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A two-stage stochastic programming model of coordinated electric bus charging scheduling for a hybrid charging scheme

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ABSTRACT

This paper proposes a coordinated charging scheduling approach for battery electric buses (BEBs) in a hybrid charging scheme, i.e., both plug-in fast charging and battery-swapping charging modes are incorporated in a single charging station. To accommodate the uncertain battery energy consumption during bus operation, a two-stage stochastic program is formulated, where the first stage decision determines the battery inventory level of each station and the second stage determines the charging mode and designs when, where, and how long each bus should be charged. Future uncertainties associated with energy consumption are captured by a set of possible discrete scenarios from historical data. A progressive hedging algorithm is developed to decompose the two-stage stochastic program into sub-problems. A case study is conducted to verify the proposed models and solution algorithms.

1. Introduction

The last two decades have witnessed a fast development of transit electrification around the world. The fast-growing adoption of the electrified transit system is mainly due to its immense potential in reducing fossil fuel consumption as well as greenhouse emissions resulting in a sustainable urban transportation system (An et al., 2020; Basma et al., 2020; Huang et al., 2022). To achieve ambitious climate goals, e.g., net-zero carbon dioxide emissions in 2060 (State Council of the People's Republic of China, 2020), many cities in China have become active to promote preferential policies for the replacement with battery electric buses (BEBs). For instance, Shenzhen had replaced all conventional fossil fuel-powered buses with BEBs by the end of 2017 (Zhang et al., 2021). Except for the decisions from planning level, designing a cost-effective BEB charging schedule is equally important by making the best use of all existing resources, including charging stations, charging piles, and power grid (Abdelwahed et al., 2020; He et al., 2020; Bie et al., 2021).

With the innovations in fast-charging technology, the opportunity charging, which allows BEBs to recharge during operation with high-power chargers installed at the charging stations along the bus lines, has been widely implemented in practice. Compared with slow overnight charging (also known as depot charging), the opportunity charging utilizes BEBs' dwelling times at bus stops or terminals to recharge. BEBs can therefore be equipped with smaller batteries, resulting in a lower vehicle battery cost (He et al., 2019). The recent development in battery swapping technology provides a more flexible way to recharge BEBs during daytime operation, while the depleted battery can be charged at night with discounted electricity prices (An et al., 2020).

One of the main challenges faced by the transit system operator in designing the BEBs' charging schedule is the unpredictable energy consumption of batteries during operation (Liu et al., 2018; Zhang et al., 2020). It can be contributed to the inherent uncertainties in driving (e.g., road condition, temperature, driving behavior) and vehicle (e.g., air conditioning, battery aging) condi-

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tions (An, 2020). For instance, the energy consumption would be higher if a bus runs at a very low speed or in stop-and-go traffic (López and Fernández, 2020). This uncertainty has significant impact on bus charging decisions including when and where a BEB should be recharged. As for a battery swapping system, the inventory management plan should accommodate the uncertain change in charging demand.

To overcome these barriers, this paper develops a coordinated bus charging scheduling approach for a hybrid charging station allowing both plug-in fast charging and battery swapping. A two-stage stochastic program is formulated to accommodate the uncertainty in energy consumption during operation, which determines the optimal charging mode and charging schedule integratedly.

1.1. Literature review

The BEB charging scheduling problem is an integration of bus timetabling and vehicle scheduling problems. In literature, two general frameworks are applied to design the optimal schedule for the conventional transit system: (1) sequential approach: first determining the timetable and then optimizing the vehicle schedule (Ceder, 2011); and (2) integrated approach: using a single optimization model to solve these two problems simultaneously (Ibarra-Rojas et al., 2014). Considering the fact that most transit systems are experiencing a switch from conventional fossil fuel-powered vehicles to BEBs, the majority of existing studies focusing on the bus charging scheduling design assume that the initial timetable is given (Abdelwahed et al., 2020; He et al., 2020; Liu and Ceder, 2020; Zhang et al., 2021). Though very few, some efforts have also been devoted to the integrated model of timetable and recharging scheduling. For example, Teng et al. (2020) develop an integrated approach to design smooth vehicle departure intervals and minimize the total charging cost simultaneously.

Compared with the overnight charging, the opportunity charging has received increasing attention because overnight charging can be seen as a special case of opportunity charging with one charging station and BEBs are not in operation (Li, 2016). The opportunity charging scheduling problem aims to optimize when, where, and how long a bus should be charged while guaranteeing the timetable punctuality (Abdelwahed et al., 2020). Liu and Ceder (2020) develop a bi-objective integer programming model to design the charging schedule by minimizing the number of required vehicles and chargers simultaneously. Yao et al. (2020) propose a new bus scheduling model which is accommodated to multiple vehicle types with respect to driving range, recharging duration, and energy consumption. Abdelwahed et al. (2020) develop two mixed-integer linear programming formulations to optimize the opportunity fast-charging schedule, both of which use discretization approaches, namely, discrete-time optimization and discrete-event optimization. The results show that the latter performs better in terms of its higher practicality and computational performance. Zhang et al. (2021) formulate the bus charging scheduling problem by a set partitioning model subject to the given trip schedule and charging facilities. Both battery degradation and non-linear charging profile are considered.

Most of the abovementioned studies are based on a basic assumption that the energy consumption of BEBs in operation is known or the consumption has a certain relationship with travel distance, resulting in a deterministic bus charging scheduling problem. However, the uncertain variation in energy consumption has been acknowledged as a challenge in designing bus charging schedules. From the perspective of data science, many efforts have been devoted to the estimation and prediction of energy consumption using data-driven approaches (Dai et al., 2014; Vepsäläinen et al., 2018; 2019). From the mathematical methods and operations research perspective, two programming approaches can be applied to deal with these uncertainties: robust optimization (Liu et al., 2018; Tang et al., 2019; An, 2020) and stochastic programming (Sachan, 2018; Asadi and Pinkley, 2021; Bie et al., 2021). The robust optimization is usually developed on an uncertain set and the optimal solution is dependent on the worst-case realization of uncertainties. For instance, Liu et al. (2018) develop a set-based robust optimization model by assuming that the uncertainty of energy consumption varies within a given set. While stochastic programming assumes a known energy consumption distribution. Bie et al. (2021) consider the stochastic volatilities in both trip travel time and energy consumption. The bus schedule is designed with the objectives of minimizing the expectation of delays, energy consumption, and bus procurement costs.

1.2. Objectives and contributions

The majority of existing studies on both deterministic and non-deterministic BEB charging scheduling focus solely on one type of charging method. The charging scheme with only one charging mode is less flexible especially in the case of high variations in energy consumption. For instance, the overlong charging time or the shortage of fully charged spare batteries would incur additional dwelling time at charging stations. Hence, there is an urgent need for constructing a multifunctional charging station which could combine various types of charging facilities to meet the diversity of BEBs' charging demand. For instance, Qingdao has built the first hybrid charging station in China by integrating fast-charging, battery-swapping, and energy storage into a coordinated charging system. It is thus necessary to develop a new charging scheduling strategy which is adapted to the novel hybrid charging system by organizing different charging modes in an efficient manner. To our best knowledge, no research has focused on the EB charging scheduling problem under the combination of plug-in charging and battery swapping.

Given the complexity of the overall problem, this paper will focus on the operational aspect of BEBs by optimizing the charging mode and schedule at hybrid charging stations, which is still an insufficiently studied problem. The contribution of this study is twofold. First, we contribute to the existing literature in proposing a novel charging scheme that contains both fast-charging and battery-swapping technologies integratedly. By optimizing the charging schedule, we optimize the charging mode, as well as when, where, and how long each bus should be charged in a day. Second, to decide the optimal battery inventory level with the consideration of unknown future energy consumption, we formulate the bus charging scheduling problem as a two-stage stochastic preprogram that determines the optimal battery inventory in the first stage and the charging mode/schedule in the second stage.

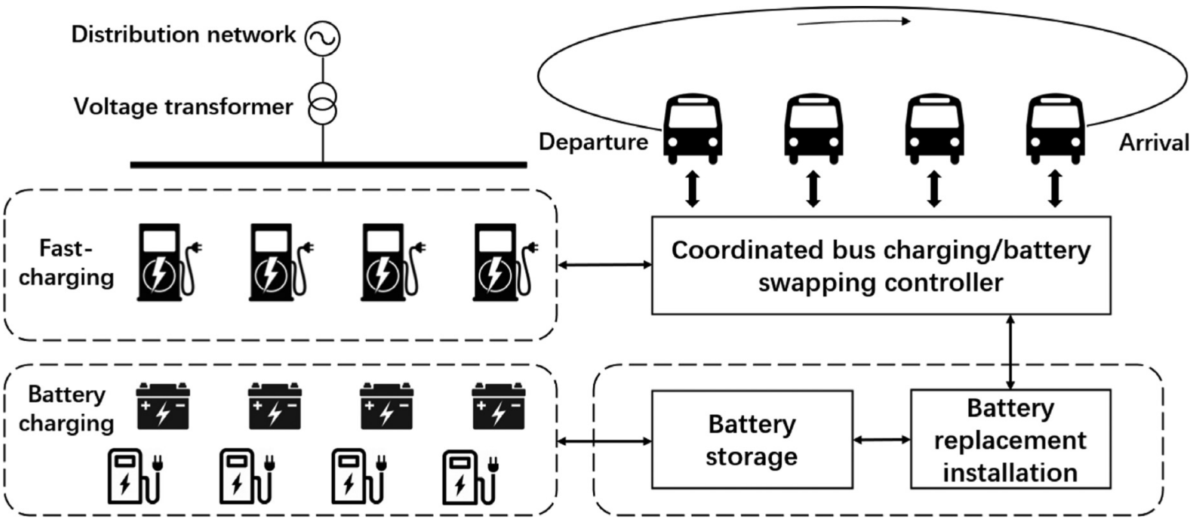


Fig. 1. Structure of the hybrid bus charging station.

The remainder of this paper is organized as follows. Section 2 gives a detailed description of the studied problem. Section 3 presents a deterministic bus charging scheduling optimization problem at first. And the stochastic formulation is formulated as follows. Section 4 develops a progressive hedging-based solution method. Section 5 presents the numerical example to verify the proposed model and algorithm. Finally, Section 6 concludes this paper and outlooks future research.

2. Problem description

In general, the structure of an electric transit network is defined by its terminal stations, timetable, and trip-assignment schedule (Abdelwahed et al., 2020). The timetable records the departure times of a bus from one terminal station, which is usually determined by the distribution of passenger demand (i.e., during peak or off-peak hours). It is important to mention that the effects of passenger demand are not considered in this study. A trip of a bus is defined by its starting time (i.e., the first departure time of the day), duration, and travel distance from one terminal to another. The relationship between buses and trips is determined by the trip-assignment schedule.

In literature, the deployment of charging facilities, including both the location of charging stations and facility capacity, is usually studied from the planning aspect at the strategic level (He et al., 2019). Whilst, this study focuses on optimizing the charging scheduling of buses at the operational level. Hence, it is assumed that all charging stations are equipped with charging facilities, and the number of chargers at each station is known in advance. Afterward, buses can only be recharged during the layover time between trips at terminal stations. In practice, the number of chargers at each terminal station is usually limited with several restrictions, such as budgeted construction cost, available land use, and negative impact on the electricity grid. In this regard, the congestion on bus charging occurs because the charging demand exceeds the number of chargers, which may cause undesirable delays for following trips. Consequently, designing an effective charging schedule is essential to maintain the feasibility of the system's daily operation by reducing the unnecessary schedule delay.

Considering time-of-use (TOU) electricity tariffs, it is always more cost-effective to charge the buses during off-peak hours with lower electricity prices (An, 2020; He et al., 2020). Though buses are usually fully charged in the overnight charging at the garage, they still need to be recharged in operation during the daytime. Meanwhile, the battery swapping provides an alternative option by exchanging the depleted battery for a fully charged one that be recharged at night. Consequently, in this work, we propose a hybrid charging scheme that contains both plug-in charging and battery swapping. As presented in Fig. 1, the studied hybrid charging station is composed of three functional areas: plug-in vehicle charging, battery charging, and battery swapping.

The hybrid charging station is organized by a coordinated bus charging/battery swapping controller, which decides the charging mode and schedule for a bus needed to be recharged. In this work, we focus on optimizing the electric transit network's charging schedule with both plug-in charging and battery swapping by deciding a bus should be charged with which mode at which station and time during the day.

Another decision that should be made is how many fully-charged batteries should be stored at each station to satisfy the charging demand on that day. In the case with fully deterministic settings, where the bus's energy consumption in trips can be measured by a deterministic function of trip distance, the charging demand can be easily obtained according to the given trip-assignment schedule. However, in practice, the energy consumption is highly uncertain depending on road conditions and battery status (e.g., degradation and aging) (An, 2020). Consequently, this work formulates the electric scheduling problem as a two-stage stochastic program, in which the future energy consumption of a bus between two charging stations is uncertain. The first stage of our scheduling problem is to make decisions on how many fully-charged batteries should be prepared at each station before the bus's daily operation, while

Table 1
List of notations.

Notation	Description
Set	
$\mathcal{J}$	Set of charging stations indexed by j.
$\mathcal{K}$	Set of buses indexed by k.
$\mathcal{T}$	Set of time slots indexed by t.
Parameter	
c_{bc}	Fixed cost of battery charging ($).
c_{bs}	Fixed cost of battery swapping service ($).
c_t	Electrical price ($/kWh) at time t.
Q	Battery capacity (kWh).
$L^t_{k,j}$	Location of buses, i.e., $L^t_{k,j} = 1$ if bus k is at station j at time t; and $L^t_{k,j} = 0$, otherwise.
LB_k	Lowest allowed state-of-charge (SoC) (%) of bus k.
M_j	Number of chargers at station j.
N_j	Inventory of batteries at station j.
$N_j^{\max}$	Maximum inventory of batteries at station j.
P_j	Power of chargers (kWh) at station j.
W^t_k	Energy consumption (%) in a trip by bus k at time t.
Δ_m	Allowed minimum charging time (min).
γ_j	Efficiency of chargers (%) at station j.
τ_e	Last minute of the planning horizon.
Variable	
SoC^t_k	An integer variable for SoC of bus k at time t.
x^t_k	A binary variable for the decision to charge, i.e., $x^t_k = 1$ if bus k is going to charge or is charging at time t; and $x^t_k = 0$, otherwise.
y^t_k	A binary variable for the decision to swap the battery, i.e., $y^t_k = 1$ if bus k swaps its battery at time t; and $y^t_k = 0$, otherwise.
u^t_k	An auxiliary variable, and $u^t_k = x^t_k \cdot x^{t-1}_k$.
v^t_k	A binary variable for the charging starting time, i.e., $v^t_k = 1$ if bus k starts charging at time t; and $v^t_k = 0$, otherwise.

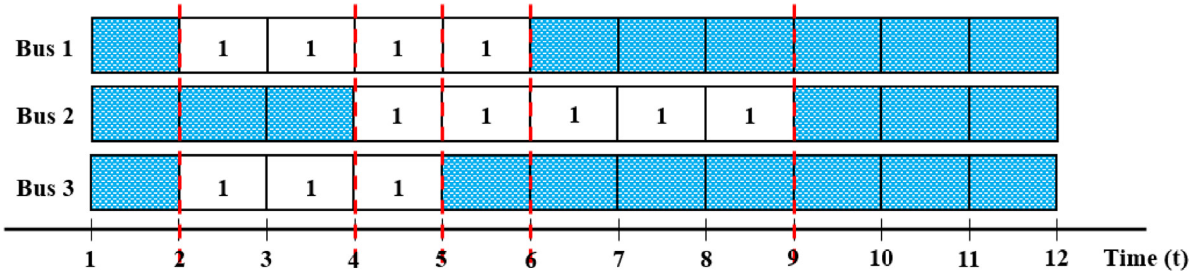


Fig. 2. Discretization of the timetable for one station with one-minute slots.

the second stage is to determine its charging mode, and where and when a bus should be charged. The total cost of the second stage (measured by the total system charging cost) is also a random variable depending on the optimal decision of the first-stage problem and the realization of the random variables in the second stage, namely, energy consumptions of buses. Our objective is to minimize the total charging cost of both modes including the cost of plug-in charging and the cost incurred during the preparation for batteries.

The notations of the sets, parameters, and variables used in the following sections are listed in Table 1.

3. Model formulation

In this section, a deterministic optimization model is formulated for the electric bus charging scheduling problem with given energy consumptions and predefined battery inventory at each terminal station. The deterministic model is then reformulated to a two-stage stochastic program by assuming that the bus's energy consumption for a trip is a random variable.

3.1. A deterministic optimization model

Similar to the classic scheduling problems in manufacturing and production systems, the optimal design of electric bus charging schedules is a time-dependent problem that the optimal solution is dependent on instantaneous state variables of the system. In this study, we adopt the discrete-time optimization framework (Abdelwahed et al., 2020) which discretizes the planning horizon (e.g., the operation time of the day) into equal time slots of minutes (see Fig. 2).

Consider an electric transit system with a given timetable and trip-assignment schedule. Let $\mathcal{K}$ denote the set of buses and $\mathcal{J}$ the set of terminal stations. The planning horizon is discretized with one-minute slots and the set of time slots is denoted by $\mathcal{T}$. The binary variable x^t_k is used to represent the charging state of bus k at time t. To track the consistency of charging process, an auxiliary binary variable u^t_k is introduced, and $u^t_k = x^t_k \cdot x^{t-1}_k$, $\forall k \in \mathcal{K}, t \in \mathcal{T}$. With a little abuse of notation, we let another binary variable v^t_k represent the starting time of the charging process of bus k at time t, and $v_k^t = x_k^t - u_k^t$, $\forall k \in \mathcal{K}$, $t \in \mathcal{T}$. Note that the vehicle's charging state variable x_k^t is defined throughout the plannin horizon $\mathcal{T}$. Additionally, bus charging activity only occurs when a bus is at the station. Hence, to avoid generating redundant variables, x_k^t is arbitrarily set to zero when bus k is in operation.

Due to the fact that the operation of battery swapping can be completed in one or two minutes (Mak et al., 2013), we assume that the swapping time is negligible compared with the plug-in charging and the physical operation finishes instantaneously. Similarly, the plug/unplug time is also neglected. The total system charging cost is composed of two parts: (1) the electricity cost of plug-in charging, and (2) the fixed cost of the battery swapping service. In sum, the deterministic optimization model of the bus charging scheduling problem can be formulated as follows:

[DO]

$$\min_{\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}} \sum_{t\in\mathcal{T}} \frac{c_t}{60} \sum_{k\in\mathcal{K}} \left(x_k^t - v_k^t\right) \sum_{j\in\mathcal{J}} L_{k,j}^t \cdot P_j + c_{bs} \cdot \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} y_k^t \sum_{j\in\mathcal{J}} L_{k,j}^t \tag{1}$$

subject to

$$SoC_k^t = \begin{cases} 100 - W_k^t, & t = 0 \\ \left(1 - y_k^t\right) \cdot \left[SoC_k^{t-1} - W_k^t + \left(x_k^t - v_k^t\right) \cdot \left(\sum_{j\in\mathcal{J}} L_{k,j}^t \cdot \frac{P_j}{60\cdot Q} \cdot \frac{\gamma_j}{100}\right)\right] + y_k^t \cdot UB_k^t, & \text{otherwise} \end{cases} \tag{2}$$

$$SoC_k^t \leq UB_k^t, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{3}$$

$$SoC_k^t \geq LB_k, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{4}$$

$$SoC_k^t \geq 100 - \sum_{j\in\mathcal{J}} L_{k,j}^t \cdot \frac{P_j}{60 \cdot Q} \cdot \frac{\gamma_j}{100}, \forall k \in \mathcal{K}, t = \tau_e, \tag{5}$$

$$x_k^t = 0, \forall k, t \in \left\{ k \in \mathcal{K}, t \in \mathcal{T} \middle| \sum_{j\in\mathcal{J}} L_{k,j}^t = 0 \right\}, \tag{6}$$

$$\sum_{k\in\mathcal{K}} \left(x_k^t \cdot L_{k,j}^t\right) \leq M_j, \forall j \in \mathcal{J}, t \in \mathcal{T}, \tag{7}$$

$$v_k^t = x_k^t - u_k^t, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{8}$$

$$u_k^t \leq x_k^{t-1}, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{9}$$

$$u_k^t \leq x_k^t, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{10}$$

$$u_k^t \geq x_k^t + x_k^{t-1} - 1, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{11}$$

$$\sum_{t' \geq t}^{t' \leq t+\Delta_m} x_k^{t'} \geq \Delta_m \cdot v_k^t, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{12}$$

$$x_k^t + y_k^t \leq 1, \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{13}$$

$$\sum_{t\in\mathcal{T}} \left(y_k^t \cdot L_{k,j}^t\right) \leq 1, \forall k \in \mathcal{K}, j \in \mathcal{J}, \tag{14}$$

$$\sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} \left(y_k^t \cdot L_{k,j}^t\right) \leq N_j, \forall j \in \mathcal{J}, \tag{15}$$

$$x_k^t, y_k^t, u_k^t, v_k^t \in \{0, 1\}, \forall k \in \mathcal{K}, t \in \mathcal{T}. \tag{16}$$

The objective function (1) minimizes the total charging cost of all charging modes. Constraint (2) defines the battery state of a certain bus with respect to a time slot. If a bus is assigned to the mode of battery swapping, the depleted battery is replaced with a prepared one immediately. Constraints (3) and (4) control the upper and lower bounds of the SoC of each bus. Constraint (5) ensures that all buses are fully charged before the beginning of the next day's operation. Constraint (6) guarantees that a bus can only be charged when it is at a charging station. Constraint (7) states that the number of charging buses at the same time is restricted by the capacity of this station in terms of the number of chargers. Constraints (8)–(11) establish the relationship among binary variables v_k^t, x_k^t, and u_k^t to guarantee a continuous charging process. Constraint (12) regulates the minimum charging time. Constraint (13) describes the decision of charging mode choice. Constraint (14) states that the battery swapping process finishes in a short time, which is neglected in the model. Constraint (15) guarantees that the number of buses chosen to swap their batteries is restricted by the battery inventory. Constraint (16) assigns binary values to the decision variables.

3.2. A two-stage stochastic programming model

As discussed in Section 2, in the real-world situation, the decision on how many fully charged batteries should be provided for the next day's operation is determined without an exact situation of the future. A common method to capture future uncertainty is to consider a set of possible scenarios (Huang et al., 2021). In this section, a scenario-based stochastic program is formulated and the uncertainty associated with energy consumption is represented by a set of discrete scenarios with a given probability of occurrence.

To extend the deterministic formulation **[DO]** to a stochastic program, we assume that the energy consumed for performing a trip is assumed to be stochastic with known distribution. Let the random vector $\boldsymbol{\xi}$ describe the uncertain energy consumptions. Each realization of $\boldsymbol{\xi}$, ξ, and its corresponding probability $p(\xi)$ define a scenario. In this study, ξ is also a random vector that contains the uncertain energy consumptions for each bus at each time interval, i.e., $\xi = \{\xi_k^t | k \in \mathcal{K}, t \in \mathcal{T}\}$.

Under this assumption, the two-stage stochastic program is stated as follows: at the first stage, a "here-and-now" decision, i.e., the number of fully-charged batteries that should be prepared at each station, needs to be made before the realization of the uncertain data ξ is known; at the second stage, after the realization of ξ becomes available, we optimize the charging mode and schedule by solving **[DO]**. The integer variable z_j, $j \in \mathcal{J}$, is introduced to represent the number of fully-charged batteries prepared in advance. And the capacity of battery inventory is $N_j^{\max}$. Let $\mathbf{z}$ denote the vector of battery inventory at each station, $\mathbf{z} = \{z_j | j \in \mathcal{J}\}$.

Consider now the case when the battery inventory decision should be made before a realization of the energy consumption becomes known. It is reasonable to view the energy consumption, ξ, as a random variable. We can further assume that the probability distribution of ξ is also known, which can be easily obtained from historical data. Hence, the total charging cost depending on the random variable ξ can be denoted by an expectation operation, denoted $\mathbb{E}[F(\mathbf{z}, \xi(\mathbf{x}, \mathbf{y}))]$, with respect to battery inventory, where $F(\mathbf{z}, \xi(\mathbf{x}, \mathbf{y}))$ is the optimal value of the second-stage problem:

$$F(\mathbf{z}, \xi(\mathbf{x}, \mathbf{y})) := \min_{\mathbf{i},\mathbf{x},\mathbf{y},\mathbf{u}} \sum_{t\in\mathcal{T}} \frac{c_t}{60} \sum_{k\in\mathcal{K}} \left(x_k^t - v_k^t\right) \sum_{j\in\mathcal{J}} L_{k,j}^t \cdot P_j + c_{bs} \cdot \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} y_k^t \sum_{j\in\mathcal{J}} L_{k,j}^t. \tag{17}$$

Consequently, the bus charging scheduling optimization problem with stochastic energy consumption can also be formulated as the following 0-1 integer program:

[SO-1]

$$\min_{\mathbf{z}} \sum_{j\in\mathcal{J}} c_{bc} z_j + \mathbb{E}\left[F(\mathbf{z}, \xi(\mathbf{x}, \mathbf{y}))\right] \tag{18}$$

subject to

$$z_j \le N_j^{\max}, \forall j \in \mathcal{J}, \tag{19}$$

$$z_j \in \mathbb{R}, \forall j \in \mathcal{J}, \tag{20}$$

$$SoC_k^t = \begin{cases} 100 - \xi_k^t, & t = 0 \\ \left(1 - y_k^t\right) \cdot \left[SoC_k^{t-1} - \xi_k^t + \left(x_k^t - v_k^t\right) \cdot \left(\sum_{j\in\mathcal{J}} L_{k,j}^t \cdot \frac{P_j}{60\cdot Q} \cdot \frac{\lambda_j}{100}\right)\right] + y_k^t \cdot UB_k^t, & \text{otherwise} \end{cases} \tag{21}$$

and constraints (3)–(16).

As aforementioned, we represent the uncertainty in energy consumption with a given finite set of consumption scenarios ξ_s, $s = 1, \dots, S$, and the probability of the occurrence of each scenario is known and denoted by $p_s = \text{Prob}\{\boldsymbol{\xi} = \xi_s\}$. As aforementioned, prior knowledge is taken into account to identify the distribution of individual scenarios. These probabilities can also be interpreted as weights that reflect the relative importance of scenarios based on prior knowledge (Watson and Woodruff, 2011). In this regard, in the case of a finite number of scenarios, the stochastic program can be rewritten as the following deterministic optimization problem:

[SO-2]

$$\min_{\mathbf{z}} \sum_{s=1}^{S} p_s \left[\mathbf{c} \cdot \mathbf{z}_s + F\left(\mathbf{z}_s, \xi_s(\mathbf{x}, \mathbf{y})\right)\right] \tag{22}$$

subject to (3)–(16), (19), and (21).

4. Solution algorithm

Solving the sub-problems defined in **[SO-2]** for all scenarios will give different scenario-dependent solutions, i.e., $\mathbf{z}_s$, $s = 1, \dots, S$. However, these solutions cannot be directly used in practice because when the decision on battery inventory is made, the transit system operator does know which scenario of energy consumption would occur. On the other hand, the decision at the first stage does not depend on the realization of a specific scenario at the second stage. Hence, to consolidate the scenario-dependent solutions to an implementable solution, the non-anticipatively constraint is imposed to avoid allowing the decision at the first stage to depend on the scenario (Rockafellar and Wets, 1991):

$$\mathbf{z}_s = \mathbf{z}_{s'}, \forall s = 1, \dots, S, s' = 1, \dots, S, s \ne s', \tag{23}$$

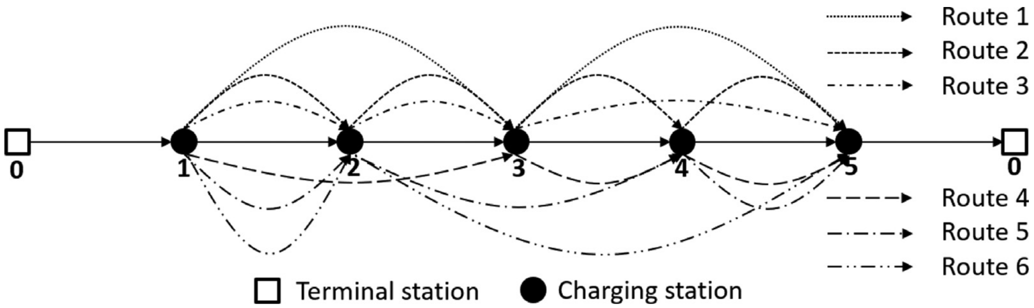


Fig. 3. The augmented bus service network.

or equivalently (Fan and Liu, 2010)

$$\mathbf{z}_s = \bar{\mathbf{z}}, \forall s = 1, \dots, S, \tag{24}$$

where $\bar{\mathbf{z}}$ is a vector of free variables.

Both constraints (23) and (24) state the fact that the battery inventory decision is independent of the realization of random variables, i.e., energy consumption, at the time this decision is made. In this paper, we impose constraint (24) as the non-anticipatively constraint on **[SO-2]**. For simplicity, the feasible region defined by constraints of **[SO-2]** as well as constraint (24) is denoted by Ω.

As aforementioned, the realization of random variables in a scenario as well as corresponding probability can be estimated from historical data (Ruszczyński and Shapiro, 2003). In the real-world situation the number of possible scenarios may be quite large, resulting in an unmanageable integer stochastic programming problem that needs effective techniques to decompose into solvable sub-problems. In this paper, the Progressive Hedging Algorithm (PHA) (Rockafellar and Wets, 1991) is applied to decompose the original stochastic program into scenario-based sub-problems. The objective function of **[SO-2]** can be rewritten as an augmented Lagrangian integrated with the relaxation of non-anticipatively constraint:

$$L(\mathbf{z}, \mathbf{x}, \mathbf{y}, \bar{\mathbf{z}}, \lambda) = \sum_{s=1}^{S} p_s \left[\mathbf{c} \cdot \mathbf{z}_s + F\left(\mathbf{z}_s, \xi_s\left(\mathbf{x}_s, \mathbf{y}_s\right)\right) + \lambda_s \mathbf{z}_s + \rho/2 \left\| \mathbf{z}_s - \bar{\mathbf{z}} \right\|^2 \right], \tag{25}$$

where λ_s is the vector of Lagrangian multiplier for scenario s, $s = 1, \dots, S$; ρ is the penalty parameter.

Note that the last term of Eq. (25), i.e., the non-separable penalty, cannot be decomposed directly. PHA provides a heuristic mechanism to fix the scenario solution $(\mathbf{z}_s, \mathbf{x}_s, \mathbf{y}_s)$ and the implementable solution $\bar{\mathbf{z}}$ by iteratively solving the penalized versions of sub-problems (Gade et al., 2016; Hu et al., 2019). The detailed procedure of the PHA is illustrated in Algorithm 1.

5. Numerical experiments

In this section, a numerical experiment is conducted based on a real-world case in Nanjing, China. The algorithm is coded in python, and the integer programs of scenario-based sub-problems are solved by Gurobi Optimizer 9.1.2. All computation experiments are conducted on an Intel Core i7-9750H CPU at 2.60 GHz with 16 GB RAM.

5.1. Experimental setup

The data is collected from six buses, each of which serves an individual bus line terminated at Nanjingnan station which is one of the largest terminals in Nanjing equipped with fast-charging facilities. The historical data of SoC of all buses are collected for 60 days. The layout of bus services is represented by a bus corridor in Fig. 3. Because the bus charging activity only occurs at the charging station, we simplify the detailed itinerary of bus lines by using bus trips. Hence, a service route for a bus within a day is composed of several round-trips, and the charging activities only take place at the stop connecting two trips. Though the charging activities of all buses occur in the same charging station, an augmented network is used to distinguish the bus's visiting sequence at the charging station. The schedule of each route is presented in Appendix A.

In practice, the transit system operator may face a wide range of possible energy consumption scenarios. A large number of possible scenarios will impose computational challenges due to the increased dimension of the problem. In literature, possible scenarios and associated probability distributions are usually extracted from historical data using statistical approaches or other generation methods such as sampling and simulation (Di Domenica et al., 2009). In this study, a clustering-based scenario generation approach, namely, the multidimensional *k*-means method, is applied. Days with a similar energy consumption pattern are clustered and these clusters then form the scenarios.

Except for the battery-related factors (such as degradation and aging), the energy consumption during a trip depends mainly on several external factors including road gradient, air-conditioning, and traffic congestion (An, 2020). Here we assume that the operational condition of all buses in one day is similar. Five possible energy consumption scenarios are applied: Scenario 1 (very low), Scenario 2 (low), Scenario 3 (medium), Scenario 4 (high), and Scenario 5 (very high). The descriptive statistics of the clustering

Table 2
Descriptive statistics of the clustering results of SoC (%).

Scenario	Description	Mean	Variance	Frequency	Probability
1	Very low	36.51	1.91	6	0.10
2	Low	40.47	1.97	11	0.19
3	Medium	49.85	0.44	26	0.43
4	High	59.47	0.05	8	0.13
5	Very high	61.62	2.99	9	0.15

Table 3
Performance of the stochastic programming and wait-and-see approaches.

	Scenario	Battery charging cost ($)	Bus charging cost ($)	Battery inventory
Wait-and-see approach	1	200	180	(0,1,3,0,0)
	2	200	222	(0,1,2,1,0)
	3	300	272	(0,3,2,1,0)
	4	350	298	(0,3,2,2,0)
	5	350	304	(0,4,1,2,0)
Stochastic approach	Case 1	350	305	(0,3,2,2,0)
	Case 2	300	282	(0,3,2,1,0)

Algorithm 1 Progressive hedging algorithm

1: $h = 0$, $\rho = 0.05$, $\varepsilon = 0.001$;
2: For all s, $\mathbf{z}_s^{(h)} = \underset{(\mathbf{z}_s,\mathbf{x}_s,\mathbf{y}_s)\in\Omega_s}{\arg\min} \ (\mathbf{c}\cdot\mathbf{z}_s + F(\mathbf{z}_s, \xi_s(\mathbf{x},\mathbf{y})))$;
3: Update: $\bar{\mathbf{z}}^{(h)} = \sum_{s=1}^{S} p_s \cdot \mathbf{z}_s^{(h)}$;
4: For all s, $\lambda_s^{(h)} = \rho \cdot (\mathbf{z}_s^{(h)} - \bar{\mathbf{z}}^{(h)})$;
5: $h = h + 1$;
6: For all s,
$\mathbf{z}_s^{(h)} = \underset{(\mathbf{z}_s,\mathbf{x}_s,\mathbf{y}_s)\in\Omega_s}{\arg\min} \ (\mathbf{c}\cdot\mathbf{z}_s + F(\mathbf{z}_s, \xi_s(\mathbf{x},\mathbf{y})) + \lambda_s^{(h-1)}\mathbf{z}_s + \rho/2\|\mathbf{z}_s - \bar{\mathbf{z}}^{(h-1)}\|^2)$;
7: Update: $\bar{\mathbf{z}}^{(h)} = \sum_{s=1}^{S} p_s \cdot \mathbf{z}_s^{(h)}$;
8: For all s, $\lambda_s^{(h)} = \lambda_s^{(h-1)} + \rho \cdot (\mathbf{z}_s^{(h)} - \bar{\mathbf{z}}^{(h)})$;
9: Check the termination criterion:
if $\sum_{s=1}^{S} p_s \cdot \|\mathbf{z}_s - \bar{\mathbf{z}}^{(h-1)}\| < \varepsilon$, terminate; otherwise, go to Line 5.

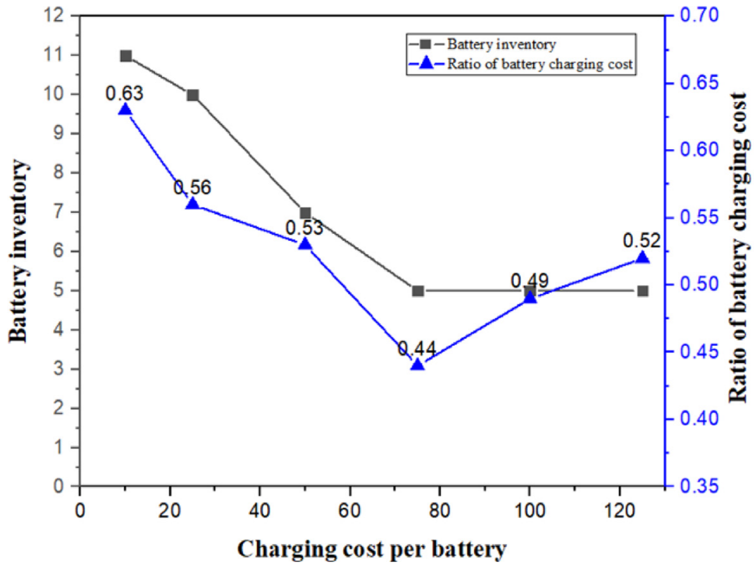
results are shown in Table 2. The corresponding probability values can be obtained by the ratio of occurrence frequency and the total 60 days.

5.2. Stochastic programming approach vs. wait-and-see approach

As stated in Section 3.2, the proposed two-stage stochastic program intends to find an expected optimal solution for all possible future realizations of energy consumptions. Whilst, the wait-and-see approach is designed to cope with the deterministic case by assuming that we could wait and see the realization of energy consumptions and then make decisions accordingly (Birge and Louveaux, 2011). Hence, the wait-and-see approach provides a set of scenario-based solutions that can be obtained by the deterministic formulation **[DO]**. In addition, the fixed costs of battery charging c_{bc} and swapping service c_{bs} are set to 50 and 20, respectively. The number of chargers M is set to 3. The penalty parameter is 0.05. The computational results of both stochastic programming and wait-and-see approaches are presented in Table 3.

As can be inferred from the table, in the case of wait-and-see, the required amount of battery inventory no doubt increases when the energy consumption becomes higher. Another observation can be drawn from Table 3: the battery inventory is unevenly distributed among stations. For instance, in Scenario 1 (very low), Station 3 stores the maximum number of batteries. It can be attributed to the fact that Station 3 is an intermediate station of the augmented bus service network and it is cost-efficient to charge in the middle of the day. In the scenarios with higher energy consumption, buses intend to choose the mode of battery-swapping in the beginning of their route to avoid the cases of battery depletion and insufficient charging time during the following trips.

As aforementioned, the optimal solution of the stochastic program depends on all possible future scenarios and the probability of each realization. To investigate the impact of scenario probabilities on the optimal solution, two cases are considered in the stochastic program: (1) Case 1: using the probability obtained from the historical data (see Table 1); and (2) Case 2: using an equal probability for all scenarios, i.e., $p_1 = \cdots = p_5 = 0.2$. The stochastic solution of Case 1 provides the largest bus charging cost compared with wait-and-see approaches. It implies that to guarantee the daily operation of the bus system, the operator intends to be conservative by



Charging cost per battery ($)	Battery inventory
10	(1,3,4,2,1)
25	(1,3,3,2,1)
50	(0,3,2,2,0)
75	(0,2,2,1,0)
100	(0,2,2,1,0)
125	(0,2,2,1,0)

Fig. 4. Effect of battery charging cost on optimal battery inventory.

applying the mode of battery swapping though the cost of battery swapping is larger than that of fast-charging. The optimal solution of Case 2 with qual probability is similar to Scenario 2 using the wait-and-see policy.

5.3. Results on the sensitivity of the solution

One of the advantages of battery swapping is that depleted batteries can be recharged during off-peak hours with lower electricity prices (Wu et al., 2017; An et al., 2020). Hence, the charging cost of depleted batteries would have significant impacts on the choice of battery swapping mode in the proposed hybrid charging scheme. A sensitivity analysis on the charging cost per battery is conducted, and results are presented in Fig. 4. As expected, the total number of required batteries decreases with increasing charging cost per battery. And the amount of optimal battery inventory remains unchanged when the charging cost is larger than 75, which indicates the minimum amount of required batteries to guarantee the bus services. The ratio of battery charging cost to total charging cost is convex with the charging cost per battery, and it reaches a minimum when the charging cost per battery equals to 75.

6. Conclusions

In this paper, a two-stage stochastic program is developed to obtain the optimal BEB charging scheduling strategy considering the uncertainty of battery energy consumption. A new hybrid charging scheme is proposed to provide a flexible charging strategy, i.e., when a BEB arrives at a charging station, if the remaining battery energy cannot guarantee the consumption for the next trip, the BEB can be recharged by fast-charging or exchanging the discharged battery with a fully charged one. Hence, the resulting problem is intrinsically a two-stage decision problem: the first stage determines the battery inventory of each charging station; and the second stage designs the charging schedule. We also compare the performance of the proposed stochastic programming approach and wait-and-see approach which models the charging scheduling as a deterministic problem. The results show that the transit operator intends to be conservative in guaranteeing the operation of the transit system by preparing more batteries for exchange the next day.

Several potential enhancements could be considered in future works: first, considering the construction cost of fast-charging and battery-swapping facilities from the planning level; second, using advanced data mining techniques to analyze the factors influencing battery consumption and then optimizing the battery inventory dynamically; third, designing tailored exact solution algorithm to solve the scenario-based scheduling optimization model.

Declaration of Competing Interest

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, “A Two-stage Stochastic Programming Model of Coordinated Electric Bus Charging Scheduling for a Hybrid Charging Scheme”.

Appendix A. Bus schedule

	Route 1			Route 2	
	Arrival	Departure		Arrival	Departure
0	8:00	8:10	**0**	8:20	8:30
1	9:10	9:40	**1**	9:30	10:00
3	10:40	11:10	**2**	11:00	11:30
5	12:10	12:40	**3**	12:30	13:00
6	13:40	13:50	**4**	14:00	14:30
			5	15:30	16:00
			6	17:00	17:10

	Route 3			Route 4	
	Arrival	Departure		Arrival	Departure
0	8:10	8:20	**0**	8:00	8:10
1	9:00	9:30	**1**	9:10	9:40
2	10:30	11:00	**3**	10:40	11:10
3	12:00	12:30	**4**	12:10	12:40
5	13:30	14:00	**5**	13:40	14:10
6	15:00	15:10	**6**	13:10	13:20

	Route 5			Route 6	
	Arrival	Departure		Arrival	Departure
0	8:00	8:10	**0**	8:10	8:20
1	9:10	9:40	**1**	9:00	9:30
2	10:40	11:10	**2**	10:30	11:00
4	12:10	12:40	**5**	12:00	12:30
5	13:40	14:10	**6**	13:30	13:40
6	13:10	13:20			

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