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Liner Container Assignment Model with Transit-Time-Sensitive Container Shipment Demand and Its Applications

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Abstract

This paper proposes a practical tactical-level liner container assignment model for liner shipping companies, in which the container shipment demand is a non-increasing function of the transit time. Given the transit-time-sensitive demand, the model aims to determine which proportion of the demand to fulfill and how to transport these containers in a liner shipping network to maximize the total profit. Although the proposed model is similar to multi-commodity network-flow (MCNF) with side constraints, unlike the MCNF with time delay constraints or reliability constraints that is NP-hard, we show that the liner container assignment model is polynomially solvable due to its *weekly schedule* characteristics by developing two link-based linear programming formulations. A number of practical extensions and applications are analyzed and managerial insights are discussed. The polynomially solvable liner container assignment model is then applied to address several important decision problems proposed by a global liner shipping company.

Key Words: Liner container assignment, Multicommodity network-flow problem with side constraints, Link-based formulation, Linear programming

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1 Introduction

Container transportation is vital to international trade and continues to grow. Approximately half of all of the sea cargos in dollar terms are containerized. In 2013, the total container trade volume amounted in 160 million twenty-foot equivalent units (TEUs) (UNCTAD, 2014). Containers are transported by container shipping lines. A liner shipping company provides regular services to transport containerized cargos (containers) from their origin ports to their destination ports over its shipping network. The shipping network comprises a number of ship routes served by ships with different capacities measured by twenty-foot equivalent units (TEUs). Given a weekly port-to-port container shipment demand pattern, it is vital for the liner shipping company to assign the containers to ship routes in the shipping network in order to maximize its weekly profit while satisfying some level-of-service (LOS) constraints. This problem is referred to as liner container assignment (LCA) (Bell et al., 2011, 2013). The importance of LCA is two-fold. At the operational level, it allows a liner shipping company to use the container paths with minimal cost to ship containers, thereby maximizing its profit. At the tactical or strategic level, it provides important evaluation criteria for the tactical and strategic-level decisions such as ship capacity utilization and profitability of a shipping network.

Global liner shipping companies usually provide weekly shipping services (see, e.g., APL, 2015; COSCO, 2015; OOCL, 2015), which means each port of call is visited on the same day every week. The establishment of the convention of weekly services in the industry has the following reasons. First, shippers prefer more frequent services and liner shipping companies wish to accumulate more cargos by providing less frequent services. A weekly service is a trade-off between the two conflicting interests. Second, weekly services imply that ships on the same ship route arrive at each port of call in different weeks but on the same day of each week (e.g., Tuesday). Consequently, container terminal operators can also allocate berth time windows on a weekly basis. If a ship route does not have a weekly service frequency, then it is difficult for container terminal operators to allocate suitable time windows as each time a ship arrives on a different day of a week. It should be mentioned that twice weekly services and thrice weekly services can easily be transformed to weekly services by repeating the port rotation twice or three times, respectively.

In the near-homogeneous liner shipping market, the most important differentiating factor is the transit time of containers from their origin port to their destination port (Notteboom, 2006). In face of the competitive pressure, liner shipping companies seek to offer short transit time, especially when the goods involved are time sensitive; typical examples are perishable goods and consumer goods with a short life cycle or elevated economic/technical depreciation, such as fashion and computers. Therefore, container shipment demand from an origin port to a destination port is dependent on the level of service in terms of origin port–to–destination port (OD) transit time. We call it the transit-time-sensitive (TTS) demand. The TTS demand is a non-increasing function of transit time, schematically shown in Fig. 1c, and the demand is 0 when the transit time exceeds a threshold value. Fig. 1a shows a case in which the demand is constant when the transit time is shorter than or equal to the threshold.

This is a special case of the general TTS demand. Fig. 1b shows a case of constant demand with no transit time requirement, which is the case assumed in most existing studies. LCA aims to determine which proportion of the TTS demand to fulfill and how to allocate the demand to container paths in order to maximize profit at the tactical level.

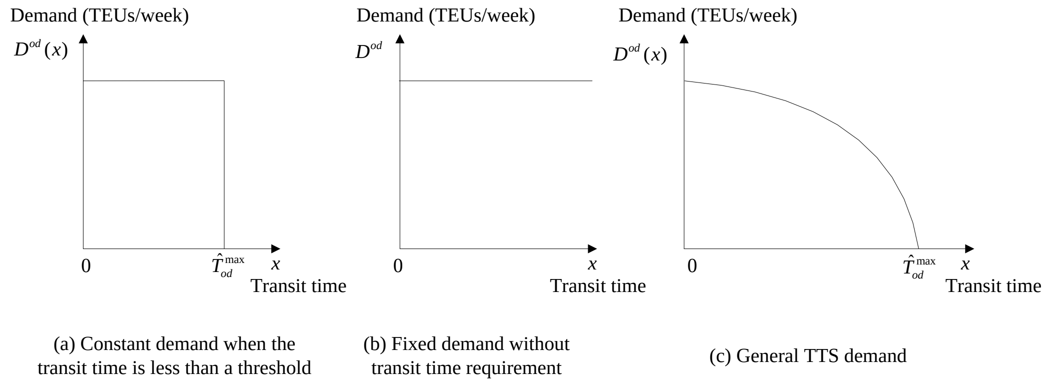

Fig. 1 TTS demands

1.1 Literature review

Most of the existing literature on liner shipping studies focus on tactical-level decisions and consider the LCA as a sub-problem (see Christiansen et al., 2004, 2013; Fransoo and Lee, 2013; Meng et al., 2014, for reviews). As a result, the number of containers can be formulated as continuous quantities (Wang, 2013).

Almost all of the prior studies assume fixed container shipment demand without transit time requirement like the one shown in Fig. 1b. One exception is Wang et al. (2013), which examined ship speed optimization for a single ship route in order to maximize profit in view of the TTS demand shown in Fig. 1c. Other exceptions are Karsten et al. (2015) and Akyüz and Lee (2016), which have incorporated the threshold transit time as shown in Fig. 1a in their container routing models by identifying proper paths.

When there is no transit time requirement, LCA becomes a multi-commodity network-flow (MCNF) problem: the link cost is the transshipment cost and the link capacity is the capacity of the ships deployed. As a result, link-based formulations can be used: the decision variables represent the flow of containers on each link. The containers on each link can either be differentiated by both the origin and destination (e.g., Agarwal and Ergun, 2008; Brouer et al., 2011; Mulder and Dekker, 2014; Dong et al., 2015; Zheng et al., 2015), or just the origin (e.g., Wang et al., 2015a, b), or just the destination (e.g., Wang and Meng, 2013; Brouer et al., 2014), or some origins and some destinations (Wang, 2014). Path-flow formulation can also be used with decision variables being the flow on each path (Ng, 2015; Wang et al., 2015c). The paths are usually dynamically generated in a column-generation scheme as the large number of paths increases exponentially with the size of the liner shipping network (e.g., Brouer et al., 2011; Karsten et al., 2015; Akyüz and Lee, 2016).

The general LCA is much more challenging than MCNF. There are some similarities between the LCA model and MCNF with side constraints (Holmberg and Yuan, 2003). For instance, in telecommunication networks, the transmission time delay of an established path depends on the links and nodes that this path contains. The time delay of a path can be measured by, for example, adding the estimated link delays. For each commodity, it is desirable that this time delay is less than a certain limit for all paths that are used for communication. Another issue in telecommunication networks is reliability. Assuming that for each link, there is a certain probability that it could fail, the failure probability of a path can be calculated. For each commodity, it is desirable that all of the used paths have a failure rate not exceeding a specified limit. By taking the logarithm of the link failure probabilities, such considerations can be formulated as additive constraints. Holmberg and Yuan (2003) proved that the MCNF with time delay constraints or reliability constraints is NP-hard and claimed that they do not believe there exists an exact formulation without using variables explicitly associated with paths. As a consequence, path-based formulations and column generation in view of an exponential number of possible paths are almost the only choice. What is worse, finding the resource-constrained shortest path in the column-generation sub-problem is also NP-hard (Gamst et al., 2010). It is not difficult to see that the special case of LCA having constant demand with a threshold transit time, as show in Fig. 1a, is almost identical to the MCNF with time delay constraints or reliability constraints. Therefore, one might conjecture that the general LCA is also NP-hard.

1.2 Objectives and contributions

The main objective and contribution of this study is to formulate the tactical-level LCA and to show its important applications. LCA with general TTD is a new research topic with practical significance for the liner shipping industry. We first demonstrate that the LCA problem is polynomially solvable by building two link-based linear programming formulations whose decision variables and constraints are polynomially bounded by the problem size. Although the structure of LCA is similar to the NP-hard MCNF with time delay constraints or reliability constraints, the underlining liner shipping network of the LCA model has a fundamental property of weekly periodicity because liner ship routes have weekly service schedules in practice. As a result of this fundamental property, in contrast to the MCNF with time delay constraints or reliability constraints, LCA is no longer NP-hard.

Some LCA instances may also be solved by path-based formulations with column generation, especially if the liner shipping network is very sparse. Our model provides a polynomial-time algorithm to find the resource-constrained shortest path in the column generation sub-problem, in contrast to the general resource-constrained shortest path problem that is NP-hard.

We further show how the model can be applied to solve several crucial decision issues currently faced by the liner shipping industry. In particular, the LCA model can help a liner shipping company in the following aspects:

(i) Provide a tangible decision support tool for the company to deal with port operators, other liner shipping companies, and shippers. In particular, the proposed LCA model can provide answers to the following questions: (a) Should the shipping company commit to a certain number of container handling operations at a particular port to enjoy a premium handling price? (b) What is a suitable price for purchasing container slots from other shipping companies? (c) What is the lowest freight rate the company should accept from a shipper with large container shipment demand?

(ii) Assist the company in strategic decision-making. For example, should a new shipping market such as East Africa be explored? Should new mega-containerships be ordered? Should a regional or global hub port be relocated?

(iii) Provide necessary and detailed information on how containers are transported in the network. This information is helpful for the liner shipping company to improve its liner shipping network, such as launching new services, changing ship fleet deployment, skipping a port, and changing port rotation directions.

(iv) Facilitate the determination of the amount of container equipment to charter or purchase by incorporating empty container repositioning and by calculating the number of empty containers required.

The remainder of the paper is organized as follows. Section 2 introduces liner shipping networks and TTS demand. Section 3 proposes two link-based linear programming formulations for LCA that are polynomially solvable. Section 4 examines how the model can incorporate a number of practical constraints and how to utilize the results obtained from the model to assist various decisions. Section 5 reports numerical experiments based on data provided by a global liner shipping company. Section 6 concludes this study. For better readability, symbols used throughout the paper are listed in Appendix 1.

2 Problem Description

Consider a liner shipping company which operates a set of ship routes $\mathcal{R}$ to transport containers over a group of ports denoted by the set $\mathcal{P}$. Each ship route has a fixed port rotation, and fixed arrival and departure times at each port of call. An illustrative liner service network is shown in Fig. 2, including three ship routes (SRs) denoted by $\mathcal{R}=\{1,2,3\}$ and seven ports denoted by $\mathcal{P}$={CC, CN, CB, HK, JK, SG, XM}. The itinerary of each ship route $r\in\mathcal{R}$ forms a loop. Let N_r represent the number of ports of call on a round trip of ship route r and $p_{ri}\in\mathcal{P}$ be the physical port corresponding to the i^{th} port of call. We can arbitrarily define one port of call as the first. For example, in Fig. 2 Hong Kong is the first port of call on Ship Route 1 (SR1), Jakarta and Singapore are the second and third ports of call, respectively, and the number of ports of call on SR1 is $N_1=3$. It should be mentioned that although Singapore is visited twice during a round trip of SR2, these two calls can easily be differentiated using the port calling sequence to refer to a port of call. Let $\mathcal{I}_r=\{1,2,\cdots,N_r\}$ be the set of port calling sequences for ship route r. We define $p_{r,N_r+1}=p_{r1}$ and call the voyage between two adjacent ports of call p_{ri} and $p_{r,i+1}$ leg i of ship route r, $i\in\mathcal{I}_r$.

Every ship route has fixed service schedules. The arrival time at each port of call i for each ship route r, denoted by t_{ri}^{arr}, is tabulated in Table 1. (The departure time does not affect the problem, and hence is not reported in Table 1.) In Table 1 we actually define the time 00:00 of a particular Sunday as time 0 (h) and focus on a ship that visits the first port of call on each ship route in week 1 (between time 0 and time 168). The arrival time of the focal ship at port of call i on the ship route is t_{ri}^{arr}.

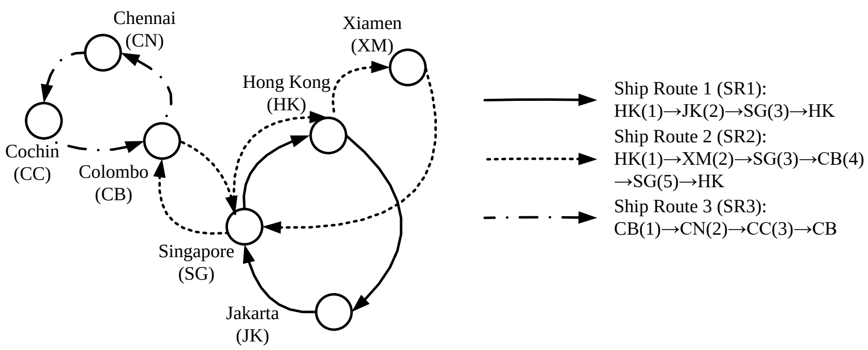

Fig. 2 A liner shipping network with three ship routes

Table 1 Schedules of the three ship routes

		Ship route 1			Ship route 2			Ship route 3
ID	Port	Arrival time (h)	ID	Port	Arrival time (h)	ID	Port	Arrival time (h)
1	HK	10 (10:00 Sun)	1	HK	0 (00:00 Sun)	1	CB	0 (00:00 Sun)
2	JK	188 (20:00 Sun)	2	XM	66 (18:00 Tue)	2	CN	60 (12:00 Tue)
3	SG	218 (02:00 Tue)	3	SG	238 (22:00 Tue)	3	CC	130 (10:00 Fri)
1	HK	346 (10:00 Sun)	4	CB	386 (02:00 Tue)	1	CB	168 (00:00 Sun)
			5	SG	514 (10:00 Sun)			
			1	HK	672 (00:00 Sun)			

2.1 Weekly service frequency

It is assumed that each ship route has a weekly service frequency, which means that every port of call (not every port) on the ship route will be visited once a week. For example, Fig. 3 plots the port rotation and transit times of American President Line (APL)'s current Asia-Europe Loop 4 service route (APL, 2015). Ships depart from each port of call on a given day every week. The round-trip time of the Asia-Europe Loop 4 is equal to 77 days. The number of ships serving this ship route, denoted by m, can be calculated as follows:

$$m = \frac{77 \text{ days}}{7 \text{ days (one week)}} = 11 \tag{1}$$

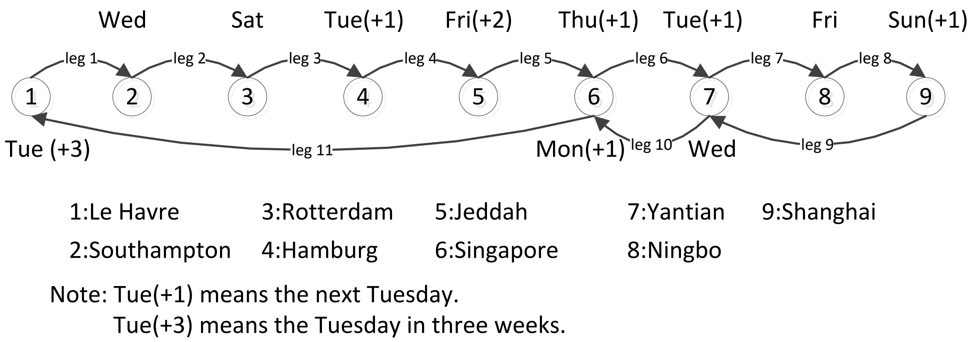

Fig. 3 Schematic representation of Asia-Europe Loop 4

A weekly service frequency has two implications. First, the round-trip journey time of a ship route, including the time at sea and the time at port, is an integer number of weeks. Second, this integer number equals the number of ships deployed on the ship route, as shown in Eq. (1). The ships deployed on ship route r form a string and have the same capacity denoted by Cap_r (TEUs).

2.2 OD transit time

To be precise, the OD transit time of containers is defined by the time interval between when the ship that is to carry the containers arrives at the origin port and when the ship that carries the containers (it can be a different ship from the one at the origin port) arrives at the destination port. The schedules of ship routes affect the OD transit time. For example, consider containers from Australia to Europe that are transshipped at the port of Singapore. If the ship from Australia to Singapore arrives at Singapore on Monday, and the ship from Singapore to Europe arrives at Singapore on Tuesday, the connection time of the containers at Singapore (or dwell time, defined as the time interval from the arrival of the ship that carries the containers from Australia to the arrival of the ship that will carry the containers to Europe) is one day. By contrast, if the ship from Australia arrives at Singapore on Tuesday, and the ship to Europe arrives at Singapore on Monday, the connection time will be six days because containers have to wait at Singapore until the next Monday as a result of the weekly service. The schedules of liner ship routes are input for LCA. At a port that is visited more than once a week, containers can be transshipped from a ship that arrives earlier to one that arrives later, but not vice versa.

2.3 Transit-time-sensitive demand

There is a set of OD pairs in the network, denoted by $\mathcal{W} \subseteq \{(o,d), o \in \mathcal{P}, d \in \mathcal{P}\}$. Represent by g'_{od} (USD/TEU) the freight rate paid by shippers for OD pair (o,d). We assume g'_{od} is independent of the real transit time. However, our models to be shown later can easily accommodate the case where g'_{od} is a function of the transit time. Since the additional cost for transporting one more container at sea is marginal compared with the handling cost, we only consider the container handling cost as the variable cost. Let $\hat{c}_p$, $\tilde{c}_p$ and $\bar{c}_p$ represent the loading, discharge, and transshipment cost (USD/TEU) at port $p \in \mathcal{P}$, respectively. The maximum profit that can be yielded from shipping one TEU for OD pair (o,d) can be calculated by

$$g_{od} := g'_{od} - \hat{c}_o - \tilde{c}_d \tag{2}$$

Therefore, the liner shipping company seeks to determine which containers to transport and assign the containers to the ship routes to maximize its total profit, which is the total revenue minus the total transshipment cost.

For each OD pair $(o,d) \in \mathcal{W}$ with a given transit time x the corresponding demand is represented by $D^{od}(x)$ (TEUs/week). It is also reasonable and practical to assume that there is an upper bound on the transit time for each OD pair (o,d), referred to as the threshold transit time and denoted by $\hat{T}_{od}^{\max}$, which satisfies $D^{od}(x) = 0, x > \hat{T}_{od}^{\max}$. For instance, $\hat{T}_{od}^{\max}$ will not exceed half a year in practice. Since in practice there are many paths to ship containers from an origin port to a destination port, the actual transit time is not a single value for an OD pair. Suppose that there are n^{od} distinct transit times for OD pair (o,d), denoted by $\tau_1^{od}, \tau_2^{od}, \cdots, \tau_{n^{od}}^{od}$, arranged in an increasing order:

$$\tau_1^{od} < \tau_2^{od} < \cdots < \tau_{n^{od}}^{od} \le \hat{T}_{od}^{\max} \tag{3}$$

Note that we are not interested in transit times longer than $\hat{T}_{od}^{\max}$. Since no container will be delivered with transit time in the interval $(\tau_{n^{od}}^{od}, \hat{T}_{od}^{\max}]$, without loss of generality, we can set $\hat{T}_{od}^{\max} = \tau_{n^{od}}^{od}$, and modify $D^{od}(x)$ such that

$$D^{od}(x) \leftarrow \begin{cases} D^{od}(x), \forall x \le \tau_{n^{od}}^{od} \\ 0, \forall x > \tau_{n^{od}}^{od} \end{cases} \tag{4}$$

Let y_i^{od} be the number of TEUs delivered for the OD pair (o,d) with transit time τ_i^{od}, $i = 1, 2, \cdots, n^{od}$. It can be seen that for any feasible container assignment, we must have

$$\sum_{j=i}^{n^{od}} y_j^{od} \le D^{od}(\tau_i^{od}), i = 1, 2, \cdots, n^{od} \tag{5}$$

Eq. (5) means that the curve of $\sum_{j=i}^{n^{od}} y_j^{od}$, as shown by the stepwise thick-dashed line in Fig. 4, should not be above the value of $D^{od}(x)$.

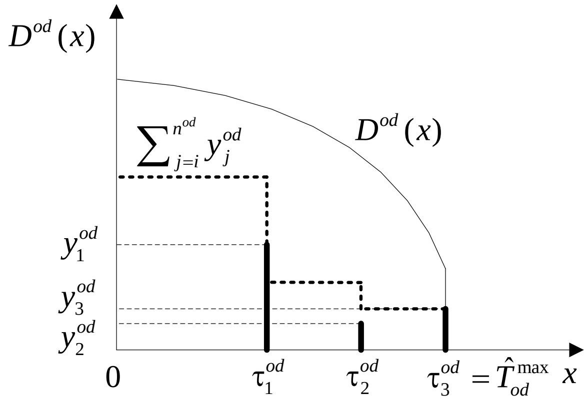

Fig. 4 Relation between a general TTS demand and the fulfilled demand

2.4 LCA model with TTS demand

We now formally state the liner container assignment model with the input—a liner shipping network $(\mathcal{R}, \mathcal{P}, \mathcal{W})$—and the following assumptions:

A1: A string of homogeneous ships with the same capacity deployed on each ship route with weekly service frequency and fixed arrival times at each port of call;

A2: Fixed loading cost, discharge cost, and transshipment cost per TEU at each port;

A3: Each OD pair (o,d) has a given freight rate and given transit-time-sensitive demand $D^{od}(x)$ (TEUs/week); $D^{od}(x)$ is the same for different weeks as we consider a tactical-level decision problem;

A4: At a transshipment port, containers can be transshipped from a ship that arrives earlier to a ship that arrives later, but not vice versa;

A5: The liner shipping company can freely choose which containers to accept and which containers to reject.

The LCA model hence determines the number of TEUs per week for each OD pair (o,d) to deliver, and how to transport the containers in the liner shipping network considering that the demand is transit-time-sensitive, in order to maximize the weekly profit.

3 Two Linear Programming Formulations

As a consequence of the TTS demand, a natural choice is to enumerate all the possible paths for each OD pair and check their transit times. However, the number of paths increases exponentially with the size of the shipping network (Holmberg and Yuan, 2003), and hence this approach is not applicable for finding exact solutions for large-scale networks. In this section we develop two linear programming models that do not need path enumeration/generation by taking advantage of the unique property of LCA that all of the ship routes have the weekly service frequency. We first construct a novel space-time network for the liner shipping network and analyze the properties of the space-time network; we then develop an OD-link-based linear programming model and an origin-link-based linear programming model.

3.1 Space-time network representation

The liner shipping network $(\mathcal{R}, \mathcal{P}, \mathcal{W})$ can be transparently represented by a space-time network built using the following procedure:

Space-time network building procedure:

Input: liner shipping network $(\mathcal{R}, \mathcal{P}, \mathcal{W})$

Output: space-time network $G = (N, A)$ where N is the set of nodes and A is the set of arcs

Step 1: (Determine the time horizon) Construct a space-time network, where the space-axis corresponds to ports of call (not ports) in the liner shipping network, and the time-axis represents the arrival time (hour) at each port of call. The time-horizon $\hat{N}_T$ (weeks) for the time-axis is calculated below:

$$\hat{N}_T = \left\lceil \max_{(o,d)\in\mathcal{W}} \hat{T}_{od}^{\max} / 168 \right\rceil + 1 \tag{6}$$

where $\lceil x \rceil$ means the smallest integer greater than or equal to x. For example, if the maximum threshold transit time for all OD pairs is 30 days (720 hours), the time-horizon should be $\lceil 720/168 \rceil + 1 = 6$ weeks. The addition of one week in Eq. (6) is because the container may be loaded at the origin port at the end of the first week.

Step 2: (Construct nodes) The number of ports of call in the liner shipping network is $\sum_{r\in\mathcal{R}} N_r$. The space-axis is hence divided into $\sum_{r\in\mathcal{R}} N_r$ segments, each segment corresponding to one port of call. For each port of call i on ship route r, we construct $\hat{N}_T$ nodes in the space-time network, and all of these $\hat{N}_T$ nodes correspond to the same port of call i on ship route r on the space-axis. The arrival times of the $\hat{N}_T$ nodes indicated by the time-axis are:

$$t_{ri}^{\text{arr}} \bmod 168,\ (t_{ri}^{\text{arr}} \bmod 168) + 168, \cdots,\ (t_{ri}^{\text{arr}} \bmod 168) + 168(\hat{N}_T - 1) \tag{7}$$

In other words, these $\hat{N}_T$ nodes represent the visit of the same port of call in week 1, week 2, … and week $\hat{N}_T$. Therefore, the node set N in the space-time network has a total of $\hat{N}_T \sum_{r\in\mathcal{R}} N_r$ nodes. A node can be represented by a triplet (r,i,t), that is, the visit at port of call i on ship route r at time t.

Step 3: (Construct a voyage arc set denoted by A^v) For each node $(r,i,t)\in N$, after a ship visits it, the ship will visit the next port of call at time $t + t_{r,i+1}^{\text{arr}} - t_{ri}^{\text{arr}}$. If $t + t_{r,i+1}^{\text{arr}} - t_{ri}^{\text{arr}} < 168\hat{N}_T$, then node $(r, i+1, t + t_{r,i+1}^{\text{arr}} - t_{ri}^{\text{arr}})$ is also in the space-time network. Hence, we construct a voyage arc from node (r,i,t) to $(r, i+1, t + t_{r,i+1}^{\text{arr}} - t_{ri}^{\text{arr}})$.

Step 4: (Construct a transshipment arc set denoted by A^t) Many nodes correspond to the same physical port and containers can be transshipped between these nodes. For each node $(r,i,t)\in N$, we check all the other nodes $(r',i',t')\in N$. If $p_{ri} = p_{r'i'}$ and $t \le t' < 168 + t$, that is, these two nodes represent the same physical port, and a ship arrives at (r',i',t') not earlier than t and not later than $168+t$, then we construct a transshipment arc from (r,i,t) to (r',i',t'). (Due to weekly services and weekly demand, the connection time at a transshipment port will not exceed 1 week.) The set of arcs in the space-time network is $A = A^v \bigcup A^t$. □

The nodes and arcs in the space-time network have attributes elaborated below.

Attribute allocation method

Step 1: (Attributes of nodes) Instead of using (r,i,t), we can also represent a node in set N by n. Node $n \in N$ contains information about ship route $r_n = r$, port of call $i_n = i$ on ship route r_n, the corresponding physical port p_n, and arrival time $t_n^{\text{arr}} = t$. Note that t_n^{arr} is the time that node n is visited, and t_n^{arr} may be different from t_{ri}^{arr}, as shown in Eq. (7).

Step 2: (Attributes of voyage arcs) We use the tail node m and head node n to represent an arc $(m,n) \in A, m,n \in N$. A voyage arc $(m,n) \in A^v$ has the attribute of transit time, denoted by $t_{mn} = t_n^{\text{arr}} - t_m^{\text{arr}}$. The cost of transporting a container on the arc, denoted by c_{mn}, is equal to 0 as the cost of transporting one more container on a ship is much smaller than the handling cost.

Step 3: (Attributes of transshipment arcs) A transshipment arc $(m,n) \in A^t$ has the attribute of connection time, denoted by $t_{mn} = t_n^{\text{arr}} - t_m^{\text{arr}}$. Its cost c_{mn} is equal to the transshipment cost $\bar{c}_{p_m}$ at port $p_m = p_n$. □

Let us illustrate how to build a space-time network using the example of the liner shipping network in Fig. 2. Suppose that $\max_{(o,d)\in \mathcal{W}} \hat{T}_{od}^{\max} = 480$ hours, the corresponding space-time network representation is shown in Fig. 5 . To construct the space-time network, we consider a time horizon of $\hat{N}_T = 4$ weeks according to Eq. (6) (168, 336, and 504 in Fig. 5 are the number of hours in one, two, and three weeks, respectively). The liner shipping network in Fig. 2 has a total of $N_{r_1} + N_{r_2} + N_{r_3} = 3+5+3 = 11$ ports of call. Hence, the space-axis is divided into 11 segments, each representing a port of call. We plot four copies for each port of call, and hence there are 44 nodes in the space-time network. Each node has attributes of ship route, port of call, port, which can be read from the space-axis, and the arrival time, which can be read from the time-axis. Fig. 5 shows the ID of a few nodes and their arrival times. For instance, the arrival time at node n_1 is 66 (the 66$^{\text{th}}$ hour, or 18:00 of Tuesday). We do not plot the arrival times for all of the nodes in Fig. 5 for better readability. Because the round trip time of SR1 is two weeks, two ships (Ship 1 and Ship 2) are deployed on SR1. Similarly, four ships (Ships 3 to 6) and one ship (Ship 7) are deployed on SR2 and SR3, respectively. Containers may be transshipped at ports visited more than once a week, i.e., Hong Kong, Singapore, and Colombo. The thick solid lines in Fig. 5 represent transshipment operations, which connect two calls at the same port. The liner services in any two weeks are identical due to the weekly service frequency. For example, the nodes and arcs in week 2 and week 3 of the space-time network are identical. Week 1 and week 4 are slightly different because arcs with head or tail not in the time horizon are not drawn.

Recall that we consider a tactical-level problem in which the container shipment demand function for an OD pair in different weeks are the same. Since the shipping services in each week are also identical, the same container routing decisions are made each week. Therefore, we only need to focus on container routing decisions of a particular week. Here we look at the containers that leave their origin ports in the first week and examine how to transport them. The transportation of containers that leave their origin ports in the second week follows the same manner as the containers in the first week. We stress that our model captures containers transported in every week, rather than just those in the first week.

We analyze the container flows in the constructed space-time network using the example of OD pair $(\mathrm{XM}, \mathrm{SG})$. Since we look at the containers that leave their origin ports in the first week, the origins of containers in the port pair $(\mathrm{XM}, \mathrm{SG})$ can be any node n in the space-time network satisfying $p_n = \mathrm{XM}$ and $0 \le t_n^{\mathrm{arr}} < 168$. In this example, only node n_1 in the space-time network can serve as the origin node of the containers. Similarly, if we are to consider the OD pair $(\mathrm{HK}, \mathrm{SG})$, then both node n_{21} and node n_{22} can serve as the origin. The set of nodes in the space-time network that can serve as origin nodes for the OD pair (o,d) is represented by $\mathcal{N}_o^{od}$. For each node $m \in \mathcal{N}_o^{od}$, we can identify the set of destination nodes that satisfy the threshold transit time constraint, represented by $\hat{\mathcal{N}}_m^{od} := \left\{ n \in N \mid p_n = d, t_n^{\mathrm{arr}} - t_m^{\mathrm{arr}} \le \hat{T}_{od}^{\max} \right\}$. For instance, assuming $\hat{T}_{\mathrm{XM,SG}}^{\max} = 330$ for OD pair $(\mathrm{XM}, \mathrm{SG})$, the corresponding destination node set for origin node n_1 is $\hat{\mathcal{N}}_{n_1}^{\mathrm{XM,SG}} = \{n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9\}$. Of course, it is not difficult to see that there is no feasible flow (or, no path) from n_1 to any node in the subset $\{n_2, n_3, n_4, n_5, n_6\}$. (In this example there will be no flow from n_1 to n_8 or n_9 in the optimal solution.) In practice, we only need a simple preprocessing step to exclude those nodes in $\hat{\mathcal{N}}_m^{od}$ that are unreachable from node m. It is possible that some origin nodes $m \in \mathcal{N}_o^{od}$ have an empty destination node set $\hat{\mathcal{N}}_m^{od}$. These origin nodes can be excluded from the set $\mathcal{N}_o^{od}$. Define $\mathcal{N}_d^{od} := \bigcup_{m \in \mathcal{N}_o^{od}} \hat{\mathcal{N}}_m^{od}$ as the set of all possible destination nodes for the OD pair.

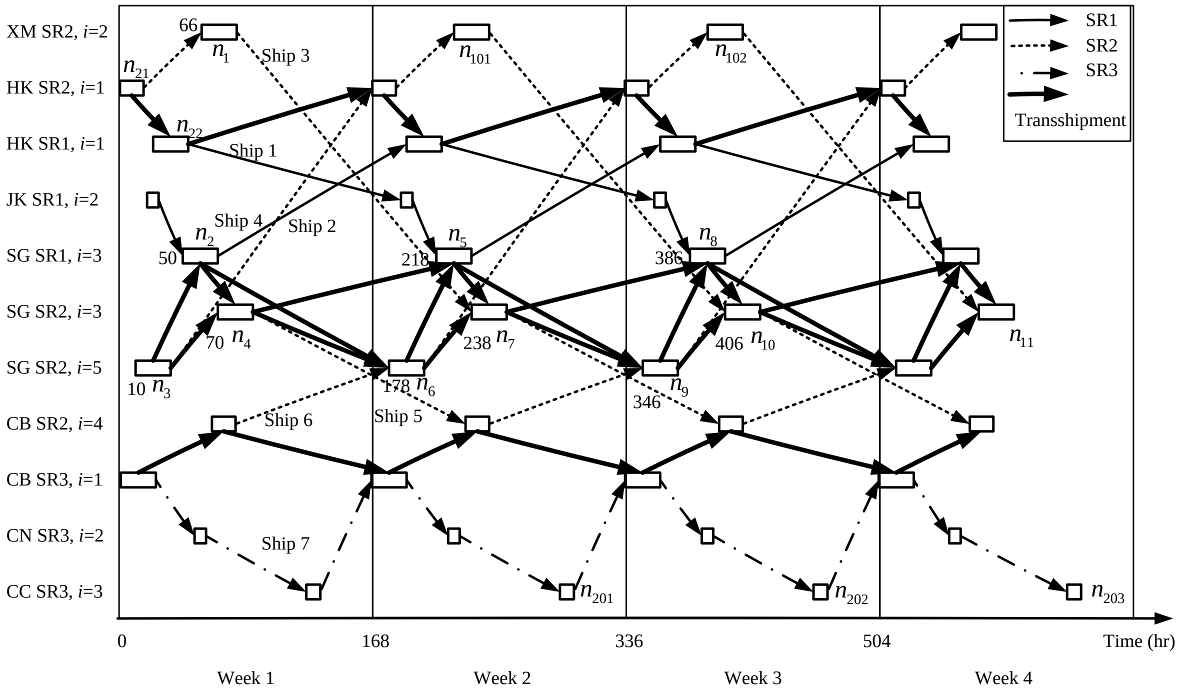

Fig. 5 Space-time network representation of liner services

We will analyze the flow of containers in the space-time network rather than the original liner shipping network. Therefore, we transform each OD pair $(o,d)\in\mathcal{W}$ to sets of new OD pairs in the space-time network denoted by $\mathcal{W}^{od}:=\{(u,v)\,|\,u\in\mathcal{N}_o^{od}, v\in\hat{\mathcal{N}}_u^{od}\}$.

New OD pair set $\mathcal{W}^{od}$ generation method:

Step 1: Construct the set $\mathcal{N}_o^{od}$, which is defined below:

$$\mathcal{N}_o^{od}=\left\{n\in N\,|\,p_n=o, 0\le t_n^{\text{arr}}<168\right\} \tag{8}$$

Step 2: For each $m\in\mathcal{N}_o^{od}$, define a set below:

$$\hat{\mathcal{N}}_m^{od}:=\{n\in N\,|\,p_n=d, t_n^{\text{arr}}-t_m^{\text{arr}}\le\hat{T}_{od}^{\max}\} \tag{9}$$

For each $n\in\hat{\mathcal{N}}_m^{od}$, if it is not reachable from node m, remove n from set $\hat{\mathcal{N}}_m^{od}$.

Step 3: For each $m\in\mathcal{N}_o^{od}$, if $\hat{\mathcal{N}}_m^{od}=\varnothing$, remove m from set $\mathcal{N}_o^{od}$.

Step 4: The set of new OD pairs $\mathcal{W}^{od}$ is defined below:

$$\mathcal{W}^{od}:=\left\{(u,v)\,|\,u\in\mathcal{N}_o^{od}, v\in\hat{\mathcal{N}}_u^{od}\right\} \tag{10}$$

For example, assuming $\hat{T}_{\text{XM,SG}}^{\max}=330$, we have the set of new OD pairs $\mathcal{W}^{\text{XM,SG}}=\left\{(n_1,n_7),(n_1,n_8),(n_1,n_9)\right\}$. The number of all of the new OD pairs for a space-time network is bounded by a polynomial expression of the size of the network:

$$\sum_{(o,d)\in\mathcal{W}}|\mathcal{W}^{od}|<|N|^2=\left(\hat{N}_T\sum\nolimits_{r\in\mathcal{R}}N_r\right)^2 \tag{11}$$

In Eq. (11), $\hat{N}_T$ is related to the threshold transit time. The threshold transit time may not increase with the size of the liner shipping network because a larger network means more ports and ship routes, but the longest distance between an OD pair will not change. Hence, the largest possible $\hat{N}_T$ is less than e.g. 26 weeks (half a year) whatever size the liner shipping network is. Hence $\hat{N}_T$ is bounded. $\sum_{r\in\mathcal{R}} N_r$ is an indication of the size of the liner shipping network.

Using new OD pairs $\mathcal{W}^{od}$ in the space-time network, the transit time is implicitly considered because the head node of an arc always corresponds to a later time than the tail node. In detail, the transit time of new OD pair $(u,v)\in\mathcal{W}^{od}$ can be calculated by $\tau_{uv} := t_v^{\text{arr}} - t_u^{\text{arr}}$. We sequence the set of new OD pairs $(u,v)\in\mathcal{W}^{od}$ such that

$$\tau_{u_1v_1} \le \tau_{u_2v_2} \le \cdots \le \tau_{u_{|\mathcal{W}^{od}|}v_{|\mathcal{W}^{od}|}}, \forall (o,d)\in\mathcal{W} \tag{12}$$

Represent by y^{uv} the volume of containers (TEUs/week) in the new OD pair (u,v) that are delivered by the liner shipping company. The TTS demand constraints which are equivalent to Eq. (5) can be restated as:

$$\sum_{i=j}^{|\mathcal{W}^{od}|} y^{u_iv_i} \le D^{od}(\tau_{u_jv_j}), j=1,2,\cdots,|\mathcal{W}^{od}|, \forall (o,d)\in\mathcal{W} \tag{13}$$

Therefore, the TTS demand can be handled by the new OD pairs $(u,v)\in\mathcal{W}^{od}$ in the space-time network with constraints (13). It should be mentioned that for constant demand with a threshold transit time as shown in Fig. 1a, Eq. (13) can be simplified as:

$$\sum_{(u,v)\in\mathcal{W}^{od}} y^{uv} \le D^{od}(0), \forall (o,d)\in\mathcal{W} \tag{14}$$

3.2 OD-link-based linear programming formulation

To formulate the ship capacity constraint, we define a new set A^{ri}, which is the set of voyage arcs in the space-time network corresponding to leg i of ship route r. The set A^{ri} may not be a singleton. For example, in Fig. 5, all of the arcs (n_1,n_7), (n_{101},n_{10}) and (n_{102},n_{11}) correspond to leg 2 of Ship Route 2. That is, $A^{r_2,2}=\{(n_1,n_7),(n_{101},n_{10}),(n_{102},n_{11})\}$. We will explain later why we need the notation A^{ri} to formulate the ship capacity constraint.

The decision variables for LCA are as follows. y^{uv} is the volume of containers (TEUs/week) between the new OD pair $(u,v)\in\mathcal{W}^{od}$, $(o,d)\in\mathcal{W}$, that are delivered; f_{mn}^{uv} is the volume of containers (TEUs/week) in the new OD pair (u,v) that flow on the arc $(m,n)\in A$. Sequencing the set of new OD pairs $(u,v)\in\mathcal{W}^{od}$ by Eq. (12), the LCA with TTS demand can be formulated as an OD-link-based linear programming (LP) model:

[OD Model] $$\max_{f_{mn}^{uv},y^{uv}} \sum_{(o,d)\in\mathcal{W}}\sum_{(u,v)\in\mathcal{W}^{od}} g_{od}y^{uv} - \sum_{(m,n)\in A^t} c_{mn}\sum_{(o,d)\in\mathcal{W}}\sum_{(u,v)\in\mathcal{W}^{od}} f_{mn}^{uv} \tag{15}$$

subject to:

$$\sum_{n,(m,n)\in A} f_{mn}^{uv} - \sum_{n,(n,m)\in A} f_{nm}^{uv} = \begin{cases} y^{uv}, \ m=u \\ -y^{uv}, m=v \\ \ \ 0, \ \text{otherwise} \end{cases} \quad ,\forall m\in N, \forall (o,d)\in \mathcal{W}, \forall (u,v)\in \mathcal{W}^{od} \tag{16}$$

$$\sum_{(m,n)\in A^{ri}} \sum_{(o,d)\in \mathcal{W}} \sum_{(u,v)\in \mathcal{W}^{od}} f_{mn}^{uv} \le \text{Cap}_r, \forall r\in \mathcal{R}, \forall i\in \mathcal{I}_r \tag{17}$$

$$\sum_{i=j}^{|\mathcal{W}^{od}|} y^{u_i v_i} \le D^{od}(\tau_{u_j v_j}), \ j=1,2\cdots |\mathcal{W}^{od}|, \forall (o,d)\in \mathcal{W} \tag{18}$$

$$y^{uv} \ge 0, \forall (o,d)\in \mathcal{W}, \forall (u,v)\in \mathcal{W}^{od} \tag{19}$$

$$f_{mn}^{uv} \ge 0, \forall (m,n)\in A, \forall (o,d)\in \mathcal{W}, \forall (u,v)\in \mathcal{W}^{od} \tag{20}$$

The objective function (15) maximizes the total weekly profit, which is the revenue minus the container handling cost. This objective function demonstrates that the model can easily handle the case where the freight rate g'_{od} in Eq. (2) depends on the transit time. Eq. (16) is the flow conservation equation. Eq. (17) requires that the sum of flows on all of the arcs corresponding to leg i of ship route r cannot exceed the capacity Cap_r of the ship route. We will explain this constraint in the next sub-section. Eq. (18) imposes the TTS demand constraint. Eqs. (19) and (20) define nonnegative decision variables.

3.2.1 Capacity constraints

We design a simple example to appreciate constraint (17). Consider a liner shipping network with three ship routes: SR1 visits HK and SG, SR2 visits HK and SG, and SR3 visits SG and CC. Suppose that the corresponding space-time network is the one shown in Fig. 6. Suppose further that there is only one OD pair (HK, CC) with $\hat{T}_{\text{HK,CC}}^{\max} = 297$. Then it has two new OD pairs $\mathcal{W}^{\text{HK,CC}} := \{(n_{11}, n_{61}), (n_{21}, n_{62})\}$ (the potential new OD pair (n_{21}, n_{61}) is infeasible as n_{61} is unreachable from n_{21}, and (n_{11}, n_{62}) violates $\hat{T}_{\text{HK,CC}}^{\max}$).

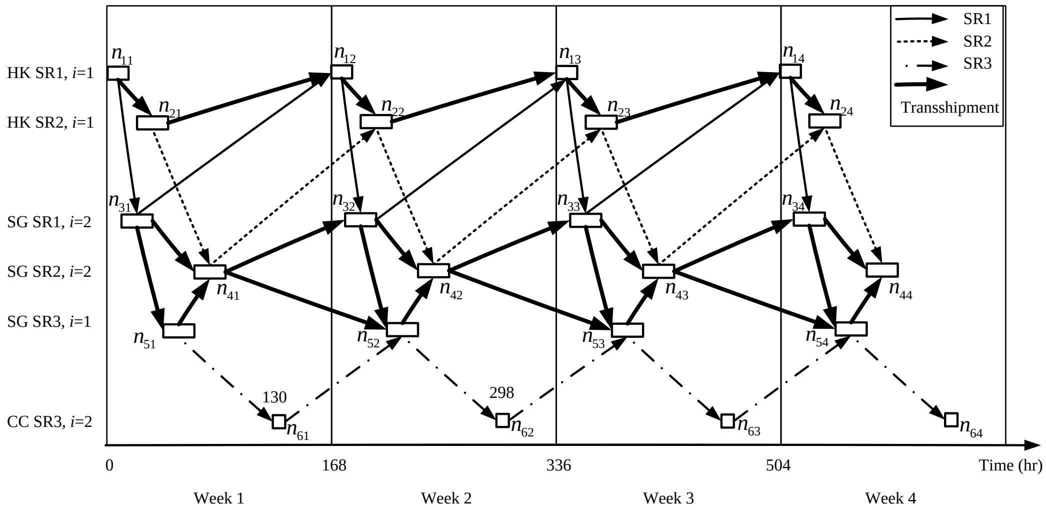

Fig. 6 Space-time network representation of a simple liner network

Table 2 Container flow in different weeks

Container flow	Containers loaded in week 1	Containers loaded in week 2	Containers loaded in week 3
Arc (n_{51}, n_{61})	$f_{n_{51}n_{61}}^{n_{11}n_{61}}$	0	0
Arc (n_{52}, n_{62})	$f_{n_{52}n_{62}}^{n_{21}n_{62}}$	Equal to $f_{n_{51}n_{61}}^{n_{11}n_{61}}$	0
Arc (n_{53}, n_{63})	0	Equal to $f_{n_{52}n_{62}}^{n_{21}n_{62}}$	Equal to $f_{n_{51}n_{61}}^{n_{11}n_{61}}$
Arc (n_{54}, n_{64})	0	0	Equal to $f_{n_{52}n_{62}}^{n_{21}n_{62}}$

We look at the capacity constraint on the first leg SG to CC of SR3. Since containers from the new OD pair (n_{11}, n_{61}) do not flow on arc (n_{52}, n_{62}) and containers from the new OD pair (n_{21}, n_{62}) do not flow on arc (n_{51}, n_{61}), we just consider decision variables $f_{n_{51}n_{61}}^{n_{11}n_{61}}$ and $f_{n_{52}n_{62}}^{n_{21}n_{62}}$. In this simple example, $f_{n_{51}n_{61}}^{n_{11}n_{61}}$ TEUs are loaded at n_{11} and transported via n_{31} and n_{51} to n_{61}; $f_{n_{52}n_{62}}^{n_{21}n_{62}}$ TEUs are loaded at n_{21} and transported via n_{41} and n_{52} to n_{62}. Note that $f_{n_{51}n_{61}}^{n_{11}n_{61}}$ and $f_{n_{52}n_{62}}^{n_{12}n_{62}}$ represent the flow of containers that leave their origin ports in the first week, and we consider a tactical problem that covers many weeks with the same demand functions. Hence, in week 2, $f_{n_{51}n_{61}}^{n_{11}n_{61}}$ TEUs are loaded at n_{12} and transported via n_{32} and n_{52} to n_{62}; $f_{n_{52}n_{62}}^{n_{21}n_{62}}$ TEUs are loaded at n_{22} and transported via n_{42} and n_{53} to n_{63}; in week 3, $f_{n_{51}n_{61}}^{n_{11}n_{61}}$ TEUs are loaded at n_{13} and transported via n_{33} and n_{53} to n_{63}; $f_{n_{52}n_{62}}^{n_{21}n_{62}}$ TEUs are loaded at n_{23} and transported via n_{43} and n_{54} to n_{64}. Table 2 summarizes the flow of containers that are loaded in the three weeks. We can see that, for instance, arc (n_{52}, n_{62}) carries $f_{n_{52}n_{62}}^{n_{21}n_{62}}$ TEUs from week 1 and $f_{n_{51}n_{61}}^{n_{11}n_{61}}$ TEUs from week 2. The sum of $f_{n_{52}n_{62}}^{n_{12}n_{62}}$ and $f_{n_{52}n_{62}}^{n_{21}n_{62}}$ must be less than or equal to the ship capacity. In sum, the left-hand side of Eq. (17) is the total number of containers from all weeks carried on any arc that corresponds to leg i of ship route r. (Note that arc (n_{51}, n_{61}) in Table 2 carries few containers because of the start-up effect.) This is the rationale behind the summation over A^{ri} in Eq. (17).

3.2.2 Computational complexity

We now formally analyze the computational complexity of the OD-link-based linear programming (LP) model. In a liner shipping network, the number of ports is $|\mathcal{P}|$, the number of voyage legs, denoted by V, is equal to $\sum_{r\in\mathcal{R}} N_r$. Evidently, $|\mathcal{P}|\le V$ because each port is visited at least once a week. Therefore, we use V as the indicator of the problem size.

The number of OD pairs $|\mathcal{W}|$ is bounded by $|\mathcal{P}|^2$, or V^2. The number of nodes in the space-time network $|N|$ is bounded by $\hat{N}_T V$, or $O(V)$. The number of new OD pairs $(u,v)\in\bigcup_{(o,d)\in\mathcal{W}}\mathcal{W}^{od}$ is bounded by $|N|^2$, or $O(V^2)$. The number of voyage arcs $|A^v|$ in the space-time network is bounded by $\hat{N}_T V$, or $O(V)$. The number of transshipment arcs can be estimated as follows. Suppose that each port is visited the same number of times in one week, calculated by $V/|\mathcal{P}|$, the total number of transshipment arcs over all the $|\mathcal{P}|$ ports in the planning horizon of $\hat{N}_T$ weeks is $|\mathcal{P}|\times\hat{N}_T\times\left[V/|\mathcal{P}|\times(V/|\mathcal{P}|-1)\right]$. This number achieves the maximum when $|\mathcal{P}|=1$, and the maximum is bounded by $O(V^2)$. Hence, the total number of arcs $|A|$ in the space-time network (both voyage and transshipment arcs) is also bounded by $O(V^2)$.

Proposition 1: The OD-link-based linear programming model has at most $O(V^4)$ decision variables and $O(V^3)$ constraints.

Proof: The number of decision variables in OD-link-based model is bounded by the number of new OD pairs $(u,v)\in\bigcup_{(o,d)\in\mathcal{W}}\mathcal{W}^{od}$ multiplied by the number of arcs $(m,n)\in A$, i.e., $O(V^4)$. The number of constraints (excluding the lower and upper bounds on decision variables) is bounded by the number of new OD pairs $(u,v)\in\bigcup_{(o,d)\in\mathcal{W}}\mathcal{W}^{od}$ multiplied by the number of nodes $m\in N$, or $O(V^3)$. □

Since both the number of decision variables and the number of constraints are bounded by polynomial expressions of the size of the problem V, and an LP problem can be solved in polynomial time with regard to its input (interior point method or ellipsoid method), the OD-link-based LP model can be solved in polynomial time with regard to the size of the liner shipping network V.

Remark 1: As mentioned in section 1, twice weekly services and thrice weekly services can easily be transformed to weekly services by repeating the port rotation twice or three times, respectively. If there are other service frequencies, for example, if a particular ship route has a six-day service frequency, then we have to transform all the services to their common service frequency, and in this case it is $6\times 7=42$ days. When all the services have the same frequency, we can apply the OD-link-based LP model.

Remark 2: If the ships deployed on a ship route are not homogeneous in terms of capacity, for example, if the round-trip journey time is 2 weeks and 2 ships of different sizes are deployed, then we can consider this service as two new services: one ship is deployed on each service, and both services have a 14-day service frequency. Now all the services are deployed with homogeneous ships and we can use the OD-link-based model.

Remark 3: The problem of finding a resource-constrained shortest path is generally NP-hard. However, finding a resource-constrained shortest path is polynomially solvable over the space-time network proposed in the study for LCA due to the weekly frequency property of liner ship routes.

3.3 Origin-link-based linear programming formulation

We can also adopt an origin-link-based (or destination-link-based) LP formulation in the space-time network. In an origin-link-based model, we need to formulate the container flow from any node u in the first week of the space-time network, i.e., $u \in N^1 := \{u \in N \mid 0 \le t_u^{\text{arr}} < 168\}$ to any node $v \in N$ in the space-time network. Represent by y^{uv} the volume of containers (TEUs/week) from node u to node v that are delivered by the liner shipping company, $(u,v) \in N^1 \times N$. We have $y^{uv} = 0$ if $(u,v) \notin \bigcup_{(o,d)\in \mathcal{W}} \mathcal{W}^{od}$. Let f_{mn}^{u} be the decision variable representing the total volume of containers (TEUs/week) with origin node u in terms of new OD pairs and any destination that flow on the arc $(m,n) \in A$. The origin-link-based LP formulation is:

[Origin Model]
$$\max_{f_{mn}^{u}, y^{uv}} \sum_{(o,d)\in \mathcal{W}} \sum_{(u,v)\in \mathcal{W}^{od}} g_{od} y^{uv} - \sum_{(m,n)\in A^t} c_{mn} \sum_{u \in N^1} f_{mn}^{u} \tag{21}$$

subject to:

$$\sum_{n,(u,n)\in A} f_{un}^{u} - \sum_{n,(n,u)\in A} f_{nu}^{u} = \sum_{v\in N} y^{uv}, \forall u \in N^1 \tag{22}$$

$$\sum_{n,(m,n)\in A} f_{mn}^{u} - \sum_{n,(n,m)\in A} f_{nm}^{u} = -y^{um}, \forall m \in N, \forall u \in N^1, m \ne u \tag{23}$$

$$\sum_{(m,n)\in A^{ri}} \sum_{u\in N^1} f_{mn}^{u} \le \text{Cap}_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{24}$$

$$\sum_{i=j}^{|\mathcal{W}^{od}|} y^{u_i v_i} \le D^{od}(\tau_{u_j v_j}), j = 1, 2, \cdots, |\mathcal{W}^{od}|, \forall (o,d) \in \mathcal{W} \tag{25}$$

$$y^{uv} \ge 0, \forall (o,d) \in \mathcal{W}, \forall (u,v) \in \mathcal{W}^{od} \tag{26}$$

$$y^{uv} = 0, \forall (u,v) \in N^1 \times N \text{ and } (u,v) \notin \bigcup_{(o,d)\in \mathcal{W}} \mathcal{W}^{od} \tag{27}$$

$$f_{mn}^{u} \ge 0, \forall (m,n) \in A, \forall u \in N^1 \tag{28}$$

Similar to Proposition 1, we have:

Proposition 2: The origin-link-based linear programming model has $O(V^3)$ decision variables and $O(V^2)$ constraints, and can be solved in polynomial time with regard to the size of the liner shipping network V .□

It should be noted that as a consequence of the construction of $\mathcal{W}^{od}$, the number of origins is much smaller than the number of destinations. Therefore, the origin-link-based formulation is preferable to destination-link-based formulation. There is no fundamental difference between the OD-link-based formulation, the origin-link-based formulation, and the destination-link-based formulation. Any one of them is valid and can be programmed to solve realistic problems.

The [OD Model] and [Origin Model] provide decision support tools for tactical-level plans. The space-time network idea can also be applied to assist operational-level plans. In particular, every week the shipping line predicts the demand functions for the next few weeks and makes the container acceptance/rejection and routing decisions for the current week. The transit time constraints in the operational-level model can also be easily captured by the space-time network.

4 Extensions and Applications

The link-based LP formulations are flexible enough to handle many practical considerations and provide useful managerial insights. In this section, we analyze some extensions and implications of the link-based LP formulations using the example of the OD-link-based model as it is more compact to present than the origin-link-based formulation. Numerical examples in practice will be reported in the next section.

4.1 Interactions with port operators, shippers and other liner shipping companies

Port operators are facing increased competition to attract shipping companies. For example, Hong Kong and Yantian are competing for the export containers from China, Singapore and Tanjung Pelepas are competing for transshipment containers in Southeast Asia. To increase the throughput, port operators usually sign confidential contracts with large liner shipping companies, offering a competitive handling price if a liner shipping company could commit a certain number of container handling operations.

For instance, suppose that the normal loading, discharge, and transshipment prices at port $p \in \mathcal{P}$ are $\hat{c}_p$, $\tilde{c}_p$, and $\bar{c}_p$, respectively. If the liner shipping company commits to at least β_p loading or discharge or transshipment operations per week, then it enjoys preferable prices of $\hat{c}'_p$, $\tilde{c}'_p$, and $\bar{c}'_p$, respectively. $\hat{c}'_p \le \hat{c}_p$, $\tilde{c}'_p \le \tilde{c}_p$, $\bar{c}'_p \le \bar{c}_p$, and at least one of $\hat{c}'_p < \hat{c}_p$, $\tilde{c}'_p < \tilde{c}_p$, $\bar{c}'_p < \bar{c}_p$ must hold. The liner shipping company needs to determine whether it should make such a commitment. To this end, we only need to solve the proposed LP model twice: one using the normal price (the values of $\hat{c}_p$ and $\tilde{c}_p$ are incorporated in g_{od}) and the other one using the preferable price while adding the constraint that the total handling operations at port $p \in \mathcal{P}$ is no less than β_p:

$$\sum_{(p,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{pd}} y^{uv} + \sum_{(o,p)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{op}} y^{uv} + \sum_{(m,n)\in A^t, p_m = p} \sum_{(o,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{od}} f_{mn}^{uv} \geq \beta_p \tag{29}$$

The first term in Eq. (29) is the loading operations at port p, the second term is the discharge operations, and the third term is the transshipment operations.

In a liner shipping network with many OD pairs and many paths to ship containers from origin to destination, it may not be an easy task to identify which potential container shipment demand is the most profitable. In the OD-link-based model, the dual variable λ_j^{od} associated with Eq. (18) reflects the change of profit with respect to a unit change of demand $D^{od}(x)$ at the transit time $x = t_{u_j v_j}$. λ_j^{od} is useful for a liner shipping company to evaluate the profitability of a potential shipping order. For constant demand with a threshold transit time in Fig. 1a, the dual variable λ^{od} associated with Eq. (14) reflects the change of profit with respect to a unit change of demand $D^{od}(0)$. Having obtained the dual variables, the liner shipping company can easily identify which OD pairs need more efforts in sales and hence the most capable sales team can be sent to attract more demand in those OD pairs.

Some large shippers may sign a confidential contract with the global liner shipping company, committing a certain container shipment volume and enjoying a preferable freight rate. The global liner shipping company may need to evaluate whether such a business is profitable, which again is no easy task without sophisticated decision support tools. To address this problem, we only need to solve an additional LP model. For simplicity, suppose that the demands for all OD pairs are constant with a threshold transit time as in Fig. 1a. A shipper has $\bar{D}^{od}(0)$ TEUs for $(o,d) \in \mathcal{W}$, and requires a freight rate of $\bar{g}'_{od}$ (USD/TEU) (note that the shipper may have containers of many OD pairs). Define $\bar{g}_{od} := \bar{g}'_{od} - \hat{c}_o - \tilde{c}_d$. We need to solve:

$$\max_{f_{mn}^{uv}, y^{uv}} \sum_{(o,d)\in\mathcal{W}} \left\{ \bar{g}_{od} \bar{D}^{od}(0) + g_{od} \left[\sum_{(u,v)\in\mathcal{W}^{od}} y^{uv} - \bar{D}^{od}(0) \right] \right\} - \sum_{(m,n)\in A^t} c_{mn} \sum_{(o,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{od}} f_{mn}^{uv} \tag{30}$$

subject to

$$\bar{D}^{od}(0) \leq \sum_{(u,v)\in\mathcal{W}^{od}} y^{uv} \leq D^{od}(0) + \bar{D}^{od}(0), \forall (o,d) \in \mathcal{W} \tag{31}$$

and constraints (16)-(17), (19)-(20). If the profit is larger than not accepting this order, the liner shipping company should sign the contract (here we are focusing solely on the profitability; it is possible that the liner shipping company accepts the business even if it loses money in order to increase its market share).

It is often the case that a ship route is operated by more than one liner shipping company in an alliance (see, Agarwal and Ergun, 2010). For example, if Orient Overseas Container Line (OOCL) and Nippon Yusen Kaisha (NYK) Line jointly operate a ship route, on which six 6000-TEU ships are deployed, and two ships belong to OOCL and four ships belong to

NYK Line, then in practice OOCL controls 2000-TEU ship capacity on each of the six ships and NYK Line manages 4000-TEU capacity. It is also the case that a global liner shipping company purchases ship slots from local shipping companies that provide feeder shipping services between small ports and large/hub ports. For example, if the total export and import volume at the port of Palembang, a small port in Indonesia near Singapore, is 20 TEUs/week, it does not make sense for the global liner shipping company to operate a ship route that visits Palembang. To earn profit, it will buy the slots of ships that provide services between Palembang and Singapore. The dual variable θ_{ri} associated with Eq. (17) reflects the change of profit with respect to a unit change of ship capacity on leg i of ship route r. If a ship route is solely operated by the liner shipping company, it is impossible to change the capacity unless the company redeploys the ships. However, if the ship route is jointly operated with other companies, or is solely operated by another liner shipping company, it is possible that the global liner shipping company buys more ship slots to transport its own containers. In the above example, OOCL may purchase additional ship capacity from NYK on one leg, a few legs, or even all legs of the ship route. θ_{ri} would provide useful information for negotiating the slot-purchasing price.

4.2 Strategic decisions about new shipping market, ship fleet planning, and hub location

The OD-link-based model can also assist for making decisions about whether a new container shipping market should be explored. For example, a liner shipping company does not serve the East Coast of Africa (East Africa) may consider whether it should deploy a feeder service connecting East Africa to a regional hub in the Middle East, e.g., Salalah or Sokhna. The company can then predict the OD demand for East Africa, and then compute the total profit by choosing one candidate regional hub each time. If the highest profit among all the candidate regional hubs (the cost of providing the feeder service should be subtracted) is larger than the current situation, the East Africa feeder service should be deployed.

The model is also helpful for ship fleet planning. For example, Maersk Line ordered a total of 10 mega-containerships with a capacity of 18,000 TEUs that will be deployed on the Asia-Europe trade lane. Other liner shipping companies are thus interested in knowing whether they should also book mega-containerships. To answer this question, a liner shipping company first plans how to deploy the mega-containerships in the network if they are booked and delivered, and then predicts the new container shipment demand in the network with the mega-containerships. The OD-link-based model can be solved for the new network and new demand, and the change of profit can be used for the ship fleet planning decision.

The model is also capable of evaluating the choice of hubs. For example, a company might be interested in whether it should shift its Southeast Asian hub from Singapore to Tanjung Pelepas, which has a lower handling cost and a smaller handling capacity. To apply the OD-link-based model, suppose that quay cranes at Tanjung Pelepas, denoted by port p, can only load or discharge θ_p TEUs per week. In the OD-link-based model for evaluating the profit of using Tanjung Pelepas as the Southeast Asian hub, we need to modify g_{od} in Eq. (2) using the handling costs at Tanjung Pelepas and include the following constraint:

$$\sum_{(p,d)\in \mathcal{W}} \sum_{(u,v)\in \mathcal{W}^{pd}} y^{uv} + \sum_{(o,p)\in \mathcal{W}} \sum_{(u,v)\in \mathcal{W}^{op}} y^{uv} + 2 \sum_{(m,n)\in A^t, p_m = p} \sum_{(o,d)\in \mathcal{W}} \sum_{(u,v)\in \mathcal{W}^{od}} f_{mn}^{uv} \le \theta_p \tag{32}$$

The number 2 in the constraint means that a transshipment operation consists of two quay crane moves (discharging a container and reloading it).

4.3 Liner shipping network improvement

The OD-link-based model could provide all the necessary information on how the containers are transported in the network. This information is helpful for the liner shipping company to improve its liner shipping network. We can also obtain the unshipped demand from the model. If the unshipped demand is significant, the liner shipping company may launch new services, or increase the freight rate for some OD pairs to increase its profit.

The capacity utilization on a leg of a ship route, defined as the ratio of the left-hand side over the right-hand side of Eq. (17), not only provides information on the quality of the liner shipping services, but also information for ship fleet deployment. For example, if 5000-TEU ships are deployed on SR1 in Fig. 2, but capacity utilizations on all the three voyage legs are lower than 80%, then deploying 4000-TEU ships on the ship route might be preferable.

The OD-link-based model also yields the container handling volume when a ship calls at a port. This information is valuable for designing ship routes and deploying ships on ship routes. For instance, if only 10 TEUs are handled at Hong Kong on SR1 in Fig. 2, the liner shipping company may need to remove Hong Kong from the ship route.

In addition, the model is capable of answering “what-if” questions from the managers of the liner shipping company, such as what if a particular service is out of operation, what if the port rotation direction of a service between Singapore and Australia is changed from clockwise to counter-clockwise, and what if another type of ships is deployed on a ship route.

4.4 Empty container repositioning and equipment management

Liner shipping companies not only transport laden containers, but also need to reposition empty containers due to the imbalance of world trade (Song and Dong, 2012, 2013). The empty container repositioning issue can easily be incorporated in the OD-link-based model. Empty containers are not as sensitive to transit time as laden containers. To simplify the notation, we assume that the threshold transit time for empty containers is equal to the longest threshold transit time for laden containers of all OD pairs so that the number of weeks in the space-time network does not need to change. Let γ_p be the number of surplus empty containers at port $p \in \mathcal{P}$ if $\gamma_p > 0$, and $-\gamma_p$ represents the number of deficit empty containers if $\gamma_p < 0$. Note that γ_p depends on how many net laden containers are shipped into port p. The additional decision variables are as follows: f_{mn}^{emp} represents the volume of empty containers (TEUs/week) that flow on the arc $(m,n) \in A$, $\hat{z}_m^{\text{emp}}$ represents the volume of empty containers (TEUs/week) that are loaded at node $m \in N$ (not including transshipment empty containers), and $\tilde{z}_m^{\text{emp}}$ represents the volume of empty containers (TEUs/week) that are discharged at node $m \in N$ (not including transshipment empty containers). Let the loading, discharge, and transshipment prices for empty containers at port $p \in \mathcal{P}$ be $\hat{c}_p^{\text{emp}}$, $\tilde{c}_p^{\text{emp}}$, and $\bar{c}_p^{\text{emp}}$, respectively. The additional cost for transporting one more empty container is also assumed to be 0. Now a transshipment arc $(m,n) \in A^t$ is associated with two costs: c_{mn} represents the transshipment cost for laden containers and c_{mn}^{emp} for empty containers. The OD-link-based model with empty container repositioning is:

$$\begin{aligned} \max_{f_{mn}^{uv}, y^{uv}, \hat{z}_m^{\text{emp}}, \tilde{z}_m^{\text{emp}}, f_{mn}^{\text{emp}}, \gamma_p} & \sum_{(o,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{od}} g_{od} y^{uv} - \sum_{(m,n)\in A^t} c_{mn} \sum_{(o,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{od}} f_{mn}^{uv} \\ & - \sum_{p\in\mathcal{P}} \sum_{m\in N, p_m = p} (\hat{c}_p^{\text{emp}} \hat{z}_m^{\text{emp}} + \tilde{c}_p^{\text{emp}} \tilde{z}_m^{\text{emp}}) - \sum_{(m,n)\in A^t} c_{mn}^{\text{emp}} f_{mn}^{\text{emp}} \end{aligned} \tag{33}$$

subject to

$$\gamma_p = \sum_{(o,p)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{op}} y^{uv} - \sum_{(p,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{pd}} y^{uv}, \forall p \in \mathcal{P} \tag{34}$$

$$\sum_{n,(m,n)\in A} f_{mn}^{\text{emp}} - \sum_{n,(n,m)\in A} f_{nm}^{\text{emp}} = \hat{z}_m^{\text{emp}} - \tilde{z}_m^{\text{emp}}, \forall m \in N \tag{35}$$

$$\sum_{(m,n)\in A^{ri}} \left[f_{mn}^{\text{emp}} + \sum_{(o,d)\in\mathcal{W}} \sum_{(u,v)\in\mathcal{W}^{od}} f_{mn}^{uv} \right] \le \text{Cap}_r, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{36}$$

$$\sum_{m\in N, p_m = p} (\hat{z}_m^{\text{emp}} - \tilde{z}_m^{\text{emp}}) = \gamma_p, \forall p \in \mathcal{P} \tag{37}$$

$$\hat{z}_m^{\text{emp}} = 0, \forall m \in N \setminus N^1 \tag{38}$$

$$\hat{z}_m^{\text{emp}} \ge 0, \tilde{z}_m^{\text{emp}} \ge 0, \forall m \in N \tag{39}$$

$$f_{mn}^{\text{emp}} \ge 0, \forall (m,n) \in A \tag{40}$$

and constraints (16), (18)-(20). The third term in the objective function (33) is the loading and discharge cost for empty containers, and the fourth term is the transshipment cost for empty containers. Eq. (34) defines the empty container repositioning requirement. Eq. (35) enforces empty container flow conservation. Eq. (36) is ship capacity constraints considering both empty and laden containers. Eq. (37) imposes that all empty container repositioning requirement must be satisfied. Eq. (38) defines that the empty containers are all loaded in the first week, which has no loss of generality because liner shipping services in each week are identical. Eqs. (39) and (40) are nonnegativity constraints. It should be mentioned that the above model is still polynomially solvable.

It happens in practice that the liner shipping company has many cargos to ship, but it does not have enough containers for these cargos. For clarity, we use CT (Container equipmenT) to refer to a container as equipment. If laden containers spend too long time on the trip from their origin to their destination, or if empty container repositioning is inefficient, the liner shipping company might be short of CTs. Assuming that the company has a total of N^{CT} CTs in its fleet, ensuring that the total number of used CTs does not exceed N^{CT} is a necessary constraint in the LCA model. Since this study is focused on maritime transportation of containers, we assume that after a laden container arrives at its destination port, on average it spends two weeks (inland transportation, packing, unpacking, etc.) before it is reloaded to ships at the port. To ensure that the used CTs does not exceed N^{CT}, we have

$$\sum_{(m,n)\in A}\left[f_{mn}^{\mathrm{emp}}+\sum_{(o,d)\in\mathcal{W}}\sum_{(u,v)\in\mathcal{W}^{od}}f_{mn}^{uv}\right]t_{mn}/168+2\sum_{(o,d)\in\mathcal{W}}\sum_{(u,v)\in\mathcal{W}^{od}}y^{uv}\le N^{\mathrm{CT}} \tag{41}$$

The dual variable π associated with Eq. (41) is the additional profit if the liner shipping company has one more CT. Comparing π with the market price for chartering a CT, the liner shipping company can decide whether it should charter more CTs from other companies, or rent CTs to other companies. Combined with other relevant information, the liner shipping company can determine whether it should purchase more CTs.

5 Numerical Examples

5.1 Problem settings

We apply the origin-link-based formulation to assign the container flow of the intra-Asia service network of a liner shipping company. This network has a total of 65 ports. The company provides 33 ship routes, with a total of 147 voyage legs, as shown in Table 3. The names of the ports shown in Table 3 use a three-letter code, however, majors ports can easily be identified, for example, MNL, SIN, and HKG represent Manila, Singapore, and Hong Kong, respectively. The transshipment cost $\hat{c}_p$ at each port is provided by the company. There are 963 OD pairs in the network. The container shipment demand in each OD pair is in the category of Fig. 1a, and the value of the demand and the threshold transit time are provided by the company. We choose the functional form of Fig. 1a rather than Fig. 1c because our discussions with liner shipping companies show that it is very hard at present for them to provide the function in Fig. 1c. They could estimate the function in Fig. 1a as only two parameters are required. Evidently, using the functional form of Fig. 1c will slightly increase the number of constraints compared with Fig. 1a; using the functional form of Fig. 1b will overestimate the profit relative to Fig. 1a. The total demand over these 963 OD pairs is 43,095 TEUs/week. As the freight rate is confidential, we assume that g_{od} equals 500+0.2×distance (n mile) from the origin port to the destination. The longest threshold transit time is 7 weeks. Therefore, the time horizon $\hat{N}_T$ of the space-time network is 8 weeks. As a consequence, the number of nodes in the space time network is 8×147=1,176. Because the space-time network only contains arcs with both head and tail nodes in it, the number of voyage arcs is actually 1,111, which is smaller than 8×147=1,176. The number of transshipment arcs in the network is 5,950. This number is much larger than the number of voyage arcs, because Singapore is visited 23 times in a week and hence the number of transshipment arcs associated with Singapore is already more than 3000.

Table 3 Ship routes in the intra-Asia network

ID	Sequence of ports of call													
1	SUB	MNL	NMP	MNL	KAO									
2	CIW	KAO	MNL	CEB	MLL	KAO	HKG							
3	DAD	DVO	SIN	DVO	HKG									
4	S4U	OOU	HH9	PUS	UBJ	JKO	PUS							
5	VLA	PUS												
6	VOS	PUS												
7	CMB	CHT												
8	CHT	SIN												
9	CHT	SIN												
10	MAA	C2M												
11	PKL	MAA	VIS	PKL	SIN									
12	HLD	CCU	HLD	SIN										
13	BLA	SIN												
14	HKG	CIW	HPZ											
15	JKT	SIN												
16	PLM	SIN												
17	PND	SIN												
18	KOE	SIN												
19	SEM	SIN												
20	SGN	AT0	SIN											
21	SUR	SIN												
22	AT0	SGN	SIN											
23	LCB	SIN												
24	UHQ	BAH	DOH	JEB										
25	AJM	JEB	DOH	JEB										
26	SCZ	SHG	BAH	JEB										
27	SHR	KHI	FUJ	JEB										
28	SGN	KAO	TYO	YOK	NGO	KOB	CIW	HKG	LCB					
29	KWY	PUS	KAO	HKG	NSS	CIW	PKL	SIN	JKT	SIN	KAO	TPS	LYG	
30	YOK	SHM	NGO	PUS	LCB	MNL	TYO							
31	KWY	TSI	NGB	SIN	JEB	DAM	BAH	SIN	NGB	SHA	PUS			
32	SIN	NSH	KHI	NSH	CMB	SIN	LCB							
33	DAI	TSI	PUS	KWY	HKG	CIW	SIN	CMB	NSH	IPX	PKL	SIN	HKG	HSN

The 963 OD pairs correspond to 31,593 new OD pairs (u, v). Each OD pair has at least one new OD pair and at most 1,155 new OD pairs, as shown in Fig. 7 . The OD pair (Hong Kong, Singapore) has 1,155 new OD pairs, because both ports are major transshipment hubs and are visited a large number of times every week. Fig. 7 shows that most OD pairs have fewer than 50 new OD pairs. In fact, 831 OD pairs have fewer than 50 new OD pairs, as shown in Fig. 8 .

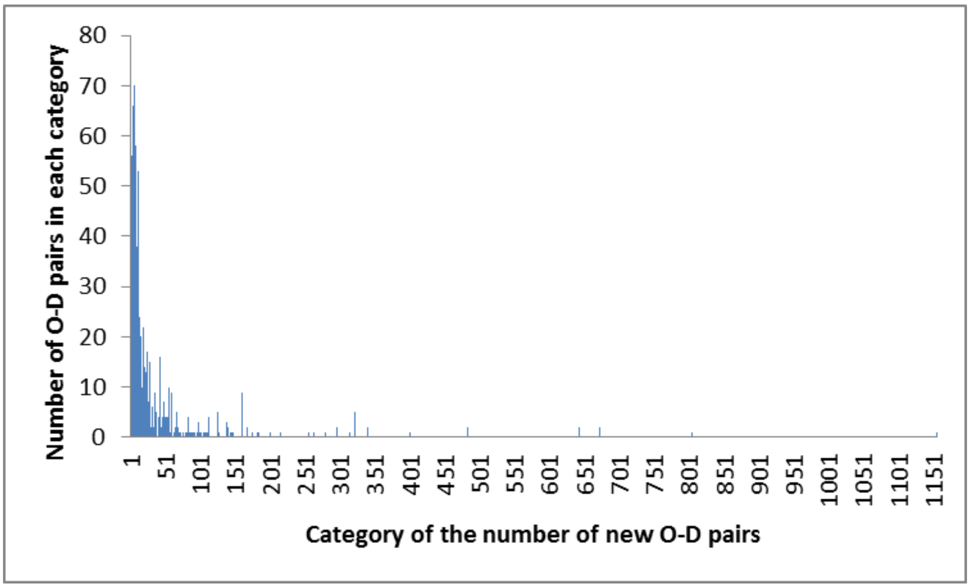

Fig. 7 Number of OD pairs according to the number of associated new OD pairs

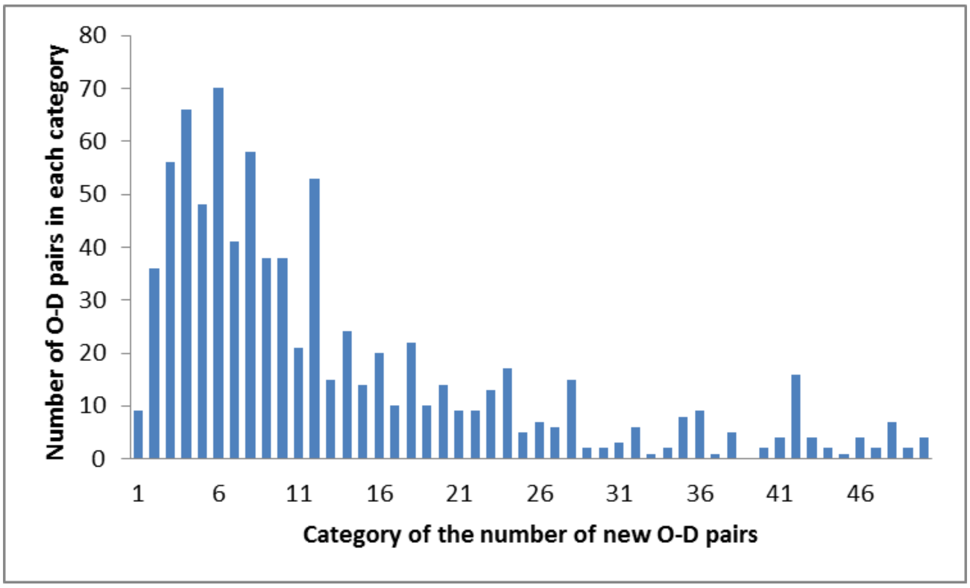

Fig. 8 Number of OD pairs according to the number of associated new OD pairs that are not greater than 50

The number of decision variables f_{mn}^{u} is 147×(1,111+5,950)= 1,037,967, the number of y^{uv} is 31,593 (excluding those y^{uv} that are set at 0), and therefore the total number of decision variables is 1,037,967+31,593=1,069,560. The constraints (25) should be replaced by (14). The total number of constraints (22)-(23) is 147×1,176=172,872, the number of constraints (24) is 147, and the number of constraints (14) is 963. Hence, the total number of constraints (excluding nonnegativity constraints) is 173,982. We solve the origin-link-based linear programming model with CPLEX 12.1 of default settings, and obtain the optimal solution in 57 seconds. A total of 31,917 TEUs are shipped, and the profit is 3.45117×10^7 USD.

5.2 Negotiation with port operators

The model provides the transshipment volume at each port. Major transshipment ports and their transshipment throughputs are shown in Fig. 9 . The liner shipping company chooses the port of Singapore as a very important hub because of its special geographical location, its capacity and efficiency, and other business considerations. Therefore, many of the intra-Asia ship routes visit Singapore. In fact, the port of Singapore is visited 23 times a week. The transshipment throughput at Pusan is 886 TEUs/week (in the case study, the transshipment price at Pusan is 97.5 USD/TEU). Now suppose that the port of Pusan provides a few options: if the liner shipping company transships more than 900 TEUs per week, the price is lowered down to 95 USD/TEU; if the volume is 1000, then the price is 90; if the volume is 2000, the price is 70. For each of these options, the liner shipping company recalculates its total profit, and the result is shown in Table 4. Therefore, the company would choose the option of 90 USD/TEU with the transshipment volume commitment of at least 1000 TEUs.

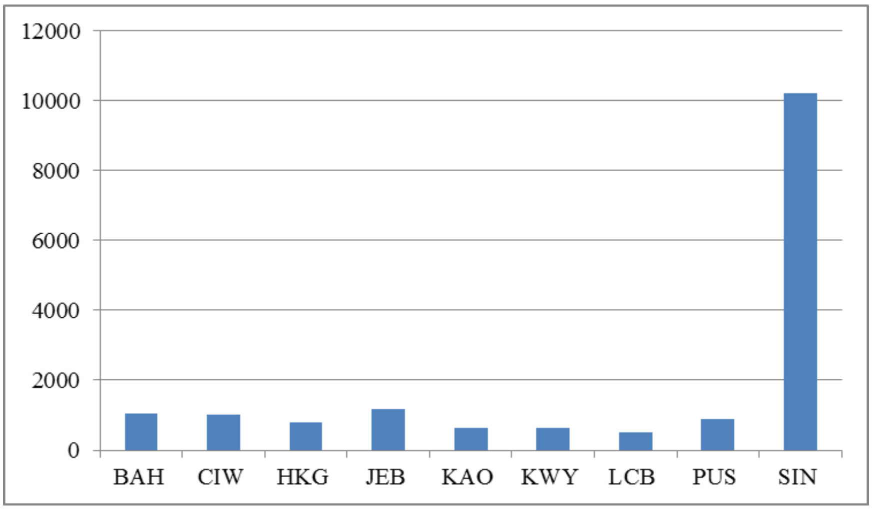

Fig. 9 Major transshipment ports and transshipment throughputs (TEUs/week)

Table 4 Scenarios for negotiating with port of Pusan regarding transshipment volume and price

Minimum commitment	Price	Total profit (1×10^7 USD/week)	Transshipped volume
0	97.5	3.45117	886
900	95	3.45134	900
1000	90	3.45151	1000
2000	70	3.45076	2000

5.3 Negotiation with shippers

The dual variable λ^{od} associated with Eq. (14) is 2627.61 for the OD pair (SUB, IPX) (Subic Bay to Pipavav), which is the largest among all the OD pairs. This means that if there is one more TEU for this OD pair to ship, the company would make an additional profit of 2,627.61 USD. The current demand is 10 TEUs for this OD pair, and the total profit is 3.45117×10^7 USD. If the demand is increased to 11 TEUs, the total profit is 3.45143×10^7 USD. If the demand is 110 TEUs, the total profit is 3.47744×10^7 USD, as shown in Table 5. Therefore, the sales team of the liner shipping company should try their best to obtain shipping demand for this OD pair so that the profitability of the company could be enhanced. It should be mentioned that when the demand for this OD pair exceeds a certain limit, a unit increase in demand will lead to an increase in profit less than 2,627.61 USD.

Table 5 Total profit for different demand scenarios of the OD pair (SUB, IPX)

Weekly demand (TEUs)	Total profit (1×10^7 USD/week)
10	3.45117
11	3.45143
110	3.47744

5.4 Negotiation with other liner shipping companies

Ship route No. 15 that visits Jakarta and Singapore is actually a feeder ship route that is not directly operated by the liner shipping company. A local (Indonesian) shipping company operates ships of 500 TEUs to provide weekly services between Jakarta and Singapore, and the global liner shipping company purchases 120 TEUs ship slots to transport its containers. The dual variable associated with the capacity of leg JKT-SIN is 729.35, which means that if the global liner shipping company has one more TEU slot on the leg JKT-SIN, it will make 729.35 USD more profit. We computed the result when the slot capacity on the leg JKT-SIN is increased to 170 TEUs, and the profit is increased by 50×729.35, which means that the dual variable value does not change within the range of 0 to 50 additional containers. As a result, the global liner shipping company needs to buy more ship slots as long as the price is not higher than 729.35 USD/TEU. Of course, if the global company finds that a large number of slots need to be purchased, for example, 1500 TEUs, then it should consider operating its own ship route.

5.5 Liner shipping network improvement

Some of the unfulfilled demand is shown in Table 6. Kaohsiung, Laem Chabang, Jakarta, Port Klang, and Singapore are all located in Southeast Asia. Therefore, it might be worthwhile to design a new ship route KAO-LCB-PKL-SIN-JKT deployed with ships of a capacity of 1000 TEUs to fulfill the demand. Karachi is far away from Kaohsiung, and fulfilling the unshipped demand from Kaohsiung to Karachi may not be economically justifiable. Of course, there are many options to design new ship routes.

Table 6 Some unfulfilled demand

OD pair	Unfulfilled demand (TEUs/week)
KAO-JKT (Kaohsiung-Jakarta)	121
KAO-KHI (Kaohsiung-Karachi)	109
KAO-LCB (Kaohsiung-Laem Chabang)	305
KAO-PKL (Kaohsiung-Port Klang)	231
KAO-SIN (Kaohsiung-Singapore)	273

Ship route No. 17 that visits Panjang and Singapore is a feeder ship route deployed with ships of 1000 TEUs operated by the liner shipping company. The container flow on the leg PND-SIN is 174 TEUs, and the flow in the other direction is 150 TEUs. Evidently, ship capacity utilization is too low on this ship route. The liner shipping company may need to deploy ships of 200 TEUs, or cancel this ship route and buy ship slots from other shipping companies providing services between Panjang and Singapore.

Ship route No. 3 (Dadiangas-Davao-Singapore-Davao-Hong Kong) is deployed with ships of 1500 TEUs operated by the liner shipping company. Only six TEUs are loaded or discharged at the second port of call. Therefore, it may not be necessary to visit Davao twice in a round trip, and this ship route should be altered.

5.6 Empty container repositioning and equipment management

We incorporate empty container repositioning in the model to see its impact on the fulfilled demand and total profit. We assume that for empty containers, the loading and discharge cost is 60 USD/TEU, and the transshipment cost is 100 USD/TEU at all ports. The result is shown in the second row of Table 7. The fulfilled demand is less than that when empty container repositioning is not considered, and the total profit is reduced. The total profit is lower because the revenue is decreased as fewer laden containers are shipped, and the container handling cost is increased because of the loading, discharge, and transshipment operations of empty containers.

We further conduct experiments to see what if there are not sufficient CTs for use. In the case with empty container repositioning, 99,086 CTs are required. If there are only 90,000 CTs, the total profit is reduced by about 1×10^5 USD/week, as shown in Table 7. Therefore, if the price of a CT is 2,000 USD and the liner shipping company decides to purchase 10,000 more CTs, it takes about four years to earn enough profit to cover the purchasing cost of the 10,000 CTs.

Table 7 Impact of empty container repositioning and the number of available CTs

Scenarios	Number of available CTs (TEUs)	Fulfilled demand (TEUs/week)	Total profit (1×10^7 USD/week)
Do not consider repositioning	infinite	31,917	3.45117
Consider repositioning	infinite	26,403	2.57174
Consider repositioning	90,000	26,199	2.56142

5.7 A case with a general TTS demand

We present a simple case with a general TTS demand as the one in Fig. 1c. We consider one origin and one destination. There are three ship routes that connect the two ports with transit times 5 days (τ_1^{od}), 6 days (τ_2^{od}), and 7 days (τ_3^{od}), and capacities all equal to 1,000. The profit for shipping one container is normalized to be 1. The TTS demand is assumed to be linear. Note that if the TTS demand is convex (concave), then the demand is very (not) sensitive to the transit time in that the demand quickly (slowly) decreases as the transit time increases. A liner demand means the sensitivity is neither very high nor very low. We assume a linear demand function also because in reality it is easier to estimate a linear function as we only need two parameters: $D^{od}(0)$ and $\hat{T}_{od}^{\max}$. We can calculate that

$$D^{od}(\tau_i^{od}) = \max\left\{0, \frac{D^{od}(0)\left(\hat{T}_{od}^{\max} - \tau_i^{od}\right)}{\hat{T}_{od}^{\max}}\right\}, i = 1,2,3 \tag{42}$$

We maximizes the total profit, which is equal to $y_1^{od} + y_2^{od} + y_3^{od}$, subject to the following constraints:

$$y_3^{od} \le D^{od}(\tau_3^{od}) \tag{43}$$

$$y_2^{od} + y_3^{od} \le D^{od}(\tau_2^{od}) \tag{44}$$

$$y_1^{od} + y_2^{od} + y_3^{od} \le D^{od}(\tau_1^{od}) \tag{45}$$

$$0 \le y_i^{od} \le 1000, i = 1,2,3 \tag{46}$$

It is not difficult to see that the optimal solution is $y_3^{od*} = \min\left\{D^{od}(\tau_3^{od}), 1000\right\}$, $y_2^{od*} = \min\left\{D^{od}(\tau_2^{od}) - y_3^{od*}, 1000\right\}$, $y_1^{od*} = \min\left\{D^{od}(\tau_1^{od}) - y_3^{od*} - y_2^{od*}, 1000\right\}$. We try different values of $\hat{T}_{od}^{\max}$ from 5 days to 10 days and $D^{od}(0)$ from 2,000 to 10,000, and plot the optimal profit in Fig. 10. As expected, the total profit increases with $\hat{T}_{od}^{\max}$ and $D^{od}(0)$. Due to limited capacities of the ships, at most 3,000 containers can be transported, and this is why we see the plateau at the top of the diagram. Moreover, because the number of container routes is finite, meaning that the possible transit times are discrete, we observe two “steps” in the figure which show that (i) the used container routes are fully loaded, (ii) there are unshipped containers, and (iii) the unused container routes’ transit times are too long to transport the containers.

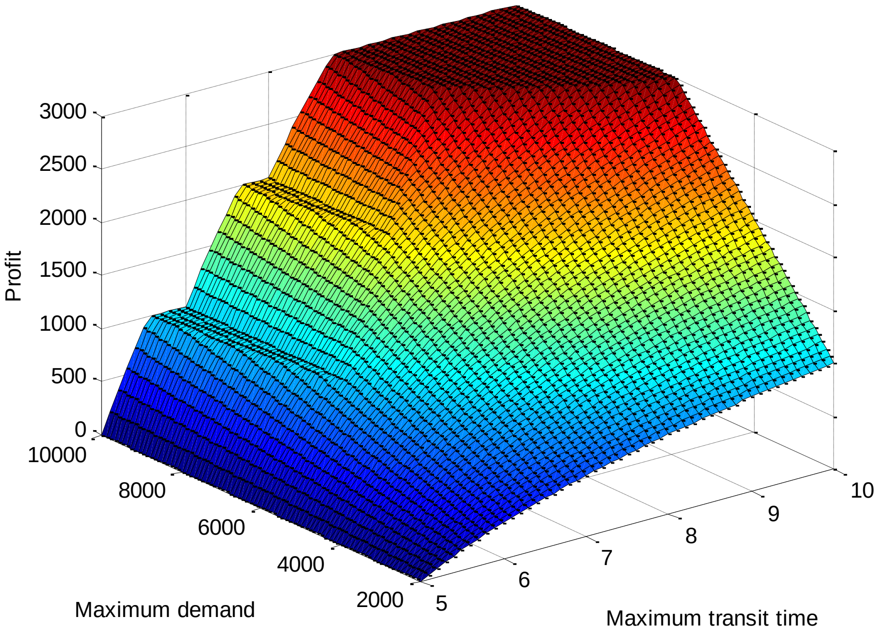

Fig. 10 Sensitivity of the profit with maximum demand and maximum transit time

6 Conclusions

This paper has investigated a practical liner container assignment model with transit-time-sensitive demand. In view of the weekly service property of liner shipping, we construct a special space-time network that implicitly incorporates the OD transit time. Based on this space-time network, two novel link-based linear programming formulations are developed to maximize the total profit. The linear programming formulations are proved to be solvable in polynomial time of the size of the liner shipping network. Practical considerations, such as empty container repositioning and limited container fleet size, can easily be incorporated in the model. Insights into the interpretation of dual variables associated with the constraints in the model are analyzed. How to use the LCA solutions to negotiate with port operators and designing liner services are discussed.

There are two future research directions worth exploring. The first should consider the dynamic and stochastic nature of container shipment demand. In dynamic LCA, the container shipment demand may vary from one week to the next; in stochastic LCA, only probabilistic information on the container shipment demand is known. Both models are more difficult than their deterministic counterpart. The second direction is to design liner services and assign containers with TTS demand in a holistic approach. This problem is NP-hard since liner shipping network design with fixed demand is already NP-hard (Agarwal and Ergun, 2008; Brouer et al., 2014; Wang and Meng, 2014). One cannot expect to find the optimal solution to general large-scale problems. Nevertheless, the space-time network proposed in this study might provide useful information for designing heuristic algorithms.

Acknowledgements

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Appendix 1: Symbols

Sets

A^{ri}:	Set of arcs in the space-time network that correspond to leg i of ship route $r \in \mathcal{R}$;
A:	Set of arcs in the space-time network;
A^{v}:	Set of voyage arcs in the space-time network;
A^{t}:	Set of transshipment arcs in the space-time network;
$\mathcal{I}_r$:	Set of port calling sequences of ship route r;
G:	The space-time network;
N:	Set of nodes in the space-time network;
N^1:	Set of nodes in the first week of the space-time network;
$\mathcal{N}_o^{od}$:	Set of nodes in the space-time network that can serve as origin nodes for the OD pair (o,d), $\mathcal{N}_o^{od} \subset N^1$;
$\hat{\mathcal{N}}_m^{od}$:	Set of destination nodes for each origin node $m \in \mathcal{N}_o^{od}$;
$\mathcal{N}_d^{od}$:	Set of nodes in the space-time network that can serve as destination nodes for the OD pair (o,d);
$\mathcal{P}$:	Set of ports;
$\mathcal{R}$:	Set of ship routes;
$\mathcal{W}$:	Set of OD pairs;
$\mathcal{W}^{od}$:	Set of new OD pairs in the space-time network for each OD pair (o,d);

Parameters

c_{mn}:	Cost per unit flow on arc (m,n) (USD/TEU);
$\hat{c}_p$:	Loading cost (USD/TEU) at port $p \in \mathcal{P}$;
$\tilde{c}_p$:	Discharge cost (USD/TEU) at port $p \in \mathcal{P}$;
$\bar{c}_p$:	Transshipment cost (USD/TEU) at port $p \in \mathcal{P}$;

Cap_r :	Capacity (TEUs) of a ship deployed on ship route $r \in \mathcal{R}$;
$D^{od}(x)$:	TTS demand (TEUs/week) for OD pair (o,d);
g'_{od} :	Freight rate paid by shippers for delivering one TEU for OD pair (o,d) (USD/TEU);
g_{od} :	g'_{od} minus the load cost at the origin port and discharge cost at the destination port;
i_n :	The corresponding port of call of node $n \in N$;
n^{od} :	Number of distinct transit time threshold values for port pair (o,d);
N_r :	Number of ports of call on ship route r;
$\hat{N}_T$:	Time horizon (weeks) of the space-time network;
p_{ri} :	The i^{th} port of call of ship route r;
p_n :	The corresponding physical port of node $n \in N$;
r_n :	The corresponding ship route of node $n \in N$;
t_{mn} :	Time (h) associated with arc (m,n);
t_{ri}^{arr} :	Arrival time (h) at the i^{th} port of call of ship route r;
τ_{uv} :	Transit time (h) of new OD pair (u,v);
$\hat{T}_{od}^{\max}$:	Threshold transit time (h) for OD pair (o,d);
V :	Total number of voyage legs in the liner shipping network;

Decision variables

f_{mn}^{u} :	Total volume of containers (TEUs) with origin node u in terms of new OD pairs that flow on the arc (m,n) in the space-time network;
f_{mn}^{uv} :	Volume of containers (TEUs/week) in the new OD pair $(u,v) \in \mathcal{W}^{od}$ that flow on arc (m,n) in the space-time network;
y^{uv} :	Volume of containers (TEUs/week) from node u to node v that are delivered in the space-time network.

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