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Column generation for the multi-port berth allocation problem with port cooperation stability

Liming Guo [a], Jianfeng Zheng [a *], Jinpeng Liang [a], Shuaian Wang [b]

[a] Transportation Engineering College, Dalian Maritime University, Dalian, China

[b] Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

Abstract:

This paper proposes a multi-port berth allocation problem (MPBAP) under a cooperative environment, which aims to determine berthing times and berthing positions for all considered vessels arriving at multiple neighboring ports. The previous studies on the MPBAP (or the BAP with multiple ports) consider that multiple ports have established stable cooperation, while the port cooperation stability problem (PCSP) has not been addressed. This paper investigates the PCSP with the MPBAP, where our MPBAP further integrates the vessel diverting issue that vessels with excessive waiting times can be diverted to neighboring ports. For the PCSP, we investigate how to group multiple neighboring ports into different stable port groups, and then determine optimal port groups. For all possible port groups, we propose a mixed integer programming model for the MPBAP, and a column generation approach is devised to solve it. Based on optimal solutions of the MPBAP for various port groups, cooperative game theory is utilized to obtain stable port groups, and then the PCSP can be formulated as a binary programming model for determining optimal port groups. Numerical experiments are carried out to account for the efficiency and effectiveness of the proposed models and solution method.

Keywords: Multi-port berth allocation; Port groups; Cooperative game; Column generation

1 Introduction

Maritime transportation plays a major role in boosting international trade and the global economy, being responsible for more than 80% of global trade (UNCTAD 2019). Port is a key component in maritime transportation, and port operators have emphasized the importance of planning and optimization for shore resources, including yard template design (Ng et al., 2010; Li, 2014; Jin et al., 2015), berth allocation (Meisel and Bierwirth, 2006; Zhen et al., 2017; Bouzekri et al., 2021), quay crane assignment (Fu et al., 2014; Diabat and Theodorou, 2014), and internal truck scheduling (Wang et al., 2014; Ladier and Alpan, 2018). Among these, the berth allocation problem (BAP) has been the most concerned optimization problem in port

* Corresponding authors.
E-mail addresses: jfzheng@dlmu.edu.cn (J. Zheng); jpliang@dlmu.edu.cn (J. Liang); wangshuaian@gmail.com (S. Wang).

planning and operations. Aiming to minimize the total port time (or generalized cost) of all considered vessels, the BAP arranges when and where vessels should berth (Imai et al., 2008a, 2008b, 2014). The port time of a vessel consists of the waiting time in anchorage and the handling time for loading/unloading cargoes, which mainly depends on the handling efficiency of the port and the number of cargoes to be handled (Zhen et al., 2015; Kramer et al., 2019; Rodrigues and Agra, 2021).

As statistics, container throughputs of the top 10 ports in 2020 were up to 256.3 million twenty-foot equivalent units (TEUs), and the growth rate is 1.5%, as compared with those in 2019. Ship board capacities of the top 10 liner shipping companies in 2021 were 21.3 million TEUs, and the growth rate is 5.1%, as compared with those in 2020. To meet the growing demands for serving vessels at ports, port operators utilize different approaches: (i) huge capital investment for upgrading port resources (Petering and Murty, 2009; Cordeau et al., 2015); (ii) new forms of technology (Henesey, 2004; Emde et al., 2014); (iii) cooperative logistics between BAP and other operations in the port, e.g., BAQCAP that integrating with BAP and QC (quay crane) assignment (Bouzekri et al., 2021; Thanos et al., 2021), BAP and yard space allocation (Karakas et al., 2021; Lin et al., 2022) and the integration BAP with the internal truck operation (Karakas et al., 2021; Chargui et al., 2021). In addition, cooperative logistics between ports have been noticed: berth sharing (Imai et al., 2008a, 2008b; Dulebenets et al., 2018), yard storage space sharing (Peng et al., 2015; Jin et al., 2019), drayage or truck sharing in inter-terminal transportation (Islam, 2018; Islam et al., 2021; Vandani et al., 2019), container sharing between ports or terminals (Zheng et al., 2015). Since berth resources are limited and the allocation plan is closely related to other port operations, the sharing of berth resources among ports is particularly important in relieving the pressure of handling operations.

While neighboring ports with overlapping hinterlands compete for cargoes or vessels visiting, some studies and realities show that some ports/terminals have incentives to cooperate by sharing information or facility resources. Hoshino (2010) and Takenayashi and Hanaoka (2021) emphasized the importance of cooperation among Japanese ports in surviving competition from ports in ASEAN. Saeed and Larsen (2010) explored the significance of port cooperation based on the real case of three terminals in Karachi Port and summarized the dynamics of cooperation between ports from three aspects: i) terminals/ports within a coalition will gain market share without more investment costs; ii) individual terminals have better use of combined capacity, which is helpful for the reduction of the average vessel waiting time; iii) service level and operational efficiency will increase. In addition, Kavirathna et al. (2020) introduced a vessel diversion policy for the cooperation among terminals and showed that the vessel diversion improved facility utilization and helped to relieve port congestion. They also pointed out that barriers of port cooperation might come from the information sharing between ports and devising benefit allocation among ports. By analyzing the evolution of ports in Liaoning Province in China, Wu and Yang (2018) pointed out that the fierce competition for cargoes between several neighboring ports similar in size and function has led to an internecine situation, which seriously restricts the development of the port competitiveness and the regional economy. With the establishment of Liaoning Port Group Co., Ltd. in November 2017, the inter-port cooperation has saved large inland transportation costs and boosted the overall revenue of ports. Following the above factual basis and literature supports, neighboring ports have a strong incentive to cooperate from the perspective of improving competitiveness and saving costs.

This study explores the BAP with port cooperation, which is called multi-port BAP (MPBAP). We consider that several neighboring ports form a coalition (or port group) and share berths for serving vessels visiting these ports. By considering weekly service frequency, vessels can only berth once at any of the considered ports. If the vessel suffers from excessive waiting time, the vessel may be diverted from the original port called to any other port within the port group. The MPBAP aims to assign berthing times and positions for vessels arriving at multiple neighboring ports. Considering that any vessel with excessive waiting time at one port can be diverted for service at any cooperative neighboring port, a key issue is to determine a proper diversion port by balancing the compensation costs for vessel diverting to the port and delay costs due to excessive waiting. For simple representation, this decision problem is called the diversion port choice problem (DPCP), which is integrated with our MPBAP. The DPCP is similar to the situations considered in Imai et al. (2008a, 2008b) and Dulebenets et al (2018), where vessels with excessive waiting times at a multi-user container terminal can be diverted to an external container terminal. For the proposed MPBAP, we should consider a cooperation agreement among multiple neighboring ports.

To the best of our knowledge, there are only four studies on the MPBAP or the BAP with multiple ports (Peng et al., 2015; Wang et al., 2015; Venturini et al., 2017; Martin-Iradi et al., 2022). Peng et al. (2015) investigated the MPBAP integrated with the yard assignment problem within neighboring ports, while no cooperation is considered among multiple ports. The rest three studies addressed the MPBAP where multiple ports are called by vessels along a shipping service route. The coordination among different ports on berth allocation is mainly based on vessel speed optimization when sailing between two successive ports along the shipping service route. While this study has the following differences from the above four studies on MPBAP: i) we focus on the cooperation among neighboring ports instead of the ports along the shipping service route considered in previous studies on MPBAP; ii) we consider vessel diversion behavior, which is not considered in previous studies on MPBAP; iii) since the handling cargoes need to be transferred with the diverting vessels, we also consider the cargo transfer problem for loading, unloading and transshipped cargoes in our MPBAP; iv) we further address the port cooperation stability and rationality issues, which have not been explored in previous studies on MPBAP. To fill this research gap, we investigate the port cooperation stability problem (PCSP), together with the MPBAP, and the port cooperation rationality issue can be reduced to the DPCP. For the PCSP at a tactical level, we investigate how to group multiple neighboring ports into different stable port groups, and then determine the optimal stable port groups. Note that, when addressing the BAP considering the cooperation among multiple terminal operators at a single port, our research framework is also applicable.

The main contributions and works of this paper are as follows.

(i) A novel MPBAP with the PCSP is proposed.

(ii) A two-stage framework is developed to solve our problem. In stage I, the MPBAP for any particular port group is formulated as a mixed integer programming (MIP) model, where all feasible port groups will be enumerated. Based on the berth allocation results of the MPBAP for various port groups obtained in stage I, stage II utilizes the Shapley value to determine revenue allocation among various ports within the same port group, and core theory is further used to analyze port group stability, and then the PCSP can be formulated as a binary programming model for determining the optimal port groups selected from the stable ones.

(iii) A column generation (CG) algorithm is devised to solve our MIP model within our proposed two-stage framework. Computational experiments are carried out to validate the efficiency and effectiveness of our solution method.

The remainder of this paper is organized as follows: Section 2 gives literature review. Section 3 provides the problem definition and formulation of the MPBAP and the PCSP. Section 4 presents the CG algorithm. Numerical experiments are carried out in Section 5. Finally, conclusions are given in Section 6.

2 Literature review

Port operations and decision problems have been greatly addressed in the literature (Stahlbock and Voß, 2008; Bierwirth and Meisel, 2010, 2015). As one of the most important issues in port operations, the BAP can be classified into the operational level, tactical level and strategic level, depending on the planning horizon. The operational level berth allocation problem (OBAP) considers the shortest time horizon, which tries to assign vessels to the berths along the quay. The tactical level berth allocation problem (TBAP), which is first mentioned by Moorthy and Teo (2006), is named the berth template planning problem covering the operation process in a time horizon from one week up to several months. The strategic level decision, with a time interval of several years, studies the issues of establishing berths and which vessels should be accepted (Imai et al., 2014; Iris et al., 2018). Besides, the BAP can be classified into two categories, namely, discrete and continuous, depending on whether the shoreline is regarded as a continuous space or multiple discrete berths (Imai et al., 2005). Based on whether vessels arrive when handling operation is in progress (Imai et al., 2001), the BAP can also be categorized into static and dynamic ones.

The MPBAP presented in this paper deals with the discrete dynamic BAP at the tactical level. Imai et al. (2001) developed a heuristic procedure based on the Lagrangian relaxation to solve the dynamic BAP in polynomial time. With the objective of minimization of the total (weighted) handling time for all ships, Cordeau et al. (2005) designed a heuristic for the continuous case. Monaco and Sammarra (2007) formulated the discrete dynamic BAP as a dynamic scheduling model solved by proposing a Lagrangean heuristic algorithm. Considering the service priority of various vessels, Golias et al. (2009) proposed a multi-objective formulation. Buhrkal et al. (2011) reviewed and compared three typical models of discrete and dynamic BAP, and the numerical results showed that the performance of the set-partitioning model is better than all other existing models. Following the work of Buhrkal et al. (2011), Kramer et al. (2019) proposed a novel time-indexed formulation and an arc-flow formulation to deal with discrete dynamic BAP. To solve the discrete dynamic BAP efficiently, Barbosa et al. (2019) proposed a hybrid evolutionary algorithm, where data envelopment analysis and free disposal hull models were adopted to select the appropriate parameters for a genetic algorithm, and then an improved genetic algorithm was presented to solve the multi-objective model.

Table 1. Overview of studies on the TBAP.

Reference	Problem	Single or multiple ports (terminals)	Port cooperation	Vessel diversion	Cargo transfer	Solution
Imai et al. (2008a)	BAP, QCAP	Single				A genetic algorithm
Imai et al. (2008b)	BAP	Multiple	√			A genetic algorithm
Han et al. (2010)	BAP, QCAP	Single				A simulation based genetic algorithm search procedure
Giallombardo et al. (2010)	BAP, QCAP	Single				A heuristic algorithm
Lee et al. (2010)	BAP	Single				A greedy randomized adaptive search procedure
Lee and Jin (2013)	BAP, YTD	Single			√	A memetic heuristic approach
Vacca et al. (2013);	BAP, QCAP	Single				An exact branch and price algorithm
Meisel and Bierwirth (2013)	BAP, QCAP	Single				Heuristics
Lalla-Ruiz et al. (2014)	BAP, QCAP	Single				A biased random key genetic algorithm
Jin et al. (2015)	BAP, YTD	Single				Heuristic methods based on column generation
Cahyono et al. (2015)	BAP, QCAP	Single				Simulation methods
Peng et al. (2015)	BAP	Multiple				Genetic algorithm
Wang et al. (2015)	BAP	Multiple	√			-
Zhen et al. (2016)	TAP	Multiple	√		√	A local branching based method and a particle swarm optimization based method
Zhen et al. (2017)	BAP, QCAP	Single				A column generation solution
Venturini et al. (2017)	BAP, SOP	Multiple	√			CPLEX
Wang et al. (2018)	BAP, QCAP, YTD	Single				A column generation-based heuristic
Heilig et al. (2019)	BAP, YTD	Multiple				-
Dulebenets et al. (2018)	BAP	Multiple	√	√		A memetic algorithm
Kavirathna et al. (2019, 2020)	BAP	Multiple	√	√		A rolling horizon approach and a general variable neighborhood search
Martin-Iradi et al. (2022)	BAP, SOP	Multiple	√			A branch-and-cut-and-price procedure

A great number of works have been carried out regarding the TBAP that are closely related to our MPBAP, as shown in Table 1. In Table 1, most studies on the TBAP aim at the single port, where quay crane assignment problem (QCAP) or yard template design (YTD) is integrated to achieve a coordinated allocation of port resources. A number of works address the multi-port (or multi-terminal) TBAP, while a few studies focus on the cooperation between ports. Note that some works on the TBAP in transshipment ports (e.g., Lee and Jin 2013) are concerned with the connection between mother and feeder vessels and the movement of cargo flow, different from the vessel diversion between multiple ports addressed in this paper. Imai et al. (2008a, 2008b), Dulebenets et al. (2018) pointed out that vessels with excessive waiting times in the ordinary port could be diverted to a fixed port or terminal, different from the DPCP proposed in this paper. Up to now, there is no study that considers both vessel diversion and cargo transfer when addressing the MPBAP with port cooperation. Hence our work fills this research gap.

Moreover, advanced heuristic algorithms have been proposed in recent years. Lee et al. (2010) developed a greedy randomized adaptive search procedure to find the suitable locations for the next vessels in the BAP. Jin et al. (2015) formulated the novel integration issue of schedule template design, berth template design and YTD as a set-partitioning model that was solved by the proposed heuristic method embedded with column generation. Lee and Jin (2013) addressed a jointed problem to determine preferred berthing positions, service times and storage yard spaces for feeder vessels in transshipment terminals, and a memetic heuristic approach was proposed to deal with this problem.

Later, the BAP has been extended to integrate with the QCAP, which is first proposed by Park and Kim (2003). Han et al. (2010) formulated the BAP integrated with the QCAP considering uncertain arrival times and handling times as a MIP model where QCs could move to other berths before the current loading and unloading tasks were completed. To achieve high resource utilization at a low cost, Meisel and Bierwirth (2013) proposed a framework with three phases to provide decision support in berth allocation, quay crane assignment, and quay crane scheduling. To address the joint optimization of the TBAP and QCAP, Vacca et al. (2013) developed an exact branch and price algorithm where some accelerating techniques were adopted to generate optimal integer solutions within the acceptable time. To achieve the coordination between the storage process of transshipment flows and the handling process of incoming vessels, Lalla-Ruiz et al. (2014) addressed the BAP and QCAP jointly and proposed a biased random-key genetic algorithm to solve the problem efficiently. Considering many berths and multiple QCs, Cahyono et al. (2015) proposed a dynamical modeling framework based on discrete-event systems to formulate the loading and unloading process. Giallombardo et al. (2010) introduced the concept of QC profiles in the TBAP to give the decisions for scheduling arriving vessels and the quay crane assignment. Following Giallombardo et al. (2010), with tidal influence and channel restrictions considered in the operational level, Zhen et al. (2017) addressed a joint BAP and QCAP where the concept of QC-profile is introduced to assign QCs for the vessel in each time period during the handling operation. Wang et al. (2018) develop a CG-based heuristic for the integration of BAP, QCAP, and yard assignment problems. Bouzekri et al. (2021) proposed and solved an integrated port management problem, including the dynamic continuous BAP, the laycan allocation problem and the QCAP with invariant time.

Recently, some scholars investigated the BAP with multiple ports (i.e., the MPBAP). Peng et al. (2015) proposed an integration of the BAP with the yard assignment problem in multiple ports, in order to minimize the handling time of vessels and equalize the assignment of yard storage. To make a more efficient and equitable berth allocation plan, Wang et al. (2015) investigated the BAP with multiple ports along the shipping service route, and proposed two collaborative mechanisms, where compensation or additional fees would be added to port operators in one mechanism and the utilities for the transshipment containers are considered in another mechanism. Venturini et al. (2017) proposed the MPBAP with vessel speed optimization problem (SOP) when sailing between ports along a given shipping service route, and a multiple depot vehicle routing problem with time window model was developed to select the optimal speed on each sailing leg and determine port arrival times. Similar to Venturini et al. (2017), Martin-Iradi et al. (2022) investigated the MPBAP with vessel speed optimization to explore the potential of the collaboration between liner shipping companies and terminal operators.

Similar to our MPBAP, Imai et al. (2008a, 2008b) and Dulebenets et al (2018) addressed the BAP with a multi-user container terminal, where vessels can be diverted from a multi-user container terminal to an external container terminal. However, only two container terminals (a multi-user container terminal and an external container terminal) are considered in these studies, and hence the container terminal (or cooperator) choice is not an issue. By considering the closeness restrictions of container groups between the blocks and the shoreline, Heilig et al. (2019) explored the impact of locations of quay and yard areas on transshipment operations in a multi-terminal port. Krimi et al. (2019, 2020) investigated the multi-quay berth allocation and crane assignment problem, and an efficient rolling horizon-based heuristic and a set of heuristics based on general variable neighborhood search were presented to solve proposed mixed-integer programming model. Zhen et al. (2016) proposed a terminal allocation problem (TAP) to investigate how to allocate the incoming vessels of a shipping alliance to the terminals in transshipment hubs. Although considering multiple quays or multiple terminals in Heilig et al. (2019), Krimi et al. (2019, 2020) and Zhen et al. (2016), the cooperation among multiple quays or multiple terminals for diverting vessels is not studied.

Different from the above studies, we investigate the PCSP, together with the MPBAP integrating the DPCP for diverting vessels. Ports are grouped into different port groups, and the PCSP is evaluated for different port groups by using cooperative game theory.

3 Problem definition and formulation

Here, we provide the notation and assumptions in subsections 3.1 and 3.2, respectively. The problem definition for the MPBAP is presented in subsection 3.3. Subsection 3.4 shows the formulation for the MPBAP with the PCSP.

3.1 Notation

Before presenting the formulations of our MPBAP, the main notations used in the MPBAP are listed as follows.

Sets	
Ω	Set of ports within a port group, $\Omega=\{1,...,i,...\}$;
B_i	Set of berths belonging to port i, $B_i=\{1,...,r,...\}$;
V	Set of arriving vessels, $V=\{1,...,k,...\}$;
P_k	Set of quay crane (QC) profiles of vessel k, $P_k=\{1,...,p,...\}$;
U	Set of service sequence for vessels in a berth, $U=\{1,...,u,...\}$.
Parameters	
w_{km}	Number of transshipment containers that are unloaded from vessel k and loaded to vessel m;
ϕ_{km}	A binary parameter which takes value 1 if there are transshipment containers transshipped from vessel k to vessel m, and 0 otherwise;
a_{ki}	Arrival time of vessel k at port i as planned;
n_k	Port number where vessel k plans to arrive at;
e_k	Estimated departure time of vessel k;
q_{kp}	Number of QCs allocated to vessel k of QC profile p;
s_{kp}	Handling time of vessel k by using QC profile p;
h_k	Draught of vessel k;
H_{ir}	Water depth of berth r belonging to port i;
$\hat{H}$	Safety threshold for length;
W_k	Waiting time limit for vessel k;
l_k	Length of vessel k;
L_{ir}	Length of berth r belonging to port i;
$\hat{L}$	Safety threshold for draught;
Q_{ir}	Total number of QCs of berth r belonging to port i;

Δd_{kij}	Distance difference associated with vessel k when diverting from port i to port j;
$g_{jj'}$	The time for transferring containers between port j to port j';
c_i	The service cost per unit hour for loading/unloading one container at port i;
c_k^1	The penalty cost per unit hour for delay of vessel k;
c_k^2	The compensation cost per unit distance for diverting vessel k;
$c_{jj'}$	The compensation cost for transferring one container from port j to port j';
M	A big positive constant.
Decision variables	
y_{kir}^u	A binary variable which takes value 1 if the uth served vessel k docks at berth r of port i, and 0 otherwise;
x_{kp}	A binary variable which takes value 1 if vessel k is served by QC profile p, and 0 otherwise;
z_{kij}	A binary variable which takes value 1 if vessel k is diverted from port i to port j;
b_{kir}^u	Berthing time of the uth served vessel k docks at berth r of port i;
$\vartheta_k^+, \vartheta_k^-$	Non-negative continuous auxiliary variables for linearizing the penalty cost function;
$\theta_{kij}^+, \theta_{kij}^-$	Non-negative continuous auxiliary variables for linearizing the compensation cost function;
μ_{kpir}^u	A binary auxiliary variable for linearization, which takes value 1 if uth served vessel k uses QC profile p and docks at berth r of port i, and 0 otherwise;
$\pi_{kmijj'}$	A binary auxiliary variable for linearization, which takes value 1 if vessel k is diverted from port i to port j and vessel m is diverted from port i to port j', and 0 otherwise.

3.2 Assumptions

To simplify our studied problem, the following assumptions are considered:

(1) The number of QCs in service is fixed during the whole period of vessel handling operations;

(2) All vessels considered arrive at the port with a fixed service frequency (i.e. the weekly arrival pattern) for a long period;

(3) The arrival information of all vessels, i.e., vessel length, handling container volume and expected arrival time etc., is known to the port in advance;

(4) Uncertainties during the sailing voyage of the vessel are not considered;

(5) We do not consider the effect of distances between berthing locations and yard spaces on the vessel handling time;

(6) Two vessels associated with transshipment containers call at the same original port.

3.3 Descriptions on the MPBAP

Here, we first provide the problem definition for the BAP with a single port on vessel-berth relationship, QC profile and time windows. Then, the descriptions on vessel diversion process, container transfer and objective function of the MPBAP are presented.

3.3.1 The BAP with a single port

As mentioned before, we will enumerate all feasible port groups when investigating the MPBAP and the PCSP. A single port can be regarded as a special port group. Here we provide descriptions of the MPBAP with a single port, which can be referred to as the BAP with a single port.

We illustrate some basic constraints of the BAP with a single port. Our problem considers the whole process from a vessel arriving at the anchorage to the completion of loading and unloading. Two main subjects of vessels and berths are involved in the BAP with a single port. Figure 1 illustrates the issue in a two-dimensional space, where the vessels are represented by rectangles. The horizontal axis and the vertical axis indicate the planning horizon and the shoreline, respectively. The key of the BAP is to insert each rectangle into the two-dimensional space without overlap. The relevant constraints can be illustrated from three aspects: vessel-berth relationship, QC profile and time windows.

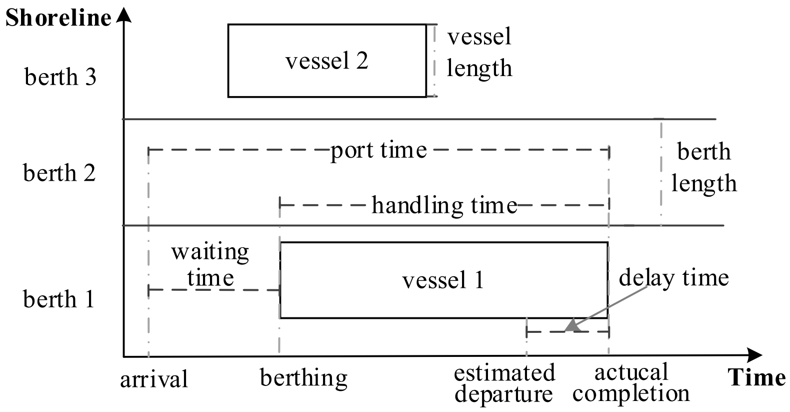

Figure 1. Two-dimensional space for the BAP with a single port.

a. Vessel-berth relationship

To make vessel k berthing at a feasible berth r, it is necessary to ensure safety thresholds in terms of both length and water depth, as shown in Figure 1. If $l_k+\hat{L} \le L_{ir}$ and $h_k+\hat{H} \le H_{ir}$, berth r can be regarded as an alternative berth for the vessel. Generally, any vessel must dock at one berth to complete the loading and unloading operations. In addition, for any berth, it is required that only one vessel can be arranged for berthing at the same time.

b. QC profile

The length of the rectangle in Figure 1 is the handling time of the vessel for loading/unloading containers, which depends on the adopted QC profile. The vessels operating by a liner shipping company usually arrive at ports with a fixed number of containers to be handled in a season (Fagerholt et al., 2015). The liner shipping company will inform the port operator about the expected turnover time as well as the number of containers to be handled for their incoming vessels. Then, the port operator assigns QC profiles for serving vessels. A complete QC profile consists of the number of QCs allocated and the corresponding handling time (Giallombardo et al., 2010; Vacca et al., 2013). The number of QCs assigned for the vessel cannot exceed the number of available QCs at the berth.

Berthing position does have an impact on the transit time of a container from the yard to the berth. In order to reduce waiting times of QCs and vessels at the port, more trucks will be assigned if the berthing position is far away from the yard. Hence, we do not consider the QC profile or handling time dependent on the berthing position, following Imai et al. (2008), Zhen et al. (2017) and Wang et al. (2018).

c. Time windows

For vessel k, the expected arrival time when a vessel arrives at the port is usually sent to the port operator in advance. As shown in Figure 1, vessel should be parked in anchorage waiting for the arranged berth. So the berthing time must be later than the arrival time, namely $\sum_{i\in\Omega}\sum_{r\in \mathrm{B}_i}\sum_{u\in \mathrm{U}} b_{kir}^{u} \geq a_{ki}$. The time spent when any vessel berths for loading and unloading operation is the handling time, which depends on the QC profile, namely, $\sum_{p\in \mathrm{P}_k} s_{kp}x_{kp}$. The time for the vessel to complete loading and unloading operations is actual completion time, expressed as $b_{kir}^{u}+\sum_{p\in \mathrm{P}_k} s_{kp}x_{kp}$. Vessel delays occur if the actual completion time is later than the estimated departure time. Besides, to avoid time overlap between two vessels berthing at the same berth r consecutively, a time-bound constraint is proposed. According to the principle of first-come-first-served (FCFS), it is required that the subsequent vessel can berth only after the handling operation of the previous vessel is completed, namely, $b_{kir}^{u}+\sum_{p\in \mathrm{P}_k} s_{kp}x_{kp}y_{kir}^{u} \leq b_{mir}^{u+1} + M\left(1-y_{mir}^{u+1}\right)$.

3.3.2 MPBAP

Based on the BAP with a single port, we propose the MPBAP for any port group. According to Imai et al (2008a, 2008b) and Kavirathna et al. (2020), we consider the cooperation between neighboring ports within the same port group by means of diverting vessels. All ports in the port group have reached an agreement that any vessel with excessive waiting time could be diverted to a partner port for loading or unloading to reduce vessel waiting time. A key issue is to determine the suitable diversion port for each vessel to be diverted, which is called DPCP as mentioned before. The decision of vessel diversion is difficult since it depends on the expected waiting time in the pre-assigned port and berth usages upon vessel arrival in other ports. Next, we introduce the vessel diversion and the resulting container transfer processes as follows.

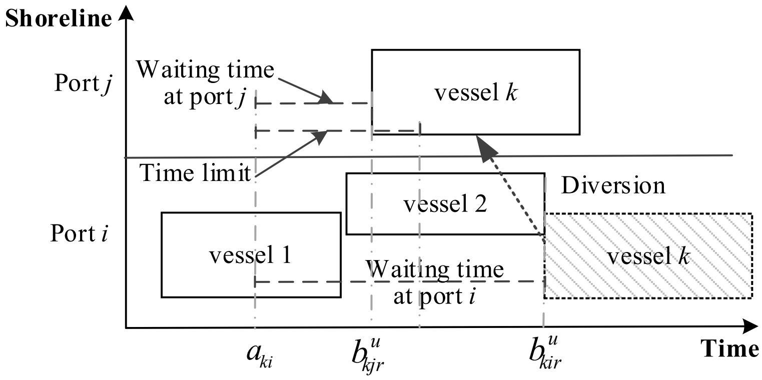

Figure 2. An illustration of vessel diversion process.

a. Vessel diversion process

Figure 2 shows a small example where vessel k is diverted from port i to port j. All berths of port i are occupied and multiple vessels waiting in line when vessel k arrives at this port, and the waiting time of this vessel is longer than that as expected. In this case, we consider two strategies. For the first strategy, vessel k keeps waiting in anchorage until a berth is available, and excessive waiting time means huge compensation costs paid by port operator. For the second strategy, vessel k can be diverted to a cooperative port j that should be properly selected, and such a port is called as the new destination port for vessel diversion. For the second strategy, we consider that vessel can be diverted to the new destination port directly without arriving at the original port. Compared with calling at the original port i, we introduce the distance difference associated with vessel k when diverting to port j, denoted by Δd_{kij}. A positive Δd_{kij} indicates that the vessel needs to travel an additional distance for the diversion, while a negative value means that the vessel diversion will shortens the distance travelled. To meet the vessel schedule, we consider that any diverted vessel arrives at the new destination port at the expected arrival time arriving at the original port. Namely, we consider a small adjustment of vessel speeds for vessels diverted between neighboring ports. To avoid invalid vessel diversion, we consider that the vessel waiting time in the new destination port for diverting vessels should not exceed the maximum time limit. Namely, we have $\sum_{r \in \mathrm{B}_j} \sum_{u \in \mathrm{U}} b_{kjr}^{u} - a_{ki} \le W_k + \left(1 - z_{kij}\right) M, \forall k \in \mathrm{V}, \forall i \in \Omega, \forall j \in \Omega, i \ne j$.

b. Container transfer

The vessel diversion also results in the container transfer problem. For the containers handled at the ports, we consider three types of containers: loading containers, unloading containers and transshipment containers, as illustrated in Figure 3. For each vessel, the numbers of containers with different types are given in advance. When vessel diversion occurs, the loading containers should be transported to the new destination port that the vessel is diverted to. Since the vessel diversion decision is made in the berth allocation plan, the loading containers can be transported directly from the hinterland to the new destination port without considering the transfer between two ports. Similarly, the unloading containers can be transported directly from the new destination port to the hinterland, so we do not consider the transfer of the unloading containers.

For the transshipment containers that are unloaded from vessel k and then loaded to vessel m, as mentioned before, the original (pre-assigned) ports called by these two vessels are the same, e.g., port i. When the vessel diversion occurs, the transshipment containers may have to be transferred. To proceed, we consider that vessels k and m associated with transshipment containers are diverted from port i to ports j and j', respectively. Note that, the transfer of transshipment containers can only start after vessel k has finished unloading operations, and vessel m cannot start loading operations until the transshipment containers have been transferred to the new destination port of vessel m (i.e., port j'). Thus, we get $\sum_{\gamma \in \mathrm{B}_{j'}} \sum_{\mu \in \mathrm{U}} b_{mj'\gamma}^{\mu} + M\left(1 - \phi_{km} z_{kij} z_{mij'}\right) \ge \sum_{r \in \mathrm{B}_j} \sum_{u \in \mathrm{U}} \left(b_{kjr}^{u} + \sum_{p \in \mathrm{P}_k} s_{kp} x_{kp} \right) + g_{jj'}$, where $\sum_{\gamma \in \mathrm{B}_{j'}} \sum_{\mu \in \mathrm{U}} b_{mj'\gamma}^{\mu}$

denotes the berthing time of vessel m, $\sum_{r\in B_j}\sum_{u\in U}\left(b_{kjr}^{u}+\sum_{p\in P_k}s_{kp}x_{kp}\right)$ denotes the completion time of vessel k, $z_{kij}z_{mij'}$ denotes whether two vessels are diverted to port j and port j', the binary parameter ϕ_{km} indicates the transshipment relationship between these two vessels, and $g_{jj'}$ is the time for transferring containers between two ports.

For two vessels associated with transshipment containers, depending on different loading and unloading ports of the transshipment containers, we consider the following four cases: (i) the vessel diversion does not occur, i.e., $i=j=j'$; (ii) the diversion of one vessel occurs, i.e., $i=j\neq j'$ (or $i=j'\neq j$), where the transshipment containers should be transferred from port i to port j' (or transferred from port j to port i); (iii) the diversion of two vessels occurs and these two vessels are diverted to two different ports, i.e., $i\neq j\neq j'$, where the transshipment containers should be transferred from port j to port j'; (iv) the diversion of two vessels occurs and these two vessels are diverted to the same port, i.e., $j=j'\neq i$, where the transfer of transshipment containers is not required.

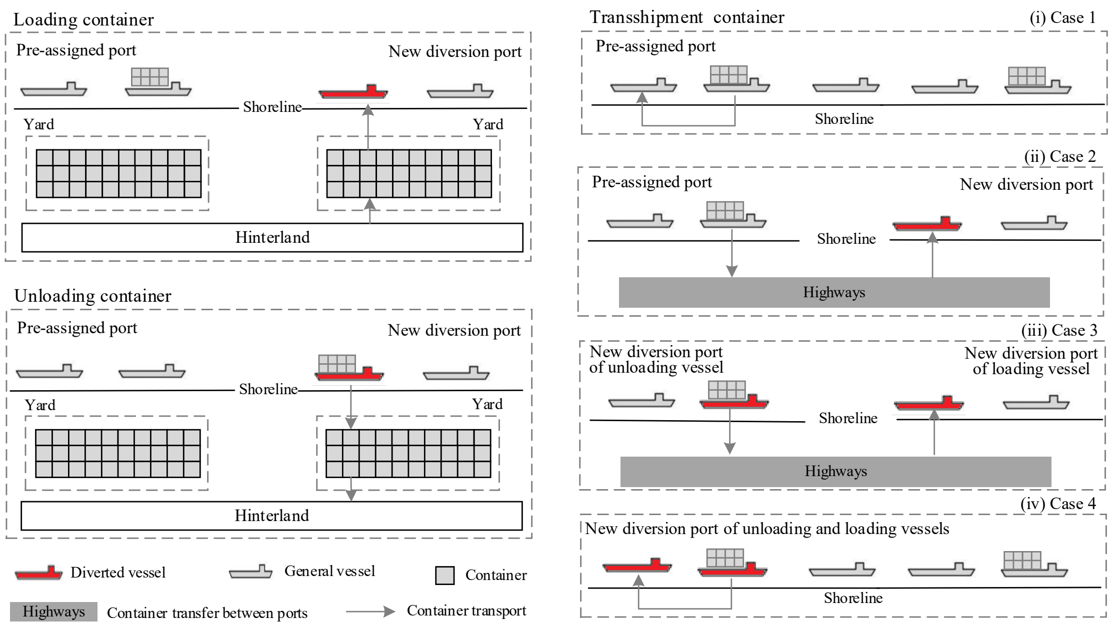

Figure 3. An illustration of container transfer due to vessel diverting.

c. Objective function of the MPBAP

Generally, the berth allocation (BA) plan is developed by the port operator who expects to satisfy the loading and unloading containers of all vessels, in order to minimize the total cost. As mentioned before, ports are grouped into various port groups, and we consider that vessels provide a weekly service frequency. Thus, the objective of the MPBAP for any particular port group is to minimize the weekly expense of the port group, which consists of the following three parts:

(i) Service cost for loading and unloading. It is assumed that all service costs come from the depletion of QCs. Based on QCs allocated to the vessel and its handling time, the service cost of vessel k is expressed as $\sum_{i\in\Omega}\sum_{p\in P_k}\sum_{r\in B_i}\sum_{u\in U} c_i s_{kp} q_{kp} x_{kp} y_{kir}^u$.

(ii) Penalty for the delay of vessels' departure. Based on the actual completion time $\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp} x_{kp}$, the delay of vessel k can be calculated by $\left[\left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp} x_{kp}\right) - e_k\right]^+$ where the operator $(\bullet)^+$ means: if $x \geq 0$, $(x)^+ = x$; otherwise, $(x)^+ = 0$. Then, we can obtain the penalty cost for vessel delay, namely, $c_k^1\left[\left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp} x_{kp}\right) - e_k\right]^+$.

(iii) Compensation for vessel diversion and container transfer. The port operator needs to pay compensation for the vessel to subsidize the additional fuel consumption cost if the vessel speeds up to arrive at the new destination port on time during the vessel diversion process. Based on the distance difference between the original port and the new destination port, the additional fuel consumption cost for vessel k can be expressed as $c_k^2\left|\Delta d_{kij} z_{kij}\right|^+$. In addition, transshipment containers may need to be transferred due to the vessel diversion, and the compensation cost for container transfer is expressed as $c_{jj'} w_{km} z_{kij} z_{mij'}$.

3.4 Formulation of the MPBAP with the PCSP

This paper investigates the MPBAP with the PCSP. As shown in Figure 4, we propose a two-stage framework for solving the MPBAP with the PCSP. In stage I, for each possible port group, a MIP model is constructed to formulate the MPBAP (see Subsection 3.4.1), and the MIP model is solved by a CG algorithm (see Section 4). Based on the costs of BA plan for all port groups obtained in stage I, stage II solves the PCSP based on cooperative game theory (see Subsection 3.4.2). Namely, cooperative game theory is used to determine revenue allocation and assess the stabilities of various port groups, and then a decision model is proposed to determine the optimal port groups selected from the stable ones.

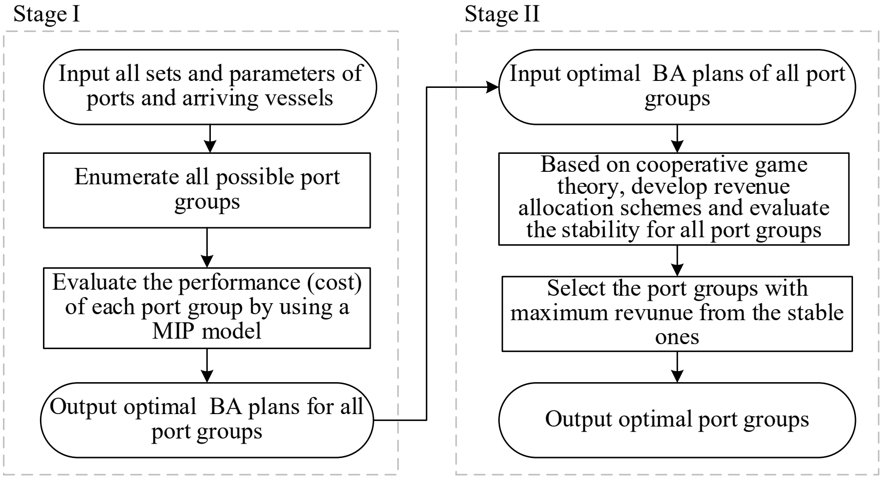

Figure 4. Two-stage framework.

3.4.1 Formulation of the MPBAP

a. The MIP model for MPBAP

With the objective of minimizing the total cost (service cost, vessel delay penalty, and compensation cost for diverting vessels and containers), the MPBAP can be formulated as the following MIP model:

$$\min \sum_{k\in V}\sum_{p\in P_k}\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} c_i s_{kp} q_{kp} x_{kp} y_{kir}^u + \sum_{k\in V} c_k^1 \left[\left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp} x_{kp} \right) - e_k \right]^+ \\ + \sum_{k\in V}\sum_{i\in\Omega}\sum_{j\in\Omega} c_k^2 \left| \Delta d_{kij} z_{kij} \right|^+ + \sum_{k\in V}\sum_{m\in V}\sum_{i\in\Omega}\sum_{j\in\Omega}\sum_{j'\in\Omega} c_{jj'} w_{km} z_{kij} z_{mij'} \tag{1}$$

subject to

$$\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} y_{kir}^u = 1, \forall k \in V; \tag{2}$$

$$\sum_{k\in V} y_{kir}^u \le 1, \forall r \in B_i, \forall i \in \Omega, u \in U; \tag{3}$$

$$\left(h_k + \hat{H}\right) \sum_{u\in U} y_{kir}^u \le H_{ir}, \forall k \in V, \forall r \in B_i, \forall i \in \Omega; \tag{4}$$

$$\left(l_k + \hat{L}\right) \sum_{u\in U} y_{kir}^u \le L_{ir}, \forall k \in V, \forall r \in B_i, \forall i \in \Omega; \tag{5}$$

$$\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u \ge a_{ki}, \forall k \in V; \tag{6}$$

$$b_{kir}^u + \sum_{p\in P_k} s_{kp} x_{kp} y_{kir}^u \le b_{mir}^{u+1} + M\left(1 - y_{mir}^{u+1}\right), k \ne m, \forall k, m \in V, \forall r \in B_i, \forall i \in \Omega, \forall u \in U; \tag{7}$$

$$b_{kir}^u \le M y_{kir}^u, \forall k \in V, \forall r \in B_i, \forall i \in \Omega, \forall u \in U; \tag{8}$$

$$\sum_{p\in P_k} x_{kp} = 1, \forall k \in V; \tag{9}$$

$$\sum_{p\in \mathrm{P}_k} x_{kp}q_{kp}\cdot\sum_{u\in \mathrm{U}} y^u_{kir} \le Q_{ir}, \forall k\in \mathrm{V}, \forall r\in \mathrm{B}_i, \forall i\in\Omega; \tag{10}$$

$$\sum_{r\in \mathrm{B}_j}\sum_{u\in \mathrm{U}} b^u_{kjr} - a_{ki} \le W_k + \left(1-z_{kij}\right)M, \forall k\in \mathrm{V}, i\ne j, \forall i,j\in\Omega; \tag{11}$$

$$\sum_{\gamma\in \mathrm{B}_{j'}}\sum_{\mu\in \mathrm{U}} b^{\mu}_{mj'\gamma} + M\left(1-\phi_{km}z_{kij}z_{mij'}\right) \ge \sum_{r\in \mathrm{B}_j}\sum_{u\in \mathrm{U}}\left(b^u_{kjr} + \sum_{p\in \mathrm{P}_k} s_{kp}x_{kp}\right) + g_{jj'} \tag{12}$$

$$, k\ne m, \forall k,m\in \mathrm{V}, \forall i,j,j'\in\Omega;$$

$$z_{kij} = \begin{cases} 0, \ if\, i\ne n_k \\ \sum_{r\in \mathrm{B}_i}\sum_{u\in \mathrm{U}} y^u_{kjr}, else \end{cases}, \forall k\in \mathrm{V}, \forall i,j\in\Omega; \tag{13}$$

$$y^u_{kir} \in \{0,1\}, \forall k\in \mathrm{V}, \forall r\in \mathrm{B}_i, \forall i\in\Omega, \forall u\in \mathrm{U}; \tag{14}$$

$$x_{kp} \in \{0,1\}, \forall k\in \mathrm{V}, \forall p\in \mathrm{P}_k; \tag{15}$$

$$z_{kij} \in \{0,1\}, \forall k\in \mathrm{V}, \forall i,j\in\Omega; \tag{16}$$

$$b^u_{kir} \ge 0, \forall k\in \mathrm{V}, \forall r\in \mathrm{B}_i, \forall i\in\Omega, \forall u\in \mathrm{U}. \tag{17}$$

The objective function (1) minimizes the total weekly costs containing three parts: the first part is the service cost, the second part is the penalty costs for vessel delay, and the last part is the compensation costs for the vessel diversion and container transfer. Constraints (2) indicate that each vessel only has one chance to berth. Constraints (3) ensure that only one vessel berths at the same berth at the same time. Constraints (4) and (5) ensure that the safety threshold is satisfied between the vessel and the berth. Constraints (6) show that the vessel cannot be served until it arrives at the port. The berthing time of the vessel is required to be greater than the arrival time. Constraints (7) ensure that more than one vessel cannot be berthed at the same berth at the same time. Constraints (8) define the relationship between berthing time and the berth location. Constraints (9) stipulate that only one QC profile is selected for each vessel. Constraints (10) guarantee that the number of QCs used cannot exceed the total number of QCs in the berth where the vessel docks at. Constraints (11) require the waiting time at the new destination port of the vessel diverted cannot exceed the time limit. Constraints (12) show the relationship between berthing time and completion time for two vessels associated with transshipment containers. Constraints (13) describe the relationship between the vessel's diversion port and the berth where the vessel docks at. Finally, Constraints (14)-(17) define the domain of decision variables.

As the four parts in the objective function and constraints (7), (10) and (12) are nonlinear, here we introduce some sets of auxiliary variables to linearize the proposed model, and the complete formulation of the linearized model is presented in the appendix.

First, we provide the linearization for the objective function. For the penalty cost function $\sum_{k\in V} c_k^1 \left[\left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp}x_{kp} \right) - e_k \right]^+$, there is a nonlinear form '$(\bullet)^+$' in the function. We define nonnegative variables ϑ_k^+ and ϑ_k^-, and then add a constraint $\left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp}x_{kp} \right) - e_k = \vartheta_k^+ - \vartheta_k^-, \forall k \in V$. Then the nonlinear function in the objective is replaced by ϑ_k^+. The objective of the above model becomes linear, i.e., $\min \sum_{k\in V} c_k^1 \vartheta_k^+$. Here ϑ_k^+ and ϑ_k^- can be defined as two non-negative continuous variables instead of integers. As in the objective, ϑ_k^+ should be as less as possible. If the value of the function $\left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp}x_{kp} \right) - e_k$ is negative, $\vartheta_k^+=0$, $\vartheta_k^+ = \left(\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} b_{kir}^u + \sum_{p\in P_k} s_{kp}x_{kp} \right) - e_k$ otherwise.

Similarly, for the compensation cost function for vessel diversion $\sum_{k\in V}\sum_{i\in\Omega}\sum_{j\in\Omega} c_k^2 \left| \Delta d_{kij} z_{kij} \right|^+$, we linearize the nonlinear form '$(\bullet)^+$' by introducing two non-negative continuous variables θ_{kij}^+ and θ_{kij}^-. If the value of the function $\Delta d_{kij} z_{kij}$ is negative, $\theta_{kij}^+=0$, and $\theta_{kij}^+=\Delta d_{kij} z_{kij}$ otherwise. Then, we add a new constraint $\left| \Delta d_{kij} z_{kij} \right|^+ = \theta_{kij}^+ - \theta_{kij}^-, \forall k \in V, \forall i \in \Omega, \forall j \in \Omega$ and get a linear function $\sum_{k\in V}\sum_{i\in\Omega}\sum_{j\in\Omega} c_k^2 \theta_{kij}^+$.

To linearize the nonlinear form $x_{kp} y_{kir}^u$ in the service cost function $\sum_{k\in V}\sum_{p\in P_k}\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} c_i s_{kp} q_{kp} x_{kp} y_{kir}^u$ and Constraints (7) and (10), we define a binary auxiliary variable $\mu_{kpir}^u = x_{kp} y_{kir}^u$. Then, the cost function, Constraints (7) and (10) can be linearized as $\sum_{k\in V}\sum_{p\in P_k}\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} c_i s_{kp} q_{kp} \mu_{kpir}^u$, $b_{kir}^u + \sum_{p\in P_k} s_{kp} \mu_{kpir}^u \le b_{mir}^{u+1} + M\left(1 - y_{mir}^{u+1}\right)$, and $\sum_{p\in P_k}\sum_{u\in U} \mu_{kpir}^u q_{kp} \le Q_{ir}$. μ_{kpir}^u takes value 1 if the u th served vessel k uses QC profile p and docks at berth r of port i, and 0 otherwise.

Similarly, we define a binary auxiliary variable $\pi_{kmijj'} = z_{kij} z_{mij'}$ to linearize the compensation cost function for transferring containers $\sum_{k\in V}\sum_{m\in V}\sum_{i\in\Omega}\sum_{j\in\Omega}\sum_{j'\in\Omega} c_{jj'} w_{km} z_{kij} z_{mij'}$

and Constraints (12). $\pi_{kmijj'}$ takes value 1 if vessel k is diverted from port i to port j and vessel m is diverted from port i to port j', and 0 otherwise. Then, the cost function and Constraints (12) would be linearized as $\sum_{k\in V}\sum_{m\in V}\sum_{i\in\Omega}\sum_{j\in\Omega}\sum_{j'\in\Omega} c_{jj'} w_{km}\pi_{kmijj'}$ and $\sum_{\gamma\in B_{j'}}\sum_{\mu\in U} b^{\mu}_{mj'\gamma} + \left(1-\phi_{km}\pi_{kmijj'}\right)M \geq \sum_{r\in B_j}\sum_{u\in U}\left(b^{u}_{kjr} + \sum_{p\in P_k} s_{kp}x_{kp}\right) + g_{jj'}$.

b. A set-partitioning model for MPBAP

The above MIP model has a large number of binary variables and big-M constraints. It is difficult to solve the MIP model by commercial linear programming solvers (e.g., CPLEX), even if for a small test case (Zhen et al., 2017). To deal with the computational difficulty, the MPBAP is reformulated as a set-partitioning model based on a set of vessel-berth plans for reducing the number of decision variables. The vessel-berth plan is regarded as a combination of a vessel and a berth. Let D_k be a set of all feasible vessel-berth plans for vessel k, including the assigned berth position, the berthing time, the allocated QCs, handling time and diversion plan. For a given vessel-berth plan d of vessel k at berth r of port i, let C_d denote the cost which can be expressed as $\min \sum_{k\in V}\sum_{p\in P_k}\sum_{i\in\Omega}\sum_{r\in B_i}\sum_{u\in U} c_i s_{kp} q_{kp} \mu^{u}_{kpir} + \sum_{k\in V} c^1_k \vartheta^+_k + \sum_{k\in V}\sum_{i\in\Omega}\sum_{j\in\Omega} c^2_k \theta^+_{kij} + \sum_{k\in V}\sum_{m\in V}\sum_{i\in\Omega}\sum_{j\in\Omega}\sum_{j'\in\Omega} c_{jj'} w_{km}\pi_{kmijj'}$, where the compensation for vessel diversion is fixed since the diversion plan (z_{kij}) can be calculated based on Constraints (13). Let $T=\{1,...t,...\}$ indicate the planning period. Define binary variable χ_d that $\chi_d=1$ if vessel-berth plan d ($d\in D_k$) is selected, otherwise $\chi_d=0$. Define binary parameter ω^{ir}_{dt} that $\omega^{ir}_{dt}=1$ if vessel-berth plan d ($d\in D_k$) occupies berth r of port i at time period t. Let q'^{ir}_{dt} denote the number of QCs used by vessel-berth plan d at time period t. Let b'^{ir}_{d} denote the berthing time in berth r of port i for vessel-berth plan d. Then, the master problem can be formulated as the following set-partitioning model.

$$\min \sum_{k\in V}\sum_{d\in D_k} C_d \chi_d \tag{18}$$

subject to

$$\sum_{d\in D_k} \chi_d = 1, \forall k \in V; \tag{19}$$

$$\sum_{k\in V}\sum_{d\in D_k}\omega_{dt}^{ir}\chi_d \le 1, \forall r\in B_i, \forall i\in\Omega, \forall t\in T; \quad (20)$$

$$\sum_{k\in V}\sum_{d\in D_k}q_{dt}'^{ir}\chi_d \le Q_{ir}, \forall r\in B_i, \forall i\in\Omega, \forall t\in T; \quad (21)$$

$$\sum_{d\in D_k}\left(\sum_{r\in B_j}b_d'^{jr}+\sum_{r\in B_j}\sum_{t\in T}\omega_{dt}^{jr}+g_{jj'}\right)\chi_d \le \sum_{v\in D_m}\sum_{r'\in B_{j'}}b_v'^{j'r'}\chi_v+M\left(1-\sum_{v\in D_m}\phi_{km}\chi_v\right) \quad (22)$$
$$,k\ne m, \forall k,m\in V, \forall j,j'\in\Omega;$$

$$\chi_d\in\{0,1\}, \forall d\in D_k, \forall k\in V. \quad (23)$$

Objective (18) aims at minimizing the total cost of all selected vessel-berth plans. The set-partitioning model emphasizes the relationship between vessel-berth plans and reformulations of several constraints in the proposed MIP model. Constraints (19) require that only one vessel-berth plan is selected for each vessel, which are equivalent to Constraints (2) in the MIP model. Constraints (20) state no vessel overlap between the selected vessel-berth plans, which are equivalent to Constraints (3) and (7) in the MIP model. Constraints (21) show that the number of the QCs allocated to all vessels at any time does not exceed the total number of available QCs in the berth, which are equivalent to Constraints (10) in the MIP model. Constraints (22) show the relationship between two vessels associated with transshipment containers, which are equivalent to Constraints (12) in the MIP model. Constraints (23) define χ_d as a binary variable.

3.4.2 Formulation of the PCSP

The PCSP aims to determine the optimal stable port groups based on cooperative game theory. Although considering the cooperation among ports within the same port group, each port aims to maximize her revenue. Hence we firstly determine revenue allocation for all port groups by using the Shapley value and assess the stability of these port groups based on core theory. Then, a binary programming model with the goal of maximizing the total revenue for all considered ports is proposed to select the optimal port groups from the stable ones. The sets, parameters and decision variables used in the PCSP are listed as follows.

Sets	
$\mathcal{L}$	Set of considered ports, and $\lvert\mathcal{L}\rvert$ is the number of considered ports;
$\mathcal{N}$	Set of stable port groups;
$\mathcal{N}(i)$	Set of stable port groups containing port i.
Parameters	

C_Ω	The weekly cost for port group Ω ($\forall\Omega \subseteq \mathcal{L}$);
$\lvert\Omega\rvert$	The number of ports in port group Ω;
w_Ω	Number of total containers to be handled for vessels served in port group Ω;
I	Price for handling one container;
R_Ω	The weekly revenue for port group Ω ($\forall\Omega \subseteq \mathcal{L}$).
Decision variables	
rev_i	The weekly revenue allocated to port i ($i \in \mathcal{L}$);
β_Ω	A binary variable which takes value 1 if port group Ω is selected as the optimal port group, and 0 otherwise.

The weekly revenue for any particular port group Ω ($\forall\Omega \subseteq \mathcal{L}$), can be calculated as follows:

$$R_\Omega = I \cdot w_\Omega - C_\Omega \tag{24}$$

where C_Ω denotes the weekly cost for port group Ω, i.e., the optimal objective value of the proposed MIP model for port group Ω. Based on the Shapley value (Shapley, 1953), we can determine revenue allocation, as follows:

$$\text{rev}_i = \sum_{\Omega \subseteq \mathcal{L}} \frac{(\lvert\Omega\rvert - 1)!(\lvert\mathcal{L}\rvert - \lvert\Omega\rvert)!}{\lvert\mathcal{L}\rvert!} \cdot \left(R_\Omega - R_{\Omega\backslash\{i\}}\right), \forall i \in \mathcal{L} \tag{25}$$

where $\left(R_\Omega - R_{\Omega\backslash\{i\}}\right)$ denotes the marginal contribution of port i in port group Ω.

Then, we compare the port's revenue allocated in the port group with the revenue of the port under non-cooperation situation and sub-coalition of ports within the port group to judge the satisfaction of the port and the stability of the port group. When all ports in the port group are happy with the allocation outcome, the port group is stable since there is no incentive to disrupt the cooperation.

The stability of each port group can be evaluated by using core theory. Namely, any port group Ω ($\forall\Omega \subseteq \mathcal{L}$) is stable if the following conditions are satisfied:

$$\sum_{i \in \Omega} \text{rev}_i = R_\Omega; \tag{26}$$

$$\sum_{i \in \Omega_1} \text{rev}_i \ge R_{\Omega_1}, \forall \Omega_1 \subset \Omega. \tag{27}$$

Stable port groups are stored in set $\mathcal{N}$, and then the PCSP can be formulated as the following binary programming model:

$$\max \sum_{\Omega \in \mathcal{N}} R_\Omega \beta_\Omega \tag{28}$$

subject to

$$\sum_{\Omega \subseteq \mathcal{N}(i)} \beta_\Omega = 1, \forall i \in \mathcal{L}; \tag{29}$$

$$\beta_\Omega \in \{0,1\}, \forall \Omega \subseteq \mathcal{N}. \tag{30}$$

Constraints (29) ensure that only one port group containing port i can be chosen as the optimal port group. Constraints (30) denote β_Ω as a binary variable. This binary programming model can be efficiently solved by using the enumeration method.

4 A column generation algorithm

It is still difficult to solve the set-partitioning model for large-scale instances, since it is intractable to enumerate all the vessel-berth plans within the acceptable time. Following Zhen et al. (2017) and Wang et al. (2018), we propose a CG (column generation) algorithm. In the CG algorithm, the original problem is decomposed into a restricted master problem (RMP) and a pricing sub-problem (PSP), where the RMP is to select vessel-berth plans with minimum total costs and the PSP finds feasible profitable vessel-berth plans for each vessel. The CG algorithm can achieve satisfactory computational efficiency since only a small subset of vessel-berth plans is generated and added to the RMP in each iteration.

Specifically, we consider the formulation of the RMP by relaxing the binary requirement of decision variables. In the RMP, we determine which vessel-berth plans are selected with the assurances that no vessel overlap between the selected vessel-berth plans and the limit that the number of QCs allocated to all vessels cannot exceed the total QC number. The PSP for each vessel is proposed to generate the optimal vessel-berth plan with the minimum negative reduced cost, under the constraints of vessel arrival time, vessel length, vessel draft, waiting time limit and QC profiles. Furthermore, we analyze the relationship between the decision variables of the PSP, and find that the PSP can be solved quickly by using the enumeration strategy.

4.1 The restricted master problem

Here, a restricted master problem (RMP) is proposed based on the set-partitioning model. Since it is difficult and not necessary to obtain the set of all feasible vessel-berth plans for any particular vessel k, we will generate a number of feasible vessel-berth plans by solving the PSP, and then the master problem is reduced to the RMP, where the binary variable (χ_d) is also relaxed as a continuous variable. The formulation of the RMP is as follows:

$$\min \sum_{k\in \mathrm{V}} \sum_{d\in \mathrm{D}_k} C_d \chi_d \tag{31}$$

subject to

(19)-(22);

$$0 \le \chi_d \le 1, \forall d \in \mathrm{D}_k, \forall k \in \mathrm{V}. \tag{32}$$

4.2 Pricing sub-problem

By solving the PSPs, we obtain a set of feasible profitable vessel-berth plans, which are added into the RMP. For a vessel-berth plan d with the fixed vessel k and the fixed berth r in port i, let Z denotes the reduced value of the objective function (i.e., reduced cost) if the vessel-berth plan is added to the RMP. Let δ_k, η_{irt}, τ_{irt}, $\psi_{kmjj'}$ be the dual variables associated with Constraints (19)-(22) in RMP, respectively. Based on the duality principle in linear programming (Zheng et al., 2017; Wang et al., 2018), we can evaluate the reduced cost of the PSP. Then, with the minimum reduced cost (i.e., $\min Z$) as the objective, the PSP can find the vessel-berth plan with the negative reduced cost, namely, the profitable vessel-berth plan. Moreover, according to the transshipment containers, vessels can be classified into three categories, i.e., the vessel without transshipment containers (ϕ_{km}=0), the vessel associated with unloading operations of transshipment containers (ϕ_{km}=1), and the vessel associated with loading operations of transshipment containers (ϕ_{km}=1). According to these three categories, we consider three kinds of the PSPs (denoted by PSP1, PSP2 and PSP3). The reduced costs for these three PSPs can be calculated as follows:

[PSP1] $Z=C_d-\delta_k-\sum_{t\in\mathrm{T}}\omega_{dt}^{ir}\eta_{irt}-\sum_{t\in\mathrm{T}}q_{dt}'^{ir}\tau_{irt}$;

[PSP2] $Z=C_d-\delta_k-\sum_{t\in\mathrm{T}}\omega_{dt}^{jr}\eta_{jrt}-\sum_{t\in\mathrm{T}}q_{dt}'^{jr}\tau_{jrt}-\left(b_d'^{jr}+\sum_{t\in\mathrm{T}}\omega_{dt}^{jr}+g_{jj'}\right)\psi_{kmjj'}$;

[PSP3] $Z=C_v-\delta_m-\sum_{t\in\mathrm{T}}\omega_{vt}^{j'r'}\eta_{j'r't}-\sum_{t\in\mathrm{T}}q_{vt}'^{j'r'}\tau_{j'r't}+\left(b_v'^{j'r'}-M\phi_{km}\right)\psi_{kmjj'}$.

For PSP1 with vessel-berth plan d with the fixed vessel k and the fixed berth r in port i, we get $\sum_{u\in\mathrm{U}}y_{kir}^{u}$=1 and z_{kij}=1 if i=j, and z_{kij}=0 otherwise, where port j denotes the original port of this vessel. The compensation cost function for vessel diversion $c_k^2\theta_{kij}^{+}$ (i.e., $c_k^2\left|\Delta d_{kij}z_{kij}\right|^{+}$) is fixed as the variable z_{kij} is given; the compensation cost for container transfer (i.e., $c_{jj'}w_{km}\pi_{kmijj'}$) equals 0 due to without transshipment containers. All parameters and variables associated with vessel-berth plan d, such as ω_{dt}^{ir}, $q_{dt}'^{ir}$ and $b_d'^{ir}$ become decision variables in the PSP1. In addition, variables x_{kp}, b_{kir}^{u}, μ_{kpir}^{u}, ϑ_k^{+} and ϑ_k^{-} are determined in the PSP1. For each PSP1, we only consider the constraints on arrival time, vessel length, vessel draft, waiting time limit and QC profiles. Without transshipment containers, for vessel-berth plan d with the fixed vessel k and fixed berth r in port i, the PSP1 can be formulated as follows:

[PSP1] $$\min c_i \sum_{p\in \mathrm{P}_k}\sum_{u\in \mathrm{U}} s_{kp} q_{kp} \mu_{kpir}^{u} + c_k^1 \vartheta_k^+ - \delta_k - \sum_{t\in \mathrm{T}} \omega_{dt}^{ir} \eta_{irt} - \sum_{t\in \mathrm{T}} q_{dt}'^{ir} \tau_{irt} \quad (33)$$

subject to

$$\sum_{u\in \mathrm{U}} b_{kir}^{u} \ge a_{kj}; \quad (34)$$

$$b_{kir}^{u} \le M y_{kir}^{u}, \forall u \in \mathrm{U}; \quad (35)$$

$$b_d'^{ir} = \sum_{u\in \mathrm{U}} b_{kir}^{u}; \quad (36)$$

$$\sum_{u\in \mathrm{U}} b_{kir}^{u} - a_{kj} \le W_k + \left(1 - z_{kij}\right) M, i \ne j; \quad (37)$$

$$\left(\sum_{u\in \mathrm{U}} b_{kir}^{u} + \sum_{p\in \mathrm{P}_k} s_{kp} x_{kp}\right) - e_k = \vartheta_k^+ - \vartheta_k^-; \quad (38)$$

$$\sum_{p\in \mathrm{P}_k} x_{kp} = 1; \quad (39)$$

$$\sum_{p\in \mathrm{P}_k} x_{kp} q_{kp} \le Q_{ir}; \quad (40)$$

$$q_{dt}'^{ir} = \omega_{dt}^{ir} \sum_{p\in \mathrm{P}_k} x_{kp} q_{kp}, \forall t \in \mathrm{T}; \quad (41)$$

$$\sum_{t\in \mathrm{T}} \omega_{dt}^{ir} = \sum_{p\in \mathrm{P}_k} x_{kp} s_{kp}; \quad (42)$$

$$\left(1 - \omega_{dt}^{ir}\right) M \ge \sum_{u\in \mathrm{U}} b_{kir}^{u} - t, \forall t \in \mathrm{T}; \quad (43)$$

$$\left(1 - \omega_{dt}^{ir}\right) M \ge t - \left(\sum_{u\in \mathrm{U}} b_{kir}^{u} + \sum_{p\in \mathrm{P}_k} x_{kp} s_{kp}\right), \forall t \in \mathrm{T}; \quad (44)$$

$$\mu_{kpir}^{u} \le y_{kir}^{u}, \forall p \in \mathrm{P}_k, \forall u \in \mathrm{U}; \quad (45)$$

$$\mu_{kpir}^{u} \le x_{kp}, \forall p \in \mathrm{P}_k, \forall u \in \mathrm{U}; \quad (46)$$

$$y_{kir}^{u} + x_{kp} - 1 \le \mu_{kpir}^{u}, \forall p \in \mathrm{P}_k, \forall u \in \mathrm{U}; \quad (47)$$

$$x_{kp} \in \{0,1\}, \forall p \in \mathrm{P}_k; \quad (48)$$

$$b_{kir}^{u} \ge 0, \forall u \in \mathrm{U}; \quad (49)$$

$$\omega_{dt}^{ir} \in \{0,1\}, \forall t \in \mathrm{T}; \quad (50)$$

$$q_{dt}'^{ir} \in Z^+, \forall t \in \mathrm{T}; \quad (51)$$

$$b_d'^{ir} \ge 0, \forall t \in \mathrm{T}; \quad (52)$$

$$\mu_{kpir}^{u} \in \{0,1\}, \forall p \in \mathrm{P}_k, \forall u \in \mathrm{U}; \quad (53)$$

$$\vartheta_k^+, \vartheta_k^- \ge 0. \quad (54)$$

The objective function (33) minimizes the total reduced cost of the vessel-berth plan. Constraints (34) show the vessel cannot be served until it arrives at the port.

Constraints (35) define the relationship between berthing time and the berth location. Constraints (36) calculate the variable $b_d'^{ir}$. Constraints (37) ensure that the waiting time of the vessel does not exceed the time limit if the vessel is diverted to the new destination port. Constraints (38) calculate the delay time of the vessel. Constraints (39) ensure that only one QC profile is selected. Constraints (40) show the restrictions on the QC profile. Constraints (41) give the QCs allocated for the vessel in each time step. Constraints (42)-(44) show the relationship between the handling process and the berth occupancy. Constraints (45)-(47) show the relationship between x_{kp}, y_{kir}^u and the auxiliary variables μ_{kpir}^u. Finally, Constraints (48)-(54) define the domain of decision variables.

As compared with the PSP1, the PSP2 and the PSP3 have identical constraints and decision variables. However, the objective functions of these three PSPs are different. For the PSP2 and the PSP3, we can obtain the values of variables $\pi_{kmijj'}$ and dual variables $\psi_{kmjj'}$ that are related to vessels k and m associated with the loading and unloading operations at the new destination ports j and $j'\left(\forall j, j' \in \Omega\right)$, respectively. For vessel-berth plan associated with vessels k and m diverted to ports j and $j'\left(\forall j, j' \in \Omega\right)$, the objective function of the PSP2 associated with vessel k and the objective function of the PSP3 associated with vessel m, can be respectively given as follows:

[PSP2]

$$\begin{aligned} \min c_j \sum_{p \in \mathrm{P}_k} \sum_{u \in \mathrm{U}} s_{kp} q_{kp} \mu_{kpjr}^u + c_k^1 \vartheta_k^+ + c_{jj'} w_{km} \pi_{kmijj'} \\ -\delta_k - \sum_{t \in \mathrm{T}} \omega_{dt}^{jr} \eta_{jrt} - \sum_{t \in \mathrm{T}} q_{dt}'^{jr} \tau_{jrt} - \left(b_d'^{jr} + \sum_{t \in \mathrm{T}} \omega_{dt}^{jr} + g_{jj'} \right) \psi_{kmjj'} \end{aligned} \tag{55}$$

[PSP3]

$$\begin{aligned} \min c_{j'} \sum_{p \in \mathrm{P}_m} \sum_{u \in \mathrm{U}} s_{mp} q_{mp} \mu_{mpj'r'}^u + c_m^1 \vartheta_m^+ + c_{jj'} w_{km} \pi_{kmijj'} \\ -\delta_m - \sum_{t \in \mathrm{T}} \omega_{vt}^{j'r'} \eta_{j'r't} - \sum_{t \in \mathrm{T}} q_{vt}'^{j'r'} \tau_{j'r't} + \left(b_v'^{j'r'} - M \phi_{km} \right) \psi_{kmjj'} \end{aligned} \tag{56}$$

4.3 Algorithm procedure

Here, the process of CG is provided. First, a heuristic, shown as follows, is employed to find an initial set of vessel-berth plans for each vessel, which is regarded as the initial columns in the RMP. To simply generate the feasible initial solution, we randomly assign the vessel to a feasible berth that meets draft and length constraints without considering vessel diverting process. A QC profile is randomly assigned to each vessel. For each berth with given vessels to be served, we can obtain the optimal service sequence and berthing times of vessels.

A heuristic algorithm for initializing the set of columns.

Input Data: Sets $\Omega, \mathrm{V}, \mathrm{U}, \mathrm{B}_i\left(\forall i \in \Omega\right), \mathrm{P}_k\left(\forall k \in \mathrm{V}\right)$ and all parameters.

For $k \in \mathrm{V}$

Randomly assign vessel k to a berth that meets both draft and length restrictions;

Randomly select a QC profile from P_k for vessel k.

End for

For $r \in \mathrm{B}_i\left(\forall i \in \Omega\right)$

Determine the service order u for each vessel based on their arrival times without considering vessel diversion;

Assign the earliest berthing time for each vessel without overlapping according to the rule of first come first service.

Output: A set of vessel scheduling plans (i.e., initial columns).

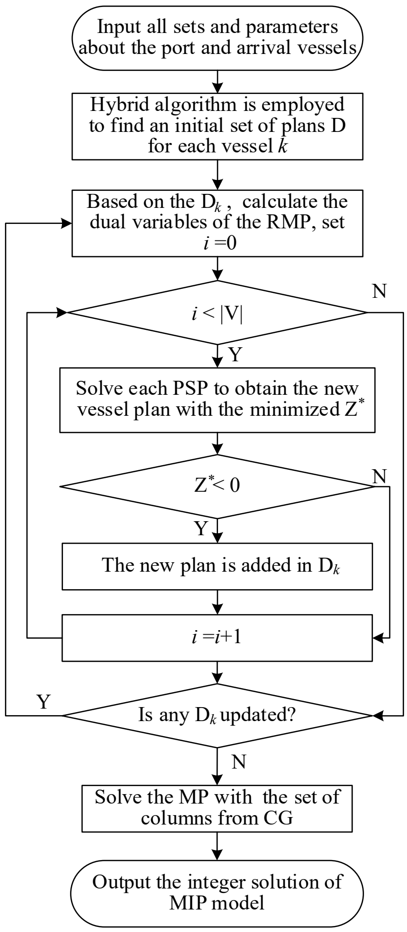

Figure 5. Framework of the CG algorithm.

Next, based on initial columns, we solve the RMP and obtain the dual variables. By solving the PSP, the new vessel-berth plan is then added as a column to improve the objective value of the RMP. During the iterative process, the new columns are continuously added to the RMP until there are no more feasible vessel-berth plans to improve the objective value. Finally, we solve the MP with a small subset of columns obtained from the CG and output the integer solution, following the previous studies (Pang et al., 2011; Liu et al., 2021). As shown in our computational experiments, we find that the results based on proposed CG are optimal or near optimal (see Subsection 5.2). The framework of the CG algorithm is shown in Figure 5.

The RMP can be solved easily by CPLEX. It is time consuming to solve the PSP. We can solve the PSP by CPLEX. Alternatively, we can enumerate all feasible vessel-berth plans. For vessel-berth plan d with the fixed vessel k and fixed berth r, there are ten variables in the PSP, i.e., ω_{dt}^{ir}, $q_{dt}'^{ir}$, $b_d'^{ir}$, b_{kir}^{u}, x_{kp}, μ_{kpir}^{u}, ϑ_k^{+} and ϑ_k^{-}. With the fixed variables b_{kir}^{u} and x_{kp}, we can directly compute other variables of the PSP and evaluate the reduced cost. Since the possible time for berthing is at most $|\mathrm{T}-a_{ki}|$ and the most number of allocated QCs is $|\mathrm{P}_k|$, we can find the minimum reduced cost in each case through $|\mathrm{P}_k|\cdot|\mathrm{T}-a_{ki}|$ calculations. The optimal vessel-berth plan of the PSP2 or PSP3 is different when vessel k and m associated with transshipment containers are diverted to various new destination ports. Thus, we provide $|\Omega|$ vessel-berth plans for a PSP to cover the various cases with different diversion ports. Then, we select the vessel-berth plans with minimum negative reduced costs from $|\Omega|$ vessel-berth plans obtained and add them to the RMP as new columns.

5 Numerical experiments

5.1 Data description

Here we provide numerical experiments considering four ports (Hong Kong, Guangzhou, Shekou and Yantian) in Pearl River Delta of China, since the possibility of the cooperation among these ports has been analyzed in Wang et al. (2012) and Yang et al. (2019). The realistic data on vessel arrival and departure time in October 2020 from four liner shipping companies (COSCO、HMM、MSK and ONE) are considered. According to the posted data of the China Ports Association, we list the number of berths and rates for loading and unloading containers for our considered four ports in Table 2. The number of arriving vessels is shown in Tables 3. We generate the number of handling containers for each vessel from the range [150 TEU, 700TEU] randomly, and we set that one third of vessels are associated with transshipment containers and the percentage of the number of transshipment containers is generated randomly in the range [15%, 25%], following Liu et al. (2016). Since liner shipping services mainly provide weekly service frequency, the planning horizon is 168 hours (i.e., number of hours per week), and the unit of time period is 1 hour. For simplicity, we consider $c_k^1\ (\forall k \in \mathrm{V})$ equals to 6000 USD per hour and $c_i\ (\forall i \in \Omega)$ are 200 USD per hour following Meisel and Bierwirth (2013) and Kavirathna et al. (2020). The average speed for diverting vessels is randomly generated from 15 to 20 knots, and the daily fuel consumption is calculated based on a cubic function (Venturini et al., 2017). With the average price of low-sulfur fuel 523 USD per ton, we can calculate the compensation cost per unit distance (i.e., $c_k^2\ (\forall k \in \mathrm{V})$). We give the distance difference among four ports for diverting vessels in Table 4, where the positive number indicates an increase in sailing distance for vessel diversion while the negative number indicates a shorter sailing distance. The average price of diesel for transporting containers is 1350 USD per ton, and we can give the compensation cost (i.e., $c_{jj'}\ (\forall j \in \Omega, \forall j' \in \Omega)$) for transferring one container between any two ports as shown in Table 5. The time for transferring containers between any two ports is provided in Table 6. Following Imai et al. (2008b), the limit of waiting time for each vessel is defined as $W_i = \min\{s_{kp}\}(\forall p \in \mathrm{P}_k, \forall k \in \mathrm{V})$. The parameter M in constraints (7)-(8) and (11)-(12) is to limit variables b^u_{kir} less than 168 (i.e., the number of hours in a week), since the vessel with the weekly arrival pattern is considered in the assumption. Hence, we set M as 168.

Table 2. Berth number and handling rates for four ports.

	Hong Kong	Guangzhou	Shekou	Yantian
Berth number	24	19	20	13
Handling rates (USD/TEU)	219	78	127	148

Table 3. The number of arriving vessels.

	COSCO	ONE	MSK	HMM
Hong Kong	18	49	38	23
Guangzhou	11	14	40	8
Shekou	17	24	10	11
Yantian	11	16	21	19

Table 4. Distance difference (Nautical mile) among four ports for diverting vessels.

	Hong Kong	Guangzhou	Shekou	Yantian
Hong Kong	-	27	20	12
Guangzhou	-27	-	-7	-3
Shekou	-20	7	-	21
Yantian	-12	3	-21	-

Table 5. Compensation cost (USD) for transferring one container among four ports.

	Hong Kong	Guangzhou	Shekou	Yantian
Hong Kong	-	18.9	10.8	24.3
Guangzhou	18.9	-	10.8	43.2
Shekou	10.8	10.8	-	31.05
Yantian	24.3	43.2	31.05	-

Table 6. Transfer time (hour) among four ports.

	Hong Kong	Guangzhou	Shekou	Yantian
Hong Kong	-	1.5	0.7	1.0
Guangzhou	1.5	-	0.7	2.1
Shekou	0.7	0.7	-	1.2
Yantian	1.0	2.1	1.2	-

All models and algorithms are implemented in C++ with the concert CPLEX library version 12.1, performed on a PC (Intel Core i7, 2.8 GHz; Memory 8G). The maximum runtime is set to be one hour. Next, we provide numerical results to show the efficiency of CG algorithm, which is used to solve our MPBAP in stage I, as shown in Subsection 5.2. To show the effectiveness of the two-stage framework, a number of comparisons under cooperation and non-cooperation situations for different port groups are provided in Subsection 5.3. Subsection 5.4 explores the assessment of port cooperation in various scenarios. Finally, we provide sensitivity analysis on stable port groups, as shown in Subsection 5.5.

5.2 Efficiency of the CG algorithm

Here we provide computational results to show the efficiency of the proposed CG algorithm. A comparison among three approaches is presented. For approach 1, our MIP model is solved by CPLEX directly. For approach 2, our MIP model is solved by our CG algorithm, where the PSP is solved by CPLEX. For approach 3, our MIP model is solved by our CG algorithm, where the PSP is solved by the enumeration strategy. Table 7 shows a comparison among these three approaches for solving our model considering three different problem sizes. For each problem size, the data on selecting ports, berths and vessels are randomly chosen from the above realistic data. To reduce the effect of randomness, five tests are considered for each problem size, and 15 cases are shown in Table 7, where case “2-5-20-1” denotes the first trial of the instance with 2 ports, 5 berths and 20 vessels. We demonstrate the details of the objective value, solution quality and runtimes of 15 cases, as shown in Table 7, where CPLEX, CG and ECG represent the above three approaches respectively. Columns “Obj.”, “#Col.” and “#Ite.” represent objective value, the number of columns, and the number of iterations, respectively. Column “Gap” is the relative difference of objective value obtained by any other approach (i.e., CG and ECG), with respect to the optimal value obtained by CPLEX. Namely, we have

$$Gap = \frac{\left(Obj_{approach} - Obj_{CPLEX}\right)}{Obj_{CPLEX}} \times 100\%$$

where $Obj_{approach}$ is the objective value of approach CG or ECG and Obj_{CPLEX} is the optimal value obtained by CPLEX. Note that we set the result to ‘-’ if CPLEX cannot give the optimal solution within the maximum runtime in a number of instances.

As shown in Table 7, three approaches can find the optimal (or near optimal) solutions, while CPLEX can only solve our MIP model directly for small-scale instances. CG can calculate all considered cases within three iterations. Compared with approach 1, the CG only generates about less 3% of all possible columns on average to obtain near optimal solutions. In addition, approach 3 outperforms approach 2 with respect to the runtimes. Our proposed CG with the enumeration strategy can reduce the number of column generation iterations by adding more columns (vessel-berth plans). Hence, our enumeration-based CG approach can deal with large-scale real problems.

Table 7. A comparison among three approaches.

Cases	Obj. (USD)			Runtimes (s)			Gap (%)			#Col.			#Ite.		
	CPLEX	CG	ECG	CPLEX	CG	ECG	CPLEX	CG	ECG	CPLEX	CG	ECG	CPLEX	CG	ECG
2-5-20-1	41126	41205	42352	3063	72	48	-	0.19	0.19	22936	384	384	-	1	1
2-5-20-2	40320	40443	40643	2615	95	36	-	0.30	0.30	21182	470	470	-	1	1
2-5-20-3	40320	40566	41066	2670	120	51	-	0.61	0.61	19966	363	363	-	1	1
2-5-20-4	41933	42055	40255	2888	99	42	-	0.29	0.29	21137	524	524	-	1	1
2-5-20-5	40320	40483	43183	3212	115	53	-	0.40	0.40	21005	400	400	-	1	1
3-20-80-1	-	189804	189804	Out of memory	172	100	-	-	-	1427306	19755	19755	-	1	1
3-20-80-2	-	201666	201666	Out of memory	228	134	-	-	-	1251819	28124	28124	-	1	1
3-20-80-3	-	207598	207598	Out of memory	203	118	-	-	-	1300801	16940	16940	-	2	2
3-20-80-4	-	197712	197712	Out of memory	199	109	-	-	-	1278173	24446	24446	-	1	1
3-20-80-5	-	193758	193758	Out of memory	269	103	-	-	-	1273485	25514	25514	-	1	1
4-40-160-1	-	467911	467911	Out of memory	265	174	-	-	-	12312661	192486	192486	-	3	3
4-40-160-2	-	463323	463323	Out of memory	374	164	-	-	-	11022763	169186	169186	-	2	2
4-40-160-3	-	444974	444974	Out of memory	296	176	-	-	-	11235524	179536	179536	-	2	2
4-40-160-4	-	444974	444974	Out of memory	347	163	-	-	-	11681908	169426	169426	-	2	2
4-40-160-5	-	477085	477085	Out of memory	365	186	-	-	-	11294357	149842	149842	-	1	1

5.3 Performance of the two-stage framework

Here we provide the results under cooperation and non-cooperation situations, in order to show the effectiveness of the two-stage framework. In the non-cooperation scenario, all ports operate independently and vessel diversions between ports are not considered. The optimal result in non-cooperation scenario for each port can be calculated by solving the MPBAP model where a single port is regarded as a group and let variables of vessel diversion $z_{kij}\left(\forall k \in \mathrm{V}, \forall i \in \Omega, \forall j \in \Omega\right)$ be 0. For the considered four ports (Hong Kong, Guangzhou, Shekou and Yantian), 11 port groups are generated. Results of the MPBAP for 11 port groups are shown in Table 8. We provide comparisons of revenues for 11 port groups under cooperation (Co.) and non-cooperation (Non-co.) situations in Table 9, as well as the revenue allocation results for these 11 port groups in Table 10. Besides, we provide a comparison of vessel delays from two aspects, i.e., the average delay time (ADT) of all delayed vessels and the percentage of vessel delays (PVD), for 11 port groups between cooperation and non-cooperation in Figures 6 and 7.

In Figures 6 and 7, the horizontal axis indicates the 11 port groups, which are in the same order as the port groups listed in Table 8. The larger the difference of the dash points (or columns) value, the greater the effect on vessel delays after port cooperation. As we can see, the ADT and PVD in any port group considering port cooperation are less than those under the non-cooperation situation, and hence port cooperation can improve port service capacity by coordinating berth resources, thus reducing the risk of vessel delays.

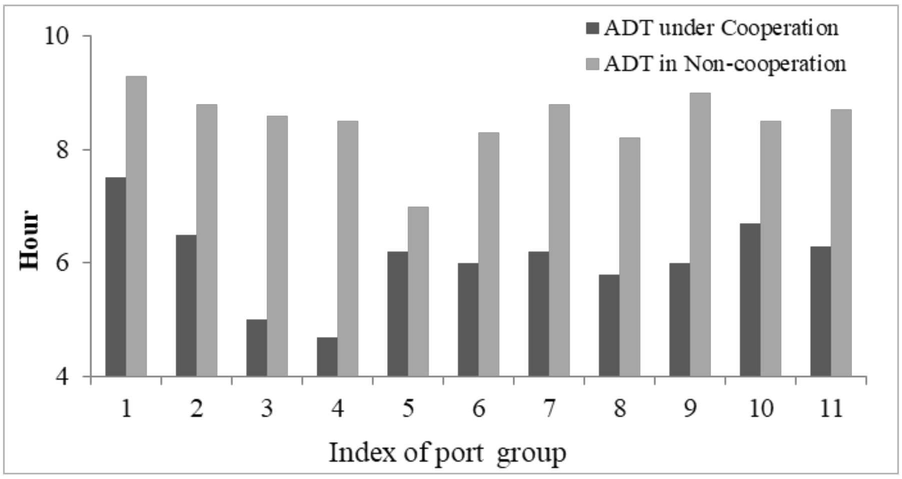

Figure 6. Comparison of ADT for 11 port groups between port cooperation and non-cooperation.

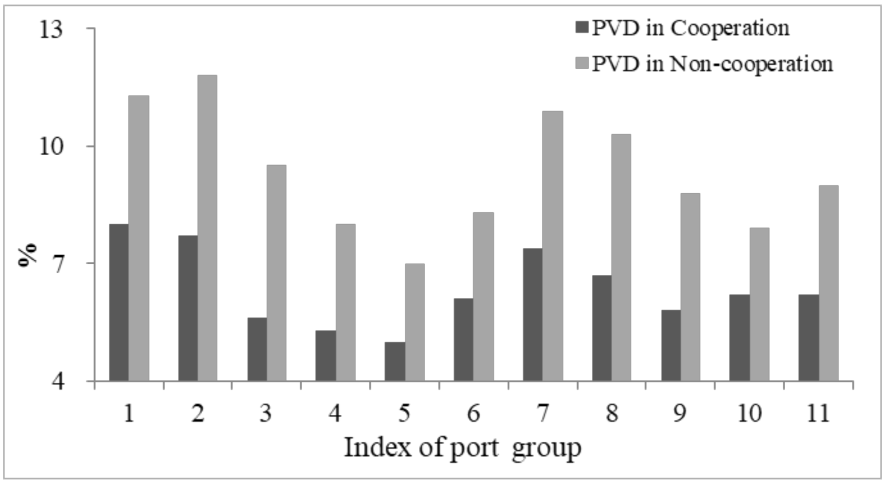

Figure 7. Comparison of PVD for 11 port groups between port cooperation and non-cooperation.

Table 8. Results of the MPBAP for 11 port groups under the cooperation situation.

Port groups	Total Cost (10^6 USD)	Number of diverting vessels
Hong Kong, Guangzhou	1.10	30
Hong Kong, Shekou	0.99	37
Hong Kong, Yantian	0.83	53
Guangzhou, Shekou	0.56	36
Guangzhou, Yantian	0.64	11

Shekou, Yantian	0.41	12
Hong Kong, Guangzhou, Shekou	1.31	41
Hong Kong, Guangzhou, Yantian	1.16	61
Hong Kong, Shekou, Yantian	1.06	65
Guangzhou, Shekou, Yantian	0.77	36
Hong Kong, Guangzhou, Shekou,Yantian	1.58	72

Table 9. Comparisons of costs and revenues for 11 port groups under cooperation and non-cooperation situations.

Port groups	Co. (10^6 USD)	Non-co. (10^6 USD)	Difference (%)
Hong Kong, Guangzhou	15.76	15.52	1.6
Hong Kong, Shekou	16.64	16.48	1.0
Hong Kong, Yantian	17.77	17.44	1.9
Guangzhou, Shekou	5.91	5.74	3.0
Guangzhou, Yantian	6.80	6.70	1.5
Shekou, Yantian	7.79	7.66	1.8
Hong Kong, Guangzhou, Shekou	19.17	18.87	1.6
Hong Kong, Guangzhou, Yantian	20.29	19.83	2.3
Hong Kong, Shekou, Yantian	21.17	20.79	1.8
Guangzhou, Shekou, Yantian	10.29	10.05	2.4
Hong Kong, Guangzhou, Shekou,Yantian	23.69	23.18	2.2

Table 10. Revenue allocation for different port groups.

Port groups	Hong Kong (10^6 USD)	Guangzhou (10^6 USD)	Shekou (10^6 USD)	Yantian (10^6 USD)
Hong Kong, Guangzhou	13.18	2.58	-	-
Hong Kong, Shekou	13.13	-	3.51	-
Hong Kong, Yantian	13.22	-	-	4.55
Guangzhou, Shekou	-	2.46	3.45	-
Guangzhou, Yantian	-	2.43	-	4.36
Shekou, Yantian	-	-	3.42	4.37
Hong Kong, Guangzhou, Shekou	13.18	2.53	3.46	-
Hong Kong, Guangzhou, Yantian	13.30	2.51	-	4.48
Hong Kong, Shekou, Yantian	13.24	-	3.44	4.48
Guangzhou, Shekou, Yantian	-	2.46	3.45	4.37
Hong Kong, Guangzhou, Shekou,Yantian	13.28	2.45	3.40	4.41
Non-cooperation	13.13	2.39	3.35	4.31

As shown in Table 8, vessel diversion occurs in all port groups. It means that ports would like to cooperate on vessel diversion. Besides, we find that the more ports in a port group, the lower the total cost, and the grand coalition has the minimum total cost, which is consistent with the conclusion provided in Saeed and Larsen (2010).

Table 9 shows that the revenue of any port group considering cooperation is higher than that under the non-cooperation situation. Under the non-cooperation situation, the revenue for Hong Kong, Guangzhou, Shekou and Yantian is 13.13×10^6 USD, 2.39×10^6 USD, 3.35×10^6 USD, 4.31×10^6 USD, respectively. Compared with revenues in non-cooperation, all port groups have an increase in revenue, resulting in a cooperative impetus for neighboring ports. By considering the cooperation situation, the revenue of port group {Guangzhou, Shekou} is increased by 3.0%, which is higher than that (2.2%) of the port group with four ports.

As shown in Table 10, we can obtain nine stable port groups from these 11 port groups, including {Hong Kong, Guangzhou}, {Hong Kong, Shekou}, {Hong Kong, Yantian}, {Guangzhou, Shekou}, {Guangzhou, Yantian}, {Shekou, Yantian}, {Hong Kong, Guangzhou, Yantian}, {Hong Kong, Shekou, Yantian} and {Guangzhou, Shekou, Yantian}. The port group {Hong Kong, Guangzhou, Shekou} is instable since the revenue allocated to Guangzhou Port is 2.53×10^6 USD, less than that (i.e., 2.58×106 USD) in port group {Hong Kong, Guangzhou}. The grand coalition, i.e., port group {Hong Kong, Guangzhou, Shekou, Yantian}, is instable since the revenues allocated to Hong Kong Port, Guangzhou Port, Yantian Port in this port group are less than those in port group {Hong Kong, Guangzhou, Yantian}. Furthermore, from stable port groups we can obtain the optimal port groups {Hong Kong, Yantian} and {Guangzhou, Shekou}, based on which we can obtain the maximum total revenue for these four ports.

Based on the above phenomenon, we can obtain the following managerial insights. 1) The port cooperation is driven by severe vessel delays at the port and the short diversion distance between ports. 2) More ports jointed in a port group will save more operating costs. Therefore, local governments should encourage more ports to join the cooperation for avoiding unwanted cost consumption and expanding total regional benefits. 3) A grand coalition is the best way to cooperate for the whole region, but it is not the best option for a number of ports.

5.4 Assessment of port groups under various scenarios

To further assess port groups under various scenarios, following Kavirathna et al. (2020), we provide three groups of experiments based on the data of four ports considered, shown as follows.

- Group 1: We consider some scenarios where only some ports in the region can participate in the cooperation. In a given scenario, we evaluate the stable and optimal port groups via the two-stage framework. The results of Hong Kong, Guangzhou, Shekou and Yantian in various scenarios are shown in Figures 8-11, where the axes represent the number of ports considered in the scenario and the allocated profit of this port, respectively.

- Group 2: Considering the different sizes and service capabilities of ports in practice, we evaluate the stability of port groups under the following scenarios: a) all ports have the same service capability and the number of incoming vessels; b) there is one busy superport (i.e., Hong Kong) and the rest three ports with similar size; c) there are two busy superports (i.e., Hong Kong and Guangzhou) and the rest two ports with similar size; d) there are three busy superports (i.e., Hong Kong, Guangzhou and Shekou) and the rest one port with small size. We consider that, for any superport the number of arriving vessels is multiplied by five times, while for other ports the number of arriving vessels is the average value of that shown in Table 3. Here, the superports can be regarded as large ports, while the other ports can be regarded as small ports. The results are shown in Table 11, where sign '√' denotes that the corresponding port group is stable.
- Group 3: Since revenue may not be the only objective of port cooperation, we provide additional metrics of the optimal BA plans for all port groups in Table 12, i.e., total profits, number of transferred containers (NTC) due to vessel diversions, percentage of vessel delays (PVD). The PVD of the port group can be calculated as follows

$$\mathrm{PVD} = \frac{\mathrm{V}'}{|\mathrm{V}|} \times 100\%$$

where V' denotes the number of delayed vessels and $|\mathrm{V}|$ is the total number of arriving vessels.

For Group 1, Figures 8-11 show that the optimal revenues allocated to the port are not increasing continuously when more ports join the port cooperation. We take Figure 8 as an example to analyze in details. For the group of Hong Kong Port and Yantian Port, Hong Kong Port prefers Guangzhou Port or Shekou Port to join this port group becoming {Hong Kong, Guangzhou, Yantian} or {Hong Kong, Shekou, Yantian}, in order for more revenues allocated. For the group of Hong Kong Port, Guangzhou Port and Yantian Port, the join of Shekou Port would reduce the revenue of Hong Kong Port. We can find that the cooperative motivation of a port changes when different ports join the port group, and a port has different attitudes when joining in other port groups in different situations.

For Group 2, as shown in Table 11, all considered port groups are stable when all ports have almost the same service capacity in case (a). Namely, in this case all ports are motivated to participate in the cooperation. In case (b), most port groups consisting of one superport and some small ports are stable, as vessel delays and associated costs are reduced in the superport via vessel diversions and the small ports can gain more revenue by serving vessels diverting from the superport. In cases (c) and (d) with two or three superports in the region, the incentive for the cooperation among ports decreases. This is because the service capacities of all supperports are approaching to saturation, resulting in hardly serving vessels diverted from other ports. Here, we can derive a managerial insight for port operators. In some cases, it is wise for the superport to establish the cooperation with small neighboring ports, rather than large ports (superports) nearby. Small ports should actively seek the cooperation with large ports to increase their activities and revenues.

For Group 3, in Table 12, we assess 11 port groups by considering different aspects. Total profit is increased with the number of ports in the port group. Hence, the grand coalition, i.e., port group {Hong Kong, Guangzhou, Shekou, Yantian} has the maximum total profit. As compared with other port groups, we can obtain minimum vessel delays for port group {Guangzhou, Yantian}. The number of transferred containers rise as more ports joining the cooperation, which results in more fuel consumptions. Hence, port cooperation may not be helpful from the perspective of environmental impact.

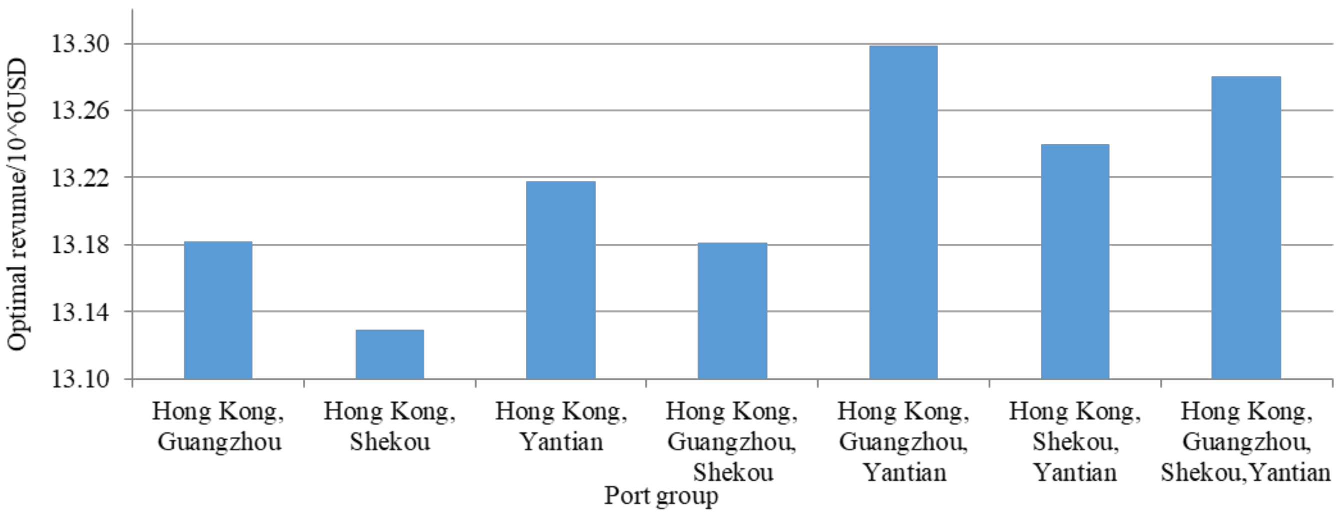

Figure 8. Optimal revenues allocated to Hong Kong in different port groups.

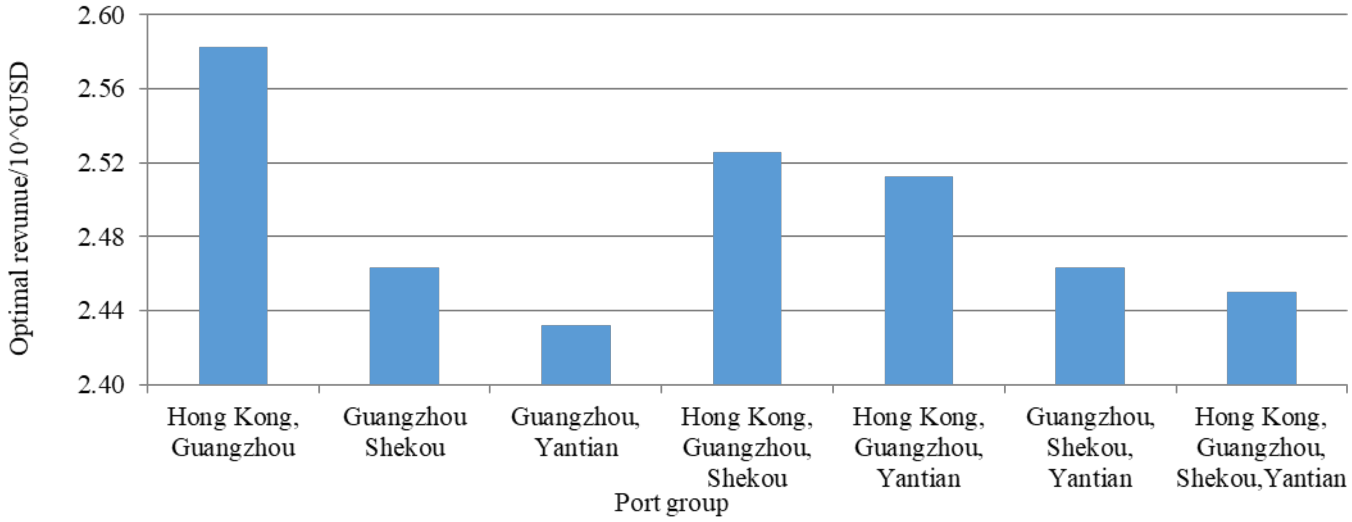

Figure 9. Optimal revenues allocated to Guangzhou in different port groups.

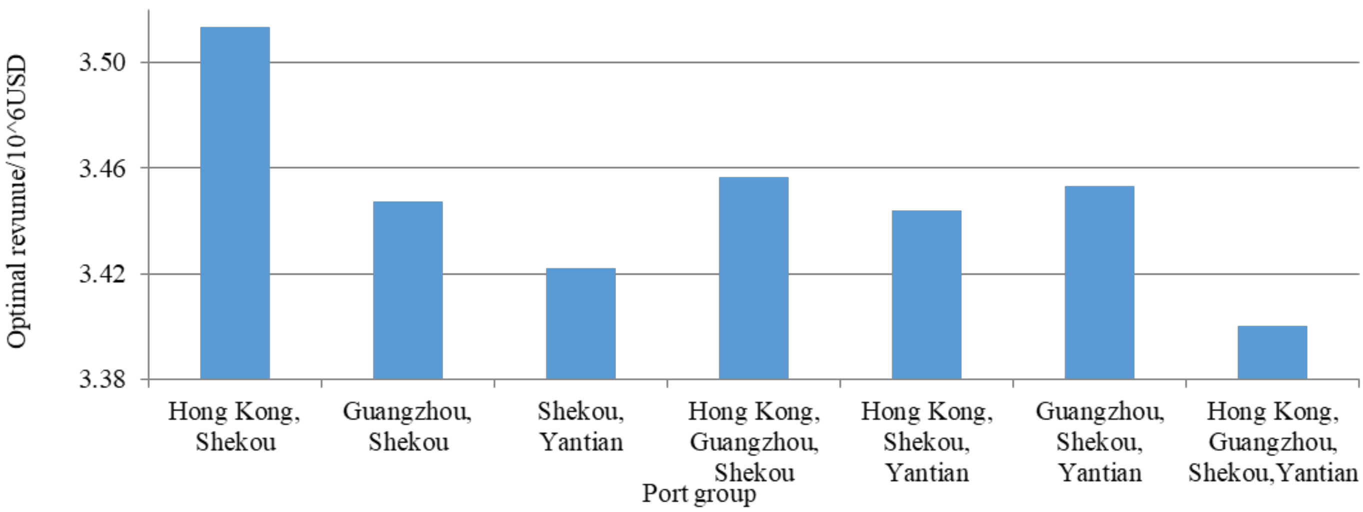

Figure 10. Optimal revenues allocated to Shekou in different port groups.

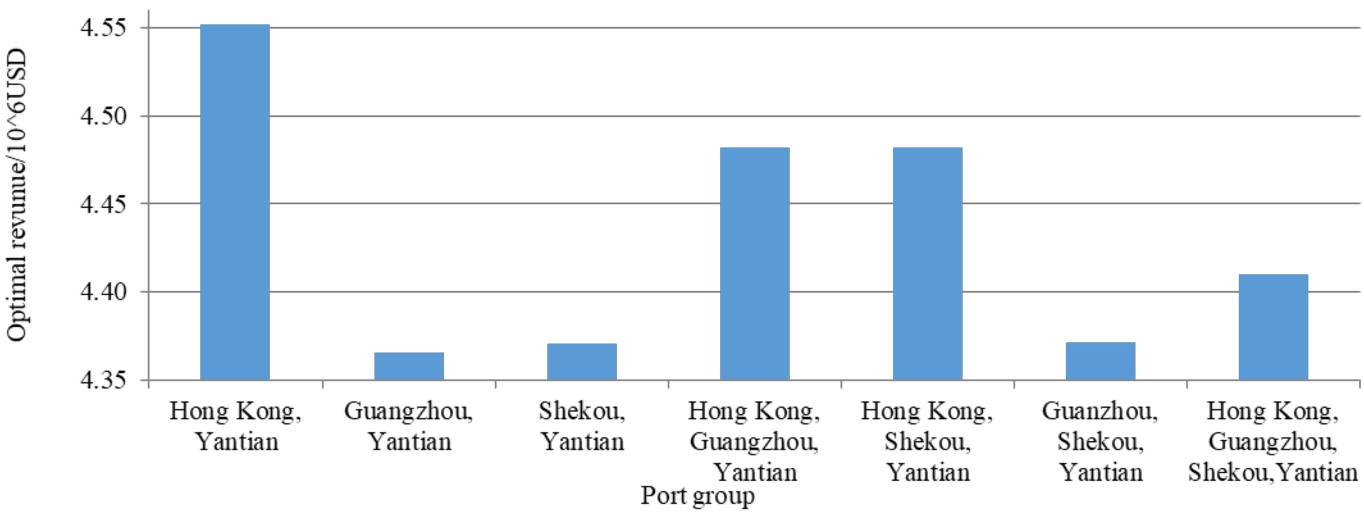

Figure 11. Optimal revenues allocated to Yantian in different port groups.

Table 11. Port group stability under four scenarios.

Port groups	a	b	c	d
Hong Kong, Guangzhou	√	√		
Hong Kong, Shekou	√	√	√	
Hong Kong, Yantian	√	√	√	√
Guangzhou, Shekou	√	√	√	
Guangzhou, Yantian	√	√	√	√
Shekou, Yantian	√	√	√	√
Hong Kong, Guangzhou, Shekou	√	√		
Hong Kong, Guangzhou, Yantian	√	√		
Hong Kong, Shekou, Yantian	√	√	√	
Guangzhou, Shekou, Yantian	√	√		
Hong Kong, Guangzhou, Shekou,Yantian	√			

Table 12. Other metrics for assessing 11 port groups.

Port groups	Profit (10^6USD)	NTC	Vessel delays (%)
Hong Kong, Guangzhou	15.76	1809	8.0
Hong Kong, Shekou	16.64	2107	7.7
Hong Kong, Yantian	17.77	2280	5.6
Guangzhou, Shekou	5.91	1831	5.3
Guangzhou, Yantian	6.80	486	5.0
Shekou, Yantian	7.79	714	6.1
Hong Kong, Guangzhou, Shekou	19.16	2064	7.4
Hong Kong, Guangzhou, Yantian	20.29	3239	6.7
Hong Kong, Shekou, Yantian	21.17	3605	5.8
Guangzhou, Shekou, Yantian	10.29	2918	6.2
Hong Kong, Guangzhou, Shekou,Yantian	23.54	4520	6.2

5.5 Sensitivity analysis on stable port groups

Here we provide sensitivity analysis on stable port groups by considering different shipping markets, which can be reflected by the number of arriving vessels. We introduce parameter p, which takes the value from {0.1, 0.5, 1, 2, 5, 10}, representing six scenarios with different sizes of arriving vessels. When $p=0.1$, it means that the number of arriving vessels for each port (see Table 3) is multiplied by 0.1 times. Then, we evaluate the stabilities of 11 port groups in six scenarios, as shown in Table 13, where sign '√' denotes that the corresponding port group is stable.

First, there is no stable port group when $p=0.1$. This is because berth resources of each port are fully sufficient to serve a small number of arriving vessels, and port cooperation is not necessary. With the increase of p, the berth resources of some ports gradually reach the saturation state, and then these ports start seeking for cooperation to save the penalty cost of vessel delay by diverting vessels. When p keeps increasing, most ports reach the saturation state, and then there are not sufficient berths available for serving the diverted vessels.

In conclusion, ports do not have incentive for the cooperation among ports when the shipping market is extremely depressed or prosperous. A normal level or slightly fluctuating in the shipping market will promote the cooperation among ports.

Table 13. Port group stability analysis in six scenarios.

Port groups	0.1	0.5	1	2	5	10
Hong Kong，Guangzhou		√	√	√		
Hong Kong，Shekou			√	√		
Hong Kong，Yantian		√	√	√	√	
Guangzhou，Shekou			√	√	√	
Guangzhou，Yantian			√	√	√	
Shekou，Yantian			√	√	√	
Hong Kong，Guangzhou，Shekou						
Hong Kong，Guangzhou，Yantian		√	√			
Hong Kong，Shekou，Yantian		√	√	√		
Guangzhou，Shekou，Yantian			√	√		
Hong Kong，Guangzhou，Shekou，Yantian						

To explore effects of fluctuation of the penalty cost for vessel delays or compensation cost for vessel diversions on the port group stability, we generate six scenarios where the penalty cost or the compensation cost for each port is multiplied by p times, and let p take the value from {0.1, 0.5, 1, 2, 5, 10}. In each scenario, we calculate the optimal BA plans of all possible port groups and evaluate the stabilities of these port groups, as shown in Tables 14 and 15, where sign ‘ √ ’ denotes that the corresponding port group is stable.

From Tables 14 and 15, one can find that the number of stable port groups increases when the penalty cost for vessel delays increases or the compensation cost for vessel diversions decreases. Hence, the compensation cost (the penalty cost) has an obviously negative (positive) effect on port cooperation stability.

Table 14. Port group stability for different penalty costs.

Port groups	0.1	0.5	1	2	5	10
Hong Kong，Guangzhou			√	√	√	√
Hong Kong，Shekou			√	√	√	√
Hong Kong，Yantian		√	√	√	√	√

Guangzhou，Shekou		√	√	√	√	√
Guangzhou，Yantian			√	√	√	√
Shekou，Yantian			√	√	√	√
Hong Kong，Guangzhou，Shekou					√	√
Hong Kong，Guangzhou，Yantian		√	√	√	√	√
Hong Kong，Shekou，Yantian		√	√	√	√	√
Guangzhou，Shekou，Yantian			√	√	√	√
Hong Kong，Guangzhou，Shekou，Yantian						

Table 15. Port group stability for different compensation costs.

Port groups	0.1	0.5	1	2	5	10
Hong Kong，Guangzhou	√	√	√			
Hong Kong，Shekou	√	√	√			
Hong Kong，Yantian	√	√	√	√		
Guangzhou，Shekou	√	√	√	√	√	
Guangzhou，Yantian	√	√	√			
Shekou，Yantian	√	√	√			
Hong Kong，Guangzhou，Shekou	√	√				
Hong Kong，Guangzhou，Yantian	√	√	√			
Hong Kong，Shekou，Yantian	√	√	√	√	√	
Guangzhou，Shekou，Yantian	√	√				
Hong Kong，Guangzhou，Shekou，Yantian						

6 Conclusions

This paper proposes a novel MPBAP with the PCSP, where multiple neighboring ports are grouped into several port groups for berth cooperation. A two-stage framework is developed to solve our problem. In stage I, for any particular port group, the MPBAP is formulated as a MIP model to evaluate the costs of various port groups. For solving the PCSP, stage II utilizes cooperative game theory to determine revenue allocation and assess port group stability, and then the PCSP is formulated as a binary programming model. Furthermore, a CG algorithm embedded with the enumeration strategy is proposed for solving the proposed MIP model. Finally, numerical experiments are carried out to validate the effectiveness of the proposed models and solution method.

Numerical results suggest the following practical observations: (1) the proposed CG algorithm can solve our MIP model for the instances with 4 ports and 160 vessels within three minutes; (2) a two-stage framework is devised to determine the cooperation among neighboring ports (i.e., forming optimal port groups), and the cooperation is helpful to increase the port revenue; (3) sensitivity analysis shows that the neighboring ports would like to seek cooperation under different shipping markets. The compensation cost (penalty cost) has a negative (positive) effect on port cooperation stability.

Furthermore, we can obtain the following managerial insights: (1) port can benefit from the cooperation with neighboring ports; (2) it is effective to establish a cooperation between a large port (superport) and a number of small ports nearby; (3) from different aspects (e.g., profit, stability), the optimal port groups to be formed may be various.

There are some works we will investigate in the future. Firstly, we will explore the effect of distances between berthing locations and yard spaces on the handling time. Secondly, we will investigate the yard template design problem integrated with the MPBAP. Thirdly, we will develop an integrated optimization model formulating the yard template design problem with the MPBAP, and then devise an efficient solution method to solve it. Finally, we will investigate the MPBAP by considering that vessel is allocated to the diversion port directly instead of arriving at the pre-assigned port first. As mentioned before, some barriers (e.g., vessel arrival information sharing between ports) associated with port cooperation are relaxed in this work. When further addressing the MPBAP, we will consider the modeling of the barriers associated with port cooperation.

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Appendix

After the linearization, our MIP model can be rewritten as follows:

$$\min \sum_{k\in \mathrm{V}}\sum_{p\in \mathrm{P}_k}\sum_{i\in\Omega}\sum_{r\in \mathrm{B}_i}\sum_{u\in \mathrm{U}} c_i s_{kp} q_{kp}\mu_{kpir}^{u} + \sum_{k\in \mathrm{V}} c_k^1 \vartheta_k^+ + \sum_{k\in \mathrm{V}}\sum_{i\in\Omega}\sum_{j\in\Omega} c_k^2\theta_{kij}^+ + \sum_{k\in \mathrm{V}}\sum_{m\in \mathrm{V}}\sum_{i\in\Omega}\sum_{j\in\Omega}\sum_{j'\in\Omega} c_{jj'} w_{km}\pi_{kmijj'} \tag{A1}$$

subject to

(2)-(6), (8)-(9), (11) and (13)-(17);

$$b_{kir}^{u} + \sum_{p\in \mathrm{P}_k} s_{kp}\mu_{kpir}^{u} \le b_{mir}^{u+1} + M\left(1 - y_{mir}^{u+1}\right), \forall k\in \mathrm{V}, \forall m\in \mathrm{V}, k\ne m, \forall i\in\Omega, \forall r\in \mathrm{B}_i, \forall u\in \mathrm{U}; \tag{A2}$$

$$\sum_{p\in \mathrm{P}_k}\sum_{u\in \mathrm{U}} \mu_{kpir}^{u} q_{kp} \le Q_{ir}, \forall k\in \mathrm{V}, \forall i\in\Omega, \forall r\in \mathrm{B}_i; \tag{A3}$$

$$\sum_{\gamma\in \mathrm{B}_{j'}}\sum_{\mu\in \mathrm{U}} b_{mj'\gamma}^{\mu} + M\left(1 - \phi_{km}\pi_{kmijj'}\right) \ge \sum_{r\in \mathrm{B}_j}\sum_{u\in \mathrm{U}}\left(b_{kjr}^{u} + \sum_{p\in \mathrm{P}_k} s_{kp}x_{kp}\right) + g_{jj'} \tag{A4}$$

$$, \forall k\in \mathrm{V}, \forall m\in \mathrm{V}, k\ne m, \forall i\in\Omega, \forall j\in\Omega, \forall j'\in\Omega;$$

$$\mu_{kpir}^{u} \le y_{kir}^{u}, \forall k\in \mathrm{V}, \forall p\in \mathrm{P}_k, \forall i\in\Omega, \forall r\in \mathrm{B}_i, \forall u\in \mathrm{U}; \tag{A5}$$

$$\mu_{kpir}^{u} \le x_{kp}, \forall k \in \mathrm{V}, \forall p \in \mathrm{P}_k, \forall i \in \Omega, \forall r \in \mathrm{B}_i, \forall u \in \mathrm{U}; \tag{A6}$$

$$y_{kir}^{u} + x_{kp} - 1 \le \mu_{kpir}^{u}, \forall k \in \mathrm{V}, \forall p \in \mathrm{P}_k, \forall i \in \Omega, \forall r \in \mathrm{B}_i, \forall u \in \mathrm{U}; \tag{A7}$$

$$\pi_{kmijj'} \le z_{kij}, \forall k \in \mathrm{V}, \forall m \in \mathrm{V}, k \ne m, \forall i \in \Omega, \forall j \in \Omega, \forall j' \in \Omega; \tag{A8}$$

$$\pi_{kmijj'} \le z_{kij'}, \forall k \in \mathrm{V}, \forall m \in \mathrm{V}, k \ne m, \forall i \in \Omega, \forall j \in \Omega, \forall j' \in \Omega; \tag{A9}$$

$$z_{kij} + z_{kij'} - 1 \le \pi_{kmijj'}, \forall k \in \mathrm{V}, \forall m \in \mathrm{V}, k \ne m, \forall i \in \Omega, \forall j \in \Omega, \forall j' \in \Omega; \tag{A10}$$

$$\left(\sum_{i \in \Omega} \sum_{r \in \mathrm{B}_i} \sum_{u \in \mathrm{U}} b_{kir}^{u} + \sum_{p \in \mathrm{P}_k} s_{kp} x_{kp} \right) - e_k = \vartheta_k^{+} - \vartheta_k^{-}, \forall k \in \mathrm{V}; \tag{A11}$$

$$\Delta d_{kij} z_{kij} = \theta_{kij}^{+} - \theta_{kij}^{-}, \forall k \in \mathrm{V}, \forall i \in \Omega, \forall j \in \Omega; \tag{A12}$$

$$\mu_{kpir}^{u} \in \{0,1\}, \forall k \in \mathrm{V}, \forall p \in \mathrm{P}_k, \forall i \in \Omega, \forall r \in \mathrm{B}_i, \forall u \in \mathrm{U}; \tag{A13}$$

$$\pi_{kmijj'} \in \{0,1\}, \forall k \in \mathrm{V}, \forall m \in \mathrm{V}, k \ne m, \forall i \in \Omega, \forall j \in \Omega, \forall j' \in \Omega; \tag{A14}$$

$$\vartheta_k^{+}, \vartheta_k^{-} \ge 0, \forall k \in \mathrm{V}; \tag{A15}$$

$$\theta_{kij}^{+}, \theta_{kij}^{-} \ge 0, \forall k \in \mathrm{V}, \forall i \in \Omega, \forall j \in \Omega. \tag{A16}$$

The objective function (A1) is a linear function. Constraints (A2)-(A4) are equivalent to Constraints (7), (10) and (12), respectively. Constraints (A5)-(A7) show the relationship between variables μ_{kpir}^{u}, x_{kp} and y_{kir}^{u}. Constraints (A8)-(A10) show the relationship between variables $\pi_{kmijj'}$, z_{kij} and $z_{mij'}$. Constraints (A11) calculate the variable ϑ_k^{+} and Constraints (A12) calculate the variable θ_{kij}^{+}. Finally, Constraints (A13)-(A16) define the domain of decision variables.

References

1. Barbosa, F., Rampazzo, B.P.C., Yamakami, A., Camanho, A.S., 2019. The use of frontier techniques to identify efficient solutions for the berth allocation problem solved with a hybrid evolutionary algorithm. Computers & Operations Research, 107, 43–60.
2. Bierwirth, C., Meisel, F., 2010. A survey of berth allocation and quay crane scheduling problems in container terminals. European Journal of Operational Research, 202 (3), 615–627.
3. Bierwirth, C., Meisel, F., 2015. A follow–up survey of berth allocation and quay

crane scheduling problems in container terminals. European Journal of Operational Research, 244, 1–15.
4. Bouzekri, H., Alpan, G., Alpan, V., 2021. Integrated laycan and berth allocation and time–invariant quay crane assignment problem in tidal ports with multiple quays. European Journal of Operational Research, 293, 892–909.
5. Buhrkal, K., Zuglian, S., Ropke, S., Larsen, J., Lusby, R., 2011. Models for the discrete berth allocation problem: a computational comparison. Transportation Research Part E, 47(4), 461–473.
6. Cahyono, R.T., Flonk, E.J., Jayawardhana, B., 2015. Dynamic berth and quay crane allocation for multiple berth positions and quay cranes. IEEE Transactions on Intelligent Transportation Systems, 21 (3), 1321–1331.
7. Chargui, K., Zouadi, T., El Fallahi, A., Reghioui, M., Aouam, T, 2021. Berth and quay crane allocation and scheduling with worker performance variability and yard truck deployment in container terminals. Transportation Research Part E 154, 102449.
8. Cordeau, J.F., Laporte, G., Legato, P., Moccia, L., 2005. Models and tabu search heuristics for the berth-allocation problem. Transportation Science, 39(4), 526–538.
9. Cordeau, J.F., Legato, P., Mazza, R., Trunfio, R., 2015. Simulation-based optimization for housekeeping in a container transshipment terminal. Computer & Operations Research 53, 81–95.
10. Diabat, A., Theodorou, E., 2014. An integrated quay crane assignment and scheduling problem. Computers & Industrial Engineering, 73, 115–123.
11. Dulebenets, M.A., Golias, M.M., Mishra, S., 2018. A collaborative agreement for berth allocation under excessive demand. Engineering Applications of Artificial Intelligence, 2018, 69, 76–92.
12. Emde, S., Boysen, N., Briskorn, D., 2014. The berth allocation problem with mobile quay walls: problem definition, solution procedures, and extensions. Journal of Scheduling, 17 (3), 289–303.
13. Fagerholt, K., Gausel, N.T., Rakke, J.G., Psaraftis, H., 2015. Maritime routing and speed optimization with Emission Control Areas. Transportation Research Part C, 52, 57-73.
14. Fu, Y.M., Diabat, A., Tsai, I.T., 2014. A multi–vessel quay crane assignment and scheduling problem: Formulation and heuristic solution approach. Expert Systems with Applications, 41 (15), 6959–6965.
15. Giallombardo, G., Moccia, L., Salani, M., Vacca, I., 2010. Modeling and solving the tactical berth allocation problem. Transportation Research Part B, 44(2), 232–245.
16. Golias, M., Boile., M., Theofanis, S., 2009. Berth scheduling by customer service differentiation: A multi-objective approach. Transportation Research Part E, 45(6), 878-892.
17. Han, X., Lu, Z., Xi, L., 2010. A proactive approach for simultaneous berth and quay crane scheduling problem with stochastic arrival and handling time. European Journal of Operational Research, 207, 1327–1340.

18. Heilig, L., Lalla-Ruiz, E., Bode, C., Yoss, S., 2019. Assessment of quay and yard transshipment operations under proximity limitations in multi-terminal container ports. Proceedings of the 52nd annual Hawaii international conference on system sciences, 1548－1557.
19. Henesey, L., 2004. Enhancing container terminal performance: The multi agent systems approach (Licentiate thesis), Blekinge Institute of Technology, Karlshamn.
20. Hoshino, H., 2010. Competition and collaboration among container ports. The Asian Journal of Shipping and Logistics, 26, 31–48.
21. Imai A., Chen, H., Nishimura E., 2008a. The simultaneous berth and quay crane allocation problem. Transportation Research Part E, 44(5), 900–920.
22. Imai, A., Nishimura, E., Papadimitriou, S., 2001. The dynamic berth allocation problem for a container port. Transportation Research Part B, 35, 401–417.
23. Imai, A., Nishimura, E., Papadimitriou, S., 2008b. Berth ships at a multi-user container terminal with a limited quay capacity. Transportation Research Part E, 44, 136–151.
24. Imai, A., Sun, X., Nishimura, E., Papadimitriou, S., 2005. Berth allocation in a container port: Using a continuous location space approach. Transportation Research Part B, 39(3), 199–221.
25. Imai, A., Yamakawa, Y., Huang, K.C., 2014. The strategic berth template problem. Transportation Research Part E, 72, 77–100.
26. Iris, C., Lalla, R.E., Lam, J.S.L., Voss, S., 2018. Mathematical programming formulations for the strategic berth template problem. Computers & Industrial Engineering, 124, 167–179.
27. Islam, S., 2018. Simulation of truck arrival process at a seaport: evaluating truck-sharing benefits for empty trips reduction. International Journal of Logistics-Research and Applications, 21 (1), 94–112.
28. Islam, S., Uddin, M.J., Shi, P., Sharif, T., Ahmed, J.U., 2021. Exploring shippers' motivations to adopt collaborative truck-sharing initiatives. International Journal of Physical Distribution & Logistics Management, 51(5), 528–550.
29. Jin, J.G., Lee, D.H., Hu, H., 2015. Tactical berth and yard template design at container transshipment terminals: A column generation based approach. Transportation Research Part E, 73, 168–184.
30. Jin, X., Park, K.T., Kim, K.H., 2019. Storage space sharing among container handling companies. Transportation Research Part E, 127, 111-131.
31. Karakas, S., Kirmizi, M., Kocaoglu, B., 2021. Yard block assignment, internal truck operations, and berth allocation in container terminals: introducing carbon-footprint minimisation objectives. Maritime Economics & Logistics 23 (4), 750-771.
32. Kavirathna, C., A., Kawasaki ,T., Hanaoka, S., 2020. Cooperation with a vessel diversion policy for coopetition among container terminals in a single port. Transport Policy, 89, 1–12.

33. Kramer, A., Lalla-Ruiz, E., Iori, M., Voßc, S., 2019. Novel formulations and modeling enhancements for the dynamic berth allocation problem. European Journal of Operational Research, 278, 170–185.
34. Krimi, I., Benmansour, R., Cadi, A., Deshayes, L., Elhachemi, N., 2019. A rolling horizon approach for the integrated multi-quays berth allocation and crane assignment problem for bulk ports. International Journal of Industrial Engineering Computations, 10, 577–591.
35. Krimi, I., Todosijevi, R., Benmansour, R., Ratli, M., Aloullal A., 2020. Modelling and solving the multi-quays berth allocation and crane assignment problem with availability constraints. Journal of Global Optimization, 78, 349–373.
36. Ladier, A.L., Alpan, G., 2018. Crossdock truck scheduling with time windows: earliness, tardiness and storage policies. Journal of Intelligent Manufacturing, 29 (3), 569–583.
37. Lalla-Ruiza, E., González-Velarde, J.L., Melián-Batista, L., Moreno-Vega, J.M., 2014. Biased random keys genetic algorithm for the tactical berth allocation problem. Applied Soft Computing, 22, 60-76.
38. Lee, D.H., Chen, J.H., Cao, J.X., 2010. The continuous berth allocation problem: a greedy randomized adaptive search solution. Transportation Research Part E, 46, 1017–1029.
39. Lee, D.H., Jin, J.G., 2013. Feeder vessel management at container transshipment terminals. Transportation Research Part E, 49, 201–216.
40. Li, M.K., 2014. A method for effective yard template design in container terminals. European Journal of Industrial Engineering, 8 (1), 1–21.
41. Lin, S.M., Zhen, L., Wang, W.C., Tan, Z.Y., 2022. Green berth and yard space allocation under carbon tax policy in tidal ports. Maritime Policy & Management.
42. Liu, B.L., Li, Z., Wang, Y.D., Sheng, D., 2021. Short-term berth planning and ship scheduling for a busy seaport with channel restrictions. Transportation Research Part E, 154, 102467.
43. Liu, M., Lee, C.Y., Zhang, Z.Z., Chu, C.B., 2016. Bi-objective optimization for the container terminal integrated planning. Transportation Research Part B, 93, 720–749.
44. Martin-Iradi, B., Pacino, D., Ropke, S., 2022. The multi-port berth allocation problem with speed optimization: Exact methods and a cooperative game analysis. Transportation Science, 56(4), 972–999.
45. Meisel, F., Bierwirth, C., 2006. Integration of berth allocation and crane assignment to improve the resource utilization at a seaport container terminal. Operations Research Proceedings, 105–110.
46. Meisel, F., Bierwirth, C., 2013. A framework for integrated berth allocation and crane operations planning in sea port container terminals. Transportation Science, 47, 131–147.

47. Monaco, M.F., Sammarra, M., 2007. The berth allocation problem: A strong formulation solved by a Lagrangean approach. Transportation Science, 41(2), 265–280.
48. Moorthy, R., Teo, C.P., 2006. Berth management in container terminal: The template design problem. OR Spectrum, 28, 495–518.
49. Ng, W.C., Mak, K.L., Li, M.K., 2010. Yard planning for vessel services with a cyclical calling pattern. Engineering Optimization, 42 (11), 1039–105.
50. Pang, K., Xu, Z., Li, C., 2011. Ship routing problem with berthing time clash avoidance constraints. International Journal of Production Economics, 131, 752–762.
51. Park, Y.M., Kim, K.H., 2003. A scheduling method for Berth and Quay cranes. OR Spectrum, 25 (1), 1–23.
52. Peng, J., Zhou, Z., Li, R., 2015. A collaborative berth allocation problem with multiple ports based on genetic algorithm. Journal of Coastal Research, 73, 290–297.
53. Petering, M., Murty, K., 2009. Effect of block length and yard crane deployment systems on overall performance at a seaport container transshipment terminal. Computer & Operations Research, 36 (5), 1711–1725.
54. Rodrigues, F., Agra, A., 2021. An exact robust approach for the integrated berth allocation and quay crane scheduling problem under uncertain arrival times. European Journal of Operational Research, 295 (2), 99–516.
55. Saeed, N., Larsen, O.I., 2010. An application of cooperative game among container terminals of one port. European Journal of Operational Research, 203, 393–403.
56. Stahlbock, R., Voss, S., 2008. Operations research at container terminals: a literature update. OR Spectrum, 30 (1), 1–52.
57. Takebayashi, M., Hanaoka, S., 2021. Efficient inter-port cooperation considering port congestion and port charge. Maritime Transport Research, 2, 100011.
58. Thanos, E., Toffolo, T., Santos, H., 2021. The tactical berth allocation problem with time-variant specific quay crane assignments. Computers & Industrial Engineering, 155, 107168.
59. UNCTAD (2019). Review of maritime transport, 2019. Technical Report. United Nations, New York and Geneva.
60. Vandani, B., Shahramfard, S., 2019. A truck scheduling problem at a cross-docking facility with mixed service mode dock doors. Engineering Computations, 36 (6), 1977–2009.
61. Vacca, I., Salani, M., Bierlaire, M., 2013. An exact algorithm for the integrated planning of berth allocation and quay crane assignment. Transportation Science, 47(2), 148–161.
62. Venturini, G., Iris, Ç., Kontovas, C.A., Larsen, A., 2017. The multi-port berth allocation problem with speed optimization and emission considerations. Transportation Research Part D, 54, 142–159.
63. Wang, A.H., Lai, S.H., Mohmand, Y.T., 2014. Evolution of inland container distribution among the cluster of ports in the Greater Pearl River Delta. Transportation

Letters-The International Journal of Transportation Research, 6 (4), 206–21.
64. Wang, K., Zhen, L., Wang, S., Laporte, G., 2018. Column generation for the integrated berth allocation, quay crane assignment, and yard assignment problem. Transportation Science, 52(4), 812–834.
65. Wang, S., Liu, Z., Qu, X., 2015. Collaborative mechanisms for berth allocation. Advanced Engineering Informatics, 29, 332–338.
66. Wang, Z.X., Chan, F.T.S., Chung, S.H., Niu, B, 2014. A decision support method for internal truck employment. Industrial Management & Data Systems, 114 (9), 1378–1395.
67. Wu, S.H., Yang, Z.Z., 2018. Analysis of the case of port co-operation and integration in Liaoning (China). Research in Transportation Business & Management, 26, 18–25.
68. Yang, Z.Z., Xiu, Q.H., Chen, D.X., 2019. Historical changes in the port and shipping industry in Hong Kong and the underlying policies. Transport Policy, 82, 138–147.
69. Zhen, L., 2015. Tactical berth allocation under uncertainty. European Journal of Operational Research, 247, 928–944.
70. Zhen, L., Liang, Z., Zhuge, D., Lee, L.H., Chew, E.P., 2017. Daily berth planning in a tidal port with channel flow control. Transportation Research Part B, 106, 193–217.
71. Zhen, L., Wang S., Wang, K., 2016. Terminal allocation problem in a transshipment hub considering bunker consumption. Naval Research Logistics, 63(7), 529–548.
72. Zheng, J.F., Sun, Z., Gao, Z.Y., 2015. Empty container exchange among liner carriers. Transportation Research Part E, 83, 58–169.