Electricity time-of-use tariff with stochastic demand

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Abstract

In this paper we study the electricity time-of-use (TOU) tariff for an electricity company with stochastic demand. The electricity company offers the flat rate (FR) and TOU tariffs to customers. Under the FR tariff, the customer pays a flat price for electricity consumption in both the peak and nonpeak periods. Under the TOU tariff, the customer pays a high price for electricity consumption in the peak period and a low price for electricity consumption in the non-peak period. The electricity company uses two technologies, namely the base-load and peak-load technologies, to generate electricity. We derive the optimal capacity investment and pricing decisions for the electricity company. Furthermore, we use real data from a case study to validate the results and derive insights for implementing the TOU tariff. We show that in almost all the cases, the electricity company needs less capacity for both technologies under the TOU tariff than under the FR tariff, even though the expected demand in the non-peak period increases. In addition, except for some extreme cases, there is essentially no signicant reduction in the total demand of the two periods, although the TOU tariff can reduce the demand in the peak period. Under the price-cap regulation, the customer pays a lower price on average under the TOU tariff than under the FR tariff. We conduct an extensive numerical study to assess the impacts of the model parameters on the optimal solutions and the robustness of the analytical results, and generate managerial implications of the research findings.

Key words : capacity; pricing; stochastic demand; time-of-use tariff

1. Introduction

Reducing the electricity demand in the peak period is a fundamental concern for electricity utilities in their drive to save electricity cost and energy (York et al. 2007). The reason is that reducing the peak period demand can reduce the electricity load in the peak period and the energy loss in transmission (Faruqui et al. 2007, Triki and Violi 2009). Currently, many electricity customers use the traditional flat-rate (FR) tariff, where the customer pays the same price for each unit of electricity consumption at any time. Under this electricity tariff, the customer has no incentive to reduce their electricity usage in the peak period.

We study in this paper another pricing mechanism, namely the time-of-use (TOU) tariff, under which the customer pays a high price for electricity consumption in the peak period and a low price for electricity consumption in the non-peak period. So the customer may change the time of electricity usage for some activities from the peak period to the non-peak period to take advantage of the low price in the non-peak period. For example, under the TOU tariff, the customer may choose to do the laundry in the non-peak period, instead of in the peak period. Thus the peak period demand will be reduced.

Nevertheless, under the TOU tariff, there will be some increase in electricity demand in the non-peak period. As such, it is interesting and important to study the effects of introducing the TOU tariff. To this end, it is necessary to understand the characteristics associated with electricity demand, generation, regulation, and tariff.

- Electricity demand is stochastic in nature and depends on the pricing scheme (Faruqui and Sergici 2010). Moreover, the customers' electricity usage pattern also affects the demand. As such, there are the peak period and non-peak period in a day. By the peak period, we mean the period in a day during which the *demand rate* (demand per unit time) is high, and by the non-peak period, we mean the period in a day during which the demand rate is low.

- Many electricity companies use the *base-load* and *peak-load technologies* to generate electricity. The base-load technology (e.g., using coal or nuclear energy to generate electricity) usually has a low production cost and a high capacity cost, and is first used to meet the demand. The peak-load technology (e.g., using natural gas to generate electricity) usually has a high production cost and a low capacity cost, and is used to meet the demand that cannot be met by the base-load technology (Crew et al. 1995, Pineau and Zaccour 2007).

• Electricity companies are usually subject to the monitoring and control of regulators under particular regulations. One regulation is called the *price-cap regulation*, which has been used in many places, e.g., the U.K. and Latin America. This regulation sets an upper bound on an index of the electricity company's price, below which the electricity company has full pricing freedom (Liston 1993, Braeutigam and Panzar 1993, Joskow 2005). Some research has shown that under this regulation, a company can reduce its costs and improve its service quality (Jamasb and Pollitt 2007). In this paper we assume that the electricity company is regulated under this regulation.

• Some electricity markets have a mixed tariff structure under which some customers use the TOU tariff while the others use the FR tariff. Examples can be found in Australia, Canada, and the U.S. (CEA 2009). However, the emerging trend is that regulators tend to favour increasing the proportion of customers using the TOU tariff over time. For example, the Department of Public Utility Control in Connecticut in the U.S. has directed all the utility companies to phase in the mandatory TOU tariff for all the customers. In other words, in each succeeding year, the mandatory TOU tariff may be applied to additional customers (Friedman 2011, Jessoe and Rapson 2014).

The TOU tariff has been implemented in some countries in Europe, some states in the U.S., and some cities in Asia (RAP 2008, CEA 2009). But some fundamental questions concerning the implementation of the TOU tariff have remained unaddressed. We set out to explore these questions in this paper. Specifically, we seek to answer the following fundamental questions: With stochastic demand, how much capacity should the electricity company build for the base-load and peak-load technologies to meet the demands in both the peak and non-peak periods? Given capacity levels, what should be the optimal prices for the TOU and FR tariffs? What are the impacts of the proportion of customers using the TOU tariff, the regulation, and costs (such as the capacity and shortage costs) on the optimal solutions and profit?

In this paper we consider a vertically integrated electricity company that seeks to determine the optimal capacity levels and prices in the non-peak and peak periods under stochastic demand. The electricity company uses two technologies, namely the base-load and peak-load technologies, to build capacity to meet the demands in the non-peak and peak periods, respectively. It offers a mixed tariff structure to customers under the price-cap regulation. As discussed above, there are real-world scenarios where the mandatory TOU tariff is applied to some customers while the regulator pushes to increase the proportion of customers using the TOU tariff over time. So in this paper we first consider the case where the proportion of customers using the TOU tariff is given and then we study the impacts of changing this proportion through a numerical study based on real data from the literature.

We derive the optimal capacity investments for both the base-load and peak-load technologies, and the optimal prices for both the TOU and FR tariffs. For the special case where the total demand is not affected by the prices, we show that the upper bound on the price in the peak period (i.e., the price cap set by the regulator) is optimal for the TOU tariff. Moreover, when the total demand under the TOU tariff is affected by the prices, the upper bound on the price in the peak period may not be optimal for the TOU tariff, and the optimal prices are determined by the cost and demand parameters. In addition, for given capacity levels, we perform sensitivity analysis of the optimal solutions with respect to changes in production and shortage costs. We also find that the probability of using the peak-load technology to meet demand is determined by the ratio of the difference between the unit capacity costs of the two technologies to the difference between the unit production costs of the two technologies.

To validate the theoretically derived insights of implementing the TOU tariff, we conduct a case study by using the cost and demand data of Ontario, Canada. We first show that the electricity company can obtain more profit if more customers use the TOU tariff. By comparing the situation where all the customers use the TOU tariff with the situation where all the customers use the FR tariff, we find that under the TOU tariff, the electricity company can obtain a profit 38.52% higher than that under the FR tariff. Intuitively, one might expect that the electricity company needs to build more capacity for the base-load technology under the TOU tariff due to an increase in the expected demand in the non-peak period. However, we show that in almost all the cases, the electricity company needs less capacity for both technologies under the TOU tariff, even though the expected demand in the non-peak period increases. This is due to the fact that the base-load technology is used to meet the demands in both periods, and a decrease in the peak period demand under the TOU tariff has a decreasing effect on the capacity of the base-load technology.

Policy makers and industry experts have long expected that the TOU tariff can reduce electricity demand (Faruqui et al. 2007, Herter at al. 2007). However, we show that the TOU tariff may not produce the desired results in terms of reducing the expected total demand of the two periods. Specifically, we show that except for some extreme cases (such as the case where the expected demand in the peak period is much higher than that in the non-peak period under the FR tariff), there is essentially no significant reduction in the total electricity demand of the two periods under the TOU tariff, although the expected demand in the peak period can be significantly reduced. Nevertheless, there are savings in cost and energy on the supply side, as the peak load is reduced under the TOU tariff.

Furthermore, we show that the price-cap regulation affects both the electricity company and customers. If the regulator sets lower price caps, both the expected demand in the peak period and the expected total demand will increase. Then the electricity company needs to build more capacity at least for one of the technologies and obtains less profit. If the price caps for the TOU tariff are set very low, the customer pays a lower price on average than that under the FR tariff, whereas in many cases the customer needs to pay a higher price on average. Thus, in order to effectively implement the TOU tariff, the regulator needs to set appropriate price caps, and/or consider designing a subsidy policy for the electricity company or customers.

We organize the rest of the paper as follows: In Section 2 we review the related literature. In Section 3 we introduce the model and the assumptions. In Section 4 we derive the optimal capacity investment and pricing decisions. In Section 5 we conduct an extensive numerical study using data from a real case to validate the results and generate managerial insights on implementing the TOU tariff. We conclude the paper and suggest topics for future research in Section 6. We provide all the proofs in the Online Appendix.

2. Literature Review

Our work is related to two streams of research. The first one is on time-varying electricity prices. In the Economics and Energy literature, some research empirically studies the impact of time-varying electricity prices on demand by experiment or simulation, such as Henley and Peirson (1994), Faruqui and George (2005), Holland and Mansur (2005), Herter at al. (2007), Faruqui et al. (2007), and Faruqui and Sergici (2010). The results show that demand is affected by time-varying electricity prices. Pineau and Zaccour (2007), and Chao (2011) modelled the effects of time-varying electricity prices, while Pineau and Zaccour (2007) focused on the capacity investment decision, and Chao (2011) focused on the effects of intermittent resources. Differing from the above studies, we explicitly model the TOU tariff problem, and derive the optimal capacity and prices for the electricity company. On the other hand, studies on peak-load pricing in electricity markets in the Economics literature are also related to time-varying electricity prices. They consider the pricing and capacity investment problems with diverse technologies and different cost characteristics. For instance, Carlton (1977), and Crew and Kleindorfer (1978) investigated some peak-load pricing problems with stochastic demand, which Chao (1983), and Kleindorfer and Fernando (1993) extended to consider supply uncertainty. Comprehensive reviews of this subject can be found in Crew et al. (1995). Research on peak-load pricing is structurally similar to our study on the TOU tariff, with the difference that the former study is from the social welfare perspective whereas our study is from the electricity company's perspective. Besides, our work explicitly models stochastic demands as functions of the prices in the two periods, and considers the FR tariff co-existing with the TOU tariff, which allows us to study the effects of the proportion of customers using the TOU tariff and other parameters.

In the OM/OR literature, there are also studies considering time-varying electricity prices, such as Garcia et al. (2005), Nogales and Conejo (2006), Triki and Violi (2009) and Banal-Estañol and Micola (2009). However, most of the above studies do not consider the customer behaviour of shifting electricity consumption from a high-price period to a low-price period. Yang et al. (2013) analyzed the TOU tariff for an electricity company taking customer behaviour into consideration. Yet unlike our model, theirs does not consider a mixed tariff structure where some customers use the TOU tariff and the rest of the customers use the FR tariff, and theirs does not consider demand uncertainty. Dong et al. (2014) studied the capacity and pricing policies for an electricity company offering the TOU tariff. However, they considered deterministic settings and assumed that the peak-load technology can only be used for the peak period demand. Our paper considers stochastic demand and assumes that the peak-load technology is used to meet the demand that cannot be met by the base-load technology, regardless whether it is the peak period demand or non-peak period demand.

The second related stream of research is on the investment of technologies. In the Economics and Energy literature, Wickart and Madlener (2007) developed an economic model to examine the optimal technology choice and investment timing with consideration of cost (e.g., input fuel cost) uncertainty. Westner and Madlener (2012) used the spread-based real option approach to study investment in a condensing power plant without heat utilization or a plant with combined heat-and-power generation. Tuthill (2008) and Schwerin (2013) investigated the effects of emission costs on investments in dirty and clean technologies.

In the OM/OR literature, many studies have modified or extended the classical newsvendor model to study strategic capacity management (e.g., Van Mieghem (1998), Harrison and Van Mieghem (1999), and Van Mieghem and Rudi (2002)). Van Mieghem (2003) provided a review of research on strategic capacity management seeking to determine the types, sizes, and timing of capacity investments and adjustments under uncertainty. The research framework has been extended to other related settings. For example, Goyal and Netessine (2007) studied the technology choices and capacity investments of two firms with stochastic price-dependent demand in a competitive environment. Boyabatli and Toktay (2011) considered a monopolistic firm that decides the technology choice and capacity level with demand uncertainty in an imperfect capital market, in which the firm is budget-constrained, which can be relaxed by borrowing money from a creditor. Kashefi (2012) investigated the effects of a non-sale capacity market on the decisions of the technology choices and capacity investments of two firms with competition and uncertain demand. Recently, there has been growing literature on technology choice and capacity investment in the energy market, and on environmental issues. For instance, Sönmez et al. (2012) studied strategic technology selection, choice of technology configuration, and capacity for incumbent and emerging technologies in the liquefied natural gas industry. By modelling the trade-off between renewable and non-renewable technologies, Aflaki and Netessine (2012) investigated the incentives for investing in renewable electricity generating capacity. Filomena et al. (2014) analyzed technology selection and capacity investment for electricity generation in a competitive market with consideration of uncertain marginal costs. Kök et al. (2015) studied the impacts of electricity pricing policy on carbon emissions and investment in renewable energy with deterministic demand. Drake et al. (2016) considered technology choice and capacity investment under emission tax and emission cap-and-trade regulation through a two-stage model, where the firm determines the capacity levels of the two technologies in the first stage and demand information is realized at a certain time between the two stages, and then the firm determines the production quantities in the second stage.

One important feature distinguishing our work from research on investment in technologies is that the latter studies consider neither the pricing issues nor the sequence of technology use as we do. The exceptions are Bish and Wang (2004), Chod and Rudi (2005), Biller et al. (2006), and Bish et al. (2012), which studied capacity investment with consideration of the pricing issue. But they focused on responsive pricing (or price postponement), where the pricing decision is made after the demands are realized, and did not consider pricing at different times. However, in our work, we determine the electricity prices before demand realization and we consider the pricing issue in two different time periods.

3. Modelling

We consider a vertically integrated electricity company that not only determines the capacity for generating electricity but also sets the electricity prices for the customers.

Capacity. Suppose that Technology 1 (the base-load technology) and Technology 2 (the peak-load technology) are to be used for generating electricity. Let k_i be the capacity of Technology i, $i \in \{1,2\}$ and $\mathbf{k} = (k_1, k_2)$. It should be noted that electricity capacity is measured in units of power, e.g., megawatt. In other words, it is measured as the maximum rate of energy per unit time. The time of electricity usage is divided into two periods, namely the peak period and the non-peak period. Let T be the total period time, e.g., one day, and τ be the proportion of the total period time that is the peak period time. Without loss of generality, we normalize $T = 1$. Then $(1-\tau)k_i$ and τk_i are the capacity levels of Technology i for the non-peak and peak period demands, respectively, $i \in \{1,2\}$. Let $k_p = k_1 + k_2$ be the total capacity of the two technologies.

Costs. Let c_i and β_i be the unit capacity cost and unit production cost of Technology i, respectively. It is well-known that the base-load technology typically has a higher unit capacity cost and a lower unit production cost than those of the peak-load technology. Thus, $c_1 > c_2$ and $\beta_1 < \beta_2$ (Crew et al. 1995, Pineau and Zaccour 2007). The shortage cost will be incurred whenever demand exceeds capacity. Let v_1 and v_2 be the unit shortage costs for the non-peak period and peak period demands, respectively. The shortage costs could be considered as the electricity prices to purchase additional electricity from outside markets. Therefore, it is reasonable to assume that $v_1 \geq \beta_2$ and $v_2 \geq \beta_2$.

Prices. The electricity company offers both the TOU and FR tariffs to the customers. A fraction $\alpha \in [0,1]$ of the customers are under the TOU tariff, so the remaining fraction $1-\alpha$ of the customers are under the FR tariff. If $\alpha = 0$, then only the FR tariff is offered to the customers; and if $\alpha = 1$, then only the TOU tariff is offered to the customers. We first assume that α is given, and later we investigate the effects of α on the optimal solutions based on some real data. Under the FR tariff, the customer pays a flat price $p_0 \in [0, \bar{p_0}]$ for electricity consumption in both the non-peak and peak periods. Under the TOU tariff, the customer pays a low price $p_1 \in [0, \bar{p_1}]$ for electricity consumption in the non-peak period, and pays a high price $p_2 \in [0, \bar{p_2}]$ for electricity consumption in the peak period. Here, $\bar{p_0}$, $\bar{p_1}$, and $\bar{p_2}$ are the upper bounds (i.e., price caps) on p_0, p_1, and p_2, respectively, which may be imposed by the regulator under the price-cap regulation. Let $\mathbf{p} = (p_0, p_1, p_2)$. We assume that $p_1 \leq p_0 \leq p_2$. Otherwise, no customer will use the TOU tariff if $p_1 > p_0$ and no customer will use the FR tariff if $p_2 < p_0$. This assumption is also justified based on actual prices used in real-life situations (Shao et al. 2010).

Demands. Let $D_{T1} \geq 0$ and $D_{T2} \geq 0$ be the demands in the non-peak and peak periods, respectively, under the TOU tariff, and $D_{F1} \geq 0$ and $D_{F2} \geq 0$ be the demands in the non-peak and peak periods, respectively, under the FR tariff. We consider stochastic demands and model them as follows: $D_{T1}(p_1,p_2) = y_{T1}(p_1,p_2) + \epsilon_{T1}$, $D_{T2}(p_1,p_2) = y_{T2}(p_1,p_2) + \epsilon_{T2}$, $D_{F1}(p_0) = y_{F1}(p_0) + \epsilon_{F1}$, and $D_{F2}(p_0) = y_{F2}(p_0) + \epsilon_{F2}$. Here, ϵ_{Ti} and ϵ_{Fi} are random variables (random noises) defined in the ranges $[A_{Ti}, B_{Ti}]$ and $[A_{Fi}, B_{Fi}]$ with mean values of $\mu_{Ti} = 0$ and $\mu_{Fi} = 0$, respectively, which indicate that the uncertainties are not affected by the prices. $y_{Ti}(p_1,p_2)$ and $y_{Fi}(p_0)$ are functions that capture the dependency between demands and prices. $y_{T1}(p_1,p_2)$ decreases with p_1 and increases with p_2, $y_{T2}(p_1,p_2)$ decreases with p_2 and increases with p_1, and both $y_{F1}(p_0)$ and $y_{F2}(p_0)$ decrease with p_0. We define $D_1 = \alpha D_{T1}(p_1,p_2) + (1-\alpha)D_{F1}(p_0)$ and $D_2 = \alpha D_{T2}(p_1,p_2) + (1-\alpha)D_{F2}(p_0)$ as the total demands in the non-peak and peak periods, respectively. Similar forms of total demand functions for two tariffs were adopted in Borenstein and Holland (2005). Let ϵ_i be the random noise in D_i. Then, $\epsilon_i = \alpha\epsilon_{Ti} + (1-\alpha)\epsilon_{Fi} \in [A_i, B_i]$, where $A_i = \alpha A_{Ti} + (1-\alpha)A_{Fi}$ and $B_i = \alpha B_{Ti} + (1-\alpha)B_{Fi}$. Using transformation, we can find the distribution of ϵ_i in terms of the joint distribution of ϵ_{Ti} and ϵ_{Fi}. This works even if ϵ_{Ti} and ϵ_{Fi} are correlated. Besides, ϵ_1 and ϵ_2 can be correlated or uncorrelated. We further let $y_i = \alpha y_{Ti}(p_1,p_2) + (1-\alpha)y_{Fi}(p_0)$. Then, $D_i = y_i + \epsilon_i$.

We consider linear demand functions in this paper. Examples of linear demand functions for electricity can be found in Pineau and Zaccour (2007), Chao (2011), and Greer (2012). Specifically, we let $y_{T1}(p_1,p_2) = a_{T1} - b_{T1}p_1 + r_1p_2$, $y_{T2}(p_1,p_2) = a_{T2} - b_{T2}p_2 + r_2p_1$, $y_{F1}(p_0) = a_{F1} - b_{F1}p_0$, and $y_{F2}(p_0) = a_{F2} - b_{F2}p_0$. Here, the base demands (a_{Ti} and a_{Fi}) and price sensitivity parameters (b_{Ti}, b_{Fi} and r_i) are all positive. Without loss of generality, we let $a_{Ti} = a_{Fi} = a_i$.

ASSUMPTION 1. *(a) $b_{T1} > r_1$ and $b_{T2} > r_2$; (b) $b_{T1} > r_2$ and $b_{T2} > r_1$.*

Part (a) of Assumption 1 states that the demand in each period is more sensitive to a unit change in its own price than a unit change in the price of the other period. Part (b) states that when the price of a period increases, the reduced demand in that period is greater than the increased demand in the other period.

In order to guarantee that the demands are positive over some ranges of prices, we assume that $A_i \geq -a_i$. Similar assumptions have been used in the literature to deal with demand realization issues for additive demand functions (Petruzzi and Dada 1999, Jiang and Anupindi 2010). Let $f_i(\cdot)$ and $F_i(\cdot)$ be the probability density function (pdf) and cumulative distribution function (cdf) of the random noise ϵ_i.

Objective function. Let $x^+ = \max\{0,x\}$ for any real number x. The electricity company's expected cost function can be expressed as follows:

$$\begin{aligned}C(\mathbf{k},\mathbf{p}) = \mathbb{E}_{\epsilon_1}\mathbb{E}_{\epsilon_2}[&c_1k_1 + c_2k_2 + \beta_1\min\{D_1,(1-\tau)k_1\} + \beta_2\min\{(D_1-(1-\tau)k_1)^+,(1-\tau)k_2\}\\ &+v_1(D_1-(1-\tau)k_1-(1-\tau)k_2)^+ + \beta_1\min\{D_2,\tau k_1\} + \beta_2\min\{(D_2-\tau k_1)^+,\tau k_2\}\\ &+v_2(D_2-\tau k_1-\tau k_2)^+]. \end{aligned} \tag{1}$$

In the cost function (1), the first and second terms are the capacity costs for Technologies 1 and 2, respectively; the third and fourth terms are the production costs associated with Technologies 1 and 2, respectively, in the non-peak period; the fifth term is the shortage cost in the non-peak period; the sixth and seventh terms are the production costs associated with Technologies 1 and 2, respectively, in the peak period; and the last term is the shortage cost in the peak period.

Our objective is to maximize the electricity company's expected profit $\Pi(\mathbf{k},\mathbf{p})$ by optimally determining the capacity for the two technologies, i.e., $\mathbf{k}=(k_1,k_2)$, and the prices of electricity for the two tariffs, i.e., $\mathbf{p}=(p_0,p_1,p_2)$, as follows:

$$\begin{aligned}\max_{\mathbf{k},\mathbf{p}}\Pi(\mathbf{k},\mathbf{p}) &= \mathbb{E}_{\epsilon_1}\mathbb{E}_{\epsilon_2}[\alpha(p_1D_{T1}+p_2D_{T2})+(1-\alpha)p_0(D_{F1}+D_{F2})]-C(\mathbf{k},\mathbf{p})\\ &= \alpha(p_1y_{T1}+p_2y_{T2})+(1-\alpha)p_0(y_{F1}+y_{F2})-C(\mathbf{k},\mathbf{p}). \end{aligned} \tag{2}$$

The objective function (2) is composed of three components. The first and second components are the expected revenues from the customers under the TOU and FR tariffs, respectively, and the third component is the electricity company's expected cost. Note that we seek to maximize the electricity company's expected profit over prices for two reasons. The first reason is that we consider the price-cap regulation. Below the price caps set by the regulator, the electricity company has full pricing freedom (Braeutigam and Panzar 1993, Liston 1993). Thus the electricity company will maximize its profit by optimally setting the prices. The second reason is that, as Crew et al. (1995) have found, the peak-load pricing literature also focuses on optimal prices. So following the peak-load pricing literature, we consider optimization of profit over prices. Nevertheless, when the prices are fixed, our objective becomes a minimization problem over capacity and all of our results hold for optimization of profit over capacity.

Note that ϵ_1 and ϵ_2 can be correlated or uncorrelated. This can be seen from the objective function (2), in which the random noises only appear in $C(\mathbf{k},\mathbf{p})$, and in the cost function (1), the terms involving ϵ_1 (or equivalently D_1) and the terms involving ϵ_2 (or equivalently D_2) in $C(\mathbf{k},\mathbf{p})$ are additively separable.

Table 1 summarizes the major notation used in this paper, where $i\in\{1,2\}$.

4. Analysis and Solution

We use the sequential decision-making approach to find the optimal solution, denoted by $(\mathbf{k}^*,\mathbf{p}^*)$, that maximizes $\Pi(\mathbf{k},\mathbf{p})$ in (2). That is, we first find the optimal response of prices, i.e., $\mathbf{p}(\mathbf{k})$, for a given $\mathbf{k}$. In the second step, we find the optimal capacity, i.e., $\mathbf{k}^*$, by maximizing $\Pi(\mathbf{k},\mathbf{p}(\mathbf{k}))$ over $\mathbf{k}$.

In the analysis, we assume that $c_1+\beta_1\le(1-\tau)v_1+\tau v_2$ and $c_2+\beta_2\le(1-\tau)v_1+\tau v_2$. Otherwise, it can be shown that if $c_1+\beta_1>(1-\tau)v_1+\tau v_2$, then $k_1^*=0$; and if $c_2+\beta_2>(1-\tau)v_1+\tau v_2$, then

Table 1 Notations

τ	the peak period time as a proportion of the total time period.
k_i	capacity of Technology i, $\mathbf{k} = (k_1, k_2)$.
k_p	total capacity of the technologies, i.e., $k_p = k_1 + k_2$.
c_i	unit capacity cost of Technology i.
β_i	unit production cost of Technology i.
v_1, v_2	unit shortage cost for the non-peak period demand and the peak period demand, respectively.
p_0	electricity price for the FR tariff, with $\bar{p_0}$ being the upper bound on p_0.
p_1, p_2	electricity prices in the non-peak and peak periods, respectively, for the TOU tariff, with $\bar{p_1}$ and $\bar{p_2}$ being the upper bounds on p_1 and p_2, respectively.
D_{T1}, D_{T2}	demands in the non-peak and peak periods, respectively, under the TOU tariff.
D_{F1}, D_{F2}	demands in the non-peak and peak periods, respectively, under the FR tariff.
y_{Ti}, y_{Fi}	expected values of D_{Ti} and D_{Fi}, respectively.
ϵ_1, ϵ_2	random noises of D_{Ti} and D_{Fi}, respectively, with pdf $f_i(\cdot)$ and cdf $F_i(\cdot)$.
D_1, D_2	total demands in the non-peak and peak periods, respectively.
y_i	expected value of D_i.
$C(\cdot)$	the electricity company's expected cost function.
$\Pi(\cdot)$	the electricity company's expected profit function.

$k_2^* = 0$. The former means that if the unit cost of generating electricity by using Technology 1 (i.e., $c_1 + \beta_1$) is larger than the average unit shortage cost (i.e., $(1-\tau)v_1 + \tau v_2$), then the electricity company should not use Technology 1. The latter can be similarly interpreted. Besides, we also assume that $c_2 + \beta_2 \geq c_1 + \beta_1$, meaning that the total unit cost of generating electricity by using Technology 2 is not less than that by using Technology 1. Otherwise, we can show that $k_1^* = 0$.

Before deriving the optimal solution, we let $F_{11} = F_1\big((1-\tau)k_1 - y_1\big)$, $F_{12} = F_1\big((1-\tau)k_1 + (1-\tau)k_2 - y_1\big)$, $F_{21} = F_2(\tau k_1 - y_2)$, $F_{22} = F_2(\tau k_1 + \tau k_2 - y_2)$, $f_{11} = f_1\big((1-\tau)k_1 - y_1\big)$, $f_{12} = f_1\big((1-\tau)k_1 + (1-\tau)k_2 - y_1\big)$, $f_{21} = f_2(\tau k_1 - y_2)$, and $f_{22} = f_2(\tau k_1 + \tau k_2 - y_2)$.

(1) Price decisions

Lemma 1. *Given $\mathbf{k}$, the objective function $\Pi(\mathbf{k}, \mathbf{p})$ is jointly concave in p_0, p_1, and p_2.*

Lemma 1 facilitates the finding of the optimal prices, given $\mathbf{k}$. In particular, without considering the boundaries of the prices, the first-order conditions can be used for finding the optimal prices, which are mainly determined by the cost and demand parameters.

Proposition 1. *Given $\mathbf{k}$, the effects of the cost parameters on the optimal prices are as follows:*

(i) $dp_1(\mathbf{k})/dc_1 = dp_1(\mathbf{k})/dc_2 = dp_2(\mathbf{k})/dc_1 = dp_2(\mathbf{k})/dc_2 = dp_0(\mathbf{k})/dc_1 = dp_0(\mathbf{k})/dc_2 = 0.$

(ii) When $\alpha = 0$, $dp_0(\mathbf{k})/d\beta_1 \geq 0$, $dp_0(\mathbf{k})/d\beta_2 \geq 0$, $dp_0(\mathbf{k})/dv_1 \geq 0$, *and* $dp_0(\mathbf{k})/dv_2 \geq 0$.

(iii) When $\alpha = 1$, $dp_1(\mathbf{k})/dv_1 \geq 0$ *and* $dp_2(\mathbf{k})/dv_2 \geq 0$*; if* $r_1 \geq r_2$*, then* $dp_1(\mathbf{k})/d\beta_1 \geq 0$, $dp_1(\mathbf{k})/d\beta_2 \geq 0$, *and* $dp_1(\mathbf{k})/dv_2 \geq 0$*; if* $r_2 \geq r_1$*, then* $dp_2(\mathbf{k})/d\beta_1 \geq 0$, $dp_2(\mathbf{k})/d\beta_2 \geq 0$, *and* $dp_2(\mathbf{k})/dv_1 \geq 0$.

Proposition 1 states the impacts of the costs on the optimal prices for given capacity levels. It can be seen from part (i) that the optimal prices are independent of the capacity costs. This result is intuitive. After the capacity levels are determined, the optimal prices will not be affected by the capacity costs. In Section 5, based on the numerical results of a case study, we find that when we determine the optimal capacity and prices together, the prices will be affected by the capacity costs. Parts (ii) and (iii) state that, given capacity levels, the optimal prices will be affected by the production and shortage costs. If the electricity company only offers the FR tariff, then it will increase the price when the production and shortage costs increase. If the electricity company only offers the TOU tariff, then it will increase the price in the non-peak period when the shortage cost for the non-peak period demand increases, and increase the price in the peak period when the shortage cost for the peak period demand increases. For the impacts of the production costs, we derive the sufficient conditions associated with the demand parameters r_1 and r_2. That is, if the increased non-peak period demand due to the price increase in the peak period is greater than the increased peak period demand due to the price increase in the non-peak period (i.e., $r_1 \geq r_2$), then the electricity company will increase the price in the non-peak period when the production costs increase; otherwise, the electricity company will increase the price in the peak period when the production costs increase. It is reasonable that the prices are increasing in the production costs.

Proposition 2. *The effects of capacity on the optimal prices are as follows:*

(i) When $\alpha = 0$, $\partial p_0(\mathbf{k})/\partial k_1 \leq 0$ *and* $\partial p_0(\mathbf{k})/\partial k_2 \leq 0$.

(ii) When $\alpha = 1$*, if* $r_1 \geq r_2$*, then* $\partial p_1(\mathbf{k})/\partial k_1 \leq 0$ *and* $\partial p_1(\mathbf{k})/\partial k_2 \leq 0$*; if* $r_2 \geq r_1$*, then* $\partial p_2(\mathbf{k})/\partial k_1 \leq 0$ *and* $\partial p_2(\mathbf{k})/\partial k_2 \leq 0$.

Proposition 2 shows how the optimal prices change with capacity. If the electricity company only offers the FR tariff, then the optimal price will decrease if the capacity increases. One reason may be that more demands will be fulfilled when the capacity increases, and in order to stimulate more demands, the prices need to be reduced. If the electricity company only offers the TOU tariff, we derive the sufficient conditions similar to those in Proposition 1. That is, if the increased non-peak period demand due to the price increase in the peak period is greater than the increased peak period demand due to the price increase in the non-peak period, then the price in the non-peak period will decrease if the capacity increases; otherwise, the price in the peak period will decrease if the capacity increases.

Suppose that $p_1(p_0, p_2)$ is the optimal p_1 for given p_0 and p_2, and $p_2(p_0, p_1)$ is the optimal p_2 for given p_0 and p_1. Following the results in the proof of Lemma 1, we obtain the following result:

PROPOSITION 3. *$\partial p_1(p_0, p_2)/\partial p_2 \geq 0$.*

Proposition 3 states that, if the price in the peak period under the TOU tariff increases, then the price in the non-peak period under the TOU tariff will also increase. However, as shown in the proof of Lemma 1, if the prices under the TOU tariff increase, then changes in the prices under the FR tariff depend on the parameters and demand distributions.

(2) Capacity decisions

LEMMA 2. *The optimal capacity* $\mathbf{k}$ *can be uniquely determined by solving the following equations:*

$$\begin{aligned}(1-\tau)(\beta_2-\beta_1)F_{11}+(1-\tau)(v_1-\beta_2)F_{12}+\tau(\beta_2-\beta_1)F_{21}+\tau(v_2-\beta_2)F_{22}\\ =(1-\tau)v_1+\tau v_2-c_1-\beta_1; \qquad (3)\end{aligned}$$

$$(1-\tau)(v_1-\beta_2)F_{12}+\tau(v_2-\beta_2)F_{22}=(1-\tau)v_1+\tau v_2-c_2-\beta_2. \qquad (4)$$

Lemma 2 shows that the optimal capacity can be determined uniquely by Equations (3) and (4). The costs, such as capacity costs, production costs, and shortage costs, play critical roles in rationing the capacity to meet the demands. The left-hand sides of Equations (3) and (4) are combinations of the probability of rationing the capacity to meet the demands in both the non-peak and peak periods, and the right-hand sides of the equations are the differences between the average unit shortage cost and the unit cost of generating electricity by a technology. Another observation is that, by combining Equations (3) and (4), we obtain

$$1-[(1-\tau)F_{11}+\tau F_{21}]=\frac{c_1-c_2}{\beta_2-\beta_1}. \qquad (5)$$

Remark 1. In Equation (5), $(1-\tau)F_{11}+\tau F_{21}$ is the probability that the demand can be fulfilled by the capacity of Technology 1. Then the left-hand side of Equation (5) is the probability of using Technology 2 to meet the demand. The right-hand side is the ratio of the difference between the unit capacity costs of the two technologies to the difference between the unit production costs of the two technologies, which is between zero and one due to $c_2+\beta_2 \geq c_1+\beta_1$, $c_1 > c_2$, and $\beta_2 > \beta_1$. Thus, it is interesting to note that the probability of using Technology 2 to meet the demand is determined by the ratio of the difference between the unit capacity costs of the two technologies to the difference between the unit production costs of the two technologies.

Note that Dong et al. (2014) considered capacity investments in two technologies with deterministic demand and showed that the optimal capacity levels are mainly determined by the demands. However, we show that, for stochastic demand, costs play a crucial role in allocating the capacity. In Section 5 we present numerical results to show the effects of costs on capacity based on some real data.

In the above we showed the optimal prices for the electricity company if it can freely set the prices. However, under the price-cap regulation, the electricity prices cannot exceed the upper bounds (i.e., price caps) set by the regulator, i.e., $\bar{p_0}$, $\bar{p_1}$, and $\bar{p_2}$. Due to the concavity property of the objective function, the global optimal electricity prices are the smaller between the optimal solutions obtained by the first-order conditions and the upper bounds set by the regulator. In Section 5 we show the effects of the price caps on the optimal solution and expected profit of the electricity company by a case study.

4.1. A Special Case

By now, we have considered our problem under the TOU tariff for the case where $b_{T1} > r_1$ and $b_{T2} > r_2$. In this sub-section we consider the special case where $b_{T1} = r_2$ and $b_{T2} = r_1$. Then $y_{T1} = a_1 - b_{T1}p_1 + b_{T2}p_2$ and $y_{T2} = a_2 - b_{T2}p_2 + b_{T1}p_1$. So we have $y_{T1} + y_{T2} = a_1 + a_2$ and $D_{T1} + D_{T2} = a_1 + a_2 + \epsilon_1 + \epsilon_2$, suggesting that the total electricity demand under the TOU tariff is not affected by the prices. Specifically, when the prices change, the expected increased demand in one period is equal to the expected decreased demand in the other period.

For this special case, the optimal capacity levels satisfy Equations (3) and (4). As regards the optimal prices, we have the following result.

PROPOSITION 4. *(i) Given p_2, the optimal p_0 and p_1 can be uniquely determined by the first-order conditions. (ii) The upper bound on p_2 is optimal for the electricity company.*

Part (i) of Proposition 4 states that, given p_2, the optimal price under the FR tariff and the optimal price in the non-peak period under the TOU tariff can be uniquely determined by the first-order conditions (see Equations (A.1) and (A.3) in the Online Appendix), which are the same as those for the case where $b_{T1} > r_1$ and $b_{T2} > r_2$. Upon substituting the optimal p_0 and p_1 back into the objective function, we can find the optimal p_2. The optimal p_2 for this special case is different from that for the case where $b_{T1} > r_1$ and $b_{T2} > r_2$. Part (ii) of Proposition 4 states that, for this special case, the upper bound on p_2 is optimal under the TOU tariff. Because the total demand for the TOU tariff is not affected by prices in this case, the company just needs to set p_2 as high as possible, for given optimal p_0 and p_1. Similarly, under the price-cap regulation, we need to guarantee that the prices should not exceed the upper bounds set by the regulator. So we first determine that the optimal price in the peak period under the TOU tariff is $\bar{p_2}$. Then we substitute $\bar{p_2}$ into Equations (A.1) and (A.3), and find the optimal solutions for p_0 and p_1. Finally, by comparing these solutions and the corresponding upper bounds set by the regulator, we can obtain the global optimal prices for p_0 and p_1.

REMARK 2. Similar to the results in Proposition 3, $dp_1(p_2)/dp_2 \geq 0$ in this case.

5. Case Study

In this section we use numerical examples to generate insights from the analytical results using data from a case study of Ontario, Canada. We first study the impacts of the proportion of customers using the TOU tariff. Then we examine the impacts of the price-cap regulation and cost parameters. In addition, we conduct a series of robustness tests of the results by varying the cost and demand parameters.

5.1. Settings

Generation technologies. Before presenting the numerical results, we introduce the settings of the numerical study. We derive the data mainly from Pineau and Zaccour (2007) for the case study of Ontario, Canada. In 2005, Ontario had a generation capacity 30,921.9 MW. Pineau and Zaccour (2007) considered two technologies in their model where one has a relatively high capacity cost and a low production cost, and the other has a relatively low capacity cost and a high production cost. Consistent with our model setting, the technologies in the Ontario case study operate in the following sequence: When the demand is low, the electricity company uses low production cost sources, such as nuclear energy and hydro, to generate electricity. When the capacity of the low production cost sources is exhausted, it turns to using sources with high production costs, such as natural gas (OEB 2015). According to Pineau and Zaccour (2007), we consider one year as the total period and set $\tau = 0.4389$. Although the cost and demand parameters are obtained from the literature, they are derived or estimated based on the real data in Ontario, Canada.

Cost data. We derived the cost parameters of the two technologies based on Tables 1 and 5 in Pineau and Zaccour (2007), i.e., $c_1 = 20.48$, $\beta_1 = 52.20$, $c_2 = 11.67$, and $\beta_2 = 87.20$. Regarding v_1 and v_2, we could not find the real data directly. We tested the effects of α, price-cap regulation, and parameters using many sets of (v_1, v_2), such as $(100, 140)$, $(150, 200)$, $(200, 300)$ etc. We found that $(150, 200)$ can typically represent the results in our tests. Therefore, in the following we illustrate the effects when (v_1, v_2) equals $(150, 200)$, where v_1 is about two times of $c_1 + \beta_1$ and v_2 is about two times of $c_2 + \beta_2$. The units of the costs are \$/MWh. In Sub-section 5.5 we show the robustness of the results by varying the values of the cost parameters over wide ranges.

Demand data. We derived the price sensitivity parameters for demands under the TOU tariff based on Table 8 in Pineau and Zaccour (2007), i.e., $a_1 = 71,480,450.48$, $b_{T1} = 181,717.23$, $r_1 = 12,118.13$, $a_2 = 60,946,977.74$, $b_{T2} = 60,645.39$, and $r_2 = 6,064.54$. Regarding the price sensitivity parameters for demands under the FR tariff, following the TOU setting for $p_1 = p_2$, we set $b_{F1} = b_{T1} - r_1 = 169599.1$ and $b_{F2} = b_{T2} - r_2 = 54,580.85$. We set the parameters associated with demand uncertainty as follows: First, we assume that the demands follow the normal distributions, i.e., $\epsilon_i \sim Normal(\mu_i, \sigma_i)$, where $i \in \{1, 2\}$ and $\mu_i = 0$. Note that in the model setting, the random variable ϵ_i is defined in the range $[A_i, B_i]$ and we assume that $A_i \geq -a_i$. In addition, for the normal distribution, 99.9999% of the values lie within five standard deviations from the mean. Thus, as an approximation, we set $\sigma_i = |A_i|/5 <= a_i/5$. We conducted many tests by setting a series of values for σ_i from $a_i/100$ to $a_i/5$. We found that $a_i/10$ can typically represent the patterns of the results in our tests. Therefore, in the following we illustrate the effects when $\sigma_i = a_i/10$. In Sub-section 5.5 we show the robustness of the results when we change the values of the demand parameters.

5.2. Impacts of Proportion of Customers Using the TOU Tariff

In this sub-section we study the impacts of the proportion of customers using the TOU tariff (α) on the optimal solution and expected profit. Table 2 shows the impacts of α. Note that here we set the price caps high enough such that the optimal prices are obtained by the first-order conditions. In Sub-section 5.3 we study the impacts of the price caps.

Table 2 **Impact Of α.**

α	k_1 (10^6)	k_2 (10^6)	k_p (10^6)	p_0	p_1	p_2	$\frac{y_1}{1-\tau}$ (10^6)	$\frac{y_2}{\tau}$ (10^6)	$\frac{y_2}{\tau}-\frac{y_1}{1-\tau}$ (10^6)	Π (10^6)
0	96	12	108	327	0	0	28.55	98.2	69.64	13939.92
0.1	92	13	105	327	250	584	31.6	94.54	62.94	14483.72
0.2	88	13	101	327	250	584	34.65	90.88	56.23	15027.67
0.3	85	12	97	327	250	584	37.7	87.23	49.53	15571.51
0.4	81	13	94	327	250	584	40.75	83.57	42.82	16115.27
0.5	78	12	90	327	250	584	43.8	79.91	36.11	16658.9
0.6	74	12	86	327	251	584	46.65	76.26	29.61	17201.52
0.7	72	11	83	327	251	583	49.65	72.7	23.05	17741.47
0.8	70	10	80	328	252	581	52.31	69.27	16.95	18275.43
0.9	69	8	77	329	254	580	54.69	65.8	11.11	18799.6
1	69	6	75	0	255	577	57.27	62.66	5.39	19309.78

Note that in Table 2 the capacity levels are in MW, and the prices and profits are in \$. The expected demand rate $y_1/(1-\tau)$ is the expected amount of energy used in the non-peak period per unit time, y_2/τ is the expected amount of energy used in the peak period per unit time, and $y_2/\tau - y_1/(1-\tau)$ is the difference in the expected demand rate between the peak and non-peak periods. They are all in MW.

From Table 2, we can see that when α increases, the electricity company's expected profit will increase. Then we can conclude that, in our setting, all the customers using the TOU tariff are optimal to the electricity company. Besides, we observe that the capacity of Technology 1 decreases with α. Although there are no clear patterns for the capacity of Technology 2, the total capacity of the two technologies still decreases with α. In other words, we can say that increasing the proportion of customers using the TOU tariff may not reduce the capacity of the peak-load technology, while it can reduce the total capacity of the two technologies and the capacity of the base-load technology. Regarding the impacts on the prices, we can see that the prices are affected by the proportion of customers using the TOU tariff. This result is different from that in Dong et al. (2014) in the setting of deterministic demand, who showed that the prices are independent of the proportion of customers using the TOU tariff.

It is worth noting that in our numerical results, the expected non-peak period demand y_1 may be greater than the expected peak period demand y_2. For example, when $\alpha = 1$, $p_1 = 255$, and $p_2 = 577$, we can calculate that $y_1 = \alpha y_{T1} + (1-\alpha) y_{F1} = y_{T1} = 32134717.84$, which is greater than $y_2 = \alpha y_{T2} + (1-\alpha) y_{F2} = y_{T2} = 27501045.41$. However, the results are reasonable because in our numerical study, the demands are presented as aggregate demands, rather than demand rates (i.e., $y_1/(1-\tau)$ and y_2/τ). As shown in Table 2, all the expected demand rates in the peak period (y_2/τ) are greater than the corresponding expected demand rates in the non-peak period ($y_1/(1-\tau)$). This result holds even if we only consider the demands of the customers under the TOU tariff, i.e., $y_{T2}/\tau > y_{T1}/(1-\tau)$. In Sub-section 5.5 we conduct a range of tests to check the robustness of the results by varying the parameters over wide ranges. Moreover, from Table 2, we observe that when α increases, the demand rate in the peak period will decrease, the demand rate in the non-peak period will increase, and their difference will decrease. This indicates that under the TOU tariff, the customers will shift some demands from the peak period to the non-peak period, which agrees with our theoretical results.

5.3. Impacts of the Price-cap Regulation

In this sub-section we study the impacts of the price-cap regulation. Note that in Sub-section 5.2, we observed that all the customers using the TOU tariff are optimal to the electricity company. Therefore, we consider the scenario where all the customers use the FR tariff as a benchmark and focus on the scenario where all the customers use the TOU tariff. We compare the performance under the TOU tariff with that of the benchmark.

Let x^{TOU} and x^{FR} be the values of x for the cases where all the customers use the TOU and FR tariffs, respectively. Except Δp, we let $\Delta x = (x^{TOU} - x^{FR})/x^{FR}$ be the relative difference between x under the TOU tariff and that under the FR tariff. For example, $\Delta \Pi = (\Pi^{TOU} - \Pi^{FR})/\Pi^{FR}$ is the relative difference between the expected profits under the two tariffs. For Δp, we let $\Delta p = ((p_1 + p_2)/2 - p_0)/p_0$, which is the relative difference between the average prices under the two tariffs.

We assume that the price caps for the FR tariff are very large and focus on the impacts of the price caps for the TOU tariff (i.e., $\bar{p_1}$ and $\bar{p_2}$). When we study the impacts of $\bar{p_2}$, we set

$\bar{p_1}$ large enough such that p_1^* is obtained by the first-order conditions. A similar setting holds when we study the impacts of $\bar{p_1}$. Before presenting the performance of the TOU tariff, we first provide the computational results for the FR tariff. When all the customers use the FR tariff, we obtain $k_1 = 96,000,000$MW, $k_2 = 12,000,000$MW, $p_0 = \$327$, $y_1 = 16,021,544.78$MWh, $y_2 = 43,099,039.79$MWh, and $\Pi = \$13,939,921,822.12$. We can verify that the demand rate in the peak period is greater than that in the non-peak period, i.e., $y_{F2}/\tau > y_{F1}/(1-\tau)$.

Table 3 **Impact Of $\bar{p_2}$.**

$\bar{p_2}$	Δk_1	Δk_2	Δk_p	Δp	Δy_1	Δy_2	$\Delta(y_1+y_2)$	$\Delta\Pi$
400	-11.46	0	-10.19	-1.99	103.06	-11.48	19.56	24.35
450	-17.71	-8.33	-16.67	6.27	102.31	-18.46	14.27	31.19
500	-21.88	-25	-22.22	14.37	102.69	-25.45	9.27	35.82
550	-26.04	-41.67	-27.78	22.78	100.8	-32.42	3.68	38.19
600	-29.17	-41.67	-30.56	27.37	99.44	-36.18	0.57	38.52

Table 3 shows the impacts of $\bar{p_2}$. Note that the optimal solution for p_2 by the first-order conditions is equal to 577. Then we find that when $\bar{p_2} = 600$, p_2^* is less than the price cap; when $\bar{p_2} \leq 550$, $p_2^* = \bar{p_2}$. We see that for all the values of $\bar{p_2}$, the demand rate in the peak period is greater than that in the non-peak period, i.e., $y_{T2}/\tau > y_{T1}/(1-\tau)$.

It is interesting to note from Table 3 that the electricity company needs less capacity for both technologies under the TOU tariff than under the FR tariff, even though the expected demand in the non-peak period will increase. One reason may be as follows: When all the customers use the TOU tariff, some peak period demand will be shifted to the non-peak period. Then the expected demand in the non-peak period will increase and the expected demand in the peak period will decrease. Technology 2 is used for both the non-peak and peak period demands, so its capacity will decrease when the peak period demand decreases. Technology 1 is also used for both non-peak and peak period demands, so the decrease in the peak period demand will cause a decrease in the capacity of Technology 1. Therefore, although the increase in the non-peak period demand will have the effect of increasing the capacity of Technology 1, it may be dominated by the decreasing effect caused by the decrease in the peak period demand. Another explanation may be that the TOU tariff can help smoothen the demands in the two periods, so the electricity company needs less capacity for at least one of the technologies, since both technologies are used to meet the demands in the two periods. Another observation is that the electricity company needs less capacity when the price caps increase. This is because when the price caps increase, the company can set a high price for the peak period demand. Then the peak period demand will decrease, which will further smoothen the demands between the two periods and lead to a decrease in capacity.

For the expected total demand of the two periods, we observe that it may increase under the TOU tariff, compared with under the FR tariff. When p_2^* is less than the price cap, the expected total demand under the TOU tariff is very close to that under the FR tariff (where $\Delta(y_1+y_2)=0.57\%$ in the above example). This shows that there is essentially no significant reduction in the total electricity demand over the whole period by introducing the TOU tariff. Henley and Peirson (1994) and Faruqui and George (2005) obtained similar results. However, as the peak period demand is reduced, cost and energy savings can still be obtained on the supply side. It can also be seen that the expected total demand decreases when the price caps increase. Besides, we also find that the average price under the TOU tariff may be lower than the price under the FR tariff. This is due to the effects of the price-cap regulation, under which the electricity company cannot set a high price under the TOU tariff. Furthermore, we observe that the expected demand in the peak period and the expected total demand decrease with the price caps, and the electricity company's expected profit increases with the price caps. The percentage increase in profit can be as high as 38.52%, which is obtained when the price is less than the price caps.

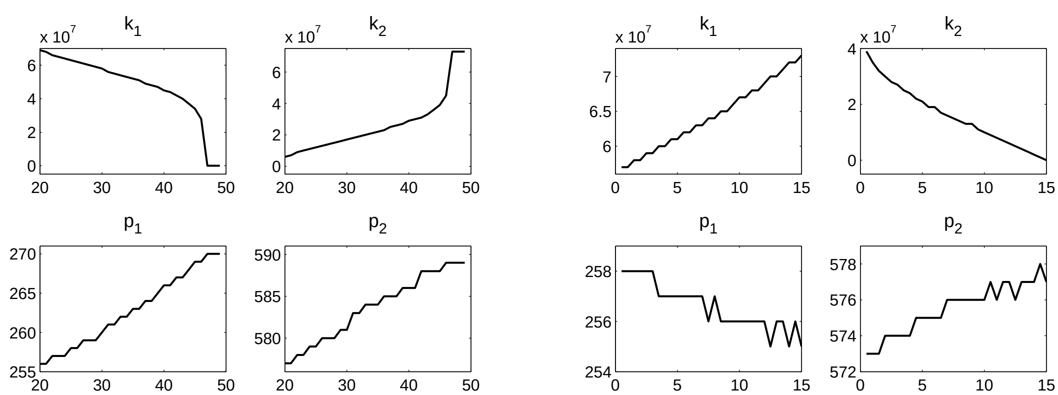
Table 4 **Impact Of $\bar{p_1}$.**

$\bar{p_1}$	Δk_1	Δk_2	Δk_p	Δp	Δy_1	Δy_2	$\Delta(y_1+y_2)$	$\Delta\Pi$
100	12.5	-66.67	3.7	-2.91	273.2	-32.46	50.37	3.78
150	-3.13	-66.67	-10.19	6.12	217.17	-33.03	34.78	22.27
200	-16.67	-66.67	-22.22	15.9	161.52	-34.29	18.77	33.97
250	-27.08	-50	-29.63	26.15	106.09	-35.98	2.52	38.47
300	-29.17	-41.67	-30.56	27.37	99.44	-36.18	0.57	38.52

Table 4 shows the impacts of $\bar{p_1}$. Here, the optimal solution for p_1 by the first-order conditions is equal to 256. Then, when $\bar{p_1}=300$, p_1^* is less than its cap; when $\bar{p_1}\leq 250$, $p_1^*=\bar{p_1}$. As shown in Table 4, Δk_1 and Δk_p may be larger than zero. This means that the capacity of Technology 1 and the total capacity of the two technologies may increase when all the customers use the TOU tariff. The reason is that the price cap for the non-peak period is set too small in that case. Then the non-peak period demand could be very large due to the small price in the non-peak period. It may lead to an increase in the capacity of Technology 1 and the total capacity of the two technologies. In fact, we find that when $\bar{p_1} \leq 200$, the demand rate in the peak period is less than that in the non-peak period, i.e., $y_{T2}/\tau < y_{T1}/(1-\tau)$. These extreme cases happen because the price cap in non-peak period is too small. The electricity company has to set a very low price in the non-peak period, under which the customers shift too much electricity from the peak period to the non-peak period. Thus, in order to effectively implement the TOU tariff, the government cannot set too small a price cap for the non-peak period. When the price cap in the non-peak period is not very small, e.g., $\bar{p_1} \geq 250$, the demand rate in the peak period is greater than that in the non-peak period, i.e., $y_{T2}/\tau > y_{T1}/(1-\tau)$, and the impacts of $\bar{p_1}$ are similar to those of $\bar{p_2}$.

5.4. Impacts of Cost Parameters

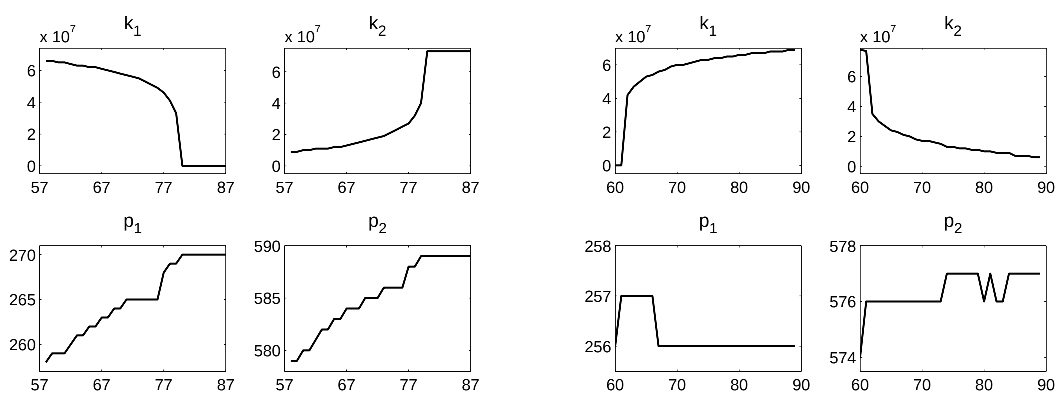
In this sub-section we study the impacts of the cost parameters, i.e., c_1, c_2, β_1, β_2, v_1, and v_2. Similar to the last sub-section, we focus on the case where all the customers use the TOU tariff. When we study the impacts of one parameter, we keep the other cost parameters unchanged.



(a)Impacts of unit capacity cost c_1 (b)Impacts of unit capacity cost c_2

Figure 1 Impacts of unit capacity costs.

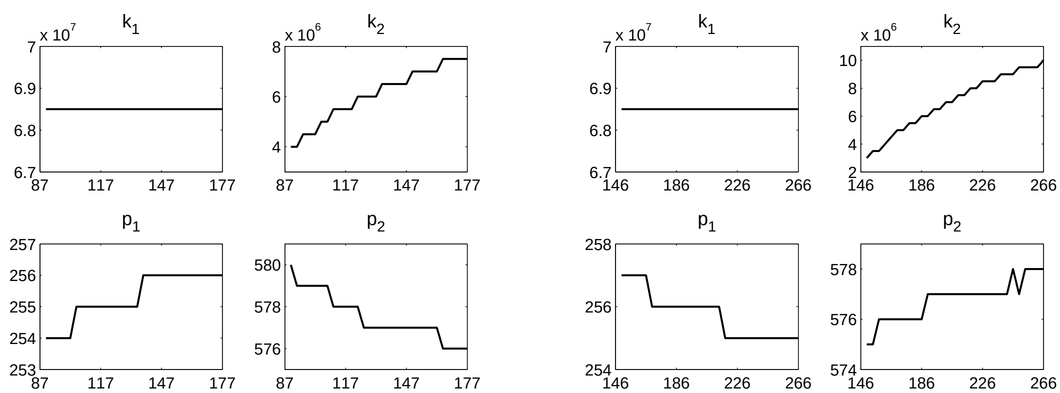
Figures 1, 2, and 3 show the impacts of the unit capacity cost, unit production cost, and unit shortage cost, respectively, on the optimal solution. In the figures, the horizontal axis is the value of a cost parameter and the vertical axis is the value of the optimal solution under the TOU tariff. It can be seen that when the capacity cost or production cost of Technology 1 (2) increases, the capacity of Technology 1 (2) will decrease and the capacity of Technology 2 (1) will increase. It is worth noting that capacity may remain unchanged, even if the costs increase. For example, as shown in Figures 1(a) and 2(a), the capacity levels remain unchanged when $c_1 \geq 47$ and $\beta_1 \geq 79$, respectively. The reason is that under those situations the costs of Technology 1 are large enough, i.e., $c_1 + \beta_1 \geq c_2 + \beta_2$, that the electricity company will only use Technology 2 to fulfill the demands. Thus, the capacity levels will remain unchanged when the costs of Technology 1 increase. This result is consistent with our analytical results. We also observe that when the shortage cost increases, the capacity of Technology 2 increases and the capacity of Technology 1 remains unchanged. This may be due to the sequence of using the technologies. Technology 2 is used if the Technology 1 cannot meet the demand and the shortage cost is incurred if both technologies cannot meet the demand. So, when the shortage cost changes, the electricity company can keep the capacity of Technology 1 unchanged and just adjusts the capacity of Technology 2.



(a)Impacts of unit production cost β_1 (b)Impacts of unit production cost β_2

Figure 2 Impacts of unit production costs.

The impacts of the cost parameters on prices are more complicated than those on capacity. It can be seen that the price in the non-peak period increases when the capacity or production cost of Technology 1 or the shortage cost in the non-peak period increases, or when the shortage cost in the peak period decreases. The price in the peak period increases when the capacity or production cost of Technology 1 increases, or when the shortage cost in the non-peak period decreases. These results are different from those in Proposition 1, where we show that for given capacity, the prices are independent of the capacity costs, and the price in the non-peak period or the price in the peak period increases with the production and shortage costs. However, we show that when we optimize the capacity and prices together, prices are affected by the capacity cost and may decrease with the production and shortage costs. We also observe that the prices change modestly when the costs increase, especially for the impacts of the costs of Technology 2 and the shortage cost in the peak period. For example, when the unit shortage cost in the peak period increases by \$120, both the prices in the non-peak and peak periods change by no more than \$5. This may be welcomed by the customers because they prefer relatively stable prices to fluctuating prices.



(a)Impacts of unit shortage cost v_1 (b)Impacts of unit shortage cost v_2

Figure 3 Impacts of unit shortage costs.

Besides, our numerical study also shows other results, which are not presented in the figures. For example, the total capacity of Technologies 1 and 2 decreases when the capacity costs or production costs increase, or when the shortage costs decrease. Regarding the expected profit, our numerical results also show that the expected profit decreases when all the cost parameters increase. For all cost values in the above numerical examples, the demand rates in the peak period are greater than the corresponding demand rates in the non-peak period, i.e., $y_{T2}/\tau > y_{T1}/(1-\tau)$.

5.5. Robustness Tests

We conducted extensive numerical work to test the robustness of the results by varying the parameters over wide ranges. In this sub-section we present the results of the robustness tests of the demand parameters, cost parameters etc.

Demand parameters. The demand parameters include the standard deviations of the two random noises σ_i, the base demand parameters a_i, and the price sensitivity parameters b_{Ti} and r_i, $i \in \{1,2\}$. For the standard deviations, we first tested many sets of (σ_1, σ_2), such as $(a_1/100, a_2/100)$, $(a_1/10, a_2/10)$, $(a_1/5, a_2/5)$ etc, to assess the impacts of α. The results are similar to those in Table 2. We also studied changes in the optimal solution and the expected profit of the electricity company by setting a series of values for σ_i from $a_i/100$ to $a_i/5$. We observe that when the standard deviations of the demands increase, the total capacity of the two technologies will increase and the expected profit will decrease, but the prices and capacity of Technology 1 and 2 may decrease or increase. By comparing the performance under the TOU tariff with that under the FR tariff, we find that the results are similar to those in Tables 3 and 4 when the optimal prices are less than their price caps. Note that in Tables 3 and 4, we show that when the prices are less than their price caps, the expected total demand under the TOU tariff is very close to that under the FR tariff, i.e., $\Delta(y_1 + y_2) = 0.57\%$. However, when σ_i varies over wide ranges, the expected total demand under the TOU tariff could be smaller than that under the FR tariff. In the numerical results, we observe that the smallest value is -0.47%, which is obtained when $\sigma_1 = a_1/5$ and $\sigma_2 = a_2/20$. With such a small value, we can still say that, when σ_i changes, there is essentially no significant reduction in the total electricity demand over the whole period by introducing the TOU tariff. Besides, in all these tests, we find that the demand rates in the peak period are greater than the corresponding demand rates in the non-peak period for both customers under the FR and TOU tariffs, i.e., $y_{F2}/\tau > y_{F1}/(1-\tau)$ and $y_{T2}/\tau > y_{T1}/(1-\tau)$.

For the base demand and price sensitivity parameters, we first define the values presented in Sub-section 5.1 as the benchmark values. That is, we let $a_1^b = 71,480,450.48$, $b_{T1}^b = 181,717.23$, $r_1^b = 12,118.13$, $a_2^b = 60,946,977.74$, $b_{T2}^b = 60,645.39$, and $r_2^b = 6,064.54$. In order to let the parameters vary over wide ranges and guarantee that $b_{Ti} > r_j, i,j \in \{1,2\}$, we let a_i change values within $\{a_i^b/100,\ a_i^b/100 + a_i^b/10,\ a_i^b/100 + 2(a_i^b/10),\ \cdots,\ a_i^b/100 + 29(a_i^b/10)\}$, let b_{Ti} change values within $\{r_1,\ r_1 + b_{Ti}^b/10,\ r_1 + 2(b_{Ti}^b/10),\ \cdots,\ r_1 + 29(b_{Ti}^b/10)\}$, and let r_i change values within $\{0,\ b_2/30,\ 2(b_2/30),\ \cdots,\ 29(b_2/30)\}$. Comparing the performance under the TOU tariff with that under the FR tariff, we still find that the results are similar to those in Tables 3 and 4. Besides, we observe that the expected total demand under the TOU tariff may be smaller than that under the FR tariff in some extreme cases, such as when a_1 is very small compared with a_2 (e.g., a_2 is several times of a_1), or when r_1 is very small, and b_1 and r_2 are very large, under which the difference in demand between the peak and non-peak periods is very large. For example, the value could be -11.16% when a_2 is about three times of a_1. Moreover, we check the demand rates and find that the demand rate in the peak period may be smaller than that in the non-peak period for the customers under the FR or TOU tariff when a_1 or r_1 is very large, or when a_2 or b_1 is very small. This may be due to the fact that under these scenarios the demand rate in the non-peak period is larger than or close to that in the peak period when only the FR tariff is offered to the customers.

Cost parameters. For the tests of the cost parameters, we changed the values to those as shown in the figures in Sub-section 5.4. By comparing the performance under the TOU and FR tariffs, we find that the results are similar to those in Tables 3 and 4. Besides, we obverse that the difference between the expected profits under the TOU and FR tariffs (i.e., $\Pi^{TOU} - \Pi^{FR}$) is concave in the capacity cost of Technology 1. This indicates that there exists a point within the range of c_1 (i.e., $c_1 = 27$) at which the TOU tariff performs the best. Regarding the other cost parameters, the TOU tariff performs better when the production cost of Technology 1 or the shortage cost in the non-peak period is smaller, or when the capacity or production cost of Technology 2 or the shortage cost in the peak period is larger. We also observe that under the TOU tariff, the electricity company needs less capacity for Technology 2 as well as less total capacity of the two technologies, but the company may need to build more capacity for Technology 1. In other words, although the TOU tariff can smoothen the demands in the two periods, building less or more capacity for Technology 1 depends on the costs of the two technologies. Moreover, for all the cost parameters we tested, the demand rate in the peak period is greater than that in the non-peak period for both the customers under the FR and TOU tariffs.

We also tested the robustness of the results regarding the parameter of the peak period time (τ). We observe that the expected total demand under the TOU tariff may be smaller than that under the FR tariff and the electricity company may need more capacity for Technology 1 when τ is very small, in which case the difference in the demand rate between the peak and non-peak periods is very large. Besides, when τ is large, the demand rates in the peak period may be smaller than those in the non-peak period for the customers under the FR or TOU tariff. This may be due to the fact that before introducing the TOU tariff, the demand rate in the non-peak period is larger than or close to that in the peak period for a given large τ.

6. Conclusions

In this paper we study the electricity TOU tariff, under which the price in the peak period is higher than that in the non-peak period. As a result, the customers will shift their electricity consumption from the peak period to the non-peak period. We consider that the demands in both the non-peak and peak periods are stochastic. The electricity company offers two tariffs to the customers, under the price-cap regulation. A fraction of the customers use the TOU tariff and the remaining fraction of the customers use the traditional FR tariff. The electricity company uses two technologies, i.e., the base-load and peak-load technologies, to generate electricity for the customers. The peak-load technology is used to meet the demand that cannot be met by the base-load technology.

We derive the optimal capacity investment and pricing policies for the electricity company. Through a numerical study based on the electricity generation and demand data from Ontario, Canada, we validate the theoretical results and generate managerial insights for the electricity company and the regulator. The TOU tariff can benefit the electricity company in terms of needing less capacity of the technologies and obtaining more profit, compared with the FR tariff. However, except for some extreme cases, there is essentially no significant reduction in the total electricity demand under the TOU tariff. The price-cap regulation significantly affects both the electricity company and customers. In order to effectively implement the TOU tariff, the regulator needs to set appropriate price caps. Besides, we also investigate the impacts of the cost parameters. We show that the total capacity of the two technologies decreases when the capacity or production costs increase, or when the shortage costs decrease. We also discuss how the capacity levels of the base-load technology and peak-load technology change when the costs change. The impacts of the cost parameters on the prices are more complicated than those on capacity. However, we observe that when the costs change, the electricity company can just change the prices over small ranges. Keeping the prices stable would be preferred by the customers to fluctuating prices. Moreover, we test the robustness of the results by varying the parameters over wide ranges. The tests show that our results hold except for a few extreme cases. For example, we show that in most cases, there is no reduction in the expected total demand of the two periods after introducing the TOU tariff.

However, when the base demand in the peak period is larger than that in the non-peak period, the expected total demand under the TOU tariff is smaller than that under the FR tariff.

The phenomenon of peak and non-peak periods exists not only in the electricity industry, but also in other industries such as the transportation and telecommunications industries. Thus, from the application perspective, the analysis of the capacity investment in various technologies, and the pricing for the peak and non-peak period demands can be applied to other industries featuring peak period and non-peak period demands.

For future research, one extension is to consider multiple electricity companies competing in the market. Some industrial examples have shown that the mandatory TOU tariff may be applied to some customers in some areas (RAP 2008, Friedman 2011, Jessoe and Rapson 2014). Based on this observation, we assume that the proportion of customers using the TOU tariff is given. However, there are examples of the case where the TOU tariff is optional to the customers (Tweed 2011). So another future research direction would be to consider the setting in which the proportion of customers using the TOU tariff is determined endogenously by the electricity prices and customers' values, based on consumers' choice behaviour. Moreover, in our model, we consider that the prices are announced at the beginning of the planning horizon and they do not change when the TOU tariff is implemented. It is interesting to extend our work to consider multiple periods and model the pricing issues as a dynamic problem in future research. Finally, it is also important to extend the study to incorporate the environmental concerns, such as CO_2 emissions, and investigate the effects of implementing the TOU tariff on the environment.

Acknowledgments

The authors thank departmental editor Qi Annabelle Feng, an anonymous senior editor, and two anonymous reviewers for their constructive comments. Ciwei Dong was supported in part by the Zhongnan University of Economics and Law (2016GS07). Chi To Ng was supported in part by the Research Grants Council of Hong Kong under grant number PolyU5012-PPR-12. T. C. E. Cheng was supported in part by The Hong Kong Polytechnic University under the Fung Yiu King - Wing Hang Bank Endowed Professorship in Business Administration.

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Online Appendix
"Electricity time-of-use tariff with stochastic demand"

Appendix A: Proofs

Proof of Lemma 1 Before showing the proof, we define $M = (\beta_1 - \beta_2)F_{11} + (\beta_2 - v_1)F_{12} \leq 0$, $N = (\beta_1 - \beta_2)F_{21} + (\beta_2 - v_2)F_{22} \leq 0$, $\hat{M} = (\beta_1 - \beta_2)f_{11} + (\beta_2 - v_1)f_{12} \leq 0$ and $\hat{N} = (\beta_1 - \beta_2)f_{21} + (\beta_2 - v_2)f_{22} \leq 0$. To prove that $\Pi(\mathbf{k}, \mathbf{p})$ is jointly concave in p_1, p_2 and p_0, we show that the Hessian matrix of the profit function $H(\Pi)$ is negative semi-definite in the following, where

$$H(\Pi) = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial p_0^2} & \frac{\partial^2 \Pi}{\partial p_0 \partial p_1} & \frac{\partial^2 \Pi}{\partial p_0 \partial p_2} \\ \frac{\partial^2 \Pi}{\partial p_1 \partial p_0} & \frac{\partial^2 \Pi}{\partial p_1^2} & \frac{\partial^2 \Pi}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi}{\partial p_2 \partial p_0} & \frac{\partial^2 \Pi}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi}{\partial p_2^2} \end{pmatrix}.$$

First, we let $|H_1^1| = \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}$, $|H_2^1| = \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}$, $|H_3^1| = \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}$,

$$|H_{12}^2| = \begin{vmatrix} \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2} & \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0 \partial p_1} \\ \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0 \partial p_1} & \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} \end{vmatrix}, |H_{13}^2| = \begin{vmatrix} \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2} & \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0 \partial p_2} \\ \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0 \partial p_2} & \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2} \end{vmatrix},$$

$$|H_{23}^2| = \begin{vmatrix} \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} & \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2} \end{vmatrix}, |H_{123}^3| = \begin{vmatrix} \frac{\partial^2 \Pi}{\partial p_0^2} & \frac{\partial^2 \Pi}{\partial p_0 \partial p_1} & \frac{\partial^2 \Pi}{\partial p_0 \partial p_2} \\ \frac{\partial^2 \Pi}{\partial p_1 \partial p_0} & \frac{\partial^2 \Pi}{\partial p_1^2} & \frac{\partial^2 \Pi}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi}{\partial p_2 \partial p_0} & \frac{\partial^2 \Pi}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi}{\partial p_2^2} \end{vmatrix}.$$

Next, we show the values of elements in the matrix. Taking the first and second partial derivatives of $\Pi(\mathbf{k}, \mathbf{p})$ with respect to p_1, p_2 and p_0, we have

$$\begin{aligned}
\frac{\partial \Pi(\mathbf{k}, \mathbf{p})}{\partial p_1} &= \alpha \Big\{ a_1 - 2b_{T1}p_1 + (r_1 + r_2)p_2 - (-b_{T1}v_1 + r_2 v_2 - b_{T1}M + r_2 N) \Big\} \\
\frac{\partial \Pi(\mathbf{k}, \mathbf{p})}{\partial p_2} &= \alpha \Big\{ a_2 - 2b_{T2}p_2 + (r_1 + r_2)p_1 - (r_1 v_1 - b_{T2}v_2 + r_1 M - b_{T2}N) \Big\} \\
\frac{\partial \Pi(\mathbf{k}, \mathbf{p})}{\partial p_0} &= (1-\alpha) \Big\{ a_1 - 2b_{F1}p_0 + a_2 - 2b_{F2}p_0 - (-b_{F1}v_1 - b_{F2}v_2 - b_{F1}M - b_{F2}N) \Big\} \\
\frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_1^2} &= \alpha \Big\{ -2b_{T1} + \alpha b_{T1}^2 \hat{M} + \alpha r_2^2 \hat{N}) \Big\} \leq 0 \\
\frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_2^2} &= \alpha \Big\{ -2b_{T2} + \alpha r_1^2 \hat{M} + \alpha b_{T2}^2 \hat{N}) \Big\} \leq 0 \\
\frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_0^2} &= (1-\alpha) \Big\{ -2b_{F1} - 2b_{F2} + (1-\alpha) b_{F1}^2 \hat{M} + (1-\alpha) b_{F2}^2 \hat{N}) \Big\} \leq 0 \\
\frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_1 \partial p_2} &= \frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_2 \partial p_1} = \alpha \Big\{ r_1 + r_2 - \alpha r_1 b_{T1} \hat{M} - \alpha r_2 b_{T2} \hat{N}) \Big\} \geq 0 \\
\frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_1 \partial p_0} &= \frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_0 \partial p_1} = \alpha(1-\alpha) \Big\{ b_{T1} b_{F1} \hat{M} - r_2 b_{F2} \hat{N}) \Big\} \\
\frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_2 \partial p_0} &= \frac{\partial^2 \Pi(\mathbf{k}, \mathbf{p})}{\partial p_0 \partial p_2} = \alpha(1-\alpha) \Big\{ -r_1 b_{F1} \hat{M} + b_{T2} b_{F2} \hat{N}) \Big\}
\end{aligned}$$

Then, combining with Assumption 1, we can obtain that

$$\begin{aligned}
|H_{12}^2| &= \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}), \mathbf{p})}{\partial p_0^2} \frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}), \mathbf{p})}{\partial p_1^2} - (\frac{\partial^2 \Pi(\mathbf{k}(\mathbf{p}), \mathbf{p})}{\partial p_0 \partial p_1})^2 \\
&= \alpha(1-\alpha) \Big\{ 4b_{T1}(b_{F1} + b_{F2}) - 2\alpha(b_{F1} + b_{F2})(b_{T1}^2 \hat{M} + r_2^2 \hat{N}) - 2(1-\alpha) b_{T1}(b_{F1}^2 \hat{M} + b_{F2}^2 \hat{N})
\end{aligned}$$

$$+\alpha(1-\alpha)(b_{F1}r_2+b_{F2}b_{T1})^2\hat{M}\hat{N}\Big\}\geq 0$$

$$\begin{aligned}
|H_{13}^2| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2})^2\\
&= \alpha(1-\alpha)\Big\{4b_{T2}(b_{F1}+b_{F2})-2\alpha(b_{F1}+b_{F2})(r_1^2\hat{M}+b_{T2}^2\hat{N})-2(1-\alpha)b_{T2}(b_{F1}^2\hat{M}+b_{F2}^2\hat{N})\\
&\quad +\alpha(1-\alpha)(b_{F1}b_{T2}+b_{F2}r_1)^2\hat{M}\hat{N}\Big\}\geq 0\\
|H_{23}^2| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2})^2\\
&= \alpha^2\Big\{4b_{T1}b_{T2}-(r_1+r_2)^2+\alpha^2(b_{T1}b_{T2}-r_1r_2)^2\hat{M}\hat{N}-2\alpha(b_{T1}b_{T2}-r_1r_2)(b_{T1}\hat{M}+b_{T2}\hat{N})\Big\}\geq 0\\
|H_{123}^3| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}[\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2})^2]\\
&\quad -\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}[\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}]\\
&\quad +\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}[\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}-\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}]\\
&= -\alpha^2(1-\alpha)\Big\{[4b_{T1}b_{T2}-(r_1+r_2)^2][2(b_{F1}+b_{F2})-(1-\alpha)(b_{F1}^2\hat{M}+b_{F2}^2)\hat{N}]\\
&\quad -4\alpha(b_{F1}+b_{F2})(b_{T1}b_{T2}-r_1r_2)(b_{T1}\hat{M}+b_{T2}\hat{N})\\
&\quad +2\alpha(b_{T1}b_{T2}-r_1r_2)[\alpha(b_{F1}+b_{F2})(b_{T1}b_{T2}-r_1r_2)+(1-\alpha)b_{F1}b_{F2}(r_1+r_2)\\
&\quad +(1-\alpha)(b_{T1}b_{F2}^2+b_{T2}b_{F1}^2)]\hat{M}\hat{N}\Big\}\leq 0
\end{aligned}$$

By now, we have proved that $|H_1^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}\leq 0$, $|H_2^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\leq 0$, $|H_3^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}\leq 0$, $|H_{12}^2|\geq 0$, $|H_{13}^2|\geq 0$, $|H_{23}^2|\geq 0$, and $|H_{123}^3|\leq 0$, indicating that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is jointly concave in p_0, p_1, and p_2. Thus, the optimal solutions can be uniquely determined by $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}=0$, $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2}=0$, and $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0}=0$, i.e.,

$$a_1-2b_{T1}p_1+(r_1+r_2)p_2 = -b_{T1}v_1+r_2v_2-b_{T1}M+r_2N, \tag{A.1}$$

$$a_2-2b_{T2}p_2+(r_1+r_2)p_1 = r_1v_1-b_{T2}v_2+r_1M-b_{T2}N, \tag{A.2}$$

$$a_1-2b_{F1}p_0+a_2-2b_{F2}p_0 = -b_{F1}v_1-b_{F2}v_2-b_{F1}M-b_{F2}N. \tag{A.3}$$

□

Proof of Proposition 1 (i) Recalling that the optimal prices can be determined by Equations (A.1), (A.2) and (A.3), which do not include c_1 and c_2. Thus, we obtain that the optimal prices are independent of c_1 and c_2, i.e., $dp_1(\mathbf{k})/dc_1=dp_1(\mathbf{k})/dc_2=dp_2(\mathbf{k})/dc_1=dp_2(\mathbf{k})/dc_2=dp_0(\mathbf{k})/dc_1=dp_0(\mathbf{k})/dc_2=0$.

(ii) In order to prove the results for $\alpha=1$, we show the values of $dp_1(\mathbf{k})/d\beta_1$, $dp_2(\mathbf{k})/d\beta_1$, $dp_1(\mathbf{k})/d\beta_2$, $dp_2(\mathbf{k})/d\beta_2$, $dp_1(\mathbf{k})/dv_1$, $dp_2(\mathbf{k})/dv_1$, $dp_1(\mathbf{k})/dv_2$, and $dp_2(\mathbf{k})/dv_2$.

We rearrange the Equations (A.1) and (A.2), and let

$$\begin{aligned}
G_1 &= a_1-2b_{T1}p_1+(r_1+r_2)p_2+b_{T1}v_1-r_2v_2+b_{T1}M-r_2N,\\
G_2 &= a_2-2b_{T2}p_2+(r_1+r_2)p_1-r_1v_1+b_{T2}v_2-r_1M+b_{T2}N.
\end{aligned}$$

By taking the first derivatives of G_1 and G_2 with respect to p_1 and p_2, we have $\frac{\partial G_1}{\partial p_1}=-2b_{T1}+b_{T1}^2\hat{M}+r_2^2\hat{N}$, $\frac{\partial G_1}{\partial p_2}=r_1+r_2-r_1b_{T1}\hat{M}-r_2b_{T2}\hat{N}$, $\frac{\partial G_2}{\partial p_1}=r_1+r_2-r_1b_{T1}\hat{M}-r_2b_{T2}\hat{N}$, and $\frac{\partial G_2}{\partial p_2}=-2b_{T2}+r_1^2\hat{M}+b_{T2}^2\hat{N}$.

Next, we first show the results for β_1. By taking the first derivatives of G_1 and G_2 with respect to β_1, we have

$$\frac{dG_1}{d\beta_1} = \frac{\partial G_1}{\partial\beta_1}+\frac{\partial G_1}{\partial p_1}\frac{dp_1(\mathbf{k})}{d\beta_1}+\frac{\partial G_1}{\partial p_2}\frac{dp_2(\mathbf{k})}{d\beta_1}=0,$$

$$\frac{dG_2}{d\beta_1} = \frac{\partial G_2}{\partial \beta_1} + \frac{\partial G_2}{\partial p_1}\frac{dp_1(\mathbf{k})}{d\beta_1} + \frac{\partial G_2}{\partial p_2}\frac{dp_2(\mathbf{k})}{d\beta_1} = 0,$$

where $\frac{\partial G_1}{\partial \beta_1} = b_{T1}F_{11} - r_2F_{21}$ and $\frac{\partial G_2}{\partial \beta_1} = -r_1F_{11} + b_{T2}F_{21}$. By solving the above two equations, we obtain that

$$\begin{aligned}\frac{dp_1(\mathbf{k})}{d\beta_1} &= \frac{(r_1-r_2)b_{T2}F_{21} + (2b_{T1}b_{T2} - r_1^2 - r_1r_2)F_{11} - (b_{T1}b_{T2} - r_1r_2)(r_1F_{21}\hat{M} + b_{T2}F_{11}\hat{N})}{4b_{T1}b_{T2} - (r_1+r_2)^2 + (b_{T1}b_{T2} - r_1r_2)[(b_{T1}b_{T2} - r_1r_2)\hat{M}\hat{N} - 2b_{T1}\hat{M} - 2b_{T2}\hat{N}]},\\ \frac{dp_2(\mathbf{k})}{d\beta_1} &= \frac{(r_2-r_1)b_{T1}F_{11} + (2b_{T1}b_{T2} - r_2^2 - r_1r_2)F_{21} - (b_{T1}b_{T2} - r_1r_2)(b_{T1}F_{21}\hat{M} + r_2F_{11}\hat{N})}{4b_{T1}b_{T2} - (r_1+r_2)^2 + (b_{T1}b_{T2} - r_1r_2)[(b_{T1}b_{T2} - r_1r_2)\hat{M}\hat{N} - 2b_{T1}\hat{M} - 2b_{T2}\hat{N}]},\end{aligned}$$

which indicate that if $r_1 \geq r_2$, then $dp_1(\mathbf{k})/d\beta_1 \geq 0$; and if $r_2 \geq r_1$, then $dp_2(\mathbf{k})/d\beta_1 \geq 0$.

Define $\Lambda = 4b_{T1}b_{T2} - (r_1+r_2)^2 + (b_{T1}b_{T2} - r_1r_2)[(b_{T1}b_{T2} - r_1r_2)\hat{M}\hat{N} - 2b_{T1}\hat{M} - 2b_{T2}\hat{N}]$. Then, by using the same approach, we can obtain the results for β_2, v_1 and v_2 as follows:

$$\begin{aligned}\frac{dp_1(\mathbf{k})}{d\beta_2} &= \frac{1}{\Lambda}\Big\{(r_1-r_2)b_{T2}(F_{22}-F_{21}) + (2b_{T1}b_{T2} - r_1^2 - r_1r_2)(F_{12}-F_{11})\\ &\quad -(b_{T1}b_{T2} - r_1r_2)[r_1(F_{22}-F_{21})\hat{M} + b_{T2}(F_{12}-F_{11})\hat{N}]\Big\},\\ \frac{dp_2(\mathbf{k})}{d\beta_2} &= \frac{1}{\Lambda}\Big\{(r_2-r_1)b_{T1}(F_{12}-F_{11}) + (2b_{T1}b_{T2} - r_2^2 - r_1r_2)(F_{22}-F_{21})\\ &\quad -(b_{T1}b_{T2} - r_1r_2)[b_{T1}(F_{22}-F_{21})\hat{M} + r_2(F_{12}-F_{11})\hat{N}]\Big\},\end{aligned}$$

which indicate that if $r_1 \geq r_2$, then $dp_1(\mathbf{k})/d\beta_2 \geq 0$, and if $r_2 \geq r_1$, then $dp_2(\mathbf{k})/d\beta_2 \geq 0$.

$$\begin{aligned}\frac{dp_1(\mathbf{k})}{dv_1} &= \frac{1}{\Lambda}\Big\{(2b_{T1}b_{T2} - r_1^2 - r_1r_2)(1-F_{12}) - (b_{T1}b_{T2} - r_1r_2)b_{T2}(1-F_{12})\hat{N}\Big\},\\ \frac{dp_2(\mathbf{k})}{dv_1} &= \frac{1}{\Lambda}\Big\{(r_2-r_1)b_{T1}(1-F_{12}) - (b_{T1}b_{T2} - r_1r_2)r_2(1-F_{12})\hat{N}\Big\},\end{aligned}$$

which indicate that $dp_1(\mathbf{k})/dv_1 \geq 0$, and if $r_2 \geq r_1$ then $dp_2(\mathbf{k})/dv_1 \geq 0$.

$$\begin{aligned}\frac{dp_1(\mathbf{k})}{dv_2} &= \frac{1}{\Lambda}\Big\{(r_1-r_2)b_{T2}(1-F_{22}) - (b_{T1}b_{T2} - r_1r_2)r_1(1-F_{22})\hat{M}\Big\},\\ \frac{dp_2(\mathbf{k})}{dv_2} &= \frac{1}{\Lambda}\Big\{(2b_{T1}b_{T2} - r_2^2 - r_1r_2)(1-F_{22}) - (b_{T1}b_{T2} - r_1r_2)b_{T1}(1-F_{22})\hat{M}\Big\}\end{aligned}$$

which indicate that $dp_2(\mathbf{k})/dv_2 \geq 0$, and if $r_1 \geq r_2$ then $dp_1(\mathbf{k})/dv_2 \geq 0$.

(iii) Next, we prove the results for $\alpha = 0$. Similar to the approach in (ii), here we take the derivatives on both sides of Equation (A.3) with respect to β_1, β_2, v_1 and v_2, respectively. By solving the resulting equations, we can obtain the values of $dp_0(\mathbf{k})/d\beta_1$, $dp_0(\mathbf{k})/d\beta_2$, $dp_0(\mathbf{k})/dv_1$, and $dp_0(\mathbf{k})/dv_2$, which are presented as follows:

$$\begin{aligned}\frac{dp_0(\mathbf{k})}{d\beta_1} &= -\frac{b_{F1}F_{11} + b_{F2}F_{21}}{-2b_{F1} - 2b_{F2} + b_{F1}^2\hat{M} + b_{F2}^2\hat{N}} \geq 0,\\ \frac{dp_0(\mathbf{k})}{d\beta_2} &= -\frac{b_{F1}(F_{12}-F_{11}) + b_{F2}(F_{22}-F_{21})}{-2b_{F1} - 2b_{F2} + b_{F1}^2\hat{M} + b_{F2}^2\hat{N}} \geq 0,\\ \frac{dp_0(\mathbf{k})}{dv_1} &= -\frac{b_{F1}(1-F_{12})}{-2b_{F1} - 2b_{F2} + b_{F1}^2\hat{M} + b_{F2}^2\hat{N}} \geq 0,\\ \frac{dp_0(\mathbf{k})}{dv_2} &= -\frac{b_{F2}(1-F_{22})}{-2b_{F1} - 2b_{F2} + b_{F1}^2\hat{M} + b_{F2}^2\hat{N}} \geq 0.\end{aligned}$$

□

Proof of Proposition 2 By using the same approach as in the proof of Proposition 1, we can obtain the following results.

(i) When $\alpha = 1$,

$$\begin{aligned}
\frac{\partial p_1(\mathbf{k})}{\partial k_1} &= \frac{1}{\Lambda}\Big\{(r_1 - r_2)b_{T2}\tau\hat{N} + (2b_{T1}b_{T2} - r_1^2 - r_1r_2)(1-\tau)\hat{M} \\
&\quad -(b_{T1}b_{T2} - r_1r_2)[r_1\tau + b_{T2}(1-\tau)]\hat{M}\hat{N}\Big\}, \\
\frac{\partial p_2(\mathbf{k})}{\partial k_1} &= \frac{1}{\Lambda}\Big\{(r_2 - r_1)b_{T1}(1-\tau)\hat{M} + (2b_{T1}b_{T2} - r_2^2 - r_1r_2)\tau\hat{N} \\
&\quad -(b_{T1}b_{T2} - r_1r_2)[b_{T1}\tau + r_2(1-\tau)]\hat{M}\hat{N}\Big\},
\end{aligned}$$

which indicate that if $r_1 \geq r_2$, then $dp_1(\mathbf{k})/dk_1 \leq 0$, and if $r_2 \geq r_1$, then $dp_2(\mathbf{k})/dk_1 \leq 0$.

$$\begin{aligned}
\frac{\partial p_1(\mathbf{k})}{\partial k_2} &= \frac{1}{\Lambda}\Big\{-(r_1 - r_2)b_{T2}\tau(v_2 - \beta_2)f_{22} - (2b_{T1}b_{T2} - r_1^2 - r_1r_2)(1-\tau)(v_1 - \beta_2)f_{12} \\
&\quad +(b_{T1}b_{T2} - r_1r_2)[r_1\tau(v_2 - \beta_2)f_{22}\hat{M} + b_{T2}(1-\tau)(v_1 - \beta_2)f_{12}\hat{N}]\Big\}, \\
\frac{\partial p_2(\mathbf{k})}{\partial k_2} &= \frac{1}{\Lambda}\Big\{-(r_2 - r_1)b_{T1}(1-\tau)(v_1 - \beta_2)f_{12} - (2b_{T1}b_{T2} - r_2^2 - r_1r_2)\tau(v_2 - \beta_2)f_{22} \\
&\quad +(b_{T1}b_{T2} - r_1r_2)[b_{T1}\tau(v_2 - \beta_2)f_{22}\hat{M} + r_2(1-\tau)(v_1 - \beta_2)f_{12}\hat{N}]\Big\},
\end{aligned}$$

which indicate that if $r_1 \geq r_2$, then $dp_1(\mathbf{k})/dk_2 \leq 0$, and if $r_2 \geq r_1$, then $dp_2(\mathbf{k})/dk_2 \leq 0$.

(ii) When $\alpha = 0$,

$$\begin{aligned}
\frac{\partial p_0(\mathbf{k})}{\partial k_1} &= -\frac{(1-\tau)b_{F1}\hat{M} + \tau b_{F2}\hat{N}}{-2b_{F1} - 2b_{F2} + b_{F1}^2\hat{M} + b_{F2}^2\hat{N}} \leq 0, \\
\frac{\partial p_0(\mathbf{k})}{\partial k_2} &= -\frac{b_{F1}(1-\tau)(\beta_2 - v_1)f_{12} + b_{F2}\tau(\beta_2 - v_2)f_{22}}{-2b_{F1} - 2b_{F2} + b_{F1}^2\hat{M} + b_{F2}^2\hat{N}} \leq 0.
\end{aligned}$$

□

Proof of Proposition 3 Note that in the proof of Lemma 1, we have obtained that $\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2} \geq 0$. Combining with the results that $\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2} \leq 0$ and $\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} \leq 0$, we obtain that $\frac{\partial p_1(p_0,p_2)}{\partial p_2} = -\frac{\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}}{\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}} \geq 0$.

□

Proof of Lemma 2 To prove the results, we show that there exists a unique global optimum for the objective function (2). Although we have obtained the optimal prices for given capacities in Lemma 1, we prove the uniqueness of the solution for the objective function in an alternative direction. That is, in the first step, we show that the objective function is jointly concave in k_1 and k_2 for given prices, and obtain the optimal capacities $\mathbf{k}(\mathbf{p})$ by the first-order conditions of $\Pi(\mathbf{k},\mathbf{p})$. In the second step, we substitute the optimal capacities back into the objective function, and show that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is jointly concave in p_0, p_1 and p_2, so the optimal prices can be uniquely determined by the first-order conditions of $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$.

(i) We first show that $\Pi(\mathbf{k},\mathbf{p})$ is jointly concave in k_1 and k_2. Taking the first and second partial derivatives to $\Pi(\mathbf{k},\mathbf{p})$ with respect of k_1 and k_2, we have

$$\begin{aligned}
\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_1} &= -\Big\{c_1 + \beta_1 - (1-\tau)v_1 - \tau v_2 + (1-\tau)(\beta_2 - \beta_1)F_{11} + (1-\tau)(v_1 - \beta_2)F_{12} \\
&\quad +\tau(\beta_2 - \beta_1)F_{21} + \tau(v_2 - \beta_2)F_{22}\Big\}, \qquad\qquad (A.4) \\
\frac{\partial^2\Pi(\mathbf{k},\mathbf{p})}{\partial k_1^2} &= -\Big\{(1-\tau)^2(\beta_2 - \beta_1)f_{11} + (1-\tau)^2(v_1 - \beta_2)f_{12} + \tau^2(\beta_2 - \beta_1)f_{21} + \tau^2(v_2 - \beta_2)f_{22}\Big\} \leq 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_2} &= -\Big\{c_2+\beta_2-(1-\tau)v_1-\tau v_2+(1-\tau)(v_1-\beta_2)F_{12}+\tau(v_2-\beta_2)F_{22}\Big\}, \qquad \text{(A.5)}\\
\frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_2^2} &= = \frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_1 \partial k_2} = -\Big\{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}\Big\} \le 0.
\end{aligned}$$

If $c_1+\beta_1>(1-\tau)v_1+\tau v_2$, then $\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_1}<0$, indicating that $\Pi(\mathbf{k},\mathbf{p})$ is decreasing in k_1, so $k_1^*=0$. Similarly, if $c_2+\beta_2>(1-\tau)v_1+\tau v_2$, then $\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_2}>0$, indicating that $\Pi(\mathbf{k},\mathbf{p})$ is decreasing in k_2, so $k_2^*=0$. Otherwise, we consider that

$$\begin{aligned}
&\frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_1^2}\frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_2^2}-\Big(\frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_1 \partial k_2}\Big)^2\\
&= \big((1-\tau)^2 f_{11}+\tau^2 f_{21}\big)(\beta_2-\beta_1)\big((1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}\big)\ge 0,
\end{aligned}$$

combining with $\frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_1^2}\le 0$ and $\frac{\partial^2 \Pi(\mathbf{k},\mathbf{p})}{\partial k_2^2}\le 0$, we conclude that $\Pi(\mathbf{k},\mathbf{p})$ is jointly concave in k_1 and k_2. Then, the optimal capacity $\mathbf{k}(\mathbf{p})$ can be uniquely determined by $\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_1}=0$ and $\frac{\partial \Pi(\mathbf{k},\mathbf{p})}{\partial k_2}=0$.

Before going to show the results in the second step, we derive the values of $\frac{\partial k_p(\mathbf{p})}{\partial p_1}$, $\frac{\partial k_p(\mathbf{p})}{\partial p_2}$, $\frac{\partial k_p(\mathbf{p})}{\partial p_0}$, $\frac{\partial k_1(\mathbf{p})}{\partial p_1}$, $\frac{\partial k_1(\mathbf{p})}{\partial p_2}$ and $\frac{\partial k_1(\mathbf{p})}{\partial p_0}$. By using the similar approach as in the proof of Proposition 1, we can obtain that

$$\begin{aligned}
\frac{\partial k_p(\mathbf{p})}{\partial p_1} &= \alpha\frac{-(1-\tau)(v_1-\beta_2)f_{12}b_{T1}+\tau(v_2-\beta_2)f_{22}r_2}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}},\\
\frac{\partial k_p(\mathbf{p})}{\partial p_2} &= \alpha\frac{(1-\tau)(v_1-\beta_2)f_{12}r_1-\tau(v_2-\beta_2)f_{22}b_{T2}}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}},\\
\frac{\partial k_p(\mathbf{p})}{\partial p_0} &= (1-\alpha)\frac{(1-\tau)(v_1-\beta_2)f_{12}b_{F1}+\tau(v_2-\beta_2)f_{22}b_{F2}}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}}\le 0,\\
\frac{\partial k_1(\mathbf{p})}{\partial p_1} &= \alpha\frac{-(1-\tau)f_{11}b_{T1}+\tau f_{21}r_2}{(1-\tau)^2 f_{11}+\tau^2 f_{21}},\\
\frac{\partial k_1(\mathbf{p})}{\partial p_2} &= \alpha\frac{(1-\tau)f_{11}r_1-\tau f_{21}b_{T2}}{(1-\tau)^2 f_{11}+\tau^2 f_{21}},\\
\frac{\partial k_1(\mathbf{p})}{\partial p_0} &= -(1-\alpha)\frac{(1-\tau)f_{11}b_{F1}+\tau f_{21}b_{F2}}{(1-\tau)^2 f_{11}+\tau^2 f_{21}}\le 0.
\end{aligned}$$

Besides, given $\mathbf{k}(\mathbf{p})$, we can obtain the values of $\frac{\partial F_{11}}{\partial p_1}$, $\frac{\partial F_{11}}{\partial p_2}$, $\frac{\partial F_{11}}{\partial p_0}$, $\frac{\partial F_{12}}{\partial p_1}$, $\frac{\partial F_{12}}{\partial p_2}$ and $\frac{\partial F_{12}}{\partial p_0}$.

$$\begin{aligned}
\frac{\partial F_{11}}{\partial p_1} &= f_{11}((1-\tau)\frac{\partial k_1(\mathbf{p})}{\partial p_1}-\frac{\partial y_1}{\partial p_1}) = \alpha\frac{f_{11}\tau f_{21}}{(1-\tau)^2 f_{11}+\tau^2 f_{21}}((1-\tau)r_2+\tau b_{T1})\ge 0,\\
\frac{\partial F_{11}}{\partial p_2} &= f_{11}((1-\tau)\frac{\partial k_1(\mathbf{p})}{\partial p_2}-\frac{\partial y_1}{\partial p_2}) = -\alpha\frac{f_{11}\tau f_{21}}{(1-\tau)^2 f_{11}+\tau^2 f_{21}}((1-\tau)b_{T2}+\tau r_1)\le 0,\\
\frac{\partial F_{11}}{\partial p_0} &= f_{11}((1-\tau)\frac{\partial k_1(\mathbf{p})}{\partial p_0}-\frac{\partial y_1}{\partial p_0}) = (1-\alpha)\frac{f_{11}\tau f_{21}}{(1-\tau)^2 f_{11}+\tau^2 f_{21}}(\tau b_{F1}-(1-\tau)b_{F2}),\\
\frac{\partial F_{12}}{\partial p_1} &= f_{12}((1-\tau)\frac{\partial k_p(\mathbf{p})}{\partial p_1}-\frac{\partial y_1}{\partial p_1})\\
&= \alpha\frac{f_{12}\tau(v_2-\beta_2)f_{22}}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}}((1-\tau)r_2+\tau b_{T1})\ge 0,\\
\frac{\partial F_{12}}{\partial p_2} &= f_{12}((1-\tau)\frac{\partial k_p(\mathbf{p})}{\partial p_2}-\frac{\partial y_1}{\partial p_2})\\
&= -\alpha\frac{f_{12}\tau(v_2-\beta_2)f_{22}}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}}((1-\tau)b_{T2}+\tau r_1)\le 0,\\
\frac{\partial F_{12}}{\partial p_0} &= f_{12}((1-\tau)\frac{\partial k_p(\mathbf{p})}{\partial p_0}-\frac{\partial y_1}{\partial p_0})\\
&= (1-\alpha)\frac{f_{12}\tau(v_2-\beta_2)f_{22}}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}}(\tau b_{F1}-(1-\tau)b_{F2}).
\end{aligned}$$

Moreover, we can find that

$$\frac{\frac{\partial F_{11}}{\partial p_1}}{\alpha[\tau b_{T1}+(1-\tau)r_2]} = -\frac{\frac{\partial F_{11}}{\partial p_2}}{\alpha[\tau r_1+(1-\tau)b_{T2}]} = \frac{\frac{\partial F_{11}}{\partial p_0}}{(1-\alpha)[\tau b_{F1}-(1-\tau)b_{F2}]}$$

$$
\begin{aligned}
&= \frac{f_{11}\tau f_{21}}{(1-\tau)^2 f_{11}+\tau^2 f_{21}} \geq 0, && \text{(A.6)}\\
\frac{\frac{\partial F_{12}}{\partial p_1}}{\alpha[\tau b_{T1}+(1-\tau)r_2]} &= -\frac{\frac{\partial F_{12}}{\partial p_2}}{\alpha[\tau r_1+(1-\tau)b_{T2}]} = \frac{\frac{\partial F_{12}}{\partial p_0}}{(1-\alpha)[\tau b_{F1}-(1-\tau)b_{F2}]}\\
&= \frac{f_{12}\tau(v_2-\beta_2)f_{22}}{(1-\tau)^2(v_1-\beta_2)f_{12}+\tau^2(v_2-\beta_2)f_{22}} \geq 0. && \text{(A.7)}
\end{aligned}
$$

Now, we are ready to show the results that the prices can be uniquely determined by the first-order conditions of $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$.

(ii) To show that the prices can be uniquely determined by the first-order conditions, we just need to show that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is jointly concave in p_0, p_1 and p_2. Thus, we will prove that the Hessian matrix of the profit function $H(\Pi)$ is negative semi-definite in the following, where

$$
H(\Pi)=\begin{pmatrix}
\frac{\partial^2\Pi}{\partial p_0^2} & \frac{\partial^2\Pi}{\partial p_0\partial p_1} & \frac{\partial^2\Pi}{\partial p_0\partial p_2}\\
\frac{\partial^2\Pi}{\partial p_1\partial p_0} & \frac{\partial^2\Pi}{\partial p_1^2} & \frac{\partial^2\Pi}{\partial p_1\partial p_2}\\
\frac{\partial^2\Pi}{\partial p_2\partial p_0} & \frac{\partial^2\Pi}{\partial p_2\partial p_1} & \frac{\partial^2\Pi}{\partial p_2^2}
\end{pmatrix}.
$$

First, we let $|H_1^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}$, $|H_2^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}$, $|H_3^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}$,

$$
|H_{12}^2|=\begin{vmatrix}
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2} & \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1} & \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}
\end{vmatrix}, |H_{13}^2|=\begin{vmatrix}
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2} & \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2} & \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}
\end{vmatrix},
$$

$$
|H_{23}^2|=\begin{vmatrix}
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} & \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2} & \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}
\end{vmatrix}, |H_{123}^3|=\begin{vmatrix}
\frac{\partial^2\Pi}{\partial p_0^2} & \frac{\partial^2\Pi}{\partial p_0\partial p_1} & \frac{\partial^2\Pi}{\partial p_0\partial p_2}\\
\frac{\partial^2\Pi}{\partial p_1\partial p_0} & \frac{\partial^2\Pi}{\partial p_1^2} & \frac{\partial^2\Pi}{\partial p_1\partial p_2}\\
\frac{\partial^2\Pi}{\partial p_2\partial p_0} & \frac{\partial^2\Pi}{\partial p_2\partial p_1} & \frac{\partial^2\Pi}{\partial p_2^2}
\end{vmatrix}.
$$

Next, we show the values of elements in the matrix. After substituting $(k_1(\mathbf{p}),k_2(\mathbf{p}))$ back into $\Pi(\mathbf{k},\mathbf{p})$ and taking the first derivative of $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ with respect to p_1, we have

$$
\begin{aligned}
\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1} &= \frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_1}\frac{\partial k_1(\mathbf{p})}{\partial p_1}+\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_2}\frac{\partial k_2(\mathbf{p})}{\partial p_1}+\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_1}\\
&= \frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial p_1}|_{\mathbf{k}=\mathbf{k}(\mathbf{p})}\\
&= \alpha\Big\{a_1-2b_{T1}p_1+(r_1+r_2)p_2-\Big\{-b_{T1}v_1+r_2v_2-b_{T1}\big((\beta_1-\beta_2)F_{11}+(\beta_2-v_1)F_{12}\big)\\
&\quad +r_2\big((\beta_1-\beta_2)F_{21}+(\beta_2-v_2)F_{22}\big)\Big\}\Big\}.
\end{aligned}
$$

The second equality holds because $\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_1}=\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_2}=0$ when $\mathbf{k}=\mathbf{k}(\mathbf{p})$.

From the first-order condition $\frac{\partial\Pi(\mathbf{k},\mathbf{p})}{\partial k_1}=0$, we can obtain that

$$
(\beta_1-\beta_2)F_{21}+(\beta_2-v_2)F_{22} = \frac{1}{\tau}(c_1+\beta_1-(1-\tau)v_1-\tau v_2)+\frac{1-\tau}{\tau}(\beta_2-\beta_1)F_{11}+\frac{1-\tau}{\tau}(v_1-\beta_2)F_{12}.
$$

Combining it with $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}$, we can obtain that

$$
\begin{aligned}
\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1} &= \alpha\Big\{a_1-2b_{T1}p_1+(r_1+r_2)p_2\\
&\quad -\Big\{\frac{r_2}{\tau}(c_1+\beta_1)+(b_{T1}+\frac{1-\tau}{\tau}r_2)\big((\beta_2-\beta_1)F_{11}+(v_1-\beta_2)F_{12}-v_1\big)\Big\}\Big\}.
\end{aligned}
$$

Combining it with Equations (A.6) and (A.7), we can obtain that

$$
\begin{aligned}
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_0}=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1} &= -\alpha(b_{T1}+\frac{1-\tau}{\tau}r_2)\big((\beta_2-\beta_1)\frac{\partial F_{11}}{\partial p_0}+(v_1-\beta_2)\frac{\partial F_{12}}{\partial p_0}\big)\\
&= -\frac{1-\alpha}{\tau}(\tau b_{F1}-(1-\tau)b_{F2})\Theta,\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2} &= \alpha\Big\{-2b_{T1}-(b_{T1}+\frac{1-\tau}{\tau}r_2)\Theta\Big\}\leq 0,\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2\partial p_1} &= \alpha\Big\{r_1+r_2-(b_{T1}+\frac{1-\tau}{\tau}r_2)\big((\beta_2-\beta_1)\frac{\partial F_{11}}{\partial p_2}+(v_1-\beta_2)\frac{\partial F_{12}}{\partial p_2}\big)\Big\}\\
&= \alpha\Big\{r_1+r_2+(r_1+\frac{1-\tau}{\tau}b_{T2})\Theta\Big\}\geq 0,
\end{aligned}
$$

where $\Theta=(\beta_2-\beta_1)\frac{\partial F_{11}}{\partial p_1}+(v_1-\beta_2)\frac{\partial F_{12}}{\partial p_1}\geq 0$.

Similarly, we can obtain that

$$
\begin{aligned}
\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2} &= \alpha\Big\{a_2-2b_{T2}p_2+(r_1+r_2)p_1-\Big\{-\frac{b_{T2}}{\tau}(c_1+\beta_1)\\
&\quad -(r_1+\frac{1-\tau}{\tau}b_{T2})\big((\beta_2-\beta_1)F_{11}+(v_1-\beta_2)F_{12}-v_1\big)\Big\}\Big\},\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2\partial p_0}=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2} &= \frac{1-\alpha}{\tau}\frac{\tau r_1+(1-\tau)b_{T2}}{\tau b_{T1}+(1-\tau)r_2}[\tau b_{F1}-(1-\tau)b_{F2}]\Theta,\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2} &= \alpha\Big\{-2b_{T2}-\frac{[\tau r_1+(1-\tau)b_{T2}]^2}{\tau[\tau b_{T1}+(1-\tau)r_2]}\Theta\Big\}\leq 0,\\
\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0} &= (1-\alpha)\Big\{a_1-2b_{F1}p_0+a_2-2b_{F2}p_0-\Big\{-\frac{b_{F2}}{\tau}(c_1+\beta_1)\\
&\quad +(b_{F1}-\frac{1-\tau}{\tau}b_{F2})\big((\beta_2-\beta_1)F_{11}+(v_1-\beta_2)F_{12}-v_1\big)\Big\}\Big\},\\
\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2} &= (1-\alpha)\Big\{-2(b_{F1}+b_{F2})-\frac{(1-\alpha)[\tau b_{F1}-(1-\tau)b_{F2}]^2}{\alpha\tau[\tau b_{T1}+(1-\tau)r_2]}\Theta\Big\}\leq 0.
\end{aligned}
$$

Then, combining with Assumption 1, we can obtain that

$$
\begin{aligned}
|H_{12}^2| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1})^2\\
&= 4\alpha(1-\alpha)b_{T1}(b_{F1}+b_{F2})+\Big\{\frac{2\alpha(1-\alpha)}{\tau}(b_{F1}+b_{F2})[\tau b_{T1}+(1-\tau)r_2]\\
&\quad +\frac{2b_{T1}(1-\alpha)^2[\tau b_{F1}-(1-\tau)b_{F2}]^2}{\tau[\tau b_{T1}+(1-\tau)r_2]})\Big\}\Theta\geq 0,\\
|H_{13}^2| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2})^2\\
&= 4\alpha(1-\alpha)b_{T2}(b_{F1}+b_{F2})+\frac{2\Theta}{\tau[\tau b_{T1}+(1-\tau)r_2]}\Big\{\alpha(1-\alpha)(b_{F1}+b_{F2})[\tau r_1+(1-\tau)b_{T2}]^2\\
&\quad +b_{T2}(1-\alpha)^2[\tau b_{F1}-(1-\tau)b_{F2}]^2\Big\}\geq 0,\\
|H_{23}^2| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2})^2\\
&= \alpha^2\Big\{4b_{T1}b_{T2}-(r_1+r_2)^2+2(b_{T1}b_{T2}-r_1r_2)(\frac{(1-\tau)[\tau r_1+(1-\tau)b_{T2}]}{\tau[\tau b_{T1}+(1-\tau)r_2]}+1)\Theta\Big\}\geq 0,\\
|H_{123}^3| &= \frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}[\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2})^2]\\
&\quad -\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}[\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}-\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}]\\
&\quad +\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}[\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1\partial p_2}-\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_2}]\\
&= -\alpha^2[4b_{T1}b_{T2}-(r_1+r_2)^2]\Big\{2(1-\alpha)(b_{F1}+b_{F2})+\frac{(1-\alpha)^2[\tau b_{F1}-(1-\tau)b_{F1}]^2}{\alpha\tau[\tau b_{T1}+(1-\tau)r_2]}\Theta\Big\}
\end{aligned}
$$

$$-4\alpha^2(1-\alpha)(b_{F1}+b_{F2})(b_{T1}b_{T2}-r_1r_2)\Big\{\frac{(1-\tau)[\tau r_1+(1-\tau)b_{T2}]}{\tau[\tau b_{T1}+(1-\tau)r_2]}+1\Big\}\Theta\le 0.$$

By now, we have proved that $|H_1^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}\le 0$, $|H_2^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}\le 0$, $|H_3^1|=\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2^2}\le 0$, $|H_{12}^2|\ge 0$, $|H_{13}^2|\ge 0$, $|H_{23}^2|\ge 0$, and $|H_{123}^3|\le 0$, indicating that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is jointly concave in p_0, p_1, and p_2. Thus, the optimal solutions can be uniquely determined by the first-order conditions of $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$.

Combining (i) and (ii), we can conclude that the optimal prices and capacities for the objective function (2) can be uniquely determined by the first-order conditions of the objective functions.

□

Proof of Proposition 4 Recalling that in the proof of Lemma 2, we show that $|H_{23}^2|\ge 0$. However, if $b_{T1}=r_2$ and $b_{T2}=r_1$, then $|H_{23}^2|=-\alpha^2(b_{T1}-b_{T2})^2\le 0$, indicating that the objective function is not jointly concave in the prices. Then we can obtain the optimal prices by a sequential decision approach. That is, we first derive the optimal p_1 and p_0 for a given p_2, then we substitute the optimal responses of p_1 and p_0 into the objective function and derive the optimal p_2.

(i) Given p_2, by following the similar approach as in the proof of Lemma 2, we can show that

$$\begin{aligned}
&\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}=\alpha\Big\{-2b_{T1}-\frac{b_{T1}}{\tau}M\Big\}\le 0,\\
&\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}=(1-\alpha)\Big\{-2(b_{F1}+b_{F2})-\frac{(1-\alpha)[\tau b_{F1}-(1-\tau)b_{F2}]^2}{\alpha\tau b_{T1}}\Theta\Big\}\le 0,\\
&\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0^2}\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1^2}-(\frac{\partial^2\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0\partial p_1})^2=4\alpha(1-\alpha)b_{T1}(b_{F1}+b_{F2})\\
&\quad+\Big\{\frac{2\alpha(1-\alpha)}{\tau}(b_{F1}+b_{F2})b_{T1}+\frac{2(1-\alpha)^2[\tau b_{F1}-(1-\tau)b_{F2}]^2}{\tau})\Big\}\Theta\ge 0.
\end{aligned}$$

It implies that $\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})$ is jointly concave in p_1 and p_0 for a given p_2. Then, the optimal p_1 and p_0 can be obtained by solving Equations (A.1) and (A.3).

(ii) Next, we prove that the upper bound on p_2 is optimal. By substituting the optimal p_1 and p_0 into the objective function, and taking the first derivative of the objective function with respect to p_2, we obtain that

$$\begin{aligned}
\frac{d\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{dp_2}&=\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}\frac{dp_1(p_2)}{dp_2}+\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0}\frac{dp_0(p_2)}{dp_2}+\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2}\\
&=\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_2}|_{\{p_1=p_1(p_2),p_0=p_0(p_2)\}}\\
&=\alpha\Big\{a_2-2b_{T2}p_2+(r_1+r_2)p_1\\
&\quad-\Big\{-\frac{b_{T2}}{\tau}(c_1+\beta_1)-(r_1+\frac{1-\tau}{\tau}b_{T2})\big((\beta_2-\beta_1)F_{11}+(v_1-\beta_2)F_{12}-v_1\big)\Big\}\Big\}.
\end{aligned}$$

The second equality holds because $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}=0$ and $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_0}=0$ when $p_1=p_1(p_2)$ and $p_0=p_0(p_2)$. Combining it with $\frac{\partial\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{\partial p_1}=0$, we obtain that

$$\begin{aligned}
\frac{d\Pi(\mathbf{k}(\mathbf{p}),\mathbf{p})}{dp_2}&=\frac{1}{b_{T1}+\frac{1-\tau}{\tau}r_2}\Big\{(b_{T1}+\frac{1-\tau}{\tau}r_2)\big(a_2-2b_{T2}p_2+(r_1+r_2)p_1+\frac{b_{T2}}{\tau}(c_1+\beta_1)\big)\\
&\quad+(r_1+\frac{1-\tau}{\tau}b_{T2})\big(a_1-2b_{T1}p_1+(r_1+r_2)p_2-\frac{r_2}{\tau}(c_1+\beta_1)\big)\Big\}.
\end{aligned}$$

Given $b_{T1} = r_2$ and $b_{T2} = r_1$, we obtain that

$$\frac{d\Pi(\mathbf{k}(\mathbf{p}), \mathbf{p})}{dp_2} = \frac{1}{b_{T1}}(b_{T1}y_2 + b_{T2}y_1) \geq 0,$$

implying that given the optimal responses of p_1 and p_0, the objective function is increasing in p_2. Thus, the upper bound on p_2 is optimal for the electricity company. $\square$