

# On the magnetic nanoparticle injection strategy for hyperthermia treatment

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## Abstract

We developed a dedicated computational framework by coupling the lattice-Boltzmann-method (LBM) modeling and the particle-swarm-optimization (PSO) algorithm to search optimal strategies of magnetic nanoparticle (MNP) injection for hyperthermia-based cancer treatment. Two simplified tumor models were considered: a circular model representing geometrically regular tumors and an elliptic model representing geometrically irregular tumors, both sharing the same area. The temperature distribution in the tumor and its surrounding healthy tissue was predicted by solving the Pennes' bio-heat transfer equation (PBHTE). Both single- and multi-site injection strategies were explored. The results suggest that the multi-site injection strategies generally work well, while the single-site injection strategy fails even on the simplest circular tumor model. The more the injection sites, the better the performance. In particular, when the number of injection sites reaches eight, all temperature requirements can be nearly 100% satisfied in both tumor models. Whether or not including the minimum dose requirement in the objective function only affects the optimization results by less than 2%. The thermal dose was also assessed by considering both temperature and heat exposure time. It was found that the optimal multi-site injection strategies perform reasonably well for both tumor models. Although the setting is only two dimensional and the optimization is on very simplified tumor models, the framework adopted in this present study

works well and can provide useful insights into magnetic hyperthermia treatment.

*Keywords:* Magnetic hyperthermia treatment, Magnetic nanoparticle, Lattice Boltzmann method, Particle swarm optimization

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## Nomenclature

### Parameters

$(x, y)$  Coordinates of injection site, m

$\alpha$  Reference thermal diffusivity, m<sup>2</sup>/s

$\beta$  Inclination angle, °

$\beta_0$  Reference inclination angle, °

$\chi_0$  Equilibrium susceptibility

$\dot{m}$  density flux, kg/m<sup>3</sup>-s

$\Psi$  Source term vector in LBM

$\mathbf{g}$  Five-dimensional distribution function vector in LBM

$\omega_1 \sim \omega_4$  Weighting factors of each basic requirement in objective function

$\Phi$  Volume fraction with time variant

$\phi$  Volume fraction without time variant

$\phi_0$  Reference volume fraction

$\rho$  Density, kg/m<sup>3</sup>

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$\sigma$	Standard deviation, m
$\sigma_0$	Reference standard deviation, m
$\tau_B$	Brownian relaxation time, s
$\tau_N$	Neel relaxation time, s
$\tau_R$	Effective relaxation time, s
$c$	Specific heat, J/(kg-K)
$C_{EM}$	Coefficient, a function of temperature
$CEM43$	Cumulative equivalent time at 43°C, min
$D$	Mass diffusivity, m <sup>2</sup> /s
$d$	distance of injection site, m
$d_0$	Reference distance
$d_{tum}$	Diameter of circular tumor, m
$Err$	Error percentage, %
$f(x)$	Test function for PSO
$J$	Objection function
$k$	Thermal conductivity, W/m-K
$k_\beta$	Inclination angle ratio
$k_\phi$	Volume fraction ratio
$k_\sigma$	Standard deviation ratio

$k_d$  Distance ratio of injection site

$L$  Length of healthy tissue, m

$l$  Number of time intervals

$N_x \times N_y$  Grid number in LBM

$Q$  Heat induced by MNPs, W/m<sup>3</sup>

$R_{CEM43}$  Percentage ratio of area for  $CEM43 \geq 60 \text{ min}$

$R_{int}$  Temperature requirement at interface between healthy tissue and tumor tissue

$R_{tis}$  Temperature requirement in healthy tissue

$R_{tum}$  Temperature requirement in tumor tissue

$S$  Area, m<sup>2</sup>

$T$  Temperature, °C

$t$  Time, min

$V$  Volume of injected MNP solution, m<sup>3</sup>

$V_H$  Hydrodynamic volume of single MNP, m<sup>3</sup>

$V_M$  Magnetic volume of single MNP, m<sup>3</sup>

### **Subscripts**

$b$  Blood

$c$  Tumor center

$i$  Injection site number



<i>mnp</i>	Magnetic nanoparticles
<i>nf</i>	Nanofluid, the mixture of tissue and injected MNPs
<i>opt</i>	Optimal value
<i>r</i>	The rightmost point on tumor boundary
<i>tis</i>	Healthy tissue
<i>tum</i>	Tumor tissue

## 1. Introduction

Hyperthermia treatment, or thermotherapy, is a cancer therapeutic procedure in which tumor tissues are locally heated to approximately above 43°C, thereby to ablate malignant cells without overheating the surrounding healthy tissue [1–3]. The damage on cells is irreversible since high temperature will cause the protein denaturation [4]. As one of hyperthermia modalities, magnetic hyperthermia treatment utilizes magnetic nanoparticles (MNPs), which are injected into tumor tissues and subjected to an alternating magnetic field (AMF), to generate the required heat at target regions [5–7]. Such a heat generation mechanism offers magnetic hyperthermia treatment several advantages over conventional approaches, such as more targeted and homogeneous heating, harmless to human body, and more effective treatment [8–10]. Although having been demonstrated to be effective, magnetic hyperthermia treatment is still facing a great challenge, i.e., how to impose fatal thermal damage to cancerous tissues, while minimizing the damage to surrounding healthy tissues? To tackle this challenge, optimal MNP concentrations and spatial distributions must be identified, which requires detailed heat and mass transfer analysis.

For the MNPs used in magnetic hyperthermia treatment, magnetite ( $\text{Fe}_3\text{O}_4$ ) has been chosen in many studies as an ideal candidate, since such iron-oxide nanoparticles bear favorable magnetic properties and low toxicity [11–13]. Nevertheless, iron-based nanoparticles are still genotoxic to

healthy skin and lung tissue [14]. According to Karponis et al. [11], iron intake in human body should be moderate, and overdose may shorten life expectancy.

Regarding the distribution of MNPs, Liangruksa et al. [15] numerically compared the performance of uniform, exponential and Gaussian distributions, and concluded that the Gaussian distribution can provide the best efficacy while minimizing the possibility of generating hot spots in the tumor. Salloum et al. [16] also confirmed that the Gaussian distribution is a good fit with the observation from MNP delivery experiments, which agrees with the analytical diffusion prediction. It was believed that, since the MNP injection rate and the blood infusion rate are slow enough, achieving the Gaussian distribution for MNPs in real situations is feasible [16, 17].

In practice, it is challenging to manually fabricate the MNP distribution for expected heating results, which is especially true for tumors with irregular shapes. Golneshan and Lahonian [18] simulated the MNP diffusion in a tumor with complex geometry, and pointed out that with the same diffusion time multi-site injection is able to create more uniform distribution and covers a larger area than single-site injection. Other studies also demonstrated that multi-site injection is a good strategy that can help enhance the effectiveness of magnetic hyperthermia treatment [19, 20].

Note that, in the above multi-site injection studies, the injection sites were arbitrarily selected. To explore the best injection sites, using optimization algorithms may be of great help. So far only a limited number of studies were reported in this aspect though. For example, Salloum et al. [21] and Tang et al. [22] applied the Nelder-Mead simplex method, and Barba et al. [23] applied the epsilon-constraint approach to optimize the temperature distribution in a tumor. However, both these two methods are local optimization algorithms, bearing the drawbacks of only finding local optima and relying very much on the initial guess of parameters, and hence are not capable of solving discrete optimization problems [24]. To address this issue, in a simple setting without considering the influence of injection number and geometrical complexity, Tang et al. [25] compared two stochastic optimization techniques, i.e., the simulated annealing (SA) and the particle swarm optimization (PSO), in exploring the best temperature distribution for hyperthermia treatment, and

illustrated that the PSO shows better ability in handling a large number of independent variables. Nevertheless, with a two-site injection strategy they still had a 19.6% injury rate in healthy tissue when achieving a 99.9% ablation rate in tumor. A similar treatment efficacy was also reported by Barba et al. [26], who applied the non-dominated sorting genetic algorithm (NSGA) on two-site injections and achieved a 91.2% ablation rate in tumor along with a 16.7% injury rate in healthy tissue.

To further enhance the treatment efficacy, in this study we aim to explore optimal MNP injection strategies for magnetic hyperthermia. To facilitate effective and efficient search, a dedicated computational framework is developed, in which the heat generation and transfer in tissues is simulated by solving the Pennes' bioheat transfer equation (PBHTE) using the lattice Boltzmann method (LBM), and the MNP injection strategies are evaluated and optimized using the PSO algorithm. This "modeling + optimization" framework is then demonstrated on two selected tumor models: one is a circle representing a geometrically regular tumor, and the other is an ellipse representing a geometrically irregular tumor. Both single- and multi-site injection strategies are evaluated. The effects of some key factors, such as the minimum dose requirement, are also studied. Moreover, the thermal dose in the optimal multi-site injection strategies is assessed by considering both temperature and heat exposure time.

## **2. Problem description and methodology**

The relevant physical and mathematical models will be introduced in this section. First, the two selected tumor models are described in Section 2.1. Then, the governing equations for simulating MNP-induced heat generation and transfer in tissues are introduced in Section 2.2. Last, the optimization algorithm adopted in the present study is presented in Section 2.3.

### *2.1. Problem description*

Two simplified tumor models were chosen in the present study, as sketched in Figure 1. One is a circle representing a geometrically regular tumor, and the other is an ellipse representing a

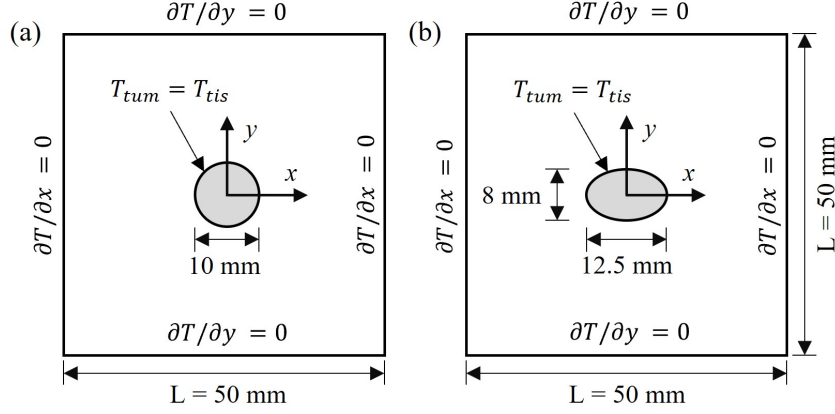


Figure 1: Schematics of two tumor models surrounded by healthy tissue: (a) a circular tumor model, and (b) an elliptical tumor model. The grey regions represent the tumor and the outer white regions represent the healthy tissue.

geometrically irregular tumor. The diameter of the circular tumor is set as 10 mm. The major and minor axes of the elliptical tumor are set as 12.5 mm and 8 mm, respectively. Hence both tumors have the same area. Surrounded by healthy tissue, each tumor is located at the center of a selected 50 mm  $\times$  50 mm tissue block. Assuming the adiabatic condition on the outside boundaries of the tissue block, the hyperthermia treatment is realized through heat generation by the AMF driven MNPs, which are injected at chosen sites in the tumor. Meanwhile, heat will also be taken away by the blood flow embedded in the tumor and the tissue.

## 2.2. Governing equations

The temperature distribution in the tumor and its surrounding healthy tissue,  $T$ , is evaluated using the Pennes' bioheat transfer equation (PBHTE) [27], which **has been widely adopted for predicting magnetic hyperthermia treatment efficacy** [28–30]. It relates the evolution of temperature in tissue with physiological and environmental factors, such as the blood perfusion and MNP-generated heat

$$(\rho c)_{nf} \frac{\partial T}{\partial t} = k_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \dot{m}_b c_b (T_b - T) + Q \quad (1)$$

where  $\rho$ ,  $c$  and  $k$  are density, specific heat and thermal conductivity, respectively. The subscript “ $nf$ ” denotes properties of nanofluid or modified tissue that is a mixture combining the tissue

Table 1: Properties of the tissue and nanoparticles [34, 37, 38]

Tissue properties	Value	MNP Properties	Value
$\rho_{tis}$ (kg/m <sup>3</sup> )	1052	$\rho_{mnp}$ (kg/m <sup>3</sup> )	5200
$c_{tis}$ (J/kg-K)	3800	$c_{mnp}$ (J/kg-K)	670
$k_{tis}$ (W/m-K)	0.5	$k_{mnp}$ (W/m-K)	6

and the injected MNPs, and “ $b$ ” denotes properties of blood. The properties of nanofluid can be determined for given MNP’s volume fraction  $\phi$  [31–33]

$$(\rho c)_{nf} = \phi(\rho c)_{mnp} + (1 - \phi)(\rho c)_{tis} \quad (2a)$$

$$k_{nf} = k_{tis} \frac{k_{mnp} + 2k_{tis} - 2\phi(k_{tis} - k_{mnp})}{k_{mnp} + 2k_{tis} + \phi(k_{tis} - k_{mnp})} \quad (2b)$$

Note that the subscript “tis” here includes the healthy tissue and the tumor tissue, both sharing the same properties for simplification since the difference in their thermal properties is only up to about 5% [34–36]. The detailed properties for the nanofluid formation are listed in Table 1.

The second term on the right hand side of Equation 1,  $\dot{m}_b c_b (T_b - T)$ , describes the rate of heat transfer from blood to tissue, where  $T_b = 37^\circ\text{C}$  is the normal temperature of blood and  $\dot{m}_b$  is the mass flow rate of perfusing blood per unit volume. According to Lang et al. [39],  $\dot{m}_b$  depends on local temperature as follows

In healthy tissue:

$$\dot{m}_b = \begin{cases} 0.45 + 3.55 \exp[-(T - 45)^2/12] & T \leq 45^\circ\text{C} \\ 4.0 & T > 45^\circ\text{C} \end{cases} \quad (3)$$

In tumor:

$$\dot{m}_b = \begin{cases} 0.833 & T < 37^\circ\text{C} \\ 0.8333 - (T - 37)^{4.8}/5438.0 & 37 < T \leq 42^\circ\text{C} \\ 0.416 & T > 42^\circ\text{C} \end{cases} \quad (4)$$

The last term in Equation 1,  $Q$ , represents the heat released from MNPs as induced by an external AMF. Its evaluation is provided in Appendix A.

Once injected at a selected site  $(x_i, y_i)$ , the evolution of MNPs' volume fraction can be determined analytically as two-dimensional diffusion

$$\Phi_i(x, y, t) = \frac{V_i}{4\pi Dt} \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{4Dt}\right] \quad (5)$$

where  $D$  is the mass diffusivity and  $V_i$  is the volume of injected MNP solution. Equation 5 clearly indicates that, at any instant, the spatial distribution of MNPs follows the Gaussian distribution with the variance  $2Dt$ .

Note that, in magnetic hyperthermia treatment the AMF is usually not applied at the time of injection. Instead, it is applied after the MNPs have sufficiently diffused in the tumor. Since the MNP diffusion is usually much slower than the thermal diffusion, which is reflected by the large Lewis number  $Le = \alpha/D = 125$  ( $\alpha$  is the thermal diffusivity) used in the present study, the temperature field governed by Equation 1 is solved by "freezing" the time for the mass diffusion [40]. That is, we only consider a temperature diffusion problem by replacing  $2Dt$  with a constant variance  $\sigma^2$ , modifying Equation 5 into

$$\phi_i(x, y) = \frac{V_i}{2\pi\sigma^2} \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{2\sigma^2}\right] \quad (6)$$

In the scenario of multi-site injections, we assume all injections are conducted at the same time and with the same dose, hence having the same MNP distribution. The total volume fraction at a given location  $(x, y)$  is then the superposition of all injections

$$\phi(x, y) = \sum_{i=1}^n \phi_i(x, y) \quad (7)$$

where  $n$  is the number of injections.

The temperature field governed by Equation 1 is solved using a lattice Boltzmann method (LBM) based computational framework. This mesoscopic approach is an alternative to conventional finite-difference, finite-element or finite-volume methods for computational fluid dynamics (CFD), with its distinctive merits of easy programming, manageable computer resources and numerical stability and accuracy [41]. Refer to Appendix B for the formulation and validation of this computational framework.

### 2.3. Optimization

The steady-state temperature distribution obtained from solving Equation 1 is then used to evaluate the performance of magnetic hyperthermia treatment and facilitate the optimization. For the optimization, the parameters to be controlled in this study include the number of injection sites  $n$ , the location of injections  $(x_i, y_i)$ , the dose applied in each injection  $V_i$ , and the variance of MNP distribution  $\sigma^2$  that reflects the timing of applying the AMF.

To enable efficient and effective searches, some of the above parameters are normalized. That is, the location of injections is measured by a distance ratio  $k_{d,i} = d_i/d_0$  where  $d_i = \sqrt{x_i^2 + y_i^2}$  is the distance of the injection site from the tumor center and  $d_0$  is a chosen reference distance from the tumor center to point  $(x_0, y_0)$ . The spatial distribution of MNPs is measured by a standard deviation ratio  $k_\sigma = \sigma/\sigma_0$  where  $\sigma_0$  is a reference standard deviation obtained 20 minutes after the MNP injection. The dose of each injection can also be measured by a dose ratio in a similar way. However, here we replace  $V_i$  by another quantity related to the volume fraction,  $k_\phi$ , through  $V_i = k_\sigma^2 k_\phi \phi_0$ , where  $\phi_0$  is a reference volume fraction, so that only one optimization parameter appears in the coefficient of the exponential function. As such, Equation 6 can be re-written as

$$\phi_i(x, y) = \frac{k_\phi \phi_0}{2\pi\sigma_0^2} \exp\left[-\frac{(x - k_{d,i}x_0)^2 + (y - k_{d,i}y_0)^2}{2k_\sigma^2\sigma_0^2}\right] \quad (8)$$

Note that  $k_\phi$  and  $k_\sigma$  do not carry the subscript  $i$  since we have assumed all injections share the same MNP dose and distribution.

In hyperthermia treatment, it is expected that the cancer tissue can be ablated at a temperature above a threshold, i.e., 43°C, while the healthy tissue stays safe by strictly keeping its temperature under this threshold. According to Hervault and Thanh [42], the temperature in both types of tissues should also be maintained under 46°C to avoid possible inflammatory responses. Moreover, overdose of iron-based MNPs may shorten life expectancy [11], so it is necessary to maintain a low level of injection quantity. Therefore, four basic requirements are formulated to evaluate the performance of magnetic hyperthermia treatment simulated in the present study, i.e.,

$$R_{tum} = \frac{S_{43^{\circ}C \leq T < 46^{\circ}C \text{ in tumor}}}{S_{tumor}} \quad (9a)$$

$$R_{tis} = \frac{S_{T < 43^{\circ}C \text{ in healthy tissue}}}{S_{healthy tissue}} \quad (9b)$$

$$R_{int} = \exp\left(-\sqrt{\frac{1}{N} \sum_{i=1}^N (T_i - 43)^2}\right) \quad (9c)$$

$$R_{dose} = 1 - \sqrt[3]{nk_{\phi}k_{\sigma}^2} \quad (9d)$$

where  $S$  represents area and  $N$  is the total number of points sampled along the tumor-healthy-tissue interface.  $R_{tum}$  defines a ratio of the tumor area with temperature between 43°C and 46°C to the entire tumor area, which evaluates how much the temperature distribution in the tumor satisfies the requirement. Similarly,  $R_{tis}$  defines a ratio of the area in the healthy tissue with temperature below 43°C over the entire healthy tissue area, evaluating how much the temperature distribution in the healthy tissue satisfies the requirement.  $R_{int}$  describes how different the temperature at the tumor-healthy-tissue interface deviates from the required 43°C. Different from the above three temperature requirements,  $R_{dose}$  evaluates the dose requirement, since the total volume of the dose is proportional to  $nk_{\phi}k_{\sigma}^2$ . Note that, to improve the optimization efficiency, the ideal maximum of all the four requirement variables are designed to be 1, the larger the better.

With the four requirement variables defined, an objective function for optimization can be



formulated as

$$J = \omega_1 R_{tum} + \omega_2 R_{tis} + \omega_3 R_{int} + \omega_4 R_{dose} \quad (10)$$

To ensure  $J$  roughly varies between 0 and 1 for good optimization performance, and to put more emphasis on the temperature distributions, the weighting factors  $\omega_1, \omega_2, \omega_3, \omega_4$  are set as 0.3, 0.3, 0.2, 0.2, respectively, after several trial runs.

The optimization is conducted to search the maximum  $J$  using the particle swarm optimization (PSO) method, so that the best MNP distribution for magnetic hyperthermia treatment can be identified. PSO is a heuristic optimization algorithm, which performs stochastic search, makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions [43]. See Appendix C for more details about this optimization method.

The optimization will generate a set of optimal ratios, i.e.,  $k_\phi, k_\sigma$  and  $k_d$ , with which the best waiting time  $t_{opt}$ , the best volume of MNPs  $V_{opt}$  and the best distance of injection sites from the tumor center  $d_{opt}$  can be determined as

$$t_{opt} = \frac{k_\sigma^2 \sigma_0^2}{2D} \quad (11a)$$

$$V_{opt} = nk_\sigma^2 k_\phi \phi_0 \quad (11b)$$

$$d_{opt} = k_d d_0 \quad (11c)$$

### 3. Results and Discussion

This section presents the optimization results and related discussions about hyperthermia treatment on the two selected tumor models under different MNP injection strategies. Sections 3.1 and 3.2 mainly focus on the optimized temperature distributions, whereas Section 3.3 further considers the time of exposure to heat in the optimization.

### 3.1. Circular tumor

For this geometrically regular tumor, we assume all MNP injections, if more than one, share the same dose and occur at the same time. In addition, all injection sites share the same distance from the center of the tumor and are evenly distributed in the azimuthal direction. As such, only the three ratios introduced in Section 2.3, i.e.,  $k_\phi$ ,  $k_\sigma$  and  $k_d$ , are involved in the optimization. Their corresponding reference values are chosen as follows:  $\phi_0 = 1.25 \text{ mm}^2$  to ensure the volume fraction of MNPs is not too large,  $\sigma_0 = 1.55 \text{ mm}$  is the standard deviation of the MNP distribution 20 minutes after the injection, which is obtained by solving Equation 5, and  $d_0 = 4.5 \text{ mm}$  is 90% the tumor radius that defines the allowed area for MNP injections.

Three injection cases are considered here, namely one-site, four-site and eight-site injections. For the one-site injection, it has to be at the tumor center due to the geometrical symmetry, and hence  $k_d = 0$ . For the other two cases, one injection is located along the  $x$  axis, while the others are evenly distributed in the azimuthal direction.

In each PSO search, 20 particles are adopted and evolve for 300 generations. Take the four-site injection case as an example, the PSO searching process is shown in Figure 2(a) and (b). Initially, the three optimization parameters for all particles in the swarm are randomly assigned between 0 and 1. After several generations, some particles gradually identify suitable parameter combinations for increasing the objective function  $J$ , and pass the experience to other particles. This effective learning process makes the three parameters of all particles converge to their respective optimal values after about 100 generations, and results in a consistent rise in the maximum  $J$  of each generation until it levels off at its optimal value (See Figure 2(b)).

Figures 2(c) to (e) show the evolution of the optimization parameters and the collectively produced maximum  $J$  in the three injection cases. It reveals that all the three cases can quickly find their optima, reflecting the appropriateness of the adopted PSO settings. Specifically, the four-site injection case finds its optimal parameter combination with the fastest pace, roughly after 50 generations, while the one-site and eight-site injection cases take a bit longer, roughly after 100

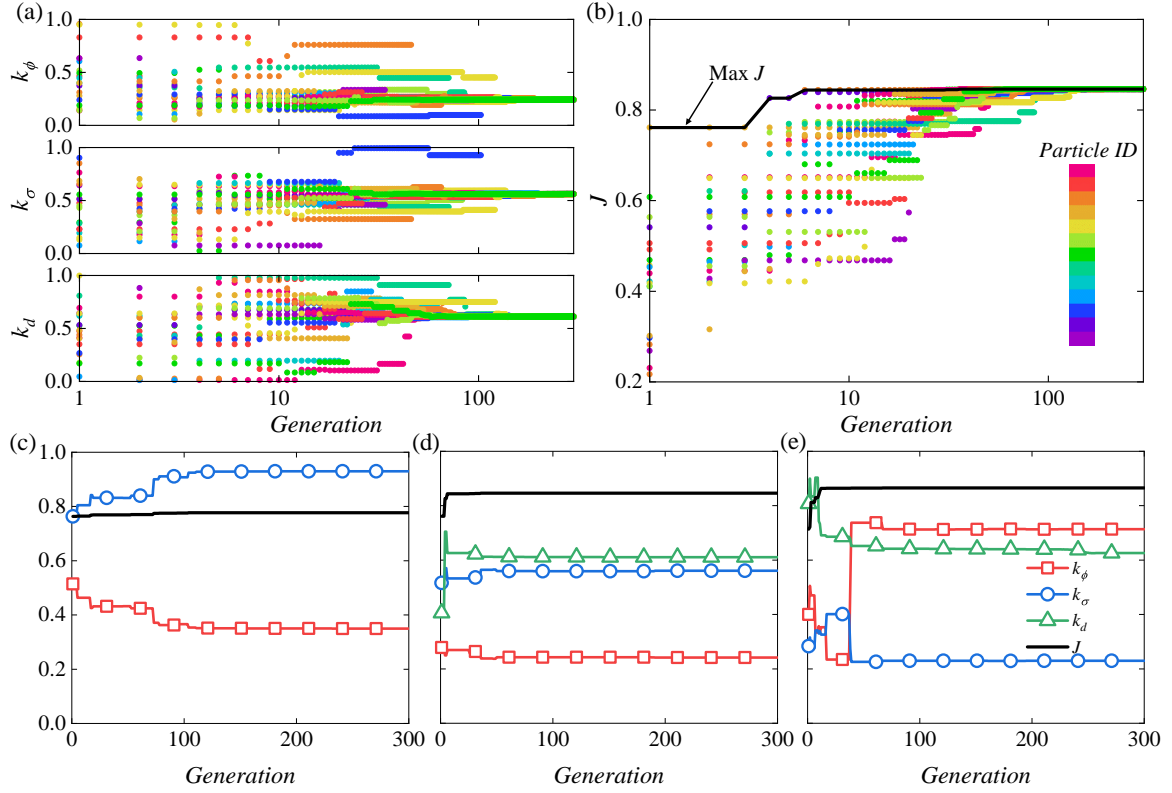


Figure 2: PSO searching process on the circular tumor model: Evolution of (a) the three ratios, i.e.,  $k_\phi$ ,  $k_\sigma$  and  $k_d$ , and (b) the objective function  $J$  in the four-site injection case; Evolution of optimal parameters and corresponding  $J$  in the cases of (c) one-site injection, (d) four-site injection, and (e) eight-site injection. The black lines depict the envelope of the maximum  $J$ .

Table 2: Optimization results for the one-, four- and eight-site injection cases on the circular tumor model

Injection Strategy	$R_{tum}$	$R_{tis}$	$R_{int}$	$R_{dose}$	$J$	$t_{opt}$ , min	$V_{opt}$ , mm <sup>2</sup>	$d_{opt}$ , mm
One-site	0.708	0.999	0.994	0.329	0.78	17.3	0.378	0
Four-site	0.987	0.999	0.925	0.328	0.85	6.3	0.380	2.75
Eight-site	1.000	0.999	0.998	0.330	0.87	1.1	0.376	2.82

and 75 generations, respectively. It is seen that, with the increase of injection number, the optimal  $k_\sigma$  monotonically reduces from 0.93 to 0.23, which, according to Equation 11(a), reflects that the optimal waiting time for the AMF significantly decreases, from 17.3 minutes for one-site injection, to 6.3 minutes for four-site injection, and then to 1.1 minutes for eight-site injection. It also shows that the optimal  $k_d$  is about 0.62 in the two multi-site injection cases, which, according to Equation 11(c), indicates that the optimal location of the injection sites is about 56% the tumor radius from the center. The most important observation made from these optimizations is that multi-site injections are beneficial to hyperthermia treatment, as confirmed by the best performance achieved in the eight-site injection case (with the largest  $J = 0.87$ ) and the worst performance achieved in the single-site injection case (with the lowest  $J = 0.78$ ).

Table 2 also lists the values at the optimal conditions for the four requirements and the corresponding physical quantities as defined in Equation 11. It is noticed that  $R_{tum}$ , the ratio of the tumor area satisfying the hyperthermia temperature requirement, is only 0.708 for the single-site injection case. Compared to those values almost or perfectly 1 in the multi-site injection cases, it clearly indicates that the single-site injection fails to fulfill the temperature requirements. Indeed, in this case a large portion at the tumor center is overheated (i.e.,  $T > 46^\circ\text{C}$ ), as revealed in the temperature contour shown in Figure 3(d). On the contrary, the two multi-site injection cases can generally fulfill all the three temperature requirements, i.e.,  $R_{tum}$ ,  $R_{tis}$  and  $R_{int}$ . The effectiveness of multi-injection strategy has also been reported by Singh et al [19]. If comparing the temperature distribution along the interface, however, the eight-site injection case slightly outperforms the four-site injection case, which is evidenced by the listed  $R_{int}$  values in Table 2 and the radial

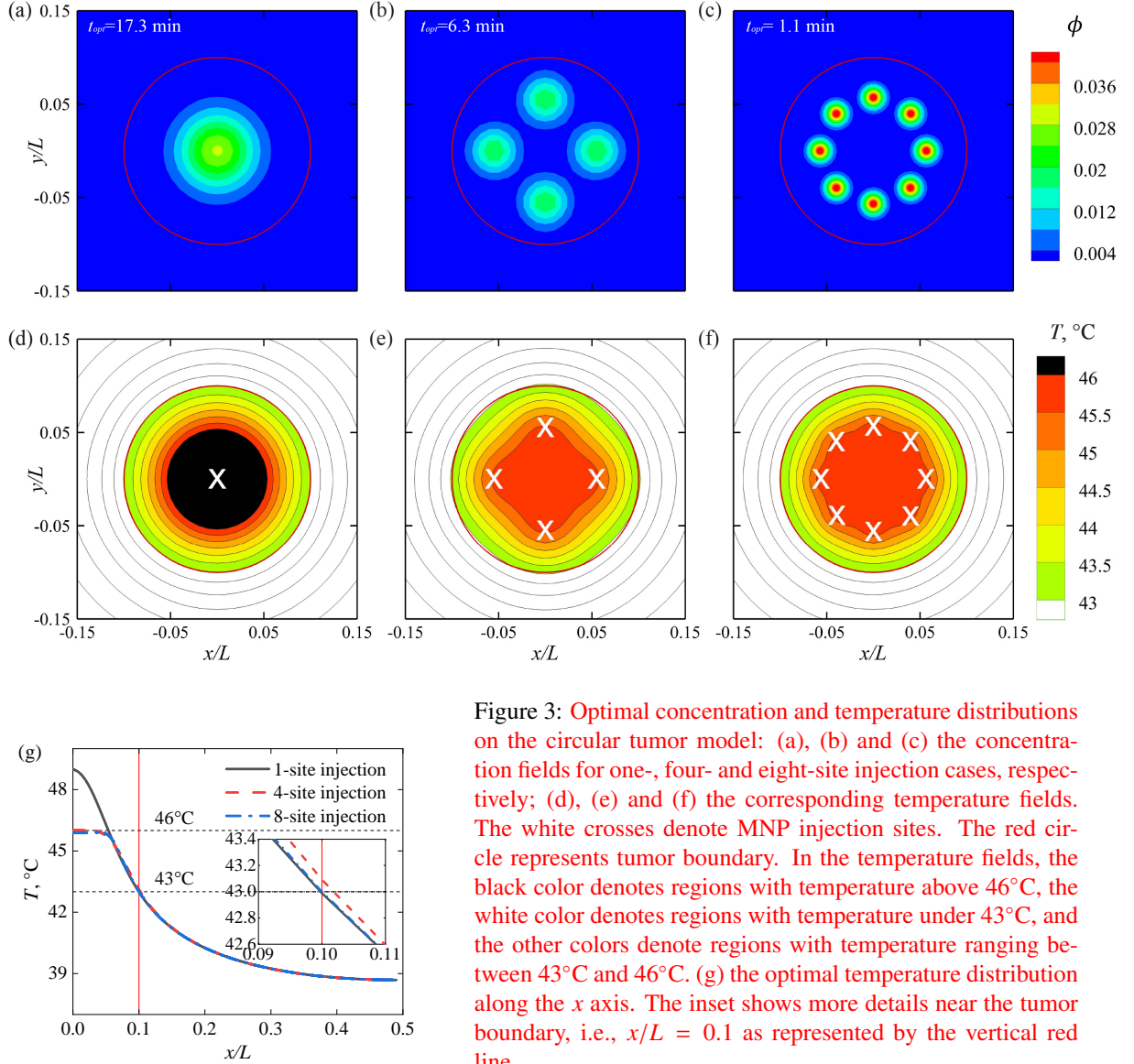
Table 3: Comparison of optimization results between cases including and not including the minimum dose requirement

Injection Strategy		$R_{tum}$	$R_{tis}$	$R_{int}$	$V_{dose}, \text{mm}^2$
One-site	w $R_{dose}$	0.708	0.999	0.994	0.378
	w/o $R_{dose}$	0.708	0.999	0.994	0.378
Four-site	w $R_{dose}$	0.987	0.999	0.925	0.379
	w/o $R_{dose}$	1.000	0.999	0.936	0.384
Eight-site	w $R_{dose}$	1.000	0.999	0.998	0.376
	w/o $R_{dose}$	1.000	0.999	0.998	0.377

temperature distribution near the interface as shown in the inset of Figure 3(g). It is this slight advantage that leads to a higher  $J$  for the eight-site injection case.

From Table 2, it is also found that the optimal total dose for MNP injections varies slightly (less than 1%) among the three cases, suggesting that the optimization may not be sensitive to the dose requirement  $R_{dose}$ , i.e., Equation 9(d). To further confirm this, we conduct another optimization study, in which  $R_{dose}$  is not included in the objective function. As revealed in Table 3, although including  $R_{dose}$  in the objective function does help reduce the total dose, the reduction is very marginal, which does not really affect the optimization results.

From Figure 3(e), one can see that the multi-site injection may result in directional difference in the temperature distribution, which then leads to nonuniform temperature distribution along the circular interface, as indicated in Table 2. This directional difference can be mitigated by adding more injection sites along the azimuthal direction, like in the eight-site injection case as shown in Figure 3(f). Since in this study the chosen computational domain is in a square shape, the direction of the four-site injections could affect the optimization results. To assess this effect, another four-site injection case is simulated, where the original four injection sites are relocated from the  $0^\circ$  (i.e.,  $x$ ) and  $90^\circ$  (i.e.,  $y$ ) axes to the  $\pm 45^\circ$  axes. The new optimization results reveal that such a rotation does not really affect the optimal distributions of MNPs and temperature, as shown in Figure 4. Meanwhile, although not listed here for brevity, the rotation has negligible effects on the hyperthermia treatment performance in terms of the temperature and dose requirements.



**Figure 3: Optimal concentration and temperature distributions on the circular tumor model:** (a), (b) and (c) the concentration fields for one-, four- and eight-site injection cases, respectively; (d), (e) and (f) the corresponding temperature fields. The white crosses denote MNP injection sites. The red circle represents tumor boundary. In the temperature fields, the black color denotes regions with temperature above  $46^{\circ}\text{C}$ , the white color denotes regions with temperature under  $43^{\circ}\text{C}$ , and the other colors denote regions with temperature ranging between  $43^{\circ}\text{C}$  and  $46^{\circ}\text{C}$ . (g) the optimal temperature distribution along the  $x$  axis. The inset shows more details near the tumor boundary, i.e.,  $x/L = 0.1$  as represented by the vertical red line.

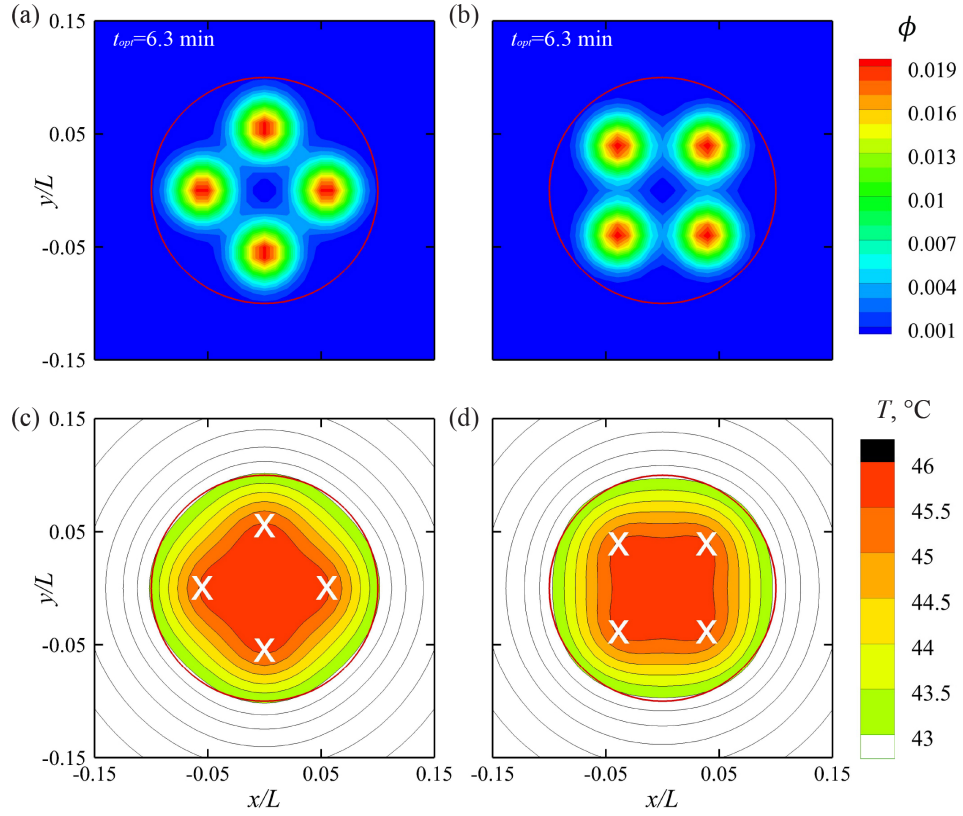


Figure 4: Optimal concentration and temperature distributions in two different four-site injection cases on the circular tumor model. In (a) and (c) the four injection sites are arranged along  $x$  and  $y$  axes, which are rotated for  $45^\circ$  in (b) and (d). The red circle represents tumor boundary. The white crosses denote the injection sites. In the temperature fields, the white color denotes regions with temperature under  $43^\circ\text{C}$ , and the other colors denote regions with temperature ranging between  $43^\circ\text{C}$  and  $46^\circ\text{C}$ .

### 3.2. Elliptical tumor

In general, tumors do not grow isotropically to form an ideal circle. Instead, they often appear in irregular shapes. To tackle the complexity caused by the geometric irregularity, another set optimization studies is conducted on an elliptical tumor. As sketched in Figure 1(b), this tumor has a 12.5-mm major axis and a 8-mm minor axis, hence sharing the same area with the circular tumor studied in Section 3.1.

Four different injection cases are considered for this elliptical tumor, namely one-site, two-site, four-site and eight-site injections. The new two-site injection case is introduced to represent the simplest injection pattern in order to address the geometric irregularity, where the injection sites are arranged along the major axis, symmetric about the tumor center. In the four-site injection case, two injection sites are symmetrically arranged along the major axis, while the other two are symmetrically arranged along the minor axis. For the eight-site injection case, four more sites are introduced, which are arranged along two lines that deviate from the major axis with an inclination angle  $\beta$ , as denoted in Figure 6(d).

To deal with the geometric irregularity, in the optimization the injection sites are allowed to vary independently along different axes. This is different from the cases for the circular tumor, where the distances of the injection sites from the tumor center along different axes are set identical. As such, the original location ratio parameter for the circular tumor,  $k_d$ , becomes  $k_{dh}$  along the major (horizontal) axis,  $k_{dv}$  along the minor (vertical) axis, and  $k_{di}$  along the inclined axes, wherever applicable, for the elliptical tumor. The corresponding ranges are set as  $d_{h0} = 1.25d_0$ ,  $d_{v0} = 0.8d_0$ , and  $d_{i0} = d_0$ , respectively, according to the given shape of the elliptical tumor. For the eight-site injection case, the angle of the inclined axes is also allowed to vary through a new ratio parameter  $k_\beta$  together with a preset range  $\beta_0 = 90^\circ$ . The arrangements for these distance parameters are also listed in Table 4. Like in the circular tumor case, all these ratio parameters vary in the range between 0 and 1 for the best searching performance.

Figure 5 shows the evolution of the optimization parameters and the objective function in all



Table 4: Displacement parameters used in the elliptical tumor cases

Strategy	$k_{dh}$	$k_{dv}$	$k_{di}$	$k_{\beta}$	$d_{h0}$	$d_{v0}$	$d_{i0}$	$\beta_0$
One-site	-	-	-	-	-	-	-	-
Two-site	✓	-	-	-	$1.25d_0$	-	-	-
Four-site	✓	✓	-	-	$1.25d_0$	$0.8d_0$	-	-
Eight-site	✓	✓	✓	✓	$1.25d_0$	$0.8d_0$	$d_0$	$90^\circ$

the four injection cases. It is seen that all cases can converge within 300 generations, confirming the capacity of PSO in dealing with a large number of optimization parameters [25]. However, if compared with the corresponding cases for the circular tumor, these cases generally require more generations to converge, reflecting the complexity of this geometrically irregular problem. Moreover, sudden jumps are observed after 100 or even 150 generations in the two-site and four-site injection cases, which were also reported in Tang et al. [25]. These sudden jumps may be attributed to the multi-extremum nature of the current optimization problem, which confirms the capability of the PSO in global optimum search.

Figure 5 also reveals that the objective function  $J$  increases from 0.66 to 0.86 as the number of injection sites increases, which is also listed in Table 5. This confirms that the more injections, the better. The optimal temperature distribution for the four cases are presented in Figure 6 (a) ~ (d). It is not surprising to see that the one-site injection fails to meet the temperature requirements again. The circular temperature contour cannot fit the tumor's elliptical shape: On one hand overheating occurs at the tumor center and in the healthy tissue near the minor axis, and on the other hand underheating occurs inside the tumor along the major axis. This intrinsic mismatch can be mitigated by using the two-site injection strategy. As shown in Figure 6(b), the two-site injection makes the  $43^\circ\text{C}$  line fit reasonably well with the tumor boundary and protects the healthy tissue from overheating, hence resulting in a significant increase in  $J$ . Unfortunately, it generates two small overheating spots inside the tumor.

The situation gets much improved if the number of injection sites are further increased to four. As shown in Figure 6(c), the two injection sites added in the minor axis help remove the

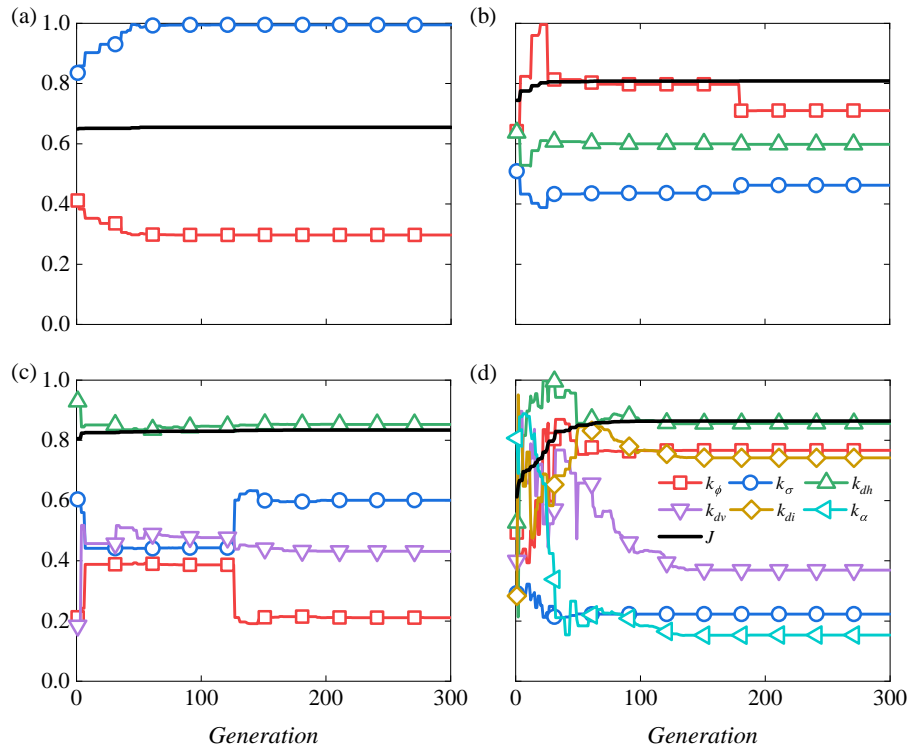


Figure 5: PSO searching process for the cases on the elliptical tumor model: (a) one-site injection, (b) two-site injection, (c) four-site injection, and (d) eight-site injection.

Table 5: Optimization results for the one-, two-, four- and eight-site injection cases on the elliptical tumor model

Strategy	$R_{tum}$	$R_{tis}$	$R_{int}$	$R_{dose}$	$J$	$t_{opt},$ min	$V_{opt},$ mm <sup>2</sup>	$d_{h,opt},$ mm	$d_{v,opt},$ mm	$d_{i,opt},$ mm	$\beta,$ °
One-site	0.620	0.996	0.516	0.335	0.66	19.8	0.367	-	-	-	-
Two-site	0.885	0.999	0.890	0.328	0.81	4.28	0.380	3.37	-	-	-
Four-site	0.974	0.999	0.884	0.327	0.83	7.23	0.381	4.80	1.55	-	-
Eight-site	1.000	1.000	0.995	0.327	0.86	0.99	0.381	4.82	1.33	3.34	13.8

overheating spots, making the temperature at the tumor center the highest. However, it requires about 3 more minutes waiting time for the operation of AMF, and there are still some temperature mismatch around the tumor boundary, as also revealed in the plots in Figure 6(e) and (f). This temperature mismatch can be further mitigated by adding four more sites in the eight-site injection case. The added four more sites help redistribute the MNPs, greatly reducing the waiting time to just about 1 minute.

It is interesting to see from Table 5 that the optimal total MNP doses in all four cases fluctuate only marginally with less than 3% difference. They are also very close to the optimal total doses suggested for the circular tumor of the same area. This indicates that the MNP dose is neither sensitive to the injection strategy nor sensitive to the shape of tumor. Instead, it seems mainly dependent on the area of the tumor, consistent with observations in clinical trials [44].

In the eight-site injection case, the optimal inclination angle for four injection sites is found to be  $\beta = 13.8^\circ$ . To check how sensitive the optimization is to this inclination angle, an additional eight-site injection case is considered, in which the inclination angle is fixed at  $\beta = 30^\circ$ . As shown and compared in Figure 7 as well as in Table 6, these two inclination angles lead to quite different results. The globally optimal injection sites in the  $\beta = 13.8^\circ$  case form an elliptical pattern alike to the shape of the tumor. On the contrary, the eight sites in the  $\beta = 30^\circ$  case form a figure “8” pattern, with the two sites in the major axis being further away and the two sites in the minor axis being much closer. The latter pattern results in a bit mismatch in temperature along the tumor boundary, leading to a slightly smaller  $J$ .

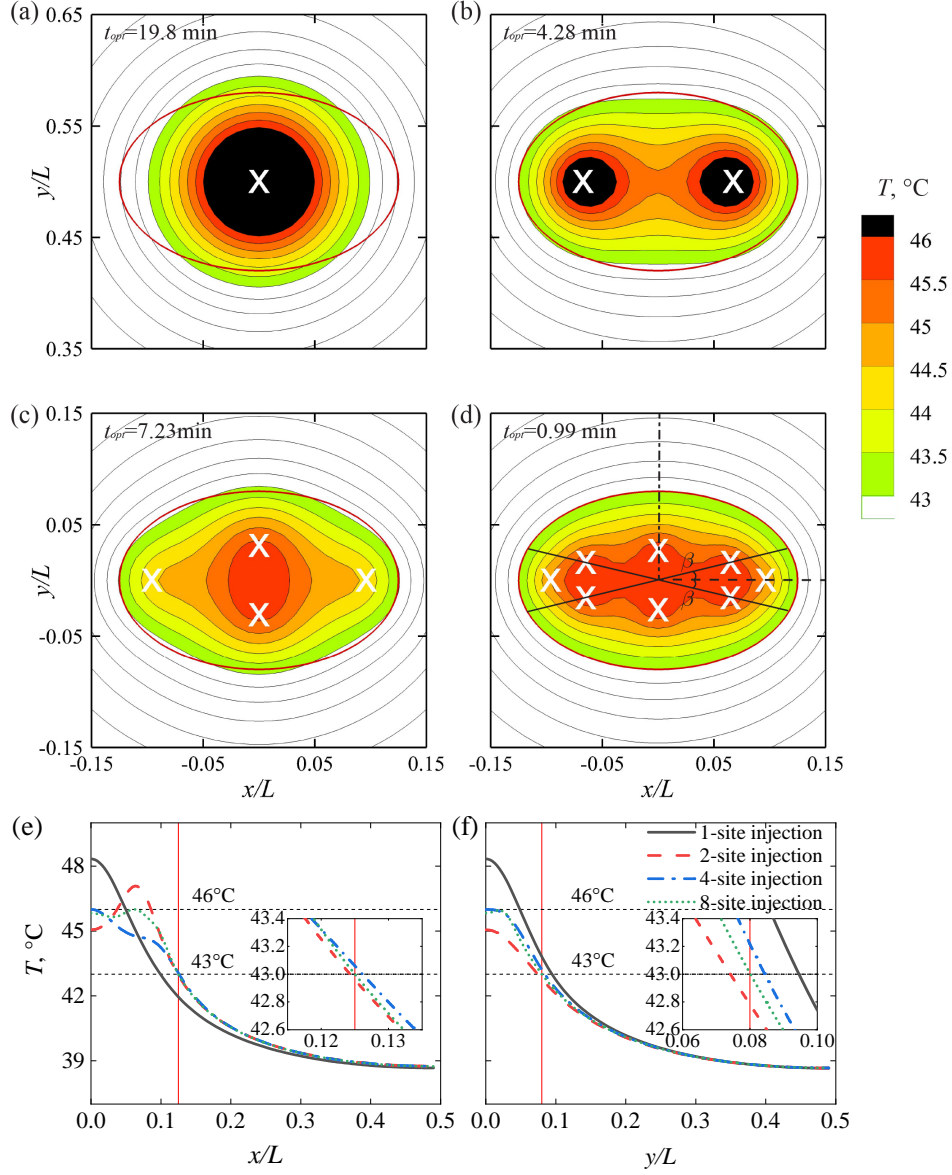


Figure 6: (a)-(d) Optimal temperature distributions on the elliptical tumor model for the one-, two-, four- and eight-site injection cases, respectively. The white crosses denote the injection sites. The red ellipse represents tumor boundary. The black color denotes regions with temperature above 46°C, the white color denotes regions with temperature under 43°C, and the other colors denote regions with temperature ranging between 43°C and 46°C. (e) and (f) Optimal temperature distribution along the  $x$  and  $y$  axes, respectively. The insets show more details near the tumor boundaries, i.e.,  $x/L = 0.125$  and  $y/L = 0.8$  as represented by the vertical red lines.

Another apparent difference between these two cases lies in the waiting time. The global optimal  $\beta = 13.8^\circ$  case only requires to wait for about 1 minute, while the  $\beta = 30^\circ$  case requires to wait for a much longer time, i.e., about 8 minutes. This long waiting time allows the MNPs to sufficiently diffuse, as confirmed in Figure 7(a), resulting in much smoother temperature distribution inside the tumor after the AMF is applied, as shown in Figure 7(c).

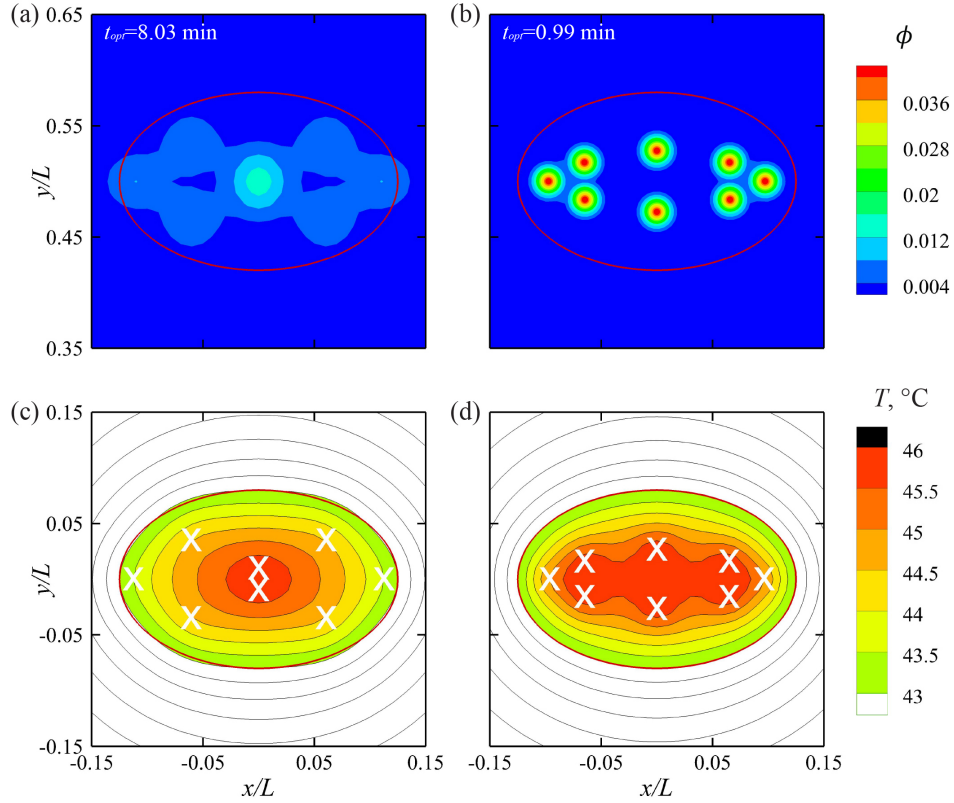


Figure 7: Optimal concentration and temperature distributions in two different eight-site injection cases on the elliptical tumor model. In (a) and (c) four injection sites are arranged along the pre-determined  $\beta = \pm 30^\circ$  axes, whereas in (b) and (d) they are arranged along the  $\beta = \pm 27.5^\circ$  axes, which is the optimization result. The red ellipse represents tumor boundary. The white crosses denote the injection sites. In the temperature fields, the white color denotes regions with temperature under  $43^\circ\text{C}$ , and the other colors denote regions with temperature ranging between  $43^\circ\text{C}$  and  $46^\circ\text{C}$ .

### 3.3. Thermal dose

In the above optimization work, the ideal outcome is a temperature field in which the temperature in the tumor is between  $43^\circ\text{C}$  and  $46^\circ\text{C}$ , while the temperature in the surrounding healthy

Table 6: Comparison of optimization results between two eight-site injection cases with different inclination angles

Inclination angle	$R_{tum}$	$R_{tis}$	$R_{int}$	$R_{dose}$	$J$	$t_{opt},$ min	$V_{opt},$ $10^{-7} \text{ m}^2$	$d_{h,opt},$ mm	$d_{v,opt},$ mm	$d_{i,opt},$ mm
$\beta = 30^\circ$	1.000	1.000	0.940	0.322	0.85	8.03	3.89	5.62	0.49	3.50
$\beta = 13.8^\circ$	1.000	1.000	0.995	0.327	0.86	0.99	3.81	4.82	1.33	3.34

tissue stays below  $43^\circ\text{C}$ . In practice, the amount of cell death in hyperthermia treatment depends not only on the temperature, but also on the time of exposure to heat, i.e., thermal dose [45]. In this section, we will check whether the optimal temperature distributions obtained for the two tumor models are still good if the exposure time is taken into account.

The cumulative-equivalent-minutes-at- $43^\circ\text{C}$  (CEM43) model is a widely used model to evaluate thermal dose, which translates different temperature–time histories to a single value representing a “thermal isoeffect dose”. Specifically, it is an equivalent exposure time expressed as minutes at the reference temperature of  $43^\circ\text{C}$ , i.e.,

$$\text{CEM43} = \sum_{i=1}^l C_{EM}^{43-T_i} \delta t \quad (12)$$

where  $T_i$  is the temperature in  $^\circ\text{C}$  at the  $i$ th time step,  $\delta t$  represents the time interval, and  $l$  denotes the total number of time steps.  $C_{EM}$  is set as 0.5 when  $T_i > 43^\circ\text{C}$  and 0.25 otherwise [46]. It was suggested that when CEM43 reaches 60 minutes, the related cells are regarded as killed by the heat [19, 45]. With this hypothesis, we assess the hyperthermia treatment efficacy through the ratio of killed cells in a tumor or in its surrounding healthy tissue

$$R_{CEM43} = \frac{S_{CEM43 \geq 60 \text{ min}}}{S_{tum}} \quad (13)$$

With this definition,  $R_{CEM43} = 1$  in tumor and 0 in healthy tissue correspond to the ideal hyperthermia treatment efficacy, i.e., all the tumor cells are killed while the healthy tissue stays safe.

By considering the effect of exposure time with the CEM43 model, the cell ablation in the tu-

mor and healthy tissue is assessed for the two tumor models, starting from the optimal temperature distributions obtained in Sections 3.1 and 3.2. Note that, in this assessment the evolution of MNP diffusion, although much slower than thermal diffusion in the present study, is taken into account since the treatment time is usually long.

Figure 8(a) presents the evolution of ablation ratios in the circular tumor model. It is not surprising to see that, for such an axisymmetric tumor model, ablation of cells in the surrounding healthy tissue starts to occur right at the instant when 100% ablation is achieved inside the tumor. Specifically, the one- and eight-site injection cases perfectly complete the tumor ablation (i.e.,  $R_{CEM43} = 1$  in tumor and 0 in healthy tissue) in 104.8 min and 126.6 min, respectively, at a similar level as in experimental observations [47]. The four-site injection case is a slight exception, which has  $R_{CEM43} = 0.32\%$  healthy tissue destroyed when 100% ablation is achieved inside the tumor. This is caused by the slight mismatch of temperature requirement along the tumor-healthy-tissue interface at the start of ablation, as depicted in Figures 3.

Although not presented here for brevity, it is also observed that in the one-site injection case an overheated (i.e.,  $T > 46^\circ\text{C}$ ) region appears at the tumor center at  $t = 6.2$  min and, due to the MNP diffusion, disappears at  $t = 38.0$  min. This 31.8-min overheat definitely influences the ablation efficacy [42]. As such, only the four- and eight-site injections can be considered to perform well for the circular tumor.

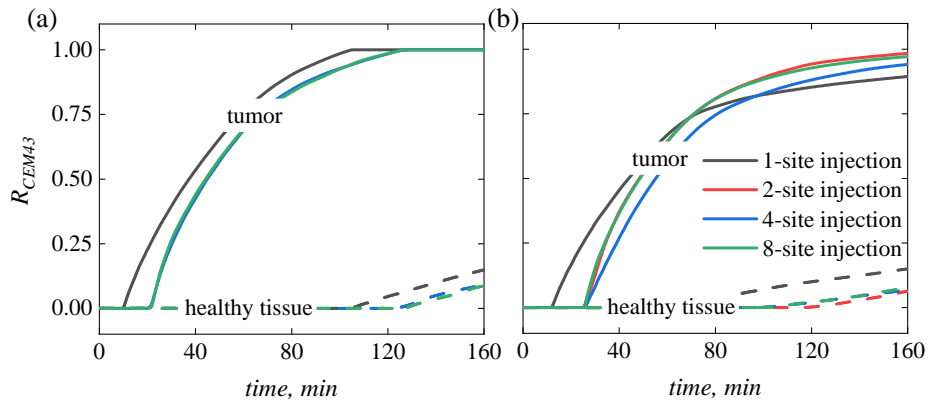


Figure 8: Evolution of ablation ratio for (a) the circular tumor model and (b) the elliptical tumor model.

Table 7:  $R_{CEM43}$  in tumor and exposure time when healthy tissue begins to be injured

Strategy	$R_{CEM43}$ in tumor	Exposure time, min
one-site	0.67	60.7
two-site	0.95	122.7
four-site	0.83	101.5
eight-site	0.90	104.3

Different from the circular tumor, the elliptical tumor does not show any perfect treatment. As shown in Figure 8(b) and listed in Table 7, in all the injection cases, not all tumor cells are killed when the healthy tissue starts to be destroyed. The one-site injection case performs the worst, in which only 67% tumor cells are killed and also overheat occurs from  $t = 9.5$  to 26.2 min. Nevertheless, in the two- and eight-site injection cases no less than 90% tumor cells are ablated.

The ablation process in the eight-site injection case is illustrated in Figure 9. At the early stage (i.e.,  $t = 30$  min), the ablated area (the white ellipse) is thinner than the tumor due to the thinner pattern of the eight injection sites determined by the optimization of temperature requirements. As time advances, the MNPs diffuse almost uniformly in all directions. Consequently, the ablated area becomes thicker and gradually evolves towards a circle. At  $t = 60$  min, it has almost the same but slightly smaller elliptic shape as the tumor. At  $t = 120$  min, it reaches out of the tumor in the minor-axis direction, but is still inside in the major-axis direction. This asynchronism reflects that the almost perfect injection site pattern optimized through temperature requirements may not work perfectly if the exposure time is considered, especially for geometrically irregular tumor models.

#### 4. Conclusions

In this study, the best MNP injection strategy for magnetic hyperthermia treatment was explored using a dedicated LBM modeling + PSO optimization framework. Two tumor models were considered, i.e., a circular model and an elliptical model that represent geometrically regular and irregular tumors, respectively. The results clearly indicated that the single-site injection strategy fails, even on the simplest circular tumor model. In this case, a significant portion of the tumor at



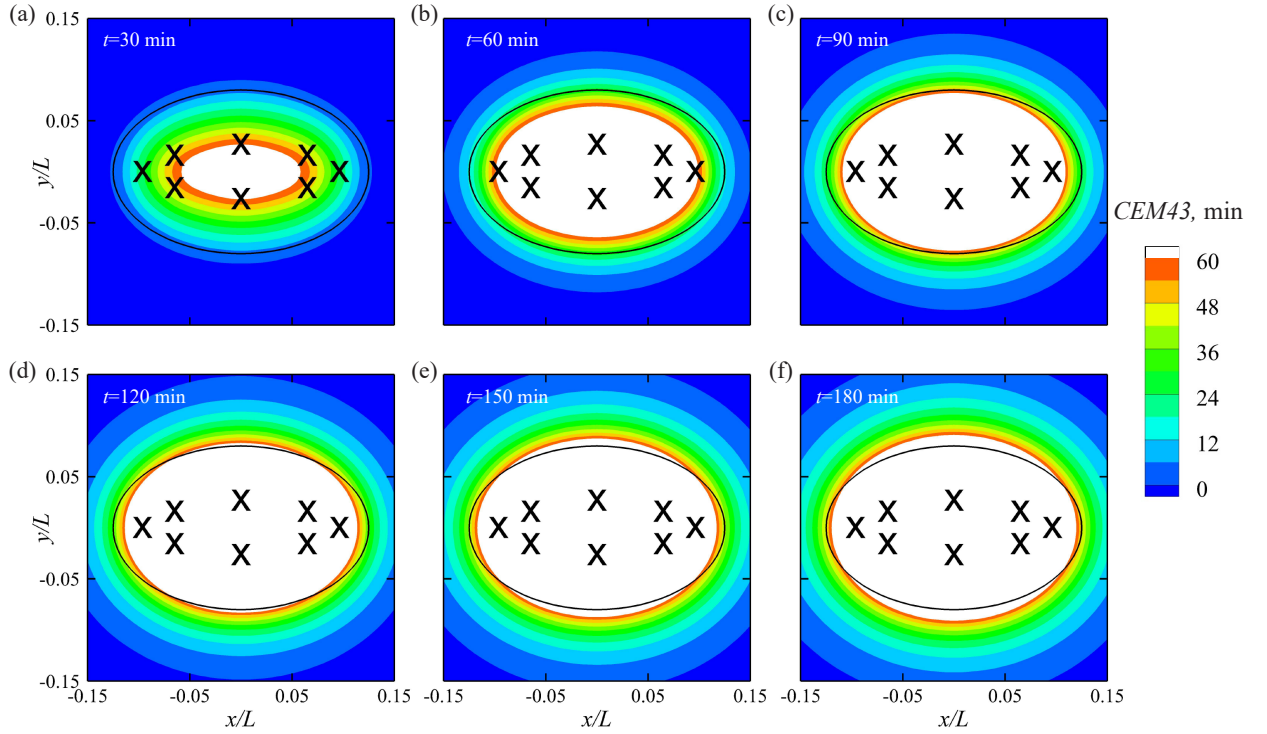


Figure 9: Evolution of the ablation process in the eight-sight injection case for the elliptical tumor at (a)  $t = 30$  min,  $R_{CEM43} = 0.15$  in tumor and 0 in healthy tissue, (b)  $t = 60$  min,  $R_{CEM43} = 0.64$  in tumor and 0 in healthy tissue, (c)  $t = 90$  min,  $R_{CEM43} = 0.85$  in tumor and 0 in healthy tissue, (d)  $t = 120$  min,  $R_{CEM43} = 0.93$  in tumor and 0.02 in healthy tissue, (e)  $t = 150$  min,  $R_{CEM43} = 0.97$  in tumor and 0.07 in healthy tissue, (f)  $t = 180$  min,  $R_{CEM43} = 0.98$  in tumor and 0.11 in healthy tissue. The black crosses denote the injection sites determined by the optimization of temperature requirements. The black ellipse represents the tumor boundary and the white area represents the ablated region.

the center overheats to more than 46°C, causing serious inflammatory responses. On the contrary, the multi-site injection strategies generally work well for both regular and irregular tumor models in terms of temperature distribution. Multi-site injection can also significantly reduce the waiting time for MNP diffusion before the AMF operation. The more the injection sites, the better the performance. However, too many injection sites may cause inconvenience. Hence, four-site injection could be an appropriate strategy that balances the temperature requirements with practical complications.

The geometrical irregularity of the tumor requires more generations for the optimum search, since it needs more parameters to describe the injection strategy. However, the required MNP dose was found insensitive to the shape of tumor. It was also found that whether including the minimum MNP dose requirement (i.e.,  $R_{dose}$ ) or not in the objective function only slightly affects the optimization results by less than 2%. This implies that the minimum dose requirement has automatically been satisfied if the other three temperature requirements are satisfied.

If the thermal dose received by cells is considered, the optimal multi-site injection strategies perform generally well for both tumor models, especially for the circular tumor. For the elliptical tumor, if the optimal two- and eight-site injection strategies are adopted, no less than 90% tumor cells are killed when the healthy tissue starts to be ablated.

Although the simulation is only two dimensional and the optimization is conducted on very simplified tumor models, the framework adopted in this present study works well and is able to provide useful insights into magnetic hyperthermia treatment. In the near future, more realistic settings will be considered for magnetic hyperthermia simulations and optimizations, in which more physics are involved, such as the dynamics of perfusing flow in tissue, the buoyancy and Lorentz forces, and time-dependent diffusion of MNPs. Moreover, an improved optimization framework will be developed to take the heat exposure time into account.

## Acknowledgement

This study was financially supported by the Research Grants Council of Hong Kong under General Research Fund (Project No. 15214418).

## Appendix A. Evaluating $Q$ in Equation 1

The last term  $Q$  in Equation 1 represents the heat released from MNPs, induced by an external AMF with amplitude  $H_0$  and frequency  $f$ . In general it includes the heat due to hysteresis loss and the heat from Neel and Brownian relaxation. According to Chang et al. [12], however, the size of MNPs used for magnetic hyperthermia is so small that the hysteresis loss can be neglected. Hence only the heat from the Neel and Brownian relaxation is considered, using the Rosensweig's model [48–50]

$$Q = \pi\mu_0\chi_0H_0^2f \frac{2\pi f\tau_R}{1 + (2\pi f\tau_R)^2} \quad (\text{A.1})$$

where  $\mu_0$  is the vacuum permeability.  $\tau_R$  is the effective relaxation time and can be evaluated with the Neel relaxation time  $\tau_N$  and the Brownian relaxation time  $\tau_B$  as

$$\frac{1}{\tau_R} = \frac{1}{\tau_N} + \frac{1}{\tau_B} \quad (\text{A.2})$$

where

$$\tau_N = \frac{\sqrt{\pi}\tau_0 \exp(\frac{KV_M}{k_B T})}{2 \sqrt{\frac{KV_M}{k_B T}}} \quad (\text{A.3})$$

$$\tau_B = \frac{3\mu V_H}{k_B T} \quad (\text{A.4})$$

Here  $\tau_0$  is a constant reference time,  $K$  is the anisotropy constant reflecting the level of directional dependence of the material's magnetic moment,  $k_B$  is the Boltzmann constant, and  $\mu$  is the fluid viscosity.  $V_M$  and  $V_H$  are magnetic volume and hydrodynamic volume of the MNP, respectively, both equal to  $\pi d_{mnp}^3/6$ .

Table A.1: Parameters for the Rosensweig's model [49]

Parameters	Value
Boltzmann constant, $k_B$ (J/K)	$1.38 \times 10^{-23}$
MNP mean diameter, $d_{mnp}$ (m)	$12 \times 10^{-9}$
Vacuum permeability, $\mu_0$ (Tm/A)	$4\pi \times 10^{-7}$
Domain magnetization, $M_d$ (A/m)	$4.46 \times 10^5$
Anisotropy constant, $K$ (J/m <sup>3</sup> )	$2.3 \times 10^4$
Viscosity of carrier fluid, $\mu$ (Pa-s)	$6.92 \times 10^{-4}$
Magnetic field frequency, $f$ (Hz)	$2.0 \times 10^5$
Magnetic field amplitude, $H_0$ (A/m <sup>2</sup> )	$8 \times 10^3$
Magnetic attempt time, $\tau_0$ (s)	$1.0 \times 10^{-9}$

$\chi_0$  in Equation A.1 accounts for equilibrium susceptibility and can be evaluated as [50]

$$\chi_0 = \chi_i \frac{3}{\xi} \left( \coth \xi - \frac{1}{\xi} \right) \quad (\text{A.5})$$

where

$$\chi_i = \frac{\mu_0 \phi M_d^2 V_M}{3k_B T} \quad (\text{A.6})$$

$$\xi = \frac{\mu_0 M_d H_0 V_M}{k_B T} \quad (\text{A.7})$$

Here  $M_d$  is the domain magnetization. The properties for calculation of source term  $Q$  are listed as Table A.1. Note that temperature  $T$  appears in Equations A.3, A.4, A.6 and A.7, making the solution of Equation 1 very challenging. To simplify the problem, in these equations  $T$  is fixed at a constant temperature 43°C, which has also been adopted in other studies such as Ref [49, 51].

## Appendix B. Solving PBHTE using LBM

The LBM is a relatively new CFD method for fluid flow and heat/mass transfer simulations. Unlike traditional CFD methods, which solve the conservation equations of macroscopic properties numerically, LBM models the fluid particles by distribution functions through consecutive streaming and collision processes over a number of square lattices [52–54]. Zhang [34] was prob-

ably the first to apply the LBM to solving PBHTE, successfully demonstrating the capability of LBM in simulating bioheat problems. This mesoscopic approach was then widely applied for bioheat studies [18, 55, 56]. In the present study, a D2Q5 (i.e., two-dimensional five-discrete-velocity) MRT (i.e., multiple-relaxation-time, a collision model that is used to improve the numerical stability [57]) LBM scheme is applied to obtain the temperature field by solving Equation 1 [58]

$$\mathbf{g}(x_k + \delta_t \mathbf{e}, t_n + \delta_t) - \mathbf{g}(x_k, t_n) = -\mathbf{N}^{-1} \mathbf{\Theta} [\mathbf{n} - \mathbf{n}^{(eq)}]_{|(x_k, t_n)} + \mathbf{N}^{-1} \delta_t (\mathbf{I} - \mathbf{\Theta}/2) \mathbf{\Psi} \quad (\text{B.1})$$

where  $\mathbf{g}(x_k, t_n) = (g_0(x_k, t_n), g_1(x_k, t_n), \dots, g_4(x_k, t_n))^T$  is a five-dimensional distribution function vector at time  $t_n$  and node  $x_k$ .  $\mathbf{e}$  describes unit velocities along 5 discrete directions

$$e_i = \begin{cases} (0, 0) & i = 0 \\ (\cos [(i-1)\pi/2], \sin [(i-1)\pi/2])c & i = 1 \sim 4 \end{cases} \quad (\text{B.2})$$

where  $c = \delta_x/\delta_t$  is the lattice speed, which is 1 since  $\delta_x = \delta_t$  in the MRT model.  $\mathbf{N}$  is a  $5 \times 5$  orthogonal transformation matrix

$$\mathbf{N} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ -4 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{pmatrix} \quad (\text{B.3})$$

$\mathbf{\Theta}$  is the diagonal relaxation matrix

$$\mathbf{\Theta} = \text{diag}(1, 1/\tau_T, 1/\tau_T, 1.5, 1.5) \quad (\text{B.4})$$

Table B.1: Studies on computational domain size and grid number

	$L/d_{tum}$	$T_c, ^\circ\text{C}$	$T_r, ^\circ\text{C}$	$Err_c, \%$	$Err_r, \%$
Domain size	5	45.88	43.00	0.36	0.39
	10	45.71	42.83	-	-
	$N_x \times N_y$	$T_c, ^\circ\text{C}$	$T_r, ^\circ\text{C}$	$Err_c, \%$	$Err_r, \%$
Grid number	$50 \times 50$	47.61	43.97	3.77	2.27
	$100 \times 100$	45.88	43.00	0.03	0.05
	$200 \times 200$	45.89	43.02	-	-

where  $\tau_T$  can be linked to thermal diffusivity of nanofluid in Chapman–Enskog analysis on Equation 1 as

$$\alpha_{nf} = c_{sT}^2(\tau_T - 0.5)\delta_t \quad (\text{B.5})$$

$\mathbf{n}$  and  $\mathbf{n}^{(eq)}$  are the velocity moments and the corresponding equilibrium velocity moments, respectively.  $\Psi$  is a source vector including the heat induced by both blood perfusion and nanoparticles. More details about the D2Q5 MRT LBM can be found in [58].

In this study, the adiabatic boundary condition is applied at the four sides of the healthy tissue block, as depicted in Figure 1. As such, the Neumann boundary condition proposed by Yoshino and Inamuro [59] is adopted, which enforces a zero normal heat flux at the four sides. At the tumor-healthy-tissue interface, the continuous temperature condition is enforced, i.e.,  $T_{tum} = T_{tis}$ .

The influence of computational domain size and grid number is also assessed. Take the eight-site injection case on the circular tumor as an example, two sets of domain size and three sets of grids are considered, as listed in Table B.1. The simulated temperatures at two selected locations, i.e., the tumor center ( $T_c$ ) and the rightmost point on the tumor-healthy-tissue interface ( $T_r$ ), are compared, and the comparison results suggest that the domain size of  $5d_{tum} \times 5d_{tum}$  and the grid number of  $100 \times 100$  are sufficient for this study.

To validate our LBM solver, we simulate the same problem studied in Zhang [34], where, as sketched in Figure B.1(a), a square tumor of  $0.25L \times 0.25L$  is surrounded by the healthy tissue that

is represented by a rectangle of  $2L \times L$ , where  $L = 50$  mm. First, three steady heat transfer scenarios are simulated and compared with the results given in Zhang [34]: 1) No tumor - the tumor behaves exactly the same as the healthy tissue; 2) Tumor I - the tumor supplies 10 times the metabolic heat and blood perfusion in the healthy tissue; 3) Tumor II - the tumor supplies 100 times the metabolic heat and blood perfusion in the healthy tissue. The simulated temperature distributions along  $x = 0$  and  $y = L$  are presented in Figure B.1(c) and (d), respectively. They agree very well with the reference. Then, a transient heat transfer process is simulated, in which the tumor generates heat with a constant rate. Good agreements are also achieved in Figure B.1(b) between the simulated temperature profiles along  $x = 0$  and the data from Zhang [34] at three selected instants. This proves the capability of our solver in solving transient heat transfer phenomena.

In another validation study, we simulated the temperature distribution in a rat limb that is subjected to magnetic hyperthermia treatment, and compared it with experimental data [17]. The distribution of injected ferrofluid is assumed to be a Gaussian shape. The reasonably good agreements under two different ferrofluid doses as shown in Figure B.2 confirm the validity of using the present 2D framework to simulate simple 3D problems.

## **Appendix C. Particle Swarm Optimization**

Particle swarm optimization (PSO) is a heuristic optimization algorithm, which performs stochastic search and does not rely on initial conditions, makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions [43]. As a population-based iterative algorithm, PSO emulates the swarm behavior of insects, animals herding, birds flocking, and fish schooling where these swarms search for food in a collaborative manner, and each member searches by learning the experience from itself and others [60]. Each member in the swarm can be treated as a particle. The movements of these particles are guided by their own best-known position in the search-space as well as the entire swarm's best-known position. As such, three mechanisms are involved in the search: inertia, self recognition, and social commu-

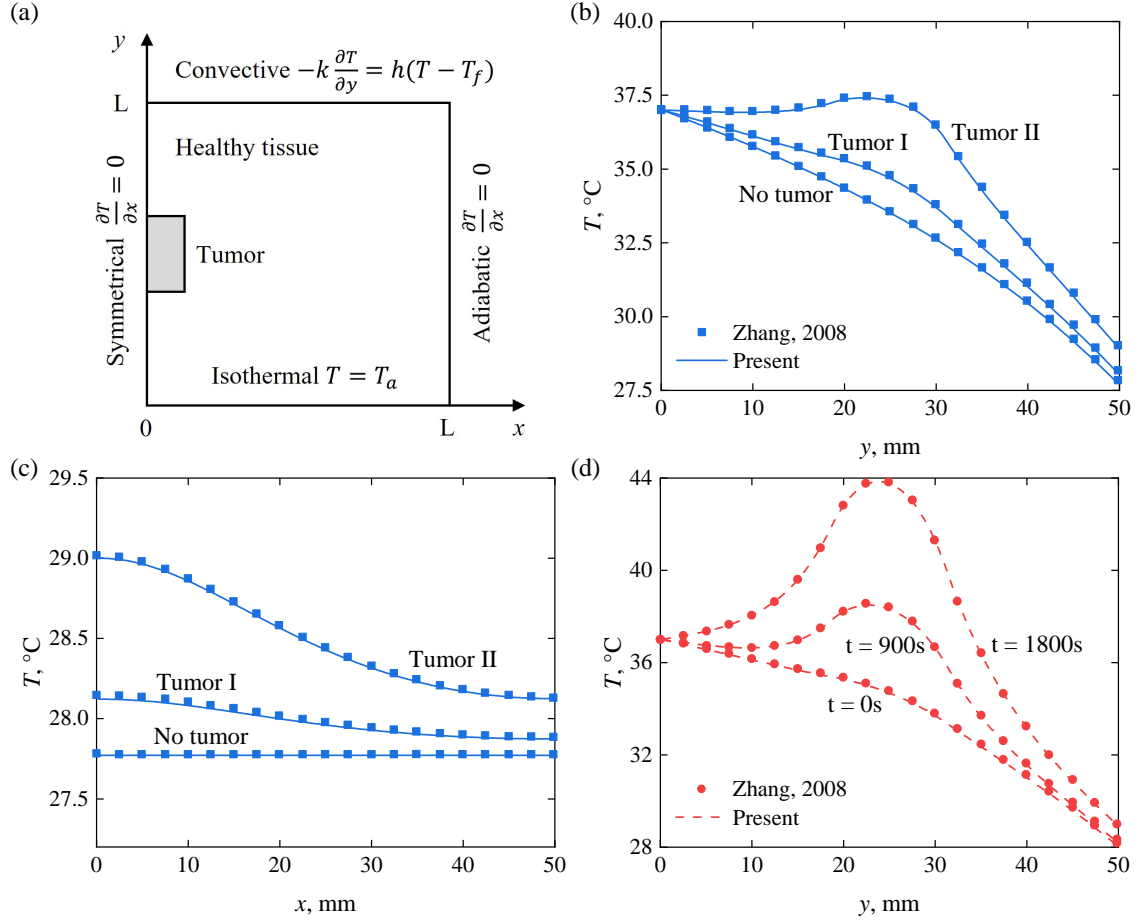


Figure B.1: Validation of the current LBM based PBHTE solver with existing simulation results: (a) Schematic of the hyperthermia problem studied in Zhang [34]; Comparison of temperature distributions in the three steady heat transfer scenarios (b) along the  $y$  axis and (c) along the  $y = L$  line; (d) Comparison of temperature distributions in a transient heat transfer process.



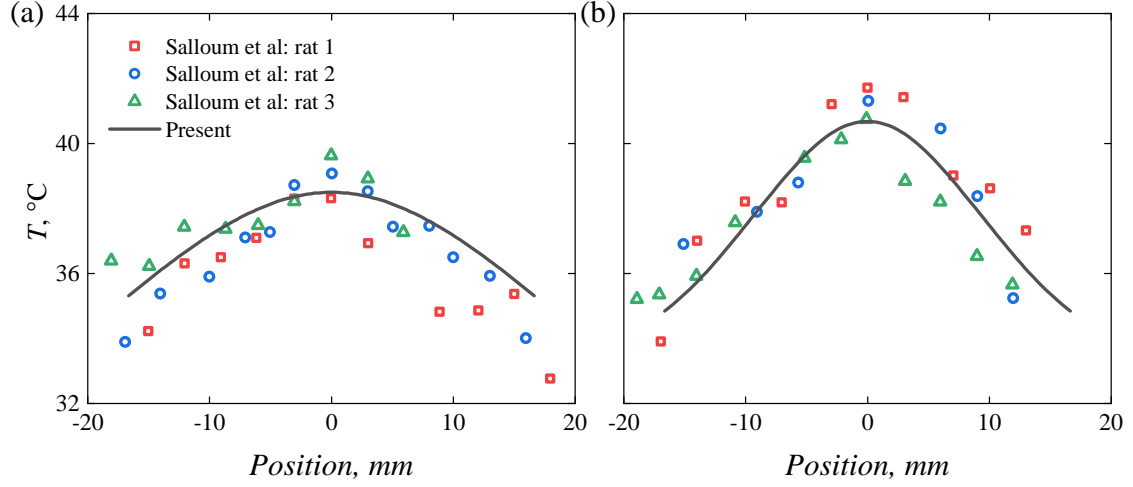


Figure B.2: Validation of the current LBM based PBHTE solver with existing experimental data obtained from rat limbs [17]: (a) 0.1 cc and (b) 0.2 cc ferrofluid injection.

nication, which enable PSO to balance local exploitation and global exploration [61]. Since it is easy to implement and capable of dealing with complex multimodal problems, after being introduced in 1995 by Kennedy and Eberhart [43], PSO has been successfully demonstrated in various applications [62–65], and evolved into many improved schemes [60, 66–71].

In the present study, the PSO with linear decreasing inertia weight is adopted

$$v^{t+1} = wv^t + c_1r_1(p^t - x^t) + c_2r_2(g^t - x^t) \quad (\text{C.1})$$

$$x^{t+1} = v^t + x^t \quad (\text{C.2})$$

where  $x$  denotes the position of particles and  $v$  denotes their velocity.  $w$  is the inertia weight.  $c_1$  and  $c_2$  are the acceleration constants describing the rate of stochastic accelerations that pull the particles toward their own and the group's best positions. In this study,  $w$  linearly reduces from 0.8 to 0 (from 0.9 to 0.4 in the cases not considering  $R_{dose}$ ), while  $c_1$  and  $c_2$  are set as 1.5.  $r_1$  and  $r_2$  are random variables in the range of  $[0, 1]$ .  $p$  and  $g$  are the historical best positions of the current particle and the whole group, respectively.

To validate our PSO model, we perform optimization on three representative benchmark func-

Table C.1: Validation of the current PSO algorithm with benchmark optimization problems

Function	Type	Dimension	Numerical Minimum	Theoretical Minimum
Sphere Function	Unimodal	5	$7.8451 \times 10^{-8}$	0
Shubert Function	Multimodal	2	$-1.8673 \times 10^2$	$-1.8673 \times 10^2$
Rastrigin Function	Multimodal	2	0	0
Rastrigin Function	Multimodal	5	$3.3657 \times 10^{-5}$	0

tions as follows [72, 73]

Sphere Function:

$$f(x) = \sum_{i=1}^n x_i^2$$

Shubert Function:

$$f(x) = \sum_{i=1}^n i \cos((i+1)x_1 + 1) \sum_{i=1}^n i \cos((i+1)x_2 + 1)$$

Rastrigin Function:

$$f(x) = 10n + \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i))$$

The sphere function is unimodal, while the Shubert and Rastrigin functions are multimodal with a large number of local extremes. The optimization results are compared with theoretical maximums as listed in Table C.1. The good performance of our PSO model ensure its validity in dealing with the optimization problem in the present study.

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