Network-level Optimization of Bus Stop Placement in Urban Areas

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ABSTRACT

Bus stops provide accessibility to public transit service, whereas they also influence the efficiency of mobility due to the extra dwell time. The decision of bus stop locations is a tradeoff between access coverage and mobility. This paper formulates a bi-objective optimization model of bus stop placement problem at the network level. Two objectives are pertinent to the improvement of mobility: minimization of total dwell time at stops and minimization of total number of bus stops. Access coverage is constrained to ensure a certain level of accessibility. The issue of stop congestion and its effect on road traffic flow is also considered in the model. The model is applied to a case study of the bus network in urban areas of Yancheng, China. The results show that the proposed model can be efficiently solved by CPLEX to obtain Pareto optimal solutions for real-case problems.

Keywords: public transport; bus stop placement; optimization

1. Introduction

Public transit is a key component in the transportation system, which is more energy efficient than other travel modes. It is generally regarded as an effective way to mitigate the problems attributed to the automobile, such as road congestion, energy consumption and air pollution (Ceder, 2007; Currie, 2010; Szeto and Jiang, Y. 2014; Tirachini et al., 2014; Chen et al., 2015; Ibarra-Rojas et al., 2015; Fan et al., 2016; Feng et al., 2016; Huang et al., 2016; among many others). To enhance attractiveness and associated ridership, public transit should provide good service to guarantee efficient mobility for all citizens within urban areas. Mobility can be expressed in terms of bus operating speed, and transit service becomes attractive when the travel time of transit modes (e.g. bus, light rail and subway) is comparable to that of the automobile. In this regard, some measures can be taken to increase bus operating speed and decrease passengers' travel time, which include signal priority at intersections, dedicated bus lanes at road segments, and the optimization of bus stop placement (McLeod and Hounsell, 2003; Currie et al., 2007; dell'Olio et al., 2011; Lin et al., 2015; Ahmed et al., 2016). The bus stop placement problem is considered an important issue with respect to the efficiency of mobility at the planning stage, which is tackled in this paper.

Bus stops increase accessibility for passengers to get to and depart from the bus service. When designing or modifying the bus network, the maximum spacing of stops needs to be constrained to ensure that bus service is accessible to passengers within a reasonable walking distance. At the same time, if stops are placed too close, bus vehicles have to dwell at excessive stops which slow down vehicles and incur an increase of passengers' in-vehicle travel time. Hence, the problem of determining bus stop placement is not trivial as it involves a balance between access coverage and mobility. This study considers these two factors simultaneously and proposes a methodology for the optimization of bus stop placement problem at the network level.

1.1. Literature Review

This section presents a review of the literature pertinent to bus stop placement. Currently, a number of studies have been conducted on the analysis of bus service access (Murray et al., 1998; Ammons, 2001; Wu and Hine, 2003; Chien and Qin, 2004; Wright and Hook, 2007; Ziari et al., 2007; Foth et al., 2013; Tirachini, 2014; Owen and Levinson, 2015). Access typically has to do with proximity to service: one is access distance from an origin to one bus stop, and the other is egress distance from one bus stop to a destination (Murray, 2003). The density and locations of bus stops are usually used to determine the access coverage of bus service. Several authors investigated the relationship between access coverage and the level of usage of bus service (Greenwald and Boarnet, 2001; Zhao et al., 2003; Cervero et al., 2009; Hess, 2009; Kim et al., 2012; Moniruzzaman and Páez, 2012). They showed that access/egress distance is negatively and significantly related to transit patronage. Consequently, the spacing of stops should have an upper bound threshold to guarantee that passengers complete their access/egress trips in a reasonable amount of time. Such maximum spacing threshold is generally location-specific, which depends on many local considerations (e.g., service region, road hierarchy, local demand). For example, bus stops in the city's central area should be placed closer than those in the suburb. The maximum spacing of stops on the arterials or trunk roads is usually set larger than that on the collectors and local roads. The above previous studies proposed several useful methods to determine the detailed access standard.

Apart from access coverage, mobility is the other salient factor which greatly influences bus system performance and level of service (Litman, 2003; Kim and Lee, 2008; Feng et al., 2010; Hahn et al., 2011; Zhu et al., 2016). Efficient mobility is closely related with bus stop placement because fewer stops generate less dwell time at stops and subsequently contribute to higher bus operating speed. For decades, numerous transport scholars have endeavored to develop different optimization models with consideration of both access coverage and mobility (Levinson, 1983; Saka, 2001; Murray and Wu, 2003; dell'Olio et al., 2006; Chien et al., 2010; Ibeas et al., 2010; Yan et al., 2013; Ceder et al., 2015; Chen et al., 2016). These models can be further divided into two categories: (1) the objective function subsumes access coverage and mobility at the same time, which is associated with the benefits of two stakeholders (passengers and bus company); and (2) the objective function focuses on the issue of mobility, and access coverage is seen as constraints. Given assumed access standard, stop placement is optimized in order to achieve the utmost improvement of mobility.

Some previous studies investigated bus stop placement at the route level, and some others addressed it as part of bus network design problem (Ceder, 2007). The general bus network design problem contains two main procedures, namely route generation and stop placement. It is usually an NP-hard problem, and most of extant studies use heuristic methods to acquire potentially good solutions (Kepaptsoglou and Karlaftis, 2009; Ibarra-Rojas et al., 2015). Once bus routes are generated and put into use for a certain period, it is often difficult to change or alter their routings. For an already existing bus network, an alternative approach to enhance bus system performance is to re-design the locations of stops along given bus routes. Compared with the general bus network design problem, network-level optimization of stop placement is easier because route generation is not needed. As a result, it may be efficiently solved by some exact solution methods or off-the-shelf solvers, such as CPLEX. Yet detailed methodologies that exclusively cope with bus stop placement at the network level are in short supply, and the present paper strives to fill this gap in the extant literature.

1.2. Objective and Contribution

The primary objective and contribution of this study is to propose a network-based methodology for the optimization of bus stop placement problem. The formulated model in this study belongs to the second category that the objective functions are pertinent to mobility, while access coverage is treated as a constraint. Bus stops increase accessibility for passengers to get to and depart from the bus service. To reduce the access/egress distance, the spacing of stops has an upper bound limit to ensure that bus service is accessible to passengers. Two objective functions are considered: minimization of total dwell time at stops and minimization of total number of bus stops. Both objectives aim to increase the average operating speed of whole bus network, and enhance the efficiency of mobility. Considering the practical circumstances, the combined service frequency at each stop should have the maximum threshold, in order to avoid bus stop congestion. The proposed bi-objective optimization model can be efficiently solved by CPLEX to garner the Pareto frontier for real-case problems.

The remainder of this paper is organized as follows. Section 2 below describes the problem and formulates the optimization model. A numerical example is presented in Section 3. Finally, conclusions are provided in the last section.

2. Problem Description and Model Development

Consider a study area with a connected bus network which contains a total of N bus routes, and let $i = 1, 2, \ldots, N$ denote each bus route. Apart from two terminals, each bus route has a set of intermediate candidate stop locations. Variables/parameters used in the model and their notation are summarized in Table 1.

Table 1 List of Notation

Indices	
i	the index of bus routes
j	the index of bus terminals and intermediate candidate stops
k	the index of intermediate candidate stops which are shared by two or more routes
Sets	
I	a set of bus routes within the study area
Γ_i	a set of stop locations along the route i including intermediate candidate stops and two terminals
Φ	a set of intermediate candidate stops which are shared by two or more routes within the study area
G_k	a set of sequence numbers that denote the identical candidate stop k
Parameters	
N	the total number of bus routes
o_i	the original number of the first terminal of route i
e_i	the ending number of the second terminal of route i
K	the number of intermediate candidate stops which are shared by two or more routes
$s_{j,j'}$	the spacing between two nodes $j \in \Gamma_i$ and $j' \in \Gamma_i$
D_j	the upper limit of the spacing between node j and its nearest succeeding selected stop
f_j	the frequency of bus route i at node $j \in \Gamma_i \setminus \{o_i, e_i\}$
F_k	the upper limit of the combined frequency of bus routes at candidate stop k
τ_j	the average dwell time per vehicle at node $j \in \Gamma_i \setminus \{o_i, e_i\}$
δ_j	a binary parameter which equals 1 if node $j \in \Gamma_i \setminus \{o_i, e_i\}$ is the candidate stop of a sole bus route, and 0 if it is shared by two or more routes
Variables	
x_j	a binary variable which equals 1 if candidate stop $j \in \Gamma_i$ is eventually selected as one stop of bus route i, and 0 otherwise
y_k	a binary variable which equals 1 if candidate shared stop $k \in \Phi$ is selected as one stop, and 0 otherwise
$\beta_{j,j'}$	a binary variable which is equal to 1 if j' equals $\min\left\{ j'' \mid j'' > j, x_{j''} = 1 \text{ or } j'' = e_i \right\}$, and 0 otherwise.

For the ease of model formulation, bus terminals and candidate stops are sequentially numbered along each route, and such sequence originates from route 1 to route N. We use o_i to denote the original number of the first bus terminal of route i, and use e_i to denote the ending number of the second terminal of route i. For an arbitrary bus route i, the sequence of intermediate candidate stops is from $o_i + 1$ to $e_i - 1$. The sequence relation between route i and route $i+1$ is $o_{i+1} = e_i + 1$, $i \le N-1$. We define set $I = \{1, 2, ..., N\}$ and set $\Gamma_i = \{o_i, o_i + 1, ..., e_i\}$ ($\forall i \in I$). Fig. 1 presents an example of four bus routes and associated sequence of bus terminals and candidate stops.

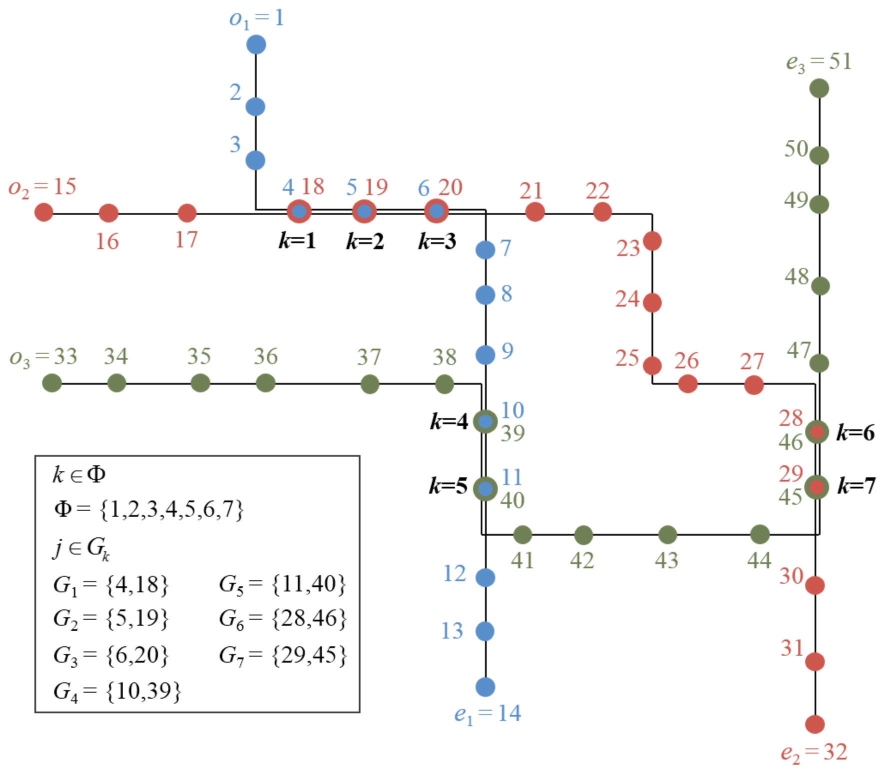


Fig. 1. An example of four bus routes.

Furthermore, some candidate stops are shared by two or more bus routes where passengers are available to transfer between various bus vehicles. In this study, we assume that the bus network has a total of K candidate shared bus stops, which constitute the set $\Phi = \{1, 2, ..., K\}$. We let G_k be the set of sequence numbers that denote the identical candidate shared stop k. In Fig. 1, as an example, set Φ has seven candidate shared stops, such as the first shared node ($k = 1$) which can be simultaneously employed by route 1 and route 2, and set G_1 equals $\{4,18\}$.

Access coverage is treated as a constraint in the model. Specifically, the spacing of stops has an upper bound limit to ensure that the bus service is accessible to passengers. We let $s_{j,j'}$ denote the spacing between two nodes $j \in \Gamma_i$ and $j' \in \Gamma_i$, and D_j denote the upper limit of the spacing between node j and its nearest succeeding selected stop. The values of D_j is location-specific, which relies on several local considerations (Ceder, 2007; Ziari et al., 2007).

Furthermore, the issue of bus stop congestion is also considered in the model. When one bus stop is used by many routes and meanwhile frequency is high, the stop inevitably gets congested and further interrupts the operation of traffic flow on the road segment (Tirachini, 2014). Therefore, it is necessary to restrict the number of bus routes at one bus stop which is mainly associated with bus frequency. We let f_j denote the frequency of route i at node j ($j \in \Gamma_i$), and F_k denote the upper limit of the combined frequency of bus routes which dwell at candidate stop k . The value of F_k depends on roadway geometry and type of bus stops (e.g., curbside, bus bay).

Then, we define x_j $(\forall j \in \Gamma_i \setminus \{o_i, e_i\})$ as the decision variable which equals 1 if candidate stop j is eventually selected as one stop of route i, and 0 otherwise. In addition, y_k ($k \in \Phi$) is defined as the decision variable which equals 1 if candidate shared stop k is selected for construction, and 0 otherwise. To formulate the model, we introduce a new binary variable, denoted by $\beta_{j,j'}$. Nodes j and j' are in set Γ_i, and j is not the bus terminal e_i. Here, $\beta_{j,j'}$ is equal to 1 if j' equals $\min\left\{ j'' \mid j'' > j, x_{j''} = 1 \text{ or } j'' = e_i \right\}$, and 0 otherwise. Fig. 2 presents the illustration of binary variable $\delta_{j,j'}$ on the basis of a single bus route.

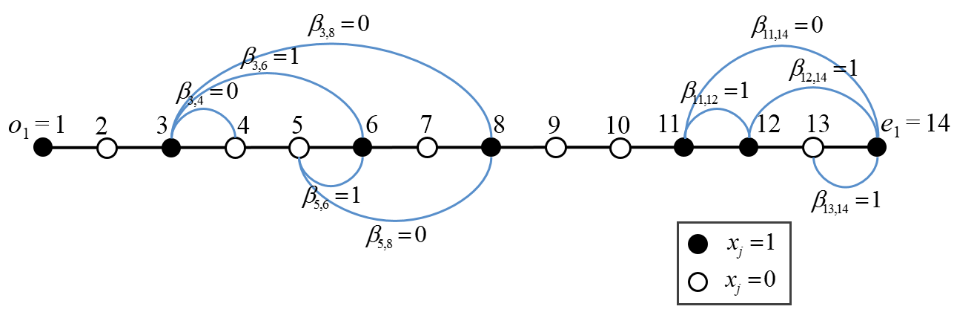


Fig. 2. An illustration of binary variable $\beta_{j,j'}$.

The bi-objective optimization model of bus stop placement problem is formulated as follows:

$$\min \ z_1 = \sum_{i \in I} \sum_{j \in \Gamma_i \setminus \{o_i, e_i\}} f_j \tau_j x_j \tag{1}$$

$$\min \ z_2 = \sum_{i \in I} \sum_{j \in \Gamma_i \setminus \{o_i, e_i\}} \delta_j x_j + \sum_{k \in \Phi} y_k \tag{2}$$

subject to:

$$\sum_{j \in G_k} f_j x_j \le F_k \quad \forall k \in \Phi \tag{3}$$

$$y_k \geq x_j \quad \forall j \in G_k, \forall k \in \Phi \tag{4}$$

$$\sum_{j'=j+1}^{e_i} \beta_{j,j'} = 1 \quad \forall j \in \Gamma_i \setminus \{e_i\}, \forall i \in I \tag{5}$$

$$\sum_{j'=j+1}^{e_i} s_{j,j'}\beta_{j,j'} \leq \min(D_j, D_{j'}) \quad \forall j \in \Gamma_i \setminus \{e_i\}, \forall i \in I \tag{6}$$

$$x_{j'} \geq \beta_{j,j'} \quad j' \geq j, \forall j, j' \in \Gamma_i \setminus \{e_i\}, \forall i \in I \tag{7}$$

$$x_{j'} \leq \sum_{j''=j+1}^{j'} \beta_{j,j''} \quad j' \geq j+1, \forall j, j' \in \Gamma_i \setminus \{e_i\}, \forall i \in I \tag{8}$$

$$x_j = 1 \quad \forall j \in \{o_i, e_i\}, \forall i \in I \tag{9}$$

$$\beta_{j,j'} = 0 \quad j' \leq j, \forall j, j' \in \Gamma_i, \forall i \in I \tag{10}$$

$$x_j \in \{0,1\} \quad \forall j \in G_i, \forall i \in I \tag{11}$$

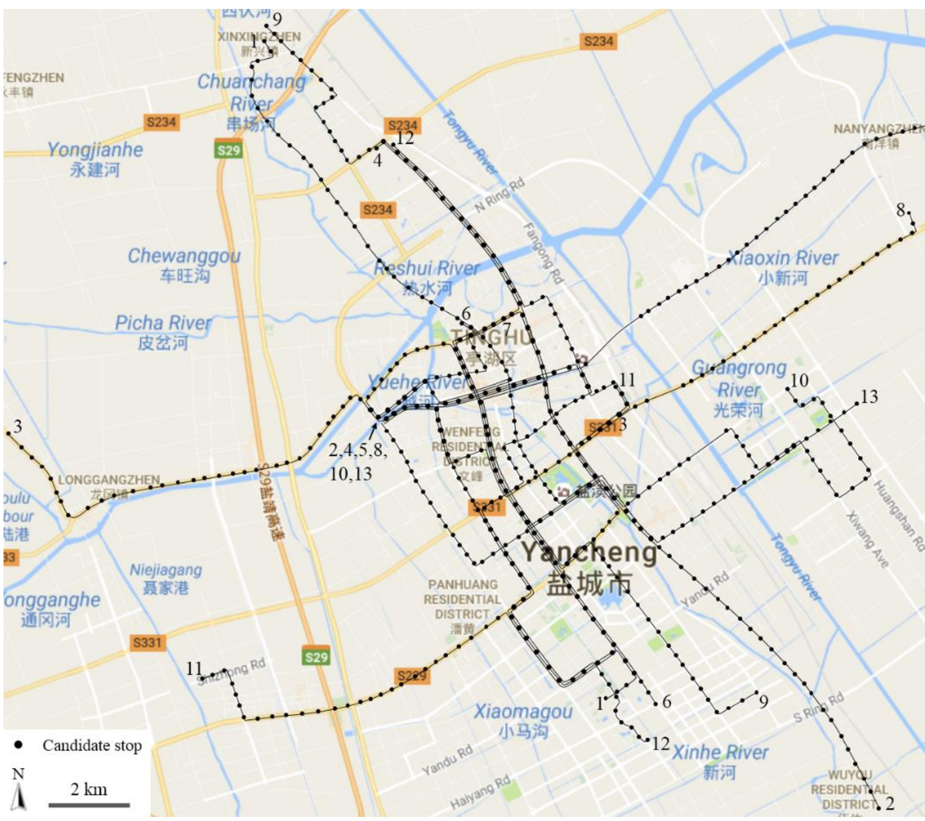
$$\beta_{j,j'} \in \{0,1\} \quad \forall j, j' \in \Gamma_i, \forall i \in I \tag{12}$$

Both objectives are associated with the enhancement of mobility. Objective (1) is to minimize the total dwell time at all the intermediate stops of N bus routes during the peak hour. τ_j denotes the average dwell time per vehicle at the intermediate stop j. Objective (2) is to minimize the total number of stops within the bus network. δ_j is a binary parameter which equals 1 if candidate stop j is owned by a sole bus route, and 0 if it is shared by two or more routes.

Constraint (3) restricts the maximum number of bus routes at candidate shared stop $k \in \Phi$ to prevent bus stop congestion. Constraint (4) ensures that candidate shared stop k is constructed if it is selected as one stop by at least one bus route. Constraint (5) determines which is the nearest succeeding selected stop for node j of route i. If none of the succeeding intermediate candidate stops are selected, β_{j,e_i} is equal to one. Constraint (6) is the stop spacing constraint, which ensures that the spacing between node j and the nearest succeeding selected stop is not more than $\min(D_j, D_{j'})$. Constraints (7) and (8) establish the relationship between decision variables x_j and $\beta_{j,j'}$, and ensure that node j' is the nearest succeeding selected stop for node j of route i if both $x_{j'}$ and $\beta_{j,j'}$ equal 1. Constraint (9) indicates that bus terminals o_i and e_i must be selected as two stops of route i. Constraint (10) indicates that $\beta_{j,j'}$ equals zero if the sequence number of node j' is not more than that of node j. Constraints (11) and (12) present that x_j and $\beta_{j,j'}$ are binary variables.

3 Numerical Example

The bi-objective optimization model described in previous sections was applied to the bus network in urban areas of Yancheng. Yancheng is a prefecture-level city in northeastern Jiangsu province, P.R. China, by the year of 2015 with a population of 7.2 million and an area of 17,000 square kilometers. Fig. 3 illustrates thirteen existing bus routes within the bus network. For each bus route, the original and ending sequence numbers, and the number of candidate stops are depicted at the lower right corner of Fig. 3. There are a total of 141 candidate stops that are shared by two or more bus routes.



i	o_i	e_i	$\Gamma_i : e_i - o_i + 1$
1	1	71	71
2	72	124	53
3	125	179	55
4	180	214	35
5	215	264	50
6	265	301	37
7	302	353	52
8	354	408	55
9	409	490	82
10	491	550	60
11	551	614	64
12	615	682	68
13	683	739	57

Fig. 3. Candidate stops of thirteen bus routes in the city of Yancheng, China.

Table 2 presents the length of each bus route and associated bus frequency during the peak period. Based on field investigations, we collected data of bus average dwell time at existing stops. We assume that candidate stops within the same road segments of existing stops shared the identical bus dwell time, and hence all the parameters τ_j in objective function Eq. (1) were obtained.

Table 2 Length and frequency of thirteen bus routes

Bus route	1	2	3	4	5	6	7	8	9
Length (km)	24.7	18.1	18.9	12.3	17.1	11.2	15.4	17.3	26.4
Frequency (min)	4.6	3.0	6.0	5.0	6.7	8.6	8.6	5.0	7.5
Bus route	10	11	12	13					
Length (km)	20.5	21.8	22.0	17.9					
Frequency (min)	5.0	5.0	6.7	7.5					

A classical technique is utilized to solve the bi-objective optimization problem, which applies the ε-constraint method; any interested readers could refer to Burke and Kendall (2005) for a detailed description of the ε-constraint method. From the perspective of efficient mobility, objective (1) can be considered as the priority, with objective (2) formulated as a constraint. When transforming two objectives into one, the model is solved using CPLEX 12.6. Based on the ε-constraint method, a set of Pareto optimal solutions are obtained by CPLEX, which constitute the Pareto frontier. The computational time for each Pareto optimal solution is generally on the order of 4-5 min. Figure 4 presents the objective values of these Pareto optimal solutions.

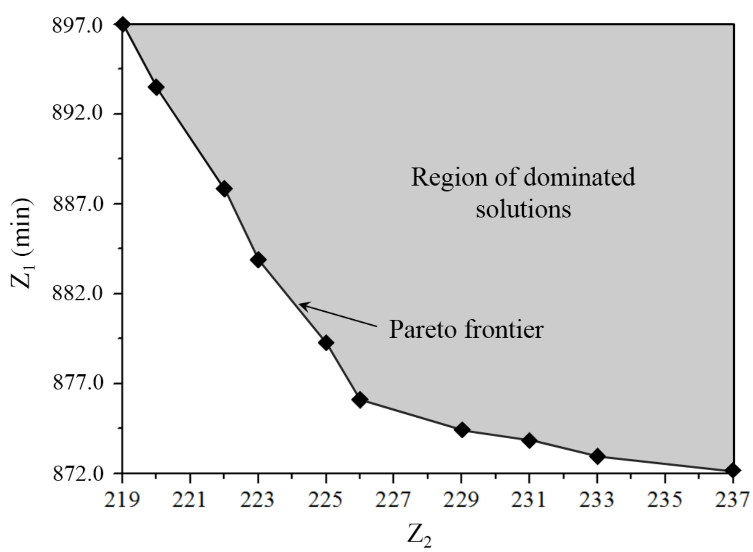


Fig. 4. Objective values of Pareto optimal solutions generated by CPLEX.

The Pareto frontier contains the set of non-dominated solutions, all of which to some extent can be seen as optimal, depending on the relative importance of the two objectives. In practice, alternative optima are beneficial because we can provide several alternative plans of bus stop placement for the local authority to perform the final optimization decision-making. Fig. 5 presents the result of optimized bus stop placement of one Pareto optimal solution.

Fig. 5. Result of optimized bus stop placement.

Table 3 depicts the comparison between optimized stop placement and existing stop placement. Results show that the adjustment of different bus routes is distinct. Among thirteen bus routes, eleven routes have larger average stop spacing compared to existing stop placement, and their performances get improved in terms of dwell time at stops along each route. At the same time, one route (i.e., route 4) retains existing stop placement with no adjustment, and the number of stops for route 7 increases by one and associated dwell time becomes longer as well.

In this study, Constraint (3) takes effect in the design of bus stop placement. For instance, a road segment of Jianjun Road contains six bus routes including route 1, 2, 3, 5, 9 and 13. Due to the consideration of stop congestion, only one stop is incapable of accommodating all the bus vehicles of six routes during the peak period. Consequently, two stops are selected at such road segment in the optimized stop placement. One bus stop is employed by five routes (i.e., route 1, 2, 3, 5 and 13) and the other stop serves route 9.

Table 3 Bus Stop Placement Characteristics and Performance

	The number of stops *			Stop spacing		Dwell time		
	Network A	Network B	Add/Delete	Network A (m)	Network B (m)	Network A (min)	Network B (min)	Improvement (%)
Sum	375	347	21/49	-	-	940.3	876.3	6.8
Mean	-	-	-	679	731	-	-	-
Routes								
1	35	33	2/4	725	771	69.6	65.8	5.5
2	27	25	1/3	696	754	31.3	29.4	6.1
3	29	26	2/5	676	757	73.2	65.5	10.6
4	21	21	0/0	615	615	37.4	37.4	0.0
5	28	24	2/6	633	743	71.4	62.5	12.5
6	17	16	1/2	700	747	61.2	56.5	7.7
7	22	23	3/2	733	700	91.8	94.9	-3.4
8	25	24	2/3	719	750	54.0	52.4	3.0
9	42	38	4/8	644	714	128.4	116.0	9.7
10	32	29	0/3	661	732	64.2	57.0	11.3
11	30	28	1/3	753	809	59.0	56.2	4.8
12	41	36	2/7	549	628	115.9	103.2	10.9
13	26	24	1/3	717	779	82.9	79.5	4.1

Note: * Network A and B denote the existing and optimized bus stop placement, respectively; Add/Delete denote two opposite measures of adjusting the existing bus stops

In general, compared with the existing stop set, the optimized stop placement has a reduction of 28 stops, which eliminates service redundancy or inefficiency in the current bus network. Fewer stops also increase the connectivity of bus service because some important stops are shared by more bus routes and passengers have more route choices when they transfer or arrive at these stops. Meanwhile, the total bus dwell time experiences a decrease of 6.8% when some stops are relocated. The network-level optimization of bus stop placement further speeds up the operation of bus vehicles, which is effective in decreasing passengers' average travel time as well as bus company's operational costs.

4. Conclusions

This paper focused on the network-level optimization of bus stop placement problem. Considering both access coverage and mobility, a bi-objective optimization model was formulated. Two objectives aimed to improve the efficiency of mobility: minimization of total dwell time at stops and minimization of total number of bus stops. Access coverage was constrained to ensure a certain level of accessibility. The effect of bus stop congestion on road traffic flow was also treated as a constraint in the model, which required that each selected stop should have an upper limit of service frequency. A case study of the bus network in urban areas of Yancheng, China was conducted based on CPLEX solver. The computational results show that Pareto optimal solutions obtained are a trade-off of two objectives, and the practical implication of the proposed model indeed improves the service efficiency of bus system.

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REFERENCES

Ahmed, B., Hounsell, N., Shrestha, B., 2016. Investigating bus priority parameters for isolated vehicle actuated junctions. *Transportation Planning and Technology*, Vol. 39, No. 1, pp. 45-58, DOI: 10.1080/03081060.2015.1108082.

Ammons, D. N., 2001. Municipal benchmarks: assessing local performance and establishing community standards, Thousand Oaks: Sage.

Burke, E. K., Kendall, G., 2005. Search Methodologies. Springer US.

Ceder, A. A., 2007. Public transit planning and operation: Theory. In: Modeling and Practice. Butterworth-Heinemann, Oxford, UK.

Ceder, A. A., Butcher, M., Wang, L. 2015. Optimization of bus stop placement for routes on uneven topography. *Transportation Research Part B: Methodological*, Vol. 74, pp. 40-61, DOI: 10.1016/j.trb.2015.01.006.

Cervero, R., Sarmiento, O. L., Jacoby, E., Gomez, L. F., Neiman, A., 2009. Influences of built environments on walking and cycling: Lessons from Bogota. *International Journal of Sustainable Transportation*, Vol. 3, No. 4, pp. 203-226, DOI: 10.1080/15568310802178314.

Chen, J., Currie, G., Wang, W., Liu, Z., Li, Z., 2016. Should optimal stop spacing vary by land use type? New methodology. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2543, pp. 34-44, DOI: 10.3141/2543-04.

Chen, J., Liu, Z., Zhu, S., Wang, W., 2015. Design of limited-stop bus service with capacity constraint and stochastic travel time. *Transportation research part E: logistics and transportation review*, 83, 1-15, DOI: 10.1016/j.tre.2015.08.007.

Chien, S. I., Qin, Z. 2004. Optimization of bus stop locations for improving transit accessibility. *Transportation Planning and Technology*, Vol. 27, No. 3, pp. 211-227, DOI: 10.1080/0308106042000226899.

Chien, S., Byun, J., Bladikas, A., 2010. Optimal stop spacing and headway of congested transit system considering realistic wait times. *Transportation Planning and Technology*, Vol. 33, No. 6, pp. 495-513, DOI: 10.1080/03081060.2010.505048.

Currie, G., Sarvi, M., Young, B., 2007. A new approach to evaluating on-road public transport priority projects: balancing the demand for limited road-space. *Transportation*, Vol. 34, No. 4, pp. 413-428, DOI: 10.1007/s11116-006-9107-3.

Currie, G., 2010. Quantifying spatial gaps in public transport supply based on social needs. *Journal of Transport Geography*, Vol. 18, No. 1, pp. 31-41, DOI: 10.1016/j.jtrangeo.2008.12.002.

dell'Olio, L., Moura, J. L., Ibeas, A., 2006. Bi-level mathematical programming model for locating bus stops and optimizing frequencies. *Transportation Research Record: Journal of the Transportation Research Board*, No. 1971, pp. 23-31, DOI: 10.3141/1971-05.

dell'Olio, L., Ibeas, A., Cecin, P., 2011. The quality of service desired by public transport users. *Transport Policy*, 18(1), 217-227, DOI: 10.1016/j.tranpol.2010.08.005.

Fan, Y., Guthrie, A., Levinson, D., 2016. Waiting time perceptions at transit stops and stations: Effects of basic amenities, gender, and security. *Transportation Research Part A: Policy and Practice*, Vol. 88, pp. 251-264, DOI: 10.1016/j.tra.2016.04.012.

Feng, S., Sun, X., Wang, D., 2016. The analysis and application of competition and cooperation between the bus lines. *KSCE Journal of Civil Engineering*, Vol. 20, No. 4, pp. 1540-1545, DOI: 10.1007/s12205-015-0030-z.

Feng, T., Zhang, J., Fujiwara, A., Timmermans, H. J., 2010. An integrated model system and policy evaluation tool for maximizing mobility under environmental capacity constraints: A case study in Dalian City, China. *Transportation Research Part D: Transport and Environment*, Vol. 15, No. 5, pp. 263-274, DOI: 10.1016/j.trd.2010.03.001.

Foth, N., Manaugh, K., El-Geneidy, A. M., 2013. Towards equitable transit: examining transit accessibility and social need in Toronto, Canada, 1996-2006. *Journal of Transport Geography*, Vol. 29, pp. 1-10, DOI: 10.1016/j.jtrangeo.2012.12.008.

Greenwald, M., Boarnet, M., 2001. Built environment as determinant of walking behavior: analyzing nonwork pedestrian travel in Portland, Oregon. *Transportation Research Record: Journal of the Transportation Research Board*, No. 1780, pp. 33-41, DOI: 10.3141/1780-05.

Hahn, J. S., Kim, H. R., Kho, S. Y., 2011. Analysis of the efficiency of Seoul Arterial Bus routes and its determinant factors. *KSCE Journal of Civil Engineering*, Vol. 15, No. 6, pp. 1115-1123, DOI: 10.1007/s12205-011-1273-y.

Hess, D. B., 2009. Access to public transit and its influence on ridership for older adults in two U.S. cities. *Journal of Transport & Land Use*, Vol. 2, No. 1, pp. 3-27, DOI: 10.5198/jtlu.v2i1.11.

Huang, D., Liu, Z., Liu, P., Chen, J., 2016. Optimal transit fare and service frequency of a nonlinear origin-destination based fare structure. *Transportation Research Part E: Logistics and Transportation Review*, 96, 1-19. DOI: 10.1016/j.tre.2016.10.004.

Ibarra-Rojas, O. J., Delgado, F., Giesen, R., Muñoz, J. C., 2015. Planning, operation, and control of bus transport systems: A literature review. *Transportation Research Part B: Methodological*, Vol. 77, pp. 38-75, DOI: 10.1016/j.trb.2015.03.002.

Ibeas, Á., dell'Olio, L., Alonso, B., Sainz, O., 2010. Optimizing bus stop spacing in urban areas. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 46, No. 3, pp. 446-458, DOI: 10.1016/j.tre.2009.11.001.

Kepaptsoglou, K., Karlaftis, M., 2009. Transit route network design problem: review. *Journal of Transportation Engineering*, Vol. 135, No. 8, pp. 491-505, DOI: 10.1061/(ASCE)0733-947X(2009)135:8(491).

Kim, K. W., Lee, D. W., 2008. A model to estimate the marginal walking time of bus users by using adaptive neuro-fuzzy inference system. *KSCE Journal of Civil Engineering*, Vol. 12, No. 3, pp. 197-204, DOI: 10.1007/s12205-008-0197-7.

Kim, K. W., Lee, D. W., Chun, Y. H., 2010. A comparative study on the service coverages of subways and buses. *KSCE Journal of Civil Engineering*, Vol. 14, No. 6, pp. 915-922, DOI: 10.1007/s12205-010-0987-6.

Levinson, H. S., 1983. Analyzing transit travel time performance. *Transportation Research Record: Journal of the Transportation Research Board*, No. 915, pp. 1-6. http://onlinepubs.trb.org/Onlinepubs/trr/1983/915/915-001.pdf

Lin, Y., Yang, X., Zou, N., Franz, M., 2015. Transit signal priority control at signalized intersections: a comprehensive review. *Transportation Letters: The International Journal of Transportation Research*, Vol. 7, No. 3, pp. 168-180, DOI: 10.1179/1942787514Y.0000000044

Litman, T., 2003. Measuring transportation: traffic, mobility and accessibility. Institute of Transportation Engineers. *ITE Journal - Institute of Transportation Engineers*, Vol. 73, No. 10, pp. 28-32.

McLeod, F., Hounsell, N., 2003. Bus priority at traffic signals - evaluating strategy options. *Journal of Public Transportation*, Vol. 6, No. 3, pp. 1-14, DOI: 10.5038/2375-0901.6.3.1.

Moniruzzaman, M., Páez, A., 2012. Accessibility to transit, by transit, and mode share: application of a logistic model with spatial filters. *Journal of Transport Geography*, Vol. 24, No. 3, pp. 198-205, DOI: 10.1016/j.jtrangeo.2012.02.006.

Murray, A. T., Davis, R., Stimson, R. J., Ferreira, L., 1998. Public transportation access. *Transportation Research Part D: Transport and Environment*, Vol. 3, No. 5, pp. 319-328, DOI: 10.1016/S1361-9209(98)00010-8.

Murray, A. T., 2003. A coverage model for improving public transit system accessibility and expanding access. *Annals of Operations Research*, Vol. 123, No. 1, pp. 143-156, DOI: 10.1023/A:1026123329433.

Murray, A. T., Wu, X., 2003. Accessibility tradeoffs in public transit planning. *Journal of Geographical Systems*, Vol. 5, No. 1, pp. 93-107, DOI: 10.1007/s101090300105.

Owen, A., Levinson, D. M., 2015. Modeling the commute mode share of transit using continuous accessibility to jobs. *Transportation Research Part A: Policy and Practice*, Vol. 74, pp. 110-

122, DOI: 10.1016/j.tra.2015.02.002.
Saka, A. A., 2001. Model for determining optimum bus-stop spacing in urban areas. *Journal of Transportation Engineering*, Vol. 127, No. 3, pp. 195-199, DOI: 10.1061/(ASCE)0733-947X(2001)127:3(195).
Szeto, W. Y., Jiang, Y. 2014. Transit route and frequency design: Bi-level modeling and hybrid artificial bee colony algorithm approach. *Transportation Research Part B: Methodological*, Vol. 67, pp. 235-263, DOI: 10.1016/j.trb.2014.05.008.
Tirachini, A., 2014. The economics and engineering of bus stops: Spacing, design and congestion. *Transportation Research Part A: Policy and Practice*. Vol. 59, pp. 37-57, DOI: 10.1016/j.tra.2013.10.010.
Tirachini, A., Hensher, D. A., Rose, J. M., 2014. Multimodal pricing and optimal design of urban public transport: The interplay between traffic congestion and bus crowding. *Transportation Research Part B: Methodological*, 61, 33-54, DOI: 10.1016/j.trb.2014.01.003.
Wright, L., Hook, W.B., 2007. Bus rapid transit planning guide. New York: Institute for Transportation & Development Policy.
Wu, B. M., Hine, J. P., 2003. A PTAL approach to measuring changes in bus service accessibility. *Transport Policy*, Vol. 10, No. 4, pp. 307-320, DOI: 10.1016/S0967-070X(03)00053-2.
Yan, Y., Liu, Z., Meng, Q., Jiang, Y., 2013. Robust optimization model of bus transit network design with stochastic travel time. *Journal of Transportation Engineering*, Vol. 139, No. 6, pp. 625-634, DOI: 10.1061/(ASCE)TE.1943-5436.0000536.
Ziari, H., Keymanesh, M. R., Khabiri, M. M., 2007. Locating stations of public transportation vehicles for improving transit accessibility. *Transport*, Vol. 22, No. 2, pp. 99-104.
Zhao, F., Chow, L. F., Li, M. T., Ubaka, I., Gan, A., 2003. Forecasting transit walk accessibility: regression model alternative to buffer method. *Transportation Research Record: Journal of the Transportation Research Board*, No. 1835, pp. 34-41, DOI: 10.3141/1835-05.
Zhu, W., Yang, X., Preston, J., 2016. Efficiency measurement of bus routes and exogenous operating environment effects on efficiency. *Transportation Planning and Technology*, Vol. 39, No. 5, pp. 464-483, DOI: 10.1080/03081060.2016.1174364.