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Gains and Losses of Key Opinion Leaders' Product Promotion in Livestream E-commerce

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Abstract: In livestream e-commerce, many brand-owners use the key opinion leaders (KOLs) for product promotion and sales, especially in the direct-selling channel. Is such a practice really beneficial to the brand-owner? In practice, we even observe that KOLs sell and then return products for a strong network externality. Therefore, to formulate the brand-owner's tradeoffs of using KOLs, we compare the brand-owner's profits with and without a KOL (i.e., brand-owner *uses a KOL for product promotion* vs. brand-owner *invests in product promotion by itself*) in a dual-channel system comprised of the brand-owner's direct-selling channel and its traditional retail channel. We find the brand-owner's gains/losses of using the KOL highly depend on the commission rate, retain rate and the network externality intensity. Interestingly, we reveal that the enhanced *KOL effect* might not be necessarily beneficial to the brand-owner and can even hurt it significantly. As the channel rival, the retailer may benefit from the brand-owner's using the KOL (because its procurement cost may be lowered) and the market expansion (because the use of KOL may spill over to the retail channel).
Keywords: Livestream operation; Dual-channel system; Network externality; Channel co-opetition

1. Introduction

Recently, because of the "see-now-buy-now" mentality of livestream e-commerce, many brand-owners are considering using key opinion leaders (KOLs) such as Michael Le, Luo Yonghao, and Li Jiaqi for product promotion and sales (WARC 2020, CEIBS 2020). In a typical livestream show, the KOL will first contract with the brand-owner on the sales quantity and commission rate, and then describe the product to the fans in a livestream room. Impulse fans will pay for the products when they are watching the show. Fans themselves will also communicate with each other and hence, form the peer pressure[1]. Famous platforms like Amazon, JD.com, Shopee, and Tiktok are also facilitating the cooperation between brand-owners and KOLs by specifying the livestream sites and events (Yahoo 2019, Forbes 2022). "JD Live"[2] is a typical one, which has expanded its live streams to include multiple categories to satisfy consumer demand and help brands grow online.

Their efforts are proven to be rewardful. In recent years, livestream e-commerce has witnessed a rapid growth worldwide, of which the market size has been valued at exceeding US$70 billion in 2019 (Bloomberg 2020) and at around US$400 billion in 2021 (Forbes 2022). Among countries, China is ranked as the biggest market, which has a market size valued at around US$300 billion in 2021 (Forbes 2022). Besides, livestream e-commerce in other countries is also growing rapidly. For example, the market size of livestream e-commerce in America was US$11 billion in 2021 and is estimated to reach US$25 billion by 2023, representing a compound annual growth rate of 50.75% (Forbes 2022, Yahoo 2022). Similarly, the boom of livestream e-commerce in Southeast Asian markets (e.g., Singapore, Vietnam, and Thailand) has been observed, which is increased by around 300% annually (Technode global 2021).

In practice, many brand-owners use KOLs by signing a revenue-sharing contract (Walkthechat 2019). KOLs are capable of significantly increasing the sales of brand-owners because the KOLs usually have many fans and even attract more ordinary consumers, which is referred to as the Internet celebrity/KOL effect (Influencer 2021). In addition, the conversion of product description, from pictures to interaction with the Internet celebrities/KOLs, is a novel experience of the consumers. This results in consumers' herd behavior and hence, a strong network externality (Cheng

1 https://smallbiztrends.com/2022/05/how-to-go-live ontiktok.html
2 https://jdcorporateblog.com/cant-sell-jds-live-streaming-makes-a-big-difference/

and Liu 2012). Having said that, consumers' impulse purchase in live room inevitably leads to high product return and refund rate. It is consumers' sense of participation that stimulates their consumption, so sometimes the return and refund rate can be as high as 60% (CGTN 2020).

One natural question arises: why not promote the product by the brand-owner itself, rather than using the KOLs? Although the network externality becomes ignorable, consumers' impulsive purchase can be effectively avoided. This helps save the return cost and more importantly, the commission fee paid to the KOLs (undoubtedly, the commission fee for Michael Le, Luo Yonghao, and Li Jiaqi can be super high). If so, the brand-owners' promotion in the direct-selling channel returns to a traditional way, which has been widely studied by the previous literature such as Taylor (2002), Krishnan et al. (2004), Tsay and Agrawal (2004), and Chen (2005).

To answer the foregoing question, we investigate a dual-channel system comprising a brand-owner's direct-selling channel and retail channel. In the direct-selling channel, it is optional for the brand-owner to use a KOL or promote the product by itself. We refer to the former as Scenario K, where the KOL and the brand-owner sign a revenue-sharing contract. The latter is referred to as Scenario E, where the brand-owner invests in product promotion by determining the promotion effort level following Tsay and Agrawal (2004) and Chen (2005). Besides, we consider a benchmark scenario (i.e., Scenario B) where the brand-owner does not promote its products. Our main findings are summarized as follows:

First, we identify the *value of self-promotion* by comparing the brand-owner's profit in Scenario E and that in Scenario B. Two driving forces account for the brand-owner's positive *value of self-promotion*: (1) The brand-owner has the incentive to charge the retailer a high wholesale price because its demand potential in the direct-selling channel is effectively expanded. So, the retailer's channel power in the retail channel is reduced; (2) the competition between the brand-owner's direct-selling channel and the retail channel is softened, because of the increased product prices and the expanded total demand potential.

Second, we define the *KOL effect* by comparing the brand-owner's profit in Scenario K and that in Scenario B. Interestingly, we find the *KOL effect* may not always benefit the brand-owner. When the retain rate in the direct-selling channel is high, the commission rate is small, and the network externality is strong, the brand-owner will be benefited by the *KOL effect*. The reason is that, when the retain rate is high and the commission rate is low, the brand-owner can benefit from the direct-selling channel a lot when the network externality is significant, which enhances its channel power compared to the retailer. Consequently, the *KOL effect* enables the brand-owner to charge the retailer a high wholesale price and enlarges the sales quantity of the direct-selling channel simultaneously. More interestingly, we find that if the *KOL effect* acts positively on the brand-owner, the bright side of the *KOL effect* will be highlighted by the enhanced network externality. However, if the *KOL effect* acts as a negative force, the dark side of this might be further amplified when the network externality is relatively weak, depending on the retain rate and the commission rate. This signifies that, there exists an "inefficient interval of network externality", that is, the enhanced network externality cannot reverse the profit loss of the brand-owner in Scenario K.

Comparing the brand-owner's profits in Scenario K and Scenario E, we find Scenario K is preferred by the brand-owner when the positive role of the *KOL effect* is more powerful than the *value of self-promotion*, i.e., when the retain rate is high, the KOL's commission rate is small, and the resulting network externality is sufficiently strong. We further find both the brand-owner and the retailer can achieve alignment in Scenario E rather than in Scenario K.

Many previous studies have examined firms' promotion strategies and the management of distribution channels. However, livestream e-commerce is believed to be a novel promotion selling channel that has emerged recently (Smartosc 2021). It has a profound impact on the upstream firms' distribution channel management and promotion strategies. Compared with the traditional promotion strategies (e.g., Tsay and Agrawal 2004, Jørgensen and Zaccour 2014, Karray and Amin 2015), the opinion influence of KOLs in the livestream e-commerce shows the unique feature that induces the network externality but also prompts returns because of consumers' impulse purchase. Therefore, we formulate the livestream e-commerce's critical features by considering the network externality and product return caused by the KOLs, which are rarely studied in the literature on distribution channel and promotion strategies. Interestingly, we reveal that the network externality and the product return may work jointly and enhance the KOL's opinion influence to hurt the brand-owner's profit. This finding can be very insightful for the brand-owner who plans to introduce the KOL selling channel. In addition, compared with the literature on agency selling (Abhishek et al. 2016, Tian et al. 2018, Ha et al. 2022), the role of the KOL is emphasized because it is authorized to decide the optimal promotion quantity in the direct-selling channel. This differs from the literature where the marketing agencies only provide the marketplaces.

The remainder of this paper is organized as follows. In Section 2, we review the related literature. In Section 3, we describe the model setting and derive the equilibrium outcomes. In Section 4, we identify the brand-owner's *value of self-promotion* and define the *KOL effect*. We investigate the preferences of the brand-owner and the retailer over the two scenarios in Section 5, based on which we identify the win-win situation. We provide several extensions in Section 6 such as the spillover effect and the consumer surplus/social welfare. Section 7 concludes this paper and discusses several future research directions. All the proofs are relegated to the Appendix.

2. Literature Review

Our paper is related to three streams of literature. The first stream is on e-commerce channel management, which is divided into two sub-streams: (a) e-commerce channel structure and (b) operational decisions (e.g., price, quantity, return rate) in e-commerce. For the literature on (a) e-commerce channel structure, early works (Chiang et al. 2003, Yoo and Lee 2011, Wang et al. 2016, Chen and Chen 2017) focus on investigating whether the upstream firms (i.e., manufacturers/suppliers) should introduce their online channels besides the existing retail channels. A notable feature of these works is that, the online channel is facilitated by the upstream firms without the participation of third-party online platforms. Differently, more recent works (e.g., Abhishek et al. 2016, Tian et al. 2018, Dong et al. 2018, and Ha et al. 2022) shed light on the upstream firms' optimal selling modes (i.e., agent-selling and reselling) when they sell goods through an online platform. For example, Abhishek et al. (2016) compare a manufacturer's profits under the agent-selling and reselling channel structures by assuming the competition between the e-commerce channel and the traditional channel has an asymmetric impact. Tian et al. (2018) study the impact of order-fulfillment cost and upstream supply competition on the manufacturer's online channel structure decisions (i.e., agent-selling vs. reselling vs. hybrid selling). For the literature on (b) operational decisions (price or service decisions) in e-commerce channel management, Cattani et al. (2006) find a uniform pricing strategy can improve a manufacturer's overall profitability when it introduces an online channel to compete with its traditional retail channel. Geng et al. (2018) study an upstream firm's optimal online channel pricing strategy under the agency and wholesale contracts. Some recent works focus on the joint price and service decisions in e-commerce channel management. For instance, Chen and Bell (2012) investigate different return policies in a dual-channel system and show that segmenting the market with differentiated prices to return can benefit the Stackelberg-leading retailer. Following Chen and Bell (2012), Chen et al. (2018) further investigate the impact of different pricing leaders on the customer returns and supply chain parties' profitability. Realizing the impact of demand changes in the luxury fashion retailing industry, Shen et al. (2017) study a supply chain system consisting of one supplier and one online retailer. They show that, when the demand change is small, adjusting the retail price alone is enough to maximize the channel profit. Otherwise, adjusting both the retail price and online retail services is necessary. Li et al. (2019) investigate the price and service decisions in a dual-channel supply chain with the showrooming effect. Qin et al. (2020) study the impact of logistics service sharing in an e-commerce supply chain. The other notable works include Niu et al. (2021), Niu et al. (2022a), and Niu et al. (2022b).

Compared with the above studies, our study is mostly related to the literature on product return such as Chen and Bell (2012) and Chen et al. (2018). Differently, we focus on the livestream e-commerce and the product return because of the KOL fans' impulse purchases. We not only study the price decisions of all the supply chain parties but also compare the brand-owner's profit by cooperating with a KOL and that under the traditional promotion strategy. We find the product return cost caused by the KOL can be compensated by the benefit of the KOL's network externality.

The second stream is on multi-channel operations. Many studies investigate the supplier encroachment issues. Among them, some papers focus on whether the supplier encroachment could lead to the win-win situations between the retailer and the encroaching supplier (e.g., Arya et al. 2007). Scholars have also studied the impact of asymmetry information on the possibility of achieving such win-win situations (Li et al. 2014, Li et al. 2015, Zhang et al. 2021 and Guan et al. 2022). Some other papers utilize the operational decisions (e.g., pricing schemes or return policies) to mitigate the channel conflicts among different supply chain parties. For example, Cai et al. (2012) explore the effectiveness of revenue-sharing schemes in coordinating the profits of the suppliers and the retailers when the suppliers' products are complementary. Chen et al. (2012) and Ding et al. (2016) evaluate a manufacturer's optimal pricing schemes to coordinate the direct-selling channel and the retailer's channel, where the former assesses three contracts (i.e., a wholesale price contract, a two-part tariff contract, and a profit-sharing contract) while the latter focuses on the hierarchical pricing schemes. Niu et al. (2015) investigate the endogenous pricing game and channel coordination opportunities in a co-opetitive supply chain where both the OEM and the ODM could act as the Stackelberg leader. Huang et al. (2019) and Huang et al. (2021) consider the impact of the grey market and demand uncertainty to study a manufacturer's optimal price decisions to coordinate a dual-channel supply chain. Under a General Nash Bargaining (GNB) framework, Zhuo et al. (2021) study two kinds of brand-owner alliance strategies to coordinate a multi-channel system. Considering an omnichannel retailer that has both online channels and physical stores, Yang and Ji (2022) investigate the impact of cross-selling on managing consumer returns. Niu et al. (2022c) study how a retailer selling both remanufactured and regular products utilize the manufacturing outsourcing decision to coordinate the in-store competition.

Different from the foregoing studies, we focus on the role of network externality in the direct-selling channel from the KOL's opinion influence, which can effectively expand the demand potential. We formulate the KOL effect and interestingly find that the KOL may not benefit the brand-owner, so the win-win situation only exists in Scenario E (traditional promotion scenario).

The third stream is on e-commerce promotion strategy, which can be divided into two sub-streams: (a) the impact of traditional sales effort; and (b) the impact of Internet celebrity and herd behavior. For the literature on (a) the impact of traditional sales effort, Li et al. (2002) investigate how cooperative advertising influences a manufacturer–retailer supply chain and examine the impact of brand name investments, local advertising, and sharing policy. Taylor (2002) finds that when demand is impacted by the retailer's sales effort, implementing both target rebate and return strategies can coordinate the supply chain. Krishnan et al. (2004) study the effectiveness of retailer promotion efforts in a newsvendor setting. Chen (2005) summarizes the studies on sales effort in the operations management field and points out that firms' production and inventory decisions can be significantly changed. Zhang (2009) studies the interactions between multichannel retailing and the retailer's price advertising, finding that price advertising helps coordinate the dual-channel system by shifting the sales from online to offline. Balachander et al. (2010) use a game-theoretic model to study how bundle discounts help increase the retailer's profit in a competitive market. Chung et al. (2013) empirically identify the economic value of celebrity endorsements. Xu et al. (2014) investigate whether the manufacturer or the dealer should make advertising and find dealer price advertising is more effective. Kumar and Tan (2015) study how online video advertising influences product demand in e-commerce. Duan et al. (2021) evaluate the impact of sales manager's effort on the supply chain members' payment arrangements and their corresponding profit performance. Niu et al. (2022d) explore competing hospitals' healthcare service investment decisions, based on which they formulate the their two-period service pricing strategies.

For the literature on (b) the impact of Internet celebrity and herd behavior, Zhang and de Seta (2018) review the history of Internet celebrities, based on which they summarize the related business models, entrepreneurism and lookism. In livestream commerce, viewers are prone to follow others, so the role of network externality should not be ignored. Cheng and Liu (2012) study firms' free trial strategy with the consideration of network externality and consumers' uncertain purchase perception. Anderson et al. (2014) examine managers' investment in each product development cycle with bilateral network externality. Following Cheng and Liu (2012), Cheng et al. (2015) further study the role of network effect under three software promotion strategies, i.e., limited version free trial, time-locked version free trial, and the hybrid. Li et al. (2020) find that the fundraising mechanism of crowdfunding can bring positive network externality, which increases the likelihood of funding success. Cao and Li (2020) study the design of group buying mechanisms on a social e-commerce platform with the consideration of network externality. Zheng et al. (2021) study the interactions between consumers' panic buying and social learning effects on the retailer's inventory ordering strategy.

Differently, we identify the KOL effect and compare the brand-owner's profits between the traditional promotion strategy (i.e., sales effort investment) and cooperating with a KOL. We find that the KOL may not benefit the brand-owner but results in random product returns because of the fans' impulse purchases. As a result, keeping the promotion strategy unchanged and improving the promotion investment efficiency (e.g., utilizing robotic advertising) can be more beneficial.

3. Model Setting

We consider a brand-owner (subscripted as b) who sells products through both the direct-selling channel (operated by itself) and the retail channel (operated by a reselling retailer, subscripted as r). In the benchmark case, the brand-owner does not promote its products (denoted by Scenario B). In the main model, we investigate two promotion strategies of the brand-owner in the direct-selling channel, i.e., (1) using a KOL (denoted as Scenario K) and (2) promotion effort investment (denoted as Scenario E). The supply chain structures of Scenarios K and E are depicted in Figure 1, and the parameters are summarized in Table 1.

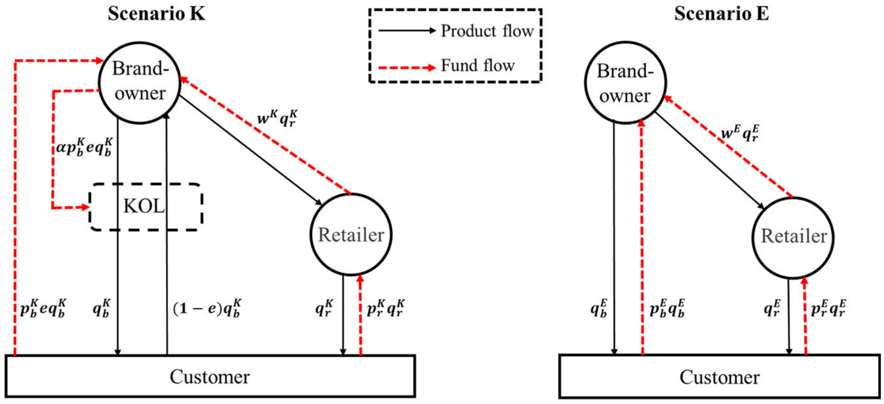


Figure 1. Supply chain structures

Table 1. Summary of Notations and Variables

Notations	
π_j^h	The profit of j in Scenario h, where $j \in \{k, r, b\}$ and $h \in \{B, K, E\}$
p_i^h	The retail price of the product sold by firm i in Scenario h, where $i \in \{r,\ b\}$ and $h \in \{B, K, E\}$
a	The demand potential
b	Channel substitutability for the same product
e	The retain rate in the direct-selling channel
k	Brand-owner's efficiency in promotion effort investment
α	The KOL's share of the revenue in the brand-owner's direct-selling channel
γ	The network externality intensity caused by the KOL
Decision variables	
q_i^h	The quantity of the product sold by firm i in Scenario h, where $i \in \{r, b\}$ and $h \in \{B, K, E\}$
w^h	The wholesale price of the brand-owner in Scenario h, where $h \in \{B, K, E\}$
θ	The promotion effort level in Scenario E

3.1 Benchmark

In the benchmark, the brand-owner does not promote its product in the direct-selling channel. Following Arya et al. (2007), Li et al. (2014), Yoon (2016), and Niu et al. (2020), the inverse demand functions of the brand-owner and the retailer are given by (superscripted by B)

$$p_b^B = a - q_b^B - bq_r^B,$$

$$p_r^B = a - q_r^B - bq_b^B,$$

where a is the demand potential and $b \in (0,1)$ represents the channel substitutability for the same product (Feng and Lu 2013, Zhuo et al. 2021). Correspondingly, the respective profit functions of the brand-owner and the retailer are

$$\underset{w^B, q_o^B}{Max}\, \pi_b^B = w^B q_r^B + p_b^B q_b^B,$$

$$\underset{q_r^B}{Max}\, \pi_r^B = (p_r^B - w^B) q_r^B.$$

The sequence of events in Scenario B is as follows. In Stage 1, the brand-owner decides the wholesale price for the retailer. In Stage 2, the brand-owner and the retailer decide the sales quantities of the direct-selling and reselling channels simultaneously. In Stage 3, the market is cleared. By backward induction, the equilibrium outcomes in Scenario B can be derived, which are summarized in Table 2.

Table 2. Equilibrium Outcomes in Scenario B

Scenario B			
w^B	$\frac{a(2-b)(4+2b-b^2)}{16-6b^2}$		
q_b^B	$\frac{a(2-b)(4+b)}{16-6b^2}$	q_r^B	$\frac{2a(1-b)}{8-3b^2}$
π_b^B	$\frac{a^2(12-8b+b^2)}{4(8-3b^2)}$	π_r^B	$\frac{4a^2(1-b)^2}{(8-3b^2)^2}$

3.2 Brand-owner's investment in promotion effort (Scenario E)

In Scenario E, the brand-owner promotes the product in the direct-selling channel by itself. We follow the literature such as Tsay and Agrawal (2004), Chen (2005), Arya and Mittendorf (2013), and Autrey et al. (2015) by assuming that the market expansion is θ and the promotion effort cost is $\frac{1}{2}k\theta^2$, where k measures the brand-owner's efficiency in promotion effort. A larger k indicates the brand-owner is less cost-efficient in product promotion. In Scenario E, the inverse demand functions and the profit functions of the brand-owner and the retailer are given by

$$p_b^E = a + \theta - q_b^E - bq_r^E,\ p_r^E = a - q_r^E - bq_b^E.$$

$$\underset{\{w^E,\theta\},q_b^E}{Max}\ \pi_b^E = w^E q_r^E + p_b^E q_b^E - \frac{1}{2}k\theta^2,$$

$$\underset{q_r^E}{Max}\ \pi_r^E = (p_r^E - w^E)q_r^E.$$

The sequence of events in Scenario E is as follows, which is illustrated in Figure 2. In Stage 1, the brand-owner decides the promotion effort level θ and the wholesale price w^E charged to the retailer. In Stage 2, the brand-owner and the retailer decide their sales quantities q_b^E and q_r^E, respectively. In Stage 3, the market is cleared.

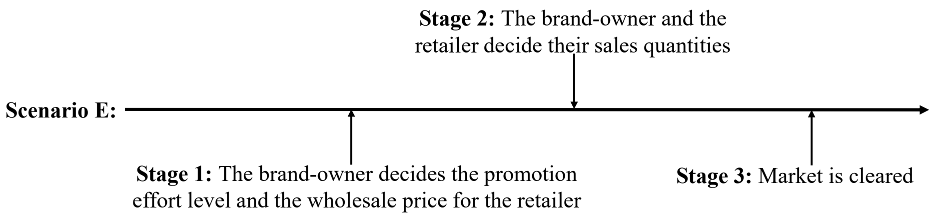


Figure 2. The sequence of events in Scenario E

We solve the profit maximization problems by backward induction and the equilibrium outcomes are listed in Table 3.

Table 3. Equilibrium Outcomes in Scenario E

Scenario E			
w^E	$\frac{a[4-(8-4b^2+b^3)k]}{8-16k+b^2(1+6k)}$	q_b^E	$\frac{a(b-8k+2bk+b^2k)}{8-16k+b^2(1+6k)}$
θ	$\frac{a(8-4b+b^2)}{16k-8-b^2(1+6k)}$	q_r^E	$\frac{2a[1-2(1-b)k]}{8-16k+b^2(1+6k)}$
π_b^E	$\frac{a^2[2-(12-8b+b^2)k]}{2[8-16k+b^2(1+6k)]}$	π_r^E	$\frac{4a[1-2(1-b)k]^2}{[8-16k+b^2(1+6k)]^2}$

3.3 Using a KOL for product promotion (Scenario K)

In Scenario K, the brand-owner relies on a KOL (subscripted by k) for product promotion and signs a revenue-sharing contract with it at a pre-determined commission rate $\alpha \in (0,1)$. The KOL promotes the products in the livestream room, which generates a strong network externality.

Following Conner (1995), Feng et al. (2020) and Niu et al. (2022a), we assume the demand potential in the direct-selling channel will be expanded to $a + \gamma q_b$, where γ denotes the network externality intensity. However, as we have discussed in the Introduction, due to the impulse purchase, the fans may return the products after they calm down, making the retained quantity sold by the KOL eq_b, where $e \in (0,1)$ denotes the retain rate[3]. Therefore, the inverse demand functions in Scenario K are given by

$$p_b^K = a + \gamma q_b^K - q_b^K - b q_r^K,$$

$$p_r^K = a - q_r^K - b q_b^K.$$

And the profit functions of the brand-owner, the retailer, and the KOL are as follows:

$$\underset{w^K}{Max}\, \pi_b^K = w^K q_r^K + (1-\alpha) p_b^K \cdot e q_b^K,$$

$$\underset{q_r^K}{Max}\, \pi_r^K = (p_r^K - w^K) q_r^K,$$

$$\underset{q_b^K}{Max}\, \pi_k^K = \alpha p_b^K \cdot e q_b^K.$$

Note that, in Scenario K, it is the KOL who decides the promotion quantity in the livestream room, which is consistent with the practice of Michael Le, Li Jiaqi and Luo Yonghao (WARC 2020, CEIBS 2020). The sequence of events in Scenario K is as follows, which is illustrated in Figure 3.

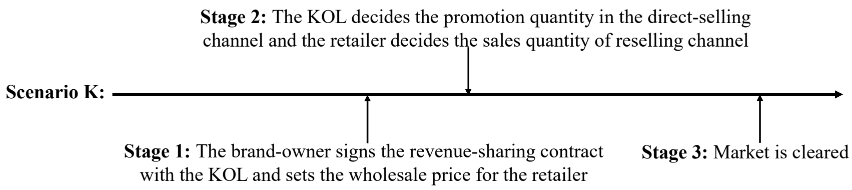


Figure 3. The sequence of events in Scenario K

In Stage 1, the brand-owner signs the revenue-sharing contract with the KOL and determines the wholesale price charged to the retailer w^K. In Stage 2, the KOL decides the promotion quantity in the direct-selling channel q_b^K and the retailer decides the sales quantity of reselling channel q_r^K. In Stage 3, the market is cleared. We solve the game by backward induction and obtain the equilibrium outcomes in Table 4.

Table 4. Equilibrium Outcomes in Scenario K

Scenario K

[3] We assume the return cost incurred by consumers is negligible. This is because we find that in the vast majority of livestream rooms, consumers are covered by the free return services offered by the platforms (e.g., Tiktok) and do not have to bear the return cost. Similar assumption has been adopted in the literature; see, e.g., Shulman et al. (2011) and Altug and Aydinliyim (2016).

w^K	$\frac{a\{b^3-2b^2[1-e(1-\alpha)](1-\gamma)+4b[1-e(1-\alpha)](1-\gamma)+8(1-\gamma)^2\}}{2\{8-b^2[2+e(1-\alpha)]-8\gamma\}(1-\gamma)}$
q_b^K	$\frac{a(8-2b-b^2-8\gamma+2b\gamma)}{2\{8-b^2[2+e(1-\alpha)]-8\gamma\}(1-\gamma)}$
q_r^K	$\frac{a\{2-b[1+e(1-\alpha)]-2\gamma\}}{8-b^2[2+e(1-\alpha)]-8\gamma}$
π_b^K	$\frac{a^2\{b^2-4b[1+e(1-\alpha)](1-\gamma)+4(1-\gamma)[1-2e(1-\alpha)+\gamma]\}}{4\{8-b^2[2+e(1-\alpha)]-8\gamma\}(1-\gamma)}$
π_r^K	$\frac{a^2\{2-b[1+e(1-\alpha)]-2\gamma\}^2}{\{8-b^2[2+e(1-\alpha)]-8\gamma\}^2}$
π_k^K	$\frac{a^2e\alpha(8-2b-b^2-8\gamma+2b\gamma)^2}{4\{8-b^2[2+e(1-\alpha)]-8\gamma\}^2(1-\gamma)}$

4. The value of self-promotion and the KOL effect

In this section, we first conduct some comparative analysis on the wholesale prices and the order/sales quantities in Scenario B, Scenario E and Scenario K, based on which we identify the brand-owner's *value of self-promotion* in Scenario E and the *KOL effect* in Scenario K. To avoid analytical triviality, we assume that $k > \frac{1}{2-2b}$ and $0 < \gamma < \frac{2-b-be(1-\alpha)}{2}$ to guarantee all the equilibrium outcomes are positive.

4.1 Brand-owner's value of self-promotion

Lemma 1. The brand-owner's wholesale price in Scenario E is higher than that in Scenario B (i.e., $w^E > w^B$).

The brand-owner has to balance the two channels' profits when it decides the wholesale price. In Scenario E, the brand-owner promotes products by itself and incurs the promotion effort cost $\frac{1}{2}k\theta^2$. As k increases, the higher promotion cost induces the brand-owner to have fewer incentives to invest in product promotion in the direct-selling channel (i.e., $\frac{\partial\theta}{\partial k} < 0$), which limits the benefit of demand potential expansion of the direct-selling channel $a + \theta$. Hence, the brand-owner has to turn to the retail channel for compensation.

Anticipating that, the brand-owner has two parallel choices: (1) increasing the wholesale price w^E for a larger profit margin; or (2) reducing the wholesale price w^E for a larger order quantity from the retailer. Note that the second choice might be harmful to the brand-owner because it leads to a fierce quantity competition between the two channels. Differently, the first choice can limit the sales quantity of the direct-selling channel, enabling the diversified competition between the two channels, which reduces the system loss and eventually benefits the brand-owner. As a result, the brand-owner prefers to increase the wholesale price charged to the retailer in Scenario E.

Lemma 2. The sales quantity of direct-selling channel in Scenario E is higher than that in Scenario B (i.e., $q_b^E > q_b^B$), while the sales quantity of retail channel in Scenario E is lower than that in Scenario B (i.e., $q_r^E < q_r^B$).

Lemma 2 immediately follows Lemma 1. That is, the brand-owner's high wholesale price w^E increases the retailer's procurement cost in Scenario E, inducing a lower sales quantity in the retail channel. Correspondingly, the sales quantity of direct-selling channel will be enlarged. We here omit repeatable explanations.

Based on the analysis of Lemma 1 and Lemma 2, we now identify the brand-owner's *value of self-promotion* by comparing the brand-owner's profits in Scenario B and Scenario E.

Proposition 1. When the brand-owner promotes products by itself in Scenario E, it can obtain a higher profit compared with that in Scenario B. We therefore have $V^{EB} = \pi_b^E - \pi_b^B > 0$ as the brand-owner's *value of self-promotion*.

In Lemma 1 and Lemma 2, we find that $w^E > w^B$ and $q_r^E < q_r^B$, which imply that the brand-owner has to downward adjust the wholesale price w^B to encourage the retailer to place a larger order quantity q_r^B if there is no product promotion (i.e., Scenario B), incurring margin profit loss. This highlights the importance of product promotion in the brand-owner's margin profit in the wholesaling business. Furthermore, we find $q_b^B < q_b^E$ (see Lemma 2), which highlights the importance of the product promotion in the brand-owner's sales quantity in the direct-selling channel. Therefore, investing in product promotion in Scenario E not only enables the brand-owner to squeeze the profit margin of the retailer by charging a higher wholesale price $w^E > w^B$ but also effectively expands the demand potential in the direct-selling channel from a to $a + \theta$, resulting in the brand-owner's positive *value of self-promotion*.

4.2 The KOL effect

Lemma 3. There exist thresholds e_1, α_1 and γ_1 (whose detailed expressions are presented in

Table 9 of the Appendix) such that the brand-owner's wholesale price in Scenario K is higher than that in Scenario B (i.e., $w^K > w^B$) when $e > e_1$, $\alpha < \alpha_1$ and $\gamma > \gamma_1$.

Compared with Scenario B, the brand-owner should balance the two channels' profits when it decides the wholesale price in Scenario E, which is impacted by the retain rate e, the commission rate α and the network externality γ. Note that, a high retain rate and a strong network externality are effective to enhance the brand-owner's channel power in the dual-channel system, whereas a high commission rate acts oppositely. Therefore, Lemma 3 confirms that the brand-owner can charge a higher wholesale price to the retailer in Scenario K when the retain rate is high (i.e., $e > e_1$), the network externality is strong (i.e., $\gamma > \gamma_1$), and the commission rate is low (i.e., $\alpha < \alpha_1$).

Lemma 4. There exist thresholds γ_2 and γ_3 (whose detailed expressions are presented in Table 9 of the Appendix) such that the sales quantity of retail channel in Scenario K is lower than that in Scenario B (i.e., $q_r^K < q_r^B$) when $\gamma > \gamma_2$, while the sales quantity of direct-selling channel in Scenario K is higher than that in Scenario B (i.e., $q_b^K > q_b^B$) when $\gamma > \gamma_3$.

Lemma 4 shows that when the network externality is strong, the sales quantity of retail (direct-selling) channel in Scenario K is lower (higher) than that in Scenario B. The sales quantity decision of the retailer is immediately resulted from the brand-owner's wholesale price decision in Lemma 3. However, we find that whether the sales quantity of direct-selling channel in Scenario K is higher than that in Scenario B depends on the network externality only. This is because the sales quantity of the direct-selling channel is determined by the KOL, and the decision only depends on its opinion influence (i.e., the network externality intensity). Therefore, when the network externality is strong, the KOL decides to sell more in the direct-selling channel (i.e., $q_b^K > q_b^B$ when $\gamma > \gamma_3$).

Based on the analysis of Lemma 3 and Lemma 4, we now identify the *KOL effect* by comparing the brand-owner's profits in Scenario B and Scenario K.

Proposition 2. There exist thresholds e_2, α_2 and γ_4 (whose detailed expressions are presented in Table 9 of the Appendix) such that the brand-owner relies on the KOL for product promotion (i.e., Scenario K), it can obtain a higher profit compared with that without product promotion (i.e., Scenario B), i.e., $V^{KB} = \pi_b^K - \pi_b^B > 0$ when $e > e_2$, $\alpha < \alpha_2$ and $\gamma > \gamma_4$. We therefore define V^{KB} as the *KOL effect*, which can be either a negative or a positive driving force for the brand-owner to prefer Scenario K.

Corollary 1. Investigating the impact of γ on the KOL effect, there exist thresholds e_2, α_2, α_3,

γ_4 and γ_5 (whose detailed expressions are presented in Table 9 of the Appendix) such that we find (a) when $e > e_2$, $\alpha < \alpha_2$ and $\gamma > \gamma_4$, $V^{KB} > 0$ and $\frac{\partial V^{KB}}{\partial \gamma} > 0$; and (b) when $\alpha > max\{0, \alpha_3\}$ and $\gamma < \gamma_5$, $V^{KB} < 0$ and $\frac{\partial V^{KB}}{\partial \gamma} < 0$, where γ_5 is a threshold with respect to γ.

Interestingly, Proposition 2 reveals that the *KOL effect* could benefit or hurt the brand-owner, depending on three factors, i.e., the retain rate e, the commission rate α and the network externality γ. The explanations are as follows.

Note that, the network externality intensity γ captures the KOL's influence. As γ becomes larger, the more fans of the KOL will be attracted to the livestreaming room (i.e., the direct-selling channel) to purchase the products. Therefore, if the retain rate is high and the commission rate is low, the brand-owner can be benefited a lot given a large γ. As a result, as shown in Lemma 3, the brand-owner has the backbone to charge the retailer a high wholesale price (i.e., $w^K > w^B$). The increased wholesale price also suppresses the sales quantity in the retail channel but enlarges the sales quantity in the direct-selling channel when the network externality is significant. Therefore, when the retain rate is high, the commission rate is low, and the network externality is significant, the *KOL effect* benefits the brand-owner. Otherwise, the using of the KOL will be too costly.

More interestingly, as shown in Corollary 1(a), when the *KOL effect* acts as a positive driving force (i.e., $V^{KB} > 0$), the *KOL effect* will be enhanced by the network externality (i.e., $\frac{\partial V^{KB}}{\partial \gamma} > 0$). However, Corollary 1 (b) shows that, when the *KOL effect* acts as a negative driving force (i.e., $V^{KB} < 0$), the dark side of the KOL effect might be further amplified (i.e., $\frac{\partial V^{KB}}{\partial \gamma} < 0$) if the network externality is small, depending on the retain rate e and the commission rate α. This signifies that there exists an "inefficient interval of network externality". That is, the enhanced network externality cannot reverse the profit loss of the brand-owner in Scenario K; see the illustration in Figure 4. We now take a close look at the "inefficient interval of network externality".

When the retain rate (return rate) is low (high) (see Figure 4(a), $e = 0.1$), the occurrence of the "inefficient interval of network externality" is mainly attributed to the unreliability of the direct-selling channel. Thus, the brand-owner has high product return even though the network externality is increasing. Conversely, when the retain rate (return rate) is relatively high (low), the occurrence of the "inefficient interval of network externality" is mainly attributed to the commission rate. A low (high) commission rate indicates that the KOL can split a small (large) proportion of the brand-owner's direct-selling profit. When the commission rate is low, the brand-owner can retain a large proportion of the direct-selling profit, making the brand-owner be benefited from the enhanced network externality. Thus, in this case, the "inefficient interval of network externality" will not arise (see Figure 4(b)). However, when the commission rate is high, it works with the product return to hurt the brand-owner's direct-selling profit. Therefore, the "inefficient interval of network externality" arises (see Figure 4(c)).

Corollary 1 can be insightful for the brand-owners who plan to use KOLs for product promotion. It reveals that when the product return rate is high or the commission rate is high, the enhanced opinion influence of KOL will backfire on the brand-owner.

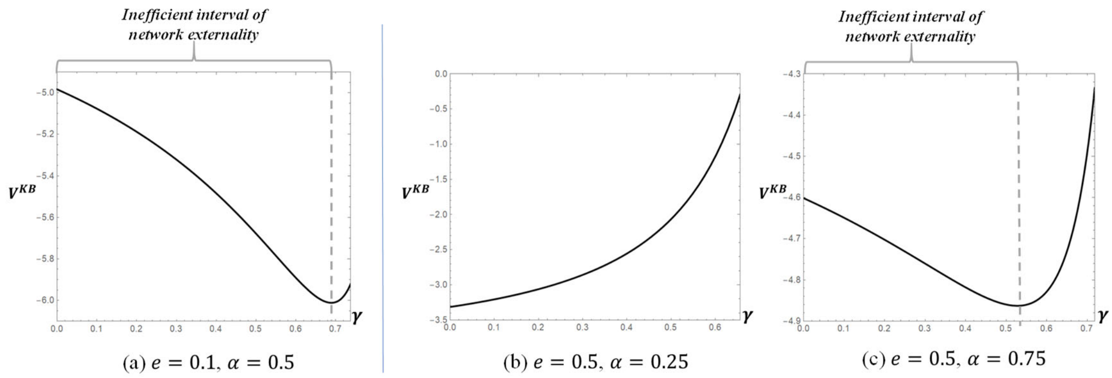


Figure 4. The impact of γ on the *KOL effect* ($a = 5,\ b = 0.5$)

5. Supply chain parties' preference analysis

We now compare the profits of the brand-owner and the retailer in Scenario K and Scenario E to investigate their preferences.

Proposition 3. There exist thresholds e_3, α_4 and γ_6 (whose detailed expressions are presented in Table 9 of the Appendix) such that the *KOL effect* in Scenario K is more powerful than the *value of self-promotion* in Scenario E (i.e., $\pi_b^K - \pi_b^B = V^{KB} > V^{EB} = \pi_b^E - \pi_b^B$) when $e > e_3$, $\alpha < \alpha_4$ and $\gamma > \gamma_6$, under which the brand-owner prefers Scenario K over Scenario E (i.e., $\pi_b^K > \pi_b^E$).

Proposition 3 is resulted from the comparison of the brand-owner's *value of self-promotion* in Scenario E and the *KOL effect* in Scenario K. We have the following explanations for this result. First, Proposition 2 indicates that, when the retain rate is high, the *KOL effect* will benefit the brand-owner if the commission rate is low (i.e., $\alpha < \alpha_2$) and the network externality is strong (i.e., $\gamma >$

γ_4). Otherwise, the *KOL effect* will act as a negative effect. Second, the *value of self-promotion* is fixed that cannot be enhanced by the network externality. However, when the *KOL effect* acts a positive effect (see Corollary 1 (a)), it will be enhanced by the network externality, making the benefit from the *KOL effect* become dominated. This induces the brand-owner to prefer Scenario K.

One natural question arises: Do there exist incentive alignment opportunities for the brand-owner and the retailer with regard to their preferences over Scenario K and Scenario E? To answer this question, we have Proposition 4.

Proposition 4. There exist thresholds e_4, α_4, γ_6 and γ_7 (whose detailed expressions are presented in Table 9 of the Appendix) such that (1) the retailer prefers Scenario E (i.e., $\pi_r^K < \pi_r^E$) when $\gamma > \gamma_7$. (2) Both the brand-owner and the retailer are better off in Scenario E (i.e., $\pi_b^K < \pi_b^E$ and $\pi_r^K < \pi_r^E$) if one of the following conditions is satisfied:

(a) $e > e_4$, $\begin{cases} \alpha < \alpha_4 \text{ and } \gamma_7 < \gamma < \gamma_6 \\ \alpha > \alpha_4 \text{ and } \gamma > \gamma_7 \end{cases}$;

(b) $e < e_4$, $\gamma > \gamma_7$.

Figure 5 shows that both the brand-owner and the retailer can be better off in Scenario E if the network externality intensity exceeds a unique threshold (i.e., $\gamma > \gamma_7$). In Proposition 4(2a), although the brand-owner has the incentive to choose Scenario K, a small γ discourages it to hire the KOL for product promotion ($\gamma_7 < \gamma < \gamma_6$). This highlights the value of self-promotion. Meanwhile, the retailer enjoys a higher sales quantity in Scenario E ($q_r^K < q_r^E$). Therefore, their preferences are aligned. Regarding Proposition 4(2b), we know that the condition comes from the brand-owner because the retailer will undoubtedly prefer Scenario E here. Taking a close look at Proposition 4(2b) and comparing it with Proposition 3, we find that the difference lies in $e < e_4$, because otherwise the profit in the direct-selling channel will significantly decrease in Scenario K. This weakens the brand-owner's channel power compared to the retailer. Therefore, the brand-owner aligns the preference with the retailer in Scenario E.

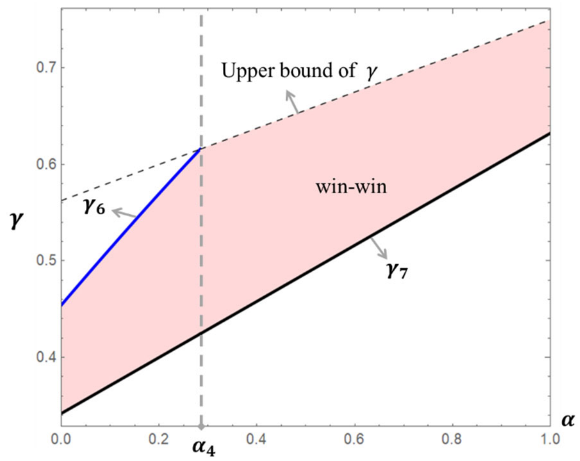


Figure 5. The win-win zone between the brand-owner and the retailer in Scenario E

$(a = 5,\ b = 0.5,\ e = 0.75)$

6. Extensions

6.1 The impact of channel spillover

In this subsection, we investigate the impact of channel spillover by allowing the retail channel to be partially benefited from the brand-owner's promotion effort in Scenario E (the KOL's network externality in Scenario K). That is, the demand potential in the retail channel is expanded by $\beta\theta$ in Scenario E ($\beta\gamma q_b^K$ in Scenario K) due to the channel spillover effect, where $\beta \in (0,1)$ captures the intensity of channel spillover (Ge et al. 2014, Abhishek et al. 2016, Wu et al. 2021, Fan et al. 2022). The inverse demand functions in Scenario E are as follows:

$$p_b^E = a + \theta - q_b^E - bq_r^E,\ p_r^E = a + \beta\theta - q_r^E - bq_b^E.$$

The inverse demand functions in Scenario K are as follows:

$$p_b^K = a + \gamma q_b^K - q_b^K - bq_r^K,\ p_r^K = a + \beta\gamma q_b^K - q_r^K - bq_b^K.$$

By backward induction, we obtain the equilibrium outcomes in Table 5 and Table 6.

Table 5. Equilibrium Outcomes in Scenario E

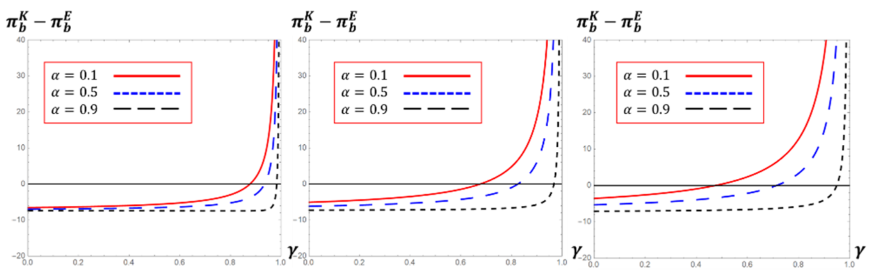
	Scenario E
w^E	$\frac{2a(1-\beta)(2-b\beta)-a(8-4b^2+b^3)k}{b^2(1+6k)-8b\beta+4(2-4k+\beta^2)}$
θ	$\frac{a[4b(1+\beta)-b^2-4(2+\beta)]}{b^2(1+6k)-8b\beta+4(2-4k+\beta^2)}$
q_b^E	$\frac{a(b-8k+2bk+b^2k-2\beta-b\beta+2\beta^2)}{b^2(1+6k)-8b\beta+4(2-4k+\beta^2)}$

q_r^E	$\frac{2a[1-2(1-b)k-\beta]}{b^2(1+6k)-8b\beta+4(2-4k+\beta^2)}$
π_b^E	$\frac{a^2[2(1-\beta)^2-(12-8b+b^2)k]}{2[b^2(1+6k)-8b\beta+4(2-4k+\beta^2)]}$
π_r^E	$\frac{4a^2[1-2(1-b)k-\beta]^2}{[b^2(1+6k)-8b\beta+4(2-4k+\beta^2)]^2}$

Table 6. Equilibrium Outcomes in Scenario K

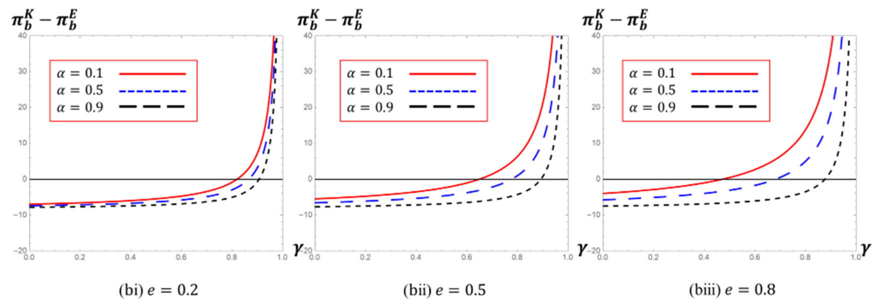
	Scenario K
w^K	$\frac{a\{b^3+4(1-\gamma)[2-\gamma(2-\beta)]-2b^2[1+e(1-\gamma)(1-\alpha)-\gamma(1-\beta)]-b[4-4e(1-\gamma)(1-\alpha)+\gamma^2(2-\beta)\beta-2\gamma(2+\beta)]\}}{2(1-\gamma)\{8-8\gamma-b^2[2+e(1-\alpha)]+2b\gamma\beta\}}$
q_b^K	$\frac{a[8(1-\gamma)+b\gamma(2+\beta)-2b-b^2]}{2(1-\gamma)\{8-8\gamma-b^2[2+e(1-\alpha)]+2b\gamma\beta\}}$
q_r^K	$\frac{a\{2-b[1+e(1-\alpha)]-\gamma(2-\beta)\}}{8-8\gamma-b^2[2+e(1-\alpha)]+2b\gamma\beta}$
π_b^K	$\frac{a^2\{b^2+8e(1-\gamma)(1-\alpha)+[2-\gamma(2-\beta)]^2-2b[2+2e(1-\gamma)(1-\alpha)-\gamma(2-\beta)]\}}{4(1-\gamma)\{8-8\gamma-b^2[2+e(1-\alpha)]+2b\gamma\beta\}}$
π_r^K	$\frac{a^2\{2-b[1+e(1-\alpha)]-\gamma(2-\beta)\}^2}{(1-\gamma)\{8-8\gamma-b^2[2+e(1-\alpha)]+2b\gamma\beta\}^2}$
π_k^K	$\frac{a^2e\alpha[2b+b^2-8(1-r)-br(2+\beta)]^2}{4(1-\gamma)\{8-8\gamma-b^2[2+e(1-\alpha)]+2b\gamma\beta\}^2}$

It is challenging to derive the closed-form comparison results, so we conduct extensive numerical studies to examine the brand-owner's preferences of Scenario K/E. Typical observations are presented in Figure 6, which shows how the channel spillover influences the brand-owner's profits in Scenario K and Scenario E.

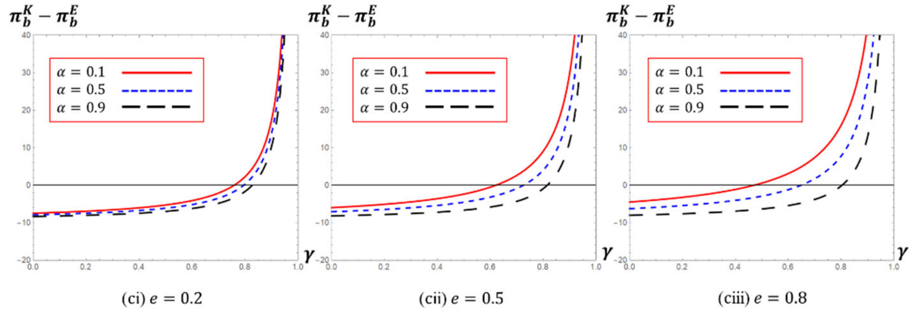


(ai) $e = 0.2$ (aii) $e = 0.5$ (aiii) $e = 0.8$

(a) $\beta = 0.25$



(b) $\beta = 0.5$



(c) $\beta = 0.75$

Figure 6. Comparison of the brand-owner's profits in Scenario K and Scenario E with considering the impact of channel spillover ($a = 5,\ b = 0.3,\ k = 2$)

Recall from Proposition 3 that the brand-owner prefers Scenario K when the retain rate (e) is high, the commission rate (α) is low and the network externality (γ) is strong. We find that when channel spillover is considered, the brand-owner's preference toward Scenario K will follow a similar pattern as that in the basic model. The difference is that, if the retain rate in the direct-selling channel is sufficiently low (i.e., $e = 0.2$ in Figure 6(ai)(bi)(ci)), the brand-owner will not overwhelmingly prefer Scenario E. This is mainly attributed to the *complementary relationship* between the channel spillover and the network externality. Note that the network externality in the direct-selling channel can directly benefit the retail channel through the channel spillover (i.e., $\beta\gamma q_b^K$). Thus, the market potential of the two channels can be significantly expanded because of the multiplication of the network externality and the channel spillover (i.e., $(1 + \beta)\gamma q_b^K$). This covers the loss from product returns and induces the brand-owner to prefer Scenario K.

We have checked the robustness of the main results in Section 6.1 by varying the values of system parameters, including channel substitutability (b), the brand-owner's efficiency in promotion effort investment (k), retain rate (e), commission rate (α), network externality (γ), and channel spillover (β). The ranges of parameter values are presented in Table 7.

Table 7. The ranges of parameter values considering the impact of channel spillover ($a = 5$)

Parameter	Lower bound	Upper bound	Step	Count of value
b	0.2	0.7	0.05	11
k	2	4	0.05	41
β	0.25	0.75	0.1	6
e	0.2	0.8	0.05	13
α	0.1	0.9	0.05	17
γ	0.1	0.9	0.1	9

In total, we obtain $11 \times 41 \times 6 \times 13 \times 17 \times 9 = 5{,}382{,}234$ value combinations. The coding was done in R, and the results of each parameter value combination are collected and sorted out by $(\beta,\ \gamma)$. We show the numerical results by data bars in Figure 7, where each data bar represents the proportion of value combinations that makes $\pi_b^K > \pi_b^E$ when $(\beta,\ \gamma)$ is fixed. Figure 7 also shows that, given a large γ, the brand-owner has more incentives to use KOL for product promotion with the consideration of the spillover effect. Therefore, our main results continue to hold and will not be qualitatively affected by the channel spillover effect (β).

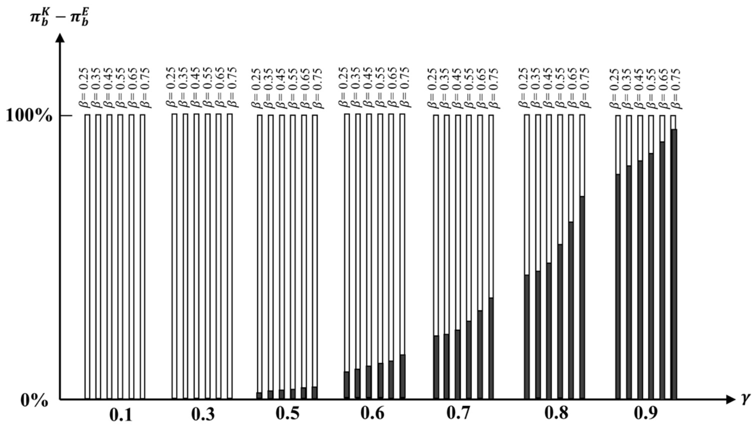


Figure 7. Comparison of the brand-owner's profits in Scenario K and Scenario E with considering the impact of channel spillover

6.2 The impact of the KOL's promotion effort

In this subsection, we investigate a scenario where the KOL makes product promotion efforts (superscripted as K'). We assume the KOL's effort level is η, and the incurred promotion cost is

$\frac{1}{2}k\eta^2$. The inverse demand functions and the profit functions of the brand-owner, the KOL, and the retailer are given by

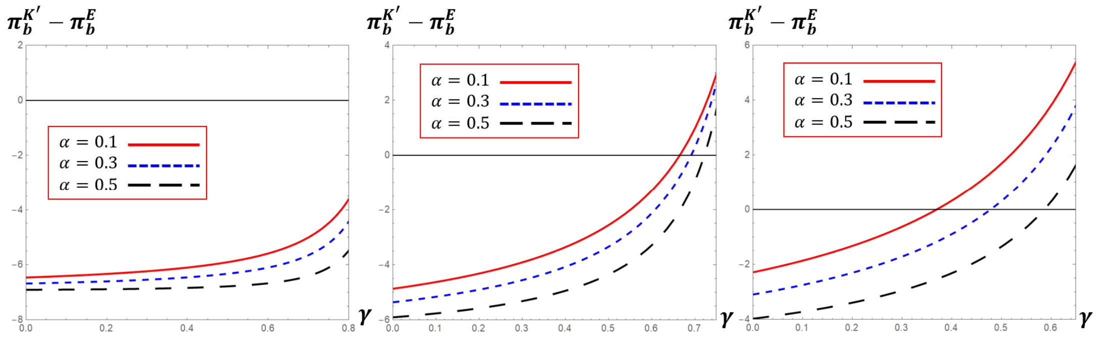
$$p_b^{K'} = a + \eta + \gamma q_b^{K'} - q_b^{K'} - bq_r^{K'},\ p_r^{K'} = a - q_r^{K'} - bq_b^{K'}.$$

$$\underset{w^{K'}}{Max}\,\pi_b^{K'} = w^{K'}q_r^{K'} + (1-\alpha)p_b^{K'}\cdot eq_b^{K'},$$

$$\underset{\{\eta, q_b^{K'}\}}{Max}\,\pi_k^{K'} = \alpha p_b^{K'}\cdot eq_b^{K'} - \frac{1}{2}k\eta^2,$$

$$\underset{q_r^{K'}}{Max}\,\pi_r^{K'} = (p_r^{K'} - w^{K'})q_r^{K'}.$$

The brand-owner decides the wholesale price $w^{K'}$ for the retailer and the KOL decides the promotion effort level η. Then the KOL and the retailer decide their quantities $q_b^{K'}$ and $q_r^{K'}$, respectively. We solve the games by backward induction and the equilibrium outcomes in Scenario K$'$ are presented in Appendix. The formulas are so complex that we rely on numerical studies to show the brand-owner's preference.



(a) $e = 0.2$ (b) $e = 0.5$ (c) $e = 0.8$

Figure 8. Comparison of the brand-owner's profits in Scenario K$'$ and Scenario E

$(a = 5,\ b = 0.2,\ k = 2)$

Figure 8 indicates that the KOL's promotion effort will not qualitatively change the brand-owner's preferences. When the retain rate in the direct-selling channel is low (see Figure 8(a)), the effective sales quantity in the direct-selling channel is low, making the *KOL effect* act negatively on the brand-owner. Thus, although the KOL makes promotion efforts, it cannot make up the brand-owner's profit loss because of the high product return rate. However, as e increases, the brand-owner's preferences may be altered from preferring Scenario E to preferring Scenario K$'$, as shown in Figures 8 (b) and (c). This is because the high product retain rate and the KOL's promotion effort interact with each other and contribute to enhance the positive force of the *KOL effect*.

6.3 The social welfare and consumer surplus

In this subsection, we analyze the social welfare and consumer surplus. According to Singh and Vives (1984), we have

$$U^B = aq_r^B + aq_b^B - \frac{(q_r^B)^2 + 2bq_r^B q_b^B + (q_b^B)^2}{2},$$

$$U^E = aq_r^E + (a+\theta)q_b^E - \frac{(q_r^E)^2 + 2bq_r^E q_b^E + (q_b^E)^2}{2},$$

$$U^K = aq_r^K + \left(a + \frac{1}{2}\gamma q_b^K\right) q_b^K - \frac{(q_r^K)^2 + 2bq_r^K q_b^K + (q_b^K)^2}{2}.$$

where U^h $(h \in \{B, K, E\})$ is the social welfare in Scenario h. Correspondingly, the consumer surplus in Scenario B, Scenario E and Scenario K are $CS^B = U^B - \pi_r^B - \pi_b^B$, $CS^E = U^E - \pi_r^E - \pi_b^E$ and $CS^K = U^K - \pi_r^K - \pi_b^K - \pi_k^K$, respectively. The formulas of social welfare and consumer surplus are lengthy, so we relegate them to the Appendix.

Next, we analyze how the brand-owner's promotion strategies (i.e., self-promotion or KOL promotion) affect the consumer surplus and social welfare. We first show that the brand-owner's product promotion (i.e., self-promotion and KOL promotion) lead to a Pareto improvement of the consumer surplus and social welfare, as stated in the following proposition.

Proposition 5. With the brand-owner's product promotion (i.e., self-promotion or KOL promotion), the social welfare and consumer surplus are higher than those without product promotion (i.e., $U^E > U^B$ and $CS^E > CS^B$, $U^K > U^B$ and $CS^K > CS^B$).

We then study which promotion strategy is more effective by conducting extensive numerical experiments. Typical curves are depicted in Figures 9 and 10.

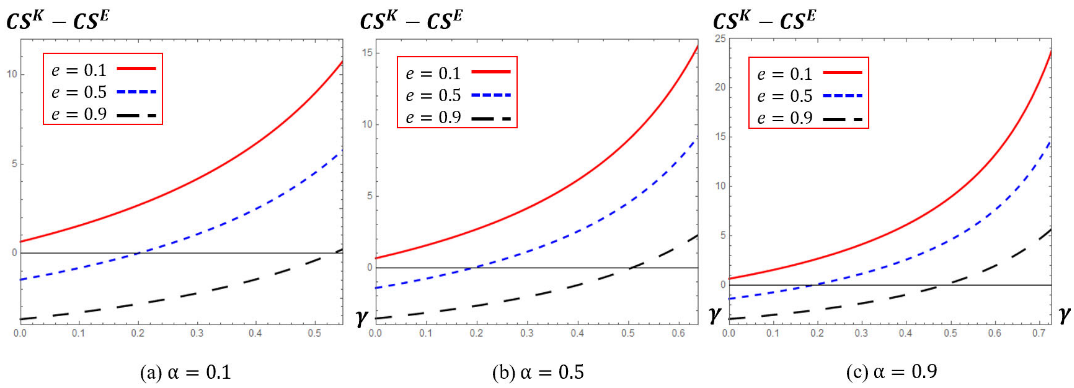


Figure 9. Comparison of the consumer surplus in Scenario K and Scenario E

$(a = 5,\ b = 0.5,\ k = 2)$

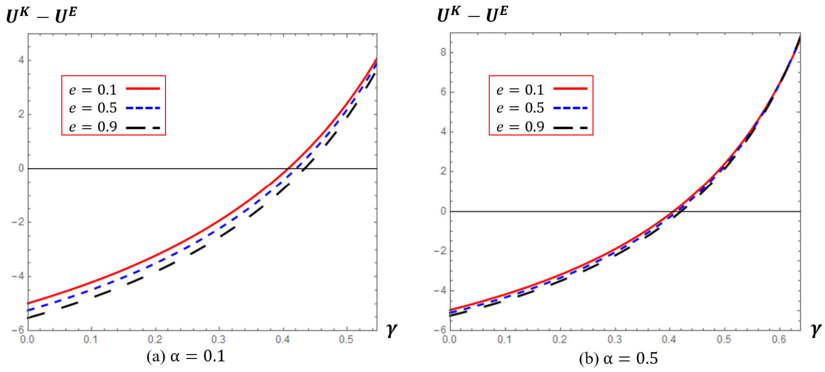


Figure 10. Comparison of the social welfare in Scenario K and Scenario E

$(a = 5,\ b = 0.5,\ k = 2)$

Figure 9 shows that, with a strong network externality (i.e., a large γ), consumers prefer the brand-owner to use the KOL for promotion. The reasons are as follows. With a strong network externality, the sales quantity in the direct-selling channel will be increased, which enlarges the total supply of the products. As a result, the competition between the direct-selling channel and the retail channel will be intensified, leading to lower product prices. Thus, the consumers are benefited from both the increased product supply and the lowered product price, resulting in a high consumer surplus in Scenario K.

We also observe that, as the retain rate e decreases, the consumer surplus is higher. We interpret it as follows. First, the quantity in the direct-selling channel in Scenario K decreases in e (i.e., $\frac{\partial q_b^K}{\partial e} < 0$), while the quantity in the retail channel in Scenario K increases in e (i.e., $\frac{\partial q_r^K}{\partial e} > 0$). So, a small e indicates the brand-owner sells more goods in the direct-selling channel. Second, the quantity in the direct-selling channel is more sensitive than that in the retail channel (i.e., $\left|\frac{\partial q_b^K}{\partial e}\right| > \left|\frac{\partial q_r^K}{\partial e}\right|$). Therefore, as e decreases, the total quantity in the dual-channel system increases, especially in the direct-selling channel. This is beneficial for the consumer surplus.

Regarding the social welfare, we find it exhibits a very similar pattern as that of the consumer surplus (see Figure 10). This indicates that, the performance of social welfare highly depends on the consumer surplus rather than the profit of the supply chain. We explain it as follows. Proposition 3 indicates that, when the network externality is strong (γ is large), both the brand-owner and the KOL can be more benefited in Scenario K. This leads to a higher profit of the supply chain even though the retailer's profit will be hurt by the KOL's promotion in Scenario K. Therefore, the social welfare in Scenario K is higher than that in Scenario E when the network externality is strong.

6.4 The network externality in Scenario B

If the brand-owner does not use the KOL but conducts livestream promotion using their employees, then there also exists a network externality γ' in Scenario B, which, however, is weaker than that from the KOL's opinion influence, i.e., $\gamma' < \gamma$. We denote this scenario as Scenario B'. In this situation, the inverse demand functions are as follows:

$$p_r^{B'} = a - q_r^{B'} - bq_b^{B'},\ p_b^{B'} = a + \gamma' q_b^{B'} - q_b^{B'} - bq_r^{B'}.$$

And the profit functions of the brand-owner and the retailer are:

$$\pi_r^{B'} = (p_r^{B'} - w^{B'})q_r^{B'},\ \pi_b^{B'} = w^{B'}q_r^{B'} + p_b^{B'}q_b^{B'}.$$

We assume that there is no product return because the consumers trust the brand-owner. By backward induction, the equilibrium outcomes are derived in Table 8.

Table 8. Equilibrium Outcomes in Scenario B'

Scenario B'			
$w^{\mathrm{B}'}$	$\frac{a[b^3-4b^2(1-\gamma')-8(1-\gamma')^2]}{2(1-\gamma')(8-3b^2-8\gamma')}$		
$q_b^{\mathrm{B}'}$	$\frac{a(8-2b-b^2-8\gamma'+2b\gamma')}{2(1-\gamma')(8-3b^2-8\gamma')}$	$q_r^{\mathrm{B}'}$	$\frac{2a(1-b-\gamma')}{8-3b^2-8\gamma'}$
$\pi_b^{\mathrm{B}'}$	$\frac{a^2\left[b^2-8b(1-\gamma')+4(3-4\gamma'+\gamma'^2)\right]}{4(1-\gamma')(8-3b^2-8\gamma')}$	$\pi_r^{\mathrm{B}'}$	$\frac{4a^2(1-b-\gamma')^2}{(8-3b^2-8\gamma')^2}$

Figure 11 shows the brand-owner's preferences over Scenario B' and Scenario K. It is worth noting that in Scenario B', the brand-owner's network externality will help expand the demand potential. Therefore, Figure 11 shows that the brand-owner will only prefer Scenario K when the KOL's network externality is sufficiently strong. On the other hand, given a small γ', the market expansion in Scenario B' is too small, so the brand-owner is motivated to prefer Scenario K when γ is large and γ' is small.

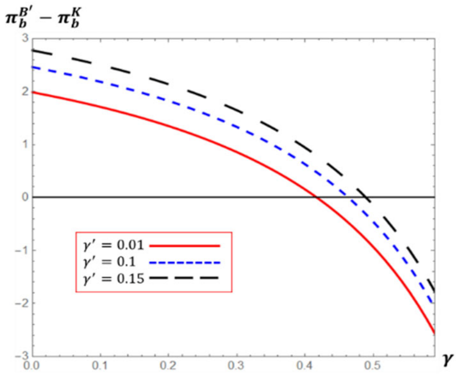


Figure 11. Comparison of the brand-owner's profits in Scenario K and Scenario B′
($a = 5,\ b = 0.5,\ e = 0.75,\ \alpha = 0.15$)

7. Conclusion

Nowadays, livestream e-commerce has become a flourishing business in the world, especially during the COVID-19 pandemic. Livestream has changed the way that consumers can only get product information from online pictures and videos before. Through interactions with the KOL and even the other consumers, the KOL fans can not only learn the product information better but also enjoy the fun and happiness of shopping in a real-time way. Usually, the more influential the KOL is, the stronger the *KOL effect* will be. These all stimulate customers' herd behavior and lead to a strong network externality (Influencer 2021). As a result, livestream e-commerce has greatly improved consumers' sense of participation and stimulated them to become more impulsive in their consumption. This results in the product return problem, so the brand-owners must seriously evaluate the tradeoffs, the gains and losses of using KOLs.

In this paper, we consider a supply chain comprised of a brand-owner, a retailer, and possibly a KOL. The brand-owner is optional to use the KOL, or, to invest in product promotion by itself. We identify the brand-owner's *value of self-promotion* and the *KOL effect*, based on which we examine and explain the supply chain parties' preferences over Scenario K (with the KOL) and Scenario E (without the KOL). The main results are as follows.

We find that in Scenario E, the *value of self-promotion* is positive, so the brand-owner always has incentives to invest in promotion efforts. However, the *KOL effect* in Scenario K is not necessarily beneficial for the brand-owner, because it expands the demand potential in the brand-owner's direct-selling channel but may incur the commission cost, product return cost, and more importantly, change the channel competition and the brand-owner's channel power.

We show that, interestingly, the *value of self-promotion* can be very powerful in the brand-owner's preference of livestream e-commerce. That is, even though the *KOL effect* is beneficial for the brand-owner, the profit of brand-owner under the traditional promotion strategy can be high, especially when the network externality is not sufficiently strong. We also examine the retailer's preference and even find incentive alignment opportunities in Scenario E. In four extensions in Section 6, we confirm the robustness of the main findings in the presence of the spillover effect, KOL's promotion effort, and etc.

We discuss two research directions to conclude this paper. First, the brand-owner may sign an alternative contract with the KOL, e.g., a revenue-sharing contract based on volume, a two-part tariff contract, and a flexible commission consignment contract (Wang et al. 2004). Then, the equilibrium outcomes will be changed, and the supply chain parties' preferences might be altered. Second, in this paper, the network externality caused by the KOL's opinion influence is assumed to be given based on historical data. But in practice, KOLs can make efforts to increase their influence, thereby increasing the network externality. We leave the KOL's efforts in network externality as an interesting and promising future study.

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Appendix

Thresholds used in the main context and their detailed expressions

Table 9. Summary of Thresholds Used

γ_1	$\frac{(2-b)\{4be(1-\alpha)-b^3[2+e(1-\alpha)]-16[1-e(1-\alpha)]-b^2[2+4e(1-\alpha)]\} + \sqrt{\begin{matrix} 4(8b-8b^2)\{(2-b)^2(8+6b+b^2)[1-e(1-\alpha)]\} \\ +(2-b)^2\{4be(1-\alpha)-b^3[2+e(1-\alpha)]-16[1-e(1-\alpha)]-b^2[2+4e(1-\alpha)]\}^2 \end{matrix}}}{2(8b-8b^2)}$
γ_2	$\frac{(8-2b-b^2)[1-e(1-\alpha)]}{16-6b}$
γ_3	$\frac{64-16b-b^2[8+8e(1-\alpha)]+b^4[2+e(1-\alpha)]-2b^3[1-e(1-\alpha)] - \sqrt{\{64-16b-b^2[8+8e(1-\alpha)]+b^4[2+e(1-\alpha)]-2b^3[1-e(1-\alpha)]\}^2-4(64-16b-8b^2)\{b^2(8-2b-b^2)[1-e(1-\alpha)]\}}}{2(64-16b-8b^2)}$
γ_4	$\frac{(8-2b-b^2)\{b^2[2+e(1-\alpha)]-2b[4+e(1-\alpha)]+8[2+e(1-\alpha)]\} - \sqrt{4(64-64b+20b^2)(8-2b-b^2)^2[e(1-\alpha)-1]+(8-2b-b^2)^2\{b^2[2+e(1-\alpha)]-2b[4+e(1-\alpha)]+8[2+e(1-\alpha)]\}^2}}{2(64-64b+20b^2)}$
γ_5	$\frac{2b^2+8e-2be+b^2e-8e\alpha+2be\alpha-b^2e\alpha-4b}{8e-2be-8e\alpha+2be\alpha-4b}$
γ_6	$\frac{16(8-2b-b^2)^2e^2k\alpha\left\{\begin{matrix}[b^3(1+e-2k)-64k+b^4k-16e(1-2k)+8b(1+e+6k)-b^2(4+3e+8k)] \\ [b^4k+b^3(4k-2\alpha)+32\alpha-6b^2(2k-\alpha)-16b(2k+\alpha)]\end{matrix}\right\} - \sqrt{\begin{matrix} 4[32k-32bk+b^2(1+10k)]\left\{\begin{matrix} b^3[4-8k+4e(1+2k)(1-\alpha)]-b^4[1+2k[1-e(1-\alpha)]]-64[2k+e(1-2k)(1-\alpha)] \\ +32b[1+2k+e(1-2k)(1-\alpha)]-4b^2[5-6k+3e(1+2k)(1-\alpha)] \end{matrix}\right\} \\ +\left\{\begin{matrix} 2b^4[ek\alpha-(2+e)k]+4b^2(1+2k)[4+3e(1-\alpha)]+64[4k+e(1-2k)(1-\alpha)] \\ -32b[1+6k+e(1-2k)(1-\alpha)]-4b^3[1-2k+e(1+2k)(1-\alpha)] \end{matrix}\right\} \end{matrix}}}{8[32k-32bk+b^2(1+10k)]}$
γ_7	$\frac{2b[3+2k-e(1-2k)(1-\alpha)]-b^2[1-2k+e(1+2k)(1-\alpha)]-8[1+2k-e(1-2k)(1-\alpha)]}{2(b-16k+6bk)}$
α_1	$\frac{2b^2(2-e)-8(1-e)-b^3(1+e)}{(8-2b^2-b^3)e}$
α_2	$\frac{12b+4(3\mathrm{be}-4e)+b^3(1+e)-2b^2(4+e)}{(12b-2b^2+b^3-16)e}$
α_3	$\frac{8e-2b(2+e)+b^2(2+e)}{(8-2b+b^2)e}$
α_4	$\frac{8e(1-2k)+b^3(1+e)k-2b(1+e)(1-6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k-2b(1-6k)]}$
e_1	$\frac{8-4b^2+b^3}{8-2b^2-b^3}$
e_2	$\frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$
e_3	$\frac{4b-2b^2}{8-2b+b^2}$
e_4	$\frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k-2b(1-6k)}$

Appendix 1: The derivation of the outcomes

Scenario B

First, we drive the outcomes in Scenario B. In this scenario, the brand-owner does not promote its products in the direct-selling channel. The expected profit functions of the brand-owner and the retailer are: $\pi_b^B = w^B q_r^B + p_b^B q_b^B$ and $\pi_r^B = (p_r^B - w^B) q_r^B$. We solve this problem by backward induction. Taking the first-order conditions of the profit functions with respect to q_r^B and q_b^B, we have the best response functions $q_r^B = \frac{1}{2}(a - w^B - bq_b^B)$ and $q_b^B = \frac{1}{2}(a - bq_r^B)$. We then obtain the optimal quantities $q_r^B = \frac{a(2-b)-2w^B}{4-b^2}$ and $q_b^B = \frac{a(2-b)+bw^B}{4-b^2}$. Substituting them into the profit function of the brand-owner and then solving for optimality with respect to w^B, we have the optimal wholesale price $w^B = \frac{a(2-b)(4+2b-b^2)}{16-6b^2}$. Substituting them back, we have the equilibrium outcomes in Table 2.

Scenario E

Now, we drive the outcomes in Scenario E. In this scenario, the brand-owner determines the effort level θ. The profit functions of the brand-owner and the retailer are $\pi_b^E = w^E q_r^E + p_b^E q_b^E - \frac{1}{2}k\theta^2$ and $\pi_r^E = (p_r^E - w^E) q_r^E$, respectively. We solve this problem by backward induction. Taking the first-order conditions of the profit functions with respect to q_r^E and q_b^E, we have the best response functions $q_r^E = \frac{1}{2}(a - w^E - bq_b^E)$ and $q_b^E = \frac{1}{2}(a + \theta - bq_r^E)$. Solving them simultaneously, we then obtain the optimal quantities $q_r^E = \frac{a(2-b)-2w^E-b\theta}{4-b^2}$ and $q_o^E = \frac{a(2-b)+bw^E+2\theta}{4-b^2}$. Substituting them into the profit function of the brand-owner and then solving for optimality with respect to w^E and θ, we have the optimal wholesale price $w^E = \frac{a[4-(8-4b^2+b^3)k]}{8-16k+b^2(1+6k)}$ and effort level $\theta = \frac{a(8-4b+b^2)}{16k-8-b^2(1+6k)}$. Substituting them back, we derive the equilibrium outcomes in Table 3.

Scenario K

Then, we drive the outcomes in Scenario K. In this scenario, the brand-owner uses a KOL with a revenue-sharing contract. The profit functions of the brand-owner and the retailer are: $\pi_b^K = w^K q_r^K + (1-\alpha) p_b^K e q_b^K$ and $\pi_r^K = (p_r^K - w) q_r^K$. We solve this problem by backward induction.

Taking the first-order conditions of the profit functions with respect to q_r^K and q_b^K, and solving them simultaneously, we have $q_r^K = \frac{a(2-b-2\gamma)-2w^K(1-\gamma)}{4-b^2-4\gamma}$ and $q_b^K = \frac{a(2-b)+bw^K}{4-b^2-4\gamma}$. Substituting them into the profit function of the brand-owner and then solving for optimality with respect to w^K, we have the optimal wholesale price $w^K = \frac{a\{b^3-2b^2[1-e(1-\alpha)](1-\gamma)+4b[1-e(1-\alpha)](1-\gamma)+8(1-\gamma)^2\}}{2\{8-b^2[2+e(1-\alpha)]-8\gamma\}(1-\gamma)}$. Substituting them back, we derive the equilibrium outcomes in Table 4.

Appendix 2: The equilibrium outcomes of Sections 6.2 and 6.3

The equilibrium outcomes in Section 6.2

Table 9. Equilibrium Outcomes in Scenario K'

	Scenario K'
$w^{K'}$	$\frac{a(4-b^2-4\gamma)\{k[b^3-(2b^2+2b^2e-\alpha 2b^2e)(1-\gamma)-(4b-4be+4\alpha b)(1-\gamma)+8(1-\gamma)^2]-4e\alpha(1-\gamma)\}}{2\{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)\}(1-\gamma)}$
η	$\frac{2ae\alpha(8-2b-b^2-8\gamma+2b\gamma)}{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)}$
$q_b^{K'}$	$\frac{ak(4-b^2-4\gamma)(8-2b-b^2-8\gamma+2b\gamma)}{2(1-\gamma)\{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)\}}$
$q_r^{K'}$	$\frac{a\{4e\alpha(1-\gamma)+k[2-b-eb(1-\alpha)-2\gamma]\}(4-b^2-4\gamma)}{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)}$
$\pi_b^{K'}$	$\frac{a^2(4-b^2-4\gamma)\left\{\begin{matrix} 32e^2\alpha^2(1-\gamma)^2-8ek\alpha[8-2b^2-e(1-\alpha)b^2-8\gamma](1-\gamma)(2-b-2\gamma) \\ -k^2[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma) \\ [b^2+(4b+4be-4be\alpha)(1-\gamma)+4(1-\gamma)(1+2e-2e\alpha-\gamma)] \end{matrix}\right\}}{4(1-\gamma)\{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)\}^2}$
$\pi_r^{K'}$	$\frac{a^2(4-b^2-4\gamma)^2\{4e\alpha(1-\gamma)+k[2-b-eb(1-\alpha)-2\gamma]\}^2}{\{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)\}^2}$
$\pi_k^{K'}$	$\frac{a^2ek\alpha(8-2b-b^2-8\gamma+2b\gamma)^2[k(4-b^2-4\gamma)^2-8e\alpha(1-\gamma)]}{4(1-\gamma)\{k[8-2b^2-e(1-\alpha)b^2-8\gamma](4-b^2-4\gamma)-2e\alpha(8-b^2-8\gamma)\}^2}$

The equilibrium outcomes in Section 6.3

Table 10. Social Welfare and Consumer Surplus in Scenario B/E/K

		Scenario B	
U^B	$\frac{a^2(304-192b-100b^2+60b^3+3b^4)}{8(8-3b^2)^2}$	CS^B	$\frac{a^2(80-76b^2+12b^3+9b^4)}{8(8-3b^2)^2}$
		Scenario E	

U^E	$\frac{a^2\left[\begin{matrix}3b^4k^2+2b^3k(7+30k)+b^2(7+28k-100k^2)\\+4(7-28k+76k^2)-192bk^2\end{matrix}\right]}{2[8-16k+b^2(1+6k)]^2}$	CS^E	$\frac{a^2\left[\begin{matrix}4+48k-96bk+80k^2+6b^3k(1+2k)\\+b^4k(1+9k)+b^2(5+36k-76k^2)\end{matrix}\right]}{2[8-16k+b^2(1+6k)]^2}$
Scenario K			
U^K	$\frac{a^2\left\{\begin{matrix}3b^4+4b^3[7+6e(1-\alpha)+2e^2(1-\alpha)^2](1-\gamma)\\-48b[3+e(1-\alpha)](1-\gamma)^2+16(1-\gamma)^2(19-7\gamma)\\-4b^2(1-\gamma)[18+e^2(1-\alpha)^2-5\gamma+2e(1-\alpha)(3-2\gamma)]\end{matrix}\right\}}{8\{8-b^2[2+e(1-\alpha)]-8\gamma\}^2(1-\gamma)}$	CS^K	$\frac{a^2\left\{\begin{matrix}b^4[7+e(2-4\alpha)]+4b^3(3-2e\alpha)(1-\gamma)\\+16b[3-e(3+\alpha)](1-\gamma)^2+16(-1+\gamma)^2(13-8e-\gamma)\\+4b^2(1-\gamma)[-20+e^2(-1+\alpha)^2+\gamma+2e(3\alpha+\gamma)]\end{matrix}\right\}}{8\{8-b^2[2+e(1-\alpha)]-8\gamma\}^2(1-\gamma)}$

Appendix 3: Proofs

Proof of Lemma 1.

The difference between w^E and w^B is: $w^E - w^B = -\frac{ab^3(8-4b+b^2)}{2(8-3b^2)[8+b^2-(16-6b^2)k]}$.

Since $k > \frac{1}{2-2b}$ and $1 > b > 0$, we find $8 + b^2 - (16 - 6b^2)k < 0$, $8 - 4b + b^2 > 0$, $8 - 3b^2 > 0$. Therefore, we have $w^E - w^B > 0$, i.e., $w^E > w^B$ for $a > 0$.

Proof of Lemma 2.

(1) The difference between q_r^E and q_r^B is: $q_r^E - q_r^B = \frac{2ab(8-4b+b^2)}{(8-3b^2)[8+b^2-(16-6b^2)k]}$.

Similar to the proof of Lemma 1, we have $q_r^E - q_r^B < 0$, i.e., $q_r^E < q_r^B$ for $a > 0$.

(2) The difference between q_b^E and q_b^B is: $q_b^E - q_b^B = -\frac{a(64-32b+4b^3-b^4)}{2(8-3b^2)[8+b^2-(16-6b^2)k]}$.

Similar to the proof of Lemma 1, we have $q_b^E - q_b^B > 0$, i.e., $q_b^E > q_b^B$ for $a > 0$.

Proof of Proposition 1.

The difference between π_b^E and π_b^B is: $\pi_b^E - \pi_b^B = -\frac{a^2(8-4b+b^2)^2}{4(8-3b^2)[8+b^2-(16-6b^2)k]}$.

Similar to the proof of Lemma 1, we have $\pi_b^E - \pi_b^B > 0$, i.e., $\pi_b^E > \pi_b^B$ for $a > 0$.

Proof of Lemma 3.

The difference between w^K and w^B is: $w^K - w^B = \frac{ag_1(\gamma)}{2(8-3b^2)(8-2b^2-b^2e+b^2e\alpha-8\gamma)(1-\gamma)}$,

where $g_1(\gamma) = -32b + 8b^2 + 12b^3 - 2b^4 - b^5 + 32be - 8b^2e - 12b^3e + 2b^4e + b^5e - 32be\alpha + 8b^2e\alpha + 12b^3e\alpha - 2b^4e\alpha - b^5e\alpha + (32b - 16b^2 + 4b^3 + 2b^4 - 2b^5 - 32be +$

$8b^2e + 12b^3e - 2b^4e - b^5e + 32be\alpha - 8b^2e\alpha - 12b^3e\alpha + 2b^4e\alpha + b^5e\alpha)\gamma + (8b^2 - 8b^3)\gamma^2$.

Subject to $1 > b > 0$ and $0 < \gamma < \frac{1}{2}(2 - b - be + be\alpha)$, it is easy to verify $8 - 3b^2 > 0$, $8 - 2b^2 - b^2e + b^2e\alpha - 8\gamma > 0$ and $1 - \gamma > 0$. Therefore, the sign of $w^K - w^B$ depends on $g_1(\gamma)$: when $g_1(\gamma) > (<)0$, $w^K > (<)w^B$. Solving $g_1(\gamma) = 0$, we have $\gamma_1 = \frac{1}{2(8b-8b^2)}\Big\{16b - 4b^2 - 2b^3 + 2b^4 + 32e - 8be - 12b^2e + 2b^3e + b^4e - 32e\alpha + 8be\alpha + 12b^2e\alpha - 2b^3e\alpha - b^4e\alpha - 32 + [4(8b - 8b^2)(32 - 8b - 12b^2 + 2b^3 + b^4 - 32e + 8be + 12b^2e - 2b^3e - b^4e + 32e\alpha - 8be\alpha - 12b^2e\alpha + 2b^3e\alpha + b^4e\alpha) + (32 - 16b + 4b^2 + 2b^3 - 2b^4 - 32e + 8be + 12b^2e - 2b^3e - b^4e + 32e\alpha - 8be\alpha - 12b^2e\alpha + 2b^3e\alpha + b^4e\alpha)^2]^{\frac{1}{2}}\Big\}$ and $\widetilde{\gamma_1} = \frac{1}{2(8b-8b^2)}\Big\{16b - 4b^2 - 2b^3 + 2b^4 + 32e - 8be - 12b^2e + 2b^3e + b^4e - 32e\alpha + 8be\alpha + 12b^2e\alpha - 2b^3e\alpha - b^4e\alpha - 32 - [4(8b - 8b^2)(32 - 8b - 12b^2 + 2b^3 + b^4 - 32e + 8be + 12b^2e - 2b^3e - b^4e + 32e\alpha - 8be\alpha - 12b^2e\alpha + 2b^3e\alpha + b^4e\alpha) + (32 - 16b + 4b^2 + 2b^3 - 2b^4 - 32e + 8be + 12b^2e - 2b^3e - b^4e + 32e\alpha - 8be\alpha - 12b^2e\alpha + 2b^3e\alpha + b^4e\alpha)^2]^{\frac{1}{2}}\Big\}$, where $\widetilde{\gamma_1} < 0$ and $\gamma_1 > 0$. We note that the quadratic coefficient is $8b^2 - 8b^3 > 0$ in the feasible region. Therefore, when $\gamma > \gamma_1$, $g_1(\gamma) > 0$, $w^K > w^B$.

Recalling that $0 < \gamma < \frac{1}{2}(2 - b - be + be\alpha)$ should be required, then we compare γ_1 with the boundary value $\frac{1}{2}(2 - b - be + be\alpha)$. We have $\gamma_1 - \frac{1}{2}(2 - b - be + be\alpha) = \frac{1}{16(1-b)b}[h_2(\alpha) - h_1(\alpha)]$, where $h_1(\alpha) = [1024 - 768b^2 - 128b^3 + 272b^4 + 48b^5 - 44b^6 - 8b^7 + 4b^8 - 2048e + 512be + 1536b^2e - 448b^3e - 192b^4e + 64b^5e - 32b^6e + 4b^7e + 4b^8e + 1024e^2 - 512be^2 - 704b^2e^2 + 320b^3e^2 + 176b^4e^2 - 64b^5e^2 - 20b^6e^2 + 4b^7e^2 + b^8e^2 + (2048e - 512be - 1536b^2e + 448b^3e + 192b^4e - 64b^5e + 32b^6e - 4b^7e - 4b^8e - 2048e^2 + 1024be^2 + 1408b^2e^2 - 640b^3e^2 - 352b^4e^2 + 128b^5e^2 + 40b^6e^2 - 8b^7e^2 - 2b^8e^2)\alpha + (1024e^2 - 512be^2 - 704b^2e^2 + 320b^3e^2 + 176b^4e^2 - 64b^5e^2 - 20b^6e^2 + 4b^7e^2 + b^8e^2)\alpha^2]^{\frac{1}{2}}$ and $h_2(\alpha) = b^3[10 - 6e(-1 + \alpha)] + b^4[-2 + e(-1 + \alpha)] + 32[1 + e(-1 + \alpha)] - 4b^2[5 + e(-1 + \alpha)] - 8be(-1 + \alpha)$. When $h_1(\alpha) > h_2(\alpha)$, i.e., $[h_1(\alpha)]^2 > [h_2(\alpha)]^2$, we have $\gamma_1 < \frac{1}{2}(2 - b - be + be\alpha)$. Solving $[h_2(\alpha)]^2 - [h_1(\alpha)]^2 = 16b^2(-32 + 48b - 28b^3 + 14b^4 - 2b^5 - 8be + 16b^2e - 16b^3e + 11b^4e - 3b^5e + 32e^2 - 40be^2 + 6b^3e^2 + 3b^4e^2 - b^5e^2) + 16b^2(8be - 16b^2e + 16b^3e - 11b^4e + 3b^5e - 64e^2 + 80be^2 - 12b^3e^2 - 6b^4e^2 + 2b^5e^2)\alpha + 16b^2(32e^2 - 40be^2 + 6b^3e^2 + 3b^4e^2 - b^5e^2)\alpha^2 = 0$, we can obtain $\alpha_1 = \frac{2b^2(2-e)-8(1-e)-b^3(1+e)}{(8-2b^2-b^3)e}$ and $\alpha_0 = \frac{4-2b+4e-be}{(4-b)e}$, where $\alpha_0 > 1$ in the feasible region.

Similarly, α_1 should be larger than 0. So, we find $0 < \alpha_1 < 1$ for $\frac{8-4b^2+b^3}{8-2b^2-b^3} < e < 1$. Due to $16b^2(32e^2 - 40be^2 + 6b^3e^2 + 3b^4e^2 - b^5e^2) > 0$, we have $[h_2(\alpha)]^2 - [h_1(\alpha)]^2 < 0$ when $\alpha < \alpha_1$ and $\frac{8-4b^2+b^3}{8-2b^2-b^3} < e < 1$. Therefore, $\gamma_1 < \frac{1}{2}(2 - b - be + be\alpha)$ for $\alpha < \alpha_1$ and $\frac{8-4b^2+b^3}{8-2b^2-b^3} < e < 1$.

As a result, we conclude that $w^K > w^B$ holds when $\alpha < \alpha_1$, $e > \frac{8-4b^2+b^3}{8-2b^2-b^3}$ and $\gamma > \gamma_1$.

Proof of Lemma 4.

(1) The difference between q_r^K and q_r^B is:

$$q_r^K - q_r^B = \frac{a[8b-2b^2-b^3-8be+2b^2e+b^3e+8be\alpha-2b^2e\alpha-b^3e\alpha+(-16b+6b^2)\gamma]}{(8-3b^2)(8-2b^2-b^2e+b^2e\alpha-8\gamma)}.$$

Subject to $1 > b > 0$ and $0 < \gamma < \frac{1}{2}(2 - b - be + be\alpha)$, it is easy to verify $8 - 3b^2 > 0$, $8 - 2b^2 - b^2e + b^2e\alpha - 8\gamma > 0$. Therefore, the sign of $q_r^K - q_r^B$ depends on $g_2(\gamma) = 8b - 2b^2 - b^3 - 8be + 2b^2e + b^3e + 8be\alpha - 2b^2e\alpha - b^3e\alpha + (-16b + 6b^2)\gamma$. $g_2(\gamma)$ is a linear function of γ, and the first-order coefficient is negative in the feasible region. Solving $g_2(\gamma) = 0$, we have $\gamma_2 = \frac{(8-2b-b^2)[1-e(1-\alpha)]}{16-6b}$ where $0 < \gamma_2 < \frac{1}{2}(2 - b - be + be\alpha)$. Therefore, when $\gamma > \gamma_2$, $g_2(\gamma) < 0$ and $q_r^K - q_r^B < 0$. Therefore, we have $q_r^K < q_r^B$ for $\gamma > \gamma_2$.

(2) The difference between q_b^K and q_b^B is: $q_b^K - q_b^B = \frac{ag_3(\gamma)}{2(8-3b^2)(8-2b^2-b^2e+b^2e\alpha-8\gamma)(1-\gamma)}$, where $g_3(\gamma) = -8b^2 + 2b^3 + b^4 + 8b^2e - 2b^3e - b^4e - 8b^2e\alpha + 2b^3e\alpha + b^4e\alpha + (64 - 16b - 8b^2 - 2b^3 + 2b^4 - 8b^2e + 2b^3e + b^4e + 8b^2e\alpha - 2b^3e\alpha - b^4e\alpha)\gamma + (-64 + 16b + 8b^2)\gamma^2$.

Similarly, we have $8 - 3b^2 > 0$, $8 - 2b^2 - b^2e + b^2e\alpha - 8\gamma > 0$ and $1 - \gamma > 0$. The sign of $q_b^K - q_b^B$ depends on $g_3(\gamma)$, where $g_3(\gamma)$ is a quadratic function of γ, and the quadratic coefficient is $-64 + 16b + 8b^2$, which is negative for $0 < b < 1$. Solving $g_3(\gamma) = 0$, we obtain $\gamma_3 = \frac{1}{2(64-16b-8b^2)}\Big\{64 - 16b - 8b^2 - 2b^3 + 2b^4 - 8b^2e + 2b^3e + b^4e + 8b^2e\alpha - 2b^3e\alpha - b^4e\alpha - [(64 - 16b - 8b^2 - 2b^3 + 2b^4 - 8b^2e + 2b^3e + b^4e + 8b^2e\alpha - 2b^3e\alpha - b^4e\alpha)^2 - 4(-64 + 16b + 8b^2)(-8b^2 + 2b^3 + b^4 + 8b^2e - 2b^3e - b^4e - 8b^2e\alpha + 2b^3e\alpha + b^4e\alpha)]^{\frac{1}{2}}\Big\}$

and $\widetilde{\gamma_3} = \frac{1}{2(64-16b-8b^2)}\Big\{64 - 16b - 8b^2 - 2b^3 + 2b^4 - 8b^2e + 2b^3e + b^4e + 8b^2e\alpha -$

$2b^3e\alpha - b^4e\alpha + [(64 - 16b - 8b^2 - 2b^3 + 2b^4 - 8b^2e + 2b^3e + b^4e + 8b^2e\alpha - 2b^3e\alpha - b^4e\alpha)^2 - 4(-64 + 16b + 8b^2)(-8b^2 + 2b^3 + b^4 + 8b^2e - 2b^3e - b^4e - 8b^2e\alpha + 2b^3e\alpha + b^4e\alpha)]^{\frac{1}{2}}\}$, where $0 < \gamma_3 < \frac{1}{2}(2 - b - be + be\alpha)$ and $\widetilde{\gamma_3} > \frac{1}{2}(2 - b - be + be\alpha)$. Therefore, when $\gamma > \gamma_3$, $g_3(\gamma) > 0$ and $q_b^K - q_b^B > 0$, we have $q_b^K > q_b^B$ for $\gamma > \gamma_3$.

Proof of Proposition 2.

The difference between π_b^K and π_b^B is: $\pi_b^K - \pi_b^B = \frac{a}{4(8-3b^2)(8-2b^2-b^2e+b^2e\alpha-8\gamma)(1-\gamma)} g_4(\gamma)$, where $g_4(\gamma) = -64 + 32b + 12b^2 - 4b^3 - b^4 + 64e - 32be - 12b^2e + 4b^3e + b^4e - 64e\alpha + 32be\alpha + 12b^2e\alpha - 4b^3e\alpha - b^4e\alpha + (128 - 96b + 16b^2 + 4b^3 - 2b^4 - 64e + 32be + 12b^2e - 4b^3e - b^4e + 64e\alpha - 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + b^4e\alpha)\gamma + (-64 + 64b - 20b^2)\gamma^2$.

Similarly, we have $8 - 3b^2 > 0$, $8 - 2b^2 - b^2e + b^2e\alpha - 8\gamma > 0$ and $1 - \gamma > 0$. The sign of $\pi_b^K - \pi_b^B$ depends on $g_4(\gamma)$. Because the quadratic coefficient is $-64 + 64b - 20b^2$, which is negative for $0 < b < 1$, so $g_4(\gamma)$ is a concave function of γ. Solving $g_4(\gamma) = 0$, we obtain $\gamma_4 = \frac{1}{2(64-64b+20b^2)}\{128 - 96b + 16b^2 + 4b^3 - 2b^4 - 64e + 32be + 12b^2e - 4b^3e - b^4e + 64e\alpha - 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + b^4e\alpha - [-4(-64 + 64b - 20b^2)(-64 + 32b + 12b^2 - 4b^3 - b^4 + 64e - 32be - 12b^2e + 4b^3e + b^4e - 64e\alpha + 32be\alpha + 12b^2e\alpha - 4b^3e\alpha - b^4e\alpha) + (128 - 96b + 16b^2 + 4b^3 - 2b^4 - 64e + 32be + 12b^2e - 4b^3e - b^4e + 64e\alpha - 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + b^4e\alpha)^2]^{\frac{1}{2}}\}$ and $\widetilde{\gamma_4} = \frac{1}{2(64-64b+20b^2)}\{128 - 96b + 16b^2 + 4b^3 - 2b^4 - 64e + 32be + 12b^2e - 4b^3e - b^4e + 64e\alpha - 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + b^4e\alpha + [-4(-64 + 64b - 20b^2)(-64 + 32b + 12b^2 - 4b^3 - b^4 + 64e - 32be - 12b^2e + 4b^3e + b^4e - 64e\alpha + 32be\alpha + 12b^2e\alpha - 4b^3e\alpha - b^4e\alpha) + (128 - 96b + 16b^2 + 4b^3 - 2b^4 - 64e + 32be + 12b^2e - 4b^3e - b^4e + 64e\alpha - 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + b^4e\alpha)^2]^{\frac{1}{2}}\}$, where $\widetilde{\gamma_4} > \frac{1}{2}(2 - b - be + be\alpha)$.

Recalling that $0 < \gamma < \frac{1}{2}(2 - b - be + be\alpha)$ should be required, then we compare γ_4 with the boundary value $\frac{1}{2}(2 - b - be + be\alpha)$. We have $\gamma_4 - \frac{1}{2}(2 - b - be + be\alpha) = \frac{1}{8(16-16b+5b^2)}[h_3(\alpha) - h_4(\alpha)]$, where $h_3(\alpha) = [(8 - 2b - b^2)^2(48b^2 - 32b^3 + 4b^4 - 64be +$

$48b^2e - 8b^3e + 4b^4e + 64e^2 - 32be^2 - 12b^2e^2 + 4b^3e^2 + b^4e^2) + (-8 + 2b + b^2)^2(64be - 48b^2e + 8b^3e - 4b^4e - 128e^2 + 64be^2 + 24b^2e^2 - 8b^3e^2 - 2b^4e^2)\alpha + (-8 + 2b + b^2)^2(64e^2 - 32be^2 - 12b^2e^2 + 4b^3e^2 + b^4e^2)\alpha^2]^{\frac{1}{2}}$ and $h_4(\alpha) = -96b + 88b^2 - 24b^3 + 2b^4 + 64e - 96be + 52b^2e - 16b^3e + b^4e + (-64e + 96be - 52b^2e + 16b^3e - b^4e)\alpha$. When $h_3(\alpha) < h_4(\alpha)$, i.e., $[h_3(\alpha)]^2 < [h_4(\alpha)]^2$, we have $\gamma_4 < \frac{1}{2}(2 - b - be + be\alpha)$. Similar to the proof of Lemma 3, we solve $[h_3(\alpha)]^2 - [h_4(\alpha)]^2 = 0$, we can obtain $\alpha_2 = \frac{12b+4(3be-4e)+b^3(1+e)-2b^2(4+e)}{(12b-2b^2+b^3-16)e}$ and $\alpha_0 = \frac{4-2b+4e-be}{(4-b)e}$, where $\alpha_0 > 1$ in the feasible region. Similarly, α_2 should be larger than 0. So, we find $0 < \alpha_2 < 1$ for $\frac{12b-8b^2+b^3}{16-12b+2b^2-b^3} < e < 1$. We have $[h_3(\alpha)]^2 - [h_4(\alpha)]^2 < 0$ when $\alpha < \alpha_2$ and $\frac{12b-8b^2+b^3}{16-12b+2b^2-b^3} < e < 1$. Therefore, $\gamma_4 < \frac{1}{2}(2 - b - be + be\alpha)$ for $\alpha < \alpha_2$ and $\frac{12b-8b^2+b^3}{16-12b+2b^2-b^3} < e < 1$.

As a result, we conclude that $\pi_b^K > \pi_b^B$ holds when $\alpha < \alpha_2$, $e > \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$ and $\gamma > \gamma_4$. Correspondingly, $\pi_b^K < \pi_b^B$ holds when the following conditions occur:

(a) $e < \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$;

(b) $e > \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$, $\begin{cases} \alpha > \alpha_2 \\ \alpha < \alpha_2 \text{ and } \gamma < \gamma_4 \end{cases}$.

Proof of Corollary 1.

(a) When $\pi_b^K > \pi_b^B$, i.e., $\alpha < \alpha_2$, $e > \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$ and $\gamma > \gamma_4$ must hold, $\frac{\partial(\pi_o^K - \pi_o^B)}{\partial\gamma} = \frac{\alpha^2(-8+2b+b^2+8\gamma-2b\gamma)}{4(8-2b^2-b^2e+b^2e\alpha-8\gamma)^2(1-\gamma)^2} g_5(\gamma)$, where $g_5(\gamma) = 4b - 2b^2 - 8e + 2be - b^2e + 8e\alpha - 2be\alpha + b^2e\alpha + (-4b + 8e - 2be - 8e\alpha + 2be\alpha)\gamma$.

Due to $0 < \gamma < \frac{1}{2}(2 - b - be + be\alpha)$, we have $-8 + 2b + b^2 + 8\gamma - 2b\gamma < 0$, so the sign of $\frac{\partial(\pi_o^K - \pi_o^B)}{\partial\gamma}$ depends on $g_5(\gamma)$. $g_5(\gamma)$ is a linear function of γ, and the first-order coefficient is $-4b + 8e - 2be - 8e\alpha + 2be\alpha > 0$ in the feasible region. This means that $g_5(\gamma)$ increases in γ. Moreover, the maximum is obtained at $\gamma = \frac{1}{2}(2 - b - be + be\alpha)$. Substituting $\gamma = \frac{1}{2}(2 - b - be + be\alpha)$ into $g_5(\gamma)$, we have

$Max[g_5(\gamma)] = g_5(\gamma)|_{\gamma=\frac{1}{2}(2-b-be+be\alpha)} = -be\{4 - b[2 + e(1-\alpha)] + 4e(1-\alpha)\}(1-\alpha) < 0.$

Hence, $g_5(\gamma) < 0$ holds in the feasible region, and $\frac{\partial(\pi_O^K - \pi_O^B)}{\partial\gamma} > 0$ is proved.

(b) When $e < \frac{12b - 8b^2 + b^3}{16 - 12b + 2b^2 - b^3}$, $\pi_b^K < \pi_b^B$. Note that the sign of $\frac{\partial(\pi_O^K - \pi_O^B)}{\partial\gamma}$ coincides with $g_5(\gamma)$. Thus, we discuss the monotonicity of the function.

We find when $0 < e \le \frac{2b}{4-b}$, $-4b + 8e - 2be - 8e\alpha + 2be\alpha < 0$, so $g_5(\gamma)$ is decreasing in γ. Moreover, the maximum is obtained at $\gamma = 0$ and the minimum is obtained at $\gamma = \frac{1}{2}(2 - b - be + be\alpha)$. Substituting $\gamma = 0$ and $\gamma = \frac{1}{2}(2 - b - be + be\alpha)$ into $g_5(\gamma)$, we have

$$Max[g_5(\gamma)] = g_5(\gamma)|_{\gamma=0} = b[4 - 2e(-1 + \alpha)] + b^2[-2 + e(-1 + \alpha)] + 8e(-1 + \alpha),$$

$$Min[g_5(\gamma)] = g_5(\gamma)|_{\gamma=\frac{1}{2}(2-b-be+be\alpha)} = be\{4 + b[-2 + e(-1 + \alpha)] - 4e(-1 + \alpha)\}(-1 + \alpha).$$

We find the $g_5(\gamma)|_{\gamma=\frac{1}{2}(2-b-be+be\alpha)} < 0$ in the feasible region, and when $0 < e \le \frac{4b-2b^2}{8-2b+b^2}$, or $\frac{4b-2b^2}{8-2b+b^2} < e \le \frac{2b}{4-b}$ and $\frac{8e-2b(2+e)+b^2(2+e)}{8e-2be+b^2e} < \alpha < 1$, $g_5(\gamma)|_{\gamma=0} > 0$. Hence, according to the zero point theorem, for the monotone decreasing function $g_5(\gamma)$, there exists a unique zero point γ in the interval of $\left(0, \frac{1}{2}(2 - b - be + be\alpha)\right)$. Solving $g_5(\gamma) = 0$, we have the unique feasible root $\gamma_5 = \frac{2b^2+8e-2be+b^2e-8e\alpha+2be\alpha-b^2e\alpha-4b}{8e-2be-8e\alpha+2be\alpha-4b}$. Thus, combined with $0 < e \le \frac{4b-2b^2}{8-2b+b^2}$, or $\frac{4b-2b^2}{8-2b+b^2} < e \le \frac{2b}{4-b}$ and $\frac{8e-2b(2+e)+b^2(2+e)}{(8-2b+b^2)e} < \alpha < 1$, we have the condition for $g_5(\gamma) > 0$ is $\gamma < \gamma_5$.

Similarly, we find when $\frac{2b}{4-b} < e < \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$ and $\frac{2b-4e+be}{be-4e} < \alpha < 1$, $-4b + 8e - 2be - 8e\alpha + 2be\alpha < 0$, so $g_5(\gamma)$ is also decreasing in γ. Moreover, the maximum is obtained at $\gamma = 0$ and the minimum is obtained at $\gamma = \frac{1}{2}(2 - b - be + be\alpha)$. Substituting $\gamma = 0$ and $\gamma = \frac{1}{2}(2 - b - be + be\alpha)$ into $g_5(\gamma)$, we have

$$Max[g_5(\gamma)] = g_5(\gamma)|_{\gamma=0} = b[4 - 2e(-1 + \alpha)] + b^2[-2 + e(-1 + \alpha)] + 8e(-1 + \alpha),$$

$$Min[g_5(\gamma)] = g_5(\gamma)|_{\gamma=\frac{1}{2}(2-b-be+be\alpha)} = be\{4 + b[-2 + e(-1 + \alpha)] - 4e(-1 + \alpha)\}(-1 + \alpha).$$

We find the $g_5(\gamma)|_{\gamma=\frac{1}{2}(2-b-be+be\alpha)} < 0$ in the feasible region, and when $\frac{2b}{4-b} < e < \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$ and $\frac{8e-2b(2+e)+b^2(2+e)}{(8-2b+b^2)e} < \alpha < 1$, $g_5(\gamma)|_{\gamma=0} > 0$. Similarly, we have $g_5(\gamma) > 0$ when $\gamma < \gamma_5$.

When $e > \frac{12b-8b^2+b^3}{16-12b+2b^2-b^3}$, $\alpha > \alpha_2$, $\pi_b^K < \pi_b^B$.

Similarly, when $\frac{12b-8b^2+b^3}{16-12b+2b^2-b^3} < e < 1$ and $\frac{2b-4e+be}{be-4e} < \alpha < 1$, we find the first-order coefficient of $g_5(\gamma)$ is negative, which means $g_5(\gamma)$ is decreasing in γ. Following the same steps as before, when $\frac{12b-8b^2+b^3}{16-12b+2b^2-b^3} < e < 1$ and $\frac{8e-2b(2+e)+b^2(2+e)}{(8-2b+b^2)e} < \alpha < 1$, we have $g_5(\gamma) > 0$ when $\gamma < \gamma_5$.

In conclusion, $\frac{\partial(\pi_o^K - \pi_o^B)}{\partial\gamma} < 0$ holds when the following conditions occur:

(a) $e < \frac{4b-2b^2}{8-2b+b^2}$ and $\gamma < \gamma_5$;

(b) $\frac{4b-2b^2}{8-2b+b^2} < e < 1$, $\alpha > \frac{8e-2b(2+e)+b^2(2+e)}{(8-2b+b^2)e}$ and $\gamma < \gamma_5$.

Proof of Proposition 3.

The difference between π_b^K and π_b^E is:

$\pi_b^K - \pi_b^E = \frac{a^2}{4(8+b^2-16k+6b^2k)(8-2b^2-b^2e+b^2e\alpha-8\gamma)(-1+\gamma)} g_6(\gamma)$, where $g_6(\gamma) = 32b - 20b^2 + 4b^3 - b^4 - 64e + 32be - 12b^2e + 4b^3e - 128k + 64bk + 24b^2k - 8b^3k - 2b^4k + 128ek - 64bek - 24b^2ek + 8b^3ek + 2b^4ek + 64e\alpha - 32be\alpha + 12b^2e\alpha - 4b^3e\alpha - 128ek\alpha + 64bek\alpha + 24b^2ek\alpha - 8b^3ek\alpha - 2b^4ek\alpha + (-32b + 16b^2 - 4b^3 + 64e - 32be + 12b^2e - 4b^3e + 256k - 192bk + 32b^2k + 8b^3k - 4b^4k - 128ek + 64bek + 24b^2ek - 8b^3ek - 2b^4ek - 64e\alpha + 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + 128ek\alpha - 64bek\alpha - 24b^2ek\alpha + 8b^3ek\alpha + 2b^4ek\alpha)\gamma + (-4b^2 - 128k + 128bk - 40b^2k)\gamma^2$.

Since $k > \frac{1}{2-2b}$, $0 < b < 1$, $0 < e < 1$, $0 < \alpha < 1$ and $0 < \gamma < \frac{1}{2}(2 - b - be + be\alpha)$, we have $8 + b^2 - 16k + 6b^2k < 0$, $8 - 2b^2 - b^2e + b^2e\alpha - 8\gamma > 0$ and $-1 + \gamma < 0$. Therefore, the sign of $\pi_b^K - \pi_b^E$ depends on $g_6(\gamma)$, which is a concave function due to the quadratic coefficient $-4b^2 - 128k + 128bk - 40b^2k < 0$. That is, if $g_6(\gamma) > (<)0$, we have $\pi_b^K > (<)\pi_b^E$. Solving $g_6(\gamma) = 0$, we can obtain $\gamma_6 = \frac{1}{2(4b^2+128k-128bk+40b^2k)}\{16(8 - 2b - b^2)^2e^2k[b^3(1 + e - 2k) - 64k + b^4k + 16e(-1 + 2k) + 8b(1 + e + 6k) - b^2(4 + 3e + 8k)]\alpha[b^4k + b^3(4k - 2\alpha) + 32\alpha + 6b^2(-2k + \alpha) - 16b(2k + \alpha)] - [4(4b^2 + 128k - 128bk + 40b^2k)(32b - 20b^2 + 4b^3 - b^4 - 64e + 32be - 12b^2e + 4b^3e - 128k + 64bk + 24b^2k - 8b^3k - 2b^4k + 128ek - 64bek - 24b^2ek + 8b^3ek + 2b^4ek + 64e\alpha - 32be\alpha +$

$12b^2e\alpha - 4b^3e\alpha - 128ek\alpha + 64bek\alpha + 24b^2ek\alpha - 8b^3ek\alpha - 2b^4ek\alpha) + (-32b + 16b^2 - 4b^3 + 64e - 32be + 12b^2e - 4b^3e + 256k - 192bk + 32b^2k + 8b^3k - 4b^4k - 128ek + 64bek + 24b^2ek - 8b^3ek - 2b^4ek - 64e\alpha + 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + 128ek\alpha - 64bek\alpha - 24b^2ek\alpha + 8b^3ek\alpha + 2b^4ek\alpha)^2]^{\frac{1}{2}}\}$ and $\widetilde{\gamma_6} = \frac{1}{2(4b^2+128k-128bk+40b^2k)}\{16(8 - 2b - b^2)^2e^2k[b^3(1+e-2k) - 64k + b^4k + 16e(-1+2k) + 8b(1+e+6k) - b^2(4 + 3e + 8k)]\alpha[b^4k + b^3(4k - 2\alpha) + 32\alpha + 6b^2(-2k+\alpha) - 16b(2k+\alpha)] + [4(4b^2 + 128k - 128bk + 40b^2k)(32b - 20b^2 + 4b^3 - b^4 - 64e + 32be - 12b^2e + 4b^3e - 128k + 64bk + 24b^2k - 8b^3k - 2b^4k + 128ek - 64bek - 24b^2ek + 8b^3ek + 2b^4ek + 64e\alpha - 32be\alpha + 12b^2e\alpha - 4b^3e\alpha - 128ek\alpha + 64bek\alpha + 24b^2ek\alpha - 8b^3ek\alpha - 2b^4ek\alpha) + (-32b + 16b^2 - 4b^3 + 64e - 32be + 12b^2e - 4b^3e + 256k - 192bk + 32b^2k + 8b^3k - 4b^4k - 128ek + 64bek + 24b^2ek - 8b^3ek - 2b^4ek - 64e\alpha + 32be\alpha - 12b^2e\alpha + 4b^3e\alpha + 128ek\alpha - 64bek\alpha - 24b^2ek\alpha + 8b^3ek\alpha + 2b^4ek\alpha)^2]^{\frac{1}{2}}\}$, where $\widetilde{\gamma_6} > \frac{1}{2}(2 - b - be + be\alpha)$.

Then we compare γ_6 with the boundary value $\frac{1}{2}(2 - b - be + be\alpha)$. The rest of proof is the same as that of Proposition 2, and we have $\gamma_6 < \frac{1}{2}(2 - b - be + be\alpha)$ for $\alpha < \frac{8e(1-2k)+b^3(1+e)k+2b(1+e)(-1+6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k+2b(-1+6k)]}$ and $\frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k+2b(-1+6k)} < e < 1$.

As a result, we conclude that $\pi_b^K > \pi_b^E$ holds when $\alpha < \frac{8e(1-2k)+b^3(1+e)k+2b(1+e)(-1+6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k+2b(-1+6k)]}$, $e > \frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k+2b(-1+6k)}$ and $\gamma > \gamma_6$.

Proof of Proposition 4.

(1) The difference between π_r^K and π_r^E is:

$\pi_r^K - \pi_r^E = \frac{a^2}{(8+b^2-16k+6b^2k)^2(8-2b^2-b^2e+b^2e\alpha-8\gamma)^2} g_7(\gamma)$, where $g_7(\gamma) = -256a^2b + 256a^2b^2 - 64a^2b^3 + 4a^2b^4 - 4a^2b^5 + a^2b^6 - 256a^2be + 192a^2b^2e - 64a^2b^3e + 16a^2b^4e - 4a^2b^5e + 2a^2b^6e + 64a^2b^2e^2 + 12a^2b^4e^2 + a^2b^6e^2 - 512a^2b^2k + 256a^2b^3k + 176a^2b^4k - 112a^2b^5k + 12a^2b^6k + 1024a^2bek - 768a^2b^2ek + 192a^2b^4ek - 112a^2b^5ek + 24a^2b^6ek - 256a^2b^2e^2k + 80a^2b^4e^2k - 16a^2b^5e^2k + 12a^2b^6e^2k + 1024a^2bk^2 - 1024a^2b^2k^2 - 256a^2b^3k^2 + 400a^2b^4k^2 - 16a^2b^5k^2 - 28a^2b^6k^2 - 1024a^2bek^2 + 768a^2b^2ek^2 + 256a^2b^3ek^2 - 192a^2b^4ek^2 - 16a^2b^5ek^2 + 8a^2b^6ek^2 +$

$256a^2b^2e^2k^2 - 208a^2b^4e^2k^2 + 32a^2b^5e^2k^2 + 20a^2b^6e^2k^2 + 256a^2be\alpha - 192a^2b^2e\alpha + 64a^2b^3e\alpha - 16a^2b^4e\alpha + 4a^2b^5e\alpha - 2a^2b^6e\alpha - 128a^2b^2e^2\alpha - 24a^2b^4e^2\alpha - 2a^2b^6e^2\alpha - 1024a^2bek\alpha + 768a^2b^2ek\alpha - 192a^2b^4ek\alpha + 112a^2b^5ek\alpha - 24a^2b^6ek\alpha + 512a^2b^2e^2k\alpha - 160a^2b^4e^2k\alpha + 32a^2b^5e^2k\alpha - 24a^2b^6e^2k\alpha + 1024a^2bek^2\alpha - 768a^2b^2ek^2\alpha - 256a^2b^3ek^2\alpha + 192a^2b^4ek^2\alpha + 16a^2b^5ek^2\alpha - 8a^2b^6ek^2\alpha - 512a^2b^2e^2k^2\alpha + 416a^2b^4e^2k^2\alpha - 64a^2b^5e^2k^2\alpha - 40a^2b^6e^2k^2\alpha + 64a^2b^2e^2\alpha^2 + 12a^2b^4e^2\alpha^2 + a^2b^6e^2\alpha^2 - 256a^2b^2e^2k\alpha^2 + 80a^2b^4e^2k\alpha^2 - 16a^2b^5e^2k\alpha^2 + 12a^2b^6e^2k\alpha^2 + 256a^2b^2e^2k^2\alpha^2 - 208a^2b^4e^2k^2\alpha^2 + 32a^2b^5e^2k^2\alpha^2 + 20a^2b^6e^2k^2\alpha^2 + (256a^2b - 256a^2b^2 + 64a^2b^3 - 8a^2b^4 + 4a^2b^5 + 256a^2be - 64a^2b^2e + 64a^2b^3e + 4a^2b^5e + 1024a^2bk - 256a^2b^3k - 96a^2b^4k + 48a^2b^5k - 1024a^2bek + 256a^2b^2ek + 48a^2b^5ek - 3072a^2bk^2 + 3072a^2b^2k^2 + 256a^2b^3k^2 - 800a^2b^4k^2 + 144a^2b^5k^2 + 1024a^2bek^2 - 256a^2b^2ek^2 - 256a^2b^3ek^2 - 256a^2b^4ek^2 + 144a^2b^5ek^2 - 256a^2be\alpha + 64a^2b^2e\alpha - 64a^2b^3e\alpha - 4a^2b^5e\alpha + 1024a^2bek\alpha - 256a^2b^2ek\alpha - 48a^2b^5ek\alpha - 1024a^2bek^2\alpha + 256a^2b^2ek^2\alpha + 256a^2b^3ek^2\alpha + 256a^2b^4ek^2\alpha - 144a^2b^5ek^2\alpha)\gamma + (64a^2b^2 + 4a^2b^4 - 1024a^2bk + 256a^2b^2k + 48a^2b^4k + 2048a^2bk^2 - 1792a^2b^2k^2 + 144a^2b^4k^2)\gamma^2.$

The sign of $\pi_r^K - \pi_r^E$ depends on $g_7(\gamma)$, which is a convex quadratic function due to the coefficient $64a^2b^2 + 4a^2b^4 - 1024a^2bk + 256a^2b^2k + 48a^2b^4k + 2048a^2bk^2 - 1792a^2b^2k^2 + 144a^2b^4k^2 > 0$ in the feasible region. That is, if $g_7(\gamma) > (<)0$, we have $\pi_r^K > (<)\pi_r^E$. Solving $g_7(\gamma) = 0$, we can obtain

$$\gamma_7 = \frac{8-6b+b^2+8e-2be+b^2e+16k-4bk-2b^2k-16ek+4bek+2b^2ek-8e\alpha+2be\alpha-b^2e\alpha+16ek\alpha-4bek\alpha-2b^2ek\alpha}{-2b+32k-12bk},$$

$$\widetilde{\gamma_7} = \frac{1}{32+2b^2-64k+32bk+12b^2k}(32 - 8b - 2b^2 - b^3 - 8be - 2b^2e - b^3e - 64k + 48bk + 20b^2k - 14b^3k + 16bek + 4b^2ek - 10b^3ek + 8be\alpha + 2b^2e\alpha + b^3e\alpha - 16bek\alpha - 4b^2ek\alpha + 10b^3ek\alpha),$$ where $\widetilde{\gamma_7} > \frac{1}{2}(2 - b - be + be\alpha) > \gamma_7 > 0$.

Therefore, when $\gamma > \gamma_7$, $g_7(\gamma) < 0$, i.e., $\pi_r^K < \pi_r^E$.

(2) From Proposition 3, $\pi_b^K < \pi_b^E$ holds when the following conditions occur:

(a) $e < \frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k+2b(-1+6k)}$,

(b) $\frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k+2b(-1+6k)} < e < 1$, $\begin{cases} \alpha > \frac{8e(1-2k)+b^3(1+e)k+2b(1+e)(-1+6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k+2b(-1+6k)]} \\ \alpha < \frac{8e(1-2k)+b^3(1+e)k+2b(1+e)(-1+6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k+2b(-1+6k)]} \text{ and } \gamma < \gamma_6 \end{cases}$.

From Proposition 4 (1), $\pi_r^K < \pi_r^E$ holds when $\gamma > \gamma_7$.

Jointly considering the conditions under which both $\pi_b^K < \pi_b^E$ and $\pi_r^K < \pi_r^E$ hold, we have the following (if one of the following conditions is satisfied, both the brand-owner and the retailer are better off in Scenario E):

(a) $e > \frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k+2b(-1+6k)}$, $\begin{cases} \alpha < \frac{8e(1-2k)+b^3(1+e)k+2b(1+e)(-1+6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k+2b(-1+6k)]} \text{ and } \gamma_7 < \gamma < \gamma_6 \\ \alpha > \frac{8e(1-2k)+b^3(1+e)k+2b(1+e)(-1+6k)+b^2(e-8k-2ek)}{e[8+b^2(1-2k)-16k+b^3k+2b(-1+6k)]} \text{ and } \gamma > \gamma_7 \end{cases}$,

(b) $e < \frac{b[2-(12-8b+b^2)k]}{8+b^2(1-2k)-16k+b^3k+2b(-1+6k)}$, $\gamma > \gamma_7$.