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Robust Optimization of Distance-based Tolls in a Network Considering Stochastic Day to Day Dynamics

Abstract

This paper investigates the nonlinear distance-based congestion pricing in a network considering stochastic day-to-day dynamics. After an implementation/adjustment of a congestion pricing scheme, the network flows in a certain period of days are not on an equilibrium state, thus it is problematic to take the equilibrium-based indexes as the pricing objective. Therefore, the concept of robust optimization is taken for the congestion toll determination problem, which takes into account the network performance of each day. First, a minimax model which minimizes the maximum regret on each day is proposed. Taking as a constraint of the minimax model, a path-based day to day dynamics model under stochastic user equilibrium (SUE) constraints is discussed in this paper. It is difficult to solve this minimax model by exact algorithms because of the implicity of the flow map function. Hence, a two-phase artificial bee colony algorithm is developed to solve the proposed minimax regret model, of which the first phase solves the minimal expected total travel cost for each day and the second phase handles the minimax robust optimization problem. Finally, a numerical example is conducted to validate the proposed models and methods.

Keywords: congestion pricing, distance-based pricing, minimax regret model, robust optimization, day-to-day dynamics

1. Introduction

Congestion pricing, as an important instrument on transportation demand management, is of great significance in ameliorating urban traffic congestions in that it encourages commuters to adjust their travel behaviors: number of trips, route, time of day, destination, mode of transport, and so on, as well as the long-term decisions on where to live, work and set up business (de Palma and Lindsey, 2011). Among all types of congestion pricing schemes (zonal-based, cordon-based, distance-based, time-based as well as congestion-based schemes), the distance-based schemes have received increasing attention both academically and practically (e.g., Lawphongpanich and Yin, 2012; Daganzo and Lehe, 2015). Due to the better equity and efficiency of distance-based pricing, the current cordon-based congestion pricing scheme in Singapore will be upgraded to the distance-based pricing scheme, which is regarded as the next generation of Electronic Road Pricing (ERP) system from 2020 onwards (Singapore LTA, 2013). The optimal toll design problem is of considerable significance for improving the efficiency of the network. Generally, system wide indexes such as the total travel cost (TTC) are taken as the objective of the optimal toll design problem.

Nearly all the existing studies use the equilibrium flow to calculate the TTC, and then evaluate each toll pattern based on the calculated TTC. However, any new toll pattern will affect travelers' route choice decisions, and the network flows cannot achieve an equilibrium state overnight. Cho and Hwang (2005) tested a small numerical network and revealed that it nearly takes 200 days to reach equilibrium state, thus it would take much longer time in a big urban area to achieve equilibrium. In addition, after such a long period, the network demand and infrastructure are largely changed, thus a new design of the optimal toll is needed again. Hence, in the whole study period of an optimal toll design problem (denoted by D), the day-to-day models can better capture the network flow conditions, rather than the final equilibrium state (He et al., 2010). Note in passing that in practice, to avoid the confusions from travelers on the toll, it is necessary to implement an unchanged toll in the whole period D; for instance, Singapore's ERP toll is adjusted every three months (Olszewski and Xie, 2005; Liu et al., 2013), and kept unchanged in-between, thus D equals three months in this case.

During the planning horizon D, the TTC is changing each day due to the change of traffic flows. Therefore, no toll pattern can give rise to a minimal TTC in each day of D. It is not reasonable to implement the toll pattern that gives rise to the minimal TTC on a certain day while neglecting the other days. From the viewpoints of policy-makers, the deterioration of some worst cases is more harmful than the loss of efficiency on the good cases, both temporally and spatially. The most desired toll pattern is the one that considers the traffic conditions of every day in the planning horizon D. This paper aims to cope with this problem of optimal toll design caused by the fluctuation of traffic flows, where the concept of robust optimization is taken for the modelling. On a particular day, each toll pattern $\boldsymbol{\tau}$ can give rise to a corresponding TTC($\boldsymbol{\tau}$). We first define the concept of *regret* for such a toll pattern on each day, which is the gap between the minimal TTC and TTC($\boldsymbol{\tau}$). Then, a minimax model which minimizes the maximum regret on each day, is proposed for the robust optimal toll design. Note that, the minimal average TTC can also be taken as an alternative objective.

Since in the planning period D the network flow is fluctuating each day, it is difficult for the travelers to have an accurate prediction on the travel time. Thus, stochastic user equilibrium (SUE) is more suitable to capture their travel behaviors, compared with user equilibrium (Meng et al., 2014). In addition, for the optimal toll design considering SUE flows, it is more rational to take the stochastic system optimum (SSO) as the objective (Liu et al., 2014a), compared with the deterministic system optimum. Hence, in this paper we assume that the flow evolution process follows day-to-day dynamics under SUE constraints, and take travelers' expected total travel cost (ETTC) as the system wide index. However, formulating and solving day-to-day models or SUE/SSO models individually are known to be very challenging. The optimal design of distance-based tolls in a network considering stochastic day to day dynamics is thus difficult to address, which is still an open question in the literature and tackled in this paper.

1.1 Literature review

Due to the inequity of flat pricing patterns which undercharge long journeys and over-restrain short journeys (Meng et al., 2012), a distance-based pricing pattern was recommended by May and Milne (2000) as an alternative for flat toll patterns. In a distance-based congestion pricing scheme, the toll is levied in terms of the travel distance, either linearly (e.g., Mitchell et al., 2005; Namdeo and Mitchell, 2008) or nonlinearly (e.g., Wang et al., 2011; Lawphongpanich and Yin, 2012). Linear models assume that the toll is linearly proportional to the travel distance, making it easier for analysis due to the additivity of the toll charge. However, according to Lawphongpanich and Yin (2012), the actual congestion toll, in most cases, is nonlinear, i.e., the total charge for a trip cannot be proportionally divided to be the charges on its component links (Meng et al., 2012). For the distance-based toll charge function, no practical data could be collected for the analysis of a proper functional form or the calibration of such a function. Hence, it is proper to assume that it is generic to any positive and nonlinear function, which includes the fixed toll rate. Thus, this paper also adopts the nonlinear function form for the distance-based tolls.

A nonlinear pricing pattern known as the two-part tariff, which can be regarded as a special case of the piecewise linear toll scheme, was adopted by Lawphongpanich and Yin (2012) to study the nonlinear pricing on transportation networks. Meng et al. (2012) and Liu et al. (2014a) extended the piecewise linear toll scheme from only two linear intervals to multiple intervals. Sun et al. (2016) investigated the equity issues of distance-based tolls. However, all these formulations are based on static traffic assignment theory, either deterministic or stochastic. Recently, Daganzo and Lehe (2015) studied the distance-dependent, time-varying congestion pricing scheme based on the macroscopic fundamental diagram theory of traffic dynamics. However, this model is a within-day dynamic model, which cannot reflect the day-to-day flow evolution process after implementing a new toll pattern.

For the congestion toll design problem with day-to-day dynamics, Wie and Tobin (1998) solved it by formulating a convex control model of the dynamic system optimal traffic assignment on general traffic networks. Sandholm (2002) proposed a dynamic congestion pricing considering road users' learning behavior and day-to-day route choice adjustment process to guarantee an efficient utilization of the entire network. Thereafter, Friesz et al. (2004) studied the day-to-day dynamic toll with the objective of maximizing the net present value of social welfare and the constraint of a minimum revenue target. Yang et al. (2007) and Wang et al. (2015) considered the convergence speed and rapidity of restoring the normal state after disruption, respectively. More recently, Guo et al. (2015) proposed a concise and practical day-to-day dynamic pricing pattern based on Friesz et al. (2004) and Yang et al. (2007), and the tolls on each day were merely determined by the flows and tolls on the previous day. However, as claimed in Ye et al. (2015), all of these day-to-day dynamic toll patterns required either an explicit mechanism for road users' route choice adjustment process or adjustable tolls. Ye et al. (2015) studied the marginal-cost pricing scheme with day-to-day dynamics, and proposed a trial-and-error method for the optimal tolls, where the information of network attributes is not required. Xu et al. (2016) also adopted a trial-and-error method to study the global convergence of traffic-restraint congestion-pricing scheme with day-to-day flow dynamics. Tan et al. (2015) investigated the day-to-day congestion pricing with the objective of minimizing the total system cost and time considering the day-to-day route flow evolution and user heterogeneity which can be captured by road users' value-of-times. However, all of the aforementioned studies focus on deterministic day-to-day dynamic pricing. Rambha and Boyles (2016) investigated the dynamic congestion pricing considering users' stochastic day-to-day route flow evolution process. Cheng et al. (2016) made a comprehensive review of urban dynamic congestion pricing and emphasized that it was an emerging research needs to investigate the dynamic congestion pricing problem.

Generally, day-to-day dynamic traffic models can be classified into two major categories: deterministic dynamics and stochastic dynamics. Since stochastic dynamics can capture the variability associated with the random nature of the day-to-day dynamic flow evolution process, they can better reflect the practical circumstances than the deterministic day-to-day dynamics (Watling and Hazelton, 2003). Many of the existing stochastic day-to-day dynamics follow Markov processes (e.g., Cascetta, 1989; Cascetta and Cantarella, 1991; Cantarella and Cascetta, 1995; Watling, 1999; Hazelton, 2002; Hazelton and Watling, 2004; Watling and Cantarella, 2013; Smith et al., 2014), and Davis and Nihan (1993) and Hazelton et al. (1996) provided a Gaussian multi-variant autoregressive process as well as a Markov Chain Monte Carlo method to solve these day-to-day dynamic models, respectively. Interested readers can refer to Watling and Cantarella (2013, 2015) for comprehensive reviews of day-to-day dynamics with Markov process. Due to the non-additive property of the nonlinear distance-based toll, path-based (instead of link-based) models are more suitable for the day to day dynamics problem in the context of distance-based toll.

As claimed before, considering the fluctuation of traffic flows, the concept of robust optimization should be taken for the congestion toll determination problem. However, most of the existing robust congestion pricing schemes (to name a few, Gardner et al., 2008; Gardner et al., 2010; Lou et al., 2010) focus on static traffic conditions, while only a few researches focus on dynamic toll problems. Recently, Chung et al. (2012) investigated the dynamic congestion pricing with demand uncertainty using a robust optimization approach, and the proposed robust dynamic solutions outperformed either the nominal dynamic or the robust static solutions according to their numerical results. Zheng et al. (2012) studied the dynamic congestion pricing with macroscopic fundamental diagram and an agent-based traffic model. The tolls are determined in terms of actual traffic dynamics, rather than the conventional models based on marginal cost and demand-supply curves. The gaps of the existing robust dynamic pricing models include: (i) the existing studies focus on a within-day time scale, rather than a day-to-day time scale; (ii) only a flat pattern is accounted for, while the more equitable and efficient distance-based tolls are not addressed and (iii) optimal pricing considering the path-based day-to-day dynamics model under SUE constraints is still an open question. Consequently, it is a timely topic to address the robust optimization of distance-based congestion pricing considering the day-to-day flow dynamics under SUE constraints.

1.2 Objectives and contributions

This paper aims to solve the optimal toll design problem in a dynamic network considering the day-to-day flow evolution process under SUE constraints. After an implementation/adjustment of a congestion pricing scheme, the network flows in a certain period of days are not on an equilibrium state, thus it is problematic to take the equilibrium-based indexes as the pricing objective. Therefore, the concept of robust optimization is taken for the congestion toll determination problem, which takes into account the network performance of each day. First, a minimax model which minimizes the maximum regret on each day is proposed (Wang et al., 2016). Taking as a constraint of the minimax model, a path-based stochastic day to day dynamics model is then proposed. It is worth noting that the proposed minimax model is also a bi-level programming model since the calculated route flows in terms of the day-to-day dynamics are deemed as the lower level of the robust optimization to minimize the maximum regret on each day.

It is difficult to handle the nonlinear distance-based toll problem because there is no specific function form to describe the toll function, thus a piecewise linear function is used as an approximation to solve the nonlinear distance-based toll problem. It is very challenging to solve the robust optimization model: not only because the bi-level programming model is an NP-hard problem, but also due to the fact that the day-to-day flow evolution mapping has no closed form. Let $d = 1, 2, \cdots, D$ refer to the dth day of the study period, and $f(\mathbf{y}, d)$ denote the path flow on day d in terms of a toll pattern $\mathbf{y}$. With a given initial flow pattern $f(\mathbf{y}, 1)$, the day-to-day flow model $f(\mathbf{y}, d)$ has no closed form when $d > 2$, which is common for all the existing day-to-day dynamics models. Due to the implicity of the flow map function, it is difficult to solve this problem using a gradient-based method. Therefore, a two-phase artificial bee colony (ABC) algorithm is developed in this paper, of which the first phase solves the minimal *ETTC* of each day and the second phase handles the minimax robust optimization problem.

To sum up, contributions of this paper are twofold: (i) a robust optimization model is built for the optimal distance-based toll in an urban network with day-to-day dynamics under SUE constraints; (ii) a two-phase ABC algorithm is developed for the highly complex problem considering the implicit day-to-day flow map function with a distance toll. This paper is organized as follows. Section 2 first introduces the nonlinear distance toll which can be approximated by a piecewise linear toll function. In Section 3, a path-based day-to-day dynamics model under SUE constraints is proposed. A minimax model for the optimal toll pattern that minimize the maximum regret on each day is introduced in Section 4, and a two-phase ABC algorithm is proposed as a solution method for solving the bi-level minimax model in Section 5. A numerical experiment is provided in Section 6 to demonstrate the application of the proposed approach, and finally conclusions are drawn in Section 7.

2. Problem Statement

Consider a strongly connected network, denoted by $G = (N, A)$, where N denotes set of nodes and A is the set of directed links. The notation in this paper mostly follows that in Liu et al. (2014a), which is summarized as follows:

Notation	Explanations
D	The total planning period for one toll pattern.
d	The number of days after the toll implementation, $d = 1, 2, \cdots, D$.

W	The set of origin-destination (OD) pairs.
R^w	The set of paths between an OD pair $w \in W$.
$\mathbf{f}$	The column vector of all the path flows over the entire network, $\mathbf{f} = \left(f_{wr}, r \in R^w, w \in W \right)^{\mathrm{T}}$.
f_{wr}	The traffic flow on path $r \in R^w$ between OD pair $w \in W$.
$\mathbf{q}$	The column vector for all the travel demands, $\mathbf{q} = \left(q^w, w \in W \right)^{\mathrm{T}}$.
q^w	The travel demand between $w \in W$.
$\mathbf{t}(\mathbf{v})$	The column vector of the link travel time functions, $\mathbf{t}(\mathbf{v}) = \left(t_a(v_a), a \in A \right)^{\mathrm{T}}$.
$t_a(v_a)$	The travel time function of link $a \in A$, assumed to be increasing, convex and continuously differentiable.
$\mathbf{v}$	The column vector of all these link flows, $\mathbf{v} = \left(v_a, a \in A \right)^{\mathrm{T}}$.
v_a	The traffic flow on link $a \in A$.
δ_{ar}^w	$\delta_{ar}^w = 1$ if path $r \in R^w$ contains link a, and $\delta_{ar}^w = 0$ otherwise.
$\mathbf{y}$	The vertex values, $\mathbf{y} = \left(y_0, y_1, y_2, \ldots y_k, \ldots y_K \right)^{\mathrm{T}}$ of the stepwise linear toll function.
$\phi(\boldsymbol{\eta})$	The toll charge function.
K	The total number of the intervals in the toll function $\phi(\boldsymbol{\eta})$.
$\boldsymbol{\eta}$	Column vector for the travel distance of all the paths in the cordon, $\boldsymbol{\eta} = \left(\eta_{wr}, r \in R^w, w \in W \right)^{\mathrm{T}}$.
$\boldsymbol{\tau}$	Column vector of the distance-based toll $\boldsymbol{\tau} = \left(\tau_{wr}, r \in R^w, w \in W \right)^{\mathrm{T}}$.

For the ease of presentation, it is assumed that there is only one cordon in the network. Yet, the proposed methodology can be easily extended to the cases with multiple cordons. Let η_{wr} denote the length portion of path $r \in R^w$ in the cordon, and that of all the paths are grouped into the column vector $\boldsymbol{\eta}$. The distance-based toll function $\phi(\boldsymbol{\eta})$ is assumed to be piecewise linear with respect to the travel distance $\boldsymbol{\eta}$, which is an approximation of any form of nonlinear functions. The function $\phi(\boldsymbol{\eta})$ is defined on the range $[\eta_0, \eta_K]$ with K equal intervals as shown in Figure 1. The maximal and minimal travel distance length of paths in all the L cordons are η_K and η_0, respectively.

We can see that the distance-based toll function is composed by K intervals. This piecewise linear toll function can be uniquely defined by the two vertexes of each interval. It should be noted that the piecewise linear approximation method of the nonlinear distance-based toll function can be easily adjusted for the case of unequal intervals from the minimal path length to the maximal path length in the pricing cordons. The distance-based toll should be a non-decreasing function of the travel distance:

$$y^{\min} = y_0 \le y_1 \le y_2 \le \ldots \le y_k \le \ldots \le y_K \le y^{\max} \qquad (1)$$

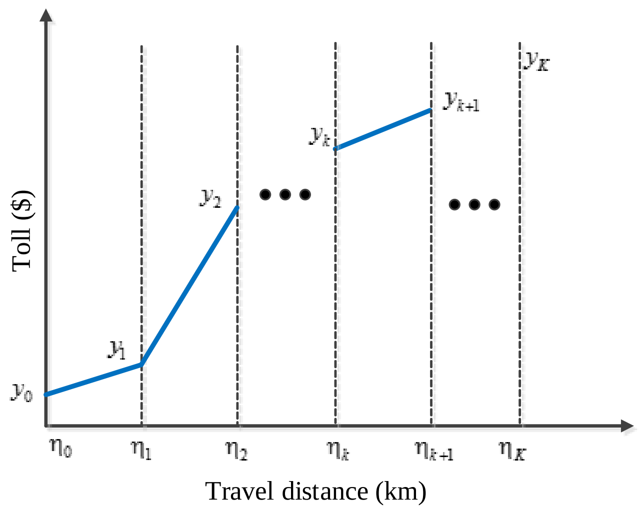


Figure 1: Piecewise linear distance-toll function

Suppose that for a particular path $r \in R^w$, its travel length in the cordon η_{wr} locates in the k th distance interval of the distance-based toll function shown in Figure 1, then the distance toll of path $r \in R^w$ can be computed by:

$$\tau_{wr} = \phi(\eta_{wr}) = y_{k-1} + \frac{\eta_{wr} - \eta_{k-1}}{\eta_k - \eta_{k-1}}(y_k - y_{k-1}) \qquad (2)$$

The total/generalized travel cost on path $r \in R^w$ between OD pair $w \in W$:

$$c_{wr} = \sum_a t_a \delta_{ar}^w + \tau_{wr} / \kappa \qquad (3)$$

where κ is the travelers' value-of-time.

From Eq. (2) we can see that each toll pattern τ is uniquely determined by the vertex values $\mathbf{y}=\left(y_0,y_1,y_2,\ldots y_k,\ldots y_K\right)^{\mathrm{T}}$. Let Ω_y be the set of all the feasible $\mathbf{y}$. Then, the toll design problem is to determine the optimal $\mathbf{y}^*\in\Omega_y$. Before introducing the model for the optimal $\mathbf{y}^*$, a path-based stochastic day-to-day dynamics model is first discussed in the next section.

3. A path-based day-to-day dynamics model

When a particular toll pattern $\mathbf{y}\in\Omega_y$ is implemented at day $d=1$, it will cause changes on the commuters' route choice decisions, thus giving rise to a new day to day flow evolution trajectory. In order to evaluate the toll pattern $\mathbf{y}$, a mechanism is needed to predict the flow evolution trajectory caused by $\mathbf{y}$. Due to the existence of nonlinear distance-based toll, the path travel cost (3) is not additive to the link costs. Therefore, path-based (instead of link-based) models are more suitable for the flow prediction problem in the context of distance-based toll.

In a day-to-day dynamics model, on any day d each traveler's route choice decisions are affected by the forecasted path travel time, denoted by $\mathbf{h}(\mathbf{y},d)=\left(h_{wr},r\in R^w,w\in W\right)^T$. $\mathbf{h}(\mathbf{y},d)$ is obtained based on his/her historical information in the long-term memory as well as limited observation to the on-going traffic conditions (Xie and Liu, 2014). Therefore, the forecasted path travel cost for the next day $\mathbf{h}(\mathbf{y},d+1)$ is usually considered as a weighted combination of the current day's actual and forecasted travel cost (Cantarella, 2013; Cantarella and Watling, 2016). Then the following expression is given for $\mathbf{h}(\mathbf{y},d+1)$:

$$\mathbf{h}(\mathbf{y},d+1)=\beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+(1-\beta)\cdot\mathbf{h}(\mathbf{y},d)\qquad(d=1,2,3,\cdots)\tag{4}$$

where $\mathbf{c}=\left(c_{wr},r\in R^w,w\in W\right)^T$ is the actual path travel costs on day d, as defined by Eq. (3). β is a weighting parameter and it satisfies $0<\beta\le 1$.

To get a general form for $\mathbf{h}(\mathbf{y},d+1)$, we expand Eq. (4) recursively:

$$
\begin{aligned}
\mathbf{h}(\mathbf{y},d{+}1) &= \beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+(1-\beta)\cdot\mathbf{h}(\mathbf{y},d) \\
&= \beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+(1-\beta)\cdot\left[\beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d-1)+(1-\beta)\cdot\mathbf{h}(\mathbf{y},d-1)\right] \\
&= \beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+\beta\cdot(1-\beta)\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d-1)+(1-\beta)^2\cdot\mathbf{h}(\mathbf{y},d-1) \\
&= \beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+\beta\cdot(1-\beta)\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d-1)+(1-\beta)^2\cdot \\
&\quad \left[\beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d-2)+(1-\beta)\cdot\mathbf{h}(\mathbf{y},d-2)\right] \\
&= \cdots\cdots \\
&= \beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+\beta\cdot\sum_{k=2}^{d-1}\left[(1-\beta)^{k-1}\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d-k+1)\right]+(1-\beta)^{d-1}\cdot\mathbf{h}(\mathbf{y},2)
\end{aligned}
\tag{5}
$$

and for day $d=2$:

$$\mathbf{h}(\mathbf{y},3)=\beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},2)+(1-\beta)\cdot\mathbf{h}(\mathbf{y},2) \tag{6}$$

As for day $d=1$, we assume that

$$\mathbf{h}(\mathbf{y},2)=\mathbf{c}(\mathbf{y},\mathbf{f},1)+\boldsymbol{\xi} \tag{7}$$

where $\boldsymbol{\xi}=\left(\xi_{wr}, r\in R^{w}, w\in W\right)^{T}$ is a vector of random variables reflecting commuters' perception errors on the path travel times (Liu et al., 2014b). Thus, Eq. (5) becomes

$$\mathbf{h}(\mathbf{y},d{+}1)=\beta\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)+\beta\cdot\sum_{k=2}^{d-1}\left[(1-\beta)^{k-1}\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d-k+1)\right]+(1-\beta)^{d-1}\cdot\mathbf{c}(\mathbf{y},\mathbf{f},1)+\boldsymbol{\xi} \tag{8}$$

Hence, $\mathbf{h}(\mathbf{y},d{+}1)$ is also a vector of random variables with the same distribution type of $\boldsymbol{\xi}$.

For the day-to-day dynamics model, a reasonable assumption is further made as follows:

Assumption 1: After any day d, only a proportion $\alpha\in(0,1]$ of travelers will reconsider their previous day's route choices, based on $\mathbf{h}(\mathbf{y},d{+}1)$; and the proportion $1-\alpha$ of travelers will insist on choosing the same routes on the previous day.

Based on Assumption 1, then we have the following dynamic route choice process (e.g., Cantarella and Watling, 2016):

$$\mathbf{f}(\mathbf{y},d+1)=\alpha\cdot\mathbf{q}\cdot\mathbf{p}\left(\mathbf{h}(\mathbf{y},d{+}1)\right)+(1-\alpha)\cdot\mathbf{f}(\mathbf{y},d) \qquad (d=1,2,3,\cdots) \tag{9}$$

where $\mathbf{f}(\mathbf{y},d+1)$ is the path flows of day $d+1$, $\mathbf{f}\left(\mathbf{y},d\right)$ denote the path flows of day d. $\mathbf{p}\left(\mathbf{h}(\mathbf{y}, d+1)\right) = \left(p_{wr}, r \in R^w, w \in W\right)^T$ is the route choice probabilities in terms of the forecasted route travel costs $\mathbf{h}(\mathbf{y}, d+1)$. In this paper, ξ is assumed to follow the Gumbel distribution, thus the route choice probabilities can be obtained by:

$$p_{wr}\left(\mathbf{h}(\mathbf{y}, d+1)\right) = \frac{\exp(-\theta h_{wr}(\mathbf{y}, d+1))}{\sum_{l \in R^w} \exp(-\theta h_{wl}(\mathbf{y}, d+1))}, r \in R^w, w \in W \tag{10}$$

where θ is a dispersion parameter.

From Eq. (8), it is clear that given an initial toll pattern and route flow on day $d=1$, we can obtain the corresponding forecasted travel costs on any day. We can see that the route choice decisions of day $d+1$ depend on *all* of the previous days' route choices, which implies that travelers never forget any experiences in the past. Such a model is regarded as an infinite learning process (Cantarella and Watling, 2016). The infinite learning process is apparently not realistic, especially when d becomes large. In fact, the commuters' route choice decisions are highly affected by the unexpected incidents occurs *more recently*; for instance, unexpected network disruptions and adverse weather conditions. Thus, a finite learning process is more suitable, as shown in Assumption 2:

Assumption 2: Travelers' route choice decisions are largely influenced by what happened in the most recent days, which is called a finite memory length *m*, and *m* is a pre-determined constant. Hence, only the most recent *m* days' route choice decisions are considered in the current day's route choice decision.

Based on the Assumption 2, then Eq. (8) is further transformed to:

$$\mathbf{h}(\mathbf{y}, d+1) = \beta \cdot \mathbf{c}(\mathbf{y}, \mathbf{f}, d) + \beta \cdot \sum_{k=2}^{m} \left[(1-\beta)^{k-1} \cdot \mathbf{c}(\mathbf{y}, \mathbf{f}, d-k+1)\right] + \xi \tag{11}$$

It is worth noting that the coefficients of $\mathbf{c}(\cdot)$ on the right hand side of Eq. (8) sum to 1, while in Eq. (11), the summation of coefficients on the right hand side does not equal to 1 due to the finite memory length *m*. In order to ensure the convergence of the proposed model, a scaling factor is imposed on the right-hand side of Eq. (11) to make the coefficients sum to 1. Hence, Eq. (11) becomes:

$$\begin{aligned} \mathbf{h}(\mathbf{y}, d+1) = & \frac{\beta}{1-(1-\beta)^m} \cdot \mathbf{c}(\mathbf{y}, \mathbf{f}, d) \\ & + \frac{\beta}{1-(1-\beta)^m} \cdot \sum_{k=2}^{m} \left[(1-\beta)^{k-1} \cdot \mathbf{c}(\mathbf{y}, \mathbf{f}, d-k+1)\right] + \xi \end{aligned} \tag{12}$$

Note that it is easy to find that $\frac{\beta}{1-(1-\beta)^m}+\frac{\beta}{1-(1-\beta)^m}\cdot\sum_{k=2}^{m}(1-\beta)^{k-1}=1$.

From the route choice process (9), we can see that the route flow on day $d+1$ is determined by two components: the first term in the right hand side of Eq. (9) is the portion of travelers who will reconsider their previous day's route choices (i.e., $\alpha\cdot\mathbf{q}$) and choose routes different from the previous day's routes in terms of the route choice probabilities $\mathbf{p}$, reflecting the *regret* of their decision making behavior; and the second term is the portion of travelers who do not change their previous day's route choices, interpreted as the *inertia* of their decision making behavior. It is worth noting that in the reconsideration part of travelers, one may change to another other route, but he/she may also repeat the previous day's route choice.

From an aggregate point of view, we define the actual route choice probabilities $\breve{\mathbf{p}}$ as follows, which is a composite of the two types of route choice probabilities $\mathbf{p}$ and $\bar{\mathbf{p}}$ on the right-hand-side of Eq. (13):

$$\breve{\mathbf{p}}\left(\mathbf{h}(\mathbf{y},d+1)\right)=\alpha\cdot\mathbf{p}\left(\mathbf{h}(\mathbf{y},d+1)\right)+(1-\alpha)\cdot\bar{\mathbf{p}}(\mathbf{y},d) \tag{13}$$

where $\bar{\mathbf{p}}(\mathbf{y},d)=\frac{\mathbf{f}(\mathbf{y},d)}{\mathbf{q}}$. Eq. (13) is in fact a transformation of Eq. (9) by dividing the demand $\mathbf{q}$ on both sides of Eq. (9). The two terms in the right hand side of Eq. (13) reflect the regret and inertia, respectively. From Eqs. (9)-(13), we can see that with a given toll pattern and route flow on day d, the next day's route flow can be obtained recursively.

The model introduced above in this section is termed as a path-based day to day dynamics model under SUE constraints. In the context of SUE, it is defined that $\mathbf{f}=\mathbf{q}\breve{\mathbf{p}}$, namely, $\mathbf{f}$ is a deterministic value. This definition is made on the basis of the weak law of the large numbers (Daganzo and Sheffi, 1977), in view that the number of commuters is large enough and also they act independently. However, if we don't consider the weak law of the large numbers here, the path flows are in fact random variables following a multinomial distribution. We use $\bar{\mathbf{f}}$ to denote the path flows in this case, it gives:

$$\bar{\mathbf{f}}(\mathbf{y},d) \sim \text{Multinomial}(\mathbf{q},\breve{\mathbf{p}}) \tag{14}$$

where $\breve{\mathbf{p}}$ is similarly defined as Eq. (13). We can see that $\mathbf{f} = E(\bar{\mathbf{f}})$.

Model (14) is consistent with many other stochastic process models for day to day dynamic traffic assignment in the literature, including Watling and Hazelton (2003), Hazelton and Watling (2004), and Cantarella and Watling (2016). The recursive traffic assignment model (12) is an m-dependent Markov chain (Hazelton and Watling, 2004).

4. A minimax regret model for the optimal toll pattern

As discussed in the Introduction, after a certain period D (say $d = D = 90$) the network environment (both supply and demand) is evidently changed. Then, a new assessment of the optimal toll should be performed, thus giving rise to a new toll pattern $\bar{\mathbf{y}}$, in which a new day-to-day flow evolution occurs and d should be reset to 1. Hence, in this paper, the study period is from $d = 1$ to $d = D$.

As an important ingredient in the travelers' route choice decisions, any toll pattern $\mathbf{y}$ would give rise to different day-to-day path flows. Let $\mathbf{f}(\mathbf{y},d)$ denote the column vector of path flows on day d in terms of a toll pattern $\mathbf{y}$, which is determined by Eq. (9). The objective of the authorities is to improve the network performance of each day rather than merely that of the equilibrium condition. Since the commuters' route choice behavior follows logit-based SUE, the optimal network performance is reflected by the one with minimal expected total travel cost (Liu et al., 2014a). On day d, the optimal toll pattern is thus given by:

$$ETTC(\mathbf{y},d) = \min_{y} \mathbf{f}(\mathbf{y},d)^{\mathrm{T}} \cdot \mathbf{c}(\mathbf{y},\mathbf{f},d) + \frac{1}{\theta}\mathbf{f}(\mathbf{y},d)^{\mathrm{T}} \cdot \ln \frac{\mathbf{f}(\mathbf{y},d)}{q^{w}} \tag{15}$$

where the objective is to minimize the expected total travel cost.

Let $\mathbf{y}(d)$ be the optimal toll pattern of day d, namely,

$$\mathbf{y}(d) \in \arg\min_{\mathbf{y}\in\Omega_y} ETTC(\mathbf{y},d) \tag{16}$$

It is unlikely that a particular toll pattern can be the optimal toll pattern of all the days/scenarios (from day 1 to day D). Thus, if an arbitrary toll pattern y is implemented, there would be a gap between the corresponding total travel cost and the optimal $ETTC(\mathbf{y},d)$. This gap is defined as the *regret* from the viewpoint of the network authorities.

For any toll pattern $\mathbf{y}\in\Omega_y$, its regret on day d equals $\mathbf{f}(\mathbf{y},d)^{\mathrm{T}}\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)-ETTC(\mathbf{y},d)$, and its maximum regret value among the whole planning period D is given as $\max_d\left[\mathbf{f}(\mathbf{y},d)^{\mathrm{T}}\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)-ETTC(\mathbf{y},d)\right]$. Therefore, to minimize the maximum regret, we propose the following robust programming model:

$$\min_{\mathbf{y}}\max_d\left[\mathbf{f}(\mathbf{y},d)^{\mathrm{T}}\cdot\mathbf{c}(\mathbf{y},\mathbf{f},d)-ETTC(\mathbf{y},d)\right] \tag{17}$$

subject to the day-to-day route flows introduced in Section 3.

The above model can be deemed as a bi-level model, where the lower level reflects the predicted network flows, which is discussed in Section 3. The optimal solution to model (17) is a robust pattern that takes into consideration the network performance on each day of the study period.

5. A two-phase ABC algorithm

The bi-level models are commonly used to formulate network design and toll design problems, which are well recognized to be an NP-hard problem and hard to solve. In the literature, some existing solution methods usually have the same techniques which is to convert the bi-level problem to a single-level one, by replacing the lower level using the first-order Taylor approximation (sensitivity analysis method, see Yang and Bell, 1998) or relaxing the lower-level and gradually adding back (system optimal relaxation method, see Wang et al., 2013) or replacing the lower level by a gap function (see, Li et al., 2012), etc.

However, none of the existing solution methods discussed above is valid to use for the proposed robust programming model (17). This is mainly caused by the complexity of the term $\mathbf{f}(\mathbf{y},d)$. With a given initial flow pattern $\mathbf{f}(\mathbf{y},1)$, the day to day flow model $\mathbf{f}(\mathbf{y},d)$ has no closed form when $d>2$, which is true for all the existing day to day dynamics models. For instance, from model (9), we can see that the first-order derivative of $\mathbf{f}(\mathbf{y},d)$ also has no closed form because we do not have a closed form of $\mathbf{f}\left(\mathbf{y}, d-1\right)$; thus the sensitivity analysis method is not applicable to the problem. To solve the optimal toll design problem under a network with day to day flow dynamics is still an open question in the literature. However, this problem is of considerable significance for the studies of congestion pricing problems, thus this paper aims to provide some initial investigations of this difficult yet important problem. With the main focus on the modelling framework, the optimization level of the final solution is to some extent compromised.

The ABC algorithm was recently used to solve transportation problems, see e.g., Szeto et al. (2011), Szeto and Jiang (2012, 2014), Chen et al. (2015) and Huang et al. (2016). Compared with extant evolutionary algorithms like genetic algorithm (GA), the ABC algorithm has a better local search mechanism that enhances the solution quality (Chen et al., 2015), because GA conducts the crossover operations to produce new or candidate solutions from the present ones, while the ABC algorithm produces the candidate solution from its parent by a simple operation based on taking the difference of parts of the parent and a randomly chosen solution from the population. This process increases the convergence speed of searching into a local minimum. In this paper, a numerical algorithm, which is called a two-phase ABC algorithm, is proposed to solve the robust programming model for the optimal toll design. The first phase is used to calculate the minimum total travel time of each day d as shown in model (15). Then, taking the $ETTC(\mathbf{y}, d)$ as an input, the second-phase is used to solve the robust programming model (17). The detailed procedures of the algorithm are summarized as follows.

Procedures of the first-phase for solving the minimum expected total travel cost of each day:

Initialization

Step 1: (Initialization of the parameters). For simplicity, we set the colony size N_c, the number of employed bees N_e, onlookers N_o; the limit, which is the predetermined number of iterations; the initial value of iteration counter $I=1$, and its maximum value $I_{\max}$. Set the interval number of the distance toll and the planning period to K and D, respectively. Set the lower bound and upper bound of congestion toll to $y_{\min}$ and $y_{\max}$, respectively. Set the initial day to $d=0$.

Iteration Procedure

Step 2: (Initial route flows). Obtain the initial route flow by averagely assigning the demand to each feasible route for each OD pair.

Step 3: (Initialization of employed bees). Generate randomly distributed initial food sources (i.e., congestion toll patterns) for every employed bee.

Step 4: (Evaluation).

Step 4.1: (Calculate the toll values). Based on the piecewise linear toll function of Eq. (2), calculate the value of toll charge on each internal path of the cordon in term of the total length of each internal path.

Step 4.2: (Calculate the route travel times). Obtain the link flows based on the route flows. Calculate the link travel times with the link travel time functions. Obtain the route travel times according to the calculated link travel times.

Step 4.3: (Network loading procedure). Conduct the stochastic network loading procedure based on the toll charge and the measured route travel time in terms of the stochastic day-to-day dynamics model.

Step 4.4: (Evaluation). Calculate the *ETTC* for all of the employed bees based on Eq. (15). Set the limit counter of each food source be zero.

Step 5: (Employed bee phase). Conduct a neighborhood search based on the food sources generated by employed bees. Evaluate the fitness for each neighbor solution. If the fitness of the neighbor solution is better than the current food source, replace the current food source by the neighbor solution, and set the limit counter be 0; otherwise, keep the current food source generated by employed bee and increase the limit counter by 1.

Step 6: (Onlooker phase). Each onlooker chooses a food source based on the quality of the solutions. A roulette wheel selection method is adopted for onlookers to determine which food source they should choose. In other words, generate a uniformly distributed random number $r \in [0,1)$, if $p_i > r$, then the onlooker will execute a neighborhood search. Evaluate the fitness of the neighbor food source. If the fitness of the neighbor food source is better, replace the current food source by the neighbor food source; otherwise, keep the current food source generated by employed bee and increase the limit counter by 1.

Step 7: (Scout bee phase). Based on the current food sources, find the best one with the highest fitness (i.e., lowest *ETTC*). If one food source cannot improve its quality within the predetermined maximal trial number limit, and it is not the best food source at the same time, then the associated employed bee becomes a scout. It will execute a neighbor search again, generate a new randomly solution and set the corresponding limit counter be zero.

Step 8: (Convergence test). Set the iteration number $I = I + 1$. If $I < I_{\max}$, then return to Step 4; otherwise, let the minimal value of $ETTC$ equal $ETTC(\mathbf{y}, d)$ on day d and record the corresponding food source, and then go to Step 9.

Step 9: (Stop test). If $d \geq D$, then stop and output the minimal $ETTC$ and its corresponding optimal toll function for each day; otherwise, set $d = d + 1$ and go to Step 2.

Then, the second stage for solving the minimax regret model is described below:

Initialization

Step 1: (Initialization of the parameters). For simplicity, we set the colony size N_c, the number of employed bees N_e, onlookers N_o; the limit, which is the predetermined number of iterations; the initial value of iteration counter $I = 1$, and its maximum value $I_{\max}$. Set the interval number of the distance toll and the planning period to K and D, respectively. Set the lower bound and upper bound of congestion toll to $y_{\min}$ and $y_{\max}$, respectively. Set the initial day to $d = 0$.

Step 2: (Initialization of employed bees). Generate randomly distributed initial food sources (i.e., congestion toll patterns) for every employed bee. Calculate the corresponding fitness based on the day-to-day dynamic mechanism introduced in this paper. Set the limit counter of each food source be zero.

Iteration Procedure

Step 3: (Employed bee phase). Conduct a neighborhood search based on the food sources generated by employed bees. Evaluate the fitness for each neighbor solution. If the fitness of the neighbor solution is better than the current food source, replace the current food source by the neighbor solution, and set the limit counter be 0; otherwise, keep the current food source generated by employed bee and increase the limit counter by 1.

Step 4: (Onlooker phase). Each onlooker chooses a food source based on the quality of the solutions. A roulette wheel selection method is adopted for onlookers to determine which food source they should choose. In other words, generate a uniformly distributed random number $r \in [0,1)$, if $p_i > r$, then the onlooker will execute a neighborhood search. Evaluate the fitness of the neighbor food source. If the fitness of the neighbor food source is better, replace the current food source by the neighbor food source; otherwise, keep the current food source generated by employed bee and increase the limit counter by 1.

Step 5: (Scout bee phase). Based on the current food sources, find the best one with the highest fitness. If one food source cannot improve its quality within the predetermined maximal trial number limit, and it is not the best food source at the same time, then the associated employed bee becomes a scout. It will execute a neighbor search again, generate a new randomly solution and set the corresponding limit counter be zero.

Step 6: (Convergence test). Set the iteration number $I = I + 1$. If $I < I_{\max}$, then return to Step 3; otherwise, terminate the algorithm and output the best solution.

6. Numerical Experiments

As shown in Figure 2, a network example proposed in Liu et al. (2014a) is used to validate the proposed model and method in this section. This network contains 13 links and 9 nodes, with a congestion toll cordon indicated by the dashed line. There are two OD pairs: $1 \to 8$ and $1 \to 9$, each of which has an OD demand of 16,000. The incidence of links and paths for the network is provided in Table 1.

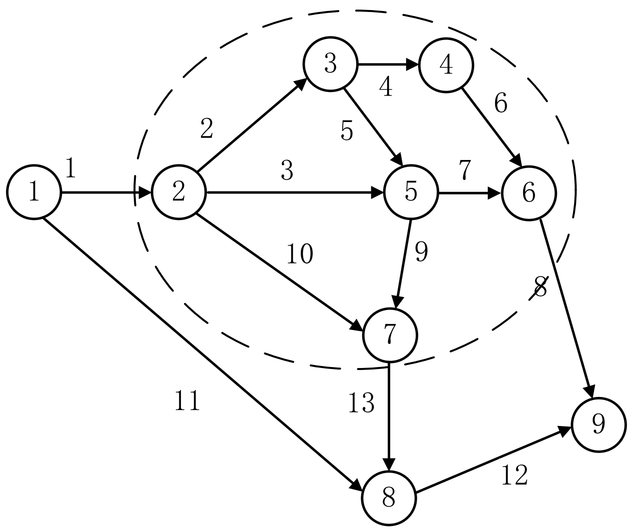


Figure 2: Network structure of the numerical example

Table 1: Link-path incidence relationship

OD pair	Path No.	Link sequence
(1,8)	1	1,2,5,9,13
	2	1,3,9,13
	3	1,10,13
	4	11

(1,9)	5	1,2,4,6,8
	6	1,2,5,7,8
	7	1,2,5,9,13,12
	8	1,3,7,8
	9	1,3,9,13,12
	10	1,10,13,12
	11	11,12

The link travel time is defined by the Bureau of Public Roads (BPR) function as follows:

$$t_a(v_a) = t_a^0 \left(1 + 0.15 \times \left(\frac{v_a}{H_a}\right)^{\rho}\right), \ a \in A \tag{18}$$

where t_a^0 is the free flow travel time on link a, H_a is the capacity of link a, and ρ is the exponent. The relevant link attributes are summarized in Table 2.

Table 2: Link data for the numerical example

Link ID	Tail	Head	Distance (km)	Free Flow Travel Time	Capacity	Exponent ρ
1	1	2	2	2	6000	4
2	2	3	7	2	4000	4
3	2	5	8	8	6000	4
4	3	4	2	2	2000	4
5	3	5	4	4	2000	4
6	4	6	6	6	1000	6
7	5	6	2	2	4000	4
8	6	9	6	6	6000	4
9	5	7	3	3	4000	4
10	2	7	9	9	2000	4
11	1	8	26	26	3000	4
12	8	9	4	4	3000	4
13	7	8	5	5	3000	4

Table 3: List of internal paths

Internal path ID	Component links	Total length
1	10	9
2	3,9	11
3	2,5,9	14
4	3,7	10

5	2,5,7	13
6	2,4,6	15

As shown in Figure 2, there are 8 links in the cordon area, including links 2, 3, 4, 5, 6, 7, 9, and 10. The 8 links compose 6 different internal paths connecting three entry nodes, which are nodes 2, 6 and 7. Table 3 provides the details of these internal paths.

The travelers' value-of-time is assumed to be $\kappa = 1.0$ in this example. According to the data of internal paths in Table 3, we can find that the minimum and maximum path distance in the cordon are 9 and 15km, and the range of travel distance difference is 6km. Hence, the piecewise linear toll function is assumed to have 6 intervals with 7 boundary distance values, and the length of each interval is 1km. It is worth noting that all the internal path distances are integers with a difference range of 6km in this example, thus 6 intervals are enough. When the number of internal paths and the difference range of internal path distances become larger, the number of intervals can also be larger to ensure a better characteristic of the nonlinear distance-based toll scheme. The upper and lower bounds of the distance toll are $y_0 = y^{\min} = 1.0$ and $y_6 = y^{\max} = 5.0$, respectively. The value of relevant parameters used in the two-phase ABC algorithm are summarized in Table 4. The numerical experiment is coded in Matlab R2016a running on a laptop with Inter(R) Core(TM) i7-5500U CPU @ 2.40GHz, 2.39GHz and 8.00G RAM.

Table 4: Parameters used in the two-phase ABC algorithm

Parameters	Value	Parameters	Value
Planning period	$D = 90$	Colony size in ABC	$N_c = 40$
Interval number of distance toll	$K = 6$	Number of employed bees in ABC	$N_e = 20$
Lower bound of congestion toll	$y^{\min} = 1.0$	Number of onlookers in ABC	$N_o = 20$
Upper bound of congestion toll	$y^{\max} = 5.0$	Limit in ABC	$\text{limit} = 2$
Memory length	$m = 3$	Maximum iteration value in ABC	$I_{\max} = 500$
Reconsideration rate	$\alpha = 0.6$	Weighting parameter	$\beta = 0.4$

As introduced in Section 5, we need to calculate the minimal *ETTC* for each day, which is an input in the second phase of the algorithm. Figure 3 depicts the minimal *ETTC* with the proposed stochastic day to day dynamics model, based on the first phase of the ABC algorithm from $d = 1$ to $d = 90$. We can find that the minimal *ETTC* has an evident fluctuation with the minimum (maximum) equal to 6.647×10^6 (6.655×10^6). Note that the corresponding optimal toll pattern of each day (see $\mathbf{y}(d)$ in Eq. (16)) is also varying.

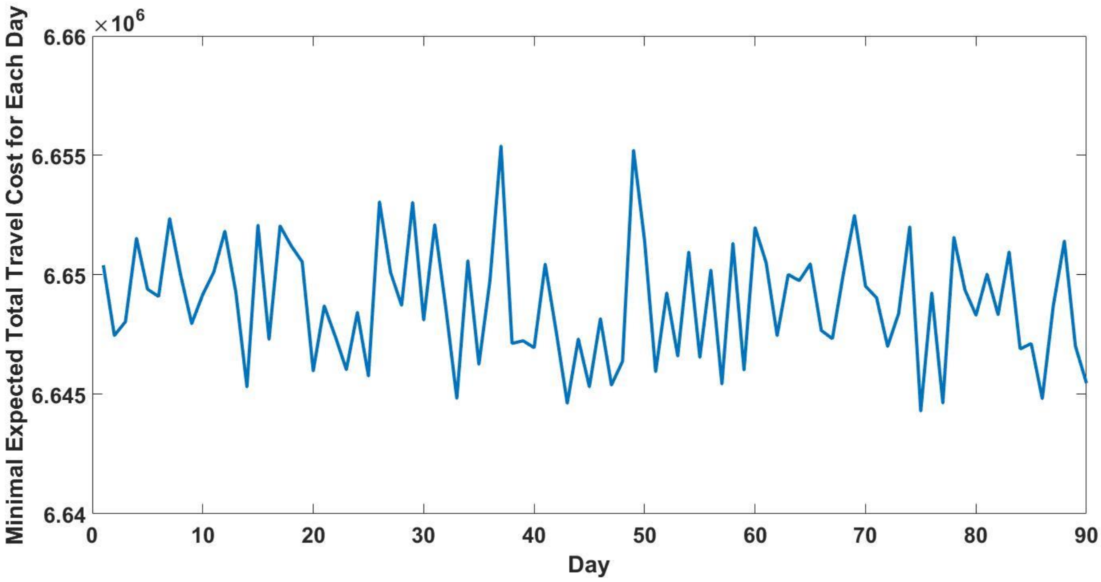


Figure 3: Minimal expected total travel cost for each day based on the first phase of the solution method

To further show the varying impacts of a particular toll pattern on the network, we use the optimal toll pattern of day 1 as an example; here $\mathbf{y}(1) = (1.08, 1.24, 1.69, 2.35, 3.01, 3.55, 4.32)$. Figure 4 then gives the value of regret from day 2 to day 90 for $\mathbf{y}(1)$ and the regret value of day 1 is zero. After the implementation of the optimal toll pattern of day 1, the regret is not a stable value because the optimal toll for day 1 is no longer optimal for other days.

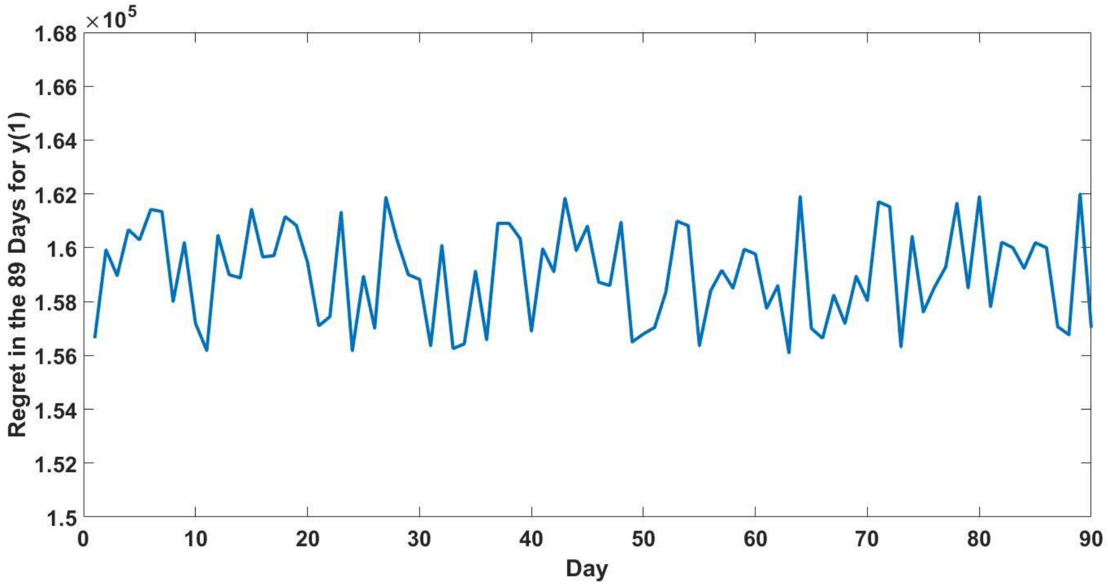


Figure 4: Value of the regret from day 2 to day 90 for $\mathbf{y}(1)$

The optimal toll can be obtained by minimizing the maximum regret in Eq. (17) and this proposed minimax regret model can be solved by the second phase of the ABC algorithm with the minimal *ETTC* for each day as an input. Figure 5 shows the convergence process, which converges after 114 iterations. The optimal toll is a robust pattern which takes into consideration the network performance on each day. Figure 6 depicts the optimal distance toll patterns for the proposed stochastic day to day dynamics model, which is clearly a nonlinear toll form: $\mathbf{y}^* = (1.45, 2.22, 3.51, 3.83, 4.20, 4.29, 4.44)$.

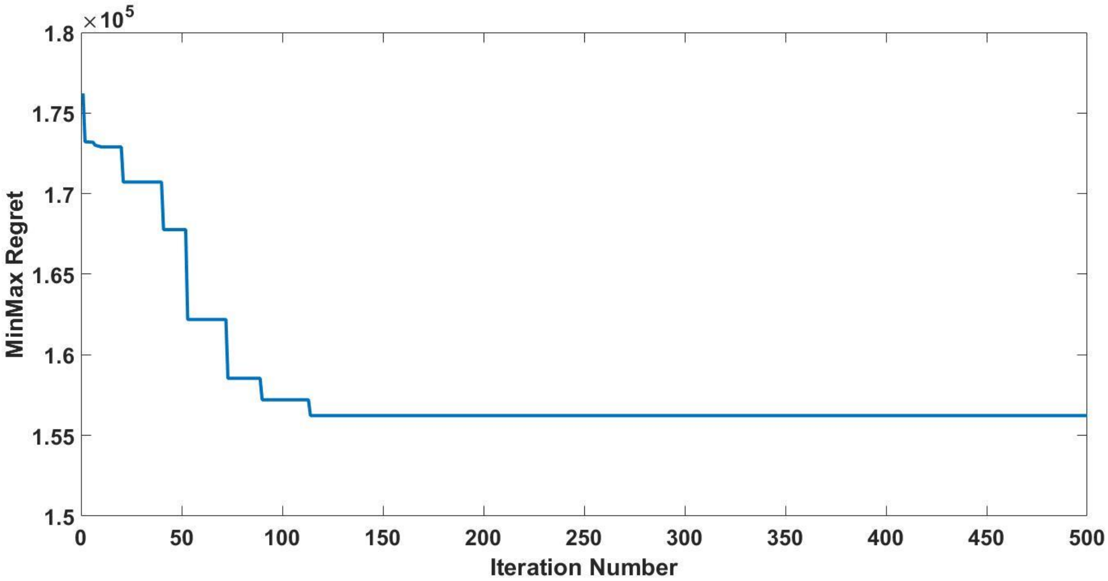


Figure 5: Convergence process of the minimax regret

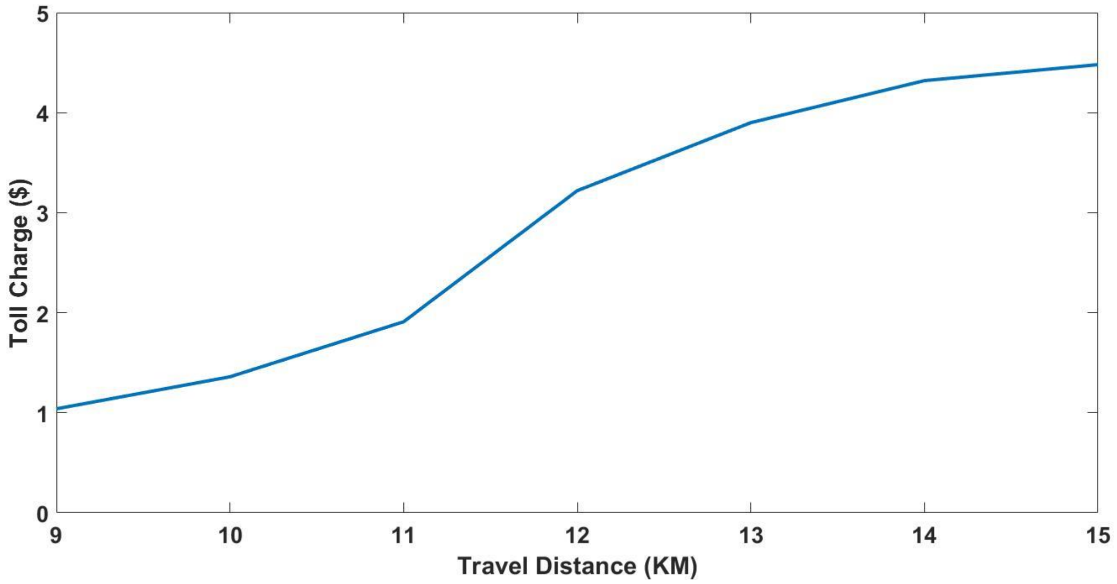
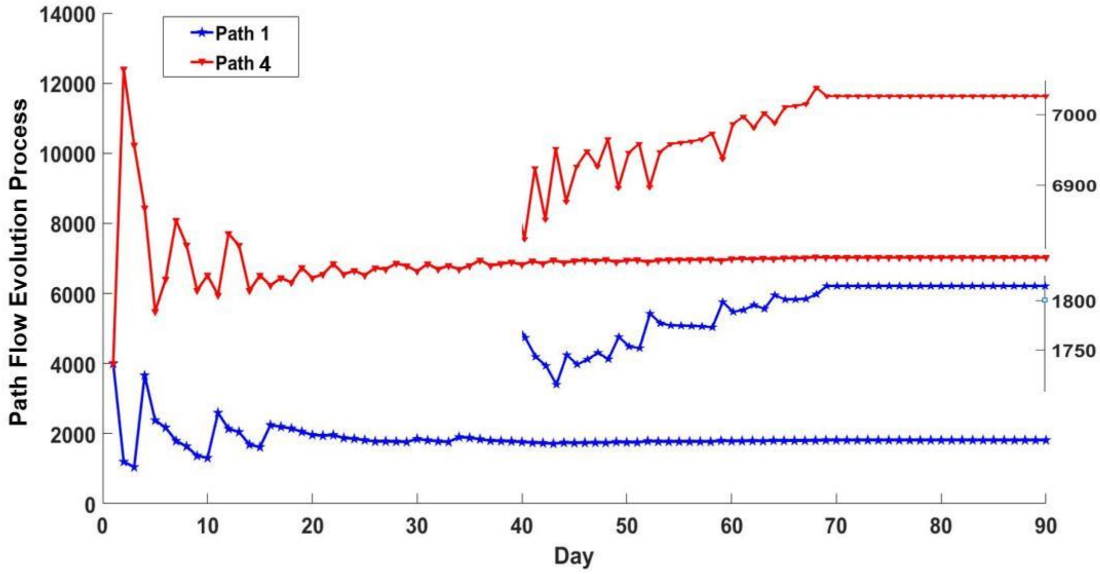
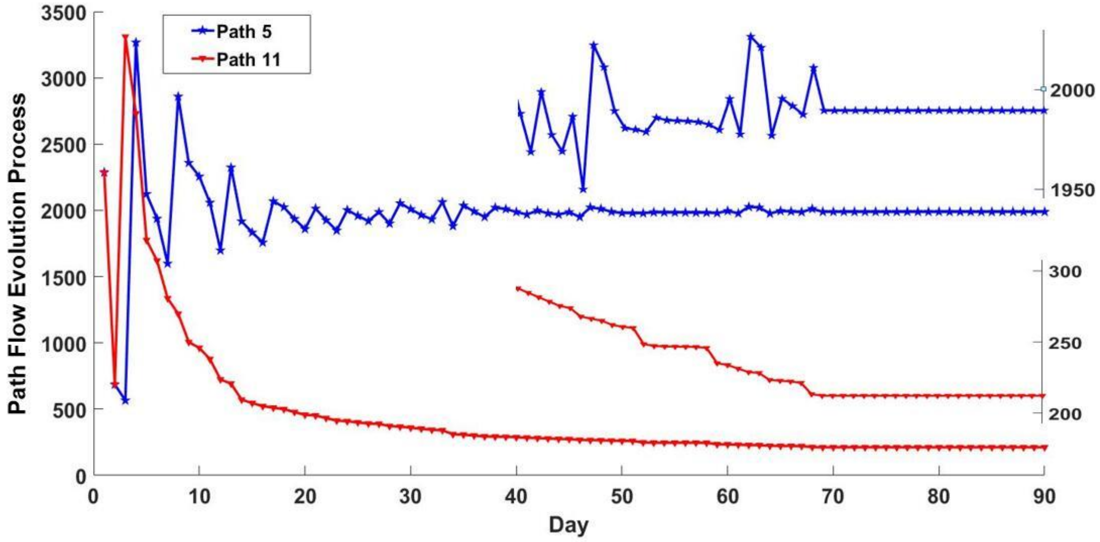


Figure 6: Optimal distance toll

We further analyze the flow evolution process in the network. For simplicity, we choose 4 typical paths which are path 1 and 4 for OD pair (1, 8) and path 5 and 11 for OD pair (1, 9). Among these four paths, path 1 and 5 are internal paths, while path 4 and 11 are external paths. Figure 7 and Figure 8 show the day-to-day flow and cost evolution trajectories over the whole planning period, and the top right corners are the partial enlarged details from day 40 to day 90. It is clear that both the day-to-day flows and costs converge to a stationary state, which follows the logit-based SUE principle, within 70 days. This evolution process also indicates the essence of the day-to-day dynamics with flows and costs evolving from disequilibrium to equilibrium states. It should be noted that the convergence speed will slow down when the network becomes larger.

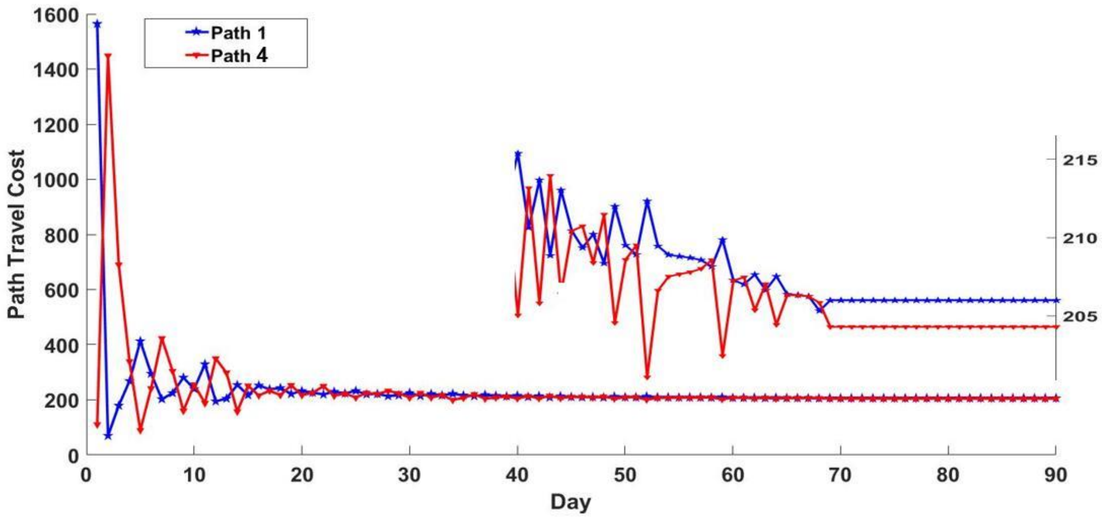


(a) Flow evolution process of path 1 and path 4

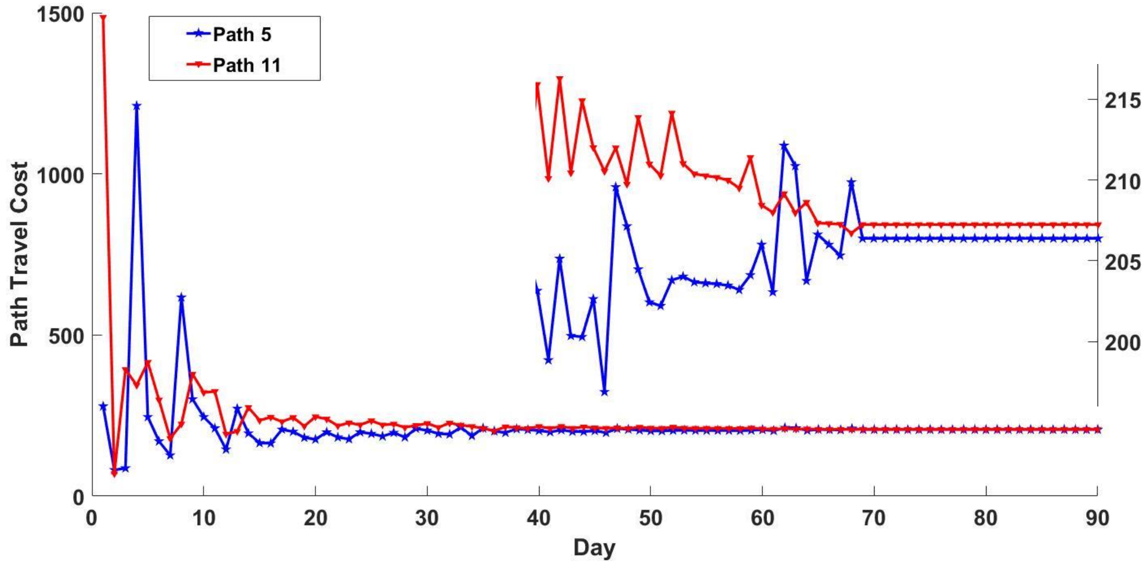


(b) Flow evolution process of path 5 and path 11

Figure 7: Path flow evolution trajectories over the planning period



(a) Travel cost of path 1 and path 4



(b) Travel cost of path 5 and path 11

Figure 8: Path travel cost evolution trajectories over the planning period

The result of this numerical test verifies the validity of the proposed model and algorithm. After the implementation/adjustment of a congestion pricing scheme, the network flows cannot achieve an equilibrium state overnight, thus it is problematic to take the equilibrium-based indexes as the pricing objective. Therefore, the concept of robust optimization is taken for the congestion toll determination problem, which takes into account the network performance of each day. For each day, it has an optimal toll pattern which minimizes the *ETTC* of that day, and thus has a relevant regret of that day. The objective is to find one robust optimal toll pattern which minimizes the maximal regret in the whole planning period. We can see from Figure 7 and Figure 8 that the path travel cost will be stable and equal for each OD pair after a certain period of days (which is 70-days in this small test network), and the flows of internal paths is much lesser than the external paths because of the tolls in the cordon areas. This result is also consistent with travelers' day-to-day route choice adjustment and learning behaviors.

7. Conclusion

This paper solves the robust optimization problem for nonlinear distance-based congestion tolls in a network considering stochastic day-to-day dynamics. After an implementation/adjustment of a congestion pricing scheme, the network flows in a certain period of days are not on an equilibrium state, thus it is problematic to take the equilibrium-based indexes as the pricing objective. Therefore, the concept of robust optimization is taken for the congestion toll determination problem, which takes into account the network performance of each day. Hence, a minimax model which minimizes the maximum regret on each day is proposed. Taking as a constraint of the minimax model, a path-based stochastic day to day dynamics model is proposed. Note that this minimax model is a bi-level programming model, with the upper level of minimizing the maximum regret and lower level of day-to-day dynamics processes. Due to the implicity of the flow map function, it is difficult to solve this minimax model by exact algorithms. Therefore, a two-phase ABC algorithm is developed to solve the bi-level model in this paper, of which the first phase solves the minimal total travel cost for each day and the second phase handles the minimax robust optimization problem.

For further researches on day-to-day dynamic pricing, several extensions need to be considered. On the one hand, the path flow adjustment ratio should be calibrated from real world data, one of which is social media data (Rashidi et al., 2017). On the other hand, efficiency, environment, as well as equity issues should be taken into consideration at the same time in the optimal toll design of congestion pricing problem. The distance-based tolls addressed in this paper are path-based, yet it is of considerable interest to further investigate the link-based distance tolls that is additive to the links. As a future work, the methodology and concepts provided in this paper are worthwhile to further study such sort of link-based distance tolls.

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Appendix

Table A: List of terminologies

ABC	Artificial bee colony
ERP	Electronic road pricing
ETTC	Expected total travel cost
GA	Genetic algorithm
SSO	Stochastic system optimum
SUE	Stochastic user equilibrium
TTC	Total travel cost