v}{L_a(w_t+w_p)^2}, \tag{C.7}$$

$$\frac{-L_e}{(L_e+t_e v)(A_u k_j - N_p - N_e)}\frac{\partial N_p^o}{\partial N_p} - \left(\frac{1}{V'} + \frac{q_e t_e L_e}{v(L_e+t_e v)(A_u k_j - N_p - N_e)}\right)\frac{\partial v}{\partial N_p}$$
$$= \frac{-(N - N_p^o - q_e t_e + N_b)}{(A_u k_j - N_p - N_e)^2}, \tag{C.8}$$

Using the matrices to express the above linear equations, we get

$$\boldsymbol{\Phi}\cdot\boldsymbol{X} = \boldsymbol{Z}, \tag{C.9}$$

where $\boldsymbol{\Phi}$ is the matrix with four columns and four rows to denote the coefficients, and each element is $\phi_{i,j}, i,j \in \{1,2,3,4\}$, $\boldsymbol{X}$ is the column matrix with $x_1 = \frac{\partial Q}{\partial N_p}$, $x_2 = \frac{\partial w_t}{\partial N_p}$, $x_3 = \frac{\partial N_p^o}{\partial N_p}$ and $x_4 = \frac{\partial v}{\partial N_p}$, and $\boldsymbol{Z}$ with each element as z_i is the column matrix to denote the constant terms of each equation.

We have

$$\phi_{1,1} = (f'\beta W'_{c,Q} - 1), \phi_{1,2} = f'\beta W'_{c,wt}, \phi_{1,3} = 0, \phi_{1,4} = -f'\gamma\frac{L_r}{v^2}, z_1 = 0, \tag{C.10}$$

$$\phi_{2,1} = w_t + w_r + \frac{L_r}{v}, \phi_{2,2} = Q, \phi_{2,3} = -\frac{1}{\frac{L_e}{v}+t_e}\left(t_e + t_s + \frac{q_e t_s^2 v t_e}{L_a}\right),$$
$$\phi_{2,4} = \frac{q_e L_e(t_e+t_s)}{v(L_e+t_e v)} + \frac{q_e^2 t_s^2 L_e t_e}{L_a(L_e+t_e v)} - \frac{q_e t_s}{v} - \frac{Q L_r}{v^2}, \tag{C.11}$$

$$\phi_{3,1} = \frac{w_t^2}{w_t+w_p}, \phi_{3,2} = \frac{Qw_t^2 + 2Qw_t w_p}{(w_t+w_p)^2}, \phi_{3,3} = -\left(1 + \frac{Qw_t^2 w_p^2 v}{L_a(w_t+w_p)^2}\right),$$
$$\phi_{3,4} = \frac{Qw_t^2 w_p^2(N_p - N_p^o)}{L_a(w_t+w_p)^2}, \tag{C.12}$$

$$\phi_{4,1} = 0, \phi_{4,2} = 0, \phi_{4,3} = \frac{-L_e}{(L_e+t_e v)(A_u k_j - N_p - N_e)},$$
$$\phi_{4,4} = -\left(\frac{1}{V'} + \frac{q_e t_e L_e}{v(L_e+t_e v)(A_u k_j - N_p - N_e)}\right), \tag{C.13}$$

$$z_1 = 0, z_2 = 0, z_3 = \frac{-Qw_t^2 w_p^2 v}{L_a(w_t + w_p)^2}, z_4 = \frac{-(N - N_p^o - q_e t_e + N_b)}{(A_u k_j - N_p - N_e)^2}, \tag{C.14}$$

Based on Cramer's Rule, and we consider the determinant of coefficient matrix $|\boldsymbol{\Phi}|$ is not equal to zero, then we can get

$$\frac{\partial Q}{\partial N_p} = \frac{|\boldsymbol{\Phi_1}|}{|\boldsymbol{\Phi}|}, \frac{\partial w_t}{\partial N_p} = \frac{|\boldsymbol{\Phi_2}|}{|\boldsymbol{\Phi}|}, \frac{\partial N_p^o}{\partial N_p} = \frac{|\boldsymbol{\Phi_3}|}{|\boldsymbol{\Phi}|}, \frac{\partial v}{\partial N_p} = \frac{|\boldsymbol{\Phi_4}|}{|\boldsymbol{\Phi}|}, \tag{C.15}$$

where $\boldsymbol{\Phi_j}$ is obtained by substituting the j-column with $\boldsymbol{Z}$ in the matrix $\boldsymbol{\Phi}$, and $|\boldsymbol{\Phi}|$ and $|\boldsymbol{\Phi_j}|$ denotes the values of the matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Phi_j}$, respectively. And we can get

$$\begin{aligned}
|\boldsymbol{\Phi}| = {} & \phi_{1,1}(\phi_{2,2}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) - \phi_{2,3}\phi_{3,2}\phi_{4,4} + \phi_{2,4}\phi_{3,2}\phi_{4,3}) \\
& - \phi_{1,2}(\phi_{2,1}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) - \phi_{2,3}\phi_{3,1}\phi_{4,4} + \phi_{2,4}\phi_{3,1}\phi_{4,3}) \\
& - \phi_{1,4}(\phi_{2,1}\phi_{3,2}\phi_{4,3} - \phi_{2,2}\phi_{3,1}\phi_{4,3}), && \text{(C.16)} \\
|\boldsymbol{\Phi_1}| = {} & z_3(\phi_{1,2}(\phi_{2,3}\phi_{4,4} - \phi_{2,4}\phi_{4,3}) + \phi_{1,4}\phi_{2,2}\phi_{4,3}) \\
& - z_4(\phi_{1,2}(\phi_{2,3}\phi_{3,4} - \phi_{2,4}\phi_{3,3}) + \phi_{1,4}(\phi_{2,2}\phi_{3,3} - \phi_{2,3}\phi_{3,2})), && \text{(C.17)} \\
|\boldsymbol{\Phi_2}| = {} & -z_3(\phi_{1,1}(\phi_{2,3}\phi_{4,4} - \phi_{2,4}\phi_{4,3}) + \phi_{1,4}\phi_{2,1}\phi_{4,3}) \\
& + z_4(\phi_{1,1}(\phi_{2,3}\phi_{3,4} - \phi_{2,4}\phi_{3,3}) + \phi_{1,4}(\phi_{2,1}\phi_{3,3} - \phi_{2,3}\phi_{3,1})), && \text{(C.18)} \\
|\boldsymbol{\Phi_3}| = {} & z_3(\phi_{1,1}\phi_{2,2}\phi_{4,4} - \phi_{1,2}\phi_{2,1}\phi_{4,4}) - z_4(\phi_{1,1}(\phi_{2,2}\phi_{3,4} - \phi_{2,4}\phi_{3,2}) \\
& - \phi_{1,2}(\phi_{2,1}\phi_{3,4} - \phi_{2,4}\phi_{3,1}) + \phi_{1,4}(\phi_{2,1}\phi_{3,2} - \phi_{2,2}\phi_{3,1})), && \text{(C.19)} \\
|\boldsymbol{\Phi_4}| = {} & -z_3(\phi_{1,1}\phi_{2,2}\phi_{4,3} - \phi_{1,2}\phi_{2,1}\phi_{4,3}) + z_4(\phi_{1,1}(\phi_{2,2}\phi_{3,3} - \phi_{2,3}\phi_{3,2}) \\
& - \phi_{1,2}(\phi_{2,1}\phi_{3,3} - \phi_{2,3}\phi_{3,1}) + \phi_{1,3}(\phi_{2,1}\phi_{3,2} - \phi_{2,2}\phi_{3,1})), && \text{(C.20)}
\end{aligned}$$

Therefore, we get:

$$\frac{\partial w_c}{\partial N_p} = \frac{1}{f'\beta}\frac{\partial Q}{\partial N_p} - \frac{\gamma}{\beta}\frac{\partial T}{\partial N_p}, \tag{C.21}$$

$$\frac{\partial q_e}{\partial N_p} = \frac{-1}{\frac{L_e}{v} + t_e}\frac{\partial N_p^o}{\partial N_p} + \frac{q_e L_e}{v(L_e + t_e v)}\frac{\partial v}{\partial N_p}, \tag{C.22}$$

$$\frac{\partial t_s}{\partial N_p} = \frac{-t_s^2 v t_e}{L_a(\frac{L_e}{v} + t_e)}\frac{\partial N_p^o}{\partial N_p} + \left(\frac{t_s^2 t_e q_e L_e}{L_a(L_e + t_e v)} - \frac{t_s}{v}\right)\frac{\partial v}{\partial N_p}, \tag{C.23}$$

$$\begin{aligned}
\frac{\partial(q_e t_e + q_e t_s)}{\partial N_p} = {} & \frac{-1}{\frac{L_e}{v} + t_e}\left(t_e + t_s + \frac{q_e t_s^2 v t_e}{L_a}\right)\frac{\partial N_p^o}{\partial N_p} \\
& + \left(\frac{q_e L_e(t_e + t_s)}{v(L_e + t_e v)} + \frac{q_e^2 t_s^2 L_e t_e}{L_a(L_e + t_e v)} - \frac{q_e t_s}{v}\right)\frac{\partial v}{\partial N_p},
\end{aligned} \tag{C.24}$$

$$\frac{\partial w_p}{\partial N_p} = \frac{w_p^2 v}{L_a}\frac{\partial N_p^o}{\partial N_p} - \frac{w_p^2(N_p - N_p^o)}{L_a}\frac{\partial v}{\partial N_p} - \frac{w_p^2 v}{L_a}, \tag{C.25}$$

$$\frac{\partial T}{\partial N_p} = -\frac{L_r}{v^2}\frac{\partial v}{\partial N_p}. \tag{C.26}$$

Comparative statics with respect to N_e. Similarly, we take partial derivatives with respect to the number of charging piles N_e on both sizes of the system of Eqs. (C.1)-(C.4), we get each element of matrix $\boldsymbol{\Phi}$. The formulations of each element is the same as those in Eqs. (C.10)-(C.13), while the constant terms are:

$$z_1 = 0, z_2 = \frac{q_e t_s^2 v}{L_a}, z_3 = 0, z_4 = \frac{-(N - N_p^o - q_e t_e + N_b)}{(A_u k_j - N_p - N_e)^2}, \tag{C.27}$$

Therefore, the determinant of matrix $|\boldsymbol{\Phi}|$ is the same as that in Eq. (C.16). The determinants $|\boldsymbol{\Phi_j}|$ are:

$$\begin{aligned}
|\boldsymbol{\Phi_1}| &= -z_2(\phi_{1,2}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) + \phi_{1,4}\phi_{3,2}\phi_{4,3}) \\
&\quad - z_4(\phi_{1,2}(\phi_{2,3}\phi_{3,4} - \phi_{2,4}\phi_{3,3}) + \phi_{1,4}(\phi_{2,2}\phi_{3,3} - \phi_{2,3}\phi_{3,2})), && \text{(C.28)} \\
|\boldsymbol{\Phi_2}| &= z_2(\phi_{1,1}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) + \phi_{1,4}\phi_{3,1}\phi_{4,3}) \\
&\quad + z_4(\phi_{1,1}(\phi_{2,3}\phi_{3,4} - \phi_{2,4}\phi_{3,3}) + \phi_{1,4}(\phi_{2,1}\phi_{3,3} - \phi_{2,3}\phi_{3,1})), && \text{(C.29)} \\
|\boldsymbol{\Phi_3}| &= -z_2(\phi_{1,1}\phi_{3,2}\phi_{4,4} - \phi_{1,2}\phi_{3,1}\phi_{4,4}) - z_4(\phi_{1,1}(\phi_{2,2}\phi_{3,4} - \phi_{2,4}\phi_{3,2}) \\
&\quad - \phi_{1,2}(\phi_{2,1}\phi_{3,4} - \phi_{2,4}\phi_{3,1}) + \phi_{1,4}(\phi_{2,1}\phi_{3,2} - \phi_{2,2}\phi_{3,1})), && \text{(C.30)} \\
|\boldsymbol{\Phi_4}| &= z_2(\phi_{1,1}\phi_{3,2}\phi_{4,3} - \phi_{1,2}\phi_{3,1}\phi_{4,3}) + z_4(\phi_{1,1}(\phi_{2,2}\phi_{3,3} - \phi_{2,3}\phi_{3,2}) \\
&\quad - \phi_{1,2}(\phi_{2,1}\phi_{3,3} - \phi_{2,3}\phi_{3,1}) + \phi_{1,3}(\phi_{2,1}\phi_{3,2} - \phi_{2,2}\phi_{3,1})), && \text{(C.31)}
\end{aligned}$$

Based on Cramer's Rule, and we consider the determinant of coefficient matrix $|\boldsymbol{\Phi}|$ is not equal to zero, then we can get $\frac{\partial Q}{\partial N_e}$, $\frac{\partial w_t}{\partial N_e}$, $\frac{\partial N_p^o}{\partial N_e}$ and $\frac{\partial v}{\partial N_e}$ following Eq. (C.15).

We then have

$$\begin{aligned}
\frac{\partial w_c}{\partial N_e} &= \frac{1}{f'\beta}\frac{\partial Q}{\partial N_e} - \frac{\gamma}{\beta}\frac{\partial T}{\partial N_e}, && \text{(C.32)} \\
\frac{\partial q_e}{\partial N_e} &= \frac{-1}{\frac{L_e}{v} + t_e}\frac{\partial N_p^o}{\partial N_e} + \frac{q_e L_e}{v(L_e + t_e v)}\frac{\partial v}{\partial N_e}, && \text{(C.33)} \\
\frac{\partial t_s}{\partial N_e} &= \frac{-t_s^2 v t_e}{L_a(\frac{L_e}{v} + t_e)}\frac{\partial N_p^o}{\partial N_e} + \left(\frac{t_s^2 t_e q_e L_e}{L_a(L_e + t_e v)} - \frac{t_s}{v}\right)\frac{\partial v}{\partial N_e} - \frac{t_s^2 v}{L_a}, && \text{(C.34)} \\
\frac{\partial(q_e t_e + q_e t_s)}{\partial N_e} &= \frac{-1}{\frac{L_e}{v} + t_e}\left(t_e + t_s + \frac{q_e t_s^2 v t_e}{L_a}\right)\frac{\partial N_p^o}{\partial N_e} \\
+ \left(\frac{q_e L_e(t_e + t_s)}{v(L_e + t_e v)} \right. &+ \left. \frac{q_e^2 t_s^2 L_e t_e}{L_a(L_e + t_e v)} - \frac{q_e t_s}{v}\right)\frac{\partial v}{\partial N_e} - \frac{q_e t_s^2 v}{L_a}, && \text{(C.35)} \\
\frac{\partial w_p}{\partial N_e} &= \frac{w_p^2 v}{L_a}\frac{\partial N_p^o}{\partial N_e} - \frac{w_p^2(N_p - N_p^o)}{L_a}\frac{\partial v}{\partial N_e}, && \text{(C.36)} \\
\frac{\partial T}{\partial N_e} &= -\frac{L_r}{v^2}\frac{\partial v}{\partial N_e}. && \text{(C.37)}
\end{aligned}$$

Comparative statics with respect to F. Similarly, we take partial derivatives with respect to trip fare F on both sizes of the system of Eqs. (C.1)-(C.4), we get each element of matrix $\boldsymbol{\Phi}$. The formulations of each element is same as that in Eqs. (C.10)-(C.13), while the constant terms are:

$$z_1 = -f', z_2 = 0, z_3 = 0, z_4 = 0, \tag{C.38}$$

Therefore, the determinant of matrix $|\boldsymbol{\Phi}|$ is same as in Eq. (C.16). But the determinants $|\boldsymbol{\Phi_j}|$ are:

$$|\boldsymbol{\Phi_1}| = z_1(\phi_{2,2}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) - \phi_{2,3}\phi_{3,2}\phi_{4,4} + \phi_{2,4}\phi_{3,2}\phi_{4,3}), \tag{C.39}$$

$$|\boldsymbol{\Phi_2}| = -z_1(\phi_{2,1}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) - \phi_{2,3}\phi_{3,1}\phi_{4,4} + \phi_{2,4}\phi_{3,1}\phi_{4,3}), \tag{C.40}$$

$$|\boldsymbol{\Phi_3}| = z_1(\phi_{2,1}\phi_{3,2}\phi_{4,4} - \phi_{2,2}\phi_{3,1}\phi_{4,4}), \tag{C.41}$$

$$|\boldsymbol{\Phi_4}| = -z_1(\phi_{2,1}\phi_{3,2}\phi_{4,3} - \phi_{2,2}\phi_{3,1}\phi_{4,3}), \tag{C.42}$$

Based on Cramer's Rule, and we consider the determinant of coefficient matrix $|\boldsymbol{\Phi}|$ is not equal to zero, then we can get $\frac{\partial Q}{\partial F}$, $\frac{\partial w_t}{\partial F}$, $\frac{\partial N_p^o}{\partial F}$ and $\frac{\partial v}{\partial F}$ following Eq. (C.15).

And we get

$$\frac{\partial w_c}{\partial F} = \frac{1}{f'\beta}\frac{\partial Q}{\partial F} - \frac{1}{\beta} - \frac{\gamma}{\beta}\frac{\partial T}{\partial F}, \tag{C.43}$$

$$\frac{\partial q_e}{\partial F} = \frac{-1}{\frac{L_e}{v} + t_e}\frac{\partial N_p^o}{\partial F} + \frac{q_e L_e}{v(L_e + t_e v)}\frac{\partial v}{\partial F}, \tag{C.44}$$

$$\frac{\partial t_s}{\partial F} = \frac{-t_s^2 v t_e}{L_a(\frac{L_e}{v} + t_e)}\frac{\partial N_p^o}{\partial F} + \left(\frac{t_s^2 t_e q_e L_e}{L_a(L_e + t_e v)} - \frac{t_s}{v}\right)\frac{\partial v}{\partial F}, \tag{C.45}$$

$$\begin{aligned}\frac{\partial(q_e t_e + q_e t_s)}{\partial F} &= \frac{-1}{\frac{L_e}{v} + t_e}\left(t_e + t_s + \frac{q_e t_s^2 v t_e}{L_a}\right)\frac{\partial N_p^o}{\partial F} \\ &+ \left(\frac{q_e L_e(t_e + t_s)}{v(L_e + t_e v)} + \frac{q_e^2 t_s^2 L_e t_e}{L_a(L_e + t_e v)} - \frac{q_e t_s}{v}\right)\frac{\partial v}{\partial F},\end{aligned} \tag{C.46}$$

$$\frac{\partial w_p}{\partial F} = \frac{w_p^2 v}{L_a}\frac{\partial N_p^o}{\partial F} - \frac{w_p^2(N_p - N_p^o)}{L_a}\frac{\partial v}{\partial F}, \tag{C.47}$$

$$\frac{\partial T}{\partial F} = -\frac{L_r}{v^2}\frac{\partial v}{\partial F}. \tag{C.48}$$

Comparative statics with respect to N. Similarly, we take partial derivatives with respect to the number of ERVs N on both sizes of the system of Eqs. (C.1)-(C.4), we get each element of matrix $\boldsymbol{\Phi}$. The formulations of each element is the same as those in Eqs. (C.10)-(C.13), while the constant terms are:

$$z_1 = 0, z_2 = 1 - \frac{t_e + t_s}{\frac{L_e}{v} + t_e} - \frac{q_e t_s^2 v t_e}{L_a\left(\frac{L_e}{v} + t_e\right)}, z_3 = 0, z_4 = \frac{-L_e}{(L_e + t_e v)(A_u k_j - N_p - N_e)}, \tag{C.49}$$

Therefore, the determinant of matrix $|\boldsymbol{\Phi}|$ is same as in Eq. (C.16). The determinants $|\boldsymbol{\Phi_j}|$ are:

$$|\boldsymbol{\Phi_1}| = -z_2(\phi_{1,2}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) + \phi_{1,4}\phi_{3,2}\phi_{4,3})$$

$$- z_4(\phi_{1,2}(\phi_{2,3}\phi_{3,4} - \phi_{2,4}\phi_{3,3}) + \phi_{1,4}(\phi_{2,2}\phi_{3,3} - \phi_{2,3}\phi_{3,2})), \tag{C.50}$$

$$|\boldsymbol{\Phi_2}| = z_2(\phi_{1,1}(\phi_{3,3}\phi_{4,4} - \phi_{3,4}\phi_{4,3}) + \phi_{1,4}\phi_{3,1}\phi_{4,3}) + z_4(\phi_{1,1}(\phi_{2,3}\phi_{3,4} - \phi_{2,4}\phi_{3,3}) + \phi_{1,4}(\phi_{2,1}\phi_{3,3} - \phi_{2,3}\phi_{3,1})), \tag{C.51}$$

$$|\boldsymbol{\Phi_3}| = -z_2(\phi_{1,1}\phi_{3,2}\phi_{4,4} - \phi_{1,2}\phi_{3,1}\phi_{4,4}) - z_4(\phi_{1,1}(\phi_{2,2}\phi_{3,4} - \phi_{2,4}\phi_{3,2}) - \phi_{1,2}(\phi_{2,1}\phi_{3,4} - \phi_{2,4}\phi_{3,1}) + \phi_{1,4}(\phi_{2,1}\phi_{3,2} - \phi_{2,2}\phi_{3,1})), \tag{C.52}$$

$$|\boldsymbol{\Phi_4}| = z_2(\phi_{1,1}\phi_{3,2}\phi_{4,3} - \phi_{1,2}\phi_{3,1}\phi_{4,3}) + z_4(\phi_{1,1}(\phi_{2,2}\phi_{3,3} - \phi_{2,3}\phi_{3,2}) - \phi_{1,2}(\phi_{2,1}\phi_{3,3} - \phi_{2,3}\phi_{3,1}) + \phi_{1,3}(\phi_{2,1}\phi_{3,2} - \phi_{2,2}\phi_{3,1})), \tag{C.53}$$

Based on Cramer's Rule, and we consider the determinant of coefficient matrix $|\boldsymbol{\Phi}|$ is not equal to zero, then we can get $\frac{\partial Q}{\partial N}$, $\frac{\partial w_t}{\partial N}$, $\frac{\partial N_p^o}{\partial N}$ and $\frac{\partial v}{\partial N}$ following Eq. (C.15).

And we get

$$\frac{\partial w_c}{\partial N} = \frac{1}{f'\beta}\frac{\partial Q}{\partial N} - \frac{\gamma}{\beta}\frac{\partial T}{\partial N}, \tag{C.54}$$

$$\frac{\partial q_e}{\partial N} = \frac{-1}{\frac{L_e}{v} + t_e}\frac{\partial N_p^o}{\partial N} + \frac{q_e L_e}{v(L_e + t_e v)}\frac{\partial v}{\partial N} + \frac{1}{\frac{L_e}{v} + t_e}, \tag{C.55}$$

$$\frac{\partial t_s}{\partial N} = \frac{-t_s^2 v t_e}{L_a(\frac{L_e}{v} + t_e)}\frac{\partial N_p^o}{\partial N} + \left(\frac{t_s^2 t_e q_e L_e}{L_a(L_e + t_e v)} - \frac{t_s}{v}\right)\frac{\partial v}{\partial N} + \frac{t_s^2 v t_e}{L_a(\frac{L_e}{v} + t_e)}, \tag{C.56}$$

$$\frac{\partial(q_e t_e + q_e t_s)}{\partial N} = \frac{-1}{\frac{L_e}{v} + t_e}\left(t_e + t_s + \frac{q_e t_s^2 v t_e}{L_a}\right)\frac{\partial N_p^o}{\partial N} + \left(\frac{q_e L_e (t_e + t_s)}{v(L_e + t_e v)} + \frac{q_e^2 t_s^2 L_e t_e}{L_a(L_e + t_e v)} - \frac{q_e t_s}{v}\right)\frac{\partial v}{\partial N} + \frac{t_e + t_s}{\frac{L_e}{v} + t_e} + \frac{q_e t_s^2 v t_e}{L_a\left(\frac{L_e}{v} + t_e\right)}, \tag{C.57}$$

$$\frac{\partial w_p}{\partial N} = \frac{w_p^2 v}{L_a}\frac{\partial N_p^o}{\partial N} - \frac{w_p^2(N_p - N_p^o)}{L_a}\frac{\partial v}{\partial N}, \tag{C.58}$$

$$\frac{\partial T}{\partial N} = -\frac{L_r}{v^2}\frac{\partial v}{\partial N}. \tag{C.59}$$

Appendix D. Comparative statics for the special case with no parking provision ($N_p = 0$) and a constant speed

To conduct comparative statics for the transportation system with cruising for charging, based on Eqs. (30a)-(30c), (30f) with $N_p^o = 0$, (30g) and (49), we take partial derivatives with respect to fare F, the number of charging piles N_e, the maximum driving range of ERVs with fully charged L_e, and average charging time t_e, respectively.

Comparative statics with respect to fare F.

$$\frac{\partial N_e^o}{\partial F} = 0, \ \frac{\partial N_e^s}{\partial F} = 0, \ \frac{\partial N_u}{\partial F} = 0, \tag{D.1}$$

$$\frac{\partial Q}{\partial F} = \frac{f'}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r + w_t + T}{Q}}, \tag{D.2}$$

$$\frac{\partial w_c}{\partial F} = \frac{f'W'_{c,Q} - f'W'_{c,w_t}\frac{w_r+w_t+T}{Q}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.3}$$

$$\frac{\partial w_t}{\partial F} = \frac{-f'\frac{w_r+w_t+T}{Q}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.4}$$

$$\frac{\partial N_{vt}}{\partial F} = \frac{-f'(w_r+T)}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.5}$$

where $W'_{c,Q} = \frac{\partial w_c}{\partial Q} = \frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q}$, and $W'_{c,w_t} = \frac{\partial w_c}{\partial w_t} = -\frac{\alpha_1}{\alpha_2}\frac{w_c}{w_t}$. As $f' < 0$ and $1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q} > 0$, we can readily have $\frac{\partial Q}{\partial F} < 0$, $\frac{\partial w_t}{\partial F} > 0$ and $\frac{\partial N_{vt}}{\partial F} > 0$. As $f'W'_{c,Q} - f'W'_{c,w_t}\frac{w_r+w_t+T}{Q} = f'\frac{1-\alpha_2}{\alpha_2}\frac{w_c}{Q} + f'\frac{\alpha_1}{\alpha_2}\frac{w_c}{Q}\frac{w_r+T}{w_t} < 0$, we have $\frac{\partial w_c}{\partial F} < 0$.

Comparative statics with respect to N_e.

$$\frac{\partial N_e^o}{\partial N_e} = 0, \frac{\partial N_e^s}{\partial N_e} = \frac{-L_a N}{(L_e + t_e v)\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2}, \tag{D.6}$$

$$\frac{\partial N_u}{\partial N_e} = \frac{L_a N}{(L_e + t_e v)\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2}, \tag{D.7}$$

$$\frac{\partial Q}{\partial N_e} = \frac{f'\beta W'_{c,w_t}\frac{1}{Q}\frac{\partial N_u}{\partial N_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.8}$$

$$\frac{\partial w_c}{\partial N_e} = \frac{W'_{c,w_t}\frac{1}{Q}\frac{\partial N_u}{\partial N_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.9}$$

$$\frac{\partial w_t}{\partial N_e} = \frac{\frac{1}{Q}\left(1 - f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\right)\frac{\partial N_u}{\partial N_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.10}$$

$$\frac{\partial N_{vt}}{\partial N_e} = \frac{\left(1 - f'\beta\frac{1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\right)\frac{\partial N_u}{\partial N_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.11}$$

We can readily have $\frac{\partial N_e^s}{\partial N_e} < 0$, $\frac{\partial N_u}{\partial N_e} > 0$, $\frac{\partial Q}{\partial N_e} > 0$, $\frac{\partial w_c}{\partial N_e} < 0$, and $\frac{\partial N_{vt}}{\partial N_e} > 0$. In particular, when $(\alpha_1 + \alpha_2) \leqslant 1$, we have $\frac{\partial w_t}{\partial N_e} > 0$; when $(\alpha_1 + \alpha_2) > 1$, if $f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q} \leqslant 1$, we have $\frac{\partial w_t}{\partial N_e} \geqslant 0$, or else, $\frac{\partial w_t}{\partial N_e} < 0$.

Comparative statics with respect to L_e.

$$\frac{\partial N_e^o}{\partial L_e} = -\frac{Nt_e}{v\left(\frac{L_e}{v}+t_e\right)^2}, \tag{D.12}$$

$$\frac{\partial N_e^s}{\partial L_e} = -\frac{N^2 t_e L_a}{v^2\left(\frac{L_e}{v}+t_e\right)^3\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2} - \frac{NL_a}{v^2\left(\frac{L_e}{v}+t_e\right)^2\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)}, \tag{D.13}$$

$$\frac{\partial N_u}{\partial L_e} = \frac{Nt_e}{v\left(\frac{L_e}{v}+t_e\right)^2} + \frac{N^2 t_e L_a}{v^2\left(\frac{L_e}{v}+t_e\right)^3\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2} + \frac{NL_a}{v^2\left(\frac{L_e}{v}+t_e\right)^2\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)}, \tag{D.14}$$

$$\frac{\partial Q}{\partial L_e} = \frac{f'\beta W'_{c,w_t}\frac{1}{Q}\frac{\partial N_u}{\partial L_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.15}$$

$$\frac{\partial w_c}{\partial L_e} = \frac{W'_{c,w_t}\frac{1}{Q}\frac{\partial N_u}{\partial L_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.16}$$

$$\frac{\partial w_t}{\partial L_e} = \frac{\frac{1}{Q}\left(1 - f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\right)\frac{\partial N_u}{\partial L_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.17}$$

$$\frac{\partial N_{vt}}{\partial L_e} = \frac{\left(1 - f'\beta\frac{1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\right)\frac{\partial N_u}{\partial L_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.18}$$

We can readily have $\frac{\partial N_e^o}{\partial L_e} < 0$, $\frac{\partial N_e^s}{\partial L_e} < 0$, $\frac{\partial N_u}{\partial L_e} > 0$, $\frac{\partial Q}{\partial L_e} > 0$, $\frac{\partial w_c}{\partial L_e} < 0$, and $\frac{\partial N_{vt}}{\partial L_e} > 0$. When $(\alpha_1+\alpha_2) \leqslant 1$, we have $\frac{\partial w_t}{\partial L_e} > 0$; when $(\alpha_1+\alpha_2) > 1$, if $f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q} \leqslant 1$, we have $\frac{\partial w_t}{\partial L_e} \geqslant 0$, or else, $\frac{\partial w_t}{\partial L_e} < 0$.

Comparative statics with respect to t_e.

$$\frac{\partial N_e^o}{\partial t_e} = \frac{NL_e}{v\left(\frac{L_e}{v}+t_e\right)^2}, \tag{D.19}$$

$$\frac{\partial N_e^s}{\partial t_e} = \frac{L_a N}{v\left(\frac{L_e}{v}+t_e\right)^2\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)}\left(-1 + \frac{N - \frac{N}{\frac{L_e}{v}+te}t_e}{N_e - \frac{N}{\frac{L_e}{v}+te}t_e}\right), \tag{D.20}$$

$$\frac{\partial N_u}{\partial t_e} = \frac{-NL_e}{v\left(\frac{L_e}{v}+t_e\right)^2} + \frac{L_a}{v\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)}\frac{N}{\left(\frac{L_e}{v}+t_e\right)^2} - \frac{N^2 L_e L_a}{v^2\left(\frac{L_e}{v}+t_e\right)^3\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2}, \tag{D.21}$$

$$\frac{\partial Q}{\partial t_e} = \frac{f'\beta W'_{c,w_t}\frac{1}{Q}\frac{\partial N_u}{\partial t_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.22}$$

$$\frac{\partial w_c}{\partial t_e} = \frac{W'_{c,w_t}\frac{1}{Q}\frac{\partial N_u}{\partial t_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.23}$$

$$\frac{\partial w_t}{\partial t_e} = \frac{\frac{1}{Q}\left(1 - f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\right)\frac{\partial N_u}{\partial t_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.24}$$

$$\frac{\partial N_{vt}}{\partial t_e} = \frac{\left(1 - f'\beta\frac{1-\alpha_2}{\alpha_2}\frac{w_c}{Q}\right)\frac{\partial N_u}{\partial t_e}}{1 - f'\beta W'_{c,Q} + f'\beta W'_{c,w_t}\frac{w_r+w_t+T}{Q}}, \tag{D.25}$$

We readily have $\frac{\partial N_e^o}{\partial t_e} > 0$. If $N > N_e$, we have $\frac{\partial N_e^s}{\partial t_e} > 0$, else if $N \leqslant N_e$, we have $\frac{\partial N_e^s}{\partial t_e} \leqslant 0$.

Considering $q_e(t_e+t_s) < N$, we get $t_s < \frac{N}{q_e} - t_e$, and $q_e = \frac{N}{\frac{L_e}{v}+te}$, so $t_s < \frac{L_e}{v}$. We have $\frac{\partial N_e^o}{\partial t_e} = \frac{\partial(q_e t_e)}{\partial t_e}$, $\frac{\partial N_e^s}{\partial t_e} = \frac{\partial(q_e t_s)}{\partial t_e} = t_s\frac{\partial q_e}{\partial t_e} + q_e\frac{\partial t_s}{\partial t_e}$, and $\frac{\partial(q_e t_e + q_e t_s)}{\partial t_e} = \frac{\partial(q_e t_e)}{\partial t_e} + t_s\frac{\partial q_e}{\partial t_e} + q_e\frac{\partial t_s}{\partial t_e}$. We have $\frac{\partial(q_e t_e)}{\partial t_e} + t_s\frac{\partial q_e}{\partial t_e} = \frac{NL_e}{v\left(\frac{L_e}{v}+t_e\right)^2} - t_s\frac{N}{\left(\frac{L_e}{v}+t_e\right)^2} > 0$ as $t_s < \frac{L_e}{v}$, and $q_e\frac{\partial t_s}{\partial t_e} = \frac{N^2 L_e L_a}{v^2\left(\frac{L_e}{v}+t_e\right)^3\left(N_e - \frac{Nt_e}{\frac{L_e}{v}+t_e}\right)^2} > 0$. Therefore, it can be readily verified that $\frac{\partial(q_e t_e + q_e t_s)}{\partial t_e} > 0$ and thus $\frac{\partial N_u}{\partial t_e} < 0$ as $N_u = N - (q_e t_e + q_e t_s)$ and $\frac{\partial N_u}{\partial t_e} = -\frac{\partial(q_e t_e + q_e t_s)}{\partial t_e}$.

We have $\frac{\partial Q}{\partial t_e} < 0$, $\frac{\partial w_c}{\partial t_e} > 0$, and $\frac{\partial N_{vt}}{\partial t_e} < 0$. When $(\alpha_1+\alpha_2) \leqslant 1$, we have $\frac{\partial w_t}{\partial t_e} < 0$; when $(\alpha_1+\alpha_2) > 1$, if $f'\beta\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w_c}{Q} \leqslant 1$, we have $\frac{\partial w_t}{\partial t_e} \leqslant 0$, or else, $\frac{\partial w_t}{\partial t_e} > 0$.

Appendix E. The impacts of parking price on the travelers' choices of cruising or parking

This section investigates the impacts of parking pricing on the travelers' choices of cruising or parking while drivers are waiting for the matching.

When the parking price p_a is charged (for parking once), the expected cost of drivers choosing to park while waiting for the matching, denoted as C_a, can be given as follows:

$$C_a = \frac{w_p}{w_t + w_p}\frac{C_e w_t v}{L_e} + \frac{w_t}{w_t + w_p} p_a, \tag{E.1}$$

where C_e is the charging price (for fully recharging the vehicle), w_p is the expected searching time for a vacant parking space, w_t is the expected ERV (or driver) waiting time to be matched with a customer by an online platform, v is the network average speed for car traffic, and L_e is the maximum driving range of an ERV fully charged. And $w_t v$ indicates the average cruising distance when the driver cruises and waits for the successful matching with the customer. Following Eq. (13), the probability of an unmatched ERV finding a parking space before being matched is $\frac{w_t}{w_t+w_p}$ and the probability of an unmatched ERV being matched before finding a parking space is $\frac{w_p}{w_t+w_p}$ accordingly. The first term on the right-hand side of Eq. (E.1) is the expected cost of drivers who have chosen to park and being matched before finding a parking space, and the second term is the expected cost of drivers who have chosen to park and finding a parking space before being matched.

In contrast, the expected cost of drivers not considering parking while waiting for the matching, denoted as C_b, is given as follows:

$$C_b = \frac{C_e w_t v}{L_e}. \tag{E.2}$$

If $C_a > C_b$, the driver will prefer to cruising on the road; if $C_a < C_b$, the driver will prefer to looking for parking; and at an interior equilibrium we should have $C_a = C_b$, i.e., the two options yield the same expected cost. The parking price by affecting the above choice equilibrium will impact the system equilibrium.

Appendix F. Proof for Proposition 3

The change of the profit of the ride-sourcing operator with respect to the number of charging piles is given as follows:

$$\begin{aligned}\frac{d\Pi}{dN_e} &= F\frac{dQ}{dN_e} + Q\frac{dF}{dN_e} - C\frac{dN}{dN_e} - C_e\frac{dq_e}{dN_e}\\ &= F\left(\frac{\partial Q}{\partial N_e} + \frac{\partial Q}{\partial N}\frac{dN}{dN_e} + \frac{\partial Q}{\partial F}\frac{dF}{dN_e}\right) + Q\frac{dF}{dN_e} - C\frac{dN}{dN_e} - C_e\left(\frac{\partial q_e}{\partial N_e} + \frac{\partial q_e}{\partial N}\frac{dN}{dN_e}\right)\end{aligned} \tag{F.1}$$

Based on Eqs. (59)-(60), we have

$$Q = -F\frac{\partial Q}{\partial F}; \tag{F.2}$$

$$C = F\frac{\partial Q}{\partial N} - C_e\frac{\partial q_e}{\partial N}. \tag{F.3}$$

We get

$$\frac{d\Pi}{dN_e} = F\left(\frac{\partial Q}{\partial N_e} + \frac{\partial Q}{\partial N}\frac{dN}{dN_e} + \frac{\partial Q}{\partial F}\frac{dF}{dN_e}\right) - F\frac{\partial Q}{\partial F}\frac{dF}{dN_e} - \left(F\frac{\partial Q}{\partial N} - C_e\frac{\partial q_e}{\partial N}\right)\frac{dN}{dN_e}$$
$$- C_e\frac{\partial q_e}{\partial N}\frac{dN}{dN_e} = F\frac{\partial Q}{\partial N_e} \tag{F.4}$$

Based on Eq. (57c), we have $\frac{\partial Q}{\partial N_e} > 0$. And we consider the interior optimal solution with $F > 0$ at profit maximization in Eq. (58). We thus have $\frac{d\Pi}{dN_e} > 0$. This completes the proof.

Appendix G. Proof for Proposition 4

The change of the social welfare with respect to the number of charging piles is given as follows:

$$\frac{dS}{dN_e} = F\frac{dQ}{dN_e} - Q\beta\frac{dw_c}{dN_e} - C\frac{dN}{dN_e} - C_e\frac{dq_e}{dN_e} - C_h$$
$$= F\left(\frac{\partial Q}{\partial N_e} + \frac{\partial Q}{\partial N}\frac{dN}{dN_e} + \frac{\partial Q}{\partial F}\frac{dF}{dN_e}\right) - \beta Q\left(\frac{\partial w_c}{\partial N_e} + \frac{\partial w_c}{\partial N}\frac{dN}{dN_e} + \frac{\partial w_c}{\partial F}\frac{dF}{dN_e}\right)$$
$$- C\frac{dN}{dN_e} - C_e\frac{\partial q_e}{\partial N}\frac{dN}{dN_e} - C_h \tag{G.1}$$

Based on Eqs. (68)-(69), we have

$$F\frac{\partial Q}{\partial F} = \beta Q\frac{\partial w_c}{\partial F}, \tag{G.2}$$

$$C = F\frac{\partial Q}{\partial N} - \beta Q\frac{\partial w_c}{\partial N} - C_e\frac{\partial q_e}{\partial N}. \tag{G.3}$$

And then we can derive that

$$\frac{dS}{dN_e} = F\frac{\partial Q}{\partial N_e} - \beta Q\frac{\partial w_c}{\partial N_e} - C_h. \tag{G.4}$$

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