

## Article

# Promoting Liquefied Natural Gas (LNG) Bunkering for Maritime Transportation: Should Ports or Ships Be Subsidized?

Jingwen Qi <sup>1,†</sup>, Hans Wang <sup>2,†</sup> and Jianfeng Zheng <sup>3,\*</sup>

<sup>1</sup> Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China; jingwen.qi@connect.polyu.hk

<sup>2</sup> Faculty of Business, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China; hans.wang@polyu.edu.hk

<sup>3</sup> Transportation Engineering College, Dalian Maritime University, Dalian 116026, China

\* Correspondence: jfzheng@dlmu.edu.cn

† These authors contributed equally to this work.

**Abstract:** Alternative fuels have been recognized as a promising method to alleviate the air emission problem of the maritime industry. LNG, as one of the most promising alternative fuels in shipping, has attracted extensive attentions, and government subsidies are extensively adopted to promote its application. We consider two-stage subsidy methods in this paper and aim to find the optimal subsidy plan under different scenarios. Distinguished from previous studies, we obtain the analytical solution to the subsidy plan optimization model. It is revealed that subsidizing ships or ports performs better in the homogeneous scenario, but a uniform subsidy amount would lead to a waste of subsidy when ships are heterogeneous. Besides, the influence of critical parameters on the optimal LNG selling price are also analyzed, and the conclusions we obtain correspond with the intuition, showing the details of as well as the logic behind such correlations.

**Keywords:** liquefied natural gas (LNG); subsidy plan design; maritime transportation



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## 1. Introduction

Ship air emissions have been considered one of the major problems in the maritime industry [1]. International Maritime Organization (IMO) has set the target to reduce ship emissions by at least 50% by 2050, compared to the level in 2008 [2]. However, with the recovery of the world economy, the global maritime trade volume is expected to increase by 4.3% in 2021 and expand at the annual rate of 2.4% between 2022 and 2026 [1]. At the same time, the world fleet keeps aging, which indicates that the average fuel efficiency tends to drop and the emission per transportation work might increase [1,3]. Combining these two reasons, one of the priorities of the industry is to find effective measures to reduce ship emissions. Alternative marine fuels, as a promising method of relieving this problem, deserve further exploration by the shipping industry, in cooperation with governments [1].

Compared with traditional fossil fuels, LNG has a high net calorific value and low impurity level. Therefore, it has been recognized as a promising alternative fuel for different transportation modes, including road, maritime, and rail transportation [4,5].

As for the application in maritime transportation, LNG has two advantages. First, are the environmental and climate benefits: LNG-fueled ships have obviously lower emission level than traditional ships. According to the estimations of the New South Wales Environment Protection Authority of Australia [6] and Wang and Notteboom [7], LNG-fueled ships reduce the NO<sub>x</sub> emission to 10–15%, CO<sub>2</sub> emission to 80–85%, and SO<sub>x</sub> and PM emission to nearly 0% of the traditional ships' emission level. Second, is the economical benefit: LNG is more cost-effective than traditional marine fuels. The study conducted by the International Maritime Organization [8] reveals that LNG has a more competitive price than marine diesel oil (MDO) and marine gas oil (MGO), which are extensively adopted by traditional ships under the new

sulfur content regulations on marine fuels [9]. Thus, LNG is recognized as a fuel in transition and leading the move to a greener future for the maritime industry [10].

However, with the two advantages, the application of LNG as a marine fuel is still limited. To date, a total of 225 LNG-fueled ships are in operation globally, and 2021 witnessed exponential growth in LNG-fueled deep sea-vessel orders. As for the infrastructure, an LNG bunkering service is available at 141 ports [10]. However, the development of the infrastructure is not geographically balanced; most of the ports with LNG bunkering service are located in Europe [10].

One of the main barriers in the areas where the LNG has not been extensively adopted as a marine fuel is the high construction cost of the LNG bunkering facility and the ship retrofitting cost. The application requires the joint effort and initial investment of the demand and the supply sides, namely the LNG-fueled ships and the LNG bunkering stations. A complete LNG bunkering system and abundant LNG-fueled ships are interdependent, which means that the absence of one side will discourage the other side's enthusiasm for investing and therefore lead to the failure of promoting LNG as a marine fuel. Currently, LNG has not been extensively adopted as a marine fuel, and both sides are still in the preliminary stage, which leads to the "chicken and egg" problem [11,12]. Therefore, in the areas without a complete LNG bunkering system, the application of LNG-fueled ships is very limited.

Being concerned with the environmental issues of the maritime industry, governments become the main impetus to break the dilemma and promote the application of LNG as marine fuel. Governmental subsidy is one of the most commonly used methods. Europe, which is the pioneer in the application of LNG as marine fuel, adopts governmental financial support as the main incentive measure. For example, the European Commission [13] announces a master plan to cover part of the initial investment for the onshore LNG infrastructure in the Rhine-Main-Danube area. According to Bajic [14], 20% of the LNG bunkering vessel building cost of the Port of Algeciras, approximately 11,000,000 EUR (approximately 13,400,000 USD), will be provided by the European Commission. China has conducted the Measures for the Administration of Subsidies for the Standardization of Inland River Ship Types [15], which regulates that newly built LNG-fueled ships with a dead weight tonnage of no less than 400 tons will receive a subsidy between 630,000–1,400,000 CNY (approximately 97,335–216,300 USD).

One of the main management problems in the application of governmental subsidies is to determine the best subsidy plan, including the recipient and the amount they receive. In practice, recipients are the ports that construct LNG bunkering stations and ships that are retrofitted to be fueled by LNG. From the study by Wang et al. [16], we know that subsidizing both the support and demand sides is more effective than focusing on one of them. Besides, the study also reveals that covering only a minor proportion of the investment would be sufficient to significantly promote the application of LNG as marine fuel. In this paper, we consider two-stage subsidy plans and establish a mathematical model to demonstrate the subsidy plan optimization problem. To better understand the relationship between the subsidy plan and the effect achieved, we obtain the analytical solution to the model under different scenarios. It turns out that the optimal subsidy plan is closely related to a group of parameters.

The main contribution of the paper is threefold. First, we propose two-stage subsidy plans that subsidize ports and ships in different ways and then develop optimization models to describe the subsidy design problem. Second, different from previous studies, this paper obtains the analytical solution to the subsidy plan optimization problem. Compared with numerical solution, the analytical solution we obtained provides more information about the correlation between the optimal solution and values of different parameters. Third, in this paper, we compare the relationship between critical parameters and the optimal solution under various scenarios obtained from the analytical solution and logical analysis. It is revealed that they corroborate each other.

The remainder of the paper is organized as follows: Section 2 gives a detailed review of the literature related to this study. Section 3 first describes the problem and presents the opti-

mization model then gives the analysis of the basic scenario. Section 4 analyzes the scenario with heterogeneous ships. Moreover, Section 5 investigates the optimal value of LNG selling price from the government's perspective. Finally the conclusions are set in Section 6.

## 2. Literature Review

In this section, we will review literature related to this paper from three perspectives: (i) the management problems in the application of LNG as marine fuel, (ii) the application of multi-level optimization model in maritime air emission reduction, and (iii) the subsidy design for the promotion of green technologies in maritime transportation.

First, is the literature regarding the management problems in the application of LNG as marine fuel. Multiple studies have been conducted to investigate the willingness of ports and ships to join in the LNG bunkering market.

From the port side, there are two main angles: the angle of a single port and the angle of an area with multiple ports. For a single port, the bunkering method selection and bunkering station layout design are extensively discussed. Tam [17] analyzes the compatibility of shore-to-ship and ship-to-ship bunkering methods from various perspectives. Considering the leaked-gas dispersion, Park et al. [18] investigate the factors impacting the safety zone in ship-to-ship LNG bunkering. Choi et al. [19] study the relationship between the layout of large storage tanks and the LNG leakage gas's flammable limits. From the angle of an area, the bunkering network design is frequently discussed. Various methods have been adopted to investigate the LNG bunkering network design, including the center of gravity method [20], grey forecast model [21], and optimization model [22,23]. Besides, studies based on the situation of a specific country or an area have been conducted to provide solutions that are more practical [24–26]. Peng et al. [27] conduct a systematic review of literature from the port side, including a series of studies related.

For ships, there are two types of main decisions related, namely whether to retrofit the vessel to be dual-fueled and when to conduct the retrofitting work. Schinas and Butler [28] generally analyze the feasibility of LNG-fueled ships from the regulatory and commercial perspectives. More specifically, there are studies focusing on specific vessel types and shipping routes. For example, Kana et al. [29] and Kana and Harrison [30] investigate the influence of the uncertainties in the economic situation, LNG supply chain, and ship emission regulations on the decision whether to retrofit a container ship; Yoo [31], meanwhile, pays attention to the economic applicability of LNG-fueled CO<sub>2</sub> carriers; Xu and Yang [32], on the other hand, focus on the Northern Sea Route and study the economic feasibility of deploying LNG-fueled container ships on it.

The second stream of literature is regarding the application of multi-level optimization model in maritime air emission reduction. In practice, governments and nonprofit organizations are the policy makers, but the shipping industry, including the ports and shipping companies, are the ones that determine the outcome of the regulations. Given the hierarchical structure, multi-level optimization models have been used in studies regarding various ship emission regulations, including Energy Efficiency Design Index (EEDI) [33,34], the Emission Control Areas (ECAs) [35], the Vessel Speed Reduction Incentive Program (VSRIP) [36,37], and the carbon tax [38,39].

The last stream is the literature investigating the subsidy design for the promotion of green technologies in maritime transportation. Wang et al. [40] optimize the subsidy amount provided by a port to improve the utilization rate of its shore power facility. Considering the network effect of the shore power facility construction work in an area, Wu and Wang [41] identify the optimal ports and shipping routes to be subsidized so that more shore power electricity can be consumed. Zhuge et al. [36] aim to design a suitable subsidy plan for the voluntary VSRIP in the proximity of the port to maximize the profit. Wang et al. [16] build a trilevel optimization model to obtain the subsidy rate for ports and ships that can maximize the social benefit.

In all previous studies, Wang et al. [16] is the most similar to this paper. Both of them investigate the subsidy design problem to promote LNG as marine fuel, and consider

the decision of ports and shipping lines as well. However, this paper is essentially distinguished from Wang et al. [16] in three different perspectives. First, in Wang et al. [16] the subsidy amount is set to be proportional to the LNG bunkering station construction cost (for ports) and ship retrofitting cost (for ships), but in this paper we consider two-stage subsidy plans in which one of the two parties is fully and the other is partially subsidized. Second, with different subsidy plan, the optimization model developed in this paper is different from that of Wang et al. [16]. Third, in this paper, based on the analysis, we obtain the analytical solution to the model originally developed. Besides, we also deduct the influence of critical parameters and then compare it with the intuition. Thus, this paper is essentially different from existing studies.

To summarize, due to the complex multi-layer structure, the existing literature seldom investigates the LNG subsidy plan optimization problem from a systematical perspective. The study that focuses on this topic builds an optimization model to capture details of the ports and ships' decision process. However, given the model complexity, the study only yields numerical solutions. The analytical solution and the logic behind the subsidy plan and the reactions of ports and ships are still to be investigated in depth. To fill this research gap, this paper originally develops two-stage subsidy plans and the corresponding optimization models. Analytical solutions to the models under various scenarios were obtained. Besides, the LNG selling price optimization problem is investigated as well. Compared with numerical solutions, analytical solutions and analyses demonstrate the correlations between decisions of different parties while facing the LNG subsidy plan and explain the logic behind them.

### 3. Basic Scenario with Homogeneous Ports and Ships

In this section, we investigate the subsidy plan design problem under the basic scenario with homogeneous ports and ships. In this paper, we consider a government that aims to promote the application of LNG as marine fuel in a shipping route and encourage all ships to consume LNG for power. To accomplish this task, the government decides to provide subsidies for ports on the route and ships sailing along the route. The government needs to make a subsidy plan consisting of the subsidy recipient and the subsidy amount to minimize the total expenditure. Given the government subsidy, the port authorities and ship operators will decide whether to construct the LNG bunkering station and retrofit the vessel according to their own benefit.

#### 3.1. Model Formulation

We consider the following setting: a total of  $M$  identical ships sail along a route (e.g., along the coastline of Australia) repeatedly and visit  $N$  ports uniformly located along the route. The sailing distance between two neighbouring ports is  $L$  nautical miles (nm) and hence the total distance of the route is  $NL$ . Each ship completes the sailing on the route in  $T$  days and then repeats the sailing. The headway between two consecutive ships on the route is  $T/M$  days, i.e., a regular service frequency is provided for the ports on the route.

The ships consume marine diesel oil (MDO) as the fuel for propulsion. The price of MDO is  $\gamma$  (USD/ton) and the consumption rate of MDO by one ship is  $Q$  tons/nm. The tank capacity of the ship is  $QL$  and can sustain the sailing for a distance of  $L$ . Therefore, the ship refuels at each of the  $n$  ports it visits, the price at which is  $\gamma$ .

The government aims to promote the use of liquefied natural gas (LNG) as marine fuel because LNG is much cleaner than MDO. To allow the ships to refuel LNG, ports have to install LNG bunkering infrastructure at the equivalent cost of  $C$  (USD) per  $T$  days. The price at which a port buys LNG, denoted by  $\alpha$  USD/ton, is determined by the LNG market. The selling price to the ship, denoted by  $\beta$  USD/ton, is pre-specified by the government. To use LNG, a ship has to be retrofitted with LNG engines at the equivalent cost of  $c$  (USD) per  $T$  days. (A ship retrofitted with LNG engines has dual-fuel engines: the traditional engines that burn MDO and the new engines that burn LNG; in other words, the ship can burn MDO along a proportion of the route and LNG along the remaining

proportion of the route.) The ships sail at the same speed when burning LNG as the speed when using MDO, and the consumption rate of LNG is  $q$  tons/nm.

We assume

- (i)  $\beta q < \gamma Q$ . That is, the fuel cost per unit sailing distance using LNG is lower than using MDO.
- (ii) After retrofitting, the LNG tank capacity of a ship is  $qL$ . Therefore, a ship will refill a full tank of LNG whenever it visits a port with the LNG bunkering infrastructure.
- (iii)  $N(\gamma QL - \beta qL) > c$ , that is, ship owners will retrofit the ships with LNG engines when all of the ports have constructed LNG bunkering infrastructure. Note that this assumption implies Assumption i.
- (iv)  $M(\beta - \alpha)qL > C$ , that is, if all of the ships have been retrofitted, then all ports will have the motivation to construct LNG bunkering infrastructure.

**Lemma 1.** *If the government fully subsidizes  $N^{\min}$  ports (i.e., offers a subsidy of  $C$  to each port) for constructing LNG bunkering infrastructure, where*

$$N^{\min} = \left\lceil \frac{c}{\gamma QL - \beta qL} \right\rceil, \quad (1)$$

*then, after the  $N^{\min}$  ports have constructed the LNG bunkering infrastructure, all ships will be retrofitted by their owners without government subsidy, and then, the remaining  $N - N^{\min}$  ports will construct the LNG bunkering infrastructure without government subsidy. (In this paper, the subsidy plan works in this way: any port that will construct LNG bunkering infrastructure can submit an application to the government to receive a subsidy of  $C$  and the first  $N^{\min}$  applications will be supported by the government.)*

Since the total number of ships  $M$  is large, we allow a fractional quantity of ships to be subsidized or retrofitted in our model.

**Lemma 2.** *If the government fully subsidizes  $M^{\min}$  ships (i.e., offers a subsidy of  $c$  to each ship) for being retrofitted with LNG engines, where*

$$M^{\min} = \frac{C}{(\beta - \alpha)qL}, \quad (2)$$

*then, after the  $M^{\min}$  ships have been retrofitted, all ports will construct the LNG bunkering infrastructure without government subsidy, and then, the remaining  $M - M^{\min}$  ships will be retrofitted by their owners without the government subsidy. (In this paper, the subsidy plan works in this way: any ship that will be retrofitted with LNG engines can submit an application to the government to receive a subsidy of  $c$  and the first  $M^{\min}$  applications will be supported by the government.)*

Lemmas 1 and 2 show that, to enable all ships to replace MDO with LNG, the minimum amount of subsidy the government has to provide does not exceed  $\min\{CN^{\min}, cM^{\min}\}$ . Next, we examine two mixed subsidy plans—both ports and ships may be subsidized in each plan—in the next two subsections.

### 3.2. Mixed Subsidy Plan 1: Fully Subsidize Ports First and Then Partially Subsidize Ships

The first mixed subsidy plan is more general than the one in Lemma 1. In the first mixed subsidy plan, the decision process is as follows.

**Step 1:** The government fully subsidizes  $n$  ports to construct LNG bunkering infrastructure,  $n = 0, \dots, N^{\min}$ , i.e., provides a subsidy of  $C$  USD to each of the  $n$  port.

**Step 2:** The government subsidizes  $M^{\min}$  ships by providing a subsidy of  $x$  USD to each ship's owner,  $0 \leq x \leq c$ , making sure that the ship owner has the motivation



to spend  $c - x$  USD in retrofitting the ship in view that  $n$  ports have already constructed the LNG bunkering infrastructure.

**Step 3:** The remaining  $N - n$  ports construct LNG bunkering infrastructure by themselves because they will make a profit from it and the remaining  $M - M^{\min}$  ships will be retrofitted with LNG engines by their owners to save fuel costs.

The government aims to design a subsidy plan that has the lowest cost. Define  $\mathbb{Z}_+$  as the set of non-negative integers. The problem faced by the government can be formulated as:

$$\text{P1:} \quad \min Cn + M^{\min}x \quad (3)$$

subject to

$$(\gamma QL - \beta qL)n \geq c - x \quad (4)$$

$$n \leq N^{\min} \quad (5)$$

$$n \in \mathbb{Z}_+ \quad (6)$$

$$x \leq c \quad (7)$$

$$x \geq 0. \quad (8)$$

**Theorem 1.** An optimal solution to model P1 is shown in Table 1.

**Proof.** Case i:  $cM^{\min} < C\frac{c}{\gamma QL - \beta qL}$ . Let  $(n^{\#}, x^{\#})$  be the optimal solution. If  $1 \leq n^{\#} \leq N^{\min} - 1$ , Equation (4) implies  $x^{\#} = (\gamma QL - \beta qL)n^{\#} - c$ . Now, consider a different solution  $(n, x) = (n^{\#} - 1, x^{\#} + \gamma QL - \beta qL)$ . This new solution is feasible and its objective value is

$$\begin{aligned} & C(n^{\#} - 1) + M^{\min}(x^{\#} + \gamma QL - \beta qL) \\ &= Cn^{\#} + M^{\min}x^{\#} + M^{\min}(\gamma QL - \beta qL) - C < Cn^{\#} + M^{\min}x^{\#}. \end{aligned} \quad (9)$$

This contradicts the optimality of  $(n^{\#}, x^{\#})$ . If  $n^{\#} = N^{\min}$ , then  $x^{\#} = 0$ . Now, consider a different solution  $(n, x) = (0, c)$ . This new solution is feasible and its objective value is  $M^{\min}c < C\frac{c}{\gamma QL - \beta qL} \leq CN^{\min}$ . This contradicts the optimality of  $(n^{\#}, x^{\#})$ .

Case ii:  $C\frac{c}{\gamma QL - \beta qL} \leq cM^{\min} < CN^{\min}$ . Let  $(n^{\#}, x^{\#})$  be an optimal solution. If  $n^{\#} = N^{\min}$ ,  $x^{\#} = 0$ . Then, its objective value is larger than that of

$$(n, x) = (N^{\min} - 1, C(\frac{c}{\gamma QL - \beta qL} - N^{\min} + 1)\frac{1}{M^{\min}}). \quad (10)$$

If  $n^{\#} \leq N^{\min} - 2$ , we can increase  $n^{\#}$  by 1 and decrease  $x^{\#}$  by  $\gamma QL - \beta qL$  and this will not increase the objective value; repeating the above procedure, we will obtain an optimal solution with  $n = N^{\min} - 1$ .

Case iii:  $CN^{\min} \leq cM^{\min}$ . Let  $(n^{\#}, x^{\#})$  be an optimal solution,  $n^{\#} \leq N^{\min} - 1$ . Now, consider a different solution  $(n, x) = (n^{\#} + 1, x^{\#} - (\gamma QL - \beta qL))$ . This new solution is feasible and its objective value is

$$\begin{aligned} & C(n^{\#} + 1) + M^{\min}(x^{\#} - (\gamma QL - \beta qL)) = Cn^{\#} + M^{\min}x^{\#} + C - M^{\min}(\gamma QL - \beta qL) \\ & \leq Cn^{\#} + M^{\min}x^{\#} + C - M^{\min}\frac{c}{N^{\min}} \leq Cn^{\#} + M^{\min}x^{\#}, \end{aligned} \quad (11)$$

showing the optimality of the new solution; repeating the above procedure, we will obtain an optimal solution with  $n = N^{\min}$ .  $\square$

Case ii of Table 1 is noteworthy. Case ii exists because the decision variable  $n$  in model P1 must be an integer. For example, if  $c/(\gamma QL - \beta qL) = 3.7$ , then  $N^{\min} = 4$ ; in this situation, if the total construction cost of LNG bunkering infrastructure at 3.7 ports is smaller than the total cost of retrofitting  $M^{\min}$  ships but the total construction cost at four

ports is larger than that of the ships, then the government should subsidize three ports and provide a proportion of the retrofitting cost to the  $M^{\min}$  ships.

**Table 1.** Optimal solution to model P1.

	Condition	$n^*$	$x^*$	Total Subsidy
Case i	$cM^{\min} < C \frac{c}{\gamma QL - \beta qL}$ , i.e., $C > (\gamma QL - \beta qL)M^{\min}$	0	$c$	$cM^{\min}$
Case ii	$C \frac{c}{\gamma QL - \beta qL} \leq cM^{\min} < CN^{\min}$	$N^{\min} - 1$	$C(\frac{c}{\gamma QL - \beta qL} - \frac{1}{N^{\min} + 1}) \frac{1}{M^{\min}}$	$C \frac{c}{\gamma QL - \beta qL}$
Case iii	$CN^{\min} \leq cM^{\min}$	$N^{\min}$	0	$CN^{\min}$

Different cases of the optimal solution to model P1 under scenario with homogeneous ships and ports.

### 3.3. Mixed Subsidy Plan 2: Fully Subsidize Ships and Then Partially Subsidize Ports

The second mixed subsidy plan is more general than the one in Lemma 2. In the second mixed subsidy plan, the decision process is as follows.

**Step 1:** The government fully subsidizes  $m$  ships to be retrofitted with LNG engines, i.e., provides a subsidy of  $c$  USD to each of the  $m$  ships. Since the total number of ships  $M$  is large, we allow  $m$  to be a fractional quantity, i.e.,  $m \in [0, M^{\min}]$ .

**Step 2:** The government provides a subsidy of  $y$  USD to each of  $N^{\min}$  ports,  $0 \leq y \leq C$ , making sure that the port has the motivation to spend  $C - y$  USD in constructing LNG bunkering infrastructure, in view that  $m$  ships have already been retrofitted with LNG engines.

**Step 3:** The remaining  $M - m$  ships will be retrofitted with LNG engines by their owners because they will thereby save fuel costs and the remaining  $N - n$  ports will construct LNG bunkering infrastructure by themselves because they will make a profit from it.

The government aims to design a subsidy plan that has the lowest cost, and this problem can be formulated as:

$$\text{P2:} \quad \min cm + N^{\min}y \quad (12)$$

subject to

$$(\beta - \alpha)QLm \geq C - y \quad (13)$$

$$m \leq M^{\min} \quad (14)$$

$$m \geq 0 \quad (15)$$

$$y \leq C \quad (16)$$

$$y \geq 0. \quad (17)$$

**Theorem 2.** An optimal solution to model P2 is shown in Table 2.

**Table 2.** Optimal solution to model P2.

	Condition	$m^*$	$y^*$	Total Subsidy
Case i	$CN^{\min} < \frac{cC}{(\beta - \alpha)QL}$ , i.e., $c > (\beta - \alpha)QLN^{\min}$	0	$C$	$CN^{\min}$
Case ii	$c \leq (\beta - \alpha)QLN^{\min}$	$M^{\min}$	0	$cM^{\min}$

Different cases of the optimal solution to model P2 under scenario with homogeneous ships and ports.

**Proof.** Case i:  $c > (\beta - \alpha)QLN^{\min}$ . Let  $(m^{\#}, y^{\#})$  be the optimal solution,  $m^{\#} > 0$ . Now, consider a different solution  $(m, y) = (0, y^{\#} + (\beta - \alpha)QLm)$ . This new solution is feasible and its objective value is

$$0 + N^{\min}(y^{\#} + (\beta - \alpha)QLm) = N^{\min}(\beta - \alpha)QLm^{\#} + N^{\min}y^{\#} < cm^{\#} + N^{\min}y^{\#}. \quad (18)$$

This contradicts the optimality of  $(m^{\#}, y^{\#})$ . Thus,  $(0, C)$  is the optimal solution in case (i).

Case ii:  $c \leq (\beta - \alpha)QLN^{\min}$ . Let  $(m^{\#}, y^{\#})$  be an optimal solution,  $m^{\#} < M^{\min}$ . Now, consider a different solution  $(m, y) = (M^{\min}, y^{\#} - (\beta - \alpha)QL(M^{\min} - m^{\#}))$ . This new solution is feasible and its objective value is

$$\begin{aligned} & cM^{\min} + N^{\min}(y^{\#} - (\beta - \alpha)QL(M^{\min} - m^{\#})) \\ & \leq cM^{\min} + N^{\min}(y^{\#} - \frac{c}{N^{\min}}(M^{\min} - m^{\#})) = cm^{\#} + N^{\min}y^{\#}, \end{aligned} \quad (19)$$

showing that the new solution is optimal and contradicting the optimality of  $(m^{\#}, y^{\#})$ . Thus, in case (ii),  $(M^{\min}, 0)$  is the optimal solution.  $\square$

### 3.4. Main Findings in the Basic Scenario

Tables 1 and 2 show that mixed subsidy plans do not significantly outperform “pure” subsidy plans in Lemmas 1 and 2. In general, the government should adopt a “pure” subsidy plan: either subsidize ports only, or subsidize ships only. In reality, however, many governments offer subsidy to both ports and ships, which is inefficient based on the above analysis. The reason is that, in reality, ships and ports are heterogeneous in various aspects including but not limited to retrofitting cost, fuel consumption rate, and remaining useful life. In the next section, the scenario with ships of different fuel consumption rates will be discussed.

## 4. Scenario with Heterogeneous Ships

However, our analysis has a drawback: it assumes the ports are homogeneous and the ships are homogeneous. Next, we will examine cases with heterogeneous ships.

Instead of identical ships, in this scenario we consider ships with different fuel consumption rates. The MDO and LNG consumption rates of ship  $j$  are denoted by  $Q_j$  and  $q_j$ , respectively, and it is assumed that  $Q_j < Q_i$  and  $q_j < q_i$  for  $j \in [0, M], i < j$ . Since we allow a fractional quantity of ships to be subsidized or retrofitted in our model, we further assume that the MDO and LNG consumption rates of different ships are averagely distributed in the domains  $[Q_M, Q_0]$  and  $[q_M, q_0]$ . As a result, the value of  $q_j$  and  $Q_j, j \in [0, M]$  can be calculated as  $q_j = q_M + (q_0 - q_M)\frac{M-j}{M}$  and  $Q_j = Q_M + (Q_0 - Q_M)\frac{M-j}{M}$ .

We assume

- (i) After retrofitting, the LNG tank capacity of ship  $j$  is  $q_jL, j = 1, \dots, M$ . Therefore, a ship will refill a full tank of LNG whenever it visits a port with LNG bunkering infrastructure.
- (ii)  $N(\gamma Q_jL - \beta q_jL) \geq N(\gamma Q_ML - \beta q_ML) > c, j = 1, \dots, M$ , that is, ship owners will retrofit the ships with LNG engines when all of the ports have constructed LNG bunkering infrastructure.
- (iii)  $M(\beta - \alpha) \int_{q_M}^{q_0} x dxL = (\beta - \alpha) \frac{q_0^2 - q_M^2}{2}L > C, j = 1, \dots, M$ , that is, if all of the ships have been retrofitted, then all ports will have the motivation to construct LNG bunkering infrastructure.

**Lemma 3.** If the government fully subsidizes  $N^{\min}$  ports (i.e., offers a subsidy of  $C$  to each port) for constructing LNG bunkering infrastructure, where

$$N^{\min} = \left\lceil \frac{c}{\gamma Q_ML - \beta q_ML} \right\rceil, \quad (20)$$



then, after the  $N^{\min}$  ports have constructed LNG bunkering infrastructure, all ships will be retrofitted by their owners without government subsidy, and then, the remaining  $N - N^{\min}$  ports will construct LNG bunkering infrastructure without the government subsidy.

**Lemma 4.** If the government fully subsidizes  $M^{\min}$  ships (i.e., offers a subsidy of  $c$  to each ship) for being retrofitted with LNG engines, where

$$q_{M^{\min}} = \sqrt{q_0^2 - \frac{2C}{(\beta - \alpha)L}}, M^{\min} = M - \frac{M}{q_0 - q_M} \left( \sqrt{q_0^2 - \frac{2C}{(\beta - \alpha)L}} - q_M \right) \quad (21)$$

then, after the  $M^{\min}$  ships have been retrofitted, all ports will construct LNG bunkering infrastructure without government subsidy, and then, the remaining  $M - M^{\min}$  ships will be retrofitted by their owners without a government subsidy. (In this paper, the subsidy plan works in this way: any ship that will be retrofitted with LNG engines can submit an application to the government to receive a subsidy of  $c$  and the applications of ship 1 to ship  $M^{\min}$  will be supported by the government since they consume more LNG and provide higher environmental benefits after retrofitting.)

Lemmas 3 and 4 show that, to enable all ships to replace MDO with LNG, the minimum amount of subsidy the government has to provide does not exceed  $\min\{CN^{\min}, cM^{\min}\}$ . Next, we examine two mixed subsidy plans—both ports and ships may be subsidized in each plan—in the next two subsections. Following Sections 3.2 and 3.3, the mixed subsidy plans with heterogeneous ships are stated as follows.

#### 4.1. Mixed Subsidy Plan 1 with Heterogeneous Ships

In the mixed subsidy plan with heterogeneous ships, the decision process is as follows.

**Step 1:** The government fully subsidizes  $n$  ports to construct LNG bunkering infrastructure,  $n = 0, \dots, N^{\min}$ , i.e., provides a subsidy of  $C$  USD to each of the  $n$  port.

**Step 2:** The government subsidizes ship 1 to ship  $M^{\min}$  by providing a subsidy of  $x$  USD to each ship's owner,  $0 \leq x \leq c$ , making sure that the ship owner has the motivation to spend  $c - x$  USD in retrofitting the ship in view that  $n$  ports have already constructed the LNG bunkering infrastructure.

**Step 3:** The remaining  $N - n$  ports construct LNG bunkering infrastructure by themselves because they will make a profit from it and the remaining  $M - M^{\min}$  ships will be retrofitted with LNG engines by their owners to save fuel costs.

Different from the subsidy plan in Section 3.2, the second step of the Mixed Subsidy Plan 1 under this scenario chooses ports with the larger fuel consumption rates, namely ship 1 to port  $M^{\min}$ , to minimize the total subsidy cost. This problem can be formulated as:

$$P1': \quad \min Cn + M^{\min}x \quad (22)$$

subject to constraints (5)–(8) and

$$(\gamma Q_{M^{\min}}L - \beta q_{M^{\min}}L)n \geq c - x. \quad (23)$$

Constraint (23) assures that ship  $M^{\min}$  will be retrofitted, which indicates that ship 1 to ship  $M^{\min} - 1$  will also be retrofitted because their LNG consumption volumes and bunker savings are higher than ship  $M^{\min}$ .

**Theorem 3.** An optimal solution to model P1' is shown in Table 3.

**Table 3.** Optimal solution to model P4.

	Condition	$n^*$	$x^*$	Total Subsidy
Case i	$cM^{\min} < C \frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L}$	0	$c$	$cM^{\min}$
Case ii	$C \frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L} \leq cM^{\min} < CN^{\min}$	$N^{\min} - 1$	$C(\frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L} - \frac{1}{N^{\min} + 1})$	$C \frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L}$
Case iii	$CN^{\min} \leq cM^{\min}$	$N^{\min}$	0	$CN^{\min}$

Different cases of the optimal solution to model P1' under scenario with heterogeneous ships.

**Proof.** Case i:  $cM^{\min} < C \frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L}$ . Let  $(n^{\#}, x^{\#})$  be the optimal solution. If  $1 \leq n^{\#} \leq N^{\min} - 1$ , Equation (23) implies  $x^{\#} = (\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L)n^{\#} - c$ . Now, consider a different solution  $(n, x) = (n^{\#} - 1, x^{\#} + \gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L)$ . This new solution is feasible and its objective value is

$$\begin{aligned} & C(n^{\#} - 1) + M^{\min}(x^{\#} + \gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L) \\ &= Cn^{\#} + M^{\min}x^{\#} + M^{\min}(\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L) - C < Cn^{\#} + M^{\min}x^{\#}. \end{aligned} \quad (24)$$

If  $n^{\#} = N^{\min}$ , then  $x^{\#} = 0$ . Now, consider a different solution  $(n, x) = (0, c)$ . This new solution is feasible and its objective value is  $M^{\min}c < C \frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L} \leq CN^{\min}$ . This contradicts the optimality of  $(n^{\#}, x^{\#})$ .

Case ii:  $C \frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L} \leq cM^{\min} < CN^{\min}$ . Let  $(n^{\#}, x^{\#})$  be an optimal solution. If  $n^{\#} = N^{\min}$ ,  $x^{\#} = 0$ . Then, its objective value is larger than that of

$$(n, x) = (N^{\min} - 1, C(\frac{c}{\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L} - N^{\min} + 1) \frac{1}{M^{\min}}). \quad (25)$$

If  $n^{\#} \leq N^{\min} - 2$ , we can increase  $n^{\#}$  by 1 and decrease  $x^{\#}$  by  $\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L$  and this will not increase the objective value; repeating the above procedure, we will obtain an optimal solution with  $n = N^{\min} - 1$ .

Case iii:  $CN^{\min} \leq cM^{\min}$ . Let  $(n^{\#}, x^{\#})$  be an optimal solution,  $n^{\#} \leq N^{\min} - 1$ . Now, consider a different solution  $(n, x) = (n^{\#} + 1, x^{\#} - (\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L))$ . This new solution is feasible and the corresponding objective value is

$$\begin{aligned} & C(n^{\#} + 1) + M^{\min}(x^{\#} - (\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L)) \\ &= Cn^{\#} + M^{\min}x^{\#} + C - M^{\min}(\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L) \\ &\leq Cn^{\#} + M^{\min}x^{\#} + C - M^{\min} \frac{c}{N^{\min}} \leq Cn^{\#} + M^{\min}x^{\#}, \end{aligned} \quad (26)$$

showing the optimality of the new solution; repeating the above procedure, we will obtain an optimal solution with  $n = N^{\min}$ .  $\square$

From constraint (23) we can see that the partial subsidy for ships makes the benefit of retrofitting ship  $M^{\min}$  equals the costs, namely  $(\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L)n = c - x$  in the optimal solution. However, for ship  $j$ ,  $1 \leq j < M^{\min}$ , we have  $(\gamma Q_j L - \beta q_j L)n > c - x$ , namely the amount of  $[(\gamma Q_j L - \beta q_j L)n] - [(\gamma Q_{M^{\min}} L - \beta q_{M^{\min}} L)n]$  is wasted in ship  $j$ . Therefore, the optimal mixed subsidy plan awards larger amount of subsidies than necessary.

#### 4.2. Mixed Subsidy Plan 2 with Heterogeneous Ships

The decision process in the second mixed subsidy plan with heterogeneous is as follows.

**Step 1:** The government fully subsidizes  $m$  ships to be retrofitted with LNG engines, i.e., provides a subsidy of  $c$  USD to each of the  $m$  ships. Since the total number of ships  $M$  is large, we allow  $m$  to be a fractional quantity, i.e.,  $m \in [0, M^{\min}]$ .

**Step 2:** The government provides a subsidy of  $y$  USD to each of  $N^{\min}$  ports,  $0 \leq y \leq C$ , making sure that the port has the motivation to spend  $C - y$  USD in constructing LNG bunkering infrastructure in view that  $m$  ships have already been retrofitted with LNG engines.

**Step 3:** The remaining  $M - m$  ships will be retrofitted with LNG engines by their owners because they will thereby save fuel costs and the remaining  $N - n$  ports will construct LNG bunkering infrastructure by themselves because they will make a profit from it.

The government aims to design a subsidy plan that has the lowest cost, and this problem can be formulated as:

$$P2': \quad \min cm + N^{\min}y \quad (27)$$

subject to constraint (14) to (17) and

$$(\beta - \alpha)Lm \int_{Q_{M^{\min}}}^{Q_0} x dx \geq C - y. \quad (28)$$

Constraint (28) can be rewritten as

$$\frac{(\beta - \alpha)Lm(Q_0^2 - Q_{M^{\min}}^2)}{2} \geq C - y. \quad (29)$$

**Theorem 4.** An optimal solution to model P2' is shown in Table 4.

**Table 4.** Optimal solution to model P2.

	Condition	$m^*$	$y^*$	Total Subsidy
Case i	$CN^{\min} < \frac{cC}{(\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)L'}$ i.e., $c > (\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)LN^{\min}$	0	C	$CN^{\min}$
Case ii	$c \leq (\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)LN^{\min}$	$M^{\min}$	0	$cM^{\min}$

Different cases of the optimal solution to model P2' under scenario with heterogeneous ships.

**Proof.** Case i:  $c > (\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)LN^{\min}$ . Let  $(m^{\#}, y^{\#})$  be the optimal solution,  $m^{\#} > 0$ . Now consider a different solution  $(m, y) = (0, y^{\#} + \frac{(\beta - \alpha)Lm^{\#}(Q_0^2 - Q_{M^{\min}}^2)}{2})$ . This new solution is feasible and its objective value is

$$\begin{aligned} & 0 + N^{\min} \left( y^{\#} + \frac{(\beta - \alpha)Lm^{\#}(Q_0^2 - Q_{M^{\min}}^2)}{2} \right) \\ &= N^{\min}(\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)Lm^{\#} + N^{\min}y^{\#} < cm^{\#} + N^{\min}y^{\#} \end{aligned} \quad (30)$$

This contradicts the optimality of  $(m^{\#}, y^{\#})$ . Thus,  $(0, C)$  is the optimal solution.

Case ii:  $c \leq (\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)LN^{\min}$ . Let  $(m^{\#}, y^{\#})$  be an optimal solution,  $m^{\#} < M^{\min}$ . Now, consider a different solution

$$(m, y) = (M^{\min}, y^{\#} - (\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)L(M^{\min} - m^{\#})). \quad (31)$$

This new solution is feasible and its objective value is

$$\begin{aligned} & cM^{\min} + N^{\min}(y^{\#} - (\beta - \alpha)(Q_0^2 - Q_{M^{\min}}^2)L(M^{\min} - m^{\#})) \\ & \leq cM^{\min} + N^{\min}(y^{\#} - \frac{c}{N^{\min}}(M^{\min} - m^{\#})) = cm^{\#} + N^{\min}y^{\#}, \end{aligned} \quad (32)$$

showing that the new solution is optimal and contradicting the optimality of  $(m^{\#}, y^{\#})$ . Thus, in case (ii),  $(M^{\min}, 0)$  is the optimal solution.  $\square$

Compared with the first subsidy plan in Section 4.1, the optimal subsidy amount does not seem to exceed the necessary level. However, because all ships receive the same amount of subsidy, the value of  $N^{\min}$  depends on the ship with the lowest fuel consumption rate, namely the least motivated ship. A more flexible subsidy plan that customizes the subsidy amount for each ship would further reduce the value of  $M^{\min}$  and the optimal total subsidy amount.

#### 4.3. Main Findings in the Scenario with Heterogeneous Ships

In this section, we investigate the scenario with heterogeneous ships. Compared with the cases of homogeneous ships, the two mixed subsidy plans in this section perform in a very similar way but with a different method to calculate the value of  $M^{\min}$ . However, considering the various retrofitting costs, the plan with uniform subsidy amount for all ships would lead to a waste of budget. Based on the analysis, it is revealed that with heterogeneous ships, a more flexible subsidy plan could further reduce the total subsidy amount.

### 5. Optimal LNG Selling Price

We have also assumed that the selling price of LNG to ships  $\beta$  is fixed in our analysis. However, the selling price would influence the subsidy amount because it determines the LNG selling profit of the ports and the bunker cost savings of ships by using LNG, which are the main driven forces behind the adoption of LNG as marine fuel. Therefore, in this section, we will examine how the government can design this price optimally to minimize the total subsidy amount.

#### 5.1. Optimal Value of $\beta$

From Table 1 in Section 3.2 we can see that in case i and case iii, the total subsidy amount is independent of  $\beta$ . In case ii, the total subsidy amount equals  $C \frac{c}{\gamma QL - \beta qL}$ , and the first derivative of it over  $\beta$  can be calculated as

$$\left( C \frac{c}{\gamma QL - \beta qL} \right)' = \frac{Cc q}{L(\gamma Q - \beta q)^2} \geq 0, \quad (33)$$

which means that the subsidy amount increases with the value of  $\beta$ . Therefore, the task is to identify the lower bound of the value of  $\beta$ , which depends on the problem assumptions and case conditions.

For assumption (i) and (iii), we have  $\beta < \frac{\gamma Q}{q}$  and  $\beta < \frac{\gamma Q_{NL} - c}{q_{NL}}$ , which impact the upper bound of the  $\beta$  value. In assumption (iv), we have  $M(\beta - \alpha)qL > C$ , which indicates  $\beta > \alpha + \frac{C}{MqL}$ . Besides, from the condition of case ii, we obtain two constraints on  $\beta$  value. The first one is  $\frac{Cc}{\gamma QL - \beta qL} \leq \frac{Cc}{(\beta - \alpha)qL}$ , which can be rewritten as  $\beta \geq \frac{\gamma Q + q\alpha}{2q}$ . The second one is  $\frac{Cc}{(\beta - \alpha)qL} < C \lceil \frac{c}{\gamma QL - \beta qL} \rceil$ , which indicates an upper bound of  $\beta$  value.

Thus, under case ii, we have

**Theorem 5.** The optimal value of  $\beta$  under case ii of model P1 is shown in Table 5.

**Table 5.** Optimal value of  $\beta$ .

	Condition	$\beta^*$	Total Subsidy
Case a	$\frac{\gamma Q}{2q} + \frac{\alpha}{2} \geq \alpha + \frac{C}{MqL}$	$\frac{\gamma Q}{2q} + \frac{\alpha}{2}$	$\frac{2Cc}{\gamma QL - \alpha qL}$
Case b	$\frac{\gamma Q}{2q} + \frac{\alpha}{2} < \alpha + \frac{C}{MqL}$	$\left(\alpha + \frac{C}{MqL}\right)^+$	$\left(\frac{cCM}{M\gamma QL - \alpha qLM - C}\right)^+$

Different cases of the optimal  $\beta$  of model P1 under scenario with homogeneous ships and ports.

**Proof.** In case (a), we have  $\frac{\gamma Q}{2q} + \frac{\alpha}{2} \geq \alpha + \frac{C}{MqL}$ , namely  $\alpha \leq \frac{\gamma QML - 2C}{qML}$ . Thus, the lower bound of  $\beta$  value equals  $\frac{\gamma Q}{2q} + \frac{\alpha}{2}$ , which is also the optimal value and the corresponding total subsidy amount equals  $\frac{2Cc}{\gamma QL - \alpha qL}$ . In case (b), we have  $\frac{\gamma Q}{2q} + \frac{\alpha}{2} < \alpha + \frac{C}{MqL}$ , namely  $\alpha < \frac{\gamma QML - 2C}{qML}$ . In this case, we have  $\beta > \alpha + \frac{C}{MqL}$ , and the optimal  $\beta$  value can be stated as  $\beta^* = \left(\alpha + \frac{C}{MqL}\right)^+$ . Meanwhile, the total subsidy amount can be calculated as  $\left(\frac{cCM}{M\gamma QL - \alpha qLM - C}\right)^+$ .  $\square$

### 5.2. Impact of $\alpha$ and $\gamma$ on Optimal $\beta$

As displayed in Table 5, the optimal LNG selling price  $\beta^*$  and the corresponding total subsidy amount are closely related to the LNG purchasing price of ports  $\alpha$  and the MDO price  $\gamma$ . In Table 6, we list the first derivative of  $\beta^*$  and the corresponding total subsidy amount. Since we cannot obtain the optimal value of  $\beta$  and total subsidy amount in case (b), we take the derivatives of the lower bounds instead.

**Table 6.** The first derivative of  $\beta^*$  and total subsidy over  $\alpha$  and  $\gamma$ .

	Derivative over $\alpha$		Derivative over $\gamma$	
	$\beta^*$	Total Subsidy	$\beta^*$	Total Subsidy
Case a	$\frac{1}{2}$	$\frac{2cCqL}{(\gamma QL - \alpha qL)^2}$	$\frac{Q}{2q}$	$\frac{-2CcQL}{(\gamma QL - \alpha qL)^2}$
Case b	1	$\frac{cCqLM^2}{(M\gamma QL - \alpha qLM - C)^2}$	0	$\frac{-cCM^2QL}{(M\gamma QL - \alpha qLM - C)^2}$

Different cases of derivatives over  $\alpha$  and  $\gamma$  under scenario with homogeneous ships and ports.

First we discuss the influence of  $\alpha$  on the optimal LNG selling price and subsidy amount. Under case (a) and case (b), according to Table 6, we know that both the value of  $\beta^*$  and the corresponding optimal total subsidy amount increase with the value of  $\alpha$ . This conclusion is intuitive. When the LNG purchasing price  $\alpha$  increases, the profit of selling one tonne of LNG for ports, which is denoted by  $\beta - \alpha$ , decreases. Responding to this, the ports require higher LNG selling price and higher subsidies to be motivated to build the LNG bunkering stations. Given the higher selling price, the ships also need larger amount of subsidies to be encouraged. As a result, both the optimal LNG selling price  $\beta^*$  and total subsidy amount increase with  $\alpha$ .

Next is the influence of the MDO price  $\gamma$ . In case (a), according to Table 6,  $\beta^*$  increases with  $\gamma$  while the total subsidy amount decreases with it. The pattern is comprehensible. When  $\gamma$  increases, the bunker cost saving of using LNG to sail one nautical mile, which equals  $Q\gamma - q\beta$ , increases too. Therefore, the ships would be retrofitted with a lower subsidy level and a higher LNG price  $\beta$ . For the ports, increased LNG selling price means higher profits, which indicates that the subsidy provided for ports can be further reduced. As a result, in case (a), the optimal selling price  $\beta^*$  increases with  $\gamma$  and the corresponding total subsidy amount decreases with it. In case (b), the logic is the same, but the value of  $\beta^*$  is independent of  $\gamma$ . The reason of this is that in case (b), the value of  $\beta^*$  is constrained by assumption iv)  $M(\beta - \alpha)qL > c$ , which assures that all ports would be motivated to build LNG bunkering stations when all ships are retrofitted. Since the assumption is not related to the price of MDO, the optimal subsidy value  $\beta^*$  is independent of  $\gamma$ .



### 5.3. Main Findings in the LNG Selling Price Optimization

In this section, we obtain the optimal LNG selling price that minimizes the total subsidy amount. Based on the result, we further analyze the impact of the LNG purchasing price of ports and the MDO price on the optimal LNG selling price. From the analysis we can see that the optimal LNG selling price increases with LNG purchasing price and decreases with the MDO bunkering price. The patterns obtained from our analytical analysis and the logical judgment corroborate each other.

## 6. Conclusions

LNG has been recognized as one of the most promising alternative fuels for maritime transportation. However, the costly LNG bunkering station and ship retrofiting makes the ports and shipping lines hesitate about making the first move. Given the current situation, government subsidies have been proofed to be an effective method to break the deadlock. The total subsidy amount and its effect depend on the specific subsidy plan, and therefore the optimization of the subsidy design deserves to be studied in depth.

This paper originally proposes two-stage subsidy plans to promote LNG as marine fuel and optimization models to describe the problem under different scenarios. Different from the existing literature, we obtain the analytical solution to the models. In the basic scenario, the subsidy plans that subsidize only the ships or ports perform better than those that subsidize both of them. In scenarios with heterogeneous ships, from the analyses, we know that subsidy plans with uniform subsidy amount for all ships will lead to a waste of budget. Thus, with the same promotion effect, customized subsidy amount for each ship would decrease the total subsidy expenditure. Moreover, the analytical results demonstrate the relationship between different parameters clearer than the numerical solutions yielded by previous studies. Besides, we also obtain the optimal LNG bunkering price and analyze the impact of LNG purchasing price for ports and the MDO bunkering price on the optimal LNG bunkering price. The result shows that the analytical analysis and the logic intuition corroborate each other.

In this paper, we consider the two-stage subsidy plan, and in further research, more flexible subsidy plans could be considered to further reduce the total subsidy amount. Moreover, the LNG supply chain can be extended, for example, the refueling of LNG storage tanks at ports can be considered in the analysis.

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## Abbreviations

The following abbreviations are used in this manuscript:

LNG	Liquefied natural gas
IMO	International Maritime Organization
MDO	Marine diesel oil
MGO	Marine gas oil

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