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An optimal charging scheduling model and algorithm for electric buses

Zhaoyao Bao [a], Jiepei Li [b], Xuehan Bai[c], Chi Xie [b, d, e], Zhibin Chen [f], Min Xu [g], Wen-Long Shang [h], Hailong Li [i]

[a] School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiaotong University, China

[b] School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiaotong University, China

[c] Cockrell School of Engineering, University of Texas at Austin, United States

[d] Key Laboratory of Road and Traffic Engineering of Ministry of Education, Tongji University, China

[e] Urban Mobility Institute, Tongji University, China

[f] Division of Engineering and Computer Science, New York University Shanghai, China

[g] Department of Industrial and Systems Engineering, Hong Kong Polytechnic University, China

[h] College of Metropolitan Transportation, Beijing University of Technology, China

[i] School of Business, Society and Technology, Mälardalen University, Sweden

ABSTRACT

Transportation electrification poses a promising low-carbon or even zero-carbon transportation solution, serving as a strategic approach to reducing carbon emissions and promoting carbon neutrality in the transportation sector. Along the transportation electrification pathway, the goal of carbon neutrality can be further accelerated with an increasing amount of electricity being generated from renewable energies. The past decade observed the rapid development of battery technologies and deployment of electricity infrastructure worldwide, fostering transportation electrification to expand from railways to light and then heavy vehicles on roadways. In China, a massive number of electric buses have been employed and operated in dozens of its metropolises and the country nowadays has the highest penetration rate of electric buses in the world. An important daily operations issue with these urban electric buses is how to coordinate their charging activities in a cost-effective manner, considering various physical, financial, institutional, and managerial constraints.

This paper addresses a general charging schedule optimization problem for an electric bus fleet with multiple bus lines and charging depots and terminals, aiming at finding an optimal set of charging location and time decisions given the available charging windows. The charging windows for each bus are predetermined in terms of its layovers at depots and terminals and each of them is discretized into a number of charging slots with the same time duration. A mixed linear integer programming model with binary charging slot choice and continuous SOC level variables is proposed to minimize the total charging cost of the bus fleet subject to individual electricity consumption rates, electricity charging rates, time-based charging windows, battery state-of-charge (SOC) level bounds, system-level time-of-use (TOU) charging tariffs, and station-specific electricity load capacities. A Lagrangian relaxation framework is employed to decouple the joint charging schedule of a bus fleet into a number of independent single-bus charging schedules, which can be efficiently addressed by a bi-criterion dynamic programming algorithm. A real-world regional electric bus fleet of 122 buses in Shanghai, China is selected for validating the effectiveness and practicability of the proposed charging scheduling model and algorithm. The numerical results demonstrate and assess the impacts of TOU tariffs, station load capacities, charging infrastructure configurations, and battery capacities on the bus system performance as well as individual recharging behaviors. Many findings can be used for improving the electric bus system operations and management, e.g., the coordination of charging activities of an electric bus fleet against the TOU tariffs and electricity load capacities, the identification of charging stations suffering from heavy charging loads, the combination use of fast and regular charging, the suitable bus battery capacity, and so on. On the computational side, comparisons between our developed algorithm and the commercial Gurobi solver are presented concerning 27 scenarios of different system parameter settings.

Keywords: Electric buses, Charging scheduling, Charging windows, Time-of-use tariffs, Electricity load capacity, Bi-criterion dynamic programming

1. INTRODUCTION

To avoid the global climate change caused by carbon emissions, many countries have launched plans and targets for energy conservation and emission reduction (Liu et al., 2022). For example, the European Union launched the European Green agreement; the United States resumed the Paris Agreement; China put forward the targets of "carbon peak and neutrality goals" and launched the largest carbon trade market in the world. Transportation activities are a significant contributor to carbon emissions and energy consumptions in most societies worldwide (Bi et al., 2021). According to the United States Environmental Protection Agency, the transportation sector accounts for almost 29% of the total carbon emissions and 27% of the total energy usage in the world (EPA, 2021).

Transportation electrification has long been regarded as an effective and reliable way of mitigating global warming and promoting carbon neutrality in the transportation sector. From a Union of Concerned Scientists (2019) report, a 40-foot electric bus emits only 347 g/mile of carbon dioxide, while a comparable diesel bus emits up to 2,680 g/mile and a natural gas-powered bus 2,364 g/mile. In addition, as more and more renewable resources can generate electricity instead of burning fossil fuels, adopting electric buses is increasingly reducing the dependence on fossil fuels. Due to their advantages of lower carbon emissions and lower fossil energy consumption, electric buses as a replacement for diesel buses have been promoted in many cities around the world, especially in Chinese metropolises (REN21, 2020; Wang et al., 2013; IEA, 2020). China is currently ranked first in the market share of electric buses around the world. According to an International Energy Agency (2020) report, around half a million electric buses had been in operations globally by 2019, 98% of which were in China. In particular, Shenzhen, the fourth largest city in China, has equipped with electric buses for all its bus services by the end of 2017 (Lu et al., 2018). The Shenzhen Municipal Transportation Commission found that the electric buses in Shenzhen in 2016 consume 72.9% less energy than diesel buses. The resulting energy savings amount to 366,000 tons of coal are replaced by 345,000 tons of alternative fuels (ITDP, 2018).

Despite all the strengths of electric buses, there are still some restrictions that hinder their adoption, among which the main concerns are driving range anxiety and long recharging time. Recently, for the most operational electric buses in China, the theoretical value of a maximum driving range is up to hundreds of kilometers. However, the actual driving range from a full charge may be influenced by a multiplicity of different factors, such as battery discharge depth, drivers' driving behaviors, traffic environments, and weather conditions, which makes it difficult to meet the required daily mileage of urban buses without en-route charging. Moreover, fully charging an electric bus requires a much longer time than refueling a diesel-powered bus. For example, it may take more than 20 hours to fully charge an urban electric bus with a 547 kWh battery pack by a charger with a charging power of 20 kW (He et al., 2020). One of the effective remedies for range anxiety caused by the limited driving range and long recharging time is to develop fast electricity-charging technologies for supporting en-route charges for electric buses during their layovers between two successive dispatches. However, recharging electric buses with fast charging equipment is a double-edged sword. On one hand, fast charging shortens recharging time, which can better meet those urgent charging needs. On the other hand, fast charging deteriorates battery safety, accelerates battery degradation, and requires advanced electricity-charging equipment. In view of these hinders, an important daily operations issue of the electric bus system is how to schedule and coordinate the charging activities of electric buses in a cost-effective manner, considering various physical, financial, institutional, and managerial constraints. Improper charging schedule and management may challenge the operation service and thus intensify range anxiety, increase the charging and operating costs and thus compromises the economic attractiveness of electric buses, and significantly impact on the power grid (Gilleran et al., 2021; Borozan et al., 2022; Jing et al., 2021). In view of all these concerns, this study aims at optimizing the charging schedule of an electric bus fleet serving multiple bus lines to minimize the charging cost of the electric bus fleet an operational day. An optimal charging schedule tells where, when, and how long for each bus in the fleet to charge in order to fulfill its assigned trips.

To address the proposed charging scheduling problem, a concise mixed linear integer programming model is developed for the proposed charging scheduling problem. The benefit of this model is its flexibility. It can accommodate multiple electric bus systems with sole or concurrent restrictions of individual electricity charging and consumption rates, charging window restrictions, battery electricity level bounds, system-level time-of-use (TOU) charging tariffs, and site-specific electricity load capacities, and can be a modeling block easily incorporated into other bus system optimization problems, e.g., the electric bus schedule and line planning problems or charging infrastructure planning problems. A Lagrangian relaxation approach is then adopted for the solution, which can decompose the bus fleet charging scheduling problem into subproblems with respect to individual buses that are efficiently addressed by a novel bi-criterion dynamic programming algorithm.

The remainder of this paper is organized as follows. Section 2 reviews previous studies on optimizing electric bus systems. Section 3 first describes our proposed charging scheduling problem and then presents a mixed linear integer programming model. In Section 4, a solution framework based on Lagrangian relaxation incorporating a bi-criterion dynamic programming algorithm is elaborated. Section 5 uses a realistic bus fleet service network from Jiading, Shanghai to demonstrate the proposed modeling and solution methods and assess the solution behaviors and algorithmic performance. Finally, highlights and future research are summarized and discussed in Section 6.

2. LITERATURE REVIEW

In terms of different planning and operations decisions of electric bus systems, researchers have been concentrating on three types of research problems: charging infrastructure planning problems, bus scheduling problems, and charging scheduling problems. The charging infrastructure planning involves locating charging stations and determining the quantity of charging piles in each station (An, 2020; An et al., 2020; Xylia et al., 2017; Liu and Song, 2017; Chen et al., 2018); the vehicle scheduling includes determining the allocation and operations of electric buses, including the size of the electric bus fleet, the operating routes, and the frequency for each electric bus to serve (Ibarra-Rojas et al., 2015; Liu and Ceder, 2020; Olsen et al., 2020; Li et al., 2019; Tang et al., 2019); and the charging scheduling refers to determining the spatial-temporal charging behaviors for electric buses in terms of the charging site, the charging time, and the charging power.

In the recent decade, a large body of studies emerge on the optimal charging scheduling problem. He et al. (2020) investigated an electric bus system charging scheduling and management problem to decide and control the charging behavior with the objective of minimizing the total charging cost. They proposed a non-linear, non-convex optimization model, where the charging power was denoted by a continuous variable associated with its certain charging durations. The original optimization model was then transformed using linearization and discretization methods into a model that can be solved directly by a solver. Qin et al. (2016) adjusted the charging threshold in the charging strategy to reduce the peak charging power demand of all buses in the bus system. Note that in their research electric bus recharging was restricted to the following two situations: one situation is when its state-of-charge (SOC) is below a predetermined charging threshold; the other is when its SOC cannot support it to return to the fast-charging station with a SOC greater than 5%. In addition, it was assumed that there is one single fast-charging pile in the system where all electric buses get recharged following a first-in-first-out (FIFO) queuing rule. You et al. (2015) scheduled the battery charging in a battery-swapping station for electric buses aiming at minimizing the sum of charging cost, battery degradation, and the utilization of swapped batteries. In their research, each electric bus would be allocated in advance to a designated battery-swapping station to replace its battery with a fully-charged one, if its SOC falls below a predetermined threshold, and then the replaced battery with low SOC would be taken for recharging. The decision variable in this charging process is the charging power for each replaced battery at each certain time slot. Bagherinezhad et al. (2020) scheduled the spatial-temporal charging events of electric buses from the perspective of minimizing the operation cost of the power grid while satisfying bus operational constraints and charging station power distribution constraints. Time-slot indexed continuous charging powers of charging stations and electric buses are optimized and the battery electricity level of buses is ensured within given maximum and minimum boundaries. Their study demonstrated that the coordinated optimization of the whole electric bus fleet's charging events can effectively stabilize the voltage of the power grid making the power distribution system more reliable. Abdelwahed et al. (2020) proposed two equivalent mixed linear integer models for optimizing the charging schedules of transit electric bus networks by using different discretization approaches. One discretized the time and decisions into equal discrete slots based on a time-expanded network; the other discretized them into irregular slots based on charging events. Their results show that the second model with fewer variables and constraints has better computational performance. Huang et al. (2022) incorporated a more realistic nonlinear charging function into the electric bus charging scheduling problem and introduced a time discretization technique to handle the nonlinearity in their modeling process. There are also a few studies handling integrated optimization problems of electric bus systems. Wang et al. (2017) and Rogge et al. (2018) respectively proposed integrated optimization frameworks for joint charging infrastructure planning and charging scheduling of the electric buses, with the objective of minimizing the total costs from strategic planning and operational scheduling.

The technique of time discretization is widely adopted in the literature related to electric bus charging scheduling problems (Wang et al., 2017; Qin et al., 2016; You et al., 2015; Bagherinezhad et al., 2020; Abdelwahed et al., 2020; Huang et al., 2022). It can advantageously specify the charging behavior of each bus in each charging slot, make it easy to track the charging time and battery electricity level and dynamically schedule the charging process and charging duration of an electric bus fleet. However, discretization of time generally leads to integer variables in the mathematical programming model and causes a computational burden when optimizing the charging schedule of a large-size electric bus fleet. It is a big challenge to develop fast and exact solution algorithms for large-scale problems. From the literature, a large body of research (e.g., Wang et al., 2017; Bagherinezhad et al., 2020; Abdelwahed et al., 2020; He et al., 2020) adopted the commercial solvers for their model solutions directly. You et al. (2015) proposed a dual decomposition algorithm with a direct projection method for solving their proposed nonlinear convex programming model. Huang et al. (2022) proposed a Lagrangian relaxation approach decoupling the bus fleet charging scheduling into individual electric bus charging scheduling subproblems. However, a k-greedy heuristic algorithm was proposed to find an approximate solution for each subproblem. In contrast, in our study, a fast and exact bi-criterion dynamic programming algorithm is developed to address each single-bus charging schedule for the first time and then incorporated into a Lagrangian relaxation solution framework for the joint charging schedule of buses in the fleet.

3. PROBLEM FORMULATION

3.1 Problem statement

The charging scheduling problem proposed in this paper aims to find out where, when, and how long for each bus in the fleet to charge in order to realize the minimum total charging cost of the electric bus fleet in an operational day, while making sure fulfilling their assigned trips, and meanwhile embracing many realistic factors and restrictions in the decision-making problems of the electric bus system. This section presents a modeling framework for the proposed problem and starts by elaborating on those modeling factors and restrictions considered in the modeling process.

First, to guarantee the operation service of electric buses, the available charging windows for each electric bus should respect its operation timetable (Bagherinezhad et al., 2020; Wang et al., 2017; He et al., 2020), which indicates that the charging activities of electric buses can only happen in their spare time except during their operations timetables. Moreover, since the operations timetables specify the spatial-temporal services of electric buses, the available charging locations and the available time durations are mutually connected for charging decisions.

Second, the TOU tariffs has been widely recognized as an important issue to be considered in the charging scheduling decisions of the electric bus system (Gallo et al., 2014; Borlaug et al., 2021). The TOU tariffs are proposed to encourage users to consume electricity in the off-peak periods by setting the electricity rate higher in the on-peak periods instead of using a single electricity rate throughout the day. When TOU tariffs are applied, the charging price of on-peak periods can be twice or three times high as that of off-peak periods and thus the electricity cost of electric buses is highly dependent on their charging profiles. It is well known that the TOU tariffs can draw heavily on the charging activity pattern of the electric bus fleet. Frequent charging at on-peak periods would be largely avoided since it increases the electric bus charging cost, and thus compromises the economic attractiveness of electric buses.

Third, the electricity load capacity of charging stations plays an important role in influencing optimal charging schedules (He et al., 2020; You et al., 2015; Bagherinezhad et al., 2020). From the aspect of station space, the maximum number of electric buses that can be charged simultaneously is restricted by the total parking space available in a charging station (van Kooten Niekerk et al., 2017).

From the aspect of the local power grid, the maximum charging power of all chargers is limited by the capability of electricity distribution systems (He et al., 2020; You et al., 2015; Bagherinezhad et al., 2020). Nowadays, it still requires costly and time-consuming upgrades to the charging power systems to support substantial electricity demand (Borlaug et al., 2021). Simultaneously charging a cluster of electric buses at a charging station leads to a charging power surge in the local power grid, which is likely to cause a series of negative effects such as transformer overload, voltage quality deterioration, and wire damage (Li et al., 2020). Thus, the total charging load of a charging station generally needs to be controlled within a predetermined bound. Actually, the electricity load capacity has a significant influence on the charging schedule of the electric bus fleet, which tends to disperse the intensive charging from the off-peak periods to on-peak periods that have a higher electricity rate.

Fourth, for better battery health, a healthy battery electricity level bound should be carefully considered in the charging scheduling decisions. The necessity of introducing the battery SOC bound is twofold. First, the battery consumption rate of electric buses is naturally stochastic due to unpredictable traffic conditions, weather conditions, passenger and cargo load, and others. To hedge against battery consumption stochasticity, the lower bound of SOC can be taken as a safe buffer to prevent the battery from running out of electricity. Second, battery degradation is heavily influenced by the depth of charging and discharging. The setting of the battery SOC bound can protect the health of the battery and thus reduce the capital cost from purchasing new batteries.

Fifth, for better coordinating charging demands, two types of electricity charging rates, i.e., fast charging and regular charging are considered in different operations scenarios and supplement each other in this paper. Generally, fast charging enables the en-route charging that often takes place at a short break between two successive dispatches. It is restricted by resource and time availability factors such as charging facility locations and capacities and operations timetables. Regular charging is commonly adopted for the at-depot charging that often takes place at the end of a day's operations so as to have sufficient charging time to replenish electricity for the next day's operations.

Figure 1 illustrates an example of a spatial-temporal operation network for electric buses, which includes the elements of depots, terminals, deadheads, assigned trips, and charging windows. Each element is associated with its corresponding geographic locations and time spans. A depot is a facility where electric buses can be stored and maintained. A bus terminal is a place, where a bus starts or ends its service of carrying passengers for a specific trip. Battery chargers are installed at depots or adjacent to selected bus terminals along these bus lines to support at-depot or at-terminal recharges. A deadhead is a bus trip between different terminals or depots without carrying passengers, including departing from a depot to a terminal to begin their trips of the day or heading to a charging station from a terminal to perform at-depot or at-terminal recharges. An assigned trip is the itinerary of a bus running from a terminal to another terminal to serve passengers. A charging window refers to a time duration when an electric bus is parking at a charging station during its layover. Within charging windows, electric buses are allowed to be charged only in specified times of day, due to physical and managerial factors.

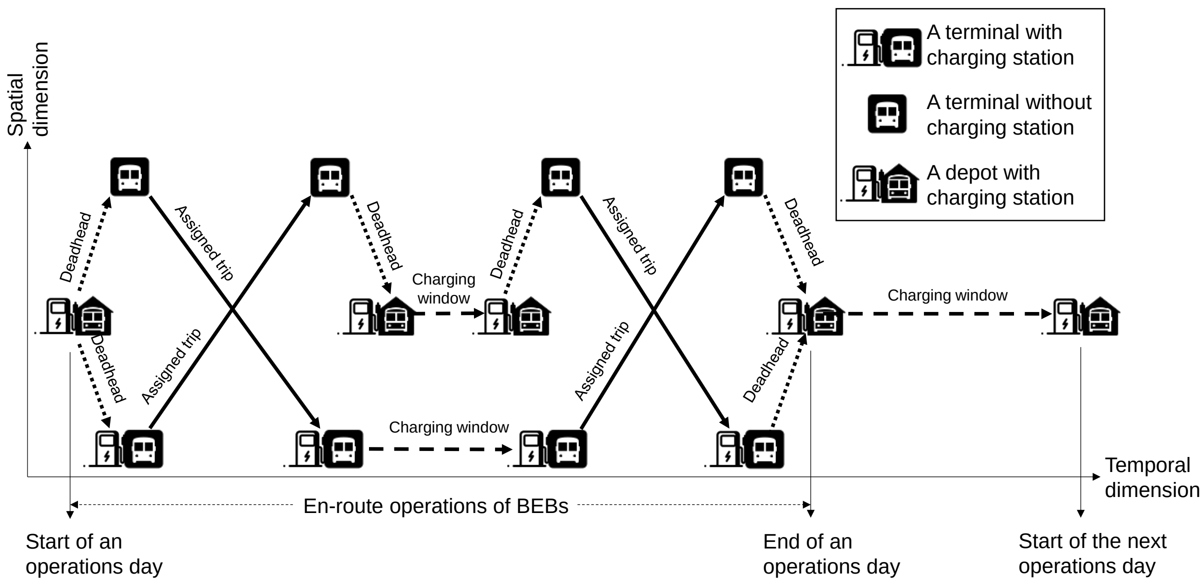


Figure 1 Illustrative example of an electric bus spatial-temporal operation network

To form a tractable model while maintaining essential problem features, a few important modeling assumptions and settings are paved below. Some of them are inherited from previous literature, while some others are unique features pertaining to our model. First, for at-depot charging, both fast charging and regular charging are available, while at-terminal charging for en-route purpose is supported only by fast-charging equipment. At a depot, performing fast charging or regular charging depends on the exact time duration of an electric bus charging window (fast charging for relatively short electric bus charging windows and regular charging for long electric bus charging windows). Fast charging and regular charging are distinguished by their charging powers. Second, each electric bus serves a certain bus trip following a given operations timetable and can be recharged during its layovers between two assigned trips at charging stations (see, for example, He et al., 2020; Wang et al., 2017). As mentioned before, the layover time duration that can be used for charging is denoted as a charging window. Third, the time discretization method is introduced to facilitate the explicit expression of objectives and constraints in terms of time-dependent variables and parameters. In a time-dependent electric bus charging system, time discretization is a method widely adopted by researchers to describe the relationship between decision variables and system parameters (see, for example, Qin et al., 2018; You et al., 2015). By discretizing each charging window into discrete charging slots of equal time duration, a charging slot becomes the smallest decision unit of charging. An electric bus charging window thus contains a set of successive charging slots. Fourth, when TOU tariffs are taken into account, each charging slot is associated with a specific charging price based on real-world TOU tariffs. Charging slots of off-peak periods may only have one half or one third of the charging price of on-peak period charging slots, which can significantly shape charging schedules in terms of economy (Gallo et al., 2014; Borlaug et al., 2021). Fifth, based on the specific configurations of each charging station, each charging slot of one charging station is associated with an electricity output upper bound to accommodate site-specific electricity load capacities. Simultaneous charging of a cluster of electric buses at one charging station should be avoided due to the limited available parking space and the negative effects of charging power surges on the local power grid (van Kooten Niekerk et al., 2017; You et al., 2015; Li et al., 2020). Sixth, the injected electricity is linearly proportional to the charging duration and the consumed electricity is linearly proportional to the driving distance. This is also a widely accepted assumption by many academic studies related to electric buses or simply electric vehicles (to name a few, Wang et al., 2017; Xie and Jiang, 2016; Xie et al., 2017; He et al., 2018; Chen et al., 2017; Chen et al., 2018; Bao and Xie, 2021). Taking into account different bus types, battery configurations, charging pile types, and road conditions, historical bus operations data can be used in this study to estimate heterogeneous charging rates and power consumption rates of different charging stations, buses, and service trips.

3.2 Model development

The proposed charging schedule problem is to minimize the total charging cost by determining whether an electric bus from the fleet should or should not be charged at each charging slot of its charging windows during an operational day while jointly considering TOU tariffs, station load capacity, and driving range requirements. The proposed charging scheduling problem can be formulated as a mixed linear integer programming model. For discussion convenience, the notation used in the model is first given in **Table 2**, followed by the model presentation and explanation.

TABLE 2 Notation

Sets	
I	Set of electric buses, $I = \{i\}$
K	Set of charging windows, $K = \{k\}$
N	Set of charging slots of an operational day, $N = \{n\}$

K_i	Set of charging window k for each electric bus i, $K_i = \{k\}$
$N_{i,k}$	Set of charging slots of charging window k for electric bus i, $N_{i,k} = \{n\}$
Y	Set of charging stations, $Y = \{y\}$
Parameters	
$p_{i,k+1}^{i,k}$	The electricity consumption for an electric bus i to travel between two adjacent charging windows k and $k+1$
$q_{i,k,n}$	The amount of electricity charged into the battery of electric bus i in charging slot n of charging window k if it gets charged
$c_{i,k,n}$	The electricity price of a charging slot n of charging window k of electric bus i
$U_{y,n}$	The upper capacity bound of the power grid load of a charging station y (i.e., the total electricity output of a charging station during the charging slot n)
$\delta_{i,k,n}^{y}$	The binary vehicle-station incidence indicator, where if charging slot n of charging window k for electric bus i can be used for recharging at charging station y, then $\delta_{i,k,n}^{y} = 1$, and $\delta_{i,k,n}^{y} = 0$ if otherwise
B_i^u	The upper bound of the battery electricity level of electric bus i
B_i^l	The lower bound of the battery electricity level of electric bus i
Variables	
$x_{i,k,n}$	Charging decision indicator, denoting whether electric bus i is charged in the charging slot n of charging window k , where $x_{i,k,n} = 1$ denotes that electric bus i is charged and $x_{i,k,n} = 0$ if otherwise
$b_{i,k}$	Battery electricity level, denoting the battery SOC of electric bus i at the start of its charging window k

We denote the set of electric buses by I, the set of charging windows by K, and the set of charging slots during an operational day by N. As different buses may have a different set of charging windows based on their respective operations timetables, K_i is used to represent the set of charging windows associated with each electric bus i, and $N_{i,k}$ represents the set of charging slots of the charging window K_i. The set of charging stations is denoted by Y. The electricity consumption of electric bus i providing bus services between two adjacent charging windows k and $k+1$ is defined as a parameter $p_{i,k+1}^{i,k}$ that can be pre-estimated based on historical data. The amount of electricity that can be charged into the battery of electric bus i in charging slot n of charging window k at a charging station is calculated by a parameter $q_{i,k,n}$. Other parameters include electricity price $c_{i,k,n}$, charging station load capacity $U_{y,n}$, and the lower and upper bounds B_i^u and B_i^l of the battery electricity level of each electric bus. $\delta_{i,k,n}^{y}$ is an incidence indicator for denoting whether a charging slot n in the charging window k of an electric bus i can be used for this bus to recharge at the charging station y, which specifies the relationship between an electric bus charging slot and the charging station y. The key decision variable is the charging activation variable $x_{i,k,n}$ that determines whether each charging slot of an electric bus charging window is activated or not for recharging while the battery electricity level variable $b_{i,k}$ is also introduced to denote the SOC level of an electric bus at the start of charging window k before recharging takes place.

Following the above notation, a mathematical programming model for the proposed charging scheduling problem is formulated. As is expressed in (1), the objective function is to minimize the total electricity cost of all used charging slots of all charging windows of the electric bus fleet during an operational day.

$$\min z(\mathbf{x}, \mathbf{b}) = \sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} c_{i,k,n} q_{i,k,n} x_{i,k,n} \tag{1}$$

Then the constraints are presented below from (2) to (6). The constraint in (2), the battery electricity level conservation constraint, specifies the battery electricity level relationship between two successive charging windows of an electric bus i. To be specific, the battery electricity level of bus i at the start of its $k+1$ charging window is equal to the sum of the battery electricity level at the start of the previous charging window k and the charged electricity in k while subtracting the electricity consumed by driving between k and $k+1$.

$$b_{i,k+1} = b_{i,k} + \sum_{n \in N_{i,k}} q_{i,k,n} x_{i,k,n} - p_{i,k+1}^{i,k} \qquad \forall i \in I, k \in K_i \tag{2}$$

The constraints in (3) and (4) ensure that the battery electricity level of an electric bus at any of its charging windows is constrained between the predetermined upper and lower bounds of battery electricity level utilization. The constraint in (3) states that after electric bus i gets charged at its charging window k, the battery electricity level cannot exceed a predetermined maximum value, while the constraint in (4) states that when electric bus i arrives at the charging window k, its battery electricity level cannot be lower than a predetermined minimum value. It should be noted that both the value of B_i^u and B_i^l can be set based on the historical operation data and adjusted dynamically during the planning horizon.

$$b_{i,k} + \sum_{n \in N_{i,k}} q_{i,k,n} x_{i,k,n} \leq B_i^u \qquad \forall i \in I, k \in K_i \tag{3}$$

$$b_{i,k} \geq B_i^l \qquad \forall i \in I, k \in K_i \tag{4}$$

The constraint in (5) is added as the electricity load capacities constraint. When each charging window is discretized into charging slots, the maximum output electricity that a charging station can load within a charging slot to charge electric buses is bounded by the charging station load capacity

$U_{y,n}$. $U_{y,n}$ can be determined according to the specific configuration of a charging station with integrated consideration of its parking capacity and power grid load ability at different charging slots. The left-hand side of the constraint in (5) sums the amount of electricity charged at the charging slot n of a charging station y by all electric buses.

$$\sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} \delta_{i,k,n}^{y} q_{i,k,n} x_{i,k,n} \leq U_{y,n} \qquad \forall y \in Y, n \in N \tag{5}$$

The constraint in (6) is simply a 0-1 binary constraint for the charging slot selection variable $x_{i,k,n}$ to indicate whether or not electric bus i chooses to be charged in charging slot n of charging window k. Note that one battery electricity level variable $b_{i,k}$ of the variable vector **b** is free and continuous and depends on the value of **x**.

$$x_{i,k,n} \in \{0,1\} \qquad \forall i \in I, k \in K_i, n \in N_{i,k} \tag{6}$$

Obviously, all the above constraints and the objective function are linear. Due to the existence of both continuous variables **b** and 0-1 integer variables **x**, the above mathematical programming model poses a mixed linear integer programming problem.

4. SOLUTION ALGORITHM

As we discussed in the last section, the proposed charging scheduling problem for an electric bus fleet is formulated into a mixed linear integer programming model with the binary charging slot activation variable as the key decision variable and the continuous battery electricity level variable as the dependent variable of the binary variable. When electricity load capacities are taken into account, the charging schedules of different electric buses are coupled together. In view of this problem feature, this section develops a solution algorithm based on Lagrangian relaxation for decoupling the bus fleet charging schedule into a number of individual bus charging schedules that can be addressed by a novel bi-criterion dynamic programming algorithm.

4.1 Lagrangian relaxation framework

Lagrangian relaxation is a general solution framework for handling optimization problems with hard constraints by relaxing hard constraints while adding them to the objective function (Ahuja et al., 1993). Each relaxed constraint added to the objective function will be associated with a Lagrangian multiplier as the penalty cost for violating this relaxed constraint. This relaxation process generates the Lagrangian multiplier problem which would become the Lagrangian relaxation problem if the value of all Lagrangian multipliers is given (Fisher, 1985). As for the model proposed in this study, the charging load capacity constraint in (5) is relaxed and penalized in the objective function. The original model can thus be reformulated as a bus fleet charging scheduling model without the charging load capacity constraint while the objective function includes penalty cost terms for preventing the charging load of each station from exceeding its bound. Each penalty term is constructed by the relaxed constraint multiplied by its associated Lagrangian multiplier. Given specific values of all Lagrangian multipliers, the reformulated model is equivalent to a bus fleet charging scheduling problem without limited electricity load capacity and with additional given Lagrangian multiplier cost on the electricity price of charging slots, which can then be decoupled into a set of individual bus charging scheduling problems and solved by the developed bi-criterion dynamic programming algorithm. By following predetermined update rules (e.g., subgradient method), the values of all Lagrangian multipliers can be adjusted according to solution results obtained from the reformulated model. The bus fleet charging scheduling problem without the electricity load capacity constraint should then be updated with the latest values of all Lagrangian multipliers and resolved by the bi-criterion dynamic programming algorithm. For each iteration, if the obtained solution is feasible to the original model with charging load capacity constraints, update the best solution ever found with this obtained solution and calculate the associated objective function value (i.e., total electricity cost) as the upper bound. The lower bound is updated with the objective function value of the Lagrangian relaxation problem that includes penalty terms. With iteratively adjusting the values of all Lagrangian multipliers and resolving the bus fleet charging scheduling problem without electricity capacity constraints, the gap between the upper and lower bounds decreases. When the gap is small enough or zero, the obtained solution is the acceptable or optimal solution.

Below the Lagrangian multipliers are introduced, denoted by $\lambda_{y,n}$ for each $y \in Y, n \in N$ of the constraint (5). Then the Lagrangian multiplier problem reads below:

$$\max_{\boldsymbol{\lambda} \geq 0} L(\boldsymbol{\lambda}, \mathbf{x}, \mathbf{b}) = \max_{\boldsymbol{\lambda} \geq 0} \left\{ \begin{array}{c} \min \sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} c_{i,k,n} q_{i,k,n} x_{i,k,n} + \\ \sum_{y \in Y} \sum_{n \in N} \lambda_{y,n} \left(\sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} \delta_{i,k,n}^{y} q_{i,k,n} x_{i,k,n} - U_{y,n} \right) \end{array} \right\} \tag{7}$$

subject to (2)-(4), (6).

Considering that each Lagrangian multiplier is given to be $\bar{\lambda}_{y,n}$, the Lagrangian relaxation problem reads:

$$L(\bar{\boldsymbol{\lambda}}, \mathbf{x}, \mathbf{b}) = \min \sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} \left(c_{i,k,n} + \sum_{y \in Y} \sum_{n \in N} \delta_{i,k,n}^{y} \bar{\lambda}_{y,n} \right) q_{i,k,n} x_{i,k,n} + \sum_{y \in Y} \sum_{n \in N} \bar{\lambda}_{y,n} U_{y,n} \tag{8}$$

subject to (2)-(4), (6)

where $\delta_{i,k,n}^{y}$ is the binary vehicle-station incidence indicator so that $\sum_{y \in Y} \sum_{n \in N} \delta_{i,k,n}^{y} \bar{\lambda}_{y,n}$ is equal to $\bar{\lambda}_{i,k,n}$. Due to both $\bar{\lambda}_{y,n}$ and $U_{y,n}$ are fixed, the objective function in (8) can be further simplified as:

$$L(\bar{\boldsymbol{\lambda}}, \mathbf{x}, \mathbf{b}) = \min \sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} (c_{i,k,n} + \bar{\lambda}_{i,k,n}) q_{i,k,n} x_{i,k,n} \tag{9}$$

The objective function in (9) ends up having the same structure as that in (1) only with an additional fixed Lagrangian multiplier cost on the charging slot n of the charging window k of the electric bus i. Such a Lagrangian relaxation problem (i.e., $L(\bar{\boldsymbol{\lambda}}, \mathbf{x}, \mathbf{b})$ s.t. (2)-(4) and (6)) decouples the correlation of charging schedules of different electric buses from the bus fleet. Therefore, we have the following charging scheduling subproblem (i.e., $L_i(\bar{\boldsymbol{\lambda}}, \mathbf{x}, \mathbf{b})$ s.t. (11)-(14)) for each single electric bus $i \in I$. The constraints in (11)-(14) are inherited from (2)-(4) and (6).

$$L_i(\bar{\boldsymbol{\lambda}}, \mathbf{x}) = \min \sum_{k \in K_i} \sum_{n \in N_{i,k}} (c_{i,k,n} + \bar{\lambda}_{i,k,n}) q_{i,k,n} x_{i,k,n} \tag{10}$$

subject to

$$b_{i,k+1} = b_{i,k} + \sum_{n \in N_{i,k}} q_{i,k,n} x_{i,k,n} - p_{i,k+1}^{i,k} \qquad \forall k \in K_i \tag{11}$$

$$b_{i,k} + \sum_{n \in N_{i,k}} q_{i,k,n} x_{i,k,n} \leq B_i^u \qquad \forall k \in K_i \tag{12}$$

$$b_{i,k} \geq B_i^l \qquad \forall k \in K_i \tag{13}$$

$$x_{i,k,n} \in \{0,1\} \qquad \forall k \in K_i, n \in N_{i,k} \tag{14}$$

4.2 Bi-criterion dynamic programming algorithm

The charge scheduling problem for each single electric bus is a binary combinatorial optimization problem providing an optimal charging schedule for each independent electric bus given Lagrangian multipliers. A bi-criterion dynamic programming algorithm is designed to address the above Lagrangian relaxation problem by solving the set of single bus charging scheduling problems. As discussed above, the charging behavior of each electric bus can only happen in its charging windows, and each charging window contains a set of charging slots. Since discrete charging processes are allowed in our study, the charging schedule is subdivided into each charging slot. For example, if the number of charging slots of a charging window is n, the number of optional charging schedules in this charging window is 2^n. The dynamic programming is to find the optimal charging schedule and the associated battery electricity level from optional charging schedules for each charging window with the objective of minimizing total charging electricity cost while respecting recharging requirements and restrictions. Below the developed bi-criterion dynamic programming algorithm is elaborated in the four subparts: introduce label pairs, regulate charging schedules, calculate label pairs, and evaluate and update label pairs. The notation used in this section is first given in **Table 3**.

TABLE 3 Additional notation for the dynamic programming algorithm

$J_{i,k}$	Set of the index of optional charging schedules in charging window k of electric bus i, $J_{i,k} = \{j\}$
$\mathbf{x}_{i,k}^{j}$	Optional charging schedule j in charging window k of electric bus i associated with the binary activation variables of all charging slots $\{1, \cdots, n\}$ in charging window k, $\mathbf{x}_{i,k}^{j} = \{x_{i,k,1}, x_{i,k,2} \dots, x_{i,k,n}\}^{j}$
h^j	Charged electricity amount of charging schedule j, $h^j = \sum_{n \in N_{i,k}} q_{i,k,n} x_{i,k,n}$
z^j	Electricity cost (including the Lagrangian multipliers as penalty cost) of adopting the charging schedule j, $z^j = \sum_{n \in N_{i,k}} (c_{i,k,n} + \bar{\lambda}_{i,k,n}) q_{i,k,n} x_{i,k,n}$
$e_{i,k}^{j}$	Cumulative battery electricity level from the start of an operational day to the charging window k when charging schedule j is adopted in charging window k
$c_{i,k}^{j}$	Cumulative electricity cost from the start of an operational day to charging window k when charging schedule j is adopted in charging window k
$(e_{i,k}^{j}, c_{i,k}^{j})$	A label pair recording the cumulative battery electricity level and cumulative electricity cost in charging window k of electric bus i when charging schedule j is adopted in charging window k
$L_{i,k,j}$	Set of label pairs of electric bus i in charging window k, when charging schedule j is adopted, $L_{i,k,j} = \left\{\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{m}\right\}, \forall\, j \in J_{i,k}$, and m is the index of label pairs since different label pairs will be derived from different previous label pairs at charging window $k-1$ even though the same charging schedule j is adopted in charging window k

4.2.1 Introduce label pairs

We introduce a label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)$ to record cumulative battery electricity level $e_{i,k}^{j}$ and cumulative electricity cost $c_{i,k}^{j}$ from the start of an operational day to charging window k of electric bus i when charging schedule j is adopted. Label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)$ will be further evaluated and updated synchronously during the course of the bi-criterion dynamic programming algorithm. A set of label pairs of charging window k will be derived from the multiple label pairs passed from different charging schedules $J_{i,k-1}$ of previous charging window $k-1$. As a result, we further introduce $L_{k,j} = \left\{\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{m}\right\}, j \in J_{i,k}$ to denote the set of label pairs of electric bus i in charging window k when charging schedule j is adopted.

The necessity of introducing label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)$ is twofold. First, the objective function (9) requires us to find the optimal charging schedule for each charging window k of an electric bus i that minimizes the total electricity cost of its charging process during the operational day, which necessitates the electricity cost label. Note that the electricity cost here includes not only the electricity cost imposed by TOU tariffs but also the penalty cost formed by Lagrangian multipliers. Obviously, the cumulative electricity cost label of the last charging window, $c_{i,k}^{j}$, is equal to the total charging cost for an optional charging schedule. This implies that the lower $c_{i,k}^{j}$ of a charging schedule for an intermediate charging window, the better the system performance. Second, the lower bound constraint of battery electricity level in (4) requires that the battery electricity level of any bus is always above a certain amount. As a result, this necessitates another label $e_{i,k}^{j}$ and implies that the higher $e_{i,k}^{j}$ of a charging schedule for an intermediate charging window, the better the system performance. However, maintaining a higher battery electricity level generally leads to charging more electricity into bus batteries, which increases $c_{i,k}^{j}$. Thus, to evaluate a charging schedule both electricity cost and battery electricity level labels are necessary to be put forward and of equal significance to consider although it is the total electricity cost that is minimized in the end. Regarding the constraint in (2), it defines how to calculate each label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)$.

4.2.2 Regulate charging schedules

For each optional charging schedule, the bi-criterion dynamic programming algorithm first abandons those schedules violating the constraints of battery electricity level bounds and only reserves the schedules that perform a charge with the lowest charging cost (i.e., the electricity cost imposed by TOU tariffs plus the penalty cost formed by Lagrangian multipliers) based on each previous label pair, i.e., $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,o}$. To this end, we first need to calculate the feasible amount of charged electricity for $\mathbf{x}_{i,k}^{j}, j \in J_{i,k}$ based on $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m}, o \in J_{i,k-1}$. It should be noted that for each label pair $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,o}, o \in J_{i,k-1}$, a feasible amount of charged electricity needs to be determined. The amount of charged electricity for an optional charging schedule $\mathbf{x}_{i,k}^{j}$ is equal to h^{j}, which is subject to the following inequality in (15) due to the constraints in (3) and (4). The inequality in (15) regulates the upper and lower bounds of the feasible amount of charged electricity for $\mathbf{x}_{i,k}^{j}, j \in J_{i,k}$ of the charging window k. This also means whether h^{j} is feasible or not also depends on the label pair $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,0}, o \in J_{i,k-1}$ considered.

$$\left\lceil \frac{p_{i,k}^{i,k-1} + p_{i,k+1}^{i,k} + B_{i}^{l} - e_{i,k-1}^{o}}{q_{i,k,n}} \right\rceil \leq h^{j} \leq \left\lfloor \frac{B_{i}^{u} + p_{i,k}^{i,k-1} - e_{i,k-1}^{o}}{q_{i,k,n}} \right\rfloor$$

$$\forall j \in J_{k}, \left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,0}, o \in J_{i,k-1} \tag{15}$$

After determining the feasible amount of charged electricity, we need to reserve the charging schedules that have a feasible amount of charged electricity and also perform charging at the lowest electricity cost for each charging window. z^{j} is used for denoting the electricity cost of the optional charging schedule $\mathbf{x}_{i,k}^{j}$. For each reserved charging schedule $\mathbf{x}_{i,k}^{\hat{j}}, \hat{j} \in J_{i,k}$, its $z^{\hat{j}}$ has to satisfy the inequality in (16). Considering $h^{\hat{j}}$ is the desired amount of charged electricity, it is easy to reserve the charging schedules satisfying the inequality in (16). In practice, for each charging window, we just activate charging slots $n \in N_{i,k}$ according to $h^{\hat{j}}$ in a successive sequence in terms of the charging cost rate, i.e., $c_{i,k,n} + \bar{\lambda}_{i,k,n}$ from low to high.

$$z^{\hat{j}} \leq z^{j} \qquad \forall h^{j} = h^{\hat{j}}, j \in J_{i,k} \tag{16}$$

4.2.3 Calculate label pairs

After regulating the charging schedules, the bi-criterion dynamic programming algorithm calculates the set of label pairs, i.e., $L_{i,k,j}$ of an optional charging schedule $\mathbf{x}_{i,k}^{j}, j \in J_{i,k}$ for a charging window k. Take $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t} \in L_{i,k,j}$ as an example. Based on a label pair $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,o}, o \in J_{i,k-1}$, the two labels, $\left(e_{i,k}^{j}\right)^{t}$ and $\left(c_{i,k}^{j}\right)^{t}$ of $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t}$, are derived according to the following recursive equations in (17) and (18), respectively. It should be highlighted that each label pair $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,o}$, $o \in J_{i,k-1}$ will produce one label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t} \in L_{i,k,j}$ at any schedule node $j \in J_{i,k}$.

$$\left(e_{i,k}^{j}\right)^{t} = \left(e_{i,k-1}^{o}\right)^{m} + h^{j} - p_{i,k}^{i,k-1} \tag{17}$$

$$\left(c_{i,k}^{j}\right)^{t} = \left(c_{i,k-1}^{o}\right)^{m} + z^{j} \tag{18}$$

4.2.4 Evaluate and update the label pairs

For each optional charging schedule $\mathbf{x}_{i,k}^{j}, j \in J_{i,k}$ of a charging window k, after a set of label pairs $L_{i,k,j}$ has been calculated, the bi-criterion dynamic programming algorithm further evaluates and updates the set of label pairs $L_{i,k,j}$ for charging window k. The process of evaluating these label pairs and abandoning the label pairs that definitely will not generate the optimal charging schedule is the key to the efficiency of the bi-criterion dynamic programming algorithm. To this end, the definition of a Pareto label pair is introduced.

It is defined that a label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t}$ is a Pareto-optimal label pair to $L_{i,k,j}$ if it is not dominated by any other existing label pairs in $L_{i,k,j}, \forall j \in J_{i,k}$. To be clear, assume $\left(\bar{e}_{i,k}^{j}, \bar{c}_{i,k}^{j}\right)^{u}$ is an existing label pair from $L_{i,k,j}$, when $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t}$ is said not to be dominated by $\left(\bar{e}_{i,k}^{j}, \bar{c}_{i,k}^{j}\right)^{u}$, it means $\left(e_{i,k}^{j}\right)^{t} > \left(\bar{e}_{i,k}^{j}\right)^{u}$ or $\left(c_{i,k}^{j}\right)^{t} < \left(\bar{c}_{i,k}^{j}\right)^{u}$. When a new label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t}$ is generated by the aforementioned recursive equations in (17) and (18), we have to judge whether this new label pair is a Pareto-optimal label pair to the set of existing label pairs $L_{i,k,j}$ by evaluating it and any other existing label pair following the dominance criteria (Brumbaugh-Smith and Shier, 1989; Xie and Waller, 2012). If a new label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t}$ is not dominated by any other existing label pair, the existing set of label pairs $L_{i,k,j}$ will be updated by adding $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t}$ into it. If the new label pair is dominated or an existing label pair is dominated, the dominated label pair will be removed from $L_{i,k,j}$.

When the update of $L_{i,k,j}$ for the last charging window is finished, the minimum charging cost of bus i can be obtained from the set of cumulative electricity cost labels of the last window, as is shown in (19). The optimal charging schedule of each charging window can be obtained eventually from $\mathbf{x}_{i,k}^{j}, \forall j \in J_{i,k}$ through label backtracking,

$$\min\left\{\left(e_{i,k}^{j}\right)^{m} \middle| \forall \left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{m} \in L_{i,k,j}, j \in J_{i,k}\right\} \tag{19}$$

where k is the index for the last charging window.

For each Lagrangian relaxation problem, the optimal charging schedule for the bus fleet is equivalent to the set of individual bus charging schedules, each of which is independently optimized by minimizing the electricity cost of a single bus.

4.3 Complete algorithmic procedure

The complete algorithmic procedure of the developed bi-criterion dynamic programming algorithm embedded in the Lagrangian relaxation framework for solving the proposed charging scheduling problem is summarized as follows:

Step 0: Initialization.

Initialize the Lagrangian multipliers $\boldsymbol{\lambda}^0 = \mathbf{0}$**.** Set the iteration counter $r = 0$, lower bound $LB = -\infty$, upper bound $UB = +\infty$, and the convergence parameter ϵ.

Step 1: Solving the relaxed Lagrangian problem by a bi-criterion dynamic programming algorithm.

Step 1.1: Initialization. Initialize the set of label pairs $L_{i,0,j}, \forall i \in I, j \in J_{i,0}$. Set $k = 0, i = 0$.

Step 1.2: Regulate the candidate charging schedules. Set $k = k + 1$. For each label pair $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m} \in L_{i,k-1,o}, o \in J_{i,k-1}$ of the charging window $k - 1$, regulate the charging schedules $\mathbf{x}_{i,k}^{j}, \forall j \in J_{i,k}$ of charging window k.

Step 1.3: Calculate the label pair. For each regulated optional charging schedule $\mathbf{x}_{i,k}^{j}$ of the charging window k based on a label pair $\left(e_{i,k-1}^{o}, c_{i,k-1}^{o}\right)^{m}$ of the previous charging window $k - 1$, generate the associated label pair $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)^{t} \in L_{i,k,j}$.

Step 1.4: Evaluate the newly generated label pair and update the set of existing label pairs. For the new $\left(e_{i,k}^{j}, c_{i,k}^{j}\right)$ generated in *Step 1.3*, evaluate it according to the dominance criteria, and update the set of existing label pairs $L_{i,k,j}, j \in J_{i,k}$.

Step 1.5: Termination. If k is the last charging window of bus i, record the optimal charging schedule for bus i through label backtracking and set $i = i + 1$, $k = 0$ and go to *Step 1.2*; go directly to *Step 1.2*, if otherwise. If bus i is the last bus of the fleet, terminate this algorithm and calculate the total charging cost $z(\mathbf{x}^r, \mathbf{b}^r)$ for the bus fleet.

Step 2: Feasibility test.

If $\sum_{i \in I} \sum_{k \in K_i} \sum_{n \in N_{i,k}} \delta_{i,k,n}^{y} q_{i,k,n} \left(x_{i,k,n}\right)^{r} \leq U_{y,n}, \forall y \in Y, n \in N$, go to **Step 3**. Otherwise, go to **Step 4**.

Step 3: Convergence test.

Calculate $LB = L(\bar{\boldsymbol{\lambda}}^r, \mathbf{x}^r, \mathbf{b}^r)$, $UB = z(\mathbf{x}^r, \mathbf{b}^r)$. If $(UB - LB)/LB < \epsilon$, let $\mathbf{x}^r$ be the optimal charging schedule and stop. Otherwise, go to **Step 4**.

Step 4: Updating the step size and Lagrangian multipliers.

If a feasible solution has not been found yet, then

- when $\sum_{i\in I}\sum_{k\in K_i}\sum_{n\in N_{i,k}}\delta_{i,k,n}^{y}q_{i,k,n}\left(x_{i,k,n}\right)^{r}\leq U_{y,n}$, $\lambda_{y,n}^{r+1}=\lambda_{y,n}^{r},\forall y\in Y,n\in N$
- when $\sum_{i\in I}\sum_{k\in K_i}\sum_{n\in N_{i,k}}\delta_{i,k,n}^{y}q_{i,k,n}\left(x_{i,k,n}\right)^{r}>U_{y,n}$: (1) $\lambda_{y,n}^{r+1}=c_1-c_{i,k,n}$ if $\lambda_{y,n}^{r}+c_{i,k,n}=c_0$; (2) $\lambda_{y,n}^{r+1}$=$c_2-c_{i,k,n}$ if $\lambda_{y,n}^{r}+c_{i,k,n}=c_1$;(3) $\lambda_{y,n}^{r+1}=\bar{c}-c_{i,k,n}$ if $\lambda_{y,n}^{r}+c_{i,k,n}=c_2$ or $\bar{c},\forall y\in Y,n\in N$, where c_0, c_1, and c_2 are TOU tariffs prices from low to high and $\bar{c}$ is an arbitrary price value larger than c_2.

If otherwise, then

- $\lambda_{y,n}^{r+1}=\max\left[0,\lambda_{y,n}^{r}+\theta_{r+1}\left(\sum_{i\in I}\sum_{k\in K_i}\sum_{n\in N_{i,k}}\delta_{i,k,n}^{y}q_{i,k,n}\left(x_{i,k,n}\right)^{r}-U_{y,n}\right)\right],\forall y\in Y,n\in N$, where $\theta_{r+1}=\frac{\alpha_1}{r+\alpha_2}$.

Let $r=r+1$, and go to **Step 1**.

5. CASE STUDY

This section presents and assesses the numerical results obtained from a real-world electric bus system as the case study. In Subsection 5.1, we describe the information of this bus system in detail. In Subsection 5.2, we first present the results of a benchmark case to validate the effectiveness and practicability of the proposed model and algorithm; further, we change the system settings in 9 different hypothesized scenarios and analyze how the system settings affect the optimal charging schedule results. Many interesting findings and insights related to charging behaviors, charging infrastructure configurations, station load capacity, TOU tariffs, as well as battery capacity are discussed. In Subsection 5.3, the computational efficiency of the proposed algorithm is evaluated by comparing its performance to that the state-of-the-art Gurobi solver. The solution algorithm was programmed in C++ and performed on a laptop computer equipped with Intel Core i7-8550U CPU and 8G RAM. The computational performance of our developed algorithm is also discussed with a comparison to the Gurobi solver.

5.1 System information

This section presents brief information and settings for the tested electric bus operation system consisting of 10 bus lines, 128 electric buses, and 13 charging stations in Jiading, Shanghai (**Figure 2**). The operations timetables of the system are constructed using real-life data from Jiading Bus Company. The system has a total of 2,121 bus trips in a single operational day with a total operating mileage of 19,170.2 km.

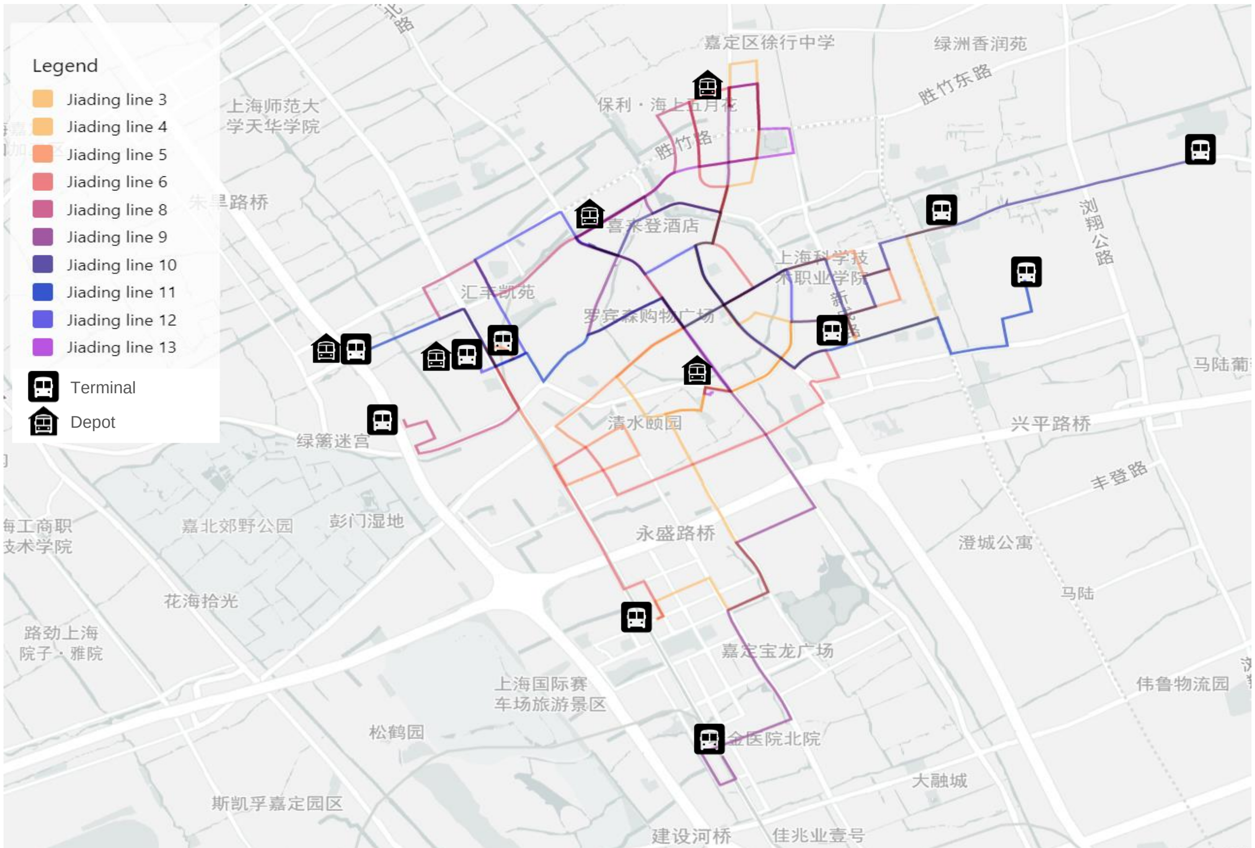


Figure 2 The electric bus network selected for the case study

The duration of one charging slot is determined to be 10 minutes so that an operational day is thus divided into 144 charging slots. If the duration of one charging slot is too long, many charging windows will be discarded. Moreover, the longer the charging slot duration is, the more inflexible the charging schedule will be since charging decisions are made slot by slot. However, if we shorten the charging slot duration, then an operations day can be divided into more charging slots which would increase the computing burden of solving the developed charging scheduling model. Besides, too short charging slot duration brings challenges to practical operations of charging. 10 minutes is chosen considering the tradeoff between solution accuracy, computing efficiency, and charging schedule attainment.

It is assumed in this case study that the electricity consumption of an electric bus is proportional to its driving distance. Accordingly, the electricity consumption rate of electric buses is set to be 1.5 kWh/km. The battery capacity of electric buses is homogeneously set to be 300 kWh for simplicity. Note that our model and solution methods allow adopting heterogenous electricity consumption rates and battery capacities as an input so that the variations of different bus service lines and different electric buses can be taken into account. The upper and lower bounds of battery electricity levels are respectively set to be 270 kWh and 60 kWh. Two types of charging stations are considered in this section, one of which is deployed at terminals, and the other is deployed at depots. The charging power of charging equipment at terminals is always set to be fast charging i.e., 160 kW, for the purpose of en-route charging only. Since the charging station at depots can provide both en-route charging and at-depot charging, the charging power of charging equipment at depots is set to be fast charging, i.e., 160 kW, for the en-route charging, and a lower power of regular charging i.e., 90 kW, for the at-depot charging. It is assumed that when the duration of a charging window at depots is shorter than 30 minutes (i.e., 3 charging slots), the depots are only used for en-route charging and the higher charging power is applied. Otherwise, we assume electric buses have sufficient layover time at depots for recharging and the lower charging power is applied at depots. To support the next day's bus service, it is assumed that each electric bus needs to be charged to a battery electricity level of over 240 kWh during its night layover at a depot after it finishes service trips. 240 kWh is also set to be the initial battery electricity level for the start of daily operations of all electric buses. Without loss of generality, the electricity load capacities of all charging stations are ubiquitously set as 1,000 kW, which constrains the total electricity charged by all the electric buses at a charging station at one charging slot. All of the above input parameters can be estimated from the real-world data and dynamically adjusted for each operations day if necessary.

The system profile information presented in **Table 4**, **Table 5**, and **Table 6** is about the charging stations, bus lines, and TOU tariffs for non-residential usage, respectively. **Table 4** gives the specific location of each charging station and the type of each charging station location. **Table 5** presents the specific electric buses operated on each bus line in terms of electric bus ID and also presents the specific charging stations that can be used for en-route or at-depot recharging for a bus line. For example, 10 electric buses from No. 52 to 61 are operated on bus line 3. Charging station 8 can provide en-route recharging and charging station 0 can provide both en-route and at-depot recharging for bus line 3. **Table 6** shows the local TOU tariffs used in a day, where the off-peak period with the lowest charging cost (i.e., 0.213 CNY/kWh) ranges from 22:00 to 6:00, the mid-peak period with a moderate charging price (i.e., 0.567 CNY/kWh) consists of three periods from 6:00 to 8:00, from 11:00 to 18:00, and from 21:00 to 22:00, and the on-peak period with the highest charging price (i.e., 0.916 CNY/kWh) ranges from 8:00 to 11:00 and from 18:00 to 21:00.

TABLE 4 Charging Station Information

Station ID	Station location	Type of location
1	Jiading Transport Center Depot	Depot and Terminal
2	East Taxin Road-Xucheng Road Terminal	Terminal

3	Pingcheng Road Depot	Depot and Terminal
4	Jiawang Road-Liang Road Terminal	Terminal
5	North Jiading Depot	Depot and Terminal
6	Xincheng Road Terminal	Terminal
7	Nanmen Depot	Depot and Terminal
8	Juyuan Depot	Depot and Terminal
9	Taxin Road-Nijiabang Road Terminal	Terminal
10	Baiyin Road Terminal	Terminal
11	West Jiading Terminal	Terminal
12	West Huicheng Road-Zhudi Road Terminal	Terminal
13	Jiading Xincheng Terminal	Terminal

TABLE 5 Bus Line Information

Line ID	Electric bus ID	Fleet size	Charging stations for terminals	Charging stations for depots	Operation mileage (km)
3	52-61	10	1, 9	1	9.525
4	62-73	12	8, 10	5	12.550
5	74-86	13	6, 11	5	10.925
6	87-99	13	8, 10	5, 8	12.700
8	100-107	8	8, 10	5, 8	12.300
9	108-128	21	8, 13	8	10.950
10	1-12	12	2, 3	1	10.950
11	13-27	15	3, 4	3	10.925
12	28-37	10	5, 6	5	8.500
13	38-51	14	7, 8	7	8.325

TABLE 6 TOU Tariffs

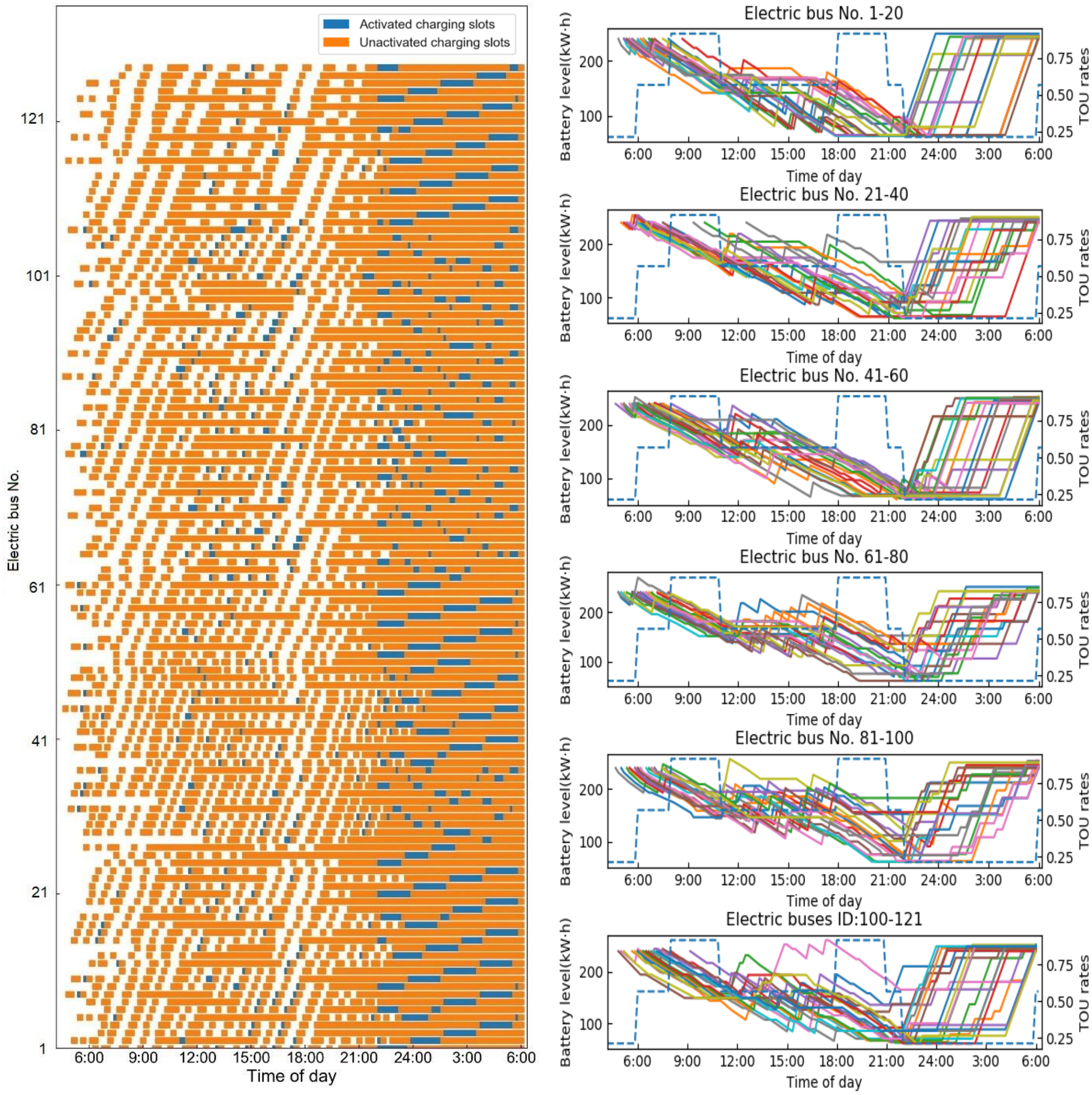
Time of day	Demand level	Price (CNY/kWh)
6:00-8:00	Mid-peak	0.567
8:00-11:00	On-peak	0.916
11:00-18:00	Mid-peak	0.567
18:00-21:00	On-peak	0.916
21:00-22:00	Mid-peak	0.567
22:00-6:00	Off-peak	0.213

5.2 Result evaluation

5.2.1 Optimal charging schedule in the benchmark case

In the benchmark case, the system parameters are set as elaborated in section 5.1. We first present the optimal charging schedules of the bus fleet in **Figure 3**. **Figure 3(a)** illustrates the activated charging slots in the optimal charging scheduling results, where the orange part denotes the unactivated charging slots of charging windows of an electric bus while the blue part denotes the activated charging slots. For the blank (or white) part, it represents electric buses are serving the trip and no charging windows are available. **Figure 3(b)** observes the change of battery electricity levels for the whole electric bus fleet against the operating time, where the blue dashed line presents the variation of TOU rate within a time of day. The change of the battery electricity level of each electric bus implicitly presents the charging and discharging behaviors. The decrease of the battery electricity level of an electric bus means that it is during the operation service; the increase of the battery electricity level of an electric bus means that it is being charged at a charging station; the unchanged status of the battery electricity level of an electric bus means that it is during a layover between two operation service trips as well as without a charging process. It should be noted that each electric bus needs to be charged to a battery electricity level of over 240 kWh by the end of the planning horizon to support the next day's bus service. Hence, the last charging window of each electric bus normally has a few activated charging slots.

The amount of charged electricity of an individual electric bus in the optimal charging scheduling result is shown in **Figure 4**. We can see that the range of charging electricity of the bus fleet is from 130 kWh to 290 kWh, which implicitly reflects the required energy consumption of each bus for completing its daily operations. In **Figure 5**, the blue and orange bars denote the number of activated charging slots of each electric bus respectively for en-route and at-depot recharging. It is obvious that the number of at-depot charges is much larger than that of en-route charges for almost all electric buses, which results from the off-peak electricity price at midnight during the last charging window of each electric bus. Hence, we can see that most charging activities of electric buses take place in their last charging windows at the depots.



(a) Activated charging slots **(b) Variations of battery electricity levels**

Figure 3 Illustration of the optimal charging schedules of the bus fleet

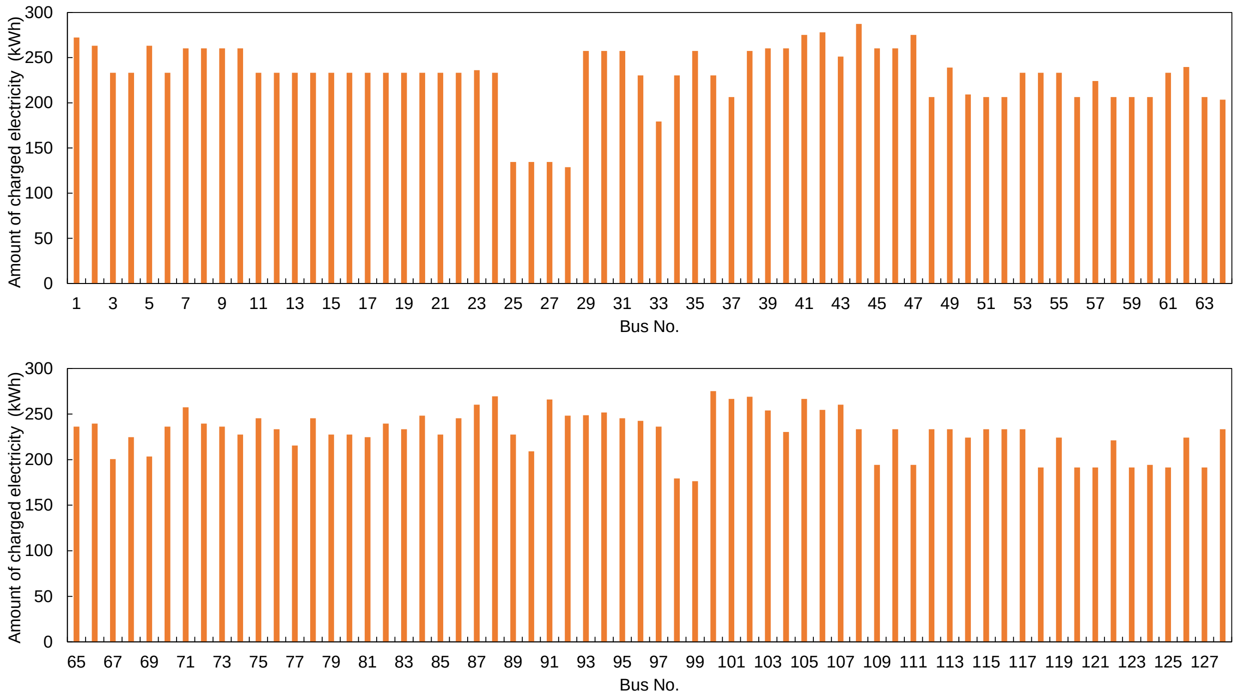


Figure 4 Charged electricity amounts of individual electric buses

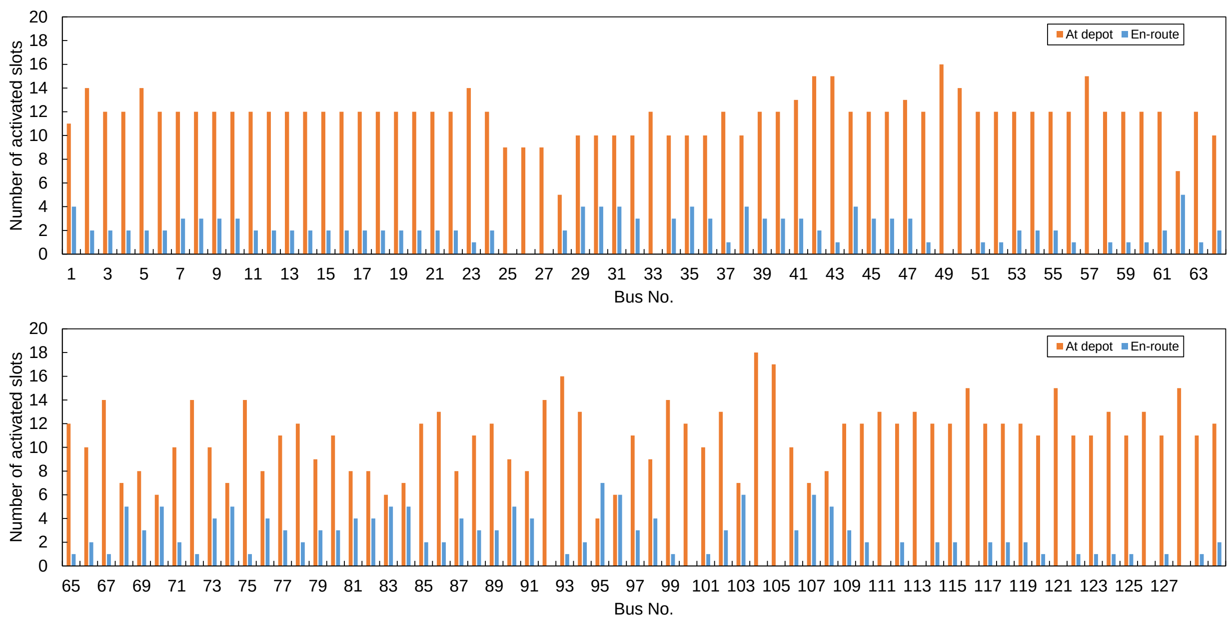


Figure 5 Activated charging slot numbers of individual electric buses for en-route and at-depot charging

5.2.2 Impacts of system settings on optimal charging schedules

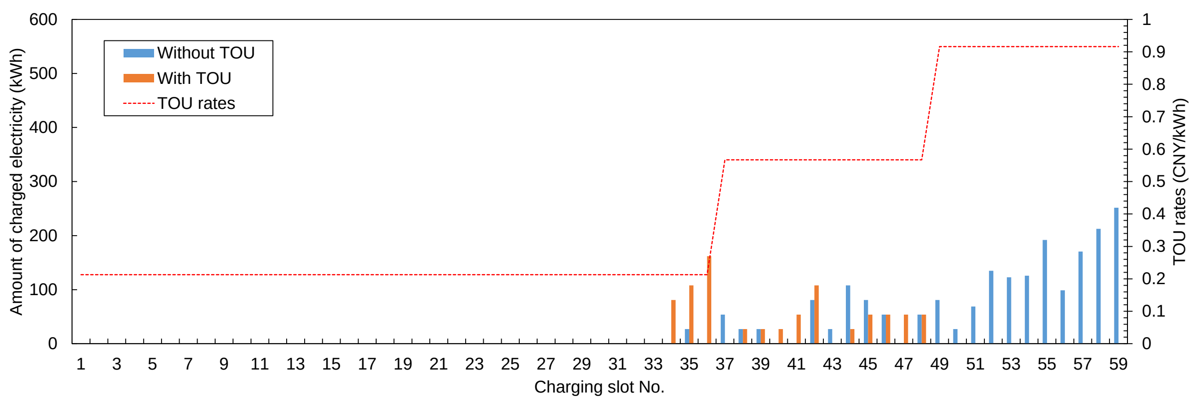
In this part, 9 different system scenarios are hypothesized, as presented in **Table 7**, to evaluate the impacts of 4 critical factors in the electric bus system on the optimal charging scheduling results. These critical factors are station load capacity, TOU tariffs, vehicle battery capacity, and charging infrastructure configurations. Scenario 1 is the benchmark scenario. For comparison purpose, we add the following settings: (a) The station load capacity constraint is removed in scenario 2. (b) TOU tariffs are not taken into account in scenario 7. (c) The charging infrastructure is configured to be "Fast Only" or "Regular Only", respectively, in scenarios 8 and 9. As we mentioned before, regular charging power is set to be 90 kW for serving at-depot charging and fast charging power is set to be 160 kW for serving en-route charging. "Fast Only" means that 160 kW is set for both at-depot and en-route charging and "Regular Only" means that 90 kW is applied to both at-depot and en-route charging. (d) From scenarios 3 to 6, the battery capacity of buses is gradually improved and so is the upper bound of battery electricity level. It can be seen the station load capacity can quite influence the total charging cost while the total charging electricity tends not to change much (compare scenarios 1 and 2). This is straightforward, as considering the load capacity constraint mainly affects when to charge but not how much to charge. However, when we ignore TOU tariffs in optimizing the charging schedule, the total expenditure needed for recharging surges by more than 70% but the total charging electricity still does not change much (compare scenarios 1 and 7). It is also observed that the optimal charging scheduling results of different vehicle battery capacities and corresponding different battery electricity level bounds (in scenarios 1, 3, 4, 5, and 6) have the same or a quite similar total charging costs and total charging electricity. This is because the total charging electricity is mainly dependent on the amount of electricity consumed during the operation while respecting the en-route charging demands instead of the vehicle battery capacities. It can also be concluded that the battery capacity of 285 kWh is quite enough for electric buses serving the current bus timetable at Jiading Shanghai. The charging infrastructure configuration has an obvious influence on both total charging cost and electricity (compare scenarios 1, 8, and 9). The impacts of these different system settings are further presented and discussed below.

TABLE 7 Optimal charging scheduling results under different system scenarios

System scenarios	Station load capacity (kW)	Battery capacity (kWh)	Battery electricity level bounds (kWh)	TOU tariffs	Charging infrastructure configuration	Total charging cost (CNY)	Total charging electricity (kWh)
1	1,000	300	60-270	Yes	Regular and Fast	9,184.55	29,552.76
2	Without limit	300	60-270	Yes	Regular and Fast	8,646.63	29,714.04
3	1,000	285	60-255	Yes	Regular and Fast	9,195.74	28,778.76
4	1,000	315	60-285	Yes	Regular and Fast	9,184.55	28,665.00

5	1,000	330	60-300	Yes	Regular and Fast	9,184.55	28,808.64
6	1,000	360	60-330	Yes	Regular and Fast	9,184.55	28,871.28
7	1,000	300	60-270	No	Regular and Fast	15,856.95	29,134.08
8	1,000	300	60-270	Yes	Fast only	9,757.80	30,780.00
9	1,000	300	60-270	Yes	Regular only	10,554.36	30,014.46

To explore the impacts of TOU tariffs on the optimal bus fleet charging schedule, two charging price scenarios, with or without considering TOU tariffs, are considered and assessed. **Figure 6** further illustrates the charged electricity amounts of the bus fleet at each charging slot under the two charging price scenarios. From **Figure 6**, we can see that without considering TOU tariffs, charging activities tend to steadily split over the operational time; while with considering TOU tariffs, recharging at slots with high TOU tariffs can largely be avoided and tend to be activated at the slots with lowest TOU tariffs. For example, considering the impact of TOU tariffs, most recharging activities happen at midnight (charging slots 134-178) instead of in the daytime for serving the daytime operations. Moreover, the total charging cost of the bus fleet decreases about 42% from 15,856.95 CNY without considering the TOU tariffs to 9,184.55 CNY, if the TOU tariffs are considered in optimizing the charging schedule.



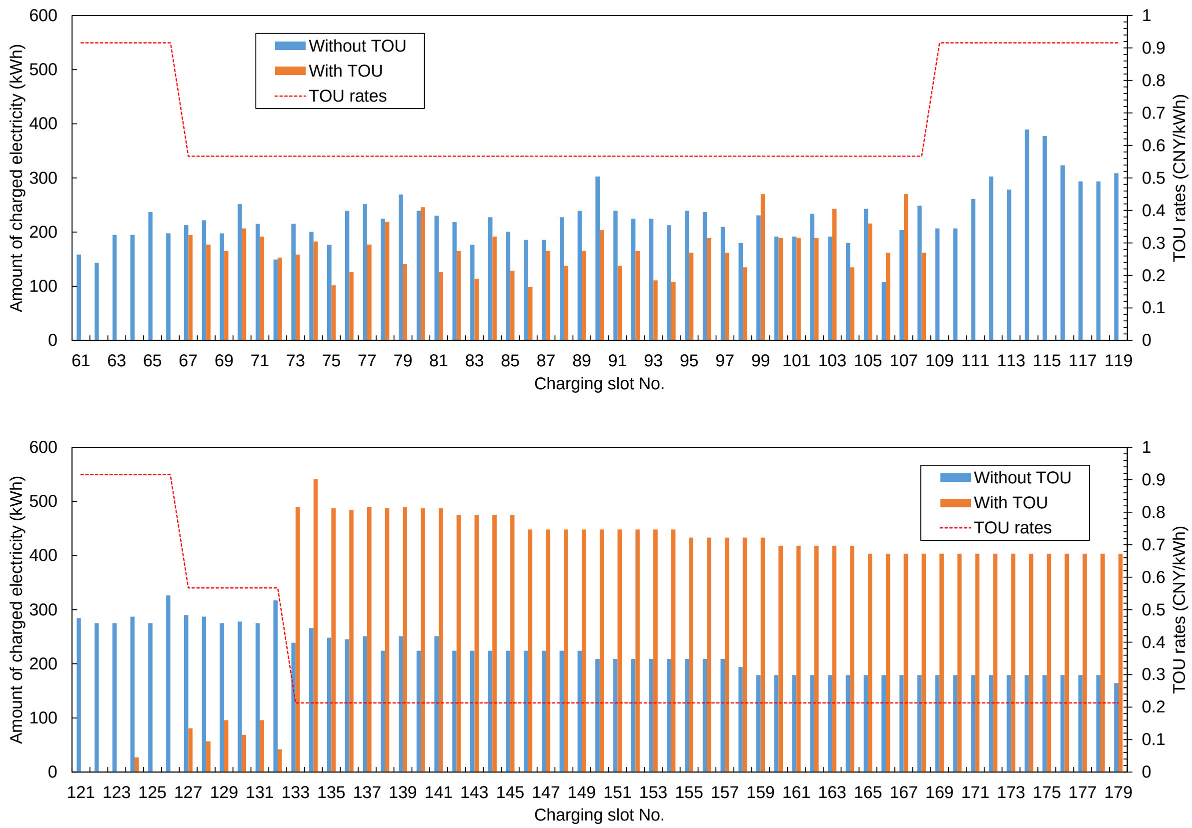
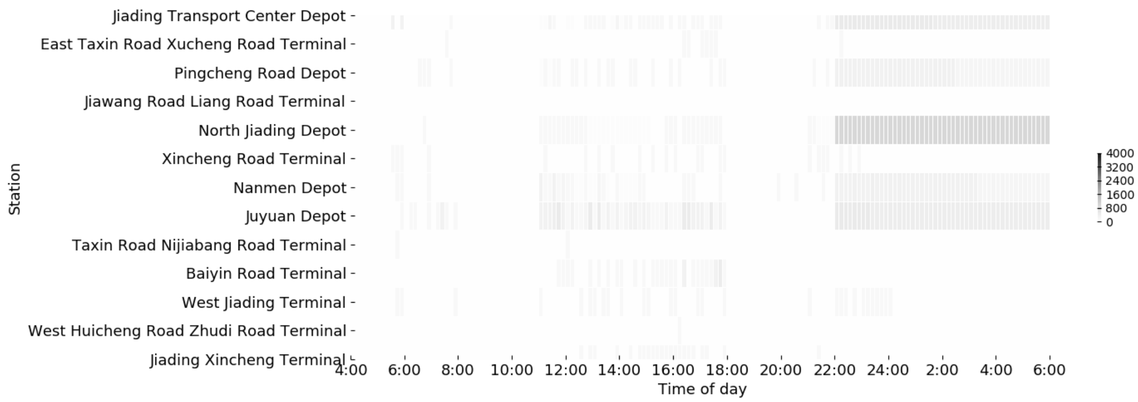
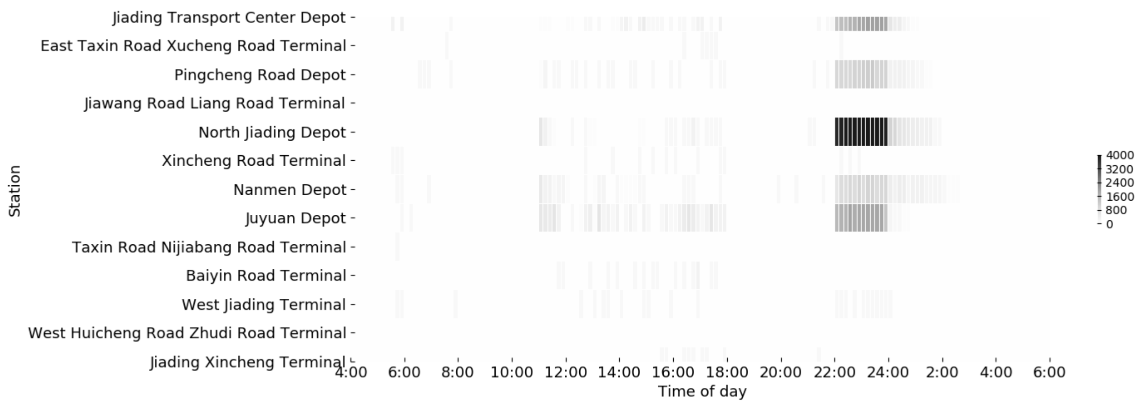


Figure 6 Optimal charged electricity amounts of charging slots with or without TOU tariffs

We evaluate the impact of station load capacity on the optimal bus fleet charging schedule by considering two typical scenarios with or without station load capacity. A heatmap is used to demonstrate the change of total charging load of each charging station over time, shown in **Figure 7**. When the station load capacity is imposed, the total charging cost of the bus fleet increases about 5.9% from 8,646.64 CNY to 9,184.55 CNY, as part of electric buses in the fleet have to switch to using on-peak charging slots, while the maximum charging load of charging stations decreases significantly from over 3,000 kW to less than 1,000 kW. The charging demands are spatiotemporally dispersed to more charging stations of more charging slots rather than being satisfied intensively by charging stations. The charging load is, therefore, balanced in both the dimensions of time and space. The heatmap also indicates that it is likely redundant to deploy charging stations at several terminals (e.g., Jiawang Road-Liang Road Terminal, West Huicheng Road-Zhudi Road Terminal) due to merely no recharges are supported by them. Depots suffering from heavy electricity loads can be recognized in **Figure 7**, such as Jiading Transport Center Depot, North Jiading Depot, and Juyuan Depot. Expanding the station load capacities of those depots to accommodate more buses can alleviate the negative effects of heavy electricity loads on the charging schedule.



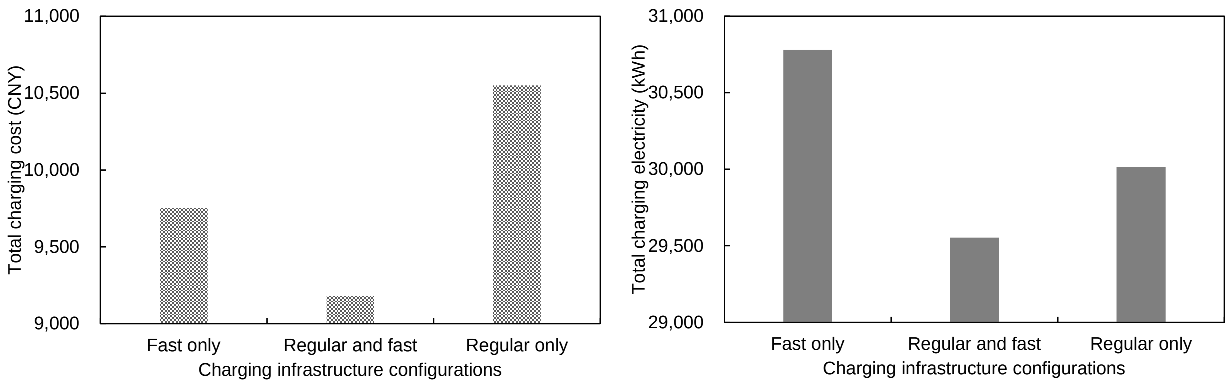
(a) With station load capacity (1,000 kW)



(b) Without station load capacity

Figure 7 Variation of the charging loads of charging stations over times of day

We examine the impact of charging infrastructure configurations on the optimal bus fleet charging schedule by hypothesizing three typical charging infrastructure configuration scenarios, i.e., with the fast-charging equipment (160 kW) only, with the regular-charging equipment (90 kW) only, or with both the regular- and fast-charging equipment. **Figure 8** briefly illustrates the variations of the total charging cost and total charging electricity with different charging infrastructure configurations. We can find that the charging infrastructure configuration with both the regular and fast charging equipment provides the least total charging cost and electricity. This implies the combination use of regular and fast charging should be preferred.



(a) Variations of total charging cost (b) Variations of total charging electricity

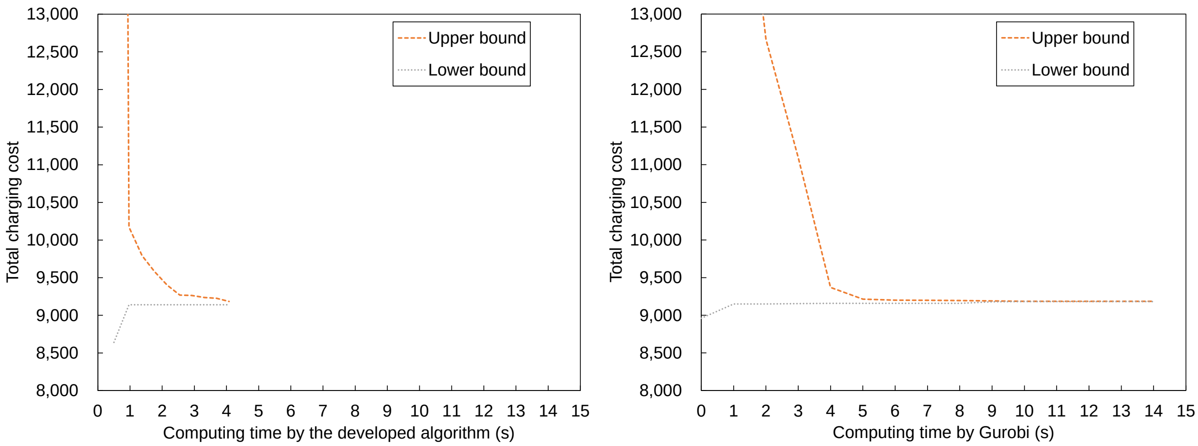
Figure 8 Variations of total charging cost and electricity with different charging infrastructure configurations

5.3 Algorithm performance

We here briefly evaluate the performance of our developed algorithm by using the Gurobi solver as a comparative reference. The comparisons focus on the optimized system cost and the consumed computing time under different system settings as shown in **Table 7**. In most scenarios, the objective function values given by our algorithm and Gurobi are the same except for a few cases. Although the objective function values in those cases given by our algorithm are slightly larger than that by the Gurobi solver, the gaps between these two results are small enough to be acceptable in practice. While our algorithm requires much less computing time than the Gurobi solver under any system scenario, especially when the electricity load capacity is relatively unrestricted. In some scenarios, the Gurobi solver consumes too much time to find the globally optimal objective function value. For example, when the initial SOC is 60% and the electricity load capacity is 800 kW Gurobi solver consumes 190.00 seconds. However, our algorithm requires only 2.23 seconds to find a near-optimal solution and the gap between the two objective function values is only 1.9%. The results in **Table 7** show that our developed algorithm may not always guarantee the convergence to the optimal solution as the Gurobi solver does; however, the suboptimality probability of the developed algorithm is very small. Moreover, **Figure 9** illustrates the convergence curves given by our algorithm and Gurobi in the aforementioned benchmark case (i.e., 80% of the initial SOC as the battery capacity and 1,000 kW electricity load capacity at each charging station).

TABLE 7 Comparisons of computational results between our algorithm and Gurobi under different system settings

Initial SOC (%)	Electricity load capacity (kW)	Gurobi		Our algorithm	
		Objective function value (CNY)	Computing time (s)	Objective function value (CN¥)	Computing time (s)
80	1,800	8,646.64	1.77	8,646.64	0.52
	1,500	8,646.64	4.43	8,646.64	0.56
	1,300	8,646.64	4.07	8,646.64	0.52
	1,200	8,745.42	7.16	9,115.80	1.89
	1,100	8,960.09	17.32	9,115.80	2.41
	1,000	9,184.55	14.97	9,184.55	3.30
	900	9,438.41	22.33	9,464.86	2.80
	800	9,946.13	14.56	9,988.44	1.90
	700	10,200.62	14.61	10,239.08	1.05
70	1,800	9,931.45	3.97	9,931.84	0.59
	1,500	9,931.45	4.48	9,931.84	0.60
	1,300	9,931.45	4.94	9,931.84	0.60
	1,200	9,931.45	5.28	9,931.84	0.61
	1,100	9,931.45	3.73	9,931.84	0.59
	1,000	9,969.66	20.17	10,322.15	2.60
	900	10,184.33	15.65	10,322.15	3.50
	800	10,665.92	19.48	10,687.08	2.10
	700	10,919.78	14.7	10,919.78	1.10
60	1,800	11,427.85	5.58	11,427.85	0.58
	1,500	11,427.85	6.16	11,427.85	0.58
	1,300	11,427.85	6.25	11,427.85	0.53
	1,200	11,427.85	7.07	11,427.85	0.61
	1,100	11,427.85	8.15	11,427.85	0.58
	1,000	11,427.85	5.83	11,427.85	0.60
	900	11,427.85	8.31	11,427.85	0.58
	800	11,612.85	190.00	11,836.46	2.23
	700	11,827.52	92.48	11,834.35	4.30



(a) Our algorithm (b) Gurobi

Figure 9 Convergence curves in solving the benchmark case

6 CONCLUSIONS

Electric buses as a green public transportation mode have been widely used in many metropolitan areas, owing to its features of zero exhaust emissions and relatively low operating and maintenance costs. However, the limited driving range and discharging/charging depths as well as relatively long recharging duration have been hindering the use of electric buses for transporting commuters so that they will not be operated as flexible as conventional gasoline or diesel buses. To tackle these restrictions and promote the adoption of electric buses, this paper proposes a general charging scheduling problem for an electric bus fleet to meet its recharging demands and manage its recharging behaviors with the aim of reducing the total electricity costs. By discretizing the time into consecutive slots, the proposed charging scheduling problem is formulated as a mixed linear integer programming model, in which decision variables are a set of binary charging slot activation variables and a set of continuous battery electricity level variables. A Lagrangian relaxation framework is constructed for decoupling the overall charging schedule of the electric bus fleet into a number of independent single-bus charging scheduling subproblems by relaxing the site-specific electricity load capacity constraint. A bi-criterion dynamic programming algorithm is put forward for solving each single-bus charging scheduling subproblem by evaluating and updating the battery electricity level and electricity cost labels and abandoning dominated partial charging schedules until an optimal charging schedule of a single bus is obtained.

It should be highlighted that the addressed problem is pretty practical by taking into account many real-world environment settings and operational requirements. Charging windows are introduced based on the bus operations timetables to regulate the temporal recharging opportunities for each bus. The driving range limit and battery SOC bounds allow every bus to get recharged in time before running out of power even if unpredictable electricity consumption occurs. The charging rates of different types of charging equipment are set to be different to accommodate both en-route and at-depot charging activities. Since TOU tariffs are considered, it is encouraged in the optimal charging schedule to recharge electric buses in the off-peak periods and avoid recharging in the on-peak periods.

To test the effectiveness and efficacy of the proposed model and algorithm, a real-world case study with 122 electric buses and 13 charging stations of 10 bus lines from Jiading, Shanghai, is conducted and analyzed. The optimal charging schedule of the bus fleet is explicitly presented in terms of the activated charging slots and the battery electricity level changes over different times of day. Furthermore, 9 different system scenarios are hypothesized, tested, and discussed to evaluate the impacts of station load capacity, TOU tariffs, vehicle battery capacity, and charging infrastructure configurations. We believe that a few findings of the following are worth to be highlighted here. First, the total expenditure needed for recharging surges more than 70% if TOU tariffs are ignored in optimizing the charging schedule. When TOU tariffs are taken into account, major recharges can be observed to take place at depots in the midnight since the off-peak electricity price is the lowest. Second, by imposing the station load capacity constraint, charging demands can be effectively dispersed to more charging stations of more charging slots rather than being satisfied intensively by a few charging stations. As a result, a few depots suffering from heavy electricity loads can be released by respecting the station load capacity, e.g., Jiading Transport Center Depot, North Jiading Depot, and Juyuan Depot. Third, the hybrid use of regular and fast chargers can help reduce both total charging cost and electricity amount. Last but not least, the battery capacity of around 285 kWh is found to be quite enough for electric buses serving the current bus timetable at Jiading Shanghai. In terms of algorithmic development, the performance of our developed algorithm is very promising: In most of the 27 tested scenarios, our algorithm hits the optimal objective function value as the Gurobi solver does while consuming much less time. It is anticipated that when used for dealing with larger-scale cases, the algorithm could still deliver optimal or highly optimal solutions in a very efficient manner.

For future research, the mixed linear integer programming model proposed in this paper is so concise and flexible that it can be used as a modeling block incorporated into more complex system optimization problems for electric bus system development and management: 1) charging infrastructure planning problems, e.g., determining charging station locations and the quantity of charging piles in each station (Xylia et al., 2017); 2) vehicle scheduling problems that optimize the allocation and operation of electric buses (Ibarra-Rojas et al., 2015). Regarding recent studies on the optimization of an electric bus system, it is common for researchers to oversimplify the charging behaviors of electric buses, which do play an important role in both the feasibility of bus operations and the reduction of operations cost.

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