How to operate ship fleets under uncertainty

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Abstract: Ships operated by a liner company are scattered around the world to transport goods. A liner company needs to adjust its shipping network every few months by repositioning its ships to respond to uncertain container shipping demand. Few studies investigate a liner company's multi-period heterogeneous fleet deployment problem under uncertainty, considering fleet repositioning, ship chartering, demand fulfillment, cargo allocation, and adaptive fleet sizes. To this end, this study formulates a mixed integer linear programming model that captures all of these elements. This study also designs a Benders-based branch-and-cut algorithm for this NP-hard problem. Two types of acceleration strategies, including approximate upper bound tightening inequalities and Pareto-optimal cuts, are applied to improve the performance of the algorithm. Extensive numerical experiments show that the proposed algorithm significantly outperforms CPLEX and its Benders decomposition framework in solving the model. We conduct an intensive analysis and find that multistage stochastic programming can lead to better solutions than two-stage stochastic programming. We also find that 10% of the benefit provided by the multistage model over the two-stage model is due to better fleet deployment decisions and that 90% of the benefit is due to better demand fulfillment and allocation decisions. By exploring three practical questions regarding driver analysis of liner company profitability, benefits analysis of adaptive fleet sizes, and the influence of the COVID-19 pandemic on liner shipping, we show how liner companies can benefit from managerial insights obtained in this study.

Keywords: Liner shipping operations management; multistage fleet deployment; fleet repositioning; heterogeneous ship fleets; Benders decomposition.

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1 Introduction

The shipping industry plays a vital role in international trade and the global economy (Fransoo and Lee, 2013; Roy et al., 2020). Supported by the recent global economic recovery, approximately 11 billion tons of goods were transported by ship in 2021 (UNCTAD, 2022). In particular, global containerized trade, which decreased by 1.3% in 2020, rebounded in 2021, reaching 165 million 20-foot equivalent units (TEUs) (UNCTAD, 2022), which is evidence that the shipping industry is currently thriving. However, fluctuations in world trade and unexpected incidents, including the COVID-19 pandemic, have brought great uncertainty to the shipping market. Hence, how liner companies operate ship fleets under uncertainty is extremely important.

For a liner company, the operation of ship fleets involves many intertwined decisions, such as the number of heterogeneous fleets (categorized by their load capacities) deployed on each route, which is related to the specific sailing sequence of these ships on each route and to the shipping demand. Liner company managers must determine whether to charter ships in or out when there is a deficit or surplus in a particular ship type, how to reposition ship fleets between different ship routes, whether to fulfill the transportation demand in the current shipment period or to postpone fulfillment to the next period, and how to adjust adaptive fleet sizes to best respond to uncertain demand. The operation of ship fleets under uncertainty is already an intractable problem (Christiansen et al., 2013); the above-intertwined decisions further complicate it.

Uncertainty in the shipping market mainly stems from the changing trends in the world economy caused by changes in seasonal demand and unexpected incidents. Hence, the demand structure may change greatly over a long period of time. For example, affected by the abrupt outbreak of COVID-19, container throughput at the Port of Shanghai fell by 8.4% from January to April 2020 (CWTN, 2021), which inevitably caused changes in the shipping network.

Thus, to remain competitive, liner companies must adjust their ship fleets every few months in response to uncertainty in shipping demand. The uncertain future may have many possible scenarios, and intertwined decisions regarding demand fulfillment and allocation must be made for every possible scenario in each time period of the planning horizon. A key long-term decision for a liner company is how to deploy a heterogeneous ship fleet in a shipping network in the first stage with uncertain demand. In addition to these deployed ships, this study allows for the possibility that a liner company may charter in additional ships for point-to-point transportation if an origin-destination (O-D) pair is in particularly high demand. When optimizing the operation plan of ship fleets, liner companies need careful evaluation and decision support based on scientific methodologies, such as multistage decision models, to comprehensively plan the deployment of ship fleets and their repositioning operations under uncertainty to compete in the growing market. However, throughout the liner shipping industry, the planning of networks, including the construction of routes and fleet movements, is still primarily performed manually, and the fleet repositioning cost is rarely factored into liner shipping models (Wang, 2013). Hence, this study proposes a multistage fleet operation optimization model that involves the first-stage decisions of determining the number of deployed regular ships of different types on ship routes in the network of the liner company, the sailing sequence of these deployed regular ships, and the numbers of chartered-in and chartered-out regular ships when there is a deficit or surplus in some ship types, as well as the decisions in the following stages of determining the numbers of additional point-to-point ships, and the numbers of accepted, delayed, and shipped containers for all O-D pairs in each time period (one time period corresponds to one stage). The objective of this decision model is to maximize the expected total profit earned by the liner company during the planning horizon.

This study is motivated by the above-mentioned real-world challenge and contributes to the literature on liner operations management by proposing a mixed integer linear programming (MILP) model and a Benders-based branch-and-cut (BBC) algorithm with two acceleration strategies. This study provides liner companies with scientific methods to integrate fleet repositioning, ship chartering, demand fulfillment, cargo allocation, and adaptive fleet sizes into their fleet deployment optimization to balance the cost-profit trade-off. Since this problem is NP-hard, we propose an exact and efficient algorithm to solve it on a practical scale.

We leverage real-world shipping routes and conduct computational experiments at different problem scales to evaluate the model without adaptive fleet sizes and the performance of the proposed BBC algorithm and the acceleration strategies. Moreover, we investigate the effect of uncertainty on the operations management of liner companies, and we show that multistage stochastic programming can lead to higher profit than two-stage stochastic programming or deterministic programming. We also conduct an intensive analysis of how multistage stochastic programming can lead to better solutions. Lastly, three practical questions regarding driver analysis of liner company profitability, benefits analysis of adaptive fleet sizes, and the influence of the COVID-19 pandemic on liner shipping are addressed. Managerial insights are obtained to guide the operations of ship fleets under uncertainty for liner companies based on the results of the computational experiments. We believe that the novel model and the algorithm, especially the quantitative decision methodology, may enable liner companies to improve their operations efficiency in an uncertain maritime transportation market.

2 Literature review and discussion

This study focuses on the multistage stochastic fleet deployment problem (FDP) and designs an exact algorithm to solve it. Thus, this section reviews the streams of related literature from three perspectives: the FDP, the design of exact algorithms for maritime-related problems, and the multistage stochastic programming framework.

The first research stream is related to the FDP. Readers interested in overviews of the FDP can refer to Christiansen et al. (2013), Meng et al. (2014), Lee and Song (2017), and Christiansen et al. (2020). As an essential planning problem for liner companies, the FDP assigns available fleets to predetermined routes to maximize the total profit or to minimize the total expense. When liner companies assign fleets to provide predetermined services, several aspects must be considered. One of the most important aspects is ship chartering. Wang and Meng (2012) propose an MILP model for an FDP that allows a liner company to deploy its own and chartered-in ships. Another important aspect lies in the different types of ships in a shipping network, which is a heterogeneous FDP (Wang et al., 2013; Ng, 2014). Tierney et al. (2015) note that the fleet repositioning problem (FRP) has received little attention in the literature related to the FDP and study the FRP with the consideration of cargo flows. Xia et al. (2015) jointly plan fleet deployment, speed optimization, and cargo allocation to maximize the total profit of a liner company. Ng (2017) develops an MILP model for an FDP with the aim of minimizing the total cost, containing the operating cost and the ship chartering cost. Wetzel and Tierney (2020) integrate the FRP into the FDP to determine fleet deployment and ship repositioning. It should be noted that most of the literature on the FDP (e.g., Kepaptsoglou et al., 2015; Zhen et al., 2019) is based on homogeneous fleets, as this assumption simplifies the model and analysis. However, this study considers heterogeneous fleets and allows different types of ships to be deployed on the same route, making our problem more realistic because liner companies often operate different types of ships and deploy these ships according to their transport needs.

Table 1 mainly reviews papers that consider heterogeneous fleets. The majority of the reviewed works only consider deterministic issues. Although a few papers consider uncertainty, they mainly use two-stage stochastic programming to deal with it. However, two-stage stochastic programming assumes that all information about uncertainty is realized after decisions on the first-stage problem are made, which may not reflect some complex and realistic environments. In our problem, liner company managers only know the exact transportation demand for one time period and the probability distributions of demand for time periods immediately beyond the decision time rather than knowing the exact transportation demand for all future time periods. Hence, to address the sequential realization of uncertainties, we use multistage stochastic programming in which uncertainty for a given stage is realized only after the decisions of the previous stage are made.

Regarding decision variables, the majority of reviewed works optimize cargo allocation, and a few consider other realistic issues, such as fleet repositioning and ship chartering. Hence, to help liner companies best respond to uncertain demand, this study examines an integrated problem of fleet deployment, fleet repositioning, ship chartering, demand fulfillment and allocation that allows for container delay, and adaptive fleet sizes. Finally, only a few works offer exact algorithms for the proposed problem, but this study does. In addition, although some papers do not emphasize algorithmic design, they offer mathematical formulations such as novel MILP models by Wang and Meng (2012), Ng (2014), and Ng (2017), and approximate solution approaches such as sample average approximation (SAA) by Wang et al. (2013).

The second stream of related works focuses on the design of exact algorithms for maritime-related problems. Many papers on maritime shipping, such as Brouer et al. (2014), Tierney et al. (2015), Xia

Table 1: Representative works on FDPs

Paper	Fleet	Model	Num. of stages	Decisions (besides deployment)				Methodology
				Fleet repositioning	*Ship chartering*	*Cargo allocation*	*Others*	
Wang and Meng (2012)	Hetero	Deter	N/A	√	√	√	Slot-purchasing, empty container repositioning	MILP
Wang et al. (2013)	Hetero	Stoch	Two		√	√	N/A	SAA
Ng (2014)	Hetero	Stoch	Two		√		N/A	MILP
Brouer et al. (2014)	Hetero	Deter	N/A			√	N/A	Heuristic
Tierney et al. (2015)	Hetero	Deter	N/A	√		√	Speed, empty equipment repositioning	Heuristic
Xia et al. (2015)	Hetero	Deter	N/A			√	Speed	Heuristic
Akyüz and Lee (2016)	Hetero	Deter	N/A			√	Speed, empty container repositioning	Exact
Ng (2017)	Hetero	Deter	N/A		√		N/A	MILP
Zhen et al. (2019)	Homo	Stoch	Two			√	Speed, berth, yard allocation	Exact
Wetzel and Tierney (2020)	Hetero	Deter	N/A	√	√	√	Speed	Heuristic
This paper	Hetero	Stoch	Multi	√	√	√	Container delay, adaptive fleet sizes	Exact

Notes: (1) "Homo" and "Hetero" denote homogeneous and heterogeneous fleet deployment, respectively; (2) "Deter" and "Stoch" denote deterministic model and stochastic model considering uncertainty, respectively; (3) "MILP" and "SAA" denote the mixed integer linear programming and sample average approximation, respectively.

et al. (2015), and Wetzel and Tierney (2020), use heuristics to solve the problems investigated. However, this study develops an exact algorithm. Therefore, we compare algorithmic features of several representative works that design exact algorithms for maritime-related problems. Table 2 presents the exact algorithms proposed in the literature for maritime-related problems. Vis and Roodbergen (2009) study a container terminal scheduling problem of container storage and retrieval, and propose a combination of the assignments in a bipartite network for parts and dynamic programming for the connections between these parts. They use dynamic programming to determine the shortest tour and design a tailored algorithm to determine the sequence of storage and retrieval requests. Engineer et al. (2012) develop a branch-price-and-cut algorithm for a maritime inventory-routing problem. Akyüz and Lee (2016) embed a column generation algorithm within the branch-and-bound algorithm to solve a simultaneous fleet deployment and container routing problem. They also apply a label-correcting algorithm to deal with the shortest path problem in column generation subproblems.

Xu and Lee (2018) develop an exact branch-and-bound algorithm for a continuous berth allocation problem. They obtain a new lower bound and incorporate it within a new heuristic, using a best-fit strategy and new dominance rules for pruning nodes to enhance the performance of the algorithm. Karsten et al. (2018) develop an exact algorithm based on Benders decomposition (BD) and column generation for a simultaneous optimization problem of ship sailing speed and container routing. They also incorporate a warm start, valid inequalities, and callbacks into the algorithm. Wang et al. (2019) design a branch-and-cut algorithm based on BD for a single intercontinental service design problem. Subtour elimination constraints, linear approximation, and valid cuts (symmetry cut, mixed integer knapsack cut, and Pareto-optimal Benders cuts) are included in the algorithm. Zhen et al. (2019) design a dynamic linearization algorithm for an integrated problem of fleet deployment and demand fulfillment. Wang and Meng (2020) use two decomposition methods (stage decomposition and scenario decomposition) to solve a mixed integer programming model of a semi-liner shipping service design problem. They apply primal-dual acceleration and multiple-cut acceleration techniques to enhance the performance of the algorithm. Lee et al. (2021) use a constraint generation approach with several pruning techniques for a two-stage robust optimization model in a liner service procurement problem with service schedules. In summary, the exact algorithm design in these papers is often based on two different methodologies to take advantage of their relative merits and is supplemented by acceleration techniques to improve the convergence of the algorithm.

Table 2: Exact algorithms for solving maritime related problems

Paper	**Basic methodologies**		**Other tactics**
	Primal	Secondary	
Vis and Roodbergen (2009)	DP	Tailored algorithm	N/A
Engineer et al. (2012)	BPC	DP	Preprocessing, boundary constraints, cuts
Akyüz and Lee (2016)	Branch-and-bound	CG	Label-correcting algorithm
Xu and Lee (2018)	Branch-and-bound	Best-fit heuristic	New lower bound, new dominance rules
Karsten et al. (2018)	BD	CG	Warm start, valid inequalities, callbacks
Wang et al. (2019)	BD	Branch-and-cut	Subtour elimination cut, linear approximation cut, and valid cuts
Zhen et al. (2019)	Dynamic linearization	N/A	N/A
Wang and Meng (2020)	Stage decomposition	Scenario decomposition	Primal-dual, multiple-cut acceleration
Lee et al. (2021)	Constraint generation	N/A	Pruning techniques
This paper	BD	Branch-and-cut	Inequalities, Pareto-optimal cuts

Note: DP: dynamic programming; BPC: Branch-price-and-cut; BD: Benders decomposition; CG: Column generation.

The last research stream is about multistage stochastic programming. Uncertainty is a key factor in many decision problems, such as the production routing problem (Adulyasak et al., 2015), the container routing problem (Dong et al., 2015), and the location selection problem (Bayram and Yaman, 2018). As is usual in stochastic programming formulations, uncertainty is represented by a finite set of scenarios, each of which is composed of collective random outcomes (Dong et al., 2015). Most of the stochastic programming methods are based on the two-stage model (Zhen et al., 2019), which is a special case of a more general method called the multistage model. The main difference between two-stage and multistage programming is that the former assumes that all information about uncertainty is known after decisions regarding the first-stage problem have been made, whereas the latter assumes that uncertainty for a given time period (stage) is only realized after the decisions for the previous time period (stage) have been made. Readers who are interested in a more detailed description of multistage stochastic programming can refer to Birge and Louveaux (2011).

In summary, the prevailing trend in the literature on the FDP is to consider homogeneous ships since this assumption simplifies the model and the analysis. Besides, the majority of studies on the heterogeneous FDP do not consider uncertainty issues. To the best of our knowledge, our study is the first to formulate the problem as a multistage stochastic programming model. Our study also incorporates into the FDP some operations decisions that are largely ignored in the literature, such as those concerning fleet repositioning, ship chartering, demand fulfillment, cargo allocation, cargo delay, and adaptive fleet sizes, because fleet repositioning and ship chartering problems are medium- to long-term decisions that have a direct impact on liner companies' operations. Moreover, an exact algorithm based on BD and branch-and-cut algorithms is developed to solve the proposed FDP. Two types of acceleration strategies, i.e., approximate upper bound tightening inequalities and Pareto-optimal cuts, are applied to improve the performance of the proposed algorithm.

3 Problem background

This study focuses on liner operations management under uncertainty. From an academic perspective, a rigorous and complete statement of the core problem in this study is *a liner company's multi-period heterogeneous FDP under uncertainty, considering fleet repositioning, ship chartering, demand fulfillment, cargo allocation, and adaptive fleet sizes*. Before we present a mathematical formulation of the problem, the following sections elaborate on five elements: (1) the shipping network with uncertainty, (2) the model's multistage feature for considering multi-period planning and multistage decision-making, (3) the heterogeneous fleet with different ship types, (4) the liner's fleet repositioning decision, (5) the ship chartering when there is a deficit or surplus of some ship types, which allows for the adaptive fleet sizes.

3.1 Shipping network with uncertainty

Weekly service for a ship route, one of the most common modes in container transport services, means that the headway between two adjacent ships serving the same route is seven days. Suppose that a liner company operates a set R of ship routes (services) that visit ports on a weekly service frequency to transport containers. Figure 1 depicts an illustrative shipping network with three routes denoted by $R = \{1, 2, 3\}$. Each ship route has fixed port rotations, and the itinerary of each route forms a loop. To maintain the weekly service frequency, a fleet of ships rather than a single ship is generally deployed on each route in the service network. For example, it takes a ship three weeks to finish a round trip of ship route 2 in Figure 1 (a round trip is an itinerary of a ship route that forms a loop in practice), and thus a fleet of three ships is required to be deployed on route 2 to maintain a weekly service frequency. Next, denoting the number of ships deployed on route r by n_r, it is easy to understand that the total time for a ship to complete a round trip of the route is $7n_r$ days. Since container ships visit ports of call on a weekly base, the minimum decision-making time unit for liner companies is one week. Here, the "weekly schedule for ships" is not an assumption, but a practical reality. However, travel times are not affected by the weekly service frequency, and we only need to know in which week each ship visits each port. Second, the "time period" considered in this study is used to indicate the decision-making time points. That is, liner companies make decisions at the beginning of time periods 0, 1, and others, but not at other time points.

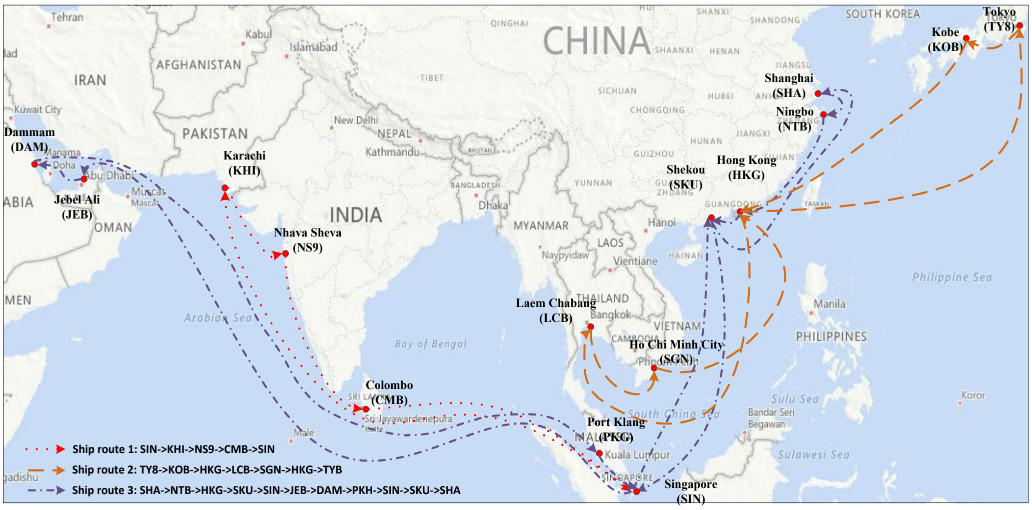

Figure 1 Example of a shipping network with three routes

The liner company operating the above network earns revenue by transporting containers between each O-D pair indexed by (o, d), and the set of all O-D pairs is denoted by D. For each O-D pair, the liner company decides the number of containers to accept, which determines the corresponding profit. In a period, the liner company may accept more containers than it can transport. For example, it may accept more containers in a period if it anticipates low demand in subsequent periods. As a result, some container shipments will be delayed, and the company will incur a penalty cost.

Over a long planning horizon, container shipping demand fluctuates constantly. For example, demand for face masks surged following the sudden outbreak of COVID-19. Hence, considering a shipping network under container transportation demand uncertainty brings our problem closer to reality. To deal with this uncertainty, we use stochastic programming to formulate the problem.

3.2 Multi-period planning and multistage decision-making

Multi-period heterogeneous fleet deployment is often determined for a long planning horizon. Therefore, the external and internal environments inevitably change under uncertainty, and liner company managers tend to divide the planning horizon into shorter time periods to reflect the uncertainty in these different periods. The decisions between adjacent time periods are also correlated because some postponed transportation demand in one time period must be fulfilled in a future time period. After the transportation demand for one time period is realized, the liner company must determine the number of accepted containers to maximize its profit. However, some accepted containers may be delayed due to capacity constraints, resulting in a corresponding penalty cost. This study allows containers to be delayed for multiple periods until the end of the planning horizon, which means that these delayed containers must be shipped during or before the last time period of the planning horizon. Hence, this study investigates how to determine the numbers of accepted, shipped, and delayed containers in each period to maximize the total profit.

In addition to the fleet deployment of regular ships, this study also allows for the possibility that a liner company may charter in additional ships for point-to-point transportation if an O-D pair is in particularly high demand. Specifically, let U represent a subset of O-D pairs (o, d) of ports that can provide additional point-to-point ships in the shipping network, and assume that each origin in U has an infinite number of point-to-point ships ready to be chartered in by the liner company. If the transportation demand for an O-D pair suddenly increases in a certain period and the ships initially deployed by the liner company cannot satisfy the surge in transportation volume, the liner company may charter in additional ships to complete the point-to-point transportation of the cargo. Therefore, frequently adjusting point-to-point ships enables liner companies to best respond to uncertain demand.

Two-stage stochastic programming based on discrete probability distributions is widely used to address uncertain transportation demand and the rental prices of chartered-in point-to-point ships. However, multistage stochastic programming may more accurately reflect the uncertain environment in this problem. The main difference between two-stage and multistage programming is demonstrated in Figure 2. Two-stage stochastic programming assumes that all information about uncertainty is known after making the first-stage decisions, which is not in line with our problem. In our problem, managers of liner companies only know the exact transportation demand and rental price of point-to-point ships for one time period, and the probability distributions of demand and price in the time periods immediately following the moment of their decision. They do not know the exact transportation demand or rental price for all future time periods. Hence, we adopt multistage stochastic programming in which uncertainty for a given stage is realized only after the decision for the previous stage has been made. Here, stage 1 corresponds to period 0, and each of periods 1, 2, ⋯ corresponds to one week. Note that stage 1 (i.e., period 0) is regarded as a few weeks before stage 2 (i.e., period 1) because liner companies usually design shipping services and advertise them for booking several months in advance (Maersk, 2022). Hence, fleet deployment of regular ships (including the available ships of different types on all ship routes in the network, the sailing sequence of these deployed ships, and the numbers of chartered-in and chartered-out regular ships) is determined only in the first stage (which is also period 0). At the end of period 0, all deployed regular ships, including repositioned ships, should be in place. In the following stages (i.e., from period 1), when the container transportation demand and the rental price of chartering in point-to-point ships become realized, the model determines, for each time period under each scenario, the number of chartered-in point-to-point ships, the numbers of accepted, and delayed containers for each O-D pair, and the numbers of shipped containers for each O-D pair by the deployed regular and point-to-point ships.

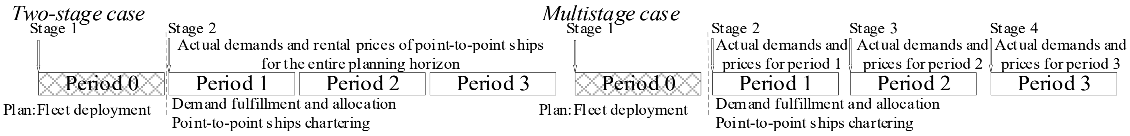

Figure 2 Differences between the two-stage and multistage cases

3.3 Heterogeneous ship fleets

A liner company usually owns heterogeneous ship fleets and therefore deploys different types of ships on each ship route (service). This study classifies ships according to their types. We consider a set K of available regular ship types indexed by k and categorized by their load capacities and costs. We use the example from Figure 1 to discuss the ship types. Suppose that the liner company owns several ships and deploys two 4,000-TEU ships, three 8,000-TEU ships, and four 4,000-TEU ships on ship routes 1, 2, and 3, respectively, and that one 4,000-TEU ship owned by another liner company is idle at the Port of Singapore. Then, the set of all available types is $K = \{\text{type } 1 = 4{,}000\text{-}\ \text{TEU ship}, \text{type } 2 = 8{,}000\text{-}\ \text{TEU ship}\}$. Moreover, since the liner company normally operates a fleet of heterogeneous ships on a given route, the sailing sequence of these ships is also important, as it directly affects the transport capacity of each round trip of the route, which restricts the transportation demand fulfillment and the allocation of containers and influences the profit of the company.

3.4 Fleet repositioning

The shipping industry operates in a competitive and dynamic environment in which liner companies may adjust their shipping networks several times a year. To this end, liner companies generally add, remove, or modify services from their networks, which inevitably requires reassignments of ships between different services of the company, which is the FRP.

If a ship used to serve a specific route is rescheduled to serve another route, a repositioning cost is incurred by fuel consumption and lost revenue. Repositioning a single ship can cost hundreds of thousands of US dollars (Tierney et al., 2015). Although, as shown in Figure 3, repositioning ships is an expensive activity due to potential losses in revenue and high fuel costs, rational fleet repositioning enables the networks to adapt to the global economy and remain competitive. Hence, optimizing the repositioning of ships is of great value to the liner shipping industry.

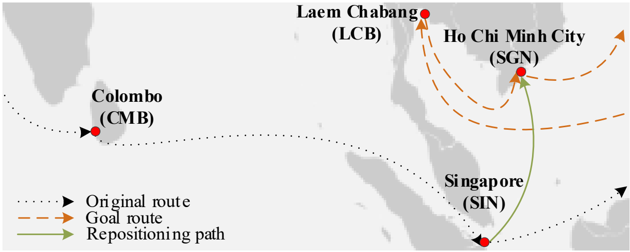

Figure 3 Fleet repositioning from an original route to the first port of call of the goal route

3.5 Ship chartering

Ship chartering is one of the most commonly used methods of maritime cargo transportation. To take part in seaborne trade, liner companies must make significant investments in ships. However, not all liner companies have enough ships to meet the adjustment needs of their shipping networks because the amount of investment capital required for container ships is extremely large. For this reason, liner companies may prefer to charter ships from other liner companies rather than to purchase new ships to handle increased container volume. In addition, when they have a surplus of a certain ship type, liner companies may seek to charter out idle ships to other companies. It is therefore necessary to consider the FRP that allows for ship chartering.

To more clearly explain the heterogeneous FRP with chartering, we use the example in Section 3.3 to characterize the repositioning of regular ships. Let H represent the set of all regular ship groups owned by the liner company and other liner companies, where ships in the same group are of the same ship type. Let H_1 and H_2 represent the subsets of regular ship groups owned by the liner company and other liner companies, respectively. Hence, H_1 has three groups, i.e., $H_1 = \{h_1, h_2, h_3\}$, which are the 4,000-TEU regular ships on route 1, 8,000-TEU regular ships on route 2, and 4,000-TEU regular ships on route 3. H_2 has only one group, $H_2 = \{h_4\}$, which is the 4,000-TEU regular ship idle at the Port of Singapore. Let f_{hr} denote the repositioning cost for a regular ship in group h to move to route r. Obviously, a reasonable fleet repositioning plan can greatly reduce the cost.

In addition to the chartering of regular ships, this study allows for the possibility that a liner company may charter in additional ships for point-to-point transportation if an O-D pair is in particularly high demand.

3.6 Summary of the problem

One point needs to be noted before formulating the mathematical model of this problem. Both the number of containers to be transported and the number of ships to sail on a route to achieve a weekly frequency are defined in weekly terms, which follows industry practice (most liner services have a weekly frequency) and related studies (Xia et al., 2015).

In sum, a shipping network operated by a global liner company should be designed in response to global economic trends and changes in cargo volumes. Liner companies, therefore, have to regularly adjust their service networks to remain competitive, thereby requiring liner companies to reoperate ship fleets during the planning horizon. To deal with these complex and intertwined decisions, this study investigates a liner company's multi-period heterogeneous FDP under uncertainty, considering fleet repositioning, ship chartering, demand fulfillment, cargo allocation, and adaptive fleet sizes. Thus, this study incorporates the first-stage decisions of determining the number of deployed regular ships of different types on ship routes in the network of the liner company, the sailing sequence of these deployed regular ships, and the numbers of chartered-in and chartered-out regular ships when there is a deficit or surplus in some ship types, and the decisions in the following stages of determining, for each time period under each scenario, the number of chartered-in point-to-point ships, the number of accepted containers for each O-D pair, the number of delayed containers for each O-D pair, and the numbers of shipped containers for each O-D pair by the deployed regular ships and point-to-point ships. The objective of the problem is to maximize the expected total profit earned by the liner company during the planning horizon, which consists of seven terms: the container shipping revenue from regular and point-to-point ships, the penalty cost of delayed container delivery, the rental cost of chartering in point-to-point ships, the repositioning cost of regular ships, the operating costs of all deployed regular ships, the rental cost of chartered-in regular ships, and the total profit from chartered-out ships.

4 Model formulation

4.1 Multistage stochastic programming with nonanticipativity constraints

Multistage stochastic programming considers that uncertainty for a given time period (stage) is only realized after the decisions for the previous time period (stage) have been made. It should be noted that stage 1 corresponds to period 0, in which fleet deployment of regular ships is determined; the decisions in the planning horizon from period 1 to the last period concern operational-level plans for point-to-point ships as well as transportation demand fulfillment and allocation.

A scenario tree, an intuitive representation of the branching process induced by the gradual observation of the stochastic progress, is usually used to construct scenarios in multistage problems. The left part of Figure 4 depicts an example with four periods and 10 scenarios to illustrate the scenario tree. One artificial root node, i.e., node 0, is in period 0, in which fleet deployment of regular ships is determined. In period 1, three different realizations of uncertain demand and rental price of chartering in point-to-point ships, represented by nodes 1–3, become known. In period 2, four different realizations, represented by nodes 4–7, become known. The above branching process goes on until the last period, i.e., period 3, where the outcome related to each leaf node in the last period in a unique path from the root to a leaf corresponds to a scenario, which results in a total of 10 scenarios. Hence, a scenario in a multistage stochastic programming model is a path from the root node to a leaf node. For multistage problems, nonanticipativity constraints must be added to ensure that the decisions in a certain period depend only on the data revealed up to that period and not on the information that will be realized in the future (Adulyasak et al., 2015). In other words, the decisions in a period in two different scenarios should be identical if the scenarios share the same parent node in the scenario tree during the period. Here, let $\boldsymbol{y}_{t,s}$ represent the decision variables that need to be decided in period t under scenario s. For the example shown in the left part of Figure 4, the nonanticipativity constraints for periods 1–3 are summarized in the right part of Figure 4.

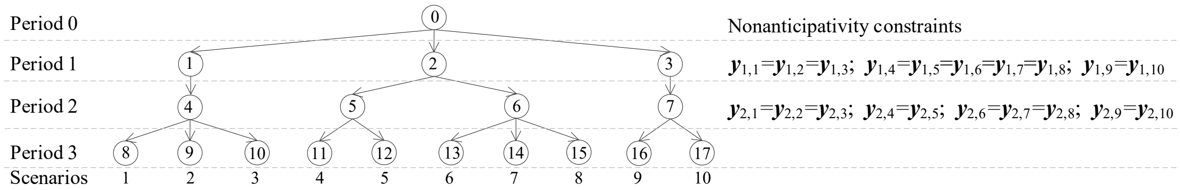

Figure 4 Illustration of a scenario tree and nonanticipativity constraints for multistage programming

4.2 Mathematical model

A multistage stochastic programming model is formulated for this problem.

Sets and indices:

R set of all ship routes in the shipping network, $r \in R$.

K set of all regular ship types, $k \in K$.

R_k subset of routes on which regular ships of type k can be deployed, $R_k \subset R$.

K_r subset of regular ship types that can be deployed on route r, $K_r \subset K$.

H set of all regular ship groups owned by the liner company and other liner companies, $h \in H$; ships in the same group are of the same ship type.

H_1 subset of regular ship groups owned by the liner company, $H_1 \subset H$.

H_2 subset of regular ship groups owned by other liner companies, $H_2 \subset H$.

D set of all O-D pairs of ports that are traversed by all routes in the shipping network, $(o, d) \in D$.

U subset of O-D pairs of ports that can provide additional point-to-point ships in the shipping network, $U \subset D$.

T set of time periods of the planning horizon starting from period 1, $t \in T$.

E_r set of round trips that are operated on route r during the planning horizon, $e \in E_r$.

I_r set of voyage legs of route r, $i \in I_r$.

S set of scenarios, $s \in S$.

Z_+ set of non-negative integers.

Parameters:

n_r number of regular ships deployed on ship route $r \in R$, which equals the number of periods for a deployed regular ship to traverse the ship route.

v_k number of containers that can be carried by a regular ship of type k, which is the capacity of ships of type k.

$f_{h,r}$ repositioning cost for a regular ship in group $h \in H$ to move to route $r \in R$.

u_h number of regular ships in group $h \in H$.

y_h ship type of regular ships in group $h \in H$.

$c_{k,r}$ operating cost of completing the voyages during the planning horizon by a regular ship of type k deployed on route r.

g_k rental cost of chartering in a regular ship of type k from other liner companies.

m_k profit of chartering out a regular ship of type k to other liner companies, $m_k < g_k$.

$\hat{g}_{o,d}^s$ rental cost of chartering in a point-to-point ship completing a specific leg from port o to port d, $(o, d) \in U$, under scenario s, $s \in S$.

$\hat{v}$ capacity (number of containers that can be carried) of a point-to-point ship.

$q_{o,d,t}^s$ newly generated container shipping demand (number of containers) in period t at port o to be transported to port d under scenario s.

$l_{o,d}$ revenue generated by each accepted container for a specific O-D pair (o, d).

$p_{o,d}$ penalty cost per period incurred by each delayed container for a specific O-D pair (o, d).

$j_{r,e}$ index (i.e., sequence position in $\{1, \dots, n_r\}$) of the regular ship that operates the e^{th} round trip of route r.

$a_{r,e,i,o,d,t}$ binary, if O-D pair $(o, d) \in D$ demand shipped in period $t \in T$ at origin port o are carried by a regular ship on voyage leg $i \in I_r$ of the e^{th} round trip on route r, it equals 1; otherwise it equals 0.

w^s probability of scenario s.

b_t^s index of node in the scenario tree at period t related to scenario s.

Decision variables:

$\alpha_{h,r}$ integer, number of regular ships from group $h \in H$ deployed on route $r \in R$.

$\beta_{k,r}$ integer, number of regular ships of type $k \in K_r$ deployed on route $r \in R$.

$\pi_{k,r,j}$ binary, if a regular ship of type $k \in K_r$ is deployed on the j^{th} ($j \in \{1, \dots, n_r\}$) sequence position of route $r \in R$ (i.e., the j^{th} regular ship on route r belongs to type k), it equals 1; otherwise, it equals 0.

$\gamma_{o,d,t}^{s}$ integer, number of point-to-point ships completing a specific voyage leg from port o to port d, $(o,d) \in U$, departing from period t, $t \in T$, under scenario s, $s \in S$.

$\theta_{o,d,t}^{s}$ continuous, number of accepted containers by ships for the demand of O-D pair $(o,d) \in D$ accumulated in period $t \in T$ under scenario s.

$\varphi_{o,d,t}^{s}$ continuous, number of delayed containers by ships for the demand of O-D pair $(o,d) \in D$ up to period $t \in T \cup \{0\}$ under scenario s, where by convention, $\varphi_{o,d,0}^{s} := 0$.

$\varepsilon_{o,d,t}^{s}$ continuous, number of shipped containers by deployed regular ships in the first stage for the demand of O-D pair (o,d) in period t (including both those accepted in period t and the delayed containers in previous periods) under scenario s.

$\delta_{o,d,t}^{s}$ continuous, total number of shipped containers transported by point-to-point ships for the demand of O-D pair (o,d), $(o,d) \in U$, in period t (including both those accepted in period t and the delayed containers in previous periods), $t \in T$, under scenario s, $s \in S$.

$\tilde{\gamma}_{o,d,t}^{b_t^s}$ integer, variable $\gamma_{o,d,t}^{s}$ related to node b_t^s.

$\tilde{\theta}_{o,d,t}^{b_t^s}$ continuous, variable $\theta_{o,d,t}^{s}$ related to node b_t^s.

$\tilde{\varphi}_{o,d,t}^{b_t^s}$ continuous, variable $\varphi_{o,d,t}^{s}$ related to node b_t^s.

$\tilde{\varepsilon}_{o,d,t}^{b_t^s}$ continuous, variable $\varepsilon_{o,d,t}^{s}$ related to node b_t^s.

$\tilde{\delta}_{o,d,t}^{b_t^s}$ continuous, variable $\delta_{o,d,t}^{s}$ related to node b_t^s.

Since this problem is a multistage problem, we need to make the decisions for stage t without knowing the demand and rental price in future periods. According to the notation introduced, the multistage stochastic programming model is formulated as follows:

$$[\mathrm{M}_1]\ \mathrm{Max} \sum_{s\in S}\sum_{t\in T}\sum_{(o,d)\in D} w^s l_{o,d}\theta_{o,d,t}^{s} - \sum_{s\in S}\sum_{t\in T}\sum_{(o,d)\in D} w^s p_{o,d}\varphi_{o,d,t}^{s} - \sum_{s\in S}\sum_{t\in T}\sum_{(o,d)\in U} w^s \hat{g}_{o,d}^{s}\gamma_{o,d,t}^{s} - \sum_{h\in H}\sum_{r\in R_{y_h}} f_{h,r}\alpha_{h,r} - \sum_{k\in K}\sum_{r\in R_k} c_{k,r}\beta_{k,r} - \sum_{h\in H_2}\sum_{r\in R_{y_h}} g_{y_h}\alpha_{h,r} + \sum_{h\in H_1} m_{y_h}\left(u_h - \sum_{r\in R_{y_h}}\alpha_{h,r}\right) \quad (1)$$

subject to:

$$\beta_{k,r} = \sum_{j\in\{1,\dots,n_r\}} \pi_{k,r,j} \qquad \forall k \in K, r \in R_k \quad (2)$$

$$\sum_{k\in K_r} \pi_{k,r,j} = 1 \qquad \forall\, r \in R, j \in \{1,\dots,n_r\} \quad (3)$$

$$\sum_{r\in R_{y_h}} \alpha_{h,r} \le u_h \qquad \forall\, h \in H \quad (4)$$

$$\sum_{h\in H, y_h=k} \alpha_{h,r} = \beta_{k,r} \qquad \forall k \in K, r \in R_k \quad (5)$$

$$\theta_{o,d,t}^{s} + \varphi_{o,d,t-1}^{s} = \varepsilon_{o,d,t}^{s} + \varphi_{o,d,t}^{s} \qquad \forall (o,d) \in D\backslash U, t \in T, s \in S \quad (6)$$

$$\theta_{o,d,t}^{s} + \varphi_{o,d,t-1}^{s} = \varepsilon_{o,d,t}^{s} + \delta_{o,d,t}^{s} + \varphi_{o,d,t}^{s} \qquad \forall (o,d) \in U, t \in T, s \in S \quad (7)$$

$$\delta_{o,d,t}^{s} \le \hat{v}\gamma_{o,d,t}^{s} \qquad \forall\, (o,d) \in U, t \in T, s \in S \quad (8)$$

$$\sum_{(o,d)\in D}\sum_{t\in T} a_{r,e,i,o,d,t}\, \varepsilon_{o,d,t}^{s} \le \sum_{k\in K_r} v_k\, \pi_{k,r,j_{re}} \qquad \forall\, r \in R, e \in E_r, i \in I_r, s \in S \quad (9)$$

$$\theta_{o,d,t}^{s} \le q_{o,d,t}^{s} \qquad \forall (o,d) \in D, s \in S,\ t \in T \quad (10)$$

$$\varphi_{o,d,0}^{s} = 0 \qquad \forall (o,d) \in D, s \in S \qquad (11)$$

$$\varphi_{o,d,|T|}^{s} = 0 \qquad \forall (o,d) \in D, s \in S \qquad (12)$$

$$\theta_{o,d,t}^{s} = \tilde{\theta}_{o,d,t}^{b_t^s} \qquad \forall (o,d) \in D, s \in S,\ t \in T \qquad (13)$$

$$\varepsilon_{o,d,t}^{s} = \tilde{\varepsilon}_{o,d,t}^{b_t^s} \qquad \forall (o,d) \in D, s \in S,\ t \in T \qquad (14)$$

$$\varphi_{o,d,t}^{s} = \tilde{\varphi}_{o,d,t}^{b_t^s} \qquad \forall (o,d) \in D, s \in S, t \in T \cup \{0\} \qquad (15)$$

$$\gamma_{o,d,t}^{s} = \tilde{\gamma}_{o,d,t}^{b_t^s} \qquad \forall (o,d) \in U, s \in S,\ t \in T \qquad (16)$$

$$\delta_{o,d,t}^{s} = \tilde{\delta}_{o,d,t}^{b_t^s} \qquad \forall (o,d) \in U, s \in S,\ t \in T \qquad (17)$$

$$\alpha_{h,r} \in Z_+ \qquad \forall\, h \in H, r \in R_{y_h} \qquad (18)$$

$$\beta_{k,r} \in Z_+ \qquad \forall k \in K, r \in R_k \qquad (19)$$

$$\pi_{k,r,j} \in \{0,1\} \qquad \forall k \in K, r \in R_k, j \in \{1, \dots, n_r\} \qquad (20)$$

$$\gamma_{o,d,t}^{s} \in Z_+, \tilde{\gamma}_{o,d,t}^{b_t^s} \in Z_+ \qquad \forall\, (o,d) \in U, t \in T, s \in S \qquad (21)$$

$$\theta_{o,d,t}^{s} \geq 0, \varepsilon_{o,d,t}^{s} \geq 0, \tilde{\theta}_{o,d,t}^{b_t^s} \geq 0, \tilde{\varepsilon}_{o,d,t}^{b_t^s} \geq 0 \quad \forall (o,d) \in D, s \in S,\ t \in T \qquad (22)$$

$$\delta_{o,d,t}^{s} \geq 0, \tilde{\delta}_{o,d,t}^{b_t^s} \geq 0 \qquad \forall (o,d) \in U, t \in T, s \in S \qquad (23)$$

$$\varphi_{o,d,t}^{s} \geq 0, \tilde{\varphi}_{o,d,t}^{b_t^s} \geq 0 \qquad \forall (o,d) \in D, s \in S, t \in T \cup \{0\}. \qquad (24)$$

Objective function (1) maximizes the expected profit earned by the liner company. Constraints (2) compute the number of regular ships of type k deployed on each route r. Constraints (3) guarantee that at each sequence position j of route r, exactly one type of regular ship is deployed. Constraints (4) require the total number of used regular ships on all routes from each regular ship group not to exceed the total number of regular ships available in the group. By constraints (5), the total number of used regular ships deployed on route r from all regular ship groups of type k equals the number of regular ships of type k deployed on route r. Constraints (6)–(7) are the balance equations for the numbers of accepted, delayed, and shipped containers between each O-D pair (o,d) in each time period $t \in T$ under each scenario s. Constraints (8) guarantee that the total number of shipped containers by point-to-point ships for O-D pair (o,d), $(o,d) \in U$, in period t under scenario s cannot exceed the total capacity of all deployed point-to-point ships completing the voyage leg from port o to port d departing from period t. Constraints (9) guarantee that for each route $r \in R$ in the shipping network, the number of shipped containers by regular ships on voyage leg $i \in I_r$ of round trip $e \in E_r$ cannot exceed the capacity of the j_{re}^{th} regular ship deployed on route r under scenario s. An example with detailed explanations of constraints (9) can be found in Appendix 1. Constraints (10) state that the number of accepted containers cannot exceed the demand under scenario s. Constraints (11) and (12) are the boundary conditions of $\varphi_{o,d,t}^{s}$ when t equals 0 and $|T|$, respectively. Specifically, constraints

(11) mean that no containers are delayed before the start of the business for the O-D pair, and constraints (12) guarantee that all of the accepted containers should be shipped before the end of the planning horizon. Constraints (13)–(17) are the nonanticipativity constraints of multistage stochastic programming. Constraints (18)–(24) define the domains of the decision variables. Our problem is NP-hard, since the 0-1 knapsack problem can be reduced into it.

5 Benders-based branch-and-cut (BBC) algorithm

The NP-hardness of our problem means that it is highly unlikely that a polynomial-time algorithm can be designed for this problem. We need an efficient and exact algorithm to solve the problem of practical scale with realistic data. Due to a large number of decision variables and constraints in the problem, solving the MILP model [M1] is difficult. BD is a partitioning algorithm for large-scale MILP models and that is shown to be efficient in solving many stochastic programming problems (Rei et al., 2009; Adulyasak et al., 2015). However, the presence of integrality constraints in master problems (MPs) in the basic BD algorithm substantially increases the solution time. Hence, we introduce a BBC algorithm to solve model [M1] that solves the linear programming (LP) relaxation of the MP at each iteration. Moreover, a tailored acceleration strategy and a standard acceleration technique from stochastic programming (Pareto-optimal cuts) are applied to improve the convergence of the BBC algorithm. The algorithm is designed for settings without adaptive fleet sizes. Once adaptive fleet sizes are considered, we also need to consider the variables related to point-to-point ships, that is, $\gamma_{o,d,t}^{s}$ as well as $\tilde{\gamma}_{o,d,t}^{b_t^s}$, and $\delta_{o,d,t}^{s}$ as well as $\tilde{\delta}_{o,d,t}^{b_t^s}$ belong to MPs and primal subproblems, respectively.

5.1 Overview of the solution approach

In brief, a BBC algorithm is a branch-and-bound algorithm in which BD is used to compute upper bounds by solving linear relaxations of MPs (for maximization problems), and Benders cuts may be added to strengthen the linear relaxations of MPs. Specifically, in a basic BD algorithm, the presence of integrality constraints in the MP makes its solution time much longer than that of dual subproblems. The computational complexity of the MP results in slow convergence of the BD algorithms. To deal with this issue, Geoffrion et al. (1974) indicate that obtaining the optimal solution to the MP at each iteration is not necessary, which means that obtaining a near-optimal solution efficiently may be beneficial. Embedding Benders cuts within a branch-and-cut framework for solving the MP is an efficient way to deal with the above issue (Pearce and Forbes, 2018). An outline of our BBC algorithm is summarized in Algorithm 1.

5.2 Benders decomposition

Algorithm 1. Benders-based branch-and-cut algorithm

1 Initialize the tree L: $L = \{\tilde{o}\}$ where $\tilde{o}$ is the original restricted master problem [M4] formulated in Section 5.2 without branching constraints.
$OBJ^* \leftarrow -\infty$ // OBJ^* records the incumbent objective function value of [M4].
$(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}, \Omega)^* \leftarrow$null // $\boldsymbol{\alpha}, \boldsymbol{\beta}$, and $\boldsymbol{\pi}$ represent vectors of α_{hr} ($h \in H, r \in R_{y_h}$), β_{kr} ($k \in K, r \in R_k$), and π_{krj} ($k \in K, r \in R_k, j \in \{1, \dots, n_r\}$), respectively; Ω is an extra decision variable defined in Section 5.2; $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}, \Omega)^*$ records the incumbent solution of corresponding decision variables in [M4].
2 **While** L is nonempty **do**
3 Select a node $o \in L$ according to the best-bound-first node selection rule.
4 $L := L \setminus \{o\}$.
5 Solve the linear relaxation of o (LP).
6 **If** LP is infeasible **then**
7 Prune the node.
8 **Else**
9 Obtain an optimal solution $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}, \Omega)$ and the optimal objective function value OBJ.
10 **If** $OBJ \le OBJ^*$ **then**
11 Prune the node.
12 **Else if** $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\pi}$ are all integer **then**
13 Solve model [M3] formulated in Section 5.2 based on $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}, \Omega)$ and generate Benders cuts.
14 **If** no cuts are generated **then**
15 Update $OBJ^* = OBJ$ and $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}, \Omega)^* \leftarrow (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\pi}, \Omega)$.
16 Prune the node.
17 **Else**
18 Add the cuts to [M4] and go to line 5.
19 **End if**
20 **Else**
21 Branch according to the maximum fractional branching rule, resulting in nodes o_1 and o_2, $L := L \cup \{o_1, o_2\}$.
22 **End if**
23 **End if**
24 **End while**
25 The algorithm terminate as the optimal solution is found.

The BD algorithm is frequently used to solve liner shipping problems (Chen et al., 2018). In our problem, once the first-stage decisions are fixed, the resulting subproblem, i.e., the primal subproblem, is a demand fulfillment and allocation problem with only continuous variables. Let $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\pi}$ denote the vectors of fixed $\alpha_{h,r}$, $\beta_{k,r}$, and $\pi_{k,r,j}$, respectively. The decision variables in the following stages are only dependent on $\bar{\pi}$ but are independent of $\bar{\alpha}$ and $\bar{\beta}$. Hence, the expected operational profit function can be computed by solving the primal subproblem formulated as follows.

$$[\text{M2}]\ \text{Max}\left\{\sum_{s\in S} w^s \sum_{(o,d)\in D} \sum_{t\in T} \left(l_{o,d}\theta^s_{o,d,t} - p_{o,d}\varphi^s_{o,d,t} \right)\right\} \tag{25}$$

subject to: constraints (6), (10)–(15), (22), (24),

$$\sum_{(o,d)\in D} \sum_{t\in T} a_{r,e,i,o,d,t}\, \varepsilon^s_{o,d,t} \le \sum_{k\in K_r} v_k \bar{\pi}_{k,r,j_{r,e}} \quad \forall\, r \in R, e \in E_r, i \in I_r, s \in S. \tag{26}$$

Due to the presence of the decision variables $\theta^s_{o,d,t}$, the above primal subproblem is always feasible because the number of accepted containers can be zero, which implies that the numbers of delayed and shipped containers are also zero. In addition, since the parameters $l_{o,d}$ and $p_{o,d}$ are finite and because of the constraints in model [M2], any feasible solution to the primal subproblem must be bounded. As a result, the dual of the primal subproblem is always feasible and bounded, which means that only optimality cuts need to be added to the MP. Let $\omega = \{\omega_{o,d,t,s} | (o,d) \in D, t \in T, s \in S\}$, $\mu = \{\mu_{o,d,t,s} | (o,d) \in D, t \in T, s \in S\}$, $\eta = \{\eta_{o,d,s} | (o,d) \in D, s \in S\}$, $\zeta = \{\zeta_{o,d,s} | (o,d) \in D, s \in S\}$, $\rho^\theta =$

$\{\rho^{\theta}_{o,d,t,s} | (o,d) \in D, t \in T, s \in S\}$, $\rho^{\varepsilon} = \{\rho^{\varepsilon}_{o,d,t,s} | (o,d) \in D, t \in T, s \in S\}$, $\rho^{\varphi} = \{\rho^{\varphi}_{o,d,t,s} | (o,d) \in D, t \in T \cup \{0\}, s \in S\}$, and $\sigma = \{\sigma_{r,e,i,s} | r \in R, e \in E_r, i \in I_r, s \in S\}$ be the dual variables related to constraints (6), (10)–(15), and (26), respectively. The dual of the primal subproblem is called the dual primal subproblem, and the polyhedron defined by the constraints of the dual primal subproblem is denoted as P_{Δ}. Hence, the dual primal subproblem can be formulated as follows.

$$\text{[M3]} \quad \text{Min}\{\sum_{s\in S} w^s [\sum_{r\in R} \sum_{e\in E_r} \sum_{i\in I_r} \sum_{k\in K_r} v_k \bar{\pi}_{k,r,j_{r,e}} \sigma_{r,e,i,s} + \sum_{(o,d)\in D} \sum_{t\in T} q^s_{o,d,t} \mu_{o,d,t,s}]\} \tag{27}$$

subject to: $(\omega, \mu, \eta, \zeta, \rho^{\theta}, \rho^{\varepsilon}, \rho^{\varphi}, \sigma) \in P_{\Delta}$. (28)

We further introduce the set of extreme points of P_{Δ} as Φ_{Δ}. We define an extra variable Ω representing the expected total profit of the primal subproblem. The previous multistage model can be reformulated as an MP formulated as follows.

$$\text{[M4]} \quad \text{Max}\left\{\Omega - \sum_{h\in H} \sum_{r\in R_{y_h}} f_{h,r} \alpha_{h,r} - \sum_{k\in K} \sum_{r\in R_k} c_{k,r} \beta_{k,r} - \sum_{h\in H_2} \sum_{r\in R_{y_h}} g_{y_h} \alpha_{h,r} + \sum_{h\in H_1} m_{y_h} (u_h - \sum_{r\in R_{y_h}} \alpha_{h,r})\right\} \tag{29}$$

subject to: constraints (2)–(5), (18)–(20),

$$\sum_{s\in S} w^s [\sum_{r\in R} \sum_{e\in E_r} \sum_{i\in I_r} \sum_{k\in K_r} v_k \pi_{k,r,j_{r,e}} \sigma_{r,e,i,s} + \sum_{(o,d)\in D} \sum_{t\in T} q^s_{o,d,t} \mu_{o,d,t,s}] \geq \Omega$$
$$\forall (\omega, \mu, \eta, \zeta, \rho^{\theta}, \rho^{\varepsilon}, \rho^{\varphi}, \sigma) \in \Phi_{\Delta}. \tag{30}$$

The BD algorithm solves the master problem and the primal subproblem repeatedly. Specifically, the BD algorithm first solves the MP to optimality, which leads to an upper bound for the original problem because this problem is a maximization problem. The dual primal subproblem is then solved given the values of $\bar{\pi}_{k,r,j}$ from the optimal solution to the master problem. One or several new Benders cuts, i.e., constraints (30) (if any), are added to the master problem at each iteration. When the decision variables $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\pi}$ in model [M4] are all integers and no Benders cuts are generated, the objective function value of the model [M4] provides a lower bound on the objective function value of the original problem. When the upper and lower bounds of the original problem converge, the algorithm terminates.

The scalability of our proposed algorithm is emphasized from three perspectives. First, the proposed algorithm can be directly applied to a number of two-stage and multistage stochastic programming models for existing liner fleet deployment problems, such as the two-stage and multistage stochastic programming models for homogeneous fleet deployment problems. Second, the proposed algorithm can be applied to complex fleet deployment problems. For example, the proposed model can handle weather uncertainty in ports, which is important because weather conditions have a direct influence on the availability of ships moored there. However, if weather disturbances are considered, the multistage optimization problem needs to include additional decision variables (e.g., the arrival time of ships) and constraints (e.g., only ships at ports under good weather conditions are available for container shipping services), which requires a slight adjustment to the current algorithm, i.e., adding weather-related constraints to the master problem. With this adjustment, the proposed algorithm could be applied as an effective algorithm to complex, real-world problems. Third, the proposed algorithm can be extended to multistage stochastic programming models for fleet deployment in various complex transportation systems, such as the air cargo shipping problem, which is similar to the liner shipping problem.

5.3 Acceleration strategies

Since BD was introduced, numerous studies have sought to improve it. For example, Magnanti and Wong (1981) introduce a new method, i.e., strong or Pareto-optimal cuts, to accelerate the convergence of the BD algorithm for MILP models. Cordeau et al. (2006) propose valid inequalities to strengthen the LP relaxation of MILP models, thereby improving the performance of the BD algorithm. Hence, this study discusses two acceleration strategies for the proposed BBC algorithm. Compared with the above papers, the acceleration strategies proposed in this section provide a more specific approach to our problem, and also a more efficient one, by considering specific features of the problem and models.

Since the quality of the upper bound in the initial solving stages is inferior, the optimality gap may be large in the initial stages of the algorithm, which results in the need for a large number of cuts. Hence, we can tighten the upper bound for the MP by using some initial cuts called upper bound tightening (UBT) inequalities. To obtain the approximate UBT inequalities, we first develop a nominal second-stage problem. Newly defined parameters and decision variables are introduced as follows.

Newly defined parameter:

$\bar{q}_{o,d,t}$ expected number of newly generated container shipping demand in period t at port o to be transported to port d, $\bar{q}_{o,d,t} = \mathbb{E}_s\left[q^s_{o,d,t}\right]$.

Newly defined decision variables:

$\bar{\theta}_{o,d,t}$ continuous, the number of accepted containers for the demand of O-D pair $(o,d) \in D$ accumulated in period $t \in T$ with the expected parameters.

$\bar{\varphi}_{o,d,t}$ continuous, the number of delayed containers for the demand of O-D pair $(o,d) \in D$ up to period $t \in T \cup \{0\}$ with the expected parameters; by convention, $\bar{\varphi}_{o,d,0} := 0$.

$\bar{\varepsilon}_{o,d,t}$ continuous, the number of shipped containers for the demand of O-D pair $(o,d) \in D$ in period $t \in T$ with the expected parameters.

The problem replacing the previous subproblem [M2] with the new second-stage problem is therefore a two-stage deterministic problem. According to the notation introduced, the second-stage deterministic programming model is then formulated:

[M5] $\mathrm{Max}\{\sum_{(o,d)\in D}\sum_{t\in T}(l_{o,d}\bar{\theta}_{o,d,t} - p_{o,d}\bar{\varphi}_{o,d,t})\}$ (31)

subject to: $\bar{\theta}_{o,d,t} + \bar{\varphi}_{o,d,t-1} = \bar{\varepsilon}_{o,d,t} + \bar{\varphi}_{o,d,t} \quad \forall (o,d) \in D, t \in T$ (32)

$\sum_{(o,d)\in D}\sum_{t\in T} a_{r,e,i,o,d,t}\,\bar{\varepsilon}_{o,d,t} \le \sum_{k\in K_r} v_k\,\bar{\pi}_{k,r,j_{r,e}} \quad \forall\, r \in R, e \in E_r, i \in I_r$ (33)

$\bar{\theta}_{o,d,t} \le \bar{q}_{o,d,t} \quad \forall (o,d) \in D, t \in T$ (34)

$\bar{\varphi}_{o,d,0} = 0 \quad \forall (o,d) \in D$ (35)

$\bar{\varphi}_{o,d,|T|} = 0 \quad \forall (o,d) \in D$ (36)

$\bar{\theta}_{o,d,t} \ge 0, \bar{\varepsilon}_{o,d,t} \ge 0 \quad \forall (o,d) \in D, t \in T$ (37)

$\bar{\varphi}_{o,d,t} \ge 0 \quad \forall (o,d) \in D, t \in T \cup \{0\}.$ (38)

Objective function (31) maximizes the operational profit earned by the liner company during the planning horizon in the second-stage deterministic problem. Constraints (32)–(38) update the related constraints for the deterministic problem.

Similarly, the dual of the second-stage problem [M5] is always feasible and bounded, which means that only optimality cuts need to be added. Let $\omega' = \{\omega'_{o,d,t} | (o,d) \in D, t \in T\}$, $\sigma' = \{\sigma'_{r,e,i} | r \in R, e \in E_r, i \in I_r\}$, $\mu' = \{\mu'_{o,d,t} | (o,d) \in D, t \in T\}$, $\eta' = \{\eta'_{o,d} | (o,d) \in D\}$, and $\zeta' = \{\zeta'_{o,d} | (o,d) \in D\}$ be the dual variables associated with constraints (32)–(36), respectively. We further denote the polyhedron defined by the constraints of the dual of the second-stage problem as P'_Δ. Finally, the dual primal subproblem for the second-stage problem [M5] can be formulated as follows:

[M6] $\mathrm{Min}\{\sum_{r\in R}\sum_{e\in E_r}\sum_{i\in I_r}\sum_{k\in K_r} v_k\bar{\pi}_{k,r,j_{re}}\sigma'_{r,e,i} + \sum_{(o,d)\in D}\sum_{t\in T}\bar{q}_{o,d,t}\mu'_{o,d,t}\}$ (39)

subject to: $(\omega', \sigma', \mu', \eta', \zeta') \in P'_\Delta$. (40)

We further introduce the set of extreme points of P'_Δ as Φ'_Δ. Hence, we can obtain the approximate UBT inequalities from the nominal second-stage problem of the original master problem:

$$\Theta[\sum_{r\in R}\sum_{e\in E_r}\sum_{i\in I_r}\sum_{k\in K_r} v_k\pi_{k,r,j_{r,e}}\sigma'_{r,e,i} + \sum_{(o,d)\in D}\sum_{t\in T}\bar{q}_{o,d,t}\mu'_{o,d,t}] \ge \Omega \quad \forall (\omega', \sigma', \mu', \eta', \zeta') \in \Phi'_\Delta, \tag{41}$$

where Θ is a parameter for dynamic adjustment in the algorithm. At first, Θ is set to 1 to make the algorithm converge quickly. When inequalities (41) become tight, Θ is set to 1.01. Finally, inequalities (41) are removed to ensure the validity of the original master problem because inequalities (41) are derived from the nominal second-stage problem [M5] and may not hold for the original master problem. Here, inequalities (41) are obtained as the approximate UBT inequalities from the nominal second-stage problem instead of Benders cuts for the original master problem.

In addition to the above introduced UBT inequalities, Pareto-optimal cuts are applied to accelerate the BBC algorithm. Since this technique is well established, we omit the details of its implementation.

In brief, we use a method similar to that of Papadakos (2008) and Bayram and Yaman (2018) to find approximate core points. We first let $\pi^0_{k,r,j} \leftarrow 1$ and update this point using the following formula:

$$\pi^0_{k,r,j} \leftarrow \frac{1}{2}\pi^0_{k,r,j} + \frac{1}{2}\bar{\pi}_{k,r,j} \qquad \forall\, k \in K, r \in R_k, j \in \{1, \dots, n_r\}. \qquad (42)$$

To generate Pareto-optimal cuts more efficiently, we use a method similar to that of Sherali and Lunday (2013) and of Wang and Jacquillat (2020), which involves solving the dual primal subproblem only once by perturbing the right-hand side of constraints (26) in the primal subproblem as follows:

$$\sum_{(o,d)\in D} \sum_{t\in T} a_{r,e,i,o,d,t}\, \varepsilon^s_{o,d,t} \leq \sum_{k\in K_r} v_k\, \bar{\pi}_{k,r,j_{re}} + \epsilon \sum_{k\in K_r} v_k\, \pi^0_{k,r,j_{re}} \forall r \in R, e \in E_r, i \in I_r, s \in S, (43)$$

where ϵ is a small perturbation coefficient that is set to 10^{-6}, which is consistent with the parameter setting used in previous works (e.g., Wang and Jacquillat, 2020).

6 Computational experiments

The setting without adaptive fleet sizes is adopted to conduct most computational experiments except the ones in the relevant part of Section 6.4. The experimental setting is provided in Appendix 2.

6.1 Computational experiments for instances of different scales

We apply three methods to solve the problem. The first is solving model [M1] by CPLEX directly to provide optimal solutions. The second is solving model [M4] by CPLEX's BD framework (callback functions). The third is solving model [M4] by applying the proposed BBC solution method. The numerical experiments are instances with various numbers of routes in a shipping network to be optimized, ports of call, time periods in the planning horizon, nodes generated from a parent node in the scenario tree for each period and the total number of scenarios, and the different route compositions of the shipping network. The algorithms' performance is measured by the computing time and objective values of the solutions obtained by the three approaches. The solution time limit for each computational instance is six hours.

We conduct 10 sets of small instances (each with two or three routes, two, three, four, or six periods, and 36 to 343 scenarios), 10 sets of medium instances (each with two or three routes, three to nine periods, and 256 to 2,187 scenarios), and 10 sets of large instances (each with five or 10 routes, four to seven periods, and 256 to 2,187 scenarios); the detailed settings and results are provided in Appendix 3. For the small-scale instances, all three methods find the optimal solution in each of the 10 instances. Hence, the accuracy of the proposed BBC algorithm for small computational instances is verified. Regarding the CPU time, although the proposed BBC algorithm is not the fastest method for the small-scale instances, it is much faster than CPLEX's BD framework. For the medium-scale instances, CPLEX and CPLEX's BD framework cannot find any solution within six hours for one and six instances, respectively. However, the proposed BBC algorithm can obtain optimal solutions for all 10 instances and can obtain an optimal solution in 1,215 seconds for the instance where CPLEX cannot find any solution within six hours. Among the instances that can be solved to optimality by CPLEX's BD framework, the CPU time of the BBC algorithm is, on average, 0.73% of that of CPLEX's BD framework. The CPU time of the BBC method is, on average, 27.98% of that of CPLEX. Hence, the accuracy and efficiency of the proposed BBC algorithm for medium-scale computational instances are verified. For the large-scale instances, CPLEX and CPLEX's BD framework cannot find any solution within six hours for six and 10 instances, respectively. However, the proposed BBC algorithm can obtain optimal solutions in nine of the 10 cases, and the CPU time of the BBC method is, on average, 29.65% of that of CPLEX among the instances that can be solved to optimality by CPLEX. Hence, the accuracy and efficiency of the proposed BBC algorithm for large-scale computational instances are verified.

6.2 Impact of the acceleration strategies

This study also investigates the impact of the acceleration strategies introduced in Section 5.3. Appendix 4 reports the performance of the BBC algorithm when the acceleration strategies, i.e., the UBT inequalities and Pareto-optimal cuts, are separately applied to the problem. The tests are performed in the above 30 small-, medium-, and large-scale instances. The results indicate that the UBT inequalities significantly reduce the solution time of the proposed algorithm by approximately 6.52%, and the generation of Pareto-optimal cuts also results in an approximate 4.30% reduction in the solution time.

6.3 Impact of uncertainty

Next, we investigate the impact of uncertainty on the operations management of liner companies. A deterministic programming model [$\mathrm{M}_{\mathrm{deter}}$], a two-stage stochastic programming model [$\mathrm{M}_{\mathrm{two}}$], and a perfect information model [$\mathrm{M}_{\mathrm{perfect}}$] are formulated in Appendix 5 and compared with the multistage stochastic programming model [M1]. In [$\mathrm{M}_{\mathrm{deter}}$], the newly generated container shipping demand in each period for an O-D pair is set to the average value of the demand in the multistage programming model over $s \in S$; in [$\mathrm{M}_{\mathrm{two}}$], under a specific scenario $s \in S$, the value of the demand of an O-D pair in each period are the same as the demand value of the O-D pair in each period in the multistage stochastic programming model. The ship chartering and fleet deployment decisions obtained by [$\mathrm{M}_{\mathrm{deter}}$] and [$\mathrm{M}_{\mathrm{two}}$] will then be evaluated by making container acceptance, shipment, and delay decisions in each period in a myopic manner elaborated in Appendix 5 to calculate the resulting expected profits, represented by Z_{deter} and Z_{two}, respectively. The optimal objective value of [M1] is denoted by Z_{multi}. As another benchmark, Appendix 5 presents a perfect information model whose average profit over all scenarios is represented by Z_{perfect}.

To compare the four models, we use the same setting as Case M8 in Table A4 of Appendix 3, and we use 10 random cases to investigate the impact of uncertainty on operations management of liner companies. The related results are presented in Table 3. The six rightmost columns show the comparison of multistage stochastic model and deterministic programming model, the comparison of the multistage stochastic model and the two-stage stochastic model, and the comparison of multistage stochastic model and perfect information model. Obviously, when decision-makers have perfect information, they can obtain the maximum expected profit. In addition, the gap between the objective values of the multistage stochastic model and the perfect information model is quite small (the average value of $\frac{\text{Gap}_{\text{perfect}}^{\text{opt}}}{Z_{\text{multi}}}$ is 0.56%). Moreover, in reality, it is almost impossible to obtain perfect information. The use of multistage stochastic programming can lead to higher profit than the use of two-stage stochastic programming (the average value of $\frac{\text{Gap}_{\text{two}}^{\text{imp}}}{Z_{\text{multi}}}$ is 6.78%) or deterministic programming (the average value of $\frac{\text{Gap}_{\text{deter}}^{\text{imp}}}{Z_{\text{multi}}}$ is 16.00%) (similar conclusions are drawn by Huang and Ahmed (2009)).

Table 3 Comparison of the multistage, deterministic, and two-stage programming models and the perfect information model

Case ID	Z_{multi} (M$)	Z_{deter} (M$)	Z_{two} (M$)	Z_{perfect} (M$)	Value of stochastic solution		Multistage vs two-stage programming		Value of perfect information	
					$\text{Gap}_{\text{deter}}^{\text{imp}}$ (M$)	$\frac{\text{Gap}_{\text{deter}}^{\text{imp}}}{Z_{\text{multi}}}$ (%)	$\text{Gap}_{\text{two}}^{\text{imp}}$ (M$)	$\frac{\text{Gap}_{\text{two}}^{\text{imp}}}{Z_{\text{multi}}}$ (%)	$\text{Gap}_{\text{perfect}}^{\text{opt}}$ (M$)	$\frac{\text{Gap}_{\text{perfect}}^{\text{opt}}}{Z_{\text{multi}}}$ (%)
1	641.52	536.52	603.08	645.29	105.00	16.37	38.44	5.99	3.77	0.59
2	637.06	537.57	542.56	640.55	99.49	15.62	94.50	14.83	3.49	0.55
3	642.96	527.96	608.85	646.81	115.00	17.89	34.11	5.31	3.85	0.60
4	626.23	526.11	588.27	629.68	100.12	15.99	37.96	6.06	3.45	0.55
5	631.78	529.31	594.33	635.01	102.47	16.22	37.45	5.93	3.23	0.51
6	643.36	540.37	605.42	647.09	102.99	16.01	37.94	5.90	3.73	0.58
7	639.79	541.18	605.12	643.09	98.61	15.41	34.67	5.42	3.30	0.52
8	627.79	532.60	589.69	631.44	95.19	15.16	38.10	6.07	3.65	0.58
9	644.65	547.04	603.34	648.01	97.61	15.14	41.31	6.41	3.36	0.52
10	638.50	535.08	601.06	642.02	103.42	16.20	37.44	5.86	3.52	0.55
Avg.	637.36	535.37	594.17	640.90	101.99	16.00	43.19	6.78	3.54	0.56

Notes: (1) $\text{Gap}_{\text{deter}}^{\text{imp}} = Z_{\text{multi}} - Z_{\text{deter}}$ ("imp" is acronym of "improvement"), $\text{Gap}_{\text{two}}^{\text{imp}} = Z_{\text{multi}} - Z_{\text{two}}$, and $\text{Gap}_{\text{perfect}}^{\text{opt}} = Z_{\text{perfect}} - Z_{\text{multi}}$; (2) the values in columns $\text{Gap}_{\text{deter}}^{\text{imp}}/Z_{\text{multi}}$ (%), $\text{Gap}_{\text{two}}^{\text{imp}}/Z_{\text{multi}}$ (%), and $\text{Gap}_{\text{perfect}}^{\text{opt}}/Z_{\text{multi}}$ (%) are calculated by $\text{Gap}_{\text{deter}}^{\text{imp}}/Z_{\text{multi}} \times 100$, $\text{Gap}_{\text{two}}^{\text{imp}}/Z_{\text{multi}} \times 100$, and $\text{Gap}_{\text{perfect}}^{\text{opt}}/Z_{\text{multi}} \times 100$, respectively ("opt" is acronym of "optimality"); (3) M$ denotes million dollars.

We next conduct an intensive analysis of how multistage stochastic programming can lead to better solutions. Detailed results of the above 10 cases are recorded in Appendix 6. The routing capacity using deterministic programming is only $17{,}135/34{,}829 \times 100 \approx 49.20\%$ and $17{,}135/35{,}529 \times 100 \approx 48.23\%$ of the capacities of two-stage stochastic programming and multistage stochastic programming, respectively, and deterministic programming is thus unable to deal effectively with demand uncertainty. The total routing capacity using multistage stochastic programming is larger than that using two-stage stochastic programming in two cases and the same in the other eight cases (but the total routing capacities of each ship route by using multistage and two-stage stochastic programming may still differ), leading to more containers accepted (an average of 841,693 TEUs accepted in the multistage model vs. 823,769 TEUs in the two-stage model). The multistage model has an average of 35,114 TEUs of delayed containers, which is much smaller than the average of 180,529 TEUs using the two-stage model.

We next examine to what extent due to different fleet deployment decisions and to what extent due to different demand fulfillment and allocation decisions that the multistage model outperforms the two-stage model. To this end, we design a multistage stochastic programming model using two-stage deployment decisions (i.e., fleet deployment decisions are first obtained using the two-stage model $[\mathrm{M_{two}}]$, and demand fulfillment and allocation decisions are then obtained using the multistage model). The difference between the expected profits of the multistage method and of the multistage stochastic programming model using two-stage deployment decisions is the benefit obtained by better fleet deployment decisions, and the difference between the expected profits obtained by the multistage stochastic programming model using two-stage deployment decisions and the two-stage method is the benefit obtained by better demand fulfillment and allocation decisions. Our computational results in Appendix 6 show that 10% of the benefit brought by the multistage model over the two-stage model is due to better fleet deployment decisions and 90% of the benefit is due to better demand fulfillment and allocation decisions with full utilization of demand information.

6.4 Managerial insights regarding liner shipping

This study then discusses three practical questions regarding the driver analysis of liner company profitability, the benefits analysis of adaptive fleet sizes, and the influence of the COVID-19 pandemic on liner shipping. Case M8 in Table A4 of Appendix 3 is selected as the computational instance in this experiment.

6.4.1 Driver analysis of liner company profitability

Table 4 reports the liner company profitability with various operational characteristics ($f_{h,r}$, $c_{k,r}$, g_k, $p_{o,d}$, and m_k) and the demand parameters ($l_{o,d}$, and $q^s_{o,d,t}$) over three random cases with three routes, nine periods, and 512 scenarios. As expected, liner company profitability depends on both operational characteristics and shipping demand. Among all operational characteristics, the operating cost of ships ($c_{k,r}$) has the greatest impact on liner company profitability. The impacts of other operational variables remain moderate. For example, even massive increases in the penalty cost for delayed containers ($p_{o,d}$) (by up to 50%) or massive decreases in the revenue of chartering out a ship (m_k) (by up to 50%) induce very moderate profit decreases (within 1%). Moreover, massive increases in the repositioning cost ($f_{h,r}$) and rental cost of chartering in a ship (g_k) (by up to 50%) result in moderate profit decreases (within 4%). However, the impact of the demand parameters ($l_{o,d}$ and $q_{o,d,t}^s$) is much larger than that of the operational characteristics. For example, a 25% decrease in the shipping demand ($q_{o,d,t}^s$) reduces profits by more than 20% in the three cases. Therefore, in addition to improving operational capabilities, liner companies must pay more attention to the demand market to manage customer expectations and attract more transportation demand by pricing strategies, public relations campaigns, and marketing campaigns. Similar conclusions are drawn by Wang et al. (2022).

Table 4 Driver analysis of liner company profitability

Parameters	Value setting	Case 1		Case 2		Case 3	
		Profit (M$)	Gap (%)	Profit (M$)	Gap (%)	Profit (M$)	Gap (%)
Operational parameters	Increase $f_{h,r}$ by 25%	625.74	−1.78	631.25	−1.82	614.92	−1.81
	Increase $f_{h,r}$ by 50%	614.43	−3.55	619.66	−3.62	603.61	−3.61
	Increase $c_{k,r}$ by 25%	621.97	−2.37	627.35	−2.43	611.15	−2.41
	Increase $c_{k,r}$ by 50%	606.89	−4.74	612.12	−4.80	596.07	−4.82
	Increase g_k by 25%	630.18	−1.08	635.86	−1.10	619.36	−1.10
	Increase g_k by 50%	623.83	−2.08	628.76	−2.21	612.48	−2.20
	Increase $p_{o,d}$ by 25%	635.62	−0.23	641.26	−0.26	624.77	−0.23
	Increase $p_{o,d}$ by 50%	634.63	−0.38	640.04	−0.45	623.81	−0.39
	Decrease m_k by 25%	634.33	−0.43	640.23	−0.42	623.50	−0.44
	Decrease m_k by 50%	631.73	−0.84	637.50	−0.85	620.77	−0.87
Demand parameter	Decrease $l_{o,d}$ by 25%	456.92	−28.28	460.60	−28.36	448.90	−28.32
	Decrease $l_{o,d}$ by 50%	280.82	−55.92	282.39	−56.08	275.42	−56.02
	Decrease $q_{o,d,t}^s$ by 25%	486.51	−23.63	491.73	−23.52	478.18	−23.64
	Decrease $q_{o,d,t}^s$ by 50%	326.27	−48.79	329.20	−48.80	319.97	−48.91

Notes: (1) The values in the "Profit" columns are objective values of model [M1], and the values in the "Gap" columns are calculated by the difference between the objective values of the original case and that of the changed value setting divided by the objective value of the original case times 100; (2) M$ denotes million dollars.

Since shipping demand is the main driver of liner company profitability and varies greatly in practice, this study next discusses the impact of uncertain container transportation demand on the operation of ship fleets. We fix the mean demand of an O-D pair in a period at 2,500 TEUs while changing its standard deviation. The detailed settings and computational results are reported in Appendix 7. We find that when the coefficient of variation of the random demand between O-D pairs in a period increases from 0, 0.05, 0.1, $\cdots$, to 0.4, the expected total profit decreases. This makes sense because the variability of demand normally means that the originally deployed ship is more likely to be unsuitable for the current demand; that is, there is sometimes a large amount of spare capacity onboard the ship, and the ship is sometimes too small to carry the required cargo, resulting in a decrease in the total profit of the liner company. Wang et al. (2013) also observe that the average total cost generally increases with the value of the coefficient of variation when all shipping demand must be fulfilled. However, equipped with the multistage stochastic programming model, the loss in profit is marginal: when the coefficient of variation increases from 0 to 0.4, the average total profit decreases by only 1.86%, indicating that the multistage stochastic programming model effectively deals with demand uncertainty. Hence, we believe that the variability of the uncertain demand has a significant effect on the planning solutions because fluctuations in container shipping demand in the shipping market greatly affect the profit of liner companies. Hence, how liner companies respond to market fluctuations initially is crucial.

Finally, we investigate how the chartering in and out of the ships affect ship repositioning plans and profitability by changing the rental cost of chartering in ships and the revenue from chartering out ships. The computational results are shown in Appendix 7. In general, the rental cost of chartering in ships and the revenue from chartering out ships do not have a significant influence on the fleet decisions. When the rental cost of chartering in ships increases or the revenue of chartering out ships decreases, the liner company tends to deploy its own ships and charter in fewer ships, which is reasonable because deploying its own ships can help the company compensate for its decline in profit.

6.4.2 Benefits analysis of adaptive fleet sizes

When the planning horizon consists of multiple periods, the liner company can react more effectively to uncertain demand by adjusting fleet capacities in view of realized demand. We consider a setting in which for certain O-D pairs, ships can be deployed to provide point-to-point shipping services on an ad hoc manner. Our detailed numerical experiments are reported in Appendix 8. Specifically, we consider a setting with charter prices independent of demand, and a setting with charter prices related to demand (i.e., the charter price is higher when demand is higher). We find that adaptive fleet sizes can increase the profit of the liner company, but the average relative gaps in company profits with charter prices independent of demand and related to demand are only 1.09% and 1.76%, respectively, even when the coefficient of variation of demand is as high as 2. When adaptive fleet sizes are available (either the setting in which charter prices are independent of demand or the setting in which charter prices are related to demand), in the face of huge fluctuations in demand, the liner company prefers to deploy ships with smaller capacity in the first stage and then to charter in more point-to-point ships to respond to increased demand more flexibly. When the coefficient of variation of demand is reduced to 1, 0.5, or 0.25, the relative gaps in company profits between the company with adaptive fleet sizes and the company with fixed fleet sizes are at most 0.63%, 0.20%, and 0.05%, respectively. Therefore, in practice, when demand fluctuation is moderate, liner companies, especially those without operations research based decision-making support, can ignore adaptive fleet sizes because the benefits brought by this factor are too small, and companies have to make many decisions manually. Moreover, it is unclear whether the liner company should charter in more point-to-point ships and whether it will earn more profits in the setting of demand-related charter prices than in the setting of demand-independent charter prices.

6.4.3 Influence of the COVID-19 pandemic on liner shipping

Finally, we investigate the influence of the COVID-19 pandemic on liner shipping. Pandemics prolong the preparation time for repositioning, thus increasing the total repositioning time and repositioning cost. Our computational results (elaborated in Appendix 9) show that the expected profit decreases as the number of days for preparation increases. Moreover, when the number of preparation days is less than 10, the expected profit is very sensitive to changes in the number of preparation days. Hence, governments need to think carefully about how to set the quarantine duration for foreign ships to balance the trade-off between containing diseases and harming the profits of liner companies.

7 Conclusions

This study has shown how to operate ship fleets under uncertainty to deal with a liner company's multi-period heterogeneous FDP in a shipping network, considering fleet repositioning, ship chartering, demand fulfillment, cargo allocation, and adaptive fleet sizes. Although this issue is crucial for liner company operations, it is rarely studied in the literature. To fill this research gap, we first introduced a multistage optimization problem and developed a mixed integer linear programming model for the problem. Since this problem is NP-hard, we designed a BBC algorithm to solve the model. Two types of acceleration strategies were applied to improve the performance of the proposed algorithm. Contributions of this paper were summarized from the following three aspects.

From the perspective of problem modeling, we formulated the problem as a multistage stochastic programming model to allow for adaptive fleet sizes, allowing liner companies to rent additional point-to-point ships for transportation in addition to deployed regular ships from the decisions in the first stage. In the first stage, the model determines the number of deployed regular ships of different types on ship routes in the network of the liner company, the sailing sequence of these deployed regular ships, and the numbers of chartered-in and chartered-out regular ships when there is a deficit or surplus in some ship types; in the following stages, when the container shipping demand and rental prices of chartering in point-to-point ships become realized, the model determines, for each time period under each scenario, the number of chartered-in point-to-point ships, the number of accepted containers for each O-D pair, the number of delayed containers for each O-D pair, and the numbers of shipped containers for each O-D pair by the deployed regular ships and point-to-point ships.

From the perspective of algorithm design, the NP-hardness of the problem did not stop us from looking for an exact and efficient algorithm to solve the problem of practical scale with realistic data. We designed a BBC algorithm to solve the formulated model. To tackle the challenge of solving the multistage optimization problem, we derived two types of acceleration strategies, including the approximate upper bound tightening inequalities and the Pareto-optimal cuts, to improve the performance of the proposed algorithm. The accuracy and efficiency of the proposed BBC algorithm for computational instances were verified.

From the perspective of managerial insights, we first investigated the impact of uncertainty on the operations management of liner companies and found that the use of multistage stochastic programming can lead to higher profit than the use of two-stage stochastic programming or deterministic programming. We next conducted an intensive analysis of how multistage stochastic programming can lead to better solutions and found that the benefits brought by multistage are divided into two categories: one is the improved deployment decisions, and the other is demand fulfillment and allocation decisions with full utilization of demand information. The latter accounts for 90% of the total benefits. Finally, we discussed three practical questions regarding driver analysis of liner company profitability, benefits analysis of adaptive fleet sizes, and the influence of the COVID-19 pandemic on liner shipping. Using our solution method, the decision-makers of liner companies can quickly obtain a plan including fleet deployment, fleet repositioning, ship chartering, demand fulfillment, and cargo allocation scheduling to deal with uncertainties in the shipping market.

As further research, we intend to explore several extensions. First, although the uncertainty of container transportation demand is considered, the uncertainty of weather in ports can also be incorporated because weather conditions directly influence the availability of ships mooring there. However, when weather disturbances are also considered, the formulations of the multistage optimization problem should include more decision variables and constraints, necessitating the development of more effective algorithms. Furthermore, in addition to fleet repositioning, ship chartering, demand fulfillment, and cargo allocation, several other operations management elements, such as empty container repositioning (Crainic et al., 1993), ship refueling (Besbes and Savin, 2009), and on-time shipment delivery (Choi et al., 2012), can be incorporated. Jointly optimizing decisions for the above integrated problem would be beneficial for liner companies, but this is more complicated to solve and merits future research.

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