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Stabilization of logical control networks: An event-triggered control approach

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Abstract This paper investigates the global stabilization problem of k-valued logical control networks (KVLCNs) via event-triggered control (ETC), where the control inputs only work at several certain individual states. Compared with the traditional state feedback control, the designed ETC not only shortens the transient period but also decreases the number of controller execution. The content of this paper is divided into two parts. In the former part, a necessary and sufficient criterion is derived for the event-triggered stabilization of KVLCNs. Meanwhile, a constructing procedure is developed to design all time-optimal event-triggered stabilizers. In the latter part, we devote to designing the switching-cost-optimal event-triggered stabilizer, that is, to minimize the number of controller execution. The labelled digraph is derived based on the dynamic of the overall system. Utilizing this digraph, we formulate a universal and unified procedure, named as the minimal spanning in-tree algorithm, to minimize the triggering event set. Finally, the effectiveness of obtained results is illustrated by several numerical examples.

Keywords Logical control network, event-triggered control, stabilization, semi-tensor product, minimal spanning in-tree.

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1 Introduction

Recently, the rapid development of DNA microarrays has set the stage for mathematical modeling of genetic regulatory networks [1]. Generally speaking, until now, numerical formal types of mathematical models have been proposed to depict, simulate, and even predict the dynamic behavior of biological networks, for instance, Markov-type genetic networks [2] and Boolean networks (BNs). Based on many experimental results, BN models, which were originally proposed by Kauffman in 1969 [3], have been proved to be capable of forecasting the dynamic sequence of protein activation patterns within genetic regulation networks [4]. A typical biological application is the *cell cycle control network in yeast* [5]. In addition, the modality of BNs has constructed a natural framework for providing minute comprehension and insights of the dynamic behavior exhibited by large-scale genetic networks.

In a Boolean model, the expression of each node on networks is approximated by two levels, namely 1 (ON) and 0 (OFF). The state update of each gene is determined by a pre-assigned logical function associated with the states of in-neighbor genes. As mentioned in [6], a recent significant discovery

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in systems biology is that exogenous perturbations, which can be described as 'control', are almost ubiquitous in many biological systems. Thereby the concept of Boolean control networks (BCNs) has been formally generated by adding binary inputs to the BNs [7]. In *capillary endothelia cells* [8] for instance, a simple BCN has been established to simulate the dynamic behavior of the signaling system, where growth factors and cell shape (spreading) are both presented by two external inputs.

Since these extensive applications for therapeutic interventions in the field of systems biology, the investigations on BNs and BCNs have aroused widespread attention of many scientists and scholars. However, although BNs and BCNs seem simple, there still lacks a systematic and valid tool to analyze and control these Boolean models. Recently, Cheng et al. have presented a generalized matrix product, named as semi-tensor product (STP) [9]. Thanks to this technique, a considerable amount of researches have been inspired and several classical control problems have been extensively investigated in BCNs, including but not limited to, stabilization and set stabilization [10–21], reachability and controllability [22–24], observability [25], synchronization [26], function perturbation [27], and several other problems [28–32]. The main idea of STP approach is to convert the logical dynamic of a BN (BCN) into a normal discrete-time linear system. Furthermore, the knowledge of matrix theory and graph theory can be applied to systematically analyze these problem. In addition, STP of matrices also provides an extreme convenience for analyzing k-valued logical control networks (KVLCNs), non-linear shift registers, finite automata and so on [33]. Indeed, KVLCNs, which can be regarded as a generalization of traditional BCNs to some extent, are more complex and have wider applications than BCNs. For example, the number of feasible choices in each player's action set may be more than two for finite evolutionary networked game as described in [34, 35], but binary Boolean model can not describe such a case. Therefore, it is more significant to consider KVLCNs in this manuscript.

The control design strategy is always an interesting topic in complex networks, naturally genetic regulatory networks. Numerical control schemes have been developed in the study of logical systems, including but not limited to, state feedback control [11], output feedback control [10], pinning control [22, 36], sampled-data control [37]. Unfortunately, in the aforementioned control paradigms, the control inputs need to be executed at each time instant. It is indeed a waste of resources if the dynamic evolution of the original network is desirable. Motivated by this, another alternative control paradigm, named as event-triggered control (ETC), has been put forward in [38]. With the advent of this triggering mechanism, substantial numbers of control cost can be reduced, so the ETC has extensively taken part in the study of logical control systems [34, 39–41], multi-agent systems [42], as well as smart grids [43]. By following the main stream of research, the event-triggered controller considered in this paper is an intermittent control strategy, which is firstly proposed in the late nineties [38]. As reported in [34, 39, 41], this typical kind of ETC consists of two parts: (1) A state feedback mechanism to determine the control inputs; (2) A set of states to decide when the control inputs should be taken into consideration.

Up to now, ETC has been formally used to address many problems of KVLCNs. In [39], two classes of event-triggered controllers have been firstly designed to deal with the disturbance decoupling problem of BCNs, and several necessary and sufficient conditions have been derived for checking whether it is solvable. Moreover, an effective event-triggered approach has been developed to realize the global stabilization of finite evolutionary networked games by some reachable sets with respect to the designated state [34]. Meanwhile, the number of control execution has been minimized by an adjustment algorithm in some special circumstances. However, since the structure of the alternative control system has been given beforehand, sometimes this approach may become invalid, such as Example 2 introduced in this paper. Inspired by this, it deserves developing a universal and unified approach to minimize the number of controller execution. Afterwards, this kind event-triggered controller has been further generalized to investigate the global stabilization of probabilistic BCNs [41]. At the same time, the design of the time-optimal event-triggered stabilizer is still open.

In this paper, the global stabilization problem of KVLCNs is realized via the time-optimal event-triggered controller and the switching-cost-optimal one, respectively. The main contributions of this manuscript are listed as follows:

- In the former part of paper, the time-optimal event-triggered stabilizer is designed. Via STP technique, the algebraic framework of the KVLCN under ETC is established, which consists of a network inherent transition matrix, an alternative network transition matrix and a triggering event set. Similar to the time-optimal state feedback stabilizer as in [11], a necessary and sufficient criterion is derived for the event-triggered stabilization of the KVLCN. Furthermore, a constructive procedure is developed to design all time-optimal event-triggered stabilizers.

• In the latter part of this manuscript, we devote to designing an event-triggered stabilizer with the minimal number of controller execution, which is called the switching-cost-optimal one. The labelled digraph is constructed to describe the dynamic behavior of an event-triggered controlled KVLCN. Moreover, by resorting to the knowledge of graph theory, the number of controller execution is minimized via a universal procedure, named as the minimal spanning in-tree algorithm. It can tackle all circumstances and overcome the constraint of the method in [34].

The remainder of this paper is structured as follows. Some preliminaries are introduced in Section 2. Section 3 presents the main results of this paper, and several illustrative examples are presented to show the effectiveness of the obtained results. A brief conclusion is given in Section 4.

2 Preliminaries and Problem Formulation

In this section, some necessary preliminaries are presented.

- $\mathbb{N}$ and $\mathbb{R}$ are the sets of all natural integers and real integers, respectively.
- $\mathbb{R}_{m\times n}$ is the set of all $m \times n$ real matrices.
- $[a, b]$ represents the set of all integers between a and b, where $a, b \in \mathbb{N}$.
- $\mathscr{D}_k := [0, k-1]$.
- $\mathscr{D}_k^n := \underbrace{\mathscr{D}_k \times \mathscr{D}_k \times \cdots \times \mathscr{D}_k}_{n}$.
- $\mathrm{Col}_i(A)$ ($\mathrm{Row}_i(A)$) is the ith column (row) of matrix A.
- $\delta_n^i := \mathrm{Col}_i(I_n)$, where I_n is the $n \times n$ identity matrix.
- Δ_n is the set of all columns of identity matrix I_n.
- $A \in \mathbb{R}_{m\times n}$ is called a Boolean matrix, if $[A]_{ij} \in \mathscr{D}_2$.
- $A \in \mathbb{R}_{m\times n}$ is called a logical matrix, if $A = [\delta_m^{i_1}, \delta_m^{i_2}, \cdots, \delta_m^{i_n}]$, simply represented as $\delta_m[i_1, i_2, \cdots, i_n]$.
- $\mathscr{L}_{m\times n}$ consists of all $m \times n$ logical matrices.
- $|\ S\ |$ is the cardinal number of elements in the set S.

2.1 STP of Matrices

Definition 1 (Cheng et al. [9]). The STP of $A \in \mathbb{R}_{m\times n}$ and $B \in \mathbb{R}_{p\times q}$ is defined as

$$A \ltimes B = (A \otimes I_{\alpha/n})(B \otimes I_{\alpha/p}),$$

where α is the least common multiple of n and p, and '$\otimes$' is the Kronecker product of matrices.

Remark 1. As a generalization of conventional matrix product, when $n = p$, $A \ltimes B = (A \otimes I_1)(B \otimes I_1) = AB$. The STP of matrices provides a method to multiply two matrices with arbitrary dimensions (see [9] for more details). In general, symbol "$\ltimes$" is omitted without any confusion.

Lemma 1 (Cheng et al. [9]). Swap matrix $W_{[m,n]}$ is an $mn \times mn$ logical matrix defined as $W_{[m,n]} = \left[I_n \otimes \delta_m^1, \cdots, I_n \otimes \delta_m^m\right]$. Based on $W_{[m,n]}$, the pseudo-commutative law of STP is concluded as follows:

(1) If $X \in \mathbb{R}_{m\times 1}$ and $B \in \mathbb{R}_{p\times q}$, then $XB = (I_m \otimes B)X$.

(2) If $X \in \mathbb{R}_{m\times 1}$ and $Y \in \mathbb{R}_{n\times 1}$, then $YX = W_{[m,n]}XY$.

Suppose that $\mathbf{x} = (x_1, x_2, \cdots, x_n) \in \mathscr{D}_k^n$, we define a bijection $\Gamma_n : \mathscr{D}_k^n \to \Delta_{k^n}$ as

$$\Gamma_n(\mathbf{x}) = \ltimes_{i=1}^n \delta_k^{k-x_i} = \delta_{k^n}^s,$$

where $s = \sum_{i=1}^n (k - x_i - 1)k^{n-i}$ and $\Gamma_n(\mathbf{x})$ is called the equivalent delta vector form of $\mathbf{x}$. Then, arbitrary logical function $f : \mathscr{D}_k^n \to \mathscr{D}_k$ can be expressed in its algebraic representation by following lemma.

Lemma 2 (Cheng et al. [9]). For a logical function $f(x_1, x_2, \dots, x_n) : \mathscr{D}_k^n \to \mathscr{D}_k$, there exists a determined matrix $M_f \in \mathscr{L}_{k\times k^n}$, called the structure matrix of f, such that

$$f(x_1, x_2, \dots, x_n) = M_f \Gamma_n\left((x_1, x_2, \cdots, x_n)\right).$$

2.2 The Dynamics of KVLCNs Under Event-Triggered Controllers

The KVLCN under ETC, presented as follows, consists of an inherent non-control k-valued logical network (KVLN) (1a), an alternative KVLCN (1b), and a triggering event set $\Lambda \subseteq \mathscr{D}_k^n$ standing for some certain individual states where the control inputs are triggered:

$$\begin{cases} x_1(t+1) = f_1(x_1(t), \cdots, x_n(t)), \\ x_2(t+1) = f_2(x_1(t), \cdots, x_n(t)), \\ \qquad\vdots \\ x_n(t+1) = f_n(x_1(t), \cdots, x_n(t)), \end{cases} \tag{1a}$$

$$\begin{cases} x_1(t+1) = f_1'(x_1(t), \cdots, x_n(t), u_1(t), \cdots, u_m(t)), \\ x_2(t+1) = f_2'(x_1(t), \cdots, x_n(t), u_1(t), \cdots, u_m(t)), \\ \qquad\vdots \\ x_n(t+1) = f_n'(x_1(t), \cdots, x_n(t), u_1(t), \cdots, u_m(t)), \end{cases} \tag{1b}$$

where $f_i : \mathscr{D}_k^n \to \mathscr{D}_k$ and $f_i' : \mathscr{D}_k^{n+m} \to \mathscr{D}_k$, $i \in [1, n]$, are logical functions, $x_i \in \mathscr{D}_k$ and $u_j \in \mathscr{D}_k$, $j \in [1, m]$, are respectively states and control inputs.

The mechanism of ETC is essentially an intermittent control strategy. In particular, when the dynamic of inherent system (1a) evolves desirably, the system maintains in the form of (1a) and the control inputs are not triggered. Otherwise, i.e., the state of system locates in the set Λ, KVLCN (1b) works and the control inputs are considered.

Next, we present the equivalent algebraic expression of the event-triggering controlled KVLCN. To facilitate the analysis, let $x(t) = \Gamma_n\left((x_1(t), x_2(t), \cdots, x_n(t))\right) \in \Delta_N$ and $u(t)=\Gamma_m\left((u_1(t), u_2(t), \cdots, u_m(t))\right) \in \Delta_M$, where $N = k^n$ and $M = k^m$. KVLN (1a) and KVLCN (1b) can be algebraically represented by Lemma 2 as follows:

$$x_i(t+1) = M_i x(t),\ i \in [1, n], \tag{2a}$$

$$x_j(t+1) = M_j' u(t) x(t),\ j \in [1, n], \tag{2b}$$

where $M_i \in \mathscr{L}_{k\times N}$ and $M_j' \in \mathscr{L}_{k\times MN}$. Then, the equations in (2a) and (2b) are further multiplied to lead that

$$x(t+1) = Lx(t), \tag{3a}$$

$$x(t+1) = L'u(t)x(t), \tag{3b}$$

where $L=M_1 * M_2 * \cdots * M_n \in \mathscr{L}_{N\times N}$ is the inherent transition matrix, and $L' = M_1' * M_2' * \cdots * M_n' \in \mathscr{L}_{N\times MN}$ is the transition matrix of alternative subsystem (1b), where '$*$' is the Khatri-Rao product [44].

Therefore, if we define $\Gamma(\Lambda) := \{\Gamma_n(\mathbf{x}) : \mathbf{x} \in \Lambda\}$, the overall dynamic of the KVLCN with ETC can be synoptically described as

$$x(t+1) = \begin{cases} Lx(t), & x(t) \in \Delta_N \backslash \Gamma(\Lambda), \\ L'u(t)x(t), & x(t) \in \Gamma(\Lambda). \end{cases} \tag{4}$$

Or equivalently, the above dynamic can be given as

$$x(t+1) = [L, L']\, \widetilde{u}(t)x(t) := \widetilde{L}\widetilde{u}(t)x(t), \tag{5}$$

where the novel control $\widetilde{u}(t) \in \Delta_{M+1}$ is constructed from $u(t)$ as follows: (1) If $x(t) \in \Delta_N \backslash \Gamma(\Lambda)$, then $\widetilde{u}(t) := \delta_{M+1}^1$; (2) If $x(t) \in \Gamma(\Lambda)$, one obtains that $\widetilde{u}(t) := [0, u(t)^\top]^\top$. Here and elsewhere, '$\top$' is the transpose of matrix.

The state trajectory of system (5) with $x(0;x_0,\widetilde{\mathbf{u}}) = x_0$ with respect to certain control sequence $\widetilde{\mathbf{u}} : \{0,1,2,\cdots\} \to \Delta_{M+1}$ is recorded as $x(t;x_0,\widetilde{\mathbf{u}})$. Then, the concept of the global event-triggered stabilization for system (5) with respect to $x^* \in \Delta_N$ is presented, where x^* is supposed to be δ_N^r without loss of generality.

Definition 2. For a given state $\delta_N^r \in \Delta_N$, system (5) is said to be globally stabilizable to δ_N^r, i.e., δ_N^r-stabilization, if for every $x_0 \in \Delta_N$, there exist a positive integer T and a control sequence $\widetilde{\mathbf{u}}$: $\{0,1,2,\cdots\} \to \Delta_{M+1}$ such that $t \geqslant T$ implies $x(t;x_0,\widetilde{\mathbf{u}})=\delta_N^r$.

Remark 2. Since system (5) contains the information of the triggering set $\Gamma(\Lambda)$, the stabilization of system (5) also can be called the event-triggered stabilization of system (4). Without raising any confusion, we simply call it stabilization in the following content.

In this paper, the control input $u(t)$ in (4) is considered as the feedback of state $x(t)$, that is,

$$u(t) = Gx(t) = \delta_M[\beta_1,\beta_2,\cdots,\beta_N]x(t), \tag{6}$$

where $G \in \mathscr{L}_{M\times N}$ is called the state feedback matrix. Response to equation (6), $\widetilde{u}(t)$ also can be regarded as a special feedback of $x(t)$ with the "state feedback matrix" $\widetilde{G}$, namely $\widetilde{u}(t) = \widetilde{G}x(t)$. In details, $\widetilde{G} = \delta_{M+1}[\gamma_1,\gamma_2,\cdots,\gamma_N]$ is built as

$$\gamma_j = \begin{cases} 1, & \delta_N^j \in \Delta_N\backslash\Gamma(\Lambda), \\ \beta_j + 1, & \delta_N^j \in \Gamma(\Lambda). \end{cases} \tag{7}$$

The objectives of this paper are to design the possible state feedback matrix $\widetilde{G} \in \mathscr{L}_{(M+1)\times N}$ such that KVLCN (5) is globally stabilizable to δ_N^r under two classes of event-triggered controllers, that is, the time-optimal stabilizer and the switching-cost-optimal one. Thereinto, the time-optimal stabilizer is to make the transient period minimal, and the switching-cost-optimal one is to minimize the triggering event set, that is, to minimize $\mid \Gamma(\Lambda) \mid$.

3 Main Results

In the section, we firstly develop the event-triggered controller for the minimum-time stabilization of KVLCN (5), which can also be called the time-optimal one. In the latter part of this paper, we develop an event-triggered controller with the minimal triggering set, called the switching-cost-optimal one, under the framework of labelled digraph.

3.1 Design of the Time-Optimal Event-Triggered Stabilizer

In this subsection, the time-optimal event-triggered stabilizers are designed. Consider KVLCN (5), a v-step reachable set with respect to state δ_N^r is defined as in [11]:

$$\mathscr{R}_v(r) = \Big\{\delta_N^j \in \Delta_N : \text{there exists } \widetilde{\mathbf{u}}(0),\widetilde{\mathbf{u}}(1),\cdots,\widetilde{\mathbf{u}}(v-1) \in \Delta_{M+1} \\ \text{such that } x(v;\delta_N^j,\widetilde{\mathbf{u}}(0),\widetilde{\mathbf{u}}(1),\cdots,\widetilde{\mathbf{u}}(v-1)) = \delta_N^r\Big\}. \tag{8}$$

On the basis of $\mathscr{R}_v(r)$ defined above, the following theorem can be obtained, whose proof is straightforward and omitted.

Theorem 1. For a given state $\delta_N^r \in \Delta_N$, system (4) can be globally δ_N^r-stabilization by event-triggered controller, if and only if, the next conditions are both satisfied:

(1) $\delta_N^r \in \mathscr{R}_1(r)$;

(2) There exists an integer $l \in [1, N-1]$ such that $\mathscr{R}_l(r) = \Delta_N$.

Without any confusion, the minimal integer satisfying condition (2) is denoted by l^*. Assume that conditions (1) and (2) in Theorem 1 are satisfied, it can be implied that $\mathscr{R}_{i+1}(r) \supseteq \mathscr{R}_i(r)$ for all $i \in$

$[0, l^*-1]$, where $\mathscr{R}_0(r) = \{\delta_N^r\}$. Then, we aim to develop a constructive procedure for the "state feedback matrix" $\widetilde{G}$, under which, the transient period of (5) is minimal.

To this end, we split Δ_N into mutually disjoint sets as

$$\Delta_N = (\mathscr{R}_{l^*}(r) \setminus \mathscr{R}_{l^*-1}(r)) \cup \cdots \cup (\mathscr{R}_2(r) \setminus \mathscr{R}_1(r)) \cup (\mathscr{R}_1(r) \setminus \mathscr{R}_0(r)) \cup \mathscr{R}_0(r). \tag{9}$$

To each $\delta_N^i \in \Delta_N$, there exists a unique integer $l_i \in [1, l^*]$ such that $\delta_N^i \in \mathscr{R}_{l_i}(r) \setminus \mathscr{R}_{l_i-1}(r)$. Denote $\alpha_i = \widetilde{L}\delta_{MN}^i$ for $i \in [1, MN]$, the 'state feedback matrix' $\widetilde{G} = \delta_{M+1}[\gamma_1, \gamma_2, \cdots, \gamma_N]$ can be given by *following procedure*:

(1) If $\alpha_r = r$, let $\gamma_r = 1$; Else, namely $\alpha_r \neq r$, let γ_r be one solution of $\alpha_{(\gamma_r-1)N+r} = r$;

(2) For $i \in [1, N]\backslash\{r\}$, if $\delta_N^{\alpha_i} \in \mathscr{R}_{l_i-1}(r)$, let $\gamma_i = 1$; Otherwise, let γ_i be one solution of $\delta_N^{\alpha_{(\gamma_i-1)N+i}} \in \mathscr{R}_{l_i-1}(r)$.

Remark 3. Under the controller constructed above, all states in Δ_N can reach δ_N^r after at most l^* steps. This time-optimal event-triggered stabilizer simultaneously reduces the number of control as small as possible. If all time-optimal stabilizers are necessary, we only need to modify the above procedure trivially. Therefore, we ignore it here.

Once matrix $\widetilde{G}$ is obtained, the triggering event set $\Gamma(\Lambda)$ can immediately be calculated as $\{\delta_N^i : \gamma_i = 1\}$, and the initial state feedback matrix G equals to $G = \delta_M[\beta_1, \beta_2, \cdots, \beta_N]$, where $\beta_i = \gamma_i - 1$ if $\gamma_i \neq 1$ and β_i can be arbitrarily selected in $[1, M]$ for $\gamma_i = 1$.

Example 1. Let $L = \delta_4[1, 1, 3, 4]$ and $L' = \delta_4[1, 3, 4, 1, 1, 3, 4, 1]$, we construct a novel system in form of (5) with state transition matrix

$$\widetilde{L} = \delta_4[1, 1, 3, 4, 1, 3, 4, 1, 1, 3, 4, 1]. \tag{10}$$

Let $r = 1$, it can be easily calculated that $\mathscr{R}_1(1) = \{\delta_4^1, \delta_4^2, \delta_4^4\}$ and $\mathscr{R}_2(1) = \Delta_4$. Since $\delta_4^1 \in \mathscr{R}_1(1)$ and $\mathscr{R}_2(1) = \Delta_4$, this system can be globally stabilizable to δ_4^1 under ETC.

According to $\widetilde{L}$, let γ_i be as in the above procedure. One has $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma_3 = 2, 3$ and $\gamma_4 = 2, 3$. Correspondingly, we can take the triggering event set $\Gamma(\Lambda) = \{\delta_4^1, \delta_4^2\}$, and β_1, β_2, β_3 and β_4 can be arbitrarily chosen from Δ_2.

Observe from above calculation, the transient period is 2. If we use the traditional state feedback control as in [11], it will be 3. Hence, it is worth formulating that the designed event-triggered controllers both reduce the control cost and transient period than traditional state feedback controllers.

3.2 Design of the Switching-Cost-Optimal Event-Triggered Stabilizer

In this subsection, assume that the conditions in Theorem 1 are satisfied, we devote to designing the event-triggered stabilizer with optimal switching cost. That is, to minimizing the triggering event set $\Gamma(\Lambda)$. In [34], an adjustment method has been formulated to minimize the triggering event set $\Gamma(\Lambda)$ in some special cases. However, it is not capable of addressing some certain generalized senses such as Example 2. Thus, a universal and unified approach to design the switching-cost-optimal stabilizer is still valuable and meaningful.

Example 2. Consider logical system (5) with transition matrices $L = \delta_8[4, 2, 1, 3, 6, 5, 8, 5]$ and $L' = \delta_8[4, 2, 1, 2, 6, 8, 3, 3, 4, 2, 1, 2, 6, 5, 3, 3]$, it is easy to confirm that this network is globally δ_8^2-stabilization under ETC by Theorem 1.

According to the approach proposed in [34], we firstly draw the attractors and basis 1) of KVLN with respect to L as follows:

1) Please see [9] for more details about attractors and basis of KVLNs.

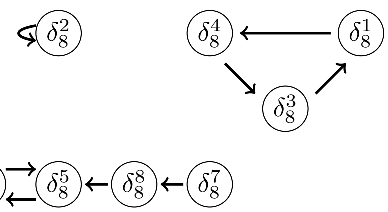

Figure 1 State transition graph of KVLN with respect to transition matrix $L = \delta_8[4, 2, 1, 3, 6, 5, 8, 5]$.

Then, we can select a possible state in the attractors $\{\delta_8^1, \delta_8^3, \delta_8^4\}$ and $\{\delta_8^5, \delta_8^6\}$, respectively, and add the feasible control inputs at these two states such that the overall system can be stablizable to δ_8^2. That is, the minimal number of controller execution is equal to 2.

However, since the transition matrix L' of KVLCN (3b) is determined, if δ_8^5 is selected, it evolves to δ_8^6 under every control input; Otherwise, δ_8^6 is chosen, it evolves to δ_8^8 under $u = \delta_2^1$ or δ_2^2. Obviously, any state in the attractor $\{\delta_8^5, \delta_8^6\}$ cannot reach δ_8^2 by this approach. Hence, this method to minimize the triggering event set $\Gamma(\Lambda)$ is not applicable for this example.

Remark 4. In fact, since the transition matrix L' is known, it is infeasible to consider L unilaterally when minimizing $\Gamma(\Lambda)$ as in [34]. In the following, based on the knowledge of graph theory, we present a universal and unified approach to minimize it.

First of all, a labelled digraph $\mathscr{G}$ is derived for equivalent graphical description of the dynamic of KVLCN (5). The labelled digraph $\mathscr{G}$ is indeed an ordered pair (V, A) consisting of a set of vertices $V := [1, N]$ and a set of directed arcs A. For every arc $(i, j) \in A$, vertices i and j are respectively named as the starting vertex and the ending one of arc (i, j).

Consider KVLN (3a), since $L \in \mathscr{L}_{N\times N}$ is a Boolean matrix, it can be associated with a labelled diagraph $\mathscr{G}_0 = (V, A_0)$. Thereinto, A_0 is a *real line arc* set, where $\mathscr{G}_0$ has a real line arc (i, j) *joining i to j* if and only if $[L]_{ji} = 1$. As for KVLCN (3b), L' is partitioned into $[L'_1, L'_2, \cdots, L'_M]$, where L'_μ, $\mu \in [1, M]$, are control-dependent transition matrices. Similar to the construction of $\mathscr{G}_0$, the labelled digraph for L_μ, denoted by $\mathscr{G}_\mu$, is associated with an order pair (V, A_μ), where A_μ is a *dashed line arc* set, when a dashed line arc $(i, j) \in A_\mu$ if and only if $\left[L'_\mu\right]_{ji} = 1$. Furthermore, by uniting these labelled digraphs $\mathscr{G}_0$ and $\mathscr{G}_\mu$, $\mu \in [1, M]$, the overall labelled digraph $\mathscr{G} = (V, A)$ is obtained as

$$\mathscr{G} = \bigcup_{\mu=0}^{M} \mathscr{G}_\mu = \left(V, \bigcup_{\mu=0}^{M} A_\mu\right).$$

Remark 5. In fact, the arc set A consists of some real line arcs and dashed line ones, corresponding to the dynamics of (3a) and (3b), respectively. For convenience, they are denoted by an identical set A without distinction from notation. From the construction of $\mathscr{G}$, it is easy to find that there may exist more than one arc in the same direction with the same starting vertex and the ending one. If we operate on the labelled digraph $\mathscr{G}$, it may cause some unnecessary issues and high time complexity. Therefore, we make some pretreatment for $\mathscr{G}$ before giving algorithm.

To facilitate the analysis, some *pretreatment* is operated on the labelled digraph $\mathscr{G}$, the labelled digraph after pretreatment is also denoted by $\mathscr{G} := (V, A)$ for convenience, here A represents the arc set of the labelled digraph after pretreatment.

1) Delete all self loops.

2) For all ordered pair $(i, j) \in [1, N] \times [1, N]$ and $i \neq j$, remain the arc with minimal weight joining i to j and delete others. If there are such two arcs, select the arbitrary one.

3) Assign each dashed line arc joining i to j by a control set $u_{(i,j)} := \{\mu : [L_\mu]_{ji} = 1,\ \mu \in [1, M]\}$.

As mentioned in [45], the stabilization problem of KVLCN can be equivalently described by the existence of *spanning in-tree* with the designated vertex r, which is call the *root* of tree. Thus, an approach to find the switching-cost-optimal stabilizer is exactly to find *a spanning in-tree at root r with the minimal number of dashed line arcs* in labelled digraph $\mathscr{G}$.

To this end, weights N and 1 are respectively assigned to each dashed line arc and real line one. Denote the weight on every arc (i,j) by $w(u,v)$, then the labelled digraph $\mathscr{G}$ with weight is denoted by $\mathscr{G} := (V, A, W)$, where W is a set of weight $w(i,j)$ for all $(i,j) \in A$. The spanning in-tree at root r with the minimal sum of weight is named as *the minimal spanning in-tree* of labelled digraph $\mathscr{G}$. In the graph theory, an effective algorithm has been proposed to find such the minimal spanning in-tree, which is called *Edmonds's Algorithm* [46]. Moreover, a universal and unified procedure is firstly derived for the switching-cost-optimal event-triggered stabilizer.

Algorithm 1 The Minimal Spanning In-Tree Algorithm.

Step 1: Initialize $i := 0$, $V_0 := V$, $E_0 := A$ and $W_0 := W$. Designate vertex r as the root.
Step 2: Calculate $J_1 = \{(v, \theta(v)) : v \in V_0 \setminus \{r\}\}$, where an order pair $(v, \theta(v))$ is the minimal weight arc among all $(v, j) \in E_0$.
Step 3: Check whether there exists directed cycles in (V_i, J_{i+1}). If do, go to Step 4; Otherwise, go to Step 7.
Step 4: Contract every cycle $\mathscr{C}$ into one new vertex to obtain a new diagraph $(V_{i+1}, E_{i+1}, W_{i+1})$, the weight set W_{i+1} is updated from W_i as follows, then $i := i+1$ and go to Step 5.

- If (u,v) is an arc joining cycle $\mathscr{C}$, remain its weight unchanged.
- If (u,v) is an arc away cycle $\mathscr{C}$, reassign its weight as $w(u,v) - w(\theta^{-1}(u), u)$.
- Keep the weight of other arcs unchanged.

Step 5: Do the pretreatment for the novel labelled digraph (V_i, E_i, W_i).
Step 6: Calculate $J_{i+1} = \{(v, \theta(v)) : v \in V_i \setminus \{r\}\}$, where an order pair $(v, \theta(v))$ is the minimal weight arc among all $(v, j) \in E_i$. Then, back to Step 3.
Step 7: Expand the contracted cycles formed during the above phase in reverse order of their contraction and remove one arc from each cycle to form a spanning in-tree.

Remark 6. The time complexity of Algorithm 1 is $O(HN)$, where $N = k^n$ and $H =\mid A \mid$.

The returned minimal spanning in-tree in Algorithm 1 is denoted by $\mathscr{G}^0 = (V, A^0, W^0)$, where $A^0 \subseteq A$ and $W_0 \subseteq W$. Once $\mathscr{G}^0$ is obtained, the corresponding event-triggered controllers can be constructed immediately. For every arc set $D \subseteq A$, $\lfloor D \rfloor$ consists of the starting vertex of each arc in D.

Algorithm 2 The Corresponding Event-Triggered Controller Design From the Minimal Spanning In-Tree.

Step 1: Construct the triggering event set $\Gamma(\Lambda)$. If $[L]_{rr} = 1$, then $\Gamma(\Lambda) = \left\{\delta_N^i : i \in \left\lfloor A^0 \backslash A_0 \right\rfloor\right\}$; Otherwise, $\Gamma(\Lambda) = \left\{\delta_N^i : i \in \left\lfloor A^0 \backslash A_0 \right\rfloor \cup \{r\}\right\}$.
Step 2: Determine the state feedback matrix G. Let β_r be randomly chosen in Δ_M if $r \notin \Gamma(\Lambda)$; Else, $\beta_r = u_{(r,r)}$. To every $j \in [1,N] \backslash \{r\}$, if $j \in \Gamma(\Lambda)$, there is a unique integer $t_j \in [1,N]$ satisfying $(j, t_j) \in A^0$, let β_j be an arbitrary integer in $u_{(j,t_j)}$; Otherwise, let β_j be an arbitrary integer in $[1, M]$. The feasible state feedback matrix can be designed as $G = \delta_M[\beta_1, \beta_2, \cdots, \beta_N]$.

Example 3. In the following, Example 2 is reconsidered by approach presented in this subsection.

Firstly, weights 8 and 1 are respectively assigned to the dashed line arcs and the real line ones. The labelled digraph after pretreatment is presented as Figure 2.

Next, calculate J_1 by Step 2 of Algorithm 1 as Figure 3.

Then, since (V_0, J_1) has cycles $\mathscr{C}_1$=$\{1,3,4\}$ and $\mathscr{C}_2$=$\{5,6\}$, then go to Step 4 of Algorithm 1. As drawn in Figure 4, cycles $\mathscr{C}_1$ and $\mathscr{C}_2$ are contracted into novel vertices U and V, respectively.

Consequently, repeating Step 5 and Step 6 of Algorithm 1, Figure 5 is obtained. There still exists a cycle in (V_1, J_2). Thus, vertices V and 8 are further recontracted and the weights of arcs are updated by Step 4 in Figure 6. By repeating Step 5 and Step 6, (V_2, J_3) is obtained without any cycle as Figure 7.

Finally, using Step 7 of Algorithm 1, we expand the contracted cycles formed during the above phase in reverse order of their contraction and remove one arc from each cycle to form a spanning in-tree. Therefore, arcs $(3,4)$, $(5,6)$ and $(5,8)$ are removed. The obtained minimal spanning in-tree $\mathscr{G}^0$ is presented as in Figure 8.

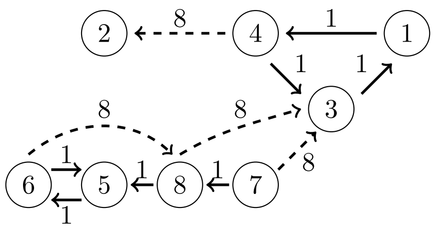

Figure 2 The labelled digraph after pretreatment (V_0, E_0, W_0).

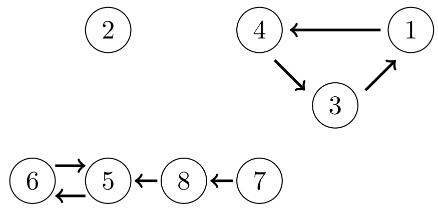

Figure 3 Calculate set $J_1=\{(v, \theta(v)) \mid v \in [1, 8]\}$ by Step 2 in Algorithm 1. That is, $\theta(1) = 4$, $\theta(3) = 1$, $\theta(4) = 3$, $\theta(5) = 6$, $\theta(6) = 5$, $\theta(7) = 8$ and $\theta(8) = 5$.

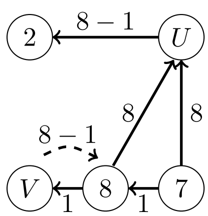

Figure 4 A new constructed weighted directed graph (V_1, E_1, W_1). By Algorithm 1, $w(U, 2) = 8 - 1$ and $w(V, 8) = 8 - 1$. The weight of other arcs keeps unchanged.

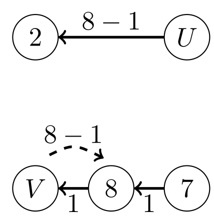

Figure 5 Find the set J_2 in Firgure 4, where $J_2 = \{(V, 8), (8, V), (7, 8), (U, 2)\}$.

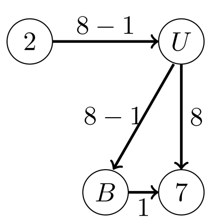

Figure 6 The vertices V and 8 are contracted into a novel vertex B. Let $w(U, B) = 8 - 1$ and the weights of other arcs be unchanged.

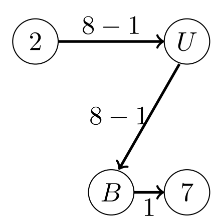

Figure 7 The set $J_3 = \{(2, U), (U, B), (B, 7)\}$.

Based on the minimal spanning in-tree $\mathscr{G}^0$, using Algorithm 2, the corresponding triggering event set is designed as $\Gamma(\Lambda) = \{\delta_8^4, \delta_8^6, \delta_8^8\}$, and the possible state feedback matrices are $G = \delta_8[*, *, *, *, *, 1, *, *]$, where $*$ is 1 or 2.

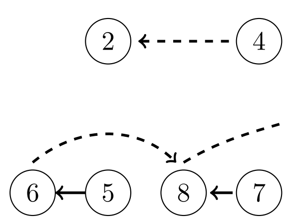

Figure 8 The minimal spanning in-tree $\mathscr{G}^0$ of Example 2.

From Figure 8, it is observed that the number of control execution is equal to 3. If we utilize the traditional state feedback control, the number of control execution will be 7 on the transient period, since all states need to be controlled.

4 Conclusion

This paper has discussed the global stabilization problem of KVLCNs with ETC. By resorting to STP of matrices, a necessary and sufficient condition has been derived for the global stabilization of event-triggered controlled KVLCNs. Meanwhile, the corresponding time-optimal event-triggered stabilizer has been realized. In the latter part of paper, we have designed the switching-cost-optimal event-triggered stabilizer. The labelled digraph of event-triggered controlled KVLCNs has been constructed. Utilizing the knowledge of graph theory, an effective algorithm, named as the minimal spanning in-tree algorithm, has been developed to minimize the number of control execution. This approach can tackle all circumstances and overcome the constraint of the method in the existing literature.

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