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Column generation for integrated berth allocation, quay crane assignment and yard assignment problem

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This study investigates an integrated optimization problem on the three main types of resources used in container terminals: berths, quay cranes, and yard storage space. It builds a mixed-integer programming model for this problem, which takes account of the decisions of berth allocation, quay crane assignment, and yard storage space unit assignment for incoming vessels. In addition, since the majority of the liner shipping services operate according to a weekly arrival pattern, the periodicity of the plan is also considered in the model and in the algorithm. In order to solve the model on large-scale problem instances, we develop a column generation-based heuristic, and we also suggest some strategies for accelerating the algorithm. Based on some realistic instances, we conduct extensive numerical experiments to validate the effectiveness of the proposed model and the efficiency of the algorithm. The results show that the column generation-based heuristic can yield a good solution with an approximate 1% optimality gap within a much shorter computation time than that of CPLEX.

Key words: Maritime logistics; Column generation; Berth allocation; Yard management; Quay crane assignment.

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1. Introduction

In port operations management, it is essential to maximize the throughput because the port operators are usually paid by a handling charge per container. The port operators usually have a great interest in berth allocation decisions since these define the first planning phase. The planned berth locations for vessels are subsequently used as the key input to yard storage, personnel, and equipment deployment planning.

When making the berth allocation decision, the quay crane (QC) assignment is usually planned at the same time because the number of QCs assigned to the vessels will affect their dwelling time in the port and will thereafter influence the berth allocation for the vessels. During a vessel's dwelling time at a port, the number of assigned QCs may change over time, which further complicates the berth allocation process. Moreover, the decision on allocating berths to vessels is intertwined with that of assigning yard space (subblocks) to vessels. The yard assignment impacts the best berth positions for vessels and hence affects the berth allocation. On the other hand, the berth positions allocated to vessels will impact the assignment of yard space to vessels. As a result, port operators face a dilemma as to which operation should be scheduled first.

Although practitioners usually plan the berth allocation before the yard assignment, ideally these two decisions should be optimized simultaneously. This study proposes an integrated model of berth allocation, QC assignment, and the yard assignment for container terminals. A column generation (CG)-based heuristic is developed to solve the problem in large-scale realistic environments. Numerical experiments are conducted to validate the model and to demonstrate the efficiency of the algorithm. For a set of real-world-like instances, our method can generate good plans within reasonable computation times.

The remainder of this paper is organized as follows. The related literature is reviewed in Section 2. Section 3 gives a detailed description of the problems. A mixed-integer mathematical model is formulated in Section 4. In Section 5, a CG procedure is developed to solve the linear programming relaxation of a proposed set covering model, while a CG-based heuristic developed to obtain feasible integer solutions is described in Section 6. Extensive computational experiments are conducted in Section 7, and conclusions are drawn in the last section.

2. Literature Review

For a comprehensive overview on container terminal operations and maritime logistics, see the review papers of Vis and de Koster (2003), Steenken et al. (2004), Stahlbock and Voß (2008), Fransoo and Lee (2013), and Meng et al. (2014).

This study is related to the berth allocation problem (BAP), which is crucial to port operations management and is also the basis for making other plans on container scheduling decisions by shipping liners. The BAP has attracted significant attention in the last two decades. Imai et al. (1997) addressed the static BAP (SBAP) in commercial ports, and Imai et al. (2001) extended the SBAP to the dynamic BAP (DBAP), while Monaco and Sammarra (2007) proposed a compact reformulation. The BAP can be classified into two types, discrete and continuous, depending on whether vessel berthing is performed in a continuous or in a discrete space (Imai et al. 2005, Mauri et al. 2016). As for the solution methodology, Ribeiro et al. (2016) developed an adaptive large neighborhood search heuristic, Kim and Moon (2003) proposed a simulated annealing method, and Park and Kim (2002) employed a subgradient optimization method. Imai et al. (2007) investigated the BAP for indented berths where mega-containerships can be served from two sides. Cordeau et al. (2005) built a BAP model based on a vehicle routing problem formulation. For the tactical level BAP, Moorthy and Teo (2006) studied a berth template planning problem, which maximizes the service level and minimizes the connectivity cost related to the transshipment container groups. Cordeau et al. (2007) studied a tactical level service allocation problem arising in the Gioia Tauro transshipment hub, based on which Giallombardo et al. (2010) investigated the tactical discrete BAP and QC assignment problem. These authors proposed a novel concept called QC-profile to facilitate the combination of the BAP and QC assignment problems. For the above problem, Vacca et al. (2013) proposed an exact branch-and-price algorithm that can solve instances with up to 20 ships and five berths. Recently, the effect of tides, which may influence the water depth of the navigation channels, has been considered in the BAP by Xu et al. (2012) and Du et al. (2015).

Following the study of Giallombardo et al. (2010), Zhen et al. (2011) integrated the tactical berth allocation planning (also known as berth template) with the yard template planning, for which Jin et al. (2015) designed a column generation-based solution method. Similar to the QC-profile, a concept of YC-profile was proposed by Jin et al. (2014) and applied to yard management. Hendriks et al. (2013) proposed a heuristic for solving a simultaneous berth allocation and yard planning problem. For bulk ports, Robenek et al. (2014) designed an exact branch-and-price algorithm to solve the integrated berth and yard planning problem.

Another stream of BAP studies concerns the integrated planning of the BAP and QC assignment. Park and Kim (2003) developed a two-phase heuristic solution procedure. Meisel and Bierwirth (2009) treated the BAP-QC assignment as a multi-mode resource constrained project scheduling problem. Imai et al. (2008) considered the constraint that QCs cannot pass or bypass from one side to the other side of a vessel whose containers are being handled. Meisel and Bierwirth (2013) proposed a framework for integrating the BAP, QC assignment, and QC scheduling. Recently, bunker fuel consumption and emissions have become more and more prevalent in some BAP related studies. Thus, Du et al. (2011) proposed a mixed-integer second-order cone programming model for a BAP by considering the fuel consumption and vessel emissions. Hu et al. (2014) further integrated QC allocation into the BAP considering fuel consumption and emissions from vessels, and developed a mixed integer second-order cone programming model. Besides the above studies which are mainly based on mathematical programming, some authors have employed discrete event simulation, e.g., Legato and Mazza (2001). A simulation optimization technique was recently applied to optimize the tactical and operational BAP decisions in an integrated way (Legato et al. 2014). Randomness in loading and unloading operations and QC assignment were also considered in Legato et al. (2014). For a comprehensive overview on the BAP, see the surveys of Bierwirth and Meisel (2010, 2015).

With respect to the related literature, this paper makes following contributions. First, it extends the traditional berth allocation and QC assignment problem, which is related to the quay side decision, to the yard side decision making (i.e., the yard storage unit assignment problem). In addition, when formulating the integrated model, this study further considers the periodicity of the plan because most liner shipping services operate on a weekly basis. Second, although a few integrated optimization problems in the fields of container port operations have been studied, the solution methods consist of metaheuristics that cannot guarantee an optimality gap. This study proposes a CG-based heuristic to solve the model on large-scale problem instances. It also conducts numerical experiments based on some realistic instances, the results of which show that the proposed algorithm exhibits a better performance than the metaheuristics previously developed.

3. Problem Background

Before formulating the integrated model for the berth allocation, the QC assignment, and the yard assignment, we provide some problem background.

3.1. QC-profiles based QC assignment decision

Normally, the shipping liners will inform the port operators about the feasible and expected turnover time interval as well as the total container handling workload for their vessels. Based on this information, the port operators will arrange a number of QCs for container handling. When more QCs are assigned to an incoming vessel, the container handing process becomes faster and the turnover time is shorter. In this context, Giallombardo et al. (2010) proposed the concept of QC-profile to facilitate the QC assignment, in which the total workload is denoted as the number of QC time steps. Here, one QC time step is the number of containers that can be handled by one QC in a time step (e.g., four hours for a time step). Based on the workload, a set of QC-profile is generated for the vessel.

Figure 1 shows three possible QC-profiles for a vessel with a workload of 20 QC $\times$ time steps. Two important parameters are defined for each QC-profile. One parameter is the handling time by using QC-profile p for Vessel i, denoted as h_{ip}. For the example of Figure 1, the handing time by using QC-profile 1 is six time steps. The other parameter is the number of QCs utilized in the m^{th} time step if QC-profile p is assigned to Vessel i, denoted as q_{ipm}. For instance, by using QC-profile 2, five QCs are utilized in the first time step (i.e., $q_{ip1} = 5$), four QCs are utilized in the second time step (i.e., $q_{ip2} = 4$), and so on.

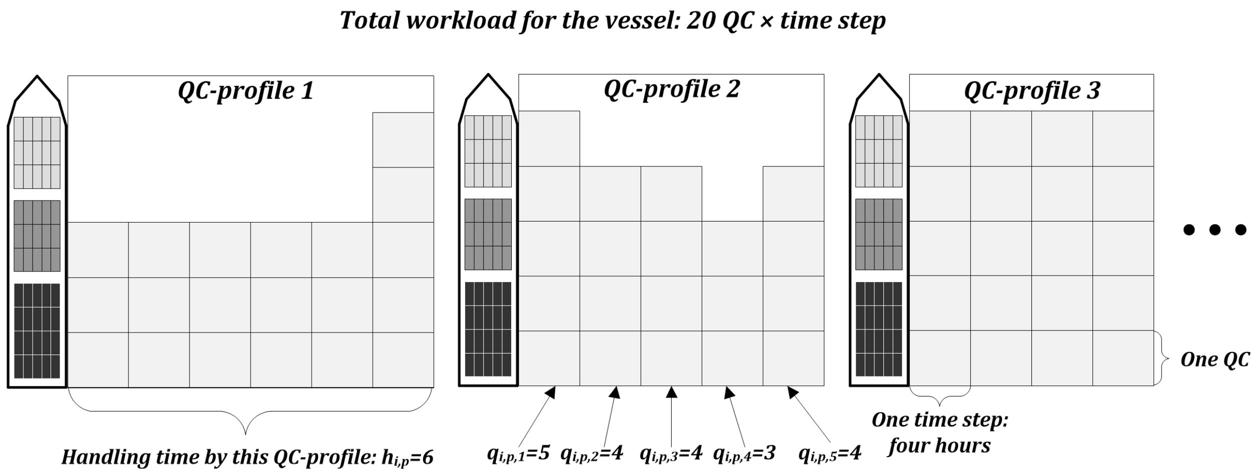

Figure 1 An example of QC-profiles for a vessel.

3.2. Integrated berth planning and yard planning

The integrated planning problem studied in this paper includes three subproblems: the berth allocation problem, the QC assignment problem (i.e., the QC-profile assignment) and the yard assignment problem, which are intertwined with each other in real-world operations. A visualization of the integrated planning problem is shown in Figure 2.

For an incoming vessel, the berth planning determines when and where the vessel moors at the terminal, as well as which QC-profile is assigned to the vessel. In Figure 2, Vessel 1 is scheduled to arrive at the terminal in time step 1 and moors at Berth 1. Meanwhile, the QC-profile selected for the vessel is such that it will moor for five time steps. Such a decision is made based on the information provided by the shipping liner. As mentioned earlier, the feasible time interval (denoted as $[a_i^f, b_i^f]$), the expected time interval (denoted as $[a_i^e, b_i^e]$) as well as the total workload for loading and unloading container are provided by the shipping liner prior to the vessel arrival. The port operators attempt to construct the berth schedules and assign the QCs in such a way that the vessel can moor at the terminal within the interval $[a_i^e, b_i^e]$. If this interval is violated, a penalty cost is charged by the shipping liner. However, the feasible time interval $[a_i^f, b_i^f]$ provided by the shipping liner cannot be violated under any circumstances.

In reality, it is extremely difficult for the terminal operators to satisfy all the shipping liners' requirements on mooring within their expected time intervals. The service quality costs (i.e., the penalty costs) charged by shipping liners are inevitable, especially when the number of incoming vessels is large with respect to berth capacity and QC resources. Thus, the objective in berth planning is to minimize the costs incurred when the expected time intervals are violated. Assume that α_i and β_i are the start time step and the end time step for the handing of Vessel i, where $\alpha_i \geq a_i^f$ and $\beta_i \geq b_i^f$. If $\alpha_i < a_i^e$ or $\beta_i > b_i^e$, a service quality cost will charged for Vessel i, which can be calculated as $c_i^p[(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+]$ (c_i^p is the penalty cost coefficient for Vessel i).

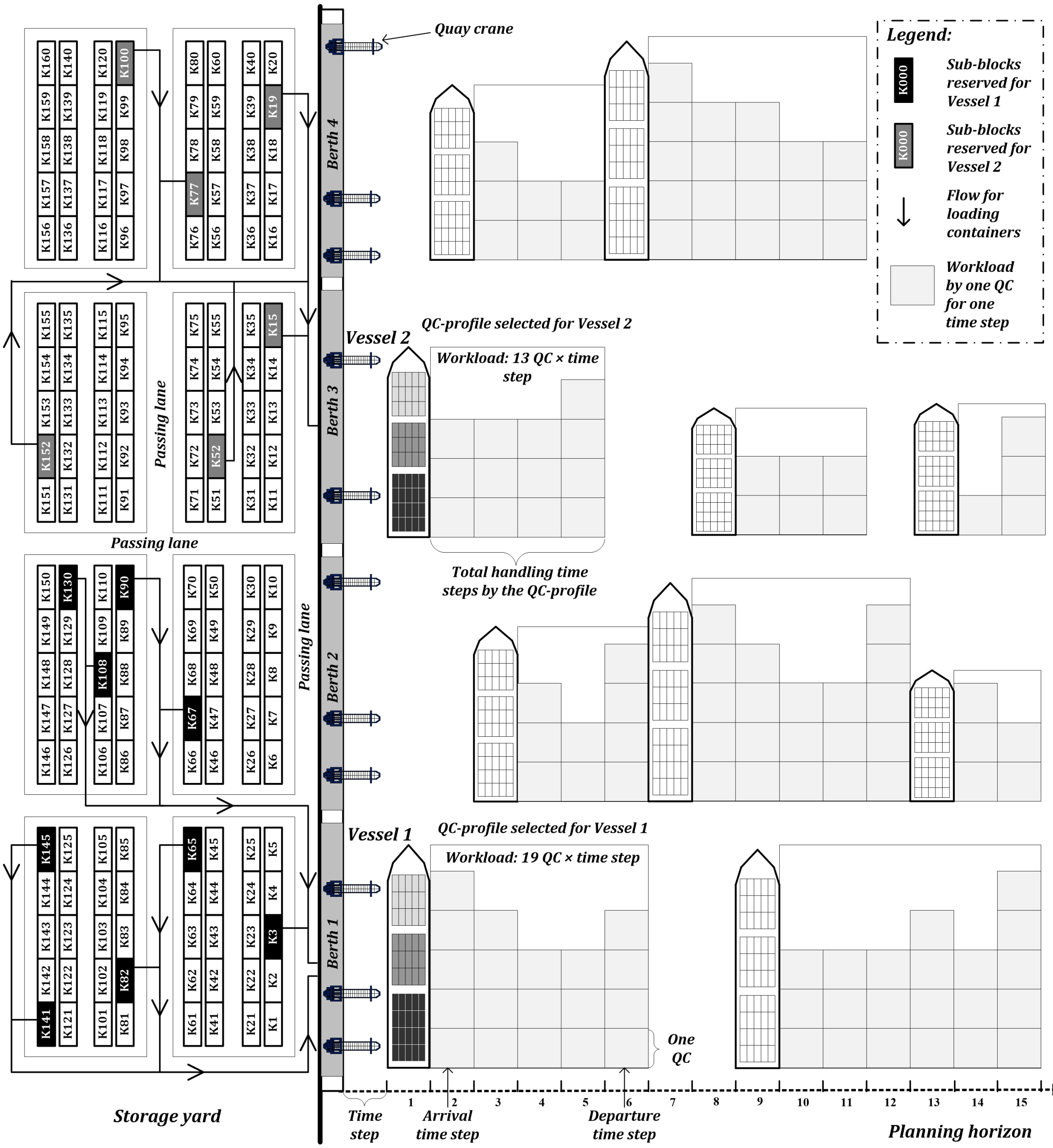

Figure 2 **Integrated berth allocation, QC assignment and yard assignment problem.**

Yard planning is affected by berth planning. Under the consignment strategy in the terminal (Lee et al. 2006, Han et al. 2008, Jiang et al. 2012), the yard is utilized for temporary container storage for the shipping liners. Some specific subblocks in the yard are reserved for each vessel. When a vessel arrives, all the containers stored are loaded from its reserved subblocks to the vessel. In the example of Figure 2, the subblocks $K15$, $K19$, $K52$, $K77$, $K100$ and $K152$ are reserved for Vessel 2, which is scheduled to moor at Berth 3. When Vessel 2 arrives at the terminal, all the containers stored in the six subblocks $K15$, $K19$, $K52$, $K77$, $K100$ and $K152$ are transported to the berth position along the solid flow lines shown. Here, we assume that each loading route between a subblock and a berth is predetermined as shown by the solid flow lines. We define D^L_{kb} as the length of the loading route between Berth b and Subblock k.

In addition to the loading process, an unloading process also occurs for an incoming vessel. The containers that need to be transshipped to other vessels are unloaded from the incoming vessel and are stored in the subblocks reserved for these vessels. Here, the unloaded containers could be transshipped to any incoming vessel, and can be then stored in any subblock. Thus, for the unloading process, we assume that if a vessel is allocated to Berth b, the route length for unloading a container is the average unloading route length between Berth b and all the subblocks in the yard, denoted as D^U_b. In yard planning, the objective is to minimize the total loading and unloading length for all the incoming vessels in terms of all the handling containers. It is easy to understand that berth allocation will impact subblock assignment, which implies that berth planning and yard planning are intertwined and cannot be optimized individually. Therefore, an integrated model for berth allocation, the QC assignment, and yard assignment is needed.

3.3. Cyclical berth planning

Since most vessels visit the port on a weekly basis, periodicity should be considered when determining the berth allocation plans. However, this brings additional challenges for the berth planning process (Moorthy and Teo 2006). The traditional BAP is usually modeled as a constrained two-dimensional bin packing problem (Lim 1998, Kim and Moon 2003). When constructing periodic schedules, the rectangle packing on a plane, as shown in Figure 2, should be extended to a packing problem on a *cylinder* with circumference equal to the length of the planning horizon. To handle periodicity in the planning process, the key idea is to enlarge the original planning horizon from H (e.g., one week) to $H+E$, where $E=\max_{\forall i\in V, p\in P_i}\{h_{ip}\}$ (P_i is the set of QC-profiles for Vessel i), which is shown in Figure 3. For each berth, we introduce the first time step (i.e., the start time step ϱ_b, to be determined) and the last time step (i.e., the end time step ς_b, to be determined) during which the berth is occupied in the planning horizon. We need to ensure that the berth cannot be occupied by any vessel before the start time step ϱ_b and after the end time step ς_b. Meanwhile, to ensure that the berth occupancy can be wrapped around the original planning horizon H, the gap between the two time steps (i.e, $\varsigma_b - \varrho_b$) cannot exceed H.

In addition, once the QC assignment is embedded within the berth planning process, the limitation for the QC utilization in each time step should be posed as follows: (i) in time step $t = \{E+1,\ E+2,..,H\}$, the total number of QCs utilized cannot exceed the number of available QCs, (ii) the sum of the number of QCs utilized in time step t, $t\in\{1,\ 2,..,E\}$, and the number of QCs utilized in its '*twin*' time step $t+H$ cannot exceed the number of available QCs.

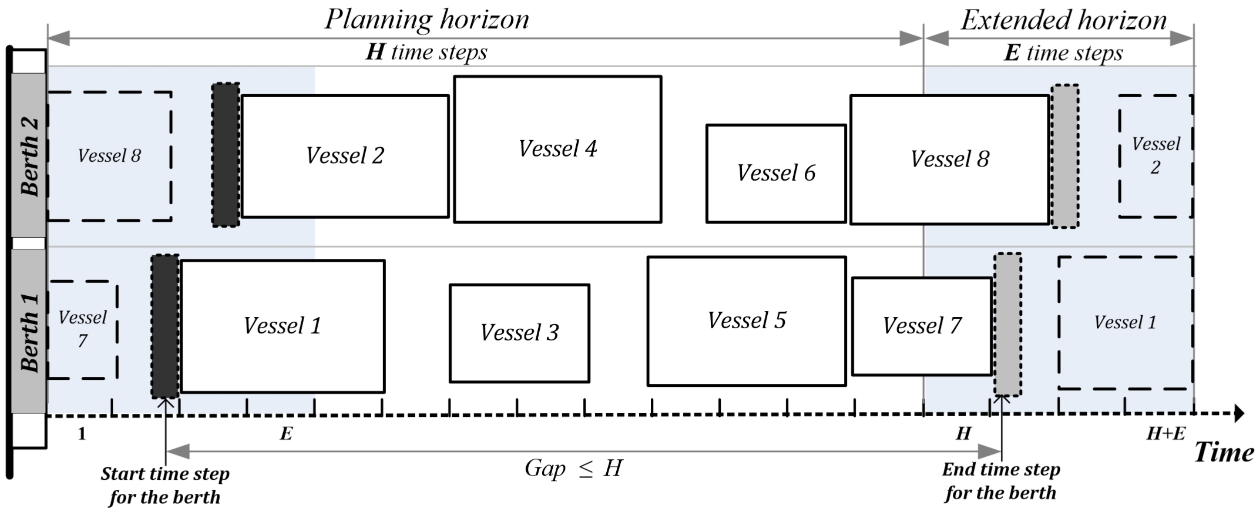

Figure 3 **Horizon extension based method for considering the periodicity of the plan.**

4. Mixed Integer Linear Programming Formulation

We now formulate a mixed integer linear programming (MILP) model for the integrated berth allocation, QC assignment and yard assignment problem. The objective of the model is to minimize the total service quality cost, including the penalty cost caused by the deviation from the vessels' expected service time, and the total operation cost related to the route length of the container transportation flows in the yard.

4.1. Notations

***Indices*:**

$i,\ j$	vessels;
k	subblocks;
b	berths;
p	QC-profiles;
t	time steps.

***Input parameters*:**

V	set of incoming vessels;
K	set of available subblocks in the yard;
B	set of berths in the quay;
H	number of time steps in the planning horizon;
E	maximum handling time of all vessels, i.e., $E = \max_{\forall i \in V, p \in P_i}\{h_{ip}\}$;
T	set of time steps, $T = \{1, \dots, H+E\}$;
P_i	set of QC-profiles for Vessel i, $i \in V$;
h_{ip}	handling time of Vessel i by using QC-profile p with unit of time step, $i \in V,\ p \in P_i$;

q_{ipm}	number of QCs used by QC-profile $p \in P_i$, $i \in V$ at the mth time step, $m \in \{1, \ldots, h_{ip}\}$;
Q_t	maximum number of QCs available at time step t, $t \in T$;
$[a_i^f, b_i^f]$	feasible service time steps for Vessel i, $i \in V$;
$[a_i^e, b_i^e]$	expected service time steps for Vessel i, $i \in V$;
r_i	number of subblocks that should be reserved for Vessel i, $i \in V$;
l_i	number of containers that should be loaded for Vessel i, $i \in V$;
u_i	number of containers that should be unloaded for Vessel i, $i \in V$;
D_{kb}^L	length of loading route from Subblock k to Berth b in the yard, $k \in K$, $b \in B$;
D_b^U	average length of unloading route from Berth b to all the subblocks in the yard, $b \in B$;
c_i^p	coefficient of the penalty cost caused by the deviation from the expected service time of Vessel i;
c^o	coefficient of the operation cost related to the route length of the container transportation flows in yard;
M	a sufficiently large positive number.

***Decision variables*:**

$\omega_{ib} \in \{0,\ 1\}$	set to one if Berth b is allocated to Vessel i, and to zero otherwise, $i \in V$, $b \in B$;
$\delta_{ijb} \in \{0,\ 1\}$	set to one if both Vessel i and Vessel j dwell at Berth b, and Vessel i dwells at the berth before Vessel j, and to zero otherwise, $i, j \in V$, $i \neq j$, $b \in B$;
$\varphi_{ik} \in \{0,\ 1\}$	set to one if Subblock k is reserved for Vessel i, and to zero otherwise, $i \in V$, $k \in K$;
$\gamma_{ip} \in \{0,\ 1\}$	set to one if Vessel i is served by QC-profile p, and to zero otherwise, $i \in V$, $p \in P_i$;
$\mu_{it} \in \{0,\ 1\}$	set to one if Vessel i begins handling in the time step t, and to zero otherwise, $i \in V$, $t \in T$;
$\eta_{ipt} \in \{0,\ 1\}$	set to one if Vessel i is served by QC-profile p and begins handling by this QC-profile in the time step t, and to zero otherwise, $i \in V$, $p \in P_i$, $t \in T$;
$\alpha_i \in T$	integer, the start time step of the handling for Vessel i, $i \in V$;
$\beta_i \in T$	integer, the end time step of the handling for Vessel i, $i \in V$;
$\varrho_b, \varsigma_b \in T$	start and end time steps for Berth b, $b \in B$;
$\sigma_t \geq 0$	integer, the number of used QCs at time step t, $t \in T$.

4.2. Mathematical model

$$[\boldsymbol{M1}]\ \text{minimize} \sum_{i \in V} c_i^p[(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+] + c^o \sum_{i \in V}\sum_{b \in B}\sum_{k \in K} [\omega_{ib}\varphi_{ik} D_{kb}^L \left(\frac{l_i}{r_i}\right)] + c^o \sum_{i \in V}\sum_{b \in B} \omega_{ib} D_b^U u_i \tag{1}$$

subject to:

$$\sum_{i \in V} \varphi_{ik} \leq 1 \quad \forall k \in K, \tag{2}$$

$$\sum_{k\in K}\varphi_{ik}=r_i \quad \forall i\in V, \tag{3}$$

$$\sum_{p\in P_i}\gamma_{ip}=1 \quad \forall i\in V, \tag{4}$$

$$\sum_{b\in B}\omega_{ib}=1 \quad \forall i\in V, \tag{5}$$

$$\sum_{t\in\{1,\dots,H\}}\mu_{it}=1 \quad \forall i\in V, \tag{6}$$

$$\sum_{t\in T}\mu_{it}t=\alpha_i \quad \forall i\in V, \tag{7}$$

$$\alpha_i+\sum_{p\in P_i}\gamma_{ip}h_{ip}-1=\beta_i \quad \forall i\in V, \tag{8}$$

$$\alpha_i+\sum_{p\in P_i}\gamma_{ip}h_{ip}\le\alpha_j+(1-\delta_{ijb})M \quad \forall i,j\in V,\ i\neq j,\ \forall b\in B, \tag{9}$$

$$\delta_{ijb}+\delta_{jib}\le\omega_{ib} \quad \forall i,j\in V,\ i\neq j,\ \forall b\in B, \tag{10}$$

$$\delta_{ijb}+\delta_{jib}\ge\omega_{ib}+\omega_{jb}-1 \quad \forall i,j\in V,\ i\neq j,\ \forall b\in B, \tag{11}$$

$$\alpha_i\ge a_i^f \quad \forall i\in V, \tag{12}$$

$$\beta_i\le b_i^f \quad \forall i\in V, \tag{13}$$

$$\eta_{ipt}\ge\gamma_{ip}+\mu_{it}-1 \quad \forall i\in V,\forall p\in P_i,\ \forall t\in T, \tag{14}$$

$$\sigma_t=\sum_{i\in V}\sum_{p\in P_i}\sum_{m=\max\{1;t-h_{ip}+1\}}^{t}\eta_{ipm}q_{ip(t-m+1)} \quad \forall t\in T, \tag{15}$$

$$\sigma_t\le Q_t \quad \forall t\in\{E+1,\dots,H\}, \tag{16}$$

$$\sigma_t+\sigma_{t+H}\le Q_t \quad \forall t\in\{1,\dots,E\}, \tag{17}$$

$$\varrho_b\le\alpha_i+(1-\omega_{ib})\cdot M \quad \forall i\in V,\forall b\in B, \tag{18}$$

$$\varsigma_b\ge\beta_i+(\omega_{ib}-1)\cdot M \quad \forall i\in V,\forall b\in B, \tag{19}$$

$$\varsigma_b-\varrho_b\le H-1 \quad \forall b\in B, \tag{20}$$

$$\omega_{ib}\in\{0,\ 1\} \quad \forall i\in V,\forall b\in B, \tag{21}$$

$$\delta_{ijb}\in\{0,\ 1\} \quad \forall i,j\in V,\ i\neq j,\ \forall b\in B, \tag{22}$$

$$\varphi_{ik}\in\{0,\ 1\} \quad \forall i\in V,\forall k\in K, \tag{23}$$

$$\gamma_{ip}\in\{0,\ 1\} \quad \forall i\in V,\forall p\in P_i, \tag{24}$$

$$\mu_{it}\in\{0,\ 1\} \quad \forall i\in V,\forall t\in T, \tag{25}$$

$$\eta_{ipt} \in \{0,\ 1\} \quad \forall i \in V, \forall p \in P_i, \forall t \in T, \tag{26}$$

$$\sigma_t \geq 0 \quad \forall t \in T, \tag{27}$$

$$\varrho_b, \varsigma_b \in T \quad \forall b \in B. \tag{28}$$

In the above model, Objective (1) minimizes the total cost, including the penalty costs, the operation costs on the loading process and the operation costs on the unloading process. Constraints (2) guarantee that each subblock is reserved for at most one vessel. Constraints (3) ensure that a given number r_i of subblocks are reserved to Vessel i. Constraints (4) stipulate that only one QC-profile is assigned to each vessel. Constraints (5) mean that each vessel can only be allocated to one berth. Constraints (6) state that each vessel starts handling in a certain time step. Constraints (7) connect the two handling start time decision variables (i.e.,π_{it} and α_i). Specifically, if Vessel i begins handling in time step t (i.e., $\pi_{it} = 1$), the start time step of the handling for Vessel i is time step t. Constraints (8) link the start time step and the end time step of the vessels. Constraints (9) ensure that for the same berth, a former dwelling vessel must end its handling activities at the berth before a late dwelling vessel starts its handling activities at the berth. Constraints (10-11) guarantee that if two vessels are allocated to the same berth, there must be a time sequence for the two vessels dwelling at the berth. Constraints (12-13) enforce the condition that the service time for each vessel must lie within its feasible service time interval. Constraints (14) link two decision variables η_{ipt} and μ_{it} that are both related to the start time of handling. Constraints (15) calculate the number of QCs used in each time step. Constraints (16) and (17) guarantee that the number of QCs used in each time step cannot exceed the capacity considering the periodicity of vessel schedules. Constraints (18) and (19) ensure that for each berth, ϱ_b (or ς_b) is no later than (or no earlier than) all the start (or end) time steps of vessels that occupy Berth b. Constraints (20) ensure that the gap between ϱ_b and ς_b does not exceed the length of the planning horizon. Constraints (21)–(28) define the domains of decision variables.

4.3. Linearization for the model

The first two parts in the objective of the above model are nonlinear, but they can be linearized. To linearized the first part, i.e., $\sum_{i\in V} c_i^p[(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+]$, we define the additional decision variables $\tau_i^{a+},\ \tau_i^{a-},\ \tau_i^{b+},\ \tau_i^{b-},\ i \in V$. By adding the following constraints, the first part in the objective can be reformulated as $c^p \sum_{i\in V} (\tau_i^{a+} + \tau_i^{b+})$:

$$a_i^e - \alpha_i = \tau_i^{a+} - \tau_i^{a-} \quad \forall i \in V, \tag{29}$$

$$\beta_i - b_i^e = \tau_i^{b+} - \tau_i^{b-} \quad \forall i \in V, \tag{30}$$

$$\tau_i^{a+},\ \tau_i^{a-},\ \tau_i^{b+},\ \tau_i^{b-} \geq 0 \quad \forall i \in V. \tag{31}$$

the second part $c^o \sum_{i\in V}\sum_{b\in B}\sum_{k\in K}[\omega_{ib}\varphi_{ik} D^L_{kb}\left(\frac{l_i}{r_i}\right)]$, can be linearized as follows:

Let $\theta_{ikb} \in \{0,\ 1\}$ equals one if and only if Vessel i dwells at Berth b and Subblock k is reserved for Vessel i, $i \in V$, $k \in K$, $b \in B$. Then,

$$\theta_{ikb} \geq \omega_{ib} + \varphi_{ik} - 1 \quad \forall i \in V,\ \forall k \in K, \forall b \in B, \tag{32}$$

$$\theta_{ikb} \in \{0,\ 1\} \quad \forall i \in V,\ \forall k \in K, \forall b \in B. \tag{33}$$

based on these new decision variables and constraints, the integrated model for the berth allocation, QC assignment and yard assignment problem can be reformulated as a mixed integer linear programming model:

$$[\boldsymbol{M2}] \quad \text{minimize} \sum_{i\in V} c_i^p(\tau_i^{a+} + \tau_i^{b+}) + c^o \sum_{i\in V}\sum_{b\in B}\sum_{k\in K}\left[\theta_{ikb} D^L_{kb}\left(\frac{l_i}{r_i}\right)\right] + c^o \sum_{i\in V}\sum_{b\in B}\omega_{ib} D^U_b u_i \tag{34}$$

subject to: Constraints (2)–(33).

5. Set Covering Model and Column Generation

The mixed-integer programming model for the integrated problem become hard to solve by some commercial solvers, such as CPLEX, when the size of problem instances become large, Therefore, in this section, we reformulate the problem as a set covering model and we apply decomposition techniques.

5.1. Set covering model

Let $\mathcal{P}_i$ be the set of all possible assignment plans of Vessel i, $i \in V$ in the given planning horizon. Each assignment plan $\mathcal{P}_i$ of Vessel i represents the allocation of a berth to the vessel in time steps, the reservation of r_i subblocks in the yard to the vessel, and the number of QCs used by Vessel i in each time step. Here, we define $\mathbb{P} = \bigcup_{i\in V}\mathcal{P}_i$ as the set of all possible assignment plans. For each assignment plan $\mathcal{P}_i$ of Vessel i, we have the following input parameters:

***Input parameters*:**

$A_{bt}^{\mathcal{P}_i}$	equals one if Berth b is allocated to Vessel i in time step t in assignment plan $\mathcal{P}_i$, and zero otherwise, $b \in B$, $t \in T$;
$R_k^{\mathcal{P}_i}$	equals one if Subblock k is reserved to Vessel i in assignment plan $\mathcal{P}_i$, and zero otherwise, $k \in K$;
$U_t^{\mathcal{P}_i}$	integer, number of QCs used by Vessel i in the time step t in assignment plan $\mathcal{P}_i$, $t \in T$.

Let $\mathcal{C}_{\mathcal{P}_i}$ be the cost constant of the assignment plan $\mathcal{P}_i$, whose calculation will be elaborated in the Section 5.3. For each feasible assignment plan $\mathcal{P}_i \in \mathcal{P}_i$, we define a binary variable $\lambda_{\mathcal{P}_i}$, equals one if and only if the assignment plan $\mathcal{P}_i$ is used by Vessel i. Based on these parameters, variables and constants, the set covering model for the problem can be formulated as follows:

$$[\boldsymbol{M3}] \ \ \text{minimize} \sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} \mathcal{C}_{\mathcal{P}_i}\lambda_{\mathcal{P}_i} \tag{35}$$

subject to:

$$\sum_{\mathcal{P}_i\in\mathcal{P}_i} \lambda_{\mathcal{P}_i} = 1 \quad \forall i\in V, \tag{36}$$

$$\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} A_{bt}^{\mathcal{P}_i}\lambda_{\mathcal{P}_i} \le 1 \quad \forall b\in B, \forall t\in T, \tag{37}$$

$$\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} R_k^{\mathcal{P}_i}\lambda_{\mathcal{P}_i} \le 1 \quad \forall k\in K, \tag{38}$$

$$\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} (U_t^{\mathcal{P}_i} + U_{t+H}^{\mathcal{P}_i})\lambda_{\mathcal{P}_i} \le Q_t \quad \forall t\in\{1,\dots,E\}, \tag{39}$$

$$\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} U_t^{\mathcal{P}_i}\lambda_{\mathcal{P}_i} \le Q_t \quad \forall t\in\{E+1,\dots,H\}, \tag{40}$$

$$t\cdot\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} A_{bt}^{\mathcal{P}_i}\lambda_{\mathcal{P}_i} + M(1-\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} A_{bt}^{\mathcal{P}_i}\lambda_{\mathcal{P}_i}) - \varrho_b \ge 0 \quad \forall b\in B, \forall t\in T, \tag{41}$$

$$t\cdot\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} A_{bt}^{\mathcal{P}_i}\lambda_{\mathcal{P}_i} + M(\sum_{i\in V}\sum_{\mathcal{P}_i\in\mathcal{P}_i} A_{bt}^{\mathcal{P}_i}\lambda_{\mathcal{P}_i} - 1) - \varsigma_b \le 0 \quad \forall b\in B, \forall t\in T, \tag{42}$$

$$\varsigma_b - \varrho_b \le H-1 \quad \forall b\in B, \tag{43}$$

$$\lambda_{\rho_i} \in \{0,\ 1\} \quad \forall i\in V, \forall \mathcal{P}_i\in\mathcal{P}_i, \tag{44}$$

$$\varrho_b, \varsigma_b \in T \quad \forall b\in B. \tag{45}$$

In the above formulation, Objective (35) minimizes the total cost of serving vessels in the port. Constraints (36) ensure that there is exactly one feasible assignment for each vessel in the solution. Constraints (37) guarantee that each berth is occupied by at most one vessel in each time step. Constraints (38) mean that each subblock can be reserved for at most one vessel. Constraints (39) and (40) state that the QCs used in each time step is within the limited capacity. Constraints (41) and (42) ensure that for each berth, ϱ_b (or ς_b) is no later than (or no earlier than) all the start (or end) time steps of vessels who occupy Berth b. Constraints (43) ensure that the gap between ϱ_b and ς_b does not exceed the length of the planning horizon. Constraints (44) and (45) define the domains of decision variables.

5.2. Restricted master problem (RMP) for the column generation procedure

The above formulation contains all the possible assignment plans for the vessels. Therefore, the size of $\mathbb{P}$ and the corresponding computational time needed to solve the problem grow exponentially with the instance size. To circumvent this difficulty, we use CG to solve the linear programming (LP) relaxation of the formulation.

In the CG procedure, we maintain a restricted master problem (RMP) with a subset of a feasible assignment plan $\mathbb{P}' = \bigcup_{i \in V} \mathcal{P}'_i \subseteq \mathbb{P}$. Initially, we derive a $\mathbb{P}'$ for the RMP by using a heuristic (Section 5.5), which ensures that an initial feasible solution exists in the RMP. The RMP is formulated as:

$$[\boldsymbol{M4}] \;\; \text{minimize} \sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}'_i} \mathcal{C}_{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \tag{46}$$

subject to:

$$\sum_{\mathcal{P}_i \in \mathcal{P}'_i} \lambda_{\mathcal{P}_i} = 1 \quad \forall i \in V, \tag{47}$$

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}'_i} A_{bt}^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \leq 1 \quad \forall b \in B, \forall t \in T, \tag{48}$$

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}'_i} R_k^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \leq 1 \quad \forall k \in K, \tag{49}$$

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}'_i} (U_t^{\mathcal{P}_i} + U_{t+H}^{\mathcal{P}_i}) \lambda_{\mathcal{P}_i} \leq Q_t \quad \forall t \in \{1, \ldots, E\}, \tag{50}$$

$$\sum_{i \in V} \sum_{\mathcal{P}_i \in \mathcal{P}'_i} U_t^{\mathcal{P}_i} \lambda_{\mathcal{P}_i} \leq Q_t \quad \forall t \in \{E+1, \ldots, H\}, \tag{51}$$

$$0 \leq \lambda_{\mathcal{P}_i} \leq 1 \quad \forall i \in V, \forall \mathcal{P}_i \in \mathcal{P}'_i. \tag{52}$$

Note that the constraints that ensure the periodicity of the berth allocation (i.e., Constraints (42) and (43)) are invalid and removed for the RMP which is an LP relaxation. In order to guarantee periodicity in feasible integer solutions, a substep is designed in a CG-based heuristic, which will be discussed in Section 6.1.

At each iteration of the CG procedure, the dual variables of the RMP are transferred to pricing problems that are used to generate new feasible assignment plans (i.e., columns). These dual variables are defined as follows:

Dual variables:

π_i	the dual variables for Constraints (47), $i \in V$;
ϖ_{bt}	the dual variables for Constraints (48) , $b \in B$, $t \in T$;
ρ_k	the dual variables for Constraints (49), $k \in K$;
ϕ_t	the dual variables for Constraints (50) and (51), $t \in T$;

The dual variables ϕ_t obtained from the RMP are $\phi_t,\ \forall t \in \{1,\ \ldots, H\}$. To ensure periodicity, the planning horizon is enlarged from H to $T = H + E$. Therefore, the dual variables ϕ_t passing to the pricing problems should be $\phi_t, \forall t \in T$, where $\phi_t = \phi_{t-H}, \forall t \in \{H+1,\ \ldots, H+E\}$. Using these dual variables, the pricing problems will generate feasible assignment plans with the lowest reduced costs (i.e., the objective values of the pricing problems). The CG procedure stops when all the minimal reduced costs are positive, which means that no feasible assignment plan can be added to the RMP.

5.3. Pricing problem (PP)

The goal of the pricing problems is to find feasible assignment plans with a negative reduced cost to be added to the RMP. At each iteration of the CG procedure, there are $|V|$ pricing problems to be solved, each of which corresponds a vessel (e.g., Vessel i), and we will generate one feasible assignment plan $\mathcal{P}_i{}^*$ for each vessel. For all the $|V|$ optimal feasible assignment plans generated by solving the pricing problems, only the feasible assignment plans with a negative reduced cost can be added to the RMP, which means that at each iteration of the CG procedure, there are at most $|V|$ columns to be added into the RMP. The formulation for the pricing problem of each vessel is given next. Note that the index $i \in V$ is removed from the formulation since the pricing problem for each vessel is solved separately.

***Input parameters*:**

$\pi, \varpi_{bt}, \rho_k, \phi_t$	the dual variables obtained from the RMP;
P	set of QC-profiles for the vessel;
h_p	handling time of the vessel by using QC-profile p with unit of time step, $p \in P$;
q_{pm}	number of QCs used by QC-profile $p \in P$ at the mth time step, $m \in \{1, \ldots, h_p\}$;
$[a^f, b^f]$	feasible service time steps for the vessel;
$[a^e, b^e]$	expected service time steps for the vessel;
r	number of subblocks that should be reserved for the vessel;
l	number of containers that should be loaded for the vessel;
u	number of containers that should be unloaded for the vessel;
c^p	coefficient of the penalty cost caused by the deviation from the vessel's expected service time.

***Decision variables*:**

$\varepsilon_{bt} \in \{0,\ 1\}$	set to one if the vessel dwells at Berth b in the time step t, and to zero otherwise, $b \in B$, $t \in T$ (corresponding to $A_{bt}^{\mathcal{P}_i}$);
$\varphi_k \in \{0,\ 1\}$	set to one if Subblock k is reserved the vessel, and to zero otherwise, $k \in K$ (corresponding to $R_k^{\mathcal{P}_i}$);
$\zeta_t \geq 0$	integer, the number of QCs used by the vessel in the time step t, $t \in T$ (corresponding to $U_t^{\mathcal{P}_i}$);

$\nu_t \in \{0,\ 1\}$	set to one if the vessel is served in the time step t, and to zero otherwise, $t \in T$;
$\omega_b \in \{0,\ 1\}$	set to one if Berth b is allocated to the vessel, and to zero otherwise, $b \in B$;
$\gamma_p \in \{0,\ 1\}$	set to one if the vessel is served by QC-profile p, and to zero otherwise, $p \in P$;
$\mu_t \in \{0,\ 1\}$	set to one if the vessel begins handling in the time step t, and to zero otherwise, $t \in T$;
$\eta_{pt} \in \{0,\ 1\}$	set to one if the vessel is served by QC-profile p and begins handling by this QC-profile in the time step t, and to zero otherwise, $p \in P$, $t \in T$;
$\theta_{kb} \in \{0,\ 1\}$	set to one if the vessel dwells at Berth b and Subblock k is reserved for the vessel, and to zero otherwise, $k \in K$, $b \in B$;
$\alpha \in T$	integer, the start time step of the handling for the vessel;
$\beta \in T$	integer, the end time step of the handling for the vessel;
$\ddot{C}_{\mathcal{P}} \geq 0$	the cost for the assignment plan of the vessel;

$\tau^{a+}, \tau^{a-},\ \tau^{b+},\ \tau^{b-}$ are additional variables for the linearization.

$$[\boldsymbol{M5}]\ \ \text{minimize}\ \mathcal{C}_{\mathcal{P}} - \left(\pi + \sum_{b \in B}\sum_{t \in T} \varpi_{bt} \cdot \varepsilon_{bt} + \sum_{k \in K} \rho_k \cdot \varphi_k + \sum_{t \in T} \phi_t \cdot \zeta_t\right) \tag{53}$$

subject to:

$$\sum_{k \in K} \varphi_k = r \tag{54}$$

$$\sum_{p \in P} \gamma_p = 1 \tag{55}$$

$$\sum_{b \in B} \omega_b = 1 \tag{56}$$

$$\sum_{t \in \{1,\ldots,H\}} \mu_t = 1 \tag{57}$$

$$\sum_{t \in T} \mu_t t = \alpha \tag{58}$$

$$\alpha + \sum_{p \in P} \gamma_p h_p - 1 = \beta \tag{59}$$

$$\alpha \geq a^f \tag{60}$$

$$\beta \leq b^f \tag{61}$$

$$\eta_{pt} \geq \gamma_p + \mu_t - 1 \quad \forall p \in P, \forall t \in T, \tag{62}$$

$$\eta_{pt} \leq \gamma_p \quad \forall p \in P, \forall t \in T, \tag{63}$$

$$\eta_{pt} \leq \mu_t \quad \forall p \in P, \forall t \in T, \tag{64}$$

$$\zeta_t = \sum_{p \in P} \sum_{m=\max\{1; t-h_p+1\}}^{t} \eta_{pm} q_{p(t-m+1)} \quad \forall t \in T, \tag{65}$$

$$t + M(1-\nu_t) \geq \alpha \quad \forall t \in T, \tag{66}$$

$$t \leq \beta + M(1-\nu_t) \quad \forall t \in T, \tag{67}$$

$$\sum_{t\in T} \nu_t = \beta - \alpha + 1 \tag{68}$$

$$\varepsilon_{bt} \geq \nu_t + \omega_b - 1 \quad \forall b \in B, \forall t \in T, \tag{69}$$

$$\varepsilon_{bt} \leq \nu_t \quad \forall b \in B, \forall t \in T, \tag{70}$$

$$\varepsilon_{bt} \leq \omega_b \quad \forall b \in B, \forall t \in T, \tag{71}$$

$$\theta_{kb} \geq \omega_b + \varphi_k - 1 \quad \forall k \in K,\ \forall b \in B, \tag{72}$$

$$a^e - \alpha = \tau^{a+} - \tau^{a-} \tag{73}$$

$$\beta - b^e = \tau^{b+} - \tau^{b-} \tag{74}$$

$$\mathcal{C}_{\mathcal{P}} = c^p\left(\tau^{a+} + \tau^{b+}\right) + c^o \sum_{b\in B}\sum_{k\in K}\left[\theta_{kb} D^L_{kb}\left(\frac{l}{r}\right)\right] + c^o \sum_{b\in B} \omega_b D^U_b u \tag{75}$$

$$\varepsilon_{bt} \in \{0,\ 1\} \quad \forall b \in B, \forall t \in T, \tag{76}$$

$$\varphi_k \in \{0,\ 1\} \quad \forall k \in K, \tag{77}$$

$$\omega_b \in \{0,\ 1\} \quad \forall b \in B, \tag{78}$$

$$\gamma_p \in \{0,\ 1\} \quad \forall p \in P, \tag{79}$$

$$\mu_t \in \{0,\ 1\} \quad \forall t \in T, \tag{80}$$

$$\eta_{pt} \in \{0,\ 1\} \quad \forall p \in P, \forall t \in T, \tag{81}$$

$$\theta_{kb} \in \{0,\ 1\} \quad \forall k \in K, \forall b \in B, \tag{82}$$

$$\eta_{pt} \in \{0,\ 1\} \quad \forall p \in P, \forall t \in T, \tag{83}$$

$$\zeta_t \geq 0 \quad \forall t \in T, \tag{84}$$

$$\alpha, \beta \in T \tag{85}$$

$$\tau^{a+}, \tau^{a-},\ \tau^{b+},\ \tau^{b-}, \mathcal{C}_{\mathcal{P}} \geq 0. \tag{86}$$

Note that $\mathcal{C}_{\mathcal{P}}$ is a decision variable of the pricing problem instead of an input parameter. Once an assignment plan $\mathcal{P}$ is chosen as the newly added column to the RMP for Vessel i, the corresponding cost $\mathcal{C}_{\mathcal{P}}$ is a cost constant of the newly added assignment plan $\mathcal{P}_i$ (i.e., $\mathcal{C}_{\mathcal{P}_i}$), which is included in the objective function of the RMP (i.e., Objective (35)). Meanwhile, the decision variables ε_{bt}, φ_k and ζ_t are transferred to the input parameters of the RMP, which are $A^{\mathcal{P}_i}_{bt}$, $R^{\mathcal{P}_i}_k$ and $U^{\mathcal{P}_i}_t$, respectively.

In the above formulation, Objective (53) minimizes the reduced cost of the optimal assignment plan. Constraints (54) states that r subblocks should be reserved for the vessel. Constraint (55) guarantees that only one QC-profile is selected for the vessel. Constraint (56) ensures that exactly one berth is allocated to the vessel. Constraint (57) states that the vessel starts handling in a certain time step. Constraint (58) connects the two handling start related decision variables (i.e., π_i and α). Constraint (59) links the start time step and the end time step of the vessel. (60) and (61) force the service time for the vessel to be within the feasible service time span. Constraints (62)–(64) link two decision variables η_{pt} and μ_t that are both related to the start time of handling. Constraint (65) calculate the number of QCs used by the vessel in each time step. Constraints (66-68) connect the three service related decision variables (i.e., α, β and ν_t). Constraints (69)–(71) links two decision variables ε_{bt} and ω_b that are both to related berth allocation. Constraints (72)–(74) are additional constraints for the linearization. Constraint (75) calculates the cost for the assignment plan of the vessel. Constraints (76)–(86) define the domains of decision variables.

After solving these pricing problems, we obtain $|V|$ optimal columns (i.e., the plans with the minimal reduced cost). The columns with the negative reduced cost are selected as the newly added columns for the RMP. The CG procedure stops if no column can be added to the RMP.

5.4. Solving the pricing problem

In this section, we propose an efficient algorithm for the pricing problem, which can compute optimal solution for the problem in pseudo-polynomial time. The basic idea of this method is as follows: for a given vessel, we list all the possible time steps at which the vessel starts to be served (i.e., $t:\ \mu_t = 1$), and all the possible number of time steps during which the vessel dwells at the port (i.e., $\beta - \alpha + 1$). Here, for the sake of simplicity, we define the time step at which the vessel starts to be served as χ, and the number of time steps that the vessel dwells at the port as ψ. Based on the input parameters, the handling time of the vessel using QC-profile p (i.e., h_p) is used to measure the efficiency of the QC-profiles. However, in order to improve the berth availability in the optimal solution, h_p can also be used to narrow down the range of ψ since the vessel will be served immediately upon arrival and can depart immediately after the service finished, which means that $\psi \in [\min(h_p), \max(h_p)]$. Regarding χ, we use another input parameter to reduce its possible range, which is $[a^f, b^f]$ (i.e., the feasible service time steps for the vessel). Given a value of ψ (i.e., the dwelling time for the vessel is given), we can further conclude that $\chi \in [a^f,\ b^f - \psi + 1]$.

We denote the combination of a given starting time step (i.e., χ) and of a dwelling time (i.e., ψ) as a scenario of the vessel. Here, note that χ can also be deemed as the arrival time step of the vessel, and $\chi + \psi - 1$ as its departure time step. The cardinality of the scenarios remains unchanged even if the size of the problem instance increases, because it is related to the service level of the port and to the flexibility of the vessel. Given a scenario, χ and ψ can be determined, which brings the following changes for the pricing problem: (i) the penalty cost (i.e., $c^p(\tau^{a+}+\tau^{b+})$) caused by the deviation can be written as $[(a^e-\chi)^+ + ((\chi+\psi-1)-b^e)^+]$, which helps avoid complex linearizations in the pricing problem; (ii) the selection of QC-profile p is isolated from the pricing problem, which can be implemented by solving $\boldsymbol{M6}$. This model can be solved very easily by an exact polynomial algorithm (denoted as **Sub-algorithm 1**). The pseudocode for this algorithm is given in Appendix A; (iii) what is left for the pricing problem is to allocate a berth and certain number of subblocks to the vessel. The berth allocation and the subblock assignment still interact with each other even in the scenario. However, we can formulate a simple model for the berth allocation and the subblock assignment, denoted as $\boldsymbol{M7}$. The exact polynomial algorithm (denoted as **Sub-algorithm 2**) for this model is also elaborated in Appendix A.

$$[\boldsymbol{M6}] \quad \text{maximize} \sum_{t\in T} \phi_t \cdot \zeta_t \tag{87}$$

subject to:

$$\sum_{p\in P} \gamma_p = 1 \tag{88}$$

$$\sum_{p\in P} \gamma_p \cdot h_p = \psi \tag{89}$$

$$\zeta_t = \sum_{p\in P} \gamma_p \cdot q_{p(t-\chi+1)} \quad \forall t \in [\chi, (\chi+\psi-1)], \tag{90}$$

$$\zeta_t = 0 \quad \forall t \in T \backslash [\chi, (\chi+\psi-1)], \tag{91}$$

$$\gamma_p \in \{0,\ 1\} \quad \forall p \in P. \tag{92}$$

In the above model, Objective (87) aims to optimize the QC related reduced cost of the scenario. Constraint (88) ensures that exactly one QC-profile is selected. Constraint (89) guarantees that the selected QC-profile must serve the vessel for exactly ψ time steps. Constraints (90) and (91) calculate the number of QCs used by the vessel in each time step. Constraints (92) define the domains of decision variables.

$$[\boldsymbol{M7}] \quad \text{minimize } c^o \sum_{b\in B}\sum_{k\in K}\left[\theta_{kb} D^L_{kb}\left(\frac{l}{r}\right)\right] + c^o \sum_{b\in B} \omega_b D^U_b u - \sum_{b\in B}\sum_{t\in T} \varpi_{bt}\cdot\varepsilon_{bt} - \sum_{k\in K} \rho_k \cdot \varphi_k \tag{93}$$

subject to:

$$\sum_{k\in K} \varphi_k = r \tag{94}$$

$$\sum_{b\in B} \omega_b = 1 \tag{95}$$

$$\theta_{kb} \geq \omega_b + \varphi_k - 1 \quad \forall k \in K, \forall b \in B, \tag{96}$$

$$\varepsilon_{bt} = \omega_b \quad \forall t \in [\chi, (\chi + \psi - 1)], \; \forall b \in B, \tag{97}$$

$$\varepsilon_{bt} = 0 \quad \forall t \in T \setminus [\chi, (\chi + \psi - 1)], \forall b \in B, \tag{98}$$

$$\varepsilon_{bt} \in \{0, \; 1\} \quad \forall b \in B, \forall t \in T, \tag{99}$$

$$\varphi_k \in \{0, \; 1\} \quad \forall k \in K, \tag{100}$$

$$\omega_b \in \{0, \; 1\} \quad \forall b \in B, \tag{101}$$

$$\theta_{kb} \in \{0, \; 1\} \quad \forall k \in K, \forall b \in B. \tag{102}$$

In the above formulation, Objective (93) minimizes the berth and subblock related reduced cost of the scenario. Constraint (94) states that r subblocks should be reserved for the vessel. Constraint (95) ensures that exactly one berth is allocated to the vessel. Constraints (96) link the two decision variables ω_b and φ_k which are related to the berth allocation and the subblock assignment, respectively. Constraints (97) and (98) aim to derive the allocation of berths to the vessel in each time step. Constraints (99)–(102) define the domains of the decision variables.

Based on above analysis, the detailed procedure of this exact algorithm for solving the pricing problem is elaborated in **Algorithm 1**:

5.5. Heuristic for the initial set of feasible assignment plans

To apply the CG procedure, we need to generate an initial set of feasible assignment plans for the RMP, so that the RMP can yield at least one feasible solution. Here, we propose a heuristic to derive an initial feasible solution. Since solving the integrated problem of berth, QC, and yard arrangement is still intractable, even heuristically, we divide the integrated problem into two stages. The berth allocation and the QC assignment are solved in the first stage, and the yard assignment is solved in the second stage given that the berth-related variables are determined.

When solving the first-stage problem (i.e., the berth allocation and the QC assignment), we apply a sequential method (Zhen et al. 2011), which consists of solving the berthing schedule for the vessels one at a time. To implement this method, a sequence of vessels must be generated at the beginning. Here, we generate this sequence in decreasing order of the c_i^p value, which reflects the priority of vessels in the sense of penalty. A berth-QC planning model denoted as $\boldsymbol{M8}$ is then solved for each vessel. After solving the model $\boldsymbol{M8}$ for a vessel, the remaining time-berth space and the number of available QCs in each time step are updated before solving the next vessel. The formulation of $\boldsymbol{M8}$ and the procedure for the first stage are given in Appendix B.

Algorithm 1 Exact algorithm for the pricing problem

Input: A given vessel
Output: An optimal assignment plan and its minimal reduced cost
for all the ψ, $\psi \in [\min(h_p), \max(h_p)]$ **do**
  for all the χ, $\chi \in [a^f,\ b^f - \psi + 1]$ **do**
    Define $V_{\chi,\psi}$ as the minimal reduced cost if the vessel starts to be served in time step χ and its dwelling time at the port is ψ
    Initialize $V_{\chi,\psi} = c^p[(a^e - \chi)^+ + ((\chi + \psi - 1) - b^e)^+]$
    Solve model ***M6*** with the objective value denoted as Z_1^*, by **Sub-algorithm 1**
    Solve model ***M7*** with the objective value denoted as Z_2^*, by **Sub-algorithm 2**
    Set $V_{\chi,\psi} = V_{\chi,\psi} - Z_1^* + Z_2^*$, which is the minimal reduced cost of the scenario
  end for
end for
Solve $\min(V_{\chi,\psi} | \forall \psi \in [\min(h_p), \max(h_p)\ ], \forall \chi \in [a^f,\ b^f - \psi + 1],) - \pi$, which is the minimal reduced cost of the pricing problem of the vessel, and the new optimal assignment plan for the vessel can be extracted from the values of the decision variables (i.e., ε_{bt}^*, φ_k^* and ζ_t^*) in the optimal scenario (the combination of the starting time step χ^* and of the dwelling time ψ^*).

In the second stage (i.e., the yard assignment), given the berth position of the vessels (i.e., ω_{ib}), we can derive the decisions for the yard assignment by solving another model denoted as ***M9***, which is formulated as follows:

$$[\boldsymbol{M9}] \quad \text{minimize } c^o \sum_{i \in V} \sum_{b \in B} \sum_{k \in K} \left[\theta_{ikb} D_{kb}^L \left(\frac{l_i}{r_i} \right) \right] \tag{103}$$

subject to: Constraints (2), (3), (23), (32) and (33).

After the two stages have been solved, a feasible solution for the problem is obtained, and an initial set of feasible assignment plans can be added into the RMP to invoke the CG procedure.

6. A Column Generation-based Heuristic

The proposed CG procedure only solves the linear relaxation of the set covering model, and does not guarantee that integer solutions will be found. Therefore, we propose a CG-based heuristic to compute near-optimal integer solutions by using different assignment plan selection strategies. The assignment plans are chosen from the subset of feasible assignment plans maintained in RMP (i.e., $\mathbb{P}'$).

6.1. Framework of the CG-based heuristic

Here, we describe the framework of our proposed CG-based heuristic. The outer procedure is the selection heuristic used to obtain an integer solution. The strategies for the selection procedure will be detailed in Section 6.2. The inner procedure is the CG procedure proposed in Section 5. Before elaborating on the framework of the algorithm, we define three port resources limited in the RMP and we initially set their values. These are $Berth\_time_{bt} = 1,\ \forall b \in B, \forall t \in T$(i.e., berth resource over time), $Subblock_k = 1,\ \forall k \in K$ (i.e., subblock resource) and $QCs_t = Q_t,\ \forall t \in \{1,\dots,H\}$ (i.e., QC resource over time). These three resources correspond to the right-hand sides of Constraints (48), (49), (50) and (51) in the RMP, respectively, and are set as input parameters for the right-hand sides of the constraints in the algorithm. The detailed framework of our algorithm is as follows:

Step 0: Initialize a vessel waiting list, which includes all the vessels that have not been designated with an assignment plan $\mathcal{P}_i$. Initialize the set Ω for the final solution plans as empty. Pass the initial three port resources (i.e., $Berth\_time_{bt} = 1$, $Subblock_k = 1$ and $QCs_t = Q_t$) to the right-hand sides of the constraints in the RMP.

Step 1: Invoke the CG procedure. When the CG procedure ends, a LP solution is obtained by solving the RMP. Update a column pool with assignment plans whose corresponding decision variables $\lambda_{\mathcal{P}_i}$ are not equal to zero.

Step 2: Test whether the assignment plans in the column pool satisfy Constraints (48), (49), (50) and (51) with the current port resources. If not, delete these assignment plans.

Step 3: Select one assignment plan $\mathcal{P}_i$ from the column pool based on the strategies proposed in Section 6.2, and pass it to the set Ω. Remove the corresponding vessel i from the vessel waiting list.

Step 4: Update the three port resources based on the selected assignment plan. For example, if the selected assignment plan $\mathcal{P}_i$ occupies Berth $b^{'}$ in time steps $t^{'}$ and $t+1^{'}$, then set $Berth\_time_{b^{'}t^{'}} = 0$ and $Berth\_time_{b^{'}t+1^{'}} = 0$.

Substep 4.1: Assume the selected assignment plan is for Vessel i, its arrival time step is $\overline{\alpha}$ (i.e., the handling start time), and its departure time step is $\overline{\beta}$ (i.e., the handling end time). To guarantee periodicity, we further update the berth resource (i.e., $Berth\_time_{bt}$) as follows: If $\overline{\beta} - (H-1) \geq 1$, we set $Berth\_time_{b\tau} = 0,\ \forall \tau \in [1, \overline{\beta} - (H-1)]$. If $\overline{\alpha} + (H-1) \leq H + E$, we set $Berth\_time_{b\tau} = 0,\ \forall \tau \in [\overline{\alpha} + (H-1), H+E]$.

After the update, pass the current three port resources to the right-hand sides of the constraints in the RMP.

Step 5: Repeat ***Steps 1-4*** until the vessel waiting list is empty. At the end of the algorithm, an integer solution for the problem can be derived from the set Ω.

Note that in Section 5.2, the berth allocation periodicity cannot be considered in the RMP since the problem is an LP relaxation. Here, we insert ***Substep 4.1*** to guarantee periodicity in the final solution. Periodicity enforces the condition that the time gap between the start time step ϱ_b for Berth b and the end time step ς_b for Berth b is less than $H-1$ (i.e., Constraint (20)), which essentially implies that the time gaps between all the arrival time steps of the vessels allocated to Berth b and all the departure time steps of the vessels allocated to Berth b are less than $H-1$. The principle behind ***Substep 4.1*** is that if a vessel has been allocated to Berth b in the solution set Ω, we must ensure that no other assignment plan can be selected if the assignment plan allocates its corresponding vessel to Berth b and the gaps between its dwelling time steps and $\overline{\alpha}$ or $\overline{\beta}$ are greater than $H-1$. Thereafter, periodicity in the final solution can be ensured.

6.2. Strategies to select the assignment plan

After ***Step 2*** of the heuristic algorithm, the column pool with feasible assignment plans is obtained. We propose four heuristic strategies to select an assignment plan from the pool.

Strategy 1: Select from the column pool the assignment plan corresponding to the largest fractional value of the decision variables $\lambda_{\mathcal{P}_i}$. If there are two assignment plans with the same fractional value, select the one with lower plan cost. The principle behind this strategy is that the assignment plan with the highest fractional value is more likely to be part of an optimal solution.

Strategy 2: Select from the column pool the assignment plan corresponding to the lowest plan cost (i.e., $\mathcal{C}_{\mathcal{P}}$). If there are two assignment plans with the same plan cost, select the one with the higher fractional value of the decision variable. The principle behind this strategy is to select the assignment plan that contributes least to the total cost under current port resources.

Strategy 3: Select from the column pool the assignment plan corresponding to the lowest reduced cost with the current values of the dual variables. The reduced cost can be calculated as $\mathcal{C}_{\mathcal{P}} - \left(\pi_i + \sum_{b\in B}\sum_{t\in T}\varpi_{bt}\cdot A_{bt}^{\mathcal{P}_i} + \sum_{k\in K}\rho_k\cdot R_k^{\mathcal{P}_i} + \sum_{t\in T}\phi_t\cdot U_t^{\mathcal{P}_i}\right)$. If there are two assignment plans with the same reduced cost, select the one with the lower cost. The principle of this strategy is to find the assignment plan that has the lowest sum of the contribution cost to the total cost and to the usage cost of port resources.

Strategy 4: To implement this strategy, we initially rank all Berths $b\in B$ from lowest to highest, based on their average distance to all the subblocks in the yard (i.e., the input parameter D_b^U). Under this strategy, we first pick all the assignment plans from the column pool that allocate its vessel to the lowest berth. If no assignment plan exists, we further check the assignment plans with the next lowest berth until the assignment plans are picked. If there is more than one assignment plan picked with the lowest berth, select the one with the lowest reduced cost. The principle of this strategy is to maximize the utilization of the berths that are close to the subblocks in the yard. Thus the transportation cost in the yard can be reduced.

6.3. Accelerating the CG procedure by dual stabilization

In the proposed algorithm, CG is the core procedure to derive an LP solution. However, CG is known to suffer from instability, which causes slow convergence. The instability of CG is due to the following reason. Suppose that we can build the master problem (MP) with all possible columns and the dual problem for the master problem (DMP). At each iteration of the CG procedure, an RMP is solved with a subset belonging to the full set of all possible columns, which means that some columns are missing from the RMP compared with MP. A column in MP denotes a constraint in DMP, which suggests that the dual problem of RMP lacks some constraints in DMP. Thus, the optimal dual solution $\Pi = (\pi_i, \varpi_{bt}, \rho_k, \phi_t, \iota_{bt}, \kappa_{bt})$ obtained by the RMP could be feasible for DMP, and thereafter optimal, or could be infeasible super-optimal for DMP.

To overcome such a problem in the CG procedure and to improve the efficiency of our algorithm, we have designed an ad hoc dual stabilization method, which is inspired from Addis et al. (2012). This method aims to pass a dual vector $\widetilde{\Pi}= (\widetilde{\pi}_i, \widetilde{\varpi}_{bt}, \widetilde{\rho}_k, \widetilde{\phi}_t, \widetilde{\iota}_{bt}, \widetilde{\kappa}_{bt})$ to the pricing problem, which is close to the optimal dual vector of DMP. To obtain a near-optimal dual vector (i.e.,$\widetilde{\Pi}$), we maintain a stability center $\overline{\Pi} = (\overline{\pi}_i, \overline{\varpi}_{bt}, \overline{\rho}_k, \overline{\phi}_t, \overline{\iota}_{bt}, \overline{\kappa}_{bt})$, which represents our incumbent best guess for the optimal dual vector. Initially, we set $\overline{\Pi}$ with zeros in all components of the vector, which is a feasible solution for DMP. At each iteration of the CG procedure, we obtain a dual vector by solving an RMP (i.e., computing Π) and pass a modified dual vector (i.e., $\widetilde{\Pi}$) to the pricing problems by the updated equation:

$$\widetilde{\Pi} = (\widetilde{\pi}_i, \widetilde{\varpi}_{bt}, \widetilde{\rho}_k, \widetilde{\phi}_t, \widetilde{\iota}_{bt}, \widetilde{\kappa}_{bt}) = a \cdot \Pi + (1-a) \cdot \ \overline{\Pi}, \tag{104}$$

where $a \in [0,1]$. Initially, we set $a = 0.5$. Given a specific a, the CG procedure is executed with all negative reduced cost columns added to the RMP. When no columns can be added with the current setting of a, this means that $\widetilde{\Pi}$ satisfies all the constraints in the dual problem and is a feasible dual solution. Thus, we update the $\overline{\Pi} = \widetilde{\Pi}$ for the incumbent best guess, and we then increase a by 0.05 for a new iteration of above process. The CG procedure terminates when $a = 1$ and no negative reduced cost columns can be found.

7. Computational experiments

We have conducted extensive numerical experiments to validate the effectiveness of the proposed model and the efficiency of the CG-based heuristic. The experiments were run on a PC equipped with 3.30GHz of Intel Core i5 CPU and 16GB of RAM. All the algorithms were programmed in C# (VS2012), and the RMP was solved by CPLEX 12.5. The time limit for all test instances was three hours (10,800 seconds).

7.1. Generation of the test instances

The planning horizon considered is one week. Each day is divided into six time steps of four hours each. In total, there are 42 time steps for the planning horizon (i.e., $H = 42$). In the computational

Table 1 Scale of instance groups in experiments

Group ID	# of vessels ($\lvert V\rvert$)	# of berths ($\lvert B\rvert$)	# of QCs (Q)	# of subblocks ($\lvert K\rvert$)	# of time steps (H)
$ISG1$	15	2	5	80	42
$ISG2$	20	3	7	120	42
$ISG3$	30	4	11	160	42
$ISG4$	35	5	12	200	42
$ISG5$	45	6	16	240	42
$ISG6$	50	7	18	280	42
$ISG7$	60	8	21	320	42

experiments, we randomly generated test instances with seven different scales. The parameter settings for the seven instance groups are listed in Table 1.

All the incoming vessels are classified into three classes, i.e., feeder vessels, medium vessels and jumbo vessels. Table 2 illustrates the QC-profile generation for the three vessel classes. The available QC-profiles for each vessel are random generated based on the table. We can calculate the average handling time for all the vessels as $(3+4+5)/3=4$, and the average workload for all the vessels as $(3.5+10.0+17.5)/3=10.3$.

Table 2 QC-profile generation for different vessel classes

Vessel		QC-profile specifications				
Class	Proportion	Range of used QCs	Range of handing time (time step)	Average handling time (time step)	Range of workload (QC × time step)	Average workload (QC × time step)
Feeder	1/3	1 to 3	2 to 4	3	2 to 5	3.5
Medium	1/3	2 to 4	3 to 5	4	6 to 14	10.0
Jumbo	1/3	3 to 5	4 to 6	5	15 to 20	17.5

Given the QC-profile generation table, it can be concluded that each vessel will occupy a berth for four time steps on average and use QC resources for 10.3 QC × time steps on average. Thereafter, for all the instance groups, the berth utilization rate and the QC utilization rate, when all incoming vessels are served, can be calculated as shown in Table 3. As can be seen, the berth utilization rate and QC utilization rate for all instance groups are in the 63%–74% range, which is realistic.

The coefficient c_i^p for the penalty cost for each vessel is randomly generated in the ranges of [2, 6], [6, 10] and [10, 14] for feeder vessels, medium vessels and jumbo vessels, respectively (Meisel and Biewirth, 2009). The coefficient for the operation cost in the yard is set as $c^o=5\times10^{-6}$ (Zhen et al., 2011). The workload of each vessel is generated based on Table 1 with the unit of QC × time step. Here, we assume that a QC can handle about 30 containers per hour. Thus, the total number of handled containers for each vessel can be calculated by multiplying its workload, four

Table 3 Berth and QC utilization rates of the instances in experiments

Group ID	Berth utilization			QC utilization		
	Vessel usage ($\lvert V\rvert \times 4$)	Port resource ($\lvert B\rvert \times H$)	Utilization rate	Vessel usage ($\lvert V\rvert \times 10.3$)	Port resource ($Q \times H$)	Utilization rate
$ISG1$	60	84	71.4%	154.5	210	73.6%
$ISG2$	80	126	63.5%	206.0	294	70.1%
$ISG3$	120	168	71.4%	309.0	462	66.9%
$ISG4$	140	210	66.7%	360.5	504	71.5%
$ISG5$	180	252	71.4%	463.5	672	69.0%
$ISG6$	200	294	68.0%	515.0	756	68.1%
$ISG7$	240	336	71.4%	618.0	882	70.1%

hours, and 30 containers. For example, the average number of handled containers for all vessels is $10.3 \times 4 \times 30 = 1236$ (10.3 is the average workload). We further assume that for each vessel, there is a random $\epsilon \in [40\%, 60\%]$ proportion of loading containers and a $1 - \epsilon$ proportion of unloading containers among all handled containers, which provide the input data for l_i and u_i. The number of subblocks that are reserved for each vessel (i.e., r_i) is generated in the sets of $\{2,\ 3\}$, $\{4,\ 5,\ 6,\ 7\}$ and $\{8,\ 9,\ 10\}$ for the three vessel classes, respectively.

7.2. Efficiency of two column generators

We initially conducted some experiments to compare the efficiency of two ways to solve the pricing problems. The first way is to use CPLEX to solve the pricing model $\boldsymbol{M5}$ directly. The second way is to use the proposed exact algorithm to solve the pricing model (i.e., **Algorithm 1**). Both ways are called the column generators for the CG procedure. To compare the efficiency of the two column generators, the RMP was solved to optimality during the CG procedure (i.e., there is no column can be added into the RMP). Based on the column generators, the optimal result of LP relaxation for the problem (i.e., LP-optimal) and the computational time (i.e., CPU time) were recorded and are listed in Table 4 by group of instances, where each group contains five instances with the same problem scale.

As can be seen from Table 4, both column generators obtain the same optimal objective values for the LP relaxation over all instances, which means that **Algorithm 1** can solve the pricing problems to optimality. However, the efficiencies of the two column generators are significantly different. According to the ‘time ratio’ in Table 4, **Algorithm 1** only needs seven percent of the CPU time of CPLEX, which demonstrates that the proposed exact algorithm is highly efficient to solve the pricing problems. The reason for this high performance is probably that solving the pricing problems by the CPLEX needs to invoke the procedure to build a model in the MILP solver, which is time-consuming. However, solving the pricing problems by **Algorithm 1** only needs a simple circulation procedure in programming without invoking any MIP solver, which leads to a higher efficiency.

Table 4 Comparison on the efficiency of two ways to solve pricing problems

Instance		Solving PP by CPLEX		Solving PP by Algorithm 1		Time ratio
Group	ID	LP-optimum	CPU time (s)	LP-optimum	CPU time (s)	
$ISG1$	4-1	43.45	159	43.45	10	0.06
	4-2	35.62	118	35.62	8	0.07
	4-3	32.65	133	32.65	9	0.07
	4-4	44.75	161	44.75	8	0.05
	4-5	31.43	150	31.43	11	0.07
$ISG2$	4-6	47.17	351	47.17	22	0.06
	4-7	44.64	247	44.64	13	0.05
	4-8	47.70	283	47.70	17	0.06
	4-9	45.05	308	45.05	23	0.07
	4-10	54.03	236	54.03	14	0.06
$ISG3$	4-11	79.70	656	79.70	54	0.08
	4-12	78.99	402	78.99	42	0.10
	4-13	77.53	566	77.53	36	0.06
	4-14	84.66	594	84.66	51	0.09
	4-15	77.38	551	77.38	40	0.07
Average			**328**		**24**	**0.07**

Notes: 'Time ratio' equals the computational time of solving PP by Algorithm 1 divided by the computational time of solving PP by CPLEX.

7.3. Comparison of the four proposed selection strategies

In Section 6.2, we proposed four assignment plan selection strategies for the CG-based heuristic. Here, we conduct extensive numerical experiment to test the efficiency and the effectiveness of the algorithm by using the four strategies. In order to test whether our proposed algorithm can identify near-optimal solutions within reasonable computational times, we also use CPLEX to solve model $\boldsymbol{M2}$ optimally. Small-scale instance groups (i.e., $ISG1$, $ISG2$ and $ISG3$) were used in this experiment.

Table 5 illustrates the comparisons between CPLEX and the proposed algorithm using different strategies. As can be seen, CPLEX can only solve the problem for some small-scale instances, i.e., Instance 5-1 to Instance 5-11. The majority of instances in $ISG3$ cannot be solved to optimality by CPLEX within three hours, which means that the optimal solution is only achievable for the instances in $ISG1$ and $ISG2$. However, all instances in the table can be solved efficiently by the proposed algorithm under different strategies. The choice of a strategy has nearly no effect on the computational time of the proposed algorithm, but has a significant effect on the quality of the solution obtained by the algorithm. **Strategy 3** and **Strategy 4** outperform **Strategy 1** and **Strategy 2** since using the former two strategies leads to average small optimality gaps of 1.02% and 1.22% compared with 5.15% and 4.39%. This demonstrates that using a tailored strategy in the CG-based heuristic can yield near-optimal solutions.

Table 5 Comparison of four assignment plan selection strategies

Instance		CPLEX		Strategy 1			Strategy 2			Strategy 3			Strategy 4		
Group	ID	Obj	Seconds	Obj	Gap	Seconds	Obj	Gap	Seconds	Obj	Gap	Seconds	Obj	Gap	Seconds
*ISG*1	5-1	59.18	74	62.40	5.44%	31	61.68	4.22%	24	59.62	0.74%	32	59.97	1.33%	34
	5-2	54.53	56	59.41	8.95%	23	57.96	6.30%	26	55.03	0.92%	29	55.21	1.26%	20
	5-3	57.64	135	60.40	4.78%	34	60.62	5.17%	40	58.44	1.39%	37	58.40	1.31%	44
	5-4	54.72	31	58.23	6.42%	19	57.38	4.86%	23	55.13	0.75%	31	55.27	1.01%	27
	5-5	46.81	34	50.26	7.38%	21	49.50	5.73%	24	46.91	0.21%	21	47.01	0.44%	30
*ISG*2	5-6	62.97	1256	65.63	4.23%	97	65.85	4.57%	89	63.48	0.81%	142	64.01	1.66%	112
	5-7	66.34	1328	68.12	2.68%	164	68.01	2.50%	148	67.36	1.53%	173	67.32	1.48%	132
	5-8	60.65	2250	62.21	2.58%	186	62.40	2.89%	202	61.10	0.75%	189	61.53	1.45%	230
	5-9	64.29	1928	68.86	7.10%	210	67.99	5.76%	231	65.45	1.81%	253	65.53	1.93%	231
	5-10	67.46	3515	69.74	3.38%	231	69.44	2.93%	240	68.32	1.28%	221	67.98	0.78%	195
*ISG*3	5-11	103.22	9775	107.09	3.75%	532	106.74	3.41%	472	104.27	1.01%	528	104.01	0.76%	547
	5-12	—	—	104.33	—	643	105.00	—	534	101.82	—	493	101.54	—	476
	5-13	—	—	99.48	—	542	99.30	—	478	97.05	—	503	97.23	—	673
	5-14	—	—	103.25	—	674	102.99	—	525	101.12	—	596	100.87	—	553
	5-15	—	—	96.29	—	540	96.02	—	609	93.78	—	525	93.70	—	601
Average					**5.15%**			**4.39%**			**1.02%**			**1.22%**	

Notes: (i) 'CPLEX' shows the solution method that solves the problem directly by CPLEX, which provides the optimal solution; (ii) 'Strategy 1', 'Strategy 2', 'Strategy 3' and 'Strategy 4' show the solution methods for the proposed CG-based heuristic by using the four proposed assignment plan selection strategies, respectively; (iii) 'Obj' is the objective value of the solution obtained by the corresponding solution method; (iv) 'Gap' is the optimality gap between the optimal solution obtained by CPLEX and the solution obtained by the CG-based heuristic by using the corresponding strategy. (v) 'Seconds' is the number of CPU seconds needed for the solution method to obtain the solution; (vi) '—' means the computational time for the instance is more than 10,800 seconds.

7.4. Effectiveness of the proposed CG-based heuristic algorithm

To validate the effectiveness of the proposed model and of the CG-based heuristic, we further conducted experiments to compare our algorithm by applying the two strategies with the FCFS (first come first served) rule and the SWO (squeaky wheel optimization) metaheuristic on large-scale instance groups (i.e., $ISG4$, $ISG5$, $ISG6$ and $ISG7$), which are commonly used in berth and yard allocation problems (Lim and Xu 2006, Meisel and Bierwirth 2009, Zhen et al. 2011). The implementations of FCFS and SWO for the problem in this paper are similar to those of Zhen et al. (2011).

Table 6 Comparison with FCFS rule and SWO metaheuristic for large-scale instances

Instance		FCFS	SWO			Strategy 3			Strategy 4		
Group	ID	Obj	Obj	Gap	Seconds	Obj	Gap	Seconds	Obj	Gap	Seconds
$ISG4$	6-1	130.91	118.80	9.25%	1386	116.53	10.98%	1119	116.86	10.73%	1073
	6-2	135.62	123.83	8.69%	1517	122.83	9.43%	1046	122.50	9.68%	1228
	6-3	137.35	123.94	9.76%	1414	122.85	10.55%	996	124.09	9.65%	1137
	6-4	134.42	120.64	10.25%	1276	119.15	11.36%	876	118.89	11.55%	1045
	6-5	129.29	118.72	8.17%	1257	117.33	9.25%	1058	117.46	9.15%	997
$ISG5$	6-6	194.56	175.32	9.89%	3782	174.43	10.35%	2750	175.48	9.81%	3012
	6-7	191.10	174.70	8.58%	3532	172.98	9.48%	2672	173.32	9.31%	2977
	6-8	186.67	168.34	9.82%	3398	162.20	13.11%	2828	162.51	12.94%	2764
	6-9	184.82	165.97	10.20%	4078	163.56	11.50%	2499	163.33	11.63%	2542
	6-10	188.98	171.21	9.40%	3123	168.46	10.86%	2375	167.93	11.14%	2212
$ISG6$	6-11	232.34	213.21	8.23%	6732	210.20	9.53%	5534	212.06	8.73%	5768
	6-12	237.37	213.47	10.07%	7071	213.81	9.92%	5774	212.52	10.47%	5423
	6-13	231.53	208.88	9.78%	7290	207.50	10.38%	4632	206.99	10.60%	4212
	6-14	233.97	211.74	9.50%	6786	207.51	11.31%	5654	207.93	11.13%	6043
	6-15	225.58	202.77	10.11%	7343	201.02	10.88%	5850	201.28	10.77%	5723
$ISG7$	6-16	292.30	—	—	—	262.61	10.16%	9190	262.25	10.28%	9289
	6-17	302.87	—	—	—	273.71	9.63%	8928	274.12	9.49%	9813
	6-18	294.21	—	—	—	264.37	10.14%	9561	265.04	9.92%	8972
	6-19	300.90	—	—	—	272.25	9.52%	9821	270.78	10.01%	9312
	6-20	302.12	—	—	—	273.24	9.56%	8722	274.33	9.20%	8834
Average				**9.45%**			**10.40%**			**10.31%**	

Notes: (i) 'SWO' shows the solution method for SWO metaheuristic; (ii) 'Strategy III' and 'Strategy IV' show the solution methods for the proposed CG-based heuristic by using the two proposed assignment plan selection strategies respectively; (iii) 'Obj' is the objective value of the solution obtained by the corresponding solution method; (iv) 'Gap' lists the objective gap between the solution obtained by FCFS rule and the solution obtained by the CG-based heuristic by using the corresponding strategy; (v) 'Seconds' is the number of CPU seconds needed for the solution method to obtain the solution; (vi) '—' means the computational time for the instance is more than 10,800 seconds (i.e., three hours).

Table 6 provides comparisons between the proposed CG-based heuristics, the FCFS rule, and the SWO-based metaheuristic. From Table 6, we can see that the CG-based heuristics and the SWO-based metaheuristic significantly outperform the commonly used FCFS decision rule. The SWO based metaheuristic can improve the objective by 9.45% on average. However, the proposed CG-based heuristics under **Strategy 3** and **Strategy 4** improve it by 10.40% and 10.31%, respectively.

The results demonstrate that the CG-based heuristic outperforms the SWO-based metaheuristic for the integrated problem with respect to both the computation time and the solution quality. The SWO-based metaheuristic cannot converge within three hours each of the instances in $ISG7$. This shows that the proposed heuristic algorithm is more efficient than the SWO-based metaheuristic.

8. Conclusions

We have considered an integrated optimization problem arising in container terminals. A MILP model was built for this problem, which takes account of the decisions of berth allocation, QC assignment, and yard subblock assignment for arrival vessels. In addition, the periodicity of the plan was also considered. A CG-based heuristic was then developed to solve the model on large-scale problem instances; some accelerating techniques for the algorithm were also investigated. We performed extensive numerical experiments based on realistic instances in order to validate the effectiveness of the proposed model and the efficiency of the algorithm. The results show that the CG-based solution algorithm can obtain a good solution with an approximate 1% optimality gap within a much shorter computation time than a direct application of CPLEX.

The contribution of this study lies mainly in the following two aspects: (i) we have proposed an integrated model on optimizing periodical plans of three key types of resources (berths, QCs and subblocks) in container terminals; (ii) a CG-based heuristic as well as some accelerating techniques can solve the model in a more efficient manner than some of metaheuristic that are commonly used for the optimization of port operations.

Appendix A: Pseudo-codes for the two sub-algorithms

Sub-algorithm 1 Exact polynomial algorithm for model $\boldsymbol{M6}$

Input: A given set of QC-profile P, a dwelling time ψ and a starting time step χ
Output: An optimal selection of a QC-profile
for all the p, $p \in P$ **do**
  Define OBJ_p^1 as the objective for QC-profile p when this profile is selected
  if $h_p \neq \psi$ **then**
    Set $OBJ_p^1 = -\infty$
    Calculate QCs used in each time step for QC-profile p, denoted as $\zeta_t,\ \forall t \in T$
  end if
  Set $OBJ_p^1 = \sum_{t \in T} \phi_t \cdot \zeta_t$
end for
Solve $\max(OBJ_p^1 | \forall p \in P)$, which is the objective for model $\boldsymbol{M6}$, and the solution is p^*

Sub-algorithm 2 Exact polynomial algorithm for model $\boldsymbol{M7}$

Input: A given set of Berth B, Subblock K, a dwelling time ψ and a starting time step χ
Output: An optimal selection of a berth and r subblocks
for all the b, $b \in B$ **do**
  Define OBJ_b^2 as the objective for Berth b when it is selected
  Set $OBJ_b^2 = c^o D_b^U u - \sum_{t\in[\chi,\ \chi+\psi-1]} \varpi_{bt}$
  for all the k, $k \in K$ **do**
    Define OBJ_k^3 as the objective for Subblock k when it is selected
    Set $OBJ_k^3 = c^o D_{kb}^L \left(\frac{l}{r}\right) - \rho_k$
  end for
  Rank OBJ_k^3 from the smallest to the largest, and record it as S_k $(S_1 \leq \cdots \leq S_K)$
  Set $OBJ_b^2 = OBJ_b^2 + \sum_{k\in[1,r]} S_k$, the objective to select Berth b and best r subblocks
end for
Solve $\min(OBJ_b^1 | \forall b \in B)$, which is the objective for model $\boldsymbol{M7}$, and the solution is b^* with the best r subblocks selected

Appendix B: Procedure for the first stage of the initial heuristic

Assume that the sequence of vessels is $(v_1, \ldots, v_n, \ldots, v_N)$, where n is the index for the sequence and N is the number of vessels. The sequential method solve the N vessels sequentially by multiple iterations. In the n^{th} iteration, the berth-QC assignment problem is solved for the n^{th} vessel in the sequence by model $\boldsymbol{M8}$. Here, we define a parameter $DS = 7$ which indicates the depth of search. In the n^{th} iteration, for model $\boldsymbol{M8}$, all the variables related to $v_1, \ldots, v_{n-1}$ are set as the input data, and all the variables related to $v_n, \ldots, v_{n+DS}$ are defined as decision variables. Once the model is solved for the n^{th} iteration, the obtained values of the decision variables for the n^{th} vessel are transferred to the input data for the $(n+1)^{st}$ iteration. In total, there are $N - DS$ iterations, and the last iteration solves model $\boldsymbol{M8}$ with the decision variables of $v_{N-DS}, \ldots, v_N$. Before formulating the model for the n^{th} iteration, two sets are defined: $V_n^F = \{v_1, \ldots, v_{n-1}\}$ and $V_n^B = \{v_n, \ldots, v_{n+DS}\}$, where $n \in \{1, \ldots,\ N - DS\}$. For the first iteration, $V_1^F = \emptyset$. The set of V_n^F provides the input data for the model $\boldsymbol{M8}$ in the n^{th} iteration, which is formulated as follows:

$$[\boldsymbol{M8}]\ \ \text{minimize} \sum_{i\in V} c_i^p(\tau_i^{a+} + \tau_i^{b+}) + c^o \sum_{i\in V}\sum_{b\in B} \omega_{ib} D_b^U u_i \tag{105}$$

subject to: Constraints (9)–(11); (15)–(20); (22); (27)–(31),

$$\sum_{p\in P_i} \gamma_{ip} = 1 \quad \forall i \in V_n^B, \tag{106}$$

$$\sum_{b\in B} \omega_{ib} = 1 \quad \forall i \in V_n^B, \tag{107}$$

$$\sum_{t\in\{1,\ldots,H\}} \mu_{it} = 1 \quad \forall i \in V_n^B, \tag{108}$$

$$\sum_{t\in T}\mu_{it}t=\alpha_i \quad \forall i\in V_n^B, \tag{109}$$

$$\alpha_i+\sum_{p\in P_i}\gamma_{ip}h_{ip}-1=\beta_i \quad \forall i\in V_n^B, \tag{110}$$

$$\alpha_i\geq a_i^f \quad \forall i\in V_n^B, \tag{111}$$

$$\beta_i\leq b_i^f \quad \forall i\in V_n^B, \tag{112}$$

$$\eta_{ipt}\geq\gamma_{ip}+\mu_{it}-1 \quad \forall i\in V_n^B, \forall p\in P_i,\ \forall t\in T, \tag{113}$$

$$\omega_{ib}\in\{0,\ 1\} \quad \forall i\in V_n^B, \forall b\in B, \tag{114}$$

$$\gamma_{ip}\in\{0,\ 1\} \quad \forall i\in V_n^B, \forall p\in P_i, \tag{115}$$

$$\mu_{it}\in\{0,\ 1\} \quad \forall i\in V_n^B, \forall t\in T, \tag{116}$$

$$\eta_{ipt}\in\{0,\ 1\} \quad \forall i\in V_n^B, \forall p\in P_i, \forall t\in T. \tag{117}$$

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