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Container Liner Fleet Deployment: A Systematic Overview

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Abstract

Container liner fleet deployment (CLFD) is the assignment of containerships to port rotations (ship routes) for efficient transport of containers. As liner shipping services have fixed schedules, the ship-related operating cost is determined at the CLFD stage. This paper provides a critical review of existing mathematical models developed for the CLFD problems. It first gives a systematic overview of the fundamental assumptions used by the existing CLFD models. The operating characteristics dealt with in existing studies are then examined, including container transshipment and routing, uncertain demand, empty container repositioning, ship sailing speed optimization and ship repositioning. Finally, this paper points out four important future research opportunities: fleet deployment considering ship surveys and inspections, service dependent demand, pollutant emissions, and CLFD for shipping alliances.

Key Words: Container liner fleet deployment; Transshipment; Green shipping; Mathematical programming models

1 Introduction

Container transportation is vital to the world trade and world economy. The total container trade volume amounted in 175 million twenty-foot equivalent units (TEUs) in 2015 (UNCTAD, 2016). Containers are usually transported by liner shipping services with fixed sequences of ports of call at a regular service frequency, which are published by liner shipping companies in advance to attract more cargoes of shippers. Shippers or freight forwarders can pick up and deliver their cargoes at the desired ports. A single shipper usually has far less than a full shipload of cargo. Containerships keep to their published departure dates even when a full payload is not available (Christiansen et al., 2004, 2013). This study focuses on container liner shipping rather than other liner shipping modes such as roll-on-roll-off (RoRo) shipping for cars (Øvstebø et al., 2011). Fig. 1 depicts a liner shipping network consisting of three ship routes with fixed port rotations. When a containership is assigned to a liner ship route, it usually serves the ship route for a period of at least three to six months. Since liner shipping services have fixed port rotations and schedules, ship-related operating cost is determined after the ship-to-route assignment. Moreover, containerships are large as liner shipping companies aim to take advantage of their economies of scale. For example, the average containership size was 3,801 TEUs at the end of July 2016 (UNCTAD, 2016). Therefore, it is important for container liner shipping companies to assign ships to port rotations in an efficient manner to transport containers. This tactical decision problem is referred to as container liner fleet deployment (CLFD).

<Insert Figure 1 here>

A number of studies have been devoted to the CLFD problem due to its importance. In this paper, we give a comprehensive overview on model building of the problem and point out future research directions. As most of the mathematical formulations are mixed-integer linear programming models, or can be transformed to mixed-integer linear programming models, with a few exceptions using decomposition approaches, they are generally solved by commercial mixed-integer linear programming solvers. Consequently, we do not discuss how to solve the models in this paper.

CLFD is often explicitly or implicitly incorporated in liner shipping service network design, which determines the routes in a network and the deployment of ships to each route. We also mention the studies on liner shipping service network design if their models are highly relevant to CLFD. Some works, such as Andersson et al. (2015), Norstad et al. (2015), Bakkehaug et al. (2016) and Chandra et al. (2016), focus on fleet deployment for ships other than containerships; these works are not reviewed.

The remainder of the paper is organized as follows. Section 2 examines the assumptions in container liner fleet deployment models so that practitioners understand the limitations of the models before putting them to use. Section 3 investigates early CLFD models in which container transshipment or routing are not incorporated. Section 4 focuses on CLFD models for modern global liner shipping companies with container transshipment and routing. Depending on how container routing is formulated, the models are classified as path-based, origin-to-destination-link-based, and origin-link-based. Section 5 reviews models for handling the uncertainty of the container shipment demand, including chance-constrained models and stochastic optimization models. Section 6 is focused on incorporating both laden and empty containers in CLFD. Section 7 analyzes how to relax the assumption of fixed sailing speed of ships. Section 8 discusses ship repositioning in CLFD. Section 9 points out future research directions. Throughout the paper, we present models with a unified notation system in Table 1. The models in the literature are classified according to practical features incorporated and the most relevant studies are summarized in Table 2.

<Insert Table 1 here>

<Insert Table 2 here>

2 Typical assumptions in existing container liner fleet deployment models

Existing studies on CLFD generally aim to minimize the total cost for transporting a given volume of containers or maximize the total profit by developing optimization models. The inputs mainly include the port rotations, the fleet and mix, and the container shipment demand. The decision variable is which ship to serve which port rotation (fleet deployment). Moreover, how the containers are transported by the ships (container routing) is also an auxiliary decision variable. Before appreciating the mathematical models, we first discuss the following typical assumptions used in most of the models.

Assumption (i): The port rotations in the container liner shipping network have been given. There are studies that investigate the design of port rotations, i.e., liner shipping network design (Agarwal and Ergun, 2008; Alvarez, 2009; Reinhardt and Pisinger, 2012). Liner shipping network design is a strategic-level decision problem (Meng et al., 2014) that needs to consider factors beyond the scope of fleet deployment, for example, the marketing strategy of the liner shipping company (e.g., whether it should target Asia-Europe trade or intra-Asia trade), the ownership of the company (e.g., APL is a Singapore-based company and hence it will use Singapore rather than Malaysian ports as its transshipment hub), the associated vertical businesses (e.g., APL operates container terminals at Kaohsiung and hence it will tranship containers at Kaohsiung rather than ports such as Xiamen), joint services with alliance members (e.g., Hapag-Lloyd, NYK and OOCL merged some of their Asia-Europe services and hence OOCL could not unilaterally decide which port to visit). Therefore, the liner shipping network design problem is significantly different from CLFD and it is reasonable to assume in the CLFD that the port rotations are given (Zacharioudakis et al., 2011). Of course, the liner shipping network design problem is also important for liner shipping companies (Brouer et al., 2014).

Assumption (ii): The fleet (size and mix) is given and ships are classified into different types: ships in each type are homogeneous in terms of capacity and cost structure. A liner shipping company has a fixed set of ships that can be deployed and hence the fleet is given. In some studies (e.g., Meng and Wang, 2010; Meng et al., 2012), the liner shipping company can charter in additional ships if needed or charter out unused ships for profit. In reality, ships in each type cannot be identical because of their different capacities, structures, ages, and past operating conditions, unless each type has only one ship. Nevertheless, the classification of ships into types is a helpful for model building. Considering that the CLFD is a tactical decision problem, the classification of ships into types is acceptable.

Assumption (iii): The container shipment demand is exogenous and independent of service factors such as the transit time and freight rate. There are two reasons behind this assumption: First, the relation between demand and transit time or freight rate is difficult to estimate. Second, it is much more challenging to model service dependent demand than exogenous demand. We will discuss this issue more in Section 9.

Assumption (iv): Ports can provide services whenever a ship arrives. CLFD models usually assume that ships sail at a constant speed during a round-trip journey, and hence based on the speed the bunker consumption and thereby bunker cost can be calculated. In reality, ports may be busy or may not work during particular days. Therefore, the sailing speed obtained from CLFD models cannot directly be applied and must be adjusted to fit into port time windows. The additional cost incurred by the adjustment of speed is insignificant compared with the proportion of operating cost determined at the CLFD stage. Hence, the assumption that ports are always ready for service is acceptable.

Assumption (v): All types of containers are converted into TEUs, and the number of TEUs to transport is formulated as a continuous variable rather than an integer. This assumption is reasonable in that the number of containers dealt with is usually several tens or several hundreds and the estimation error in container shipment demand is much larger than the error caused by rounding up the number of containers to integers. The numerical experiments of Wang (2013) substantiate the argument that treating the volume of containers as a continuous number of TEUs does not have a significant impact on the fleet deployment decisions.

Assumption (vi): Ships can immediately serve any port rotation. In reality, ships owned by a liner shipping company are scattered all over the world. If a ship located in America needs to serve an Asia-Europe service, it has to be repositioned to the Asia-Europe trade lane, which requires cost and time. We will discuss this issue in Section 8.

Assumption (vii): All relevant parameters are known, such as bunker price, port charges, freight rate, currency exchange rate, canal dues, and container shipment demand. In reality, these parameters may change every day. However, for modeling purposes, estimated values, either fixed or stochastic with known probability distribution, are used.

3 Fleet deployment without container transshipment or routing

The majority of the pioneering studies on CLFD made by Perakis and his research collaborators (Perakis, 1985, 2002; Papadakis and Perakis, 1989; Jaramillo and Perakis, 1991; Perakis and Jaramillo, 1991; Cho and Perakis, 1996; Powell and Perakis, 1997) do not take into account container transshipment operations. They assume that each ship route has to fulfill its container shipment demand and provide at least a given number of voyages in a planning horizon. The basic model of these studies is presented below.

Consider a set $\mathcal{R}$ of port rotations (ship routes), regularly serving a group of ports denoted by the set $\mathcal{P}$. Port rotation $r \in \mathcal{R}$ can be expressed as:

$$p_{r1} \to p_{r2} \to \cdots \to p_{rN_r} \to p_{r1} \tag{1}$$

where N_r is the number of ports of call and p_{ri} is the physical port corresponding to the ith port of call, $i = 1, 2, \cdots, N_r$. Define $\mathcal{I}_r := \{1, 2, \cdots, N_r\}$. The voyage from port i to port $i+1$ is called leg i and leg N_r is the voyage from port N_r to port 1. In Fig. 1 three port rotations are shown: port rotation 1 has three legs, port rotation 2 has five legs, and port rotation 3 has three legs. The fleet deployment plan covers a horizon of T days. The container shipment demand from port i to port j on ship route r in the planning horizon is denoted by q_r^{ij} (TEUs). The types of ships in the fleet is denoted by $\mathcal{V}$. The number of ships in type $v \in \mathcal{V}$ is m_v and the container capacity of a ship is V_v (TEUs). A ship in type v can complete n_{rv} round trips if it is deployed on port rotation r and the operating cost of one round trip is $\hat{c}_{rv}$. At least n_r round trips must be completed on port rotation r in the planning horizon. Once a ship is deployed on a port rotation, it cannot serve other port rotations in the planning horizon. The container liner shipping company needs to determine how many ships of each type to deploy on each port rotation, and how many voyages the ships in each type should complete on each port rotation, to transport all containers at minimum cost.

Let x_{rv} be the decision variable of the number of ships in type v deployed on port rotation r and y_{rv} be the decision variable of the number of round trips completed by ships in type v

on port rotation r. Define $\mathbb{Z}^+$ as the set of nonnegative integers. This basic CLFD model can be formulated as an integer linear programming problem:

[P1]
$$\min_{x_{rv}, y_{rv}} \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} \hat{c}_{rv} y_{rv} \tag{2}$$

subject to:

$$y_{rv} \le n_{rv} x_{rv}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V} \tag{3}$$

$$\sum_{v \in \mathcal{V}} y_{rv} \ge n_r, \forall r \in \mathcal{R} \tag{4}$$

$$\sum_{r \in \mathcal{R}} x_{rv} \le m_v, \forall v \in \mathcal{V} \tag{5}$$

$$\sum_{j \le i} \sum_{k \ge i+1} q_r^{jk} + \sum_{j \le i} \sum_{k < j} q_r^{jk} + \sum_{j \ge i+2} \sum_{i+1 \le k < j} q_r^{jk} \le \sum_{v \in \mathcal{V}} V_v y_{rv}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6}$$

$$x_{rv}, y_{rv} \in \mathbb{Z}^+, \forall r \in \mathcal{R}, \forall v \in \mathcal{V} \tag{7}$$

The objective function (2) minimizes the total operating cost in the planning horizon. Constraint (3) limits number of voyages that a ship can complete. Constraint (4) enforces the minimum number of voyages required on each port rotation. Constraint (5) requires that the number of ships used cannot exceed the available number in the fleet. Constraint (6) imposes ship capacity constraint on each leg of each port rotation, where the left-hand side is the total volume of containers on leg i of port rotation r. Constraint (7) defines x_{rv} and y_{rv} as nonnegative integer variables. Note that constraint (6) can be rewritten as:

$$\bar{V}_r \le \sum_{v \in \mathcal{V}} V_v y_{rv}, \forall r \in \mathcal{R} \tag{8}$$

where $\bar{V}_r$ is the shipping capacity required on port rotation r:

$$\bar{V}_r := \max_{i \in \mathcal{I}_r} \left\{ \sum_{j \le i} \sum_{k \ge i+1} q_r^{jk} + \sum_{j \le i} \sum_{k < j} q_r^{jk} + \sum_{j \ge i+2} \sum_{i+1 \le k < j} q_r^{jk} \right\}, \forall r \in \mathcal{R} \tag{9}$$

In addition to the basic model (2)-(7), some studies also required a minimum number of ship layup days in the planning horizon, incorporated the possibility of ship chartering and examined simple cases of speed optimization. In particular, Cho and Perakis (1996) allowed containers from their origin port to their destination port to be split among several ship routes. For example, In Fig. 1, some containers from Singapore to Hong Kong can be transported on ship route 1, and the others are transported on ship route 2. Ng (2017) took into account that, in a finite horizon, it is possible that e.g. 4.3 trips are completed but that 0.3 trip can still be used to transport some containers. In model [P1], it is assumed that the sailing speed of each ship is given a priori. Nowadays because of the high bunker fuel price, sailing speed has become an important planning decision (Bell and Bichou, 2008; Du et al., 2011, Norstad et al., 2011). We will discuss how to incorporate sailing speed optimization in CLFD models in Section 7.

4 Fleet deployment with container transshipment and routing

In recent years, the sizes of containerships are increased and large ships have to connect with feeder services. Consequently, more and more containers are transshipped. Transshipment complicates the formulation and solution of models for delivery of containers because in a liner shipping network there are many paths (routes) to transport a container from its origin port to its destination port. For example, a container from Colombo to Hong Kong in Fig. 1 can be transported on ship route 2, or first transported to Singapore on ship route 2, and then transported from Singapore to Hong Kong on ship route 1. Hence, container transshipment and routing are incorporated in most of the recent studies. There are a few studies that consider transshipment but no routing because there is only one path for each port pair (e.g., Fagerholt, 1999, 2004; Mourão et al., 2001; Fagerholt et al., 2009; Gelareh and Pisinger, 2011). From the fleet deployment point of view, these studies are similar to [P1]. Another issue that is included in recent studies is weekly service frequency because large liner shipping companies generally provide at least a weekly service frequency to ensure the level of service (Brouer et al., 2013).

To formulate the complex container transshipment and routing operations and weekly frequency, we need more notation. Represent by $\mathcal{W}$ the set of origin-to-destination (O-D) port pairs, $\mathcal{W} \subset \mathcal{P} \times \mathcal{P}$. The demand for O-D pair $(o,d) \in \mathcal{W}$ is denoted by q^{od} (TEUs/week). The penalty cost for not shipping a container is g^{od} (USD/TEU). Containers can be transshipped at any port from origins to destinations. The load, transshipment and discharge costs (USD/TEU) at port $p \in \mathcal{P}$ are denoted by $\hat{c}_p$, $\overline{c}_p$ and $\tilde{c}_p$, respectively. In practice $\overline{c}_p < \hat{c}_p + \tilde{c}_p$ because less paper work is needed in transshipment and shipping lines have more freedom to choose transshipment ports. A total of m_{rv} ships in type v are needed to maintain a weekly frequency of port rotation r and the weekly cost of operating such a string is c_{rv}

(USD/week). If port rotation r is operated, then at least a weekly service frequency must be maintained, i.e., a weekly frequency, a twice-weekly frequency, or a thrice-weekly frequency. The objective is to determine how to deploy containerships and how to transport containers to minimize the sum of ship operating cost, container handling cost, and penalty for not fulfilling the demand.

Depending on how to formulate container routing, there are three types of CLFD models: path-based, O-D-link-based, and origin-link-based. We elaborate on these three types of models below. It should be mentioned that container routing is also an auxiliary decision in other planning problems such as network design and schedule construction.

4.1 Path-based fleet deployment model

The most straightforward approach to formulate container flow is to use container paths. For example, Liu et al. (2011) and Meng and Wang (2012) have used a path flow formulation in the CLFD. Song and Dong (2012) used a path flow formulation for container flow optimization. Here a container path is a route on which containers are transported by ships. For example, the followings are three container paths with respect to the ship routes shown in Fig. 1:

$$h_1 = p_{1,3}(\text{SG}) \xrightarrow{\text{Ship Route 1}} p_{1,1}(\text{HK}) \tag{10}$$

$$h_2 = p_{2,5}(\text{SG}) \xrightarrow{\text{Ship Route 2}} p_{2,1}(\text{HK}) \tag{11}$$

$$h_3 = p_{2,2}(\text{XM}) \xrightarrow{\text{Ship Route 2}} p_{2,4}(\text{CB}) \mapsto p_{3,1}(\text{CB}) \xrightarrow{\text{Ship Route 3}} p_{3,2}(\text{CN}) \tag{12}$$

Container path h_1 is used to directly deliver containers from Singapore to Hong Kong which are loaded at the 3rd port of call of the ship route 1 (Singapore) and discharged at the 1st port of call of the ship route 1 (Hong Kong). Containers along the container path h_2 are delivered by the ship route 2. Container path h_3 involves container transshipment operations: containers are first loaded at the 2nd port of call of the ship route 2 (Xiamen) and delivered to the 4th port of call of the ship route 2 (Colombo). At Colombo, these containers are discharged and reloaded (transshipped) to a ship deployed on ship route 3, and transported to their destination, Chennai.

The set of container paths for O-D $(o,d) \in \mathcal{W}$ is denoted by $\mathcal{H}^{od}$. The container handling cost of $h \in \mathcal{H}^{od}$ is c_h (USD/TEU). For instance, in Eq. (12), $c_{h_3} = \hat{c}_{\text{XM}} + \bar{c}_{\text{CB}} + \tilde{c}_{\text{CN}}$.

Define $\mathcal{H} := \bigcup_{(o,d)\in\mathcal{W}} \mathcal{H}^{od}$ to be the set of all container paths for all the O-D port pairs. We further let binary coefficient ρ_h^{ri} be 1 if containers on container path h are transported on leg i of ship route r, and 0 otherwise. For example, the container path h_3 consists of the 2nd and the 3rd legs of the ship route 2 and the 1st leg of the ship route 3. We hence have $\rho_{h_3}^{2,2} = 1$, $\rho_{h_3}^{2,3} = 1$, and $\rho_{h_3}^{3,1} = 1$.

The decision variables are as follows. x_{rv} is a nonnegative integer variable representing the number of ships in type v deployed on port rotation r; y_h is the volume of containers transported on container path $h \in \mathcal{H}$; and z^{od} is the unfulfilled demand for $(o,d) \in \mathcal{W}$. The CLFD problem with container routing can be formulated as a mixed-integer linear programming model:

[P2]
$$\min_{x_{rv},y_h,z^{od}} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} c_{rv}\frac{x_{rv}}{m_{rv}} + \sum_{h\in\mathcal{H}} c_h y_h + \sum_{(o,d)\in\mathcal{W}} g^{od} z^{od} \tag{13}$$

subject to:

$$\sum_{h\in\mathcal{H}} \rho_h^{ri} y_h \le \sum_{v\in\mathcal{V}} V_v x_{rv}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{14}$$

$$\sum_{h\in\mathcal{H}^{od}} y_h + z^{od} = q^{od}, \forall (o,d) \in \mathcal{W} \tag{15}$$

$$\sum_{r\in\mathcal{R}} x_{rv} \le m_v, \forall v \in \mathcal{V} \tag{16}$$

$$\frac{x_{rv}}{m_{rv}} \in \mathbb{Z}^+, \forall r \in \mathcal{R}, \forall v \in \mathcal{V} \tag{17}$$

$$y_h \ge 0, \forall h \in \mathcal{H} \tag{18}$$

$$z^{od} \ge 0, \forall (o,d) \in \mathcal{W} \tag{19}$$

The objective function (13) minimizes the sum of ship operating cost, container handling cost, and penalty cost in a week. x_{rv} / m_{rv} is the number of weekly services provided by ships of type v on ship route r. Constraint (14) imposes ship capacity constraint on each leg of each port rotation. Constraint (15) defines the container shipment demand. Constraint (16) requires that the number of ships used cannot exceed the number of ships in the fleet. Constraints (17)-(19) define the domains for the decision variables.

The path-based formulation [P2] is very elegant. Moreover, side constraints on container routing can also be easily accommodated, for example, maximum transit time of containers

(Meng and Wang, 2012) and maritime cabotage. However, [P2] needs the set of container paths $\mathcal{H}$. In reality, $\mathcal{H}$ can either be designed a priori by experienced planners, generated a priori using optimization algorithms (Meng and Wang, 2012), or generated dynamically by column generation (Brouer et al., 2011). Nevertheless, the cardinality of $\mathcal{H}$ increases exponentially with network size and some potentially good container paths may not be found for a large-scale network.

4.2 O-D-link-based fleet deployment model

A compact model that does not need path enumeration or generation is O-D-link-based. Agarwal and Ergun (2008) applied such a formulation in network design. The decision variables are as follows. x_{rv} is a nonnegative integer variable representing the number of ships in type v deployed on port rotation r; $\hat{z}_{ri}^{od}$ and $\tilde{z}_{ri}^{od}$ are the volume of containers from $(o,d)\in\mathcal{W}$ loaded and discharged at port of call i on ship route r, respectively (note that when calculating $\hat{z}_{ri}^{od}$ and $\tilde{z}_{ri}^{od}$, a transshipped container is considered as being discharged once and being loaded once); f_{ri}^{od} is the volume of containers from $(o,d)\in\mathcal{W}$ flowing on leg i on ship route r (we define $f_{r0}^{od} := f_{rN_r}^{od}$). y^{od} and z^{od} are the fulfilled and unfulfilled demand for $(o,d)\in\mathcal{W}$, respectively; $\hat{z}_p$, $\tilde{z}_p$, and $\overline{z}_p$ are the total volume of loaded, discharged, and transshipped containers at port $p\in\mathcal{P}$, respectively. The O-D-link-based CLFD model is a mixed-integer linear programming problem:

[P3]
$$\min_{x_{rv},\hat{z}_{ri}^{od},\tilde{z}_{ri}^{od},f_{ri}^{od},y^{od},z^{od},\hat{z}_p,\overline{z}_p,\tilde{z}_p} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} c_{rv}\frac{x_{rv}}{m_{rv}} + \sum_{p\in\mathcal{P}}\left(\hat{z}_p\hat{c}_p + \overline{z}_p\overline{c}_p + \tilde{z}_p\tilde{c}_p\right) + \sum_{(o,d)\in\mathcal{W}} g^{od}z^{od} \tag{20}$$

subject to:

$$f_{r,i-1}^{od} + \hat{z}_{ri}^{od} = f_{ri}^{od} + \tilde{z}_{ri}^{od}, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r, \forall (o,d)\in\mathcal{W} \tag{21}$$

$$\hat{z}_p = \sum_{(p,d)\in\mathcal{W}} y^{pd}, \forall p\in\mathcal{P} \tag{22}$$

$$\tilde{z}_p = \sum_{(o,p)\in\mathcal{W}} y^{op}, \forall p\in\mathcal{P} \tag{23}$$

$$\overline{z}_p = \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r, p_{ri}=p}\sum_{(o,d)\in\mathcal{W}} \hat{z}_{ri}^{od} - \hat{z}_p, \forall p\in\mathcal{P} \tag{24}$$

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r, p_{ri}=p}\left(\hat{z}_{ri}^{od}-\tilde{z}_{ri}^{od}\right)=\begin{cases} y^{od}, p=o \\ -y^{od}, p=d \\ 0, \text{ otherwise}\end{cases}, \forall(o,d)\in\mathcal{W}, \forall p\in\mathcal{P} \quad (25)$$

$$\sum_{(o,d)\in\mathcal{W}} f_{ri}^{od} \le \sum_{v\in\mathcal{V}} V_v \frac{x_{rv}}{m_{rv}}, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \quad (26)$$

$$y^{od}+z^{od}=q^{od}, \forall(o,d)\in\mathcal{W} \quad (27)$$

$$\hat{z}_{ri}^{od}\ge 0, \tilde{z}_{ri}^{od}\ge 0, f_{ri}^{od}\ge 0, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r, \forall(o,d)\in\mathcal{W} \quad (28)$$

$$y^{od}\ge 0, z^{od}\ge 0, \forall(o,d)\in\mathcal{W} \quad (29)$$

and constraints (16)-(17).

The objective function (20) minimizes the sum of weekly ship operating cost, container handling cost, and penalty cost. Constraint (21) is container flow conservation equation. Constraints (22)-(24) define the total volume of loaded, discharged, and transshipped containers at port $p\in\mathcal{P}$, respectively. Constraint (25) computes the fulfilled demand. Constraint (26) imposes ship capacity constraint on each leg of each port rotation. Constraint (27) defines the container shipment demand. Constraints (28)-(29) define the domains for the decision variables.

The number of flow variables (e.g. f_{ri}^{od}) in the O-D-link-based model [P3] has the magnitude of $|\mathcal{W}|\sum_{r\in\mathcal{R}} N_r$, which, in theory, is much smaller than the path-based formulation [P2] in the worst case. In [P3] containers can be transshipped at any port, and therefore, the transshipment properties of butterfly ship routes in Reinhardt and Pisinger (2012) and the cycle-based long-haul ship routes in Song and Dong (2013) could be correctly captured. However, [P3] suffers from the deficiency that characteristics associated with paths cannot be represented. For example, it would be extremely hard to impose the maximum transit time of containers, maritime cabotage, or the requirement that a container can be transshipped at most twice based on model [P3].

4.3 Origin-link-based fleet deployment model

A more compact model is origin-link-based, which is applied in network design by Alvarez (2009) and fleet deployment by Wang and Meng (2012). Brouer et al. (2014) and Bell et al. (2011, 2013) have used a similar destination-link-based formulation. The difference in the origin-link-based model compared with O-D-link-based model is that: (i) we use $\hat{z}_{ri}^{o}$ and $\tilde{z}_{ri}^{o}$ to represent the total volume of containers with origin port $o \in \mathcal{P}$ and any destination loaded and discharged at port of call i on ship route r, respectively (transshipped containers are also considered) and use f_{ri}^{o} to denote the total volume of containers with origin port $o \in \mathcal{P}$ and any destination flowing on leg i of ship route r; and (ii) we define $\bar{\mathcal{W}} := \mathcal{P} \times \mathcal{P}$ and $q^{od} = 0$ if there is no demand from port o to port d. The origin-link-based model for CLFD is a mixed-integer linear programming problem:

[P4] $$\min_{x_{rv}, \hat{z}_{ri}^{o}, \tilde{z}_{ri}^{o}, f_{ri}^{o}, y^{od}, z^{od}, \hat{z}_p, \bar{z}_p, \tilde{z}_p} \sum_{r\in\mathcal{R}} \sum_{v\in\mathcal{V}} c_{rv} \frac{x_{rv}}{m_{rv}} + \sum_{p\in\mathcal{P}} \left(\hat{z}_p \hat{c}_p + \bar{z}_p \bar{c}_p + \tilde{z}_p \tilde{c}_p \right) + \sum_{(o,d)\in\bar{\mathcal{W}}} g^{od} z^{od} \tag{30}$$

subject to:

$$f_{r,i-1}^{o} + \hat{z}_{ri}^{o} = f_{ri}^{o} + \tilde{z}_{ri}^{o}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{31}$$

$$\bar{z}_p = \sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}_r, p_{ri}=p} \sum_{o\in\mathcal{P}} \hat{z}_{ri}^{o} - \hat{z}_p, \forall p \in \mathcal{P} \tag{32}$$

$$\sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}_r, p_{ri}=p} \left(\hat{z}_{ri}^{o} - \tilde{z}_{ri}^{o} \right) = \begin{cases} \sum_{(o,d)\in\bar{\mathcal{W}}} y^{od}, p = o \\ -y^{op}, p \neq d \end{cases}, \forall o \in \mathcal{P}, \forall p \in \mathcal{P} \tag{33}$$

$$\sum_{o\in\mathcal{P}} f_{ri}^{o} \le \sum_{v\in\mathcal{V}} V_v \frac{x_{rv}}{m_{rv}}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{34}$$

$$\hat{z}_{ri}^{o} \ge 0, \tilde{z}_{ri}^{o} \ge 0, f_{ri}^{o} \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{35}$$

and constraints (16)-(17), (22)-(23), (27) and (29).

In the origin-link-based model [P4], the number of flow variables (e.g. f_{ri}^{o}) has the magnitude of $|\mathcal{P}| \sum_{r\in\mathcal{R}} N_r$, which is one order smaller than the O-D-link-based model [P3]. Similar to [P3], [P4] also allows containers to be transshipped at any port any number of times and suffers from the deficiency that characteristics associated with paths cannot be represented. [P4] combines the origins of the containers and thereby requires fewer variables, however, the side effect is that [P4] has less flexibility in formulating some constraints of container routing. For example, we may impose in [P3] that containers from Shanghai (SH) to Los Angeles (LA) should not visit Rotterdam (RD) by adding the constraint that $f_{ri}^{\text{SH, LA}} = 0$ if $p_{r,i-1} = \text{RD}$ or $p_{ri} = \text{RD}$. Such a constraint cannot be incorporated in model [P4]. That is, we cannot add to [P4] the following constraint $f_{ri}^{\text{SH}} = 0$ if $p_{r,i-1} = \text{RD}$ or $p_{ri} = \text{RD}$, because containers from Shanghai to Hamburg may visit Rotterdam. Similarly, [P3] can also incorporate the requirement that containers of a particular O-D cannot be transshipped at a particular port, whereas [P4] cannot.

5 Fleet deployment with uncertain container shipment demand

In reality, the container shipment demand cannot be predicted accurately. Therefore, some studies assume that the demand is a random variable following a known probability distribution. Chance-constrained models and stochastic optimization models have been developed. For simplicity, we use the path-based model [P2] to describe the developed models with uncertain demand.

5.1 Chance-constrained models

Meng and Wang (2010) assumed that the container shipment demand is a random variable. A certain minimum probability that the demand of each O-D port pair can be fulfilled must be maintained. Their objective is to minimize the total cost. The original model of Meng and Wang (2010) is formulated based on model [P1]. However, to be consistent with other models with uncertain demand, we reformulate the model of Meng and Wang (2010) based on model [P2]. Suppose that the demand for O-D pair $(o,d) \in \mathcal{W}$ is a random variable ξ^{od} (TEUs/week) whose cumulative density function is given as $F_{od}(x)$. It is required that the fleet must be able to fulfill the demand for each O-D with a probability of at least α. Therefore, the individual chance-constrained model for CLFD is:

[P5]
$$\min_{x_{rv}, y_h} \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} c_{rv} \frac{x_{rv}}{m_{rv}} + \sum_{h \in \mathcal{H}} c_h y_h \tag{36}$$

subject to:

$$\Pr\left(\sum_{h\in\mathcal{H}^{od}} y_h \geq \xi^{od}\right) \geq \alpha, \forall (o,d)\in\mathcal{W} \tag{37}$$

and constraints (14) and (16)-(18).

Eq. (37) is the chance constraint to ensure the probability of fulfilling all containers in each O-D. Since the chance constraint is imposed individually on each O-D, it can be transformed to a deterministic constraint:

$$\sum_{h\in\mathcal{H}^{od}} y_h \geq F_{od}^{-1}(\alpha), \forall (o,d)\in\mathcal{W} \tag{38}$$

where $F_{od}^{-1}(\cdot)$ is the inverse function of $F_{od}(x)$.

It should be mentioned that since constraint (37) is imposed individually on each O-D port pair in Meng and Wang (2010), the probability that the container shipment demand of at least one O-D is not fulfilled may be large. For instance, if there are 100 O-D pairs and $\alpha = 0.99$, then the probability that all containers of all O-D pairs are transported is only $0.99^{100} \approx 0.37$. To overcome this problem, we propose a joint chance-constrained model:

[P6]
$$\min_{x_{rv}, y_h} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} c_{rv}\frac{x_{rv}}{m_{rv}} + \sum_{h\in\mathcal{H}} c_h y_h \tag{39}$$

subject to:

$$\Pr\left(\sum_{h\in\mathcal{H}^{od}} y_h \geq \xi^{od}, \forall (o,d)\in\mathcal{W}\right) \geq \alpha \tag{40}$$

and constraints (14) and (16)-(18).

The only difference is Eq. (40): it guarantees that the probability that the demands from all O-D pairs can simultaneously be fulfilled is at least α. Eq. (40) may be of greater interest to shipping lines than Eq. (37). However, Eq. (40) poses considerable difficulties for computation as the set of decision variables satisfying Eq. (40) may not be convex.

5.2 Stochastic optimization models

Meng et al. (2012) proposed a stochastic optimization model to minimize the expected total cost for fulfilling the demand. It can be assumed that the container shipment demand has a limited number of scenarios represented by set Ω. The probability of scenario $\omega\in\Omega$ is $p^{\omega} > 0$, $\sum_{\omega\in\Omega} p^{\omega} = 1$. In scenario $\omega\in\Omega$, the demand for O-D pair $(o,d)\in\mathcal{W}$ is q_{od}^{ω}

(TEUs/week). Note that if there are a large number of scenarios, or if the demand is modeled as a continuous random variable, then the sample average approximation approach used by Meng et al. (2012) can be applied. Now there are two types of decision variables: the fleet deployment variables x_{rv} are here-and-now decisions and cannot be adjusted with the realization of the uncertain demand; the container flow variables represented by y_h^{ω} and z_{od}^{ω} are wait-and-see decisions and can be determined after observing the uncertain demand. Hence, the CLFD model minimizing the expected total cost is:

[P7]
$$\min_{x_{rv}, y_h^{\omega}, z_{od}^{\omega}} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} c_{rv}\frac{x_{rv}}{m_{rv}} + \sum_{\omega\in\Omega} p^{\omega}\left[\sum_{h\in\mathcal{H}} c_h y_h^{\omega} + \sum_{(o,d)\in\mathcal{W}} g^{od} z_{od}^{\omega}\right] \tag{41}$$

subject to:

$$\sum_{h\in\mathcal{H}} \rho_h^{ri} y_h^{\omega} \le \sum_{v\in\mathcal{V}} V_v \frac{x_{rv}}{m_{rv}}, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r, \forall\omega\in\Omega \tag{42}$$

$$\sum_{h\in\mathcal{H}^{od}} y_h^{\omega} + z_{od}^{\omega} = q_{od}^{\omega}, \forall (o,d)\in\mathcal{W}, \forall\omega\in\Omega \tag{43}$$

$$y_h^{\omega} \ge 0, \forall h\in\mathcal{H}, \forall\omega\in\Omega \tag{44}$$

$$z_{od}^{\omega} \ge 0, \forall (o,d)\in\mathcal{W}, \forall\omega\in\Omega \tag{45}$$

and constraints (16) and (17).

The objective function (41) minimizes the sum of weekly ship operating cost and the expected container handling cost and penalty cost. The stochastic optimization model [P7] minimizes the expected total cost and nests the deterministic counterpart [P2] as a special case. It can be proved that the optimal fleet deployment that minimizes the expected total cost may be different from the fleet deployment that minimizes the total cost under the average demand of all demand scenarios. This is the value of using the more complex model [P7] rather than [P2].

However, [P7] ignores the variability of the total cost in different scenarios. The fixed cost of 1 million dollars is different from the cost of 0.5 million dollars with a probability of 0.5 and 1.5 million dollars with a probability of 0.5. If the company is risk averse, the fixed cost of 1 million dollars is better; if it is risk seeking, the latter case is preferable; if it is risk neutral, both cases are the same. To reflect the risk attitude in the models, Wang et al. (2012) developed a stochastic optimization model that minimizes the expected cost plus weighting λ times the absolute deviation of the total cost:

$$\text{[P8]} \quad \begin{aligned} &\min_{x_{rv}, y_h^{\omega}, z_{od}^{\omega}} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} c_{rv}\frac{x_{rv}}{m_{rv}} + \sum_{\omega\in\Omega} p^{\omega}\left[\sum_{h\in\mathcal{H}} c_h y_h^{\omega} + \sum_{(o,d)\in\mathcal{W}} g^{od} z_{od}^{\omega}\right] \\ &+\lambda\sum_{\omega\in\Omega} p^{\omega}\left|\sum_{h\in\mathcal{H}} c_h y_h^{\omega} + \sum_{(o,d)\in\mathcal{W}} g^{od} z_{od}^{\omega} - \sum_{\omega\in\Omega} p^{\omega}\left[\sum_{h\in\mathcal{H}} c_h y_h^{\omega} + \sum_{(o,d)\in\mathcal{W}} g^{od} z_{od}^{\omega}\right]\right| \end{aligned} \quad (46)$$

subject to the same constraints as [P7].

In Eq. (46), the second term excluding λ is the average absolute deviation of the cost under all the scenarios relative to the expected cost, and λ is a parameter: $\lambda > 0$ means that the company is risk averse, $\lambda < 0$ indicates a risk seeking company, and $\lambda = 0$ leads to model [P7]. Hence, model [P8] successfully captures the risk attitude of the decision maker. Of course, incorporating risks in decision making will to some extent sacrifice the average total cost. Wang et al. (2012) proved that the average total cost of the fleet deployment in [P8] is at least as high as that of [P7].

In addition to the modeling approach in [P8], one can also consider the worst-case scenario with limited information on the distribution function of the demand (Chen et al., 2007). For instance, Ng (2014, 2015) considered fleet deployment problems in which the distribution function of the demand is unknown but its mean and variance are known.

6 Fleet deployment with empty container repositioning

Because of the imbalance in world trade, a large number of empty containers are accumulated at import-oriented countries and must be repositioned to export-oriented countries. As pointed out by Shintani et al. (2007), Brouer et al. (2011), Wang (2013), and Huang et al. (2015), it is important to consider both laden and empty containers in planning models. We use the O-D-link-based model [P3] to demonstrate how to incorporate empty container repositioning in CLFD. We need the following parameters: The load, transshipment and discharge cost for an empty container (USD/TEU) at port $p\in\mathcal{P}$ is denoted by $\hat{c}_p^e$, $\bar{c}_p^e$ and $\tilde{c}_p^e$, respectively, $\bar{c}_p^e < \hat{c}_p^e + \tilde{c}_p^e$; the penalty cost for not repositioning an empty container to deficit port $p\in\mathcal{P}$ is g_p^{+e} (USD/TEU); and the penalty cost for not repositioning an empty container from surplus port $p \in \mathcal{P}$ is g_p^{-e} (USD/TEU). We also need the following additional variables: $\hat{z}_{ri}^e$ and $\tilde{z}_{ri}^e$ are the volume of empty containers loaded and discharged at port of call i on ship route r, respectively (including transshipped empty containers); $\overline{f}_{ri}^e$ is the volume of empty containers flowing on leg i on ship route r (we define $\overline{f}_{r0}^e := \overline{f}_{rN_r}^e$); y_p^{+e} and y_p^{-e} are the volume of empty containers un-repositioned to deficit port $p \in \mathcal{P}$ and volume of empty containers un-repositioned from surplus port $p \in \mathcal{P}$, respectively; $\hat{z}_p^e$, $\tilde{z}_p^e$, and $\overline{z}_p^e$ are the total volume of loaded, discharged, and transshipped empty containers at port $p \in \mathcal{P}$, respectively. The CLFD problem with empty container repositioning can be formulated as:

[P9]

$$\begin{aligned}
&\min_{x_{rv}, \hat{z}_{ri}^{od}, \tilde{z}_{ri}^{od}, f_{ri}^{od}, y^{od}, z^{od}, \hat{z}_p, \overline{z}_p, \tilde{z}_p, \hat{z}_{ri}^e, \tilde{z}_{ri}^e, \overline{f}_{ri}^e, y_p^{+e}, y_p^{-e}, \hat{z}_p^e, \tilde{z}_p^e, \overline{z}_p^e} \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} c_{rv} \frac{x_{rv}}{m_{rv}} \\
&+ \sum_{p \in \mathcal{P}} \left(\hat{z}_p \hat{c}_p + \overline{z}_p \overline{c}_p + \tilde{z}_p \tilde{c}_p \right) + \sum_{(o,d) \in \mathcal{W}} g^{od} z^{od} \\
&+ \sum_{p \in \mathcal{P}} \left(\hat{z}_p^e \hat{c}_p^e + \overline{z}_p^e \overline{c}_p^e + \tilde{z}_p^e \tilde{c}_p^e \right) + \sum_{p \in \mathcal{P}} \left(g_p^{+e} y_p^{+e} + g_p^{-e} y_p^{-e} \right)
\end{aligned} \tag{47}$$

subject to:

$$\overline{f}_{r,i-1}^e + \hat{z}_{ri}^e = \overline{f}_{ri}^e + \tilde{z}_{ri}^e, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{48}$$

$$\hat{z}_p^e = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r, p_{ri} = p} \hat{z}_{ri}^e - \overline{z}_p^e, \forall p \in \mathcal{P} \tag{49}$$

$$\tilde{z}_p^e = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r, p_{ri} = p} \tilde{z}_{ri}^e - \overline{z}_p^e, \forall p \in \mathcal{P} \tag{50}$$

$$\overline{z}_p^e = \min \left\{ \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r, p_{ri} = p} \hat{z}_{ri}^e, \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r, p_{ri} = p} \tilde{z}_{ri}^e \right\}, \forall p \in \mathcal{P} \tag{51}$$

$$\sum_{(o,d) \in \mathcal{W}} f_{ri}^{od} + \overline{f}_{ri}^e \le \sum_{v \in \mathcal{V}} V_v \frac{x_{rv}}{m_{rv}}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{52}$$

$$\sum_{(o,p) \in \mathcal{W}} y^{op} - y_p^{+e} - \tilde{z}_p^e = \sum_{(p,d) \in \mathcal{W}} y^{pd} - y_p^{-e} - \hat{z}_p^e, \forall p \in \mathcal{P} \tag{53}$$

$$\hat{z}_{ri}^e \ge 0, \tilde{z}_{ri}^e \ge 0, f_{ri}^e \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{54}$$

$$y_p^{+e} \ge 0, y_p^{-e} \ge 0, \forall p \in \mathcal{P} \tag{55}$$

and constraints (16)-(17), (21)-(25), (27)-(29).

The fourth term of the objective function (47) is empty container handling cost and the fifth term is penalty cost for not repositioning all empty containers. Eq. (48) is empty container flow conservation equation. Eqs. (49)-(51) define the volume of empty containers handled at each port. In Eq. (52), the left-hand side is the sum of the volumes of laden and empty containers on leg i on ship route r . Eq. (53) calculates the volume of empty containers that should be repositioned at each port. Comparing [P3] and [P9], it can be seen that incorporating empty containers only slightly increases the number of variables and constraints as the number of flow variables for empty containers (e.g. f_{ri}^{e}) has the magnitude of only $\sum_{r\in\mathcal{R}} N_r$.

7 Fleet deployment with ship sailing speed optimization

In models [P1] – [P9], the ship sailing speed is considered as exogenous. In fact, the sailing speed can also be considered as a decision variable in the CLFD. For example, Perakis and Jaramillo (1991) and Meng and Wang (2010) incorporated some forms of speed optimization; Gelareh and Meng (2010) discretized and optimized the sailing speed in CLFD models. Alvarez (2009) considered ships of different speeds as different types in network design. Xia et al. (2015) optimized the speed for each leg for network design where the range of possible speeds is discretized. We use model [P2] to elaborate on the approach by Alvarez (2009).

To maintain a weekly frequency, the higher the speed is, the smaller the number of ships is. Therefore, optimizing speed is equivalent to determining the number of ships m in a port rotation. A larger m implies a lower speed and vice versa. If m ships of type v are deployed to maintain a weekly frequency, the weekly operating cost of such a string can be calculated and is denoted by c_{rvm} (USD/week). Moreover, in reality, based on limits of ship speed and operating rules, one can easily estimate a lower and upper bound for m, represented by $m_{rv}^{\min}$ and $m_{rv}^{\max}$, respectively.

The decision variables are as follows. x_{rvm} is a nonnegative integer variable representing the number of ships in type v whose speed satisfies that exactly m ships are needed to maintain a weekly service frequency on port rotation r ; y_h is the volume of containers transported on container route $h\in\mathcal{H}$; and z^{od} is the unfulfilled demand for $(o,d)\in\mathcal{W}$. Based on [P2], the CLFDP with sailing speed optimization can be formulated as:

[P10]
$$\min_{x_{rvm},y_h,z^{od}} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}}\sum_{m=m_{rv}^{\min}}^{m_{rv}^{\max}} c_{rvm}\frac{x_{rvm}}{m}+\sum_{h\in\mathcal{H}} c_h y_h+\sum_{(o,d)\in\mathcal{W}} g^{od} z^{od} \tag{56}$$

subject to:

$$\sum_{h \in \mathcal{H}} \rho_h^{ri} y_h \le \sum_{v \in \mathcal{V}} V_v \sum_{m=m_{rv}^{\min}}^{m_{rv}^{\max}} \frac{x_{rvm}}{m}, \forall r \in \mathcal{R}, \forall i \in I_r \tag{57}$$

$$\sum_{r \in \mathcal{R}} \sum_{m=m_{rv}^{\min}}^{m_{rv}^{\max}} x_{rvm} \le m_v, \forall v \in \mathcal{V} \tag{58}$$

$$\frac{x_{rvm}}{m} \in \mathbb{Z}^+, \forall r \in \mathcal{R}, \forall v \in \mathcal{V}, \forall m = m_{rv}^{\min}, m_{rv}^{\min}+1, \cdots, m_{rv}^{\max} \tag{59}$$

and constraint (15) and (18)-(19).

The optimal sailing speed of ships is already implicitly included in the decision variable x_{rvm} in model [P10]. In addition to speed, Wang et al. (2015) and Xia et al. (2015) have considered the impact of vessel displacement on fuel costs. Interested readers can further refer to Wang et al. (2013) and Psaraftis and Kontovas (2014) for reviews on shipping speed optimization models.

8 Fleet deployment with ship repositioning

The ships operated by liner shipping companies are scattered all over the world to transport containers. If a ship that used to serve a particular ship route is re-scheduled to serve another ship route, then a repositioning cost is incurred. If these two ship routes have common ports of call, the ship can phase out from one ship route at a common port and phase into another ship route. In this situation, the repositioning cost mainly involves discharging the remaining containers on the ship. If these two ship routes have no common port of call, the ship has to sail from the itinerary of the first ship route to the itinerary of the second ship route, and the repositioning cost may be much higher.

Wang (2013) formulated the ship repositioning problem in the context of CLFD. We use model [P2] to illustrate this formulation. Let S_v be the set of ship groups in ship type $v \in \mathcal{V}$. Ships in ship group $s \in S_v$ not only belong to the same type v, but also have the same repositioning cost to any ship route $r \in \mathcal{R}$, denoted by $\breve{c}_{rs}$ (USD/week). It should be mentioned because in model [P2] we minimize the weekly cost while ship repositioning is a one-off activity, $\breve{c}_{rs}$ is actually the total repositioning cost divided by the number of weeks in the planning horizon. Define $S = \bigcup_{v \in \mathcal{V}} S_v$ and let $\bar{m}_s$ represent the number of ships in group $s \in S$. We further need a new decision variable z_{rs} representing the number of ships from group $s \in S$ deployed on ship route r. The CLFDP with ship repositioning can be formulated as follows

[P11]
$$\min_{x_{rv}, y_h, z^{od}, z_{rs}} \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} c_{rv} \frac{x_{rv}}{m_{rv}} + \sum_{h \in \mathcal{H}} c_h y_h + \sum_{(o,d) \in \mathcal{W}} g^{od} z^{od} + \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} \sum_{s \in S_v} \breve{c}_{rs} z_{rs} \quad (60)$$

subject to

$$\sum_{r \in \mathcal{R}} z_{rs} \leq \bar{m}_s, \forall s \in S \quad (61)$$

$$\sum_{s \in S_v} z_{rs} = x_{rv}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V} \quad (62)$$

$$z_{rs} \in \mathbb{Z}^+, \forall r \in \mathcal{R}, \forall s \in S \quad (63)$$

and constraints (14)-(19).

The last term in the objective function (60) is the ship repositioning cost. Constraint (61) requires that the total used ships in each ship group cannot exceed the total number of ships available in the group. Constraint (62) enforces that the total used ships from all ship groups of type v equals the number of ships deployed. Constraint (63) defines z_{rs} as nonnegative integer variables. It is clear that a number of integer variables z_{rs} are introduced for formulating ship repositioning. Nevertheless, Wang (2013) proved that the ship repositioning sub-problem has the totally unimodularity property. Hence, the decision variables z_{rs} can be modeled as continuous variables. That means that the additional computational burden caused by incorporating ship repositioning is trivial.

9 Future research directions

The advancement of CLFD models mainly focuses on modeling more factors that are relevant in practice. We expect that future CLFD models will also follow this trend. In this regard, we classify future research into two types: modeling CLFD while accounting for realistic factors that have existed for a long time yet not been modelled, and modeling new factors that arise recently. In the first direction, fleet deployment with ship surveys and inspections, and service dependent demand are worthwhile future research topics; in the second direction, CLFD considering emissions (green shipping) and CLFD for shipping alliances are new topics that should be explored.

9.1 Fleet deployment with ship surveys and inspections

Ships need regular surveys and inspections to ensure satisfactory operating conditions. For example, according to IACS (2005), periodical hull survey is of prime importance as far as structural assessment of cargo holds and adjacent tanks is concerned. Periodical hull survey consists of annual, intermediate and special surveys and inspections. At annual surveys and inspections, overall survey is required. At intermediate surveys and inspections, in addition to the surveys required for annual surveys, examination of cargo holds and ballast tank may be required depending on the age of the ship. Special surveys and inspections of the hull structure are carried out at five-year intervals to confirm that the structural integrity is satisfactory and will remain fit for its intended purpose until the next special survey.

When a ship in service is to be surveyed and inspected, it will phase out from a particular ship route. This will destroy the weekly frequency of the ship route, and hence another ship (for example, one that is in lay-up) will phase into this ship route. Consequently, in the fleet deployment stage, the survey and inspection requirement of each ship should be taken into account. Perakis and Jaramillo (1991) and Meng and Wang (2010) required that there are at least a certain number of days in the planning horizon during which a ship cannot be put to service. Other than this simplified consideration, the ship survey and inspection requirement is neither incorporated in CLFD models, nor in other liner ship planning problems in the literature.

9.2 Fleet deployment with service dependent demand

It is evident that the container shipment demand depends on the service in terms of transit time and freight rate. As the freight rate is to a large extent confidential, has a number of elements, and may change every day, it is difficult to evaluate the impact of the freight rate on the demand. At the same time, it is widely acknowledged in the liner shipping industry the negative correlation between transit time and demand. Increased transit time because of slow steaming may not be acceptable for customers, depending on the sensitivity of demand to transit time. To attract more demand, Daily Maersk (2013) has been advertising that its total transportation time from Asia to Europe is five days shorter than other Asia-Europe services.

Despite the negative correlation between transit time and demand, most CLFD studies assumed that the demand is independent of the services. A few works have adopted a simplified relation between transit time and demand. Mourão et al. (2001) incorporated the inventory cost of containers. Meng and Wang (2012) required that there is a maximum allowable transit time of containers in each O-D pair. Cheaitou and Cariou (2012) examined the effect of slow steaming under semi-elastic demand where containerized perishable product is sensitive to transit time and frozen and dry products are not. General CLFD models with service dependent demand need to be developed to capture the demand in a more reasonable manner. There are two challenges for such general models: first, how to obtain the relation between service factors and demand; second, to formulate the exact transit time, the schedule of each ship route (that is, the arrival and departure day at each port of call) must be designed. However, schedule design depends on the availability of ports and incorporating the availability of ports in liner planning models is extremely difficult due to the combinatorial nature of the problem.

9.3 Fleet deployment considering pollutant emissions

Shipping was estimated to have accounted for 2.2 per cent of the global greenhouse gas (GHG) emissions in 2012 (UNCTAD, 2016). Therefore, the greenhouse gas (GHG) emissions from shipping have been dominating substantive discussions at the International Maritime Organization (IMO). IMO has adopted a number of new regulations on energy efficiency for ships, for example, the Energy Efficiency Design Index (EEDI) is mandatory for new ships and the Ship Energy Efficiency Management Plan (SEEMP) is mandatory for all ships (UNCTAD, 2016). Market-based measures (MBMs) for the reduction of GHG emissions are under extensive debates in IMO (Psaraftis, 2012). The MBM proposals under review range from those envisaging a levy on all GHG emissions from all ships, or only those generated by ships not meeting the EEDI requirement, to emissions trading schemes. In addition to carbon emissions, more stringent regulations are also enforced to control sulfur and nitrogen emissions. For example, since 1 January 2015, ships trading in the emission control areas in Europe and North America have to switch to bunker fuel with sulfur content of at most 0.1%; moreover, to reduce ship emissions at ports, a number of ports require ships to use low-sulfur fuel while berthing and/or provide shore power so that ships can turn off auxiliary engines at berth. These newly enforced regulations will affect all levels of decision making including CLFD. How to systematically model the CLFDP at the network level while considering regulations on emissions is a fruitful research arena.

9.4 Fleet deployment for shipping alliances

The overcapacity of container liner shipping and the resulting low freight rates have led to a number of major mergers and acquisitions to lower cost and increase competitiveness. For instance, China Ocean Shipping (Group) Company and China Shipping (Group) Company merged to form China COSCO Shipping Corporation (COSCO Shipping); CMA CGM acquired of Singapore's Neptune Orient Lines (NOL), which owned American President Lines (APL). Moreover, shipping companies form shipping alliances to increase service frequency, take advantage of economies of scale in ship size, and expand service scope. Currently, there are three large shipping alliances, namely Ocean alliance, 2M alliance, and THE alliance, which control nearly 90% of Asia-North America trade. CLFD for shipping alliances not only involves many more ships than that for one shipping company, but should also account for how much capacity each shipping company in the alliance should contribute to each route, and how to allocate the container slot capacity of a ship to the shipping companies. This is a timely and relevant topic for the container shipping industry.

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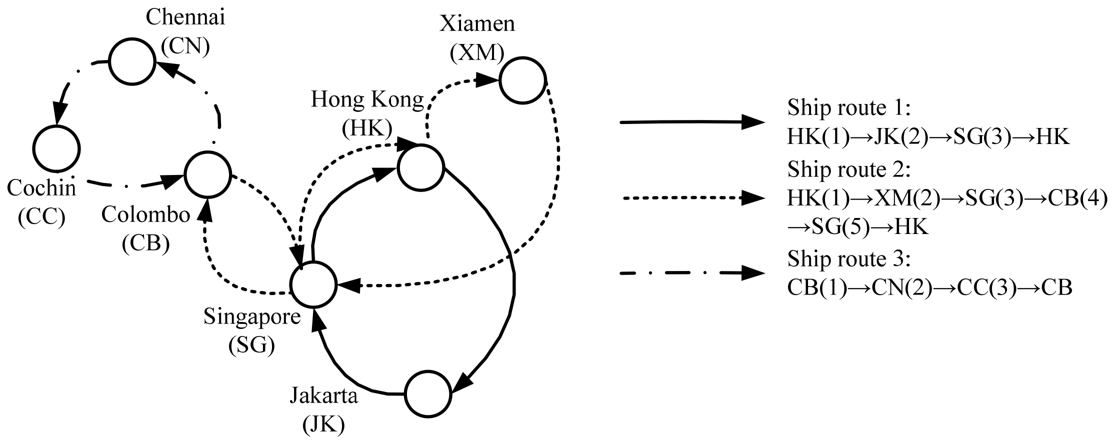

Fig. 1 An illustrative liner shipping network (Wang, 2013)

List of Tables:

Table 1. Symbols

Sets	
Ω	Set of container shipment demand scenarios
$\mathcal{H}^{od}$	Set of container paths for O-D $(o,d)\in\mathcal{W}$
$\mathcal{H}^{od}$	Set of all container paths, $\mathcal{H}:=\bigcup_{(o,d)\in\mathcal{W}}\mathcal{H}^{od}$
$\mathcal{I}_r$	Set of ports of call on ship route $r\in\mathcal{R}$, $\mathcal{I}_r:=\{1,2,\cdots,N_r\}$
$\mathcal{P}$	Set of ports
$\mathcal{R}$	Set of port rotations (ship routes)
S_v	Set of ship groups in ship type $v\in\mathcal{V}$
S	Set of all ship groups, $S=\bigcup_{v\in\mathcal{V}}S_v$
$\mathcal{V}$	Set of types of ships in the fleet
$\mathcal{W}$	Set of origin-to-destination (O-D) port pairs
$\bar{\mathcal{W}}$	Set of all port pairs, $\bar{\mathcal{W}}:=\mathcal{P}\times\mathcal{P}$
$\mathbb{Z}^+$	Set of nonnegative integers
Indices	
$\omega\in\Omega$	A container shipment demand scenario
$h\in\mathcal{H}$	A container path
$(o,d)\in\mathcal{W}$	An O-D pair
$r\in\mathcal{R}$	A ship route
$s\in S$	A ship group
$p\in\mathcal{P}$	A port
$v\in\mathcal{V}$	A ship type
Parameters	
α	The container shipment demand for each O-D must be fulfilled with a probability of at least α
ρ_h^{ri}	Binary coefficient that is 1 if containers on container path h are transported on leg i of ship route r, and 0 otherwise
λ	Weight for the absolute deviation of the total cost

ξ^{od}	Random variable representing the container shipment demand for O-D pair $(o,d) \in \mathcal{W}$ (TEUs/week)
$\hat{c}_{rv}$	Operating cost of a round trip on port rotation r completed by a ship in type v (USD)
c_{rv}	Weekly cost of operating a string of ships in type v to maintain a weekly frequency of port rotation r (USD/week)
c_{rvm}	Weekly cost of operating m ships in type v to maintain a weekly frequency of port rotation r (USD/week)
c_h	Total handling cost of a container delivered on path $h \in \mathcal{H}$ (USD/TEU)
$\hat{c}_p$	Load cost of a container at port $p \in \mathcal{P}$ (USD/TEU)
$\bar{c}_p$	Transshipment cost of a container at port $p \in \mathcal{P}$ (USD/TEU)
$\tilde{c}_p$	Discharge cost of a container at port $p \in \mathcal{P}$ (USD/TEU)
$\hat{c}_p^e$	Load cost of an empty container at port $p \in \mathcal{P}$ (USD/TEU)
$\bar{c}_p^e$	Transshipment cost of an empty container at port $p \in \mathcal{P}$ (USD/TEU)
$\tilde{c}_p^e$	Discharge cost of an empty container at port $p \in \mathcal{P}$ (USD/TEU)
$\breve{c}_{rs}$	Cost of repositioning a ship in group $s \in S_v$ to ship route $r \in \mathcal{R}$ divided by the number of weeks in the planning horizon (USD/week)
$F_{od}(x)$	Cumulative density function of random variable ξ^{od}
g^{od}	Penalty cost for not shipping a container for O-D pair $(o,d) \in \mathcal{W}$ (USD/TEU)
g_p^{+e}	Penalty cost for not repositioning an empty container to deficit port $p \in \mathcal{P}$ (USD/TEU)
g_p^{-e}	Penalty cost for not repositioning an empty container from surplus port $p \in \mathcal{P}$ (USD/TEU)
m_v	Number of ships in type $v \in \mathcal{V}$ in the fleet
m_{rv}	Number of ships in type v needed to maintain a weekly frequency of port rotation r at given sailing speed
$m_{rv}^{\min}$	Lower bound on the number of ships of type v required to maintain a weekly frequency of port rotation r
$m_{rv}^{\max}$	Upper bound on the number of ships of type v required to maintain a weekly frequency of port rotation r
$\bar{m}_s$	Number of ships in ship group $s \in S$

N_r	Number of ports of call on ship route $r \in \mathcal{R}$
n_{rv}	Number of round trips a ship in type v can complete in the planning horizon if it is deployed on port rotation r
n_r	Lower bound on the number of trips that must be completed on port rotation r in the planning horizon
p^{ω}	Probability of container shipment demand scenario $\omega \in \Omega$
p_{ri}	Port corresponding to the i th port of call on ship route $r \in \mathcal{R}$
q_r^{ij}	Container shipment demand from port i to port j on ship route r in the planning horizon (TEUs)
q^{od}	Container shipment demand for O-D pair $(o,d) \in \mathcal{W}$ modeled as a fixed value (TEUs/week)
q_{od}^{ω}	Container shipment demand for O-D pair $(o,d) \in \mathcal{W}$ in scenario $\omega \in \Omega$ (TEUs/week)
T	Length of the planning horizon (days)
V_v	Container capacity of a ship in type $v \in \mathcal{V}$ (TEUs)
$\bar{V}_r$	Shipping capacity required on port rotation r (TEUs)
Decision Variables	
f_{ri}^{od}	Number of containers from $(o,d) \in \mathcal{W}$ flowing on leg i on ship route r (TEUs/week)
f_{ri}^{o}	Number of containers with origin port $o \in \mathcal{P}$ and any destination flowing on leg i on ship route r (TEUs/week)
$\bar{f}_{ri}^{e}$	Number of empty containers flowing on leg i on ship route r (TEUs/week)
x_{rv}	Number of ships in type v deployed on port rotation r
x_{rvm}	Number of ships in type v whose speed satisfies that exactly m ships are needed to maintain a weekly service frequency on port rotation r
y_{rv}	Number of round trips completed by ships in type v on port rotation r in the planning horizon
y_h	Number of containers transported on container path $h \in \mathcal{H}$ (TEUs/week)
y_h^{ω}	Number of containers transported on container path $h \in \mathcal{H}$ in scenario $\omega \in \Omega$ (TEUs/week)

y_p^{+e}	Number of empty containers un-repositioned to deficit port $p \in \mathcal{P}$ (TEUs/week)
y_p^{-e}	Number of empty containers un-repositioned from surplus port $p \in \mathcal{P}$ (TEUs/week)
y^{od}	Fulfilled demand for $(o,d) \in \mathcal{W}$ (TEUs/week)
z^{od}	Unfulfilled demand for $(o,d) \in \mathcal{W}$ (TEUs/week)
z_{od}^{ω}	Unfulfilled demand for $(o,d) \in \mathcal{W}$ in scenario $\omega \in \Omega$ (TEUs/week)
$\hat{z}_{ri}^{od}$	Number of containers from $(o,d) \in \mathcal{W}$ loaded at port of call i on ship route, including transshipment containers (TEUs/week)
$\tilde{z}_{ri}^{od}$	Number of containers from $(o,d) \in \mathcal{W}$ discharged at port of call i on ship route, including transshipment containers (TEUs/week)
$\hat{z}_{ri}^{o}$	Number of containers with origin port $o \in \mathcal{P}$ and any destination loaded at port of call i on ship route, including transshipment containers (TEUs/week)
$\tilde{z}_{ri}^{o}$	Number of containers with origin port $o \in \mathcal{P}$ and any destination discharged at port of call i on ship route, including transshipment containers (TEUs/week)
$\hat{z}_{ri}^{e}$	Number of empty containers loaded at port of call i on ship route, including transshipment containers (TEUs/week)
$\tilde{z}_{ri}^{e}$	Number of empty containers discharged at port of call i on ship route, including transshipment containers (TEUs/week)
$\hat{z}_p$	Number of loaded containers at port $p \in \mathcal{P}$ (TEUs/week)
$\overline{z}_p$	Number of transshipped containers at port $p \in \mathcal{P}$ (TEUs/week)
$\tilde{z}_p$	Number of discharged containers at port $p \in \mathcal{P}$ (TEUs/week)
$\hat{z}_p^{e}$	Number of loaded empty containers at port $p \in \mathcal{P}$ (TEUs/week)
$\overline{z}_p^{e}$	Number of transshipped empty containers at port $p \in \mathcal{P}$ (TEUs/week)
$\tilde{z}_p^{e}$	Number of discharged empty containers at port $p \in \mathcal{P}$ (TEUs/week)
z_{rs}	Number of ships from group $s \in S$ deployed on ship route r

Table 2. Summary of the studies on CLFD

	Transshipment	Container routing	Uncertain demand	Empty containers	Speed optimization	Other features
Perakis (1985)					Y	
Papadakis and Perakis (1989)					Y	
Jaramillo and Perakis (1991)					Y	
Perakis and Jaramillo (1991)					Y	
Cho and Perakis (1996)		Y				
Powell and Perakis (1997)						
Fagerholt (1999)	Y					
Fagerholt (2004)	Y					
Fagerholt et al. (2009)	Y					
Gelareh and Meng (2010)					Y	
Meng and Wang (2010)			Y			
Liu et al. (2011)	Y	Y		Y		Nonlinear revenue function
Meng and Wang (2012)	Y	Y				Transit time, dynamic demand
Meng et al. (2012)	Y	Y	Y			
Wang and Meng (2012)	Y	Y				
Wang et al. (2012)	Y	Y	Y			
Wang (2013)	Y	Y		Y		Ship repositioning
Ng (2014)			Y			
Branchini et al. (2015)						Spot voyages
Ng (2015)			Y			
Ng (2017)						Partial trips