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# Development of Connected and Automated Vehicle Platoons with Combined Spacing Policy

Yuan Zheng, Min Xu, Shining Wu, Shuaian Wang

***Abstract*—**Vehicle platoon has the potential to significantly improve traffic throughput and reduce fuel consumption and emissions and thus has attracted extensive attention recently. In this study, we propose a vehicle platoon of connected and automated vehicles (CAVs) with a combined spacing policy to enhance traffic performance. First, a combined spacing policy composed of the constant time gap (CTG) and constant spacing (CS) is formulated for the proposed vehicle platoon, where the leader adopts the CTG and the followers use the CS policy. Based on the $h_2$-norm string stability criteria, the notion of exogenous-head-to-tail string stability is newly introduced, and the sufficient conditions of the local stability and string stability in the frequency domain are derived using the Routh-Hurwitz criterion and Laplace transform respectively. Numerical experiments are conducted to validate the string stability. The effectiveness of the proposed vehicle platoons is verified by theoretical analysis and numerical experiments using two typical scenarios and several measurements of effectiveness (MOE) in various performance aspects, including efficiency, safety, energy, and emission. The results show that the proposed vehicle platoon performs better than the CS-based vehicle platoon in all aspects except for efficiency. It also indicates that the proposed vehicle platoon has obvious advantages over the CTG-based vehicle platoon in efficiency and safety aspects. The findings have demonstrated the merits of the combined application of CTG and CS policies for the vehicle platoon in enhancing stability and traffic performance.

***Key Words* —Connected and automated car-following; Vehicle platoon; Constant time gap; Constant spacing; Stability analysis; Traffic performance.**

## I. Introduction

Connected and automated vehicles (CAVs) have received considerable attention due to their potential to drastically improve traffic efficiency, traffic safety, fuel consumption, and emissions [1]–[4]. As a representative application of CAVs, vehicle platoon allows multiple CAVs to operate cooperatively in the form of a platoon [5]–[7], thanks to the advanced sensing and vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication technologies. Vehicle platoon enables the CAVs to automatically maintain stable and safe spacings with the predecessors and resolve traffic disturbances even a small inter-vehicle spacing is adopted [8]–[10], which is also potentially applied in dedicated lane operation and congested car-following scenarios. Therefore, much attention has been paid to the vehicle platoon for pursuing higher traffic throughput and better flow stability and reduced fuel consumption and emissions with shorter car-following spacing and more exchanged state information [11]–[13].

Regarding the vehicle platoon, local stability and string stability are the most significant properties and should be addressed before practical application [8], [14]–[16]. It aims to suppress the error of motion states caused by disturbances from propagating to upstream traffic and thus mitigate the adverse impacts of the disturbances on traffic performance. Local stability refers to the attenuation ability of a single vehicle against the disturbance from its preceding vehicle, while string stability represents the attenuation ability through a string of CAVs under disturbances [4], [17]–[19]. Another important component of the vehicle platoon is the car-following spacing policy, which is to determine the desired spacing that a CAV attempts to maintain with respect to the preceding vehicle. The spacing policy plays a significant role since it affects the car-following behavior, stability, and traffic performance (e.g., throughput, fuel consumption, and emissions) of the vehicle platoons to a large extent.

Specifically, two spacing policies, i.e., constant time gap (CTG) and constant spacing (CS), are commonly used in vehicle platoon systems [8], [10], [15], [16], [20]–[22]. As indicated by review studies (introduced in Section II), the CTG policy can improve the string stability of the vehicle platoon but it requires more intensive inter-vehicular communication for the improvements in throughput, and the throughput will decrease with the increase of driving speed since the inter-vehicle distance could linearly become larger. On the contrary, the CS policy can potentially lead to higher traffic throughput than the CTG policy. However, it can only achieve string stability on spacing error with at least a leader-predecessor-follower communication topology, and thus the operation performance in fuel consumption and emissions cannot be guaranteed. Moreover, the applications of pure CS-based vehicle platoon are rather limited in practice since the leading vehicle with the CS policy is difficult to deal with varying car-

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Yuan Zheng is with the School of Transportation, Southeast University, Nanjing, China.
Min Xu* is with the Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hung Hom, Hong Kong, China. (Email: xumincee@gmail.com; min.m.xu@polyu.edu.hk)
Shining Wu and Shuaian Wang are with the Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Hong Kong, China.

following situations. Therefore, it is desirable to design a vehicle platoon that achieves better traffic performance without sacrificing the stability, which is important for the implementation of CAVs towards future mobility. In addition, although many studies have examined the traffic operation performance of CTG-based vehicle platoons [13], [23], the evaluation of the other vehicle platoons is limited, such as CS-based vehicle platoons. Moreover, the traffic performance comparisons among different types of vehicle platoons remain to be explored.

To bridge the above gaps, this study develops a vehicle platoon system that integrates CTG and CS policies in one vehicle platoon to enhance stability and traffic performance. We introduce a novel notion of exogenous-head-to-tail string stability for the proposed vehicle platoon and then derive the sufficient conditions of local and string stability for the vehicle platoon. Extensive numerical experiments are conducted to justify the stability of the proposed vehicle platoon and evaluate its traffic performance compared to the CTG-based and CS-based vehicle platoons. The main contributions of the study are shown as follows:

(1) the study mathematically formulates a vehicle platoon with a combined spacing policy that synergizes the CTG and CS policies in which the leading vehicle follows CTG policy and the following vehicles are regulated according to CS policy, which opens a new perspective to design a vehicle platoon for the improvements of stability and traffic performance.

(2) the study first introduces a notion of exogenous -head-to-tail string stability concerning the attenuation ability of the last vehicle in the vehicle platoon to the exogenous vehicle and then derives the sufficient conditions of local stability based on the Routh-Hurwitz stability criterion and the sufficient conditions of string stability in the frequency domain by the Laplace transform.

(3) the study makes a systematical evaluation of the proposed vehicle platoon compared with the prevalent vehicle platoons (e.g., CTG-based and CS-based vehicle platoons) from various performance aspects, and analyzes the merits of the proposed vehicle platoon, which can provide a useful guide for the implementation of vehicle platoon technology.

## II. Literature Review

Over the past decades, many studies have focused on the development of novel spacing policies and the refinement of existing spacing policies to enhance the stability and traffic operation performance of vehicle platoon systems. The earliest studies for CTG policy focused on the adaptive cruise control dedicated to a single vehicle only [24]. The recent development of V2V/V2I communication has motivated more and more efforts on the cooperative adaptive cruise control for a group of CAVs such as a convoy under the CTG policy [5], [8], [16], [18], [25], referred to as CTG-based vehicle platoon thereafter. In a CTG-based vehicle platoon, the state information of predecessors can be exchanged and applied to formulate the vehicle controllers through V2V or V2I [16], [18], [25] and this formulation can guarantee the string stability of the CTG-based vehicle platoon with a simple predecessor-follower communication topology based on linear feedforward and feedback controllers. Previous experiments and simulations have demonstrated that the CTG-based vehicle platoons with a variety of feedback and feedforward structures can improve traffic throughput and fuel consumption with guaranteed string stability [1], [4], [11], [25], [26]. For example, compared with the predecessor-follower communication topology, the multiple-predecessor-follower communication topology will result in a tighter vehicle platoon, in which a smaller time gap can be adopted to ensure string stability resort to more state information, and thus the traffic throughput of platoon systems is further enhanced [5], [8]. Furthermore, the communication delay and sensing delay, etc., which are the intrinsic characteristics of the automation and communication systems, can be detrimental to the string stability performance of a CTG-based vehicle platoon since those delays have been found to enlarge the required time gaps to ensure string stability [1], [5], [8], [16], [20], [26]. As such, the traffic throughput is also adversely affected by those delays. To address this problem, many methods have been proposed in the literature [16], [25], [27], among which the delay compensating method, which is to synchronize the data information collected from the sensing and communication devices and implements the coordinated historical state information of the predecessors to formulate a new car-following controller, was demonstrated to have better and robust performance against the uncertainties of those delays in the vehicle platoon and the potential to achieve better string stability and throughput [25].

As for CS-based vehicle platoons, the initial attempt of research aimed to design the controller to regulate a fixed inter-vehicle spacing, using the state information (i.e. position, speed, acceleration) of the preceding vehicle collected as a reference [15], [28]. However, the vehicle platoon systems often suffer from string instability problems. To improve stability, the state information of the leading vehicle of the vehicle platoon is also incorporated into the vehicle controller to maintain constant spacing. In this way, the CS-based vehicle platoon using a leader-predecessor-follower communication topology can achieve string stability on spacing error [15]. Some studies further illustrated that the CS-based vehicle platoon can significantly enhance the throughput with guaranteed string stability [19], [20], [29]. However, analogous to the CTG-based vehicle platoon, the string stability performance of the CS-based vehicle platoon is inevitably affected by the sensing delay and communication delay, etc. [19], [29]–[31]. Again, the adverse impacts of delays can be mitigated by the delay compensating method that synchronizes the historical state information of the predecessors. Zhang et al. (2020) found that the delay compensating method can significantly improve the string stability performance of the CS-based vehicle platoons. However, most studies investigated the string stability on spacing error only without considering the stability performance on acceleration, which suggests that CS-based vehicle platoons may have poor performance, e.g., in fuel consumption and emission aspects. Moreover, how the pure CS-based vehicle platoon effectively follows the exogenous vehicles on roads, especially in the varying car-following situations, has not been adequately addressed. Consequently, real-world applications of the CS-based vehicle platoon are rather limited since the leading vehicle is hard to deal with the varying car-following behaviors with CS policy.

In addition, there are vehicle platoon developments with other spacing policies proposed in the existing studies, such as variable time gap (VTG) [32]–[34], traffic flow stability [35], and safety distance [36]. The application of the VTG has received considerable attention among those spacing policies, since it can be effectively extended for the platoon control of CAVs. The core idea of the VTG policy is to determine the desired spacing/time gap by a non-linear function of speed, e.g., a univariate quadratic function, which is flexible to adapt to complex driving conditions. For example, Chen et al. (2021) proposed a consensus-based control approach based on the VTG policy. The asymptotic and string stability conditions of the control system are derived under fixed and switching topologies with time-varying communication delays [32]. Numerical results verified the effectiveness of the proposed control approach, showing that the inter-distance increases under the VTG compared to the CTG at high-speed conditions for safety improvement, whereas the inter-vehicle distance reduces at low-speed conditions to improve road utilization. Xiao et al. (2021) developed a dynamic control framework for vehicle platoon embedding an event-triggered transmission mechanism under random communication topologies and various spacing policies of CS, CTG, and VTG [33]. Based on the co-design conditions of the stable platoon control and communication bandwidth-aware management, numerical experiments are conducted to present the enlarged distance under VTG compared to CTG in high-speed situations. Li et al. (2021) proposed a novel consensus-based connected vehicle platoon controller using the VTG policy considering time delays and external disturbances [34]. The asymptotic stability for the platoon is then provided by theoretical analysis. Extensive simulations are performed to validate the effectiveness of the controller in dealing with inflexible spacing adjustment and adverse impacts of external disturbances. Although the VTG-based vehicle platoon can be more flexible to adjust the spacing and thus enables the trade-off analysis between the improvements in road utilization and safety, the spacing policy design brings a great challenge for the rigorous string stability analysis of the CAV platoon.

As mentioned above, the CTG and CS spacing policies with simple linear formulations can be easy to be applied in vehicle platoons, which is the focus of the study. Moreover, we can see that the CTG and CS policies have distinct advantages and shortcomings. For example, the CTG-based vehicle platoon can achieve improvements in traffic throughput with guaranteed string stability but requires a complex communication topology (e.g., multiple predecessor-follower) and the throughput will decrease with the increase of driving speed since the inter-vehicle distance could become larger. The CS-based vehicle platoon can ensure large and constant traffic throughput but it only achieves string stability on spacing error and leads to poor operation performance, e.g., high fuel consumption and emissions. However, all the aforementioned studies focused on either pure CTG-based or pure CS-based vehicle platoons. To the best of our knowledge, few studies have ever formulated and analyzed the traffic performance of a vehicle platoon with a combined spacing policy that integrates the CTG and CS policies and accordingly inherits the benefits of the two policies in one vehicle platoon system. The traffic performance among the CTG-based, CS-based, and vehicle platoons with a combined spacing policy needs to be further analyzed.

The remainder of this study is organized as follows. Section III presents the assumptions and problem description. The proposed vehicle platoon is formulated in Section IV. Section V introduces the novel notions of string ability and rigorously demonstrates their validation in proposed vehicle platoons. Section VI elaborates on the experiment settings and the experimental results in detail. Section VII concludes this paper with future research directions.

## III. Problem Description

As reviewed in Section II, we can find that most studies focused on either CS-based or CTG-based vehicle platoon systems. Fig. 1(a) and Fig. 1(b) illustrate a prevalent CS-based vehicle platoon with a leader-predecessor-follower communication topology and a CTG-based vehicle platoon with a two-predecessor-follower communication topology. Let $r$ denote the number of the predecessors with which the subject vehicle can communicate in the communication topology, and obviously we have $r = 2$ in Fig. 1. In this section, we propose a novel vehicle platoon based on a combined spacing policy, in which the leading vehicle employs the CTG policy and the following vehicles use the CS policy as illustrated in Fig. 1(c). In this design, the application of CTG policy for the leading vehicle can assist the vehicle platoon to mitigate the exogenous disturbances for stability improvements, whereas the adoption of CS policy for the following vehicles can lead to better traffic efficiency for the vehicle platoon. As a result, the proposed vehicle platoon with the combined formulation that synergizes the CTG and CS policies and inherits the benefits of the two policies in one vehicle platoon can improve stability and traffic performance. For ease of presentation, the vehicles involved in the vehicle platoon are indexed by $i \in \{0, 1, 2, \dots, n\}$ in terms of their longitudinal sequence. Specifically, $i = 0$ denotes the exogenous vehicle of the vehicle platoon (See the black vehicle in Fig. 1 (c)), $i = 1$ denotes the leading vehicle in the proposed vehicle platoon that follows the CTG policy, and $i = 2, \dots, n$ denote the following vehicles in the proposed vehicle platoon governed by the CS policy.

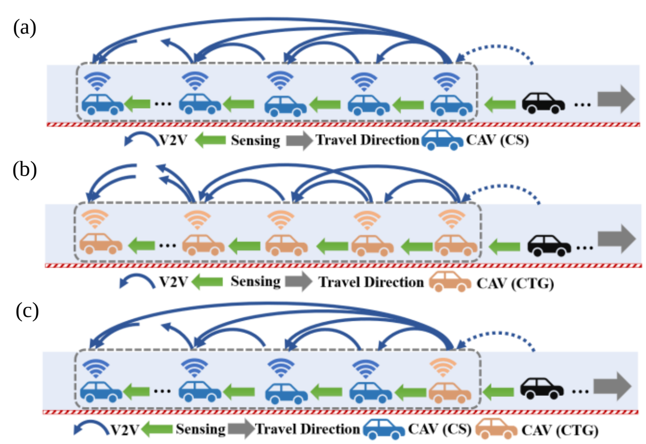



Fig. 1. Illustration of three types of vehicle platoons with different spacing policies: (a) CS; (b) CTG; (c) CTG+CS.

As for the communication topology, we assume that depending on the type of exogenous vehicle $i$ = 0, the leading vehicle of the proposed vehicle platoon, i.e., vehicle $i$ = 1, either employs the predecessor-follower communication (if vehicle $i$ = 0 is a CAV) or has no communication with the other vehicles (if vehicle $i$ = 0 is an automated vehicle (AV) or human-driven vehicle (HDV)). For any subject vehicle $i \in \{1, 2, \ldots, n\}$ in the proposed vehicle platoon, the position and speed information of its preceding vehicle are perceived by the on-board sensor of the subject vehicle with a sensing delay $\delta_i$ [1], [26]. The acceleration information of the preceding vehicle ($i$-1) sent to the subject vehicle $i$ is exchanged via V2V communication with a communication delay $\theta_{i-1,i}$ , which is assumed to be time-varying and bounded in $\left[\underline{\theta_{i-1,i}}, \overline{\theta_{i-1,i}}\right]$[25], [31]. In addition to the information of the immediate preceding vehicle, the state information, i.e., position, speed, and acceleration, of all the other predecessors $j$ sent to the subject vehicle $i$ will be obtained via V2V communication with distinct communication delays, which is denoted by $\theta_{j,i}$. The difference between $\theta_{j,i-1}$ and $\theta_{j,i}$ is denoted by $\triangle\theta_{j,i}$ , and assumed to be time-varying and bounded in $\left[\underline{\theta_{j,i}-\theta_{j,i-1}}, \overline{\theta_{j,i}-\theta_{j,i-1}}\right]$. Moreover, for simplicity, we assume that the following vehicles in the proposed platoon, i.e., vehicle $i \in \{2, 3, \ldots, n\}$, have the same control law and controller parameters following the CS policy, whereas the leading vehicle $i$ =1 adopts the control law and controller parameters obeying the CTG policy.

The objective of this study is to mathematically formulate the proposed vehicle platoon with a combined spacing policy, introduce the notations of stability for the new vehicle platoon, identify the sufficient conditions for its stability by rigorous proof, and analyze the traffic performance of the proposed vehicle platoon. Details can be found in the next sections.

## IV. Model Formulation

Following the conversion of literature [8], [16], [18], [37], the longitudinal vehicle dynamics for any vehicle $i \in \{1, 2, \ldots, n\}$ in the proposed vehicle platoon and the exogenous vehicle can be formulated by the following linear third-order model

$$\dot{p}_i(t) = v_i(t), \forall i \in \{0,1,2,\ldots,n\} \tag{1}$$

$$\dot{v}_i(t) = a_i(t), \forall i \in \{0,1,2,\ldots,n\} \tag{2}$$

$$\varphi \dot{a}_i(t) = u_i(t) - a_i(t), \forall i \in \{0,1,2,\ldots,n\} \tag{3}$$

where $p_i(t)$, $v_i(t)$, $a_i(t)$ are the rear bumper position, speed, and realized acceleration of vehicle $i$ at time $t$ in the vehicle proposed platoon, respectively, $u_i(t)$ represents the desired acceleration, and $\varphi$ is the driveline time-lag to realize the desired acceleration. Since the leading vehicle $i$ = 1 and the following vehicles $i \in \{2, 3, \ldots, n\}$ in the proposed vehicle platoon follow different spacing policies and control laws, we will formulate the car-following controls of them respectively and the state-space of the platoon in the next subsections.

### A. Car-following Control of the Leading Vehicle

To mitigate the adverse impacts of the delays on controller performance, we assume that the leading vehicle in the proposed vehicle platoon, i.e. vehicle $i$ = 1, follows the delay compensating-based CTG policy proposed by Zhang et al. (2020), to regulate the inter-distance between vehicles. Therefore, the target spacing between the exogenous vehicle $i$ = 0 and the subject vehicle $i$ = 1 is formulated as follows:

$$s_1^*(t) = v_1(t)h + \int_{\tau=t-g_1}^{t} v_0(\tau)d\tau + d_i \tag{4}$$

where $h$ is the pre-defined constant time gap, $d$ is the desired inter-vehicle distance in the standstill condition between the front bumper of the following vehicle and the rear bumper of the preceding vehicle, and $g_1$ is the time delay that satisfies $g_1 \geq max(\delta_1, \overline{\theta_{0,1}})$, which is no less than the sensor delay $\delta_1$ and the upper bound of communication delay $\overline{\theta_{0,1}}$ of the leading vehicle $i$ = 1.

The spacing error between the exogenous vehicle $i$ = 0 and the subject vehicle $i$ = 1 is given by

$$\begin{aligned}\Delta s_1(t) &= s_1(t) - s_1^*(t) \\ &= \left[p_0(t) - p_1(t) - L\right] - \left[v_1(t)h + \int_{\tau=t-g_1}^{t} v_0(\tau)d\tau + d_i\right] \\ &= p_0(t-g_1) - p_1(t) - L - v_1(t)h - d_i\end{aligned} \tag{5}$$

where $s_1(t)$ denotes the actual spacing between the subject vehicle $i$ = 1 and its preceding vehicle $i$ = 0, i.e., $\Delta s_1(t) = p_0(t) - p_1(t) - L$ , and $L$ is the length of the vehicle, which is assumed to be the same for all vehicles.

In the spirit of the previous studies [5], [8], the desired acceleration of the leading vehicle $i$ = 1 is formulated by

$$\begin{aligned}u_1(t) &= k_s \Delta s_1(t) + k_v\left[v_0(t-g_1) - v_1(t)\right] \\ &\quad + k_a\left[a_0(t-g_1) - a_1(t)\right] \\ &= k_s\left[p_0(t-g_1) - p_1(t) - L - v_1(t)h - d_i\right] \\ &\quad + k_v\left[v_0(t-g_1) - v_1(t)\right] + k_a[a_0(t-g_1) - a_1(t)]\end{aligned} \tag{6}$$

where $k_s$, $k_v$, and $k_a$ denote the controller parameters of the subject vehicle $i$ = 1, which are the coefficients of the spacing error, speed difference, and acceleration difference acquired via V2V communication, respectively. Note that the acceleration information of the preceding vehicle is not available if the preceding vehicle, i.e., vehicle $i$ = 0, is an AV or an HDV.

### B. Car-following Control of the Following Vehicles

For the following vehicles in the proposed vehicle platoon, i.e., vehicle $i \in \{2, 3, \ldots, n\}$, the delay compensating-based CS policy is applied with a leader-predecessor-follower communication topology. Therefore, the target spacing is calculated by

$$s_i^*(t) = d_i + \int_{\tau=t-g_i}^{t} v_{i-1}(\tau)d\tau, \forall i \in \{2,3,\ldots,n\} \tag{7}$$

where $g_i$ is the time delay no smaller than the sensor delay $\delta_i$, the upper bound of communication delay $\overline{\theta_{i-1,i}}$ , and the upper bound of the communication delay difference $\overline{\Delta\theta_{1,i}}$ between two consecutive vehicles relative to the leading vehicle, namely, $g_i \geq max(\delta_i, \overline{\theta_{i-1,i}}, \overline{\Delta\theta_{1,i}})$ . Note that $\overline{\Delta\theta_{1,i}}$ is calculated by $\overline{\Delta\theta_{1,i}} = \overline{\theta_{1,i} - \theta_{1,i-1}}$ , where $\theta_{1,i}$ denotes communication delay between the leading vehicle $i$ = 1 and the subject vehicle $i$.

The spacing error between the subject vehicle $i$ and the preceding vehicle ($i$-1) is calculated by

$$
\begin{aligned}
\Delta s_i(t) &= s_i(t) - s_i^*(t) \\
&= \left[ p_{i-1}(t) - p_i(t) - L \right] - \left[ d_i + \int_{\tau=t-g_i}^{t} v_{i-1}(\tau) d\tau \right] \\
&= \left[ p_{i-1}(t) - \int_{\tau=t-g_i}^{t} v_{i-1}(\tau) d\tau \right] - p_i(t) - L - d_i \\
&= p_{i-1}(t-g_i) - p_i(t) - L - d_i, \forall i \in \{2,3,...,n\}
\end{aligned} \tag{8}
$$

and the leading vehicle $i$ = 1 is given by

$$
\begin{aligned}
\Delta s_{1,i}(t) &= \sum\nolimits_{k=2}^{i} \Delta s_k(t + \sigma_k - \sigma_i) \\
&= \sum\nolimits_{k=2}^{i} \left[ p_{k-1}(t + \sigma_k - \sigma_i - g_k) - p_k(t + \sigma_k - \sigma_i) - L - d_i \right] \\
&= p_1(t - \sigma_i) - p_i(t) - \sum\nolimits_{k=2}^{i} (L + d_i), \forall i \in \{2,3,...,n\}
\end{aligned} \tag{9}
$$

where $\sigma_i$ is the accumulated time delay between the leading vehicle and the subject vehicle, namely, $\sigma_i = \sum\nolimits_{k=2}^{i} g_k$ .

On basis of the CS policy, the desired acceleration is formulated as follows aiming to control the subject vehicle to track the position of both the preceding vehicle with desired distance and the leading vehicle with desired accumulated distance [15], [29]:

$$
\begin{aligned}
u_i(t) &= \frac{1}{1+q_3} \left[ a_{i-1}(t-g_i) + q_3 a_1(t-\sigma_i) + (q_1+\lambda)\Delta\dot{s}_i(t) \right] \\
&\quad + \frac{1}{1+q_3} \left[ q_1 \lambda \Delta s_i(t) + (q_4 + \lambda q_3) \Delta \dot{s}_{1,i}(t) + \lambda q_4 \Delta s_{1,i}(t) \right] \\
&= \frac{1}{1+q_3} \left[ a_{i-1}(t-g_i) + q_3 a_1(t-\sigma_i) \right] \\
&\quad + \frac{1}{1+q_3} \left\{ (q_1+\lambda) \left[ v_{i-1}(t-g_i) - v_i(t) \right] \right\} \\
&\quad + \frac{1}{1+q_3} \left\{ q_1 \lambda \left[ p_{i-1}(t-g_i) - p_i(t) - d_i - L \right] \right\} \\
&\quad + \frac{1}{1+q_3} \left\{ (q_4 + \lambda q_3) \left[ v_1(t-\sigma_i) - v_i(t) \right] \right\} \\
&\quad + \frac{\lambda q_4}{1+q_3} \left[ p_1(t-\sigma_i) - p_i(t) - \sum_{k=2}^{i} (L + d_i) \right], \forall i \in \{2,...,n\}
\end{aligned} \tag{10}
$$

where $q_1$, $q_3$, $q_4$, and $\lambda$ denote the controller parameters of the following vehicles $i \in \{2, 3, …, n\}$ in the proposed vehicle platoon. It can be found that the desired acceleration of the subject vehicle is dependent on the acceleration of both the leading vehicle and the preceding vehicle, and the speed difference and spacing error of the subject vehicle with respect to both the leading vehicle and the preceding vehicle.

### C. State Space Formulation

The vehicle platoon system can be formulated as the state-space system. The control objective of the vehicle platoon system is to take the state information of the exogenous vehicle as the reference trajectory for tracking. However, the following vehicles may not receive the state information of the exogenous vehicle in practice. Actually, the control objective can be implemented with the distributed controllers formulated in the above subsections utilizing the local state information. According to Bian's study [8], the state of the vehicle $i \in \{1, 2, …, n\}$ is formulated as $x_i(t) = \left[ \bar{p}_i(t), \bar{v}_i(t), \bar{a}_i(t) \right]^T$ with:

$$
\begin{cases}
\bar{p}_i(t) = p_i(t) - p_0(t-\psi_i) + \left[ v_1(t-\sigma_i)h + d + L + \sum\nolimits_{k=2}^{i} (d+L) \right] \\
\bar{v}_i(t) = v_i(t) - v_0(t-\psi_i) \\
\bar{a}_i(t) = a_i(t) - a_0(t-\psi_i)
\end{cases} \tag{11}
$$

where $\psi_i$ is the accumulated traceable time along with the vehicle platoon, i.e., $\psi_i = \sigma_i + g_1 = \sum\nolimits_{k=2}^{i} g_k + g_1$ , which can be used to track the coordinated historical state information (i.e., position, speed, acceleration) of the exogeneous vehicle as a reference with the implementation of delay compensating strategy. Note that the time instant $t$ is not presented in the rest of this subsection for the sake of readability.

By lumping the state of each vehicle in the proposed vehicle platoon, the state of the vehicle platoon system can be formulated as follows:

$$
\boldsymbol{x} = \left[ \boldsymbol{p}^T, \boldsymbol{v}^T, \boldsymbol{a}^T \right]^T \tag{12}
$$

with

$$
\begin{cases}
\boldsymbol{p} = \left[ \bar{p}_1, ..., \bar{p}_i, ..., \bar{p}_n \right]^T \\
\boldsymbol{v} = \left[ \bar{v}_1, ..., \bar{v}_i, ..., \bar{v}_n \right]^T \\
\boldsymbol{a} = \left[ \bar{a}_1, ..., \bar{a}_i, ..., \bar{a}_n \right]^T
\end{cases} \tag{13}
$$

Based on the Eq. (11), we have

$$
\begin{cases}
\dot{\bar{p}}_i = \bar{v}_i + h a_1 = \bar{v}_i + h \bar{a}_1(t-\sigma_i) + h a_0(t-\psi_i) \\
\dot{\bar{v}}_i = \bar{a}_i \\
\dot{\bar{a}}_i = -\bar{a}_i / \varphi + u_i / \varphi
\end{cases} \tag{14}
$$

Therefore, the closed-loop dynamics of the vehicle platoon system can be described as follows:

$$
\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{C} a_0 (t - \psi) \tag{15}
$$

where $a_0$ is the acceleration of the exogenous vehicle of the vehicle platoon, $\boldsymbol{A}^{3n\times 3n} = \begin{bmatrix} 0^{n\times n} & \boldsymbol{I}^{n\times n} & \boldsymbol{H}^{n\times n} \\ 0^{n\times n} & 0^{n\times n} & \boldsymbol{I}^{n\times n} \\ -\boldsymbol{k}_2\boldsymbol{k}_1 & -\boldsymbol{k}_3\boldsymbol{k}_1 & -\boldsymbol{k}_4\boldsymbol{k}_1 - \boldsymbol{k}_1 \end{bmatrix}$,

$$
\boldsymbol{H}^{n\times n} = \begin{bmatrix} h & 0 & 0 & 0 \\ h & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ h & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{k}_1^{n\times n} = \begin{bmatrix} 1/\tau & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1/\tau \end{bmatrix},
$$

$$
\boldsymbol{k}_2^{n\times n} = \begin{bmatrix}
k_s & 0 & 0 & 0 & 0 & 0 \\
\frac{-\lambda q_4 - \lambda q_1}{1+q_3} & \frac{\lambda q_4 + \lambda q_1}{1+q_3} & 0 & 0 & 0 & 0 \\
\frac{-\lambda q_4}{1+q_3} & \frac{-q_1\lambda}{1+q_3} & \frac{\lambda q_4 + \lambda q_1}{1+q_3} & 0 & 0 & 0 \\
\frac{-\lambda q_4}{1+q_3} & 0 & \frac{-q_1\lambda}{1+q_3} & \frac{\lambda q_4 + \lambda q_1}{1+q_3} & 0 & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & 0 \\
\frac{-\lambda q_4}{1+q_3} & 0 & \cdots & 0 & \frac{-q_1\lambda}{1+q_3} & \frac{\lambda q_4 + \lambda q_1}{1+q_3}
\end{bmatrix},
$$

$$
\boldsymbol{k}_3^{n\times n}=\begin{bmatrix}
k_v & 0 & 0 & 0 & 0 & 0\\
\frac{-\lambda q_4-\lambda q_1}{1+q_3} & \lambda+\frac{q_4+q_1}{1+q_3} & 0 & 0 & 0 & 0\\
\frac{-q_4-\lambda q_3}{1+q_3} & \frac{-q_1-\lambda}{1+q_3} & \lambda+\frac{q_4+q_1}{1+q_3} & 0 & 0 & 0\\
\frac{-q_4-\lambda q_3}{1+q_3} & 0 & \frac{-q_1-\lambda}{1+q_3} & \lambda+\frac{q_4+q_1}{1+q_3} & 0 & 0\\
\vdots & \vdots & \ddots & \ddots & \ddots & 0\\
\frac{-q_4-\lambda q_3}{1+q_3} & 0 & \cdots & 0 & \frac{-q_1-\lambda}{1+q_3} & \lambda+\frac{q_4+q_1}{1+q_3}
\end{bmatrix},
$$

$$
\boldsymbol{k}_4^{n\times n}=\begin{bmatrix}
k_a & & & & & \\
-1 & 0 & & & & \\
\frac{-q_3}{1+q_3} & \frac{-1}{1+q_3} & 0 & & & \\
\frac{-q_3}{1+q_3} & 0 & \frac{-1}{1+q_3} & 0 & & \\
\vdots & \vdots & \ddots & \ddots & \ddots & \\
\frac{-q_3}{1+q_3} & 0 & \cdots & 0 & \frac{-1}{1+q_3} & 0
\end{bmatrix},\ \boldsymbol{C}^{n\times 1}=\begin{bmatrix} h\\ 1+h\\ \vdots\\ 1+h\end{bmatrix}.
$$

## V. Stability Analysis

Local stability and string stability are the critical properties serving the vehicle platoons [14], [38]. In what follows, we will derive the sufficient conditions for the local stability, the string stability on spacing error, and introduce novel string stability on acceleration for the proposed vehicle platoon system.

### A. Local Stability Analysis

**Definition 1** [12]: A vehicle platoon is linearly local stable if and only if all eigenvalues of the characteristic equation in the closed-loop system have strictly negative real parts.

**Proposition 1**: A vehicle platoon with a combined spacing policy is local stable if the following conditions are satisfied:

$$
\begin{cases}
1+k_a>0\\
k_p>0\\
(1+k_a)(k_v+hk_p)-\varphi k_p>0\\
1+q_3>0\\
(q_1+q_4)\lambda>0\\
\lambda(1+q_3)>(\lambda\varphi-1)(q_1+q_4)
\end{cases}\tag{16}
$$

**Proof.** According to Definition 1, the local stability of a linear control system is determined by the matrix $\boldsymbol{A}$. In particular, local stability requires that all eigenvalues denoted by $\beta$ have the negative real parts. To investigate the local stability, we first calculate the characteristic equation below:

$$
\begin{aligned}
&\left|\beta\boldsymbol{I}^{3n}-\boldsymbol{A}^{3n}\right|\\
&=\begin{vmatrix}
\beta\boldsymbol{I}^n & -\boldsymbol{I}^n & -\boldsymbol{H}^n\\
0^n & \beta\boldsymbol{I}^n & -\boldsymbol{I}^n\\
\boldsymbol{k}_2\boldsymbol{k}_1 & \boldsymbol{k}_3\boldsymbol{k}_1 & \beta\boldsymbol{I}^n+(\boldsymbol{k}_4\boldsymbol{k}_1+\boldsymbol{k}_1)
\end{vmatrix}\\
&=\left|\beta^3\boldsymbol{I}^n+\beta^2\boldsymbol{k}_1(\boldsymbol{I}^n+\boldsymbol{k}_4)+\beta\boldsymbol{k}_1(\boldsymbol{k}_2\boldsymbol{H}+\boldsymbol{k}_3)+\boldsymbol{k}_1\boldsymbol{k}_2\right|\\
&=\left[\beta^3+\beta^2\frac{1}{\varphi}\left(1+k_a\right)+\beta\frac{1}{\varphi}\left(k_sh+k_v\right)+\frac{1}{\varphi}k_s\right]\times\\
&\prod_{i=2}^{n}\left[\beta^3+\frac{1}{\varphi}\beta^2+\frac{1}{\varphi}\beta\left(\frac{q_4+\lambda q_3+q_1+\lambda}{1+q_3}\right)+\frac{1}{\varphi}\left(\frac{\lambda q_4+\lambda q_1}{1+q_3}\right)\right]
\end{aligned}\tag{17}
$$

The derivation outcome in Eq. (17) is attributed to the fact that the matrix $\boldsymbol{k}_1$ is diagonal and the matrices $\boldsymbol{k}_2$, $\boldsymbol{k}_3$, $\boldsymbol{k}_4$, and $\boldsymbol{H}$ are lower-triangular. The structure of these matrices offers an opportunity to decouple the platoon system in Eq. (15) into $n$ subsystems corresponding to the leader and followers in the platoon considered in our study.

To facilitate the local stability analysis of the proposed platoon system, we define the following two polynomials. The first polynomial is closely related to the controller parameters of the leading vehicle $i$ = 1 in the platoon, which are shown as follows:

$$
y_1(\beta)=\varphi\beta^3+\beta^2\left(1+k_a\right)+\beta\left(k_sh+k_v\right)+k_s\tag{18}
$$

The second polynomial is related to the controller parameters of the following vehicle $i\in\{2, \ldots, n\}$ in the platoon shown below:

$$
y_i(\beta)=\varphi\beta^3+\beta^2+\beta\left(\frac{q_4+\lambda q_3+q_1+\lambda}{1+q_3}\right)+\left(\frac{\lambda q_4+\lambda q_1}{1+q_3}\right)\tag{19}
$$

Since the vehicle platoon is local stable if and only if all eigenvalues of the characteristic polynomials $y_i(\beta)$ have negative real parts, we can derive the sufficient conditions of local stability with respect to the first polynomial based on the Routh-Hurwitz stability criterion:

$$
\begin{cases}
1+k_a>0\\
k_p>0\\
k_v+hk_p>0\\
(1+k_a)(k_v+hk_p)-\varphi k_p>0
\end{cases}\tag{20}
$$

We can also derive the sufficient conditions of local stability for the second polynomial:

$$
\begin{cases}
1+q_3>0\\
(q_1+q_4)\lambda>0\\
\lambda(1+q_3)>(\lambda\varphi-1)(q_1+q_4)
\end{cases}\tag{21}
$$

Combining the derived inequality conditions in Eqs. (20) and (21), we can obtain sufficient conditions for the local stability of the proposed vehicle platoon. By simple manipulations, we can obtain the stability conditions in Eq. (16). This completes the proof of Proposition 1. □

### B. String Stability Analysis

This subsection analyses the string stability of the proposed vehicle platoon, particularly the $h_2$-norm string stability, both on spacing error and acceleration. We will introduce the definitions of the string stability for the proposed vehicle platoon and derive sufficient conditions by rigorous mathematical proofs.

#### 1) String Stability on Spacing Error

**Definition 2** [19]: A vehicle platoon is $h_2$-norm string stable on spacing error if and only if

$$
\frac{\left\|\Delta s_i(z)\right\|_2}{\left\|\Delta s_{i-1}(z)\right\|_2}\le 1,\forall w>0, z=jw,\forall i\in\left\{2,3,\ldots,n\right\}\tag{22}
$$

where ||.|| denotes the $h_2$ norm, $\Delta s_i(z)$ is the Laplace transform of the spacing error of $\Delta s_i(t)$, $j$ is the imaginary unit, and $w$ is the frequency.

**Proposition 2** [19]: A vehicle platoon is string stable on spacing error if the following conditions are satisfied:

$$\left\| \frac{z^2 + (q_1+\lambda)z + q_1\lambda}{(1+q_3)z^2(1+z\varphi) + (q_1+\lambda+q_4+q_3\lambda)z + (q_1+q_4)\lambda} \right\| \le 1, \forall w > 0, z = jw \tag{23}$$

*2) String Stability on Acceleration*

**Definition 3** [16]: A vehicle platoon is $h_2$-norm string stable on acceleration if and only if

$$\frac{\|a_i(z)\|_2}{\|a_{i-1}(z)\|_2} \le 1, \forall w > 0, z = jw, \forall i \in \{2,3,...,n\} \tag{24}$$

where $a_i(z)$ is the Laplace transform of the acceleration $a_i(t)$.

We can see from Eq. (24) that the string stability criteria are strict since they require disturbance attenuation for every vehicle in a platoon. In fact, Ge and Orosz (2014) have found that it is hard to achieve the string stability on acceleration for the vehicle platoons in a connected cruise control with complex communication topologies whose control law additionally depends on the acceleration information of multiple predecessors [17]. Therefore, they proposed the notion of head-to-tail string stability on acceleration and derived the sufficient and necessary stability conditions for the proposed vehicle platoons. The head-to-tail string stability on acceleration focuses on the attenuation of acceleration of the last vehicle with respect to the leading vehicle in the platoon as follows:

**Definition 4** [17]: A vehicle platoon is $h_2$-norm head-to-tail string stable on acceleration if and only if

$$\frac{\|a_n(z)\|_2}{\|a_1(z)\|_2} \le 1, \forall w > 0, z = jw \tag{25}$$

where $a_1(z)$ and $a_n(z)$ are the Laplace transform of the acceleration of the leading vehicle $a_1(t)$ and the last vehicle $a_n(t)$ in the platoon, respectively.

It can be seen that the head-to-tail string stability criteria on acceleration in Eq. (25) are weaker than the string stability criteria on acceleration in Eq. (24). However, we find that the proposed vehicle platoons cannot ensure the head-to-tail string stability on acceleration as demonstrated in Appendix B and the same applies for the CS-based vehicle platoons. Nevertheless, unlike the CS-based vehicle platoon, the leader in the proposed vehicle platoon follows the CTG policy, which is likely to assist the vehicle platoon system to attenuate the exogenous disturbances. To illustrate the performance of the proposed vehicle platoon in car-following situations, we introduce an exogenous vehicle for ease of presentation. In light of this, we introduce a novel notion of string stability for the proposed vehicle platoons as follows:

**Definition 5**: A vehicle platoon is $h_2$-norm exogenous-head-to-tail string stable if and only if

$$\frac{\|a_n(z)\|_2}{\|a_0(z)\|_2} \le 1, \forall w > 0, z = jw \tag{26}$$

where $a_0(z)$ is the Laplace transform of the exogenous vehicle's acceleration $a_0(t)$ of the platoon.

Kindly note that the above definition is contingent on the existence of an exogenous vehicle, namely the vehicle platoon is in the car-following situation. It can guarantee that the acceleration of the last vehicle in the platoon is smaller than that of the exogenous vehicle. Compared with the head-to-tail string stability concerning the attenuation of the last vehicle to the leading vehicle, the exogenous-head-to-tail string stability indicates the attenuation of the last vehicle to the exogenous vehicle. It implies the attenuation ability of the last vehicle in the platoon against exogenous disturbances, which can be regarded as a novel string stability on acceleration. Note that the exogenous-head-to-tail string stable (stability) is referred to as ex-head-to-tail string stable (stability) in the rest of the paper. In the following proposition, we will derive sufficient conditions for the ex-head-to-tail string stability on acceleration for the proposed vehicle platoons.

**Proposition 3:** A vehicle platoon is ex-head-to-tail string stable on acceleration if the following conditions are satisfied:

$$\left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\| \ge max\left\{ \frac{\|C(z)\|}{\|A(z)\|-\|B(z)\|}, 1 \right\}, \forall w > 0, z = jw \tag{27}$$

where $G(z)$, $K(z)$, and $F(z)$ are the vehicle dynamic model, controller terms (except the time gap), and time gap in the platoon, respectively, and $A(z)$, $B(z)$, and $C(z)$ are the controller terms related to the longitudinal positions of the vehicles in the platoon, which are shown in Appendix B in details.

**Proof.** We will first prove that the following inequalities hold if Eq. (27) is satisfied:

$$\frac{\|a_1(z)\|}{\|a_0(z)\|} \le 1, \forall w > 0, z = jw \tag{28}$$

Specifically, we will examine the leading vehicle $i = 1$. This vehicle follows the CTG policy in the proposed vehicle platoon. The relation of the acceleration between the vehicle $i = 1$ and the exogenous vehicle $i = 0$ can be expressed as follows based on Eq. (6) using the Laplace transform:

$$\frac{a_1(z)}{a_0(z)} = \frac{G(z)K(z)}{1+G(z)K(z)+G(z)F(z)h} e^{-g_1 z} \tag{29}$$

with

$$\begin{cases} G(z) = 1/(z^2 + z^3\varphi) \\ K(z) = k_a z^2 + k_v z + k_p \\ F(z) = k_p z \end{cases} \tag{30}$$

It follows from Eq. (29) that

$$\begin{aligned} \frac{\|a_1(z)\|}{\|a_0(z)\|} &\le \left\| \frac{G(z)K(z)}{1+G(z)K(z)+G(z)F(z)h} \right\| \left\| e^{-g_1 z} \right\| \\ &\le \left\| \frac{G(z)K(z)}{1+G(z)K(z)+G(z)F(z)h} \right\| \le 1, \forall w > 0, z = jw \end{aligned} \tag{31}$$

where the last inequality follows from Eq. (27). Therefore, we have proved that Eq. (28) is satisfied under condition (27).

Based on the Eq. (10), as for the vehicles $i \in \{2, 3, \ldots, n\}$, we have:

$$A(z)p_i(z) = B(z)p_{i-1}(z)e^{-g_i z} + C(z)p_1 e^{-\sigma_i z}, \forall i \in \{2,...,n\} \quad (32)$$

Next, we will use mathematical induction to prove that if Eqs. (27) are satisfied, the following inequalities hold, $\forall$ $i \in \{2, \ldots, n\}$.

$$\frac{\|p_i(z)\|}{\|p_1(z)\|} \le \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\|, \forall w > 0, z = jw \quad (33)$$

Particularly, for $i$ = 1, we have

$$\frac{\|p_1(z)\|}{\|p_1(z)\|} = 1 \le \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\|, \forall w > 0, z = jw \quad (34)$$

Suppose that the following inequalities hold for the vehicle ($i$-1), $\forall$ $i \in \{2, 3, \ldots, n\}$.

$$\frac{\|p_{i-1}(z)\|}{\|p_1(z)\|} \le \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\|, \forall w > 0, z = jw \quad (35)$$

It follows from Eq. (27) that

$$\left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\| \ge \frac{\|C(z)\|}{\|A(z)\| - \|B(z)\|}, \forall w > 0, z = jw \quad (36)$$

By simple manipulation, we can further obtain

$$\frac{\|C(z)\|}{\|A(z)\|} \le (1 - \frac{\|B(z)\|}{\|A(z)\|}) \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\|, \forall w > 0, z = jw \quad (37)$$

Therefore, we have

$$\begin{aligned} \frac{\|p_i(z)\|}{\|p_1(z)\|} &= \left\| \frac{B(z)p_{i-1}(z)e^{-g_i z}}{A(z)p_1(z)} + \frac{C(z)p_1 e^{-\sigma_i z}}{A(z)p_1(z)} \right\| \\ &\le \frac{\|B(z)\|}{\|A(z)\|} \frac{\|p_{i-1}(z)\|}{\|p_1(z)\|} \left\| e^{-g_i z} \right\| + \frac{\|C(z)\|}{\|A(z)\|} \left\| e^{-\sigma_i z} \right\| \\ &\le \frac{\|B(z)\|}{\|A(z)\|} \frac{\|p_{i-1}(z)\|}{\|p_1(z)\|} + \frac{\|C(z)\|}{\|A(z)\|} \\ &\le \frac{\|B(z)\|}{\|A(z)\|} \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\| \\ &\quad + (1 - \frac{\|B(z)\|}{\|A(z)\|}) \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\| \\ &= \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\|, \forall i \in \{2,3,...,n\} \end{aligned} \quad (38)$$

where the first equality follows from Eq. (32) and the third inequality follows from Eq. (35) (for the first term) and Eq. (37) (for the second term).

The above recursive principle indicates that $\frac{\|p_i(z)\|}{\|p_1(z)\|} \le \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\|$ always holds for any vehicle $i \in \{2, 3, \ldots, n\}$.

According to the property of Laplace transform, we have $\frac{\|p_i(z)\|_2}{\|p_1(z)\|_2} = \frac{\|a_i(z)\|_2}{\|a_1(z)\|_2}$. Hence, based on Eqs. (31) and (33), we can prove as follows that Eq. (26) in Definition 5 is satisfied for the proposed vehicle platoon:

$$\begin{aligned} \frac{\|a_n(z)\|}{\|a_0(z)\|} &= \frac{\|a_n(z)\|}{\|a_1(z)\|} \times \frac{\|a_1(z)\|}{\|a_0(z)\|} \\ &\le \left\| \frac{1+G(z)K(z)+G(z)F(z)h}{G(z)K(z)} \right\| \left\| \frac{G(z)K(z)}{1+G(z)K(z)+G(z)F(z)h} \right\| = 1 \end{aligned} \quad (39)$$

This completes the proof of Proposition 3. □

We further define the string stability for a vehicle platoon system and summarize the sufficient conditions for string stability of the proposed vehicle platoon as follows.

**Definition 6:** A vehicle platoon with a combined spacing policy is string stable on spacing error and ex-head-to-tail string stable on acceleration if and only if Eqs. (22) and (26) are satisfied.

The above stability definition is to guarantee that the spacing error can attenuate in the platoon and the acceleration of the last vehicle in the platoon is also no larger than the exogenous vehicle. The notion emphasizes the attenuation ability on both acceleration and spacing error against the disturbance of the exogenous vehicle. In this way, it can enhance the string stability performance and has the potential to improve the traffic operation performance of the vehicle platoon.

**Proposition 4:** A vehicle platoon with a combined spacing policy is string stable on spacing error and ex-head-to-tail string stable on acceleration if Eqs. (23) and (27) are satisfied.

## VI. EXPERIMENTAL RESULTS

In this section, numerical experiments are conducted to verify the effectiveness of the proposed vehicle platoon system from various aspects in comparison with the CTG-based and CS-based vehicle platoons. We will first present two instances to validate the mathematical proof of stability analysis. The traffic performance of the three types of vehicle platoons will be compared extensively. Finally, we summarize the rankings of the three types of vehicle platoons in several performance aspects. The parameter settings of the vehicle platoons for numerical experiments are illustrated in Table I according to the previous studies [5], [8], [19], [20].

Table I Parameter settings for numerical experiments

| Parameter | Notation | Value |
|---|---|---|
| Simulation time | $T$ | 150 s |
| Time step | $\Delta t$ | 0.01 s |
| Vehicle length | $L$ | 5 m |
| Standstill distance (CTG/CS) | $d$ | 5 m/15 m |
| Driveline time-lag | $\varphi$ | 0.5 s |
| Sensor delay | $\delta$ | 0.02 s |
| Communication delay | $\theta$ | $U$(0.02, 0.1) s |

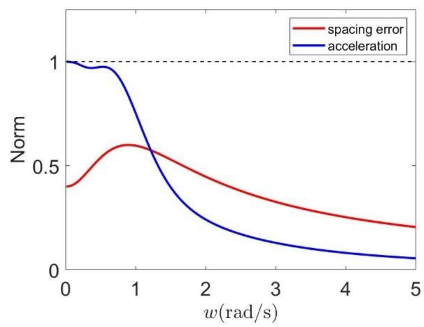
### A. Stability Verification

In this subsection, numerical experiments will be first conducted to illustrate the stability analysis. Particularly, we will present two instances of the proposed vehicle platoon with string stability (i.e., the stability conditions are fully satisfied) and string instability (i.e., the stability conditions are not fully satisfied). Both platoon instances are composed of five CAVs with the same time gap $h$ set at 1.4 s and the same controller parameters set as follows: $q_1$ = 0.4, $q_3$ = 0.9, $q_4$ = 0.6, $k_s$ = 0.1, $k_v$ = 0.7, $k_a$ = 0.84 except that $\lambda$ = 0.1 in the former instance while

$\lambda$ = 0.3 in the latter instance. It can be verified that the parameters in the former instance satisfy the stability conditions in Proposition 4, suggesting that they are located in the feasible region of the string stability on spacing error and ex-head-to-tail string stability on acceleration, whereas the parameters in the latter instance satisfy all conditions in Proposition 4 except for Eq. (27), indicating that the parameters in the latter instance are within the feasible region of string stability on spacing error but outside of the feasible region of ex-head-to-tail string stability on acceleration.

Recall that by Definition 6, a vehicle platoon with a combined spacing policy is string stable if both the two norms in Eqs. (22) and (26) are smaller than 1 for any *w*; otherwise, it is string unstable. We visualize the norm of $e_i/e_{i\text{-}1}$ in Eq. (22) and the norm of $a_5/a_0$ under frequency *w* in Eq. (26) in the above two instances in Fig. 2(a) and (b), respectively. As expected, it shows that the proposed vehicle platoon is string stable in the former instance, i.e., string stable on spacing error and ex-head-to-tail string stable on acceleration, since both norms are less than 1 for any *w*, whereas the vehicle platoon becomes string unstable in the latter instance because the condition in Proposition 3, i.e., Eq. (27), is not satisfied.

(a)




(b)

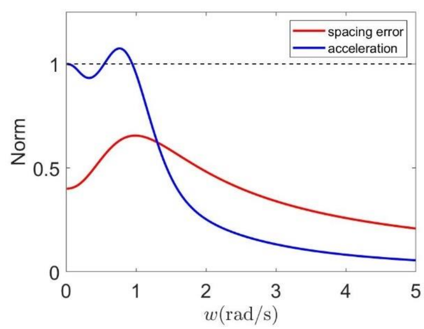
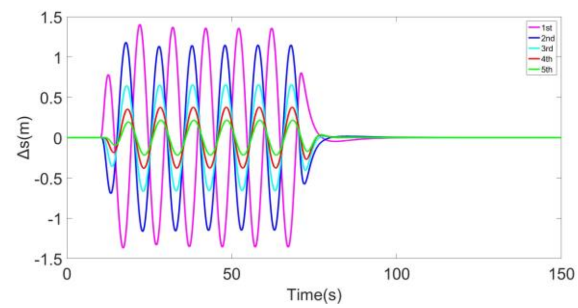


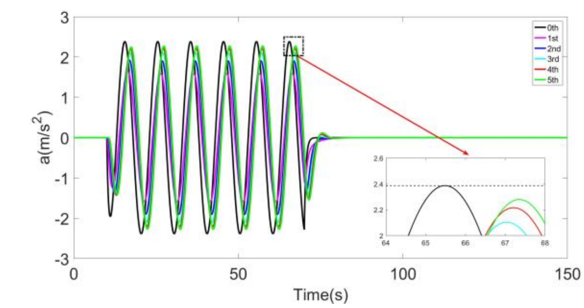

Fig. 2. Norms of $e_i/e_{i\text{-}1}$ and $a_5/a_0$ in the proposed vehicle platoon: (a) string stability; (b) string instability.

We further illustrate the temporal spacing error and acceleration of the above two instances by numerical simulation experiments in Fig. 3. It is assumed that the exogenous vehicle of the proposed vehicle platoon performs the periodical acceleration and deceleration ranging from 2.5 m/s$^2$ and -2.5 m/s$^2$ from 10 s to 70 s. For the former instance, as shown in Fig. 3(a), the spacing error decreases or converges to zero when the disturbance propagates to the tail vehicle in the vehicle platoon. It is found that the leading vehicle using the CTG policy in the platoon has an opposite phase of spacing error compared with the followers. This is because the magnitude of its deceleration is larger than that of actual spacing when the disturbance propagates from the exogenous vehicle to the leading vehicle. It suggests that the target spacing of the leading vehicle is smaller than the actual spacing relative to the exogenous vehicle. Therefore, there exists some extra space resulting in the opposite phase of the spacing error. In addition, we can observe from Fig. 3(b) that the tail vehicle in the vehicle platoon has a smaller acceleration than the exogenous vehicle, even if the magnitude of the acceleration of the followers gradually becomes larger. It has demonstrated that the exogenous disturbance can attenuate through the whole vehicle platoon and the proposed vehicle platoon can guarantee the convergence of spacing error and the gradual attenuation of the acceleration at a platoon level. On the contrary, for the latter instance, although the spacing error converges in Fig. 3(c), the acceleration of the tail vehicle in the platoon is amplified relative to that of the exogenous vehicle, as shown in Fig. 3(d). In conclusion, the results of the numerical experiments are in accordance with the theoretical analysis in Section V.

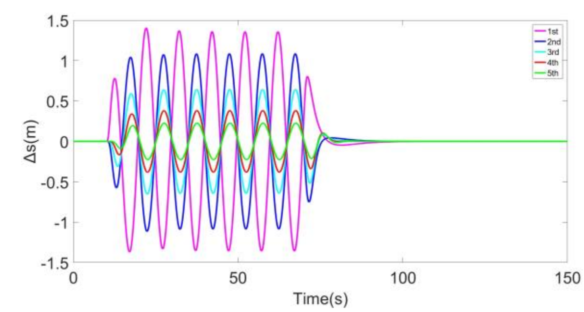
(a)




(b)




(c)




(d)

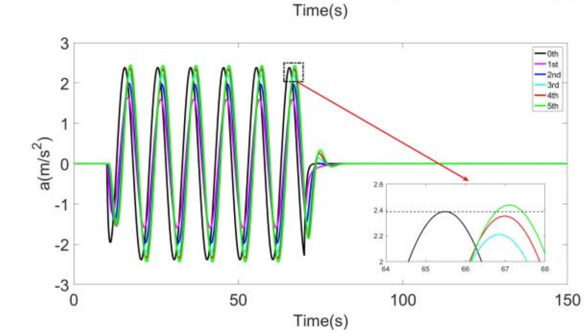


Fig. 3. Motion profile of the proposed vehicle platoon: (a) spacing error with string stability; (b) acceleration with string stability; (c) spacing error with string instability; (d) acceleration with string instability.

### *B. Performance Comparisons of Different Vehicle Platoons*

In this subsection, we first conduct the theoretical analysis for the advantages of the proposed vehicle platoons compared with CS-based and CTG-based platoons in the stability (measured by ex-head-to-tail string stability on acceleration) and throughput aspects.

#### *1) Theoretical Analysis*

##### *a) Ex-head-to-tail String Stability on Acceleration*

As for the CS-based vehicle platoons, we first derive the norm of the last vehicle to the leading vehicle which is known as the head-to-tail string stability, and the norm of the leading vehicle of the platoon to the exogenous vehicle, for the convenience of deriving the ex-head-to-tail string stability on acceleration.

Based on Proposition B in Appendix B, the norm of the last vehicle to the leading vehicle has been obtained. Then, it is assumed that the leading vehicle can receive the acceleration information of the exogenous vehicle for the CS-based vehicle platoon, based on the Eq. (50), we have:

$$A(z)p_1(z)e^{-g_1 z} = B(z)p_0(z)e^{-g_1 z} + C(z)p_0 e^{g_1 z} = [B(z)+C(z)]p_1 e^{-g_1 z} \tag{40}$$

The norm of the leading vehicle of the platoon to the exogenous vehicle is calculated as follows:

$$\begin{aligned}\frac{\|a_n\|}{\|a_1\|} = \frac{\|p_n\|}{\|p_1\|} &= \left\|\frac{B(z)e^{-g_1 z}+C(z)e^{-g_1 z}}{A(z)}\right\| = \left\|\frac{B(z)+C(z)}{A(z)}\right\|\left\|e^{-g_1 z}\right\| \\ &= \left\|\frac{z^2+(q_1+\lambda)z+q_1\lambda+q_3z^2+(q_4+q_3\lambda)z+q_4\lambda}{(1+q_3)z^2(1+z\tau)+(q_1+\lambda+q_4+q_3\lambda)z+(q_1+q_4)\lambda}\right\| \\ &= \left\|\frac{(1+q_3)z^2+(q_1+\lambda+q_4+q_3\lambda)z+(q_1+q_4)\lambda}{(1+q_3)\tau z^3+(1+q_3)z^2+(q_1+\lambda+q_4+q_3\lambda)z+(q_1+q_4)\lambda}\right\|, \forall w>0, z=jw\end{aligned} \tag{41}$$

It is intuitive to observe that the norm in Eq. (41) is larger than 1 for sufficiently small frequencies *w* regardless of controller parameters. Therefore, we can conclude that the leading vehicle using the CS policy is not string stable on acceleration.

Since we have $\frac{\|a_n\|}{\|a_0\|} = \frac{\|a_n\|}{\|a_1\|}\frac{\|a_1\|}{\|a_0\|}$, it can be further derived that the CS-based vehicle platoon is not ex-head-to-tail string stable on acceleration since both the two norms on the right side are larger than 1 for any controller parameters and index *n* through numerical calculations.

Together with Proposition 3, we can conclude that the proposed vehicle platoon with a combined spacing policy has a better performance than the pure CS-based vehicle platoon in terms of ex-head-to-tail string stability on acceleration based on the rigorous mathematical analysis, since the feasible control parameters for ensuring the ex-head-to-tail string stability can be found.

##### *b) Theoretical Throughput*

The theoretical throughput of the vehicle platoon depends on the average time headway between the vehicles in the vehicle platoon with guaranteed stability. The throughput *TQ* (veh/s) of the vehicle platoons can be calculated as follows:

$$TQ = \frac{1}{\left[h_l + (d_l+L)/v + (n-1)\left(h_f + (d_f+L)/v\right)\right]/n} \tag{42}$$

where $h_l$ (s) and $d_l$ (m) is the time gap and standstill distance of the leader respectively, $h_f$ (s) and $d_f$ (m) is the time gap and standstill distance of the followers in the vehicle platoon respectively.

For ease of comparison in the theoretical throughput of the proposed vehicle platoon with the other two kinds of vehicle platoons, we can formulate the following equation:

$$\begin{aligned}\Delta TQ &= TQ^* - TQ^c \\ &= \frac{1}{\left[h_l^* + \frac{d_l^*+L}{v} + (n-1)\left(h_f^* + \frac{d_f^*+L}{v}\right)\right]/n} - \frac{1}{\left[h_l^c + \frac{d_l^c+L}{v} + (n-1)\left(h_f^c + \frac{d_f^c+L}{v}\right)\right]/n} \\ &= \frac{n\left[h_l^c + \frac{d_l^c+L}{v} + (n-1)\left(h_f^c + \frac{d_f^c+L}{v}\right)\right] - n\left[h_l^* + \frac{d_l^*+L}{v} + (n-1)\left(h_f^* + \frac{d_f^*+L}{v}\right)\right]}{\left[h_l^* + \frac{d_l^*+L}{v} + (n-1)\left(h_f^* + \frac{d_f^*+L}{v}\right)\right]\left[h_l^c + \frac{d_l^c+L}{v} + (n-1)\left(h_f^c + \frac{d_f^c+L}{v}\right)\right]} \\ &= \frac{n(h_l^c - h_l^*) + n(d_l^c - d_l^*)/v + n(n-1)(h_f^c - h_f^*) + n(n-1)(d_f^c - d_f^*)/v}{\left[h_l^* + \frac{d_l^*+L}{v} + (n-1)\left(h_f^* + \frac{d_f^*+L}{v}\right)\right]\left[h_l^c + \frac{d_l^c+L}{v} + (n-1)\left(h_f^c + \frac{d_f^c+L}{v}\right)\right]}\end{aligned} \tag{43}$$

where $TQ^*$ (veh/s) denotes the throughput of the proposed vehicle platoon and $TQ^c$ (veh/s) denotes the throughput of the CTG-based or CS-based vehicle platoon. Note that the vehicle length and speed are identical for the vehicles in all types of vehicle platoons.

The comparison results are determined by the numerator in Eq. (43) because the denominator is rationally positive in this equation, which is re-written as the following equation:

$$\begin{aligned}y &= n(h_l^c - h_l^*) + n(d_l^c - d_l^*)/v + n(n-1)(h_f^c - h_f^*) \\ &\quad + n(n-1)(d_f^c - d_f^*)/v\end{aligned} \tag{44}$$

First, we compare the throughput between the proposed and CS-based vehicle platoons. It can be intuitively found that the Eq. (44) can reduce to $y = n(h_l^{CS} - h_l^*) + n(d_l^{CS} - d_l^*)/v$ because $h_f^{CS}$ and $h_f^*$ are 0 and the standstill distances of the followers are the same for the two types of vehicle platoons. For the throughput comparison between the proposed vehicle platoon and the CTG-based vehicle platoon, we can see that the Eq. (44) can become $y = n(n-1)(h_f^{CTG} - h_f^*) + n(n-1)(d_f^{CTG} - d_f^*)/v$ for the same parameter setting of leader in the vehicle platoons.

Moreover, an example is used to illustrate the theoretical comparisons. The standstill distance *d* is 5 m for the vehicles using the CTG policy and 15 m for the vehicles using the CS policy and the speed is set at 20 m/s, as well the number of vehicles is ranged from 3 to 10 for the theoretical calculation. The controller parameters for the three types of vehicle platoons are set as follows [5], [8], [19]: (i) CS-based vehicle platoon: $\lambda$ = 0.1, $q_1$ = 0.4, $q_3$ = 0.9, and $q_4$ = 0.6; (ii) CTG-based vehicle platoon: $k_s$ = 0.1, $k_v$ = 0.7, and $k_a$ = 0.84; (iii) CTG and CS-based vehicle platoon: $\lambda$ = 0.1, $q_1$ = 0.4, $q_3$ = 0.9, $q_4$ = 0.6, $k_s$ = 0.1, $k_v$ = 0.7, and $k_a$ = 0.84. Based on the derived sufficient conditions in Proposition 4 for guaranteeing the string stability on spacing error and ex-head-to-tail string stability on acceleration, the critical time gap of the leader with CTG policy $h_l^*$ is 1.4 s and the follower $h_f^*$ is 0 s for the proposed vehicle platoon. For the consistency of comparison, the time gap of the leader $h_l^{CTG}$ is 1.4 s and the follower $h_f^{CTG}$ is 0.7 s for ensuring the stability of the CTG-based vehicle platoon.

Combining the parameter settings of the leader and the followers in the vehicle platoon, the comparison results among three types of vehicle platoons are illustrated in Fig. 4. It can be observed that the throughput of the proposed vehicle platoon is greater than the CTG-based vehicle platoon. However, both of them are not as good as the CS-based vehicle platoon. In a conclusion, the results demonstrate that the proposed vehicle platoon is better than the CTG-based platoon but weaker than the CS-based platoon in terms of theoretical throughput.

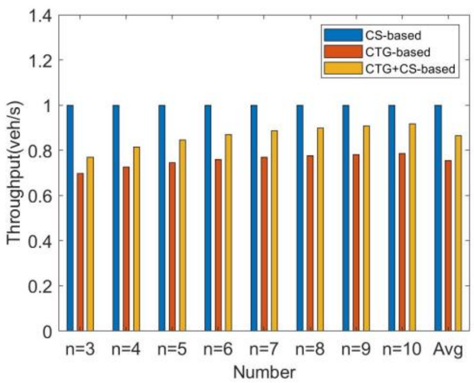



Fig. 4. Throughput of the three types of vehicle platoons.

*2) Numerical Experiments*

In what follows, we will further compare the performance of three types of vehicle platoon systems in various performance aspects, i.e., efficiency, safety, energy, and emission, by numerical experiments. To evaluate the effectiveness of the proposed vehicle platoon in a more realistic simulation scenario, we conduct the numerical simulation experiments using the accurate vehicle dynamic model applied in [26], [39]–[42], and the parameter setting of the model are referential to previous studies [38]–[40]. The other parameters in numerical experiments for the three types of vehicle platoons are the same as those in the above section of theoretical analysis.

Several measurements of effectiveness (MOE) in the literature including the traffic outflow, modified time-to-collision, fuel consumption, and emission measurements will be adopted to evaluate the effectiveness of the proposed vehicle platoon system in the aforementioned aspects. The following two disturbance scenarios with a CAV as the exogenous vehicle of three types of vehicle platoon systems are used for the tests [23], [25], [38]:

- **Scenario 1-Periodic Fluctuation**: The exogenous vehicle of the platoon performs periodic acceleration and deceleration from 10s, and the disturbance period lasts for 75 s with the acceleration ranging from -2.5 m/s$^2$ and 2.5 m/s$^2$, which is shown in Fig. 5(a).
- **Scenario 2-Large Deceleration and Acceleration**: The exogenous vehicle of the platoon moves at a constant speed of 20 m/s for 30 s and then decelerates to 5 m/s with a constant deceleration of -4.5 m/s$^2$, and then the vehicle restores to the original speed with a constant acceleration of 4.5 m/s$^2$ at the 60s, which is shown in Fig. 5(b).

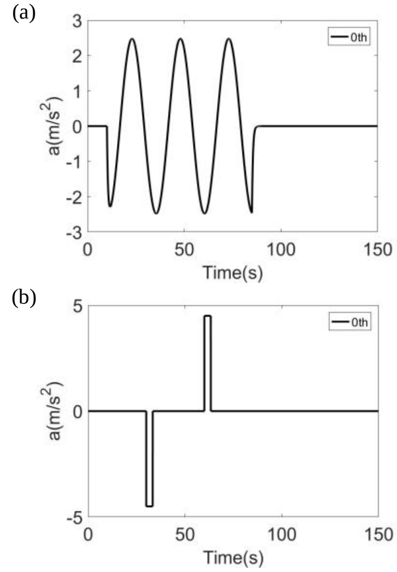



Fig. 5. Acceleration profile of the exogenous vehicle: (a) periodic fluctuation; (b) large deceleration and acceleration.

*a) Efficiency*

We will compare the efficiency of different vehicle platoon systems in terms of traffic outflow by simulations under the above two scenarios. The traffic outflow $Q$ (veh/s) is defined as the ratio of the total vehicles $n$ to the time interval between the first vehicle $t_1$ and the last vehicle $t_n$ passing the downstream measurement position of the same section as follows [43]:

$$Q = n/(t_n - t_1) \tag{45}$$

To explore the impact of the platoon size (i.e., the number of CAVs in a platoon), we simulate the vehicle platoons with the number of vehicles ranging from 3 to 10 and report the traffic outflow of the vehicle platoons for each specific number of CAVs, e.g., '$n$ = 4', and the average traffic outflow of all vehicle platoons with different sizes for each type of vehicle platoon, i.e., 'Avg', in Fig. 6. It shows that the proposed vehicle platoon using CTG and CS policy has better performance in traffic efficiency than the CTG-based one as it has a larger average traffic outflow. However, both of them are not as good as the CS-based vehicle platoon under Scenario 1. Moreover, we find that the traffic outflow of the proposed vehicle platoon is greater than the CTG-based one under Scenario 1. For the CTG-based and proposed vehicle platoons, the traffic outflows become larger as the number of CAVs increases, implying that the increment of following CAV members can enhance the traffic outflow, especially for the proposed vehicle platoon. Although the CS-based vehicle platoon has the largest traffic flow for all vehicle platoons, the traffic outflow is not enlarged with the increments in the number of CAVs. Generally speaking, the proposed vehicle platoon is superior to the CTG-based vehicle platoon but weaker than the CS-based vehicle platoon in terms of traffic efficiency. Under Scenario 2, similar findings can be observed in terms of the traffic outflows for the comparisons of the three types of vehicle platoons. In summary, the results have demonstrated the advantages of the CS policy on maximizing traffic outflow, followed by the proposed and CTG-based vehicle platoons in order. It also verifies the effectiveness of the theoretical throughput analysis.

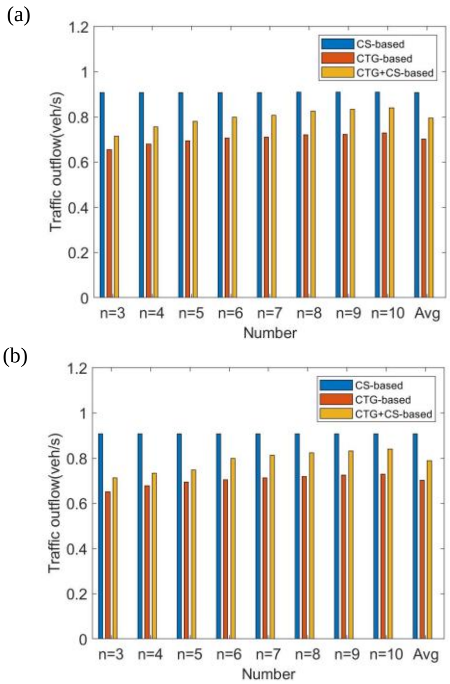


Fig. 6. Traffic outflows of vehicle platoons under different Scenarios: (a) Scenario 1; (b) Scenario 2.

*b) Safety*

This subsection aims to evaluate the safety performance of the three types of vehicle platoons in terms of two surrogate safety measurements named Time Exposed Time-to-collision (TET) and Time Integrated Time-to-collision (TIT). Both measurements are regarded as the modified surrogate safety measures of Time-to-collision (TTC) and have been widely used to evaluate safety performance [23], [44]. A larger TET or TIT value indicates a higher risk of a vehicle approaching oscillations. The detailed calculations are shown as follows:

$$TET=\sum_{t=1}^{T}\sum_{i=1}^{n}\gamma_t^i\Delta t,\ \gamma_t^i=\begin{cases}1, \forall t \text{ s.t. } 0<TTC_i(t)\le TTC^* \\ 0,\ \text{otherwise}\end{cases} \tag{46}$$

$$TIT=\sum_{t=1}^{T}\sum_{i\in\{1,\dots,n|0<TTC_i(t)\le TTC^*\}}\left(\frac{1}{TTC_i(t)}-\frac{1}{TTC^*}\right)\Delta t \tag{47}$$

where the TTC of vehicle $i$ at time $t$, i.e. $TTC_i(t)$, is time that remains until a collision between the consecutive vehicles occurs if they maintain the current speeds when the subject vehicle $i$ move faster than the preceding vehicle ($i$-1), and it is set at infinity when the vehicle $i$ is slower than the preceding vehicle ($i$-1), and $TTC^*$ is the safety threshold value.

Table II presents all (average) TET and TIT values of the vehicle platoons in the two scenarios, in which $TTC^*$ are set to be 5 s and 3 s for Scenarios 1 and 2 respectively for the case of safety comparison. It can be seen that both the TET and TIT values of the proposed and CS-based vehicle platoons are zero under Scenario 1, whereas the TET and TIT values of the CTG-based vehicle platoon are 0.98 and 0.01, respectively. The results suggest that the proposed and CS-based vehicle platoons have better safety performance as they have smaller TET or TIT values. This is because the proposed (except for the leader) and CS-based vehicle platoons aim to maintain constant spacing, which is not affected by the driving speed. Therefore, the magnitude variation of the actual spacing is far smaller than the speed difference between the consecutive vehicles in the proposed and CS-based vehicle platoons. On the contrary, the CTG-based vehicle platoon exhibits a higher collision risk under Scenario 1, since the ratio of speed difference to the actual spacing can easily exceed the threshold value according to Eqs. (46) and (47) under the disturbances. Moreover, the safety performance appears insensitive to the number of CAVs in the vehicle platoons. Under Scenario 2, similar findings can be found that the safety performance of the CTG-based vehicle platoon is still inferior to the other vehicle platoons. In conclusion, the findings demonstrate that both the proposed and CS-based vehicle platoons are superior to the CTG-based vehicle platoons in improving safety performance.

Table II Surrogate safety measurements of platoons under Scenarios 1 and 2

| Scenario | Measurement | System | $n$ = 3 | $n$ = 4 | $n$ = 5 | $n$ = 6 | $n$ = 7 | $n$ = 8 | $n$ = 9 | $n$ = 10 | Avg |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Scenario 1 | TET | CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | CTG-based | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
| | | CTG+CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | TIT | CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | CTG-based | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| | | CTG+CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Scenario 2 | TET | CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | CTG-based | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| | | CTG+CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | TIT | CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | CTG-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | CTG+CS-based | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Fig. 7 illustrates the 1/TTC results of the vehicles for the three types of vehicle platoons incorporating 5 CAVs. The black dotted lines represent the critical 1/TTC threshold values for Scenarios 1 and 2. It can be seen that the 1/TTC values periodically vary under the periodic disturbance of Scenario 1 and change during the deceleration and acceleration process under Scenario 2 for the three types of vehicle platoons. Additionally, for the CTG-based vehicle platoon, the 1/TTC values larger than the critical one occur at the 2nd vehicle, namely, the first follower within the platoon, under the two scenarios in Figs. 7(c) and 7(d). This is mainly because the time gap of the follower used is smaller than that of the leader in the platoon, which results in high collision risks for the first follower in the platoon. The large 1/TTC values happen at the 1st vehicle for the proposed vehicle platoon under Scenarios 1 and 2 respectively in Figs. 7(e) and 7(f), and the 2nd vehicle for

CS-based vehicle platoon under Scenarios 1 and 2 respectively in Figs. 7(a) and 7(b), since the 1st vehicle in the proposed vehicle platoon adopts the CTG policy and the 2nd vehicle in the CS-based vehicle platoon only utilizes the leader's state information.

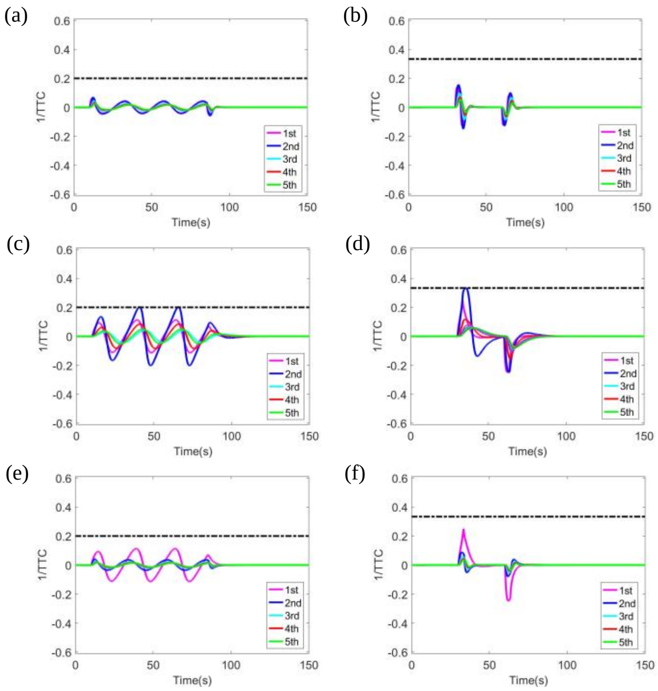



Fig. 7. 1/TTC results under different Scenarios: (a) Scenario 1 for CS-based platoon; (b) Scenario 2 for CS-based platoon; (c) Scenario 1 for CTG-based platoon; (d) Scenario 2 for CTG-based platoon; (e) Scenario 1 for CTG+CS-based platoon; (f) Scenario 2 for CTG+CS-based platoon.

### *c) Energy and Emissions*

In this subsection, we will first evaluate the stability performance against the exogenous disturbances for the three types of vehicle platoons, because it is closely related to the energy and emission performance. In particular, the dampening ratio (DR) is used to measure the stability performance of the vehicle platoons [12] as shown below:

$$DR = \frac{\|a_n(z)\|_2}{\|a_0(z)\|_2} \tag{48}$$

The aforementioned two disturbance scenarios will be employed in the simulations. Likewise, both the DR and the average DR of all vehicle platoons with different sizes for each type of vehicle platoon are reported in Fig. 8. It can be found that the DR and average DR are smaller than 1 for the proposed and CTG-based vehicle platoons, while they are larger than 1 for the CS-based vehicle platoon under Scenario 1. The results suggest that compared with the CS-based vehicle platoon, the proposed and CTG-based vehicle platoons have better stability performance that makes the disturbances attenuate from the exogenous vehicle to the last vehicle in the vehicle platoon. The advantage of the proposed vehicle platoon over the CS-based vehicle platoon should be attributed to the relatively large time gap of the leader in the proposed vehicle platoon, which essentially acts as a buffer against periodical disturbances from the exogenous vehicle. As a result, the combined formulation of spacing policy shows a comparative stability performance to the CTG-based vehicle platoon with commonly-accepted good stability performance evidenced by smaller DR and average DR values. However, the superiority in stabilizing traffic flow for the proposed vehicle platoon is found to slightly deteriorate when the number of CAVs in the platoon increase. This can be seen from the increased value of the DR in the proposed vehicle platoon in Scenario 1. Under Scenario 2, all the DR and average DR are still smaller than 1 for the proposed and CTG-based vehicle platoons and larger than 1 for the CS-based vehicle platoon. The results also indicate that the proposed vehicle platoon remains effective in guaranteeing stability through the whole platoon under large deceleration and acceleration conditions with all DR values less than 1, even if it is slightly inferior to the CTG-based vehicle platoon.

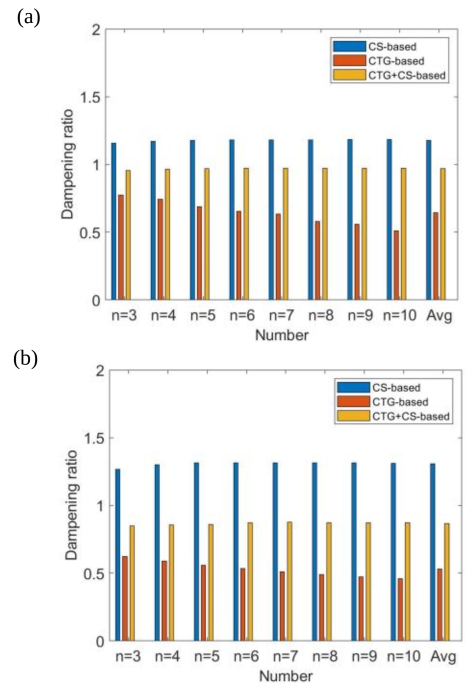



Fig. 8. Dampening ratios of vehicle platoons under different Scenarios : (a) Scenario 1; (b) Scenario 2.

Moreover, we evaluate the energy and emission performance of the three types of vehicle platoons by aggregating the fuel consumption and emissions of each vehicle in the vehicle platoon. The values of fuel consumption and emissions can be calculated based on the following VT-micro model [45], [46]:

$$In(MOE_i^k) = \sum_{c_1=0}^{3}\sum_{c_2=0}^{3} e_{c_1,c_2}^k v_i^{c_1} a_i^{c_2} \tag{49}$$

where $k$ denotes the category of energy and emission measurements, i.e., fuel consumption, HC, CO, $NO_x$, and $CO_2$, $MOE_i^k$ is the fuel consumption and emissions of vehicle $i$ at category $k$ (L/s or mg/s), $c_1$ and $c_2$ is the power of speed and acceleration ranging from 0 to 3, respectively, and $e_{c_1,c_2}^k$ is the regression coefficient, which is distinct for different categories under the speed power $c_1$ and acceleration power $c_2$. The values $e_{c_1,c_2}^k$ used are adopted from Ahn et al. (2002).

Table III summarizes the fuel consumption and emissions of the vehicle platoons under the aforementioned two scenarios. The results show that the proposed vehicle platoon has better performance in reducing fuel consumption and emissions than the CS-based vehicle platoon under Scenarios 1 and 2. The improvements are attributed to the great synergy between the CTG and CS policies, which has effectively reduced the oscillatory magnitude of the acceleration and speed, thus leading to smaller fuel consumption and emissions. However, the fuel consumption and emission performances of the proposed vehicle platoon are not good to the CTG-based vehicle platoon due to the inferior asymptotic stability on the acceleration of the proposed vehicle platoon compared with the CTG-based vehicle platoon. Of course, it is expected that the fuel consumption and emissions are positively affected by the number of CAVs in the vehicle platoon.

*3) Summary and Discussions*

For ease of overall comparison, we rank the three types of vehicle platoon systems in various performance aspects based on the average MOE values in Table IV, where "1" represents the best performance. Note that we use the representative measurement "$CO_2$" as the reference for the rankings in the emission aspect. We can find that, under both two scenarios, the proposed vehicle platoon performs the best in the safety aspect and it is ranked second place for the efficiency and energy and emission aspects following the CS-based and CTG-based vehicle platoon, respectively. Although the CS-based vehicle platoon has the best performance and predominant advantage in efficiency, it is the last place in many other aspects such as stability, energy, and emissions. This is due to the string instability on acceleration aforementioned in the above subsection of string stability analysis. As a result, fuel consumption and emissions can be negatively affected. Despite the energy and emission performances of the proposed vehicle platoon failing to surpass the CTG-based vehicle platoon due to the inferior asymptotic stability, the proposed vehicle platoon has large advantages in efficiency and safety over the CTG-based vehicle platoon due to the application of CS policy. The proposed vehicle platoon can also enhance the stability since the dampening ratio is smaller than 1, although the stability performance of the proposed vehicle platoon is inferior to the CTG-based vehicle platoon. The overall ranking results demonstrate the superiority of great synergy between the CTG and CS policies in the proposed vehicle platoon.

Overall speaking, the CS-based and CTG-based vehicle platoons show unbalanced performance in these aspects. Most importantly, the proposed vehicle platoon systems show visible advantages over other types of vehicle platoons from a point of view of overall performance, which has demonstrated the merits and great potential of combined formulations.

Table IV Rankings of three types of vehicle platoons in different aspects

| Scenario | Type | Efficiency | Safety | Energy | Emission |
|---|---|---|---|---|---|
| Scenario 1 | CS-based | 1 | 1 | 3 | 3 |
| | CTG-based | 3 | 3 | 1 | 1 |
| | CTG+CS-based | 2 | 1 | 2 | 2 |
| Scenario 2 | CS-based | 1 | 1 | 3 | 3 |
| | CTG-based | 3 | 3 | 1 | 1 |
| | CTG+CS-based | 2 | 1 | 2 | 2 |

Table III Fuel and emissions of vehicle platoons under Scenarios 1 and 2

| Scenario | Measurement | System | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ | $n = 7$ | $n = 8$ | $n = 9$ | $n = 10$ | Avg |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Scenario 1 | Fuel | CS-based | 3.68 | 4.92 | 6.16 | 7.40 | 8.64 | 9.88 | 11.12 | 12.37 | 8.02 |
| | | CTG-based | 3.53 | 4.69 | 5.84 | 6.99 | 8.13 | 9.27 | 10.40 | 10.96 | 7.47 |
| | | CTG+CS-based | 3.57 | 4.77 | 5.97 | 7.17 | 8.37 | 9.57 | 10.77 | 11.97 | 7.77 |
| | HC | CS-based | 3.25 | 4.34 | 5.44 | 6.53 | 7.63 | 8.73 | 9.82 | 10.92 | 7.08 |
| | | CTG-based | 3.24 | 4.30 | 5.37 | 6.43 | 7.49 | 8.54 | 9.59 | 10.11 | 6.88 |
| | | CTG+CS-based | 3.24 | 4.33 | 5.41 | 6.49 | 7.58 | 8.66 | 9.74 | 10.83 | 7.04 |
| | CO | CS-based | 32.39 | 43.30 | 54.24 | 65.19 | 76.13 | 87.08 | 98.03 | 108.98 | 70.67 |
| | | CTG-based | 31.15 | 41.39 | 51.59 | 61.75 | 71.88 | 81.97 | 92.03 | 97.05 | 66.10 |
| | | CTG+CS-based | 31.50 | 42.05 | 52.62 | 63.19 | 73.76 | 84.34 | 94.91 | 105.49 | 68.48 |
| | $NO_x$ | CS-based | 2.79 | 3.79 | 4.80 | 5.81 | 6.84 | 7.86 | 8.88 | 9.91 | 6.33 |
| | | CTG-based | 2.674 | 3.45 | 4.21 | 4.92 | 5.60 | 6.24 | 6.82 | 7.10 | 5.13 |
| | | CTG+CS-based | 2.75 | 3.65 | 4.57 | 5.49 | 6.41 | 7.34 | 8.26 | 9.19 | 5.96 |
| | $CO_2$ | CS-based | 8,564 | 11,446 | 14,333 | 17,220 | 20,109 | 22,997 | 25,886 | 28,775 | 18,666 |
| | | CTG-based | 8,192 | 10,887 | 13,560 | 16,220 | 18,864 | 21,496 | 24,117 | 25,422 | 17,345 |
| | | CTG+CS-based | 8,296 | 11,081 | 13,869 | 16,658 | 19,447 | 22,236 | 25,025 | 27,814 | 18,053 |
| Scenario 2 | Fuel | CS-based | 3.15 | 4.21 | 5.28 | 6.34 | 7.41 | 8.47 | 9.54 | 10.60 | 6.88 |
| | | CTG-based | 3.08 | 4.11 | 5.13 | 6.16 | 7.18 | 8.20 | 9.23 | 10.25 | 6.67 |
| | | CTG+CS-based | 3.09 | 4.12 | 5.15 | 6.19 | 7.22 | 8.26 | 9.291 | 10.32 | 6.70 |
| | HC | CS-based | 3.15 | 4.22 | 5.29 | 6.36 | 7.42 | 8.49 | 9.56 | 10.62 | 6.89 |
| | | CTG-based | 2.96 | 3.96 | 4.95 | 5.94 | 6.94 | 7.94 | 8.95 | 9.96 | 6.45 |
| | | CTG+CS-based | 3.01 | 4.03 | 5.05 | 6.08 | 7.10 | 8.12 | 9.14 | 10.16 | 6.59 |
| | CO | CS-based | 29.00 | 38.97 | 48.98 | 58.98 | 68.97 | 78.96 | 88.94 | 98.92 | 63.96 |
| | | CTG-based | 26.86 | 35.80 | 44.71 | 53.63 | 62.54 | 71.45 | 80.37 | 89.30 | 58.08 |
| | | CTG+CS-based | 27.23 | 36.40 | 45.58 | 54.75 | 63.93 | 73.11 | 82.28 | 91.46 | 59.34 |
| | $NO_x$ | CS-based | 2.26 | 3.07 | 3.88 | 4.70 | 5.51 | 6.33 | 7.14 | 7.96 | 5.11 |
| | | CTG-based | 1.12 | 1.55 | 1.96 | 2.40 | 2.85 | 3.32 | 3.82 | 4.37 | 2.67 |
| | | CTG+CS-based | 1.46 | 2.06 | 2.67 | 3.28 | 3.89 | 4.51 | 5.12 | 5.73 | 3.59 |
| | $CO_2$ | CS-based | 7,257 | 9,700 | 12,148 | 14,598 | 17,048 | 19,499 | 21,949 | 24,400 | 15,825 |
| | | CTG-based | 7,128 | 9,497 | 11,864 | 14,229 | 16,592 | 18,953 | 21,312 | 23,669 | 15,405 |
| | | CTG+CS-based | 7,132 | 9,511 | 11,892 | 14,274 | 16657 | 19,041 | 21,424 | 23,808 | 15,467 |

### C. Performance Comparisons of Proposed and VTG-based Vehicle Platoons

Numerical simulation experiments have been further conducted to better evaluate the effectiveness of the proposed vehicle platoon combing the CTG and CS spacing policies compared with the VTG-based vehicle platoon in various performance aspects under the two scenarios of periodic disturbances and large deceleration and acceleration. The control model and control parameter settings for the VTG-based vehicle platoon in previous studies [32], [34] were used. The performance results of the VTG-based vehicle platoon in terms of traffic outflows, modified time-to-collision (i.e., TET and TIT), fuel consumption, and emissions under the two scenarios are elaborated in Table V. As indicated by the comparisons of the results in Table V and Section VI-B2-Numerical experiments, we can see that the VTG-based vehicle platoon has similar overall performance in efficiency, safety, fuel consumption, and emission aspects to that of the CTG-based vehicle platoon under the two scenarios. Therefore, the performance comparison results between the proposed and VTG-based vehicle platoons are similar to that of the proposed and CTG-based vehicle platoons (see the results in Table IV). Notably, the proposed vehicle platoon using CTG and CS policies has much better performance in traffic efficiency than the VTG-based vehicle platoon under the two scenarios because it has the larger average traffic outflows (see the results in Table V and Fig. 6). The safety performance of the VTG-based and proposed vehicle platoons are comparable under the two scenarios as they have smaller TET and TIT values (see the results in Tables II and V).

Table V Performance results of VTG-based vehicle platoon under Scenarios 1 and 2

| Scenario | Measurement | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ | $n = 7$ | $n = 8$ | $n = 9$ | $n = 10$ | Avg |
|---|---|---|---|---|---|---|---|---|---|---|
| Scenario 1 | Q | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 |
| | TET | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | TIT | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | Fuel | 3.34 | 4.44 | 5.54 | 6.63 | 7.72 | 8.80 | 9.87 | 10.94 | 7.16 |
| | HC | 3.19 | 4.22 | 5.25 | 6.28 | 7.31 | 8.34 | 9.37 | 10.41 | 6.79 |
| | CO | 31.02 | 41.00 | 50.98 | 60.95 | 70.94 | 80.95 | 90.97 | 101.01 | 65.98 |
| | $NO_x$ | 2.15 | 2.48 | 2.80 | 3.09 | 3.38 | 3.67 | 3.94 | 4.22 | 3.22 |
| | $CO_2$ | 8,264 | 10,817 | 13,354 | 15,878 | 18,388 | 20,886 | 23,372 | 25,849 | 17,101 |
| Scenario 2 | Q | 0.72 | 0.72 | 0.72 | 0.72 | 0.72 | 0.72 | 0.72 | 0.72 | 0.72 |
| | TET | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | TIT | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | Fuel | 3.08 | 4.11 | 5.13 | 6.15 | 7.17 | 8.20 | 9.22 | 10.24 | 6.67 |
| | HC | 3.03 | 4.04 | 5.07 | 6.10 | 7.13 | 8.14 | 9.17 | 10.17 | 6.61 |
| | CO | 27.12 | 36.12 | 45.14 | 54.12 | 63.08 | 71.98 | 80.94 | 89.76 | 58.53 |
| | $NO_x$ | 1.57 | 2.15 | 2.83 | 3.49 | 4.15 | 4.69 | 5.44 | 5.87 | 3.77 |
| | $CO_2$ | 7,112 | 9,474 | 11,829 | 14,183 | 16,535 | 18,888 | 21,233 | 23,587 | 15,355 |

## VII. Conclusion And Future Work

In this study, we propose a vehicle platoon of connected and automated vehicles (CAVs) jointly using the constant time gap (CTG) and constant spacing (CS) policy to enhance traffic performance. The vehicle platoon with a combined spacing policy is firstly formulated, where the leader adopts the CTG policy and the followers use the CS policy in the platoon based on the linear feedback and feedforward controllers. Based on the combined formulation, the $h_2$-norm string stability criteria related to spacing error and acceleration are used for the stability analysis. The sufficient conditions of the local stability of the proposed vehicle platoon are derived based on the Routh-Hurwitz criterion. The notion of ex-head-to-tail string stability on acceleration for the proposed vehicle platoon is introduced and the sufficient conditions of string stability on spacing error and ex-head-to-tail string stability on acceleration in the frequency domain are derived using the Laplace transform. Numerical experiments are conducted to validate the mathematical proofs of stability analysis. Moreover, theoretical analysis is conducted to demonstrate the advantages of the proposed vehicle platoon in the stability and theoretical throughput. Several measurements of effectiveness under two typical scenarios, i.e., periodical fluctuation and large deceleration and acceleration, are adopted to further verify the effectiveness of the proposed vehicle platoon in the aspects of efficiency, safety, energy, and emissions. The results show that the proposed vehicle platoon has better overall performance compared with the CS-based and CTG-based vehicle platoons. Our study may serve as a useful guide for the applications of vehicle platoon technology in practice. The proposed vehicle platoon combining two spacing policies can be designated to form in advance or determined by the vehicle/traffic operation center and implemented for the improvements of stability and traffic performance in congested car-following conditions under traffic scenarios like intersections and on-ramp merging.

In a conclusion, some key results through the systematic analysis are summarized: (i) the proposed vehicle platoon can achieve string stability that guarantees the string stability on spacing error and ex-head-to-tail string stability on acceleration. The combined spacing policy can theoretically enhance the string stability of a vehicle platoon compared to one using the CS policy only. It also provides a potential opportunity to implement vehicle systems with different spacing policies on the platoon level. (ii) the proposed vehicle platoon has much better traffic performance than the CS-based vehicle platoon in most aspects, including stability, energy, and emissions. (iii) the proposed vehicle platoon has better traffic performance compared to the CTG-based vehicle platoon in efficiency and safety aspects under periodical fluctuation and large deceleration and acceleration scenarios. Essentially, it has neutralized the poor performance in terms of efficiency and safety and the good performance regarding the fuel consumption and emissions brought by the better stability of the CTG-based vehicle platoon.

Further research work can be conducted in the following several aspects. First, other vehicle controllers with distinct formulations using these spacing policies can be further explored to make a comparison of the performance among the proposed, CTG-based, and CS-based vehicle platoons. Moreover, the performance of the proposed vehicle platoon is largely affected by the heterogeneity and uncertainty in vehicle dynamics and controller parameters, etc. It is thus necessary to propose more efficient robust methods for the future implementation of the proposed vehicle platoon.

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## APPENDIX A. NOTATION

| | |
|---|---|
| $i$ | Vehicle longitudinal sequence |
| $n$ | Total number of the platoon system |
| $t$, $\tau$ | Time instant |
| $p$ | Front bumper position |
| $v$ | Speed |
| $a$ | Realized acceleration |
| $u$ | Desired acceleration |
| $r$ | Number of predecessors |
| $s$ | Actual spacing |
| $s^*$ | Target spacing |
| $\Delta s$ | Spacing error |
| $h$ | Constant time gap |
| $d$ | Inter-vehicle distance in standstill condition |
| $L$ | Vehicle length |
| $\delta$ | Sensor delay |
| $\theta$ | Communication delay |
| $\varphi$ | Actuation time-lag |
| $g$ | Time delay |
| $\sigma$ | Accumulated time delay |
| $T$ | Simulation time |
| $\Delta t$ | Simulation time step |
| $k_s$, $k_v$, $k_a$ | Controller parameters of the leading vehicle |
| $q_1$, $q_3$, $q_4$, $\lambda$ | Controller parameters of the following vehicles |

## APPENDIX B. PROOF OF PROPOSITION B

**Proposition B**: The vehicle platoon with a combined spacing policy is not head-to-tail string stable on acceleration.

**Proof.** As for the vehicles $i \in \{2, 3, \ldots, n\}$, Eq. (10) in the frequency domain using the Laplace transform can be written as follows:

$$A(z)p_i(z) = B(z)p_{i-1}(z)e^{-g_i z} + C(z)p_1 e^{-\sigma_i z}, \forall i \in \{2,...,n\} \quad (50)$$

with

$$\begin{cases} A(z) = (1+q_3)z^2(1+z\varphi) + (q_1+\lambda+q_4+q_3\lambda)z + (q_1+q_4)\lambda \\ B(z) = z^2 + (q_1+\lambda)z + q_1\lambda \\ C(z) = q_3 z^2 + (q_4+q_3\lambda)z + q_4\lambda \end{cases} \quad (51)$$

where $p_i(z)$, $p_{i-1}(z)$, and $p_1(z)$ are the Laplace transform of the position of vehicle $i$, vehicle ($i$-1), and leading vehicle $i$ = 1 in the platoon, respectively.

By substituting the index ($i$-1) in Eq. (50), we have

$$A(z)p_{i-1}(z)e^{-g_i z} = B(z)p_{i-2}(z)e^{-g_i z - g_{i-1} z} + C(z)p_1 e^{-\sigma_i z}, \forall i \in \{3,...,n\} \quad (52)$$

According to Eq. (52), we substitute the index ($i$-1) of the right-hand term in Eq. (50) as $i$ = 1 and obtain

$$\begin{aligned} p_i(z) &= \frac{B(z)}{A(z)} p_{i-1}(z)e^{-g_i z} + \frac{C(z)}{A(z)} p_1(z)e^{-\sigma_i z} \\ &= \left(\frac{B(z)}{A(z)}\right)^{i-1} p_1(z)e^{-\sigma_i z} + \frac{C(z)}{A(z)-B(z)}\left[1-\left(\frac{B(z)}{A(z)}\right)^{i-1}\right] p_1(z)e^{-\sigma_i z}, \forall i \in \{2,...,n\} \end{aligned} \quad (53)$$

Then we can obtain the norm of the head-to-tail string stability on acceleration for the proposed vehicle platoon:

$$\frac{\|a_n\|}{\|a_1\|} = \frac{\|p_n\|}{\|p_1\|} = \left\| \left(\frac{B(z)}{A(z)}\right)^{n-1} + \frac{C(z)}{A(z)-B(z)}\left[1-\left(\frac{B(z)}{A(z)}\right)^{n-1}\right] \right\| \left\| e^{-\sigma_n z} \right\|, \forall w > 0, z = jw \quad (54)$$

The norm of the right-hand term on the Eq. (54) is always larger than or equal to 1 for any controller parameters and index $n$ through numerical calculations. In other words, we cannot find the feasible controller parameters and index $n$ that Eq. (25) in Definition 4 holds based on the local stability conditions (i.e., Eq. (16)). Therefore, we can conclude that the proposed vehicle platoon is not head-to-tail string stable on acceleration. This completes the proof of Proposition B. □

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**Dr. Yuan Zheng** received his Ph.D. degree (2020) from the School of Transportation at the Southeast University in Nanjing, China. He is currently a Post-doc Fellow at the Hong Kong Polytechnic University. His interests include connected automated vehicle and platoon control and microscopic traffic modeling.

**Dr. Min Xu** is currently an Assistant Professor in the Department of Industrial and Systems Engineering at the Hong Kong Polytechnic University. She received her Ph.D. degree from the Ph.D. degree in Transportation Engineering from National University of Singapore, Singapore. She focuses on the modeling and optimization problem in urban transportation and logistics systems with emerging technologies. She serves on editorial advisory board of Transportation Research Part C and Transportation Research Part E.

**Dr. Shining Wu** is currently an Associate Professor in the Department of Logistics and Maritime Studies at the Hong Kong Polytechnic University. He received his Ph.D. in Industrial Engineering and Logistics Management from the Hong Kong University of Science and Technology, Hong Kong. His research interests include the sharing economy, behavioral operations management, queueing theory and its applications, and data-driven optimization.

**Dr. Shuaian Wang** is currently a Professor in the Department of Logistics and Maritime Studies at the Hong Kong Polytechnic University. He received his Ph.D. in Transportation Engineering from National University of Singapore, Singapore. His research interests include the sharing economy, strategic consumer behavior, queueing theory and its applications, and data-driven optimization. He is the editor or editorial advisory board member of many transportation journals including Transportation Research Part B, Transportation Research Part E.