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Market Driven Ship Investment Decision Using ROA

Meifeng LUO [a] and Ying KOU [b]*

[a] Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hong Kong

Email: Meifeng.Luo@polyu.edu.hk

[b] Sino-US Global Logistics Institute, Shanghai Jiao Tong University, 1954 Huashan Road, Shanghai 200032, PR China

Email: maggiekou@gmail.com

[*]Corresponding Author

Abstract:

This paper analyzes ship investment behavior where the main driving force is future freight earnings. The shipping market is cyclic and uncertain, and the decision is whether to invest now or later. Theoretically, we have found the trigger rates—the necessary freight rate for profitable ship investment—for both the net present value (NPV) and real option approach (ROA), assuming that future freight rates follow a mean reverting stochastic process. The combination of these trigger rates provides the necessary and sufficient conditions for immediate investment. Empirically, we estimated the trigger rates for the whole sample from January 1976 to July 2014, as well as sub-samples that account for structural changes in the shipping market. Both theoretical and empirical results show that ship investment decisions can be made based on the relationships between the current freight rate and the trigger rates from NPV and ROA. If the freight rate cannot make NPV positive, no investment should be considered. Immediate investment is only recommended when the freight rate is higher than the trigger rate using ROA. Sensitivity analysis shows that the trigger rate is most sensitive to changes in the long-run mean of the freight rate. A simple regression analysis indicates that our model can explain market driven ship investment activities in the past. In addition, results that incorporate structural breaks in the shipping market are closer to actual investment behavior in the market.

Keywords: Ship investment, Real Option Approach, Mean-reverting Stochastic Process

1. Introduction

Ship investment is one of the critical decisions that all shipping companies must make. A decision over ship investment can be market-driven, i.e., motivated by the expected earnings of the ship in the future freight market. It can also be motivated by other strategies, such as hedging on low ship prices (Alizadeh & Nomikos, 2007; Lorange, 2001), and strategic capacity competition in a competitive market (Kou & Luo, 2015b). Compared with the latter, market-driven ship investment decisions face considerable challenges due to the cyclical and uncertain nature of the future shipping market. A new ship requires a long time to build, and its huge capital investment entails a long payback period. Choosing the optimal time to invest in new ships not only benefits a company's capital cost savings, but also promotes its future performance over the long run. Whether to and when to invest in a new ship are two essential decisions that are critical to the success of a shipping company.

The most common measure in project evaluation is the Net Present Value (NPV), which is defined as the sum of all future incomes valued at the current time net of the investment cost. Once the NPV is positive, the project is deemed to be profitable and should go ahead. However, this method fails to take into account the value of delay due to uncertainties in future incomes. The Real Option Approach (ROA) has therefore been widely applied in financial analysis when the future cash flow is uncertain (Dixit, 1989; Abel, 1983; Dias & Rocha, 1999). It evaluates various options that an investor can take, but is not obligated to take, in the future, such as delaying, canceling, or expanding. For example, on June 3, 2015, Maersk, the number one liner operator in the world, signed a $1.8 billion contract for eleven 19,630 TEU vessels with Daewoo Shipbuilding & Marine Engineering, with an option of six more such vessels in the future (Lloydslist, 2015).

This research analyzes market-driven ship investment decisions with the option of delay. Specifically, when it is possible to delay an investment, and how to judge whether it is better to invest now or later. The shipping market is notorious for its cyclic and uncertain nature. To model future changes of the shipping freight rate, the mean-reverting process, which is also called the Ornstein-Uhlendbeck (OU) process, is more appropriate (Schwartz, 1997; Sarkar, 2003), as it can take into account both the general trend of the market and the market volatility, as well as other unknown variations in the freight rate process.

The objective of this paper is to develop the critical conditions, i.e., the trigger rates, which can be used to determine whether to invest now or later based on the current market conditions. Theoretically, we derive the decision making rules based on the trigger rates for NPV and ROA. We found that, with the option of delay, NPV criteria are necessary, but not sufficient, for immediate investment. This situation only occurs when the ROA is also satisfied, i.e., the current freight rate is higher than the trigger rate under ROA, which again is higher than the trigger rate under NPV. Empirically, we estimate the trigger rates for the whole sample (1976.01-2014.07), and then for sub-samples that take into account structural breaks in the shipping market. Our decision rule can explain about half of the new orders in the past, which is to be expected, as other behaviors in ship investment, such as asset play and strategic capacity investment, are not considered.

This is the first attempt to study the conditions for ship investment when taking into account the future uncertainties and cyclical nature of the shipping market using ROA. It fills the gap by theoretically analyzing the trigger rate for ship investment using the more accurate OU assumption on the freight rate process, and empirically calculates the trigger rates by considering the shipping market's cyclical nature. It contributes to the investment decision theory applied to a project with a huge capital cost, long lifespan and cyclical market conditions. In a practical way, it gives shipping companies a clear rule to follow when making ship investment decisions, and helps them to make investment decisions under different market conditions. More importantly, by taking into consideration the value of delay, the capacity investment will be more conservative, which can help to ease the overcapacity problems that exist in the current shipping market.

The next section reviews the current literature on investment decision making using ROA, as well as studies of the stochastic property of shipping freight rates. After that, a theoretical model on ship investment decisions using both NPV and ROA will be provided, together with the conditions necessary for either investing immediately or postponing, assuming that future freight rates follow an OU process. Following this, an empirical section develops the trigger rates using actual data on time-charter rates and new-building prices, and this section also includes a sensitivity analysis of the main parameters, a regression analysis on the predictability of the model, and a prediction of future shipping investments. The main results and findings are summarized in the last section.

2. Literature Review

The ROA has been widely applied to evaluating investment decisions under uncertainty. The pioneering work was done by Tourinho (1979), who used the concept of option value to evaluate natural resource conservation facing price uncertainty. Since then, many research works have been developed, with notable contributions being made by Brennan and Schwartz (1985), McDonald and Siegel (1986), and Dixit and Pindyck (1994). Brennan and Schwartz (1985) used stochastic dynamic programming to evaluate the closure option of a copper mine investment, in which the present value of this project was determined by the stochastic spot prices and inventory levels. They suggested solving the model numerically, since no analytic solution was provided. McDonald and Siegel (1986) examined the investment decisions on an irreversible project taking into account the value of delay. They considered uncertainties in both cost and benefit of investment by developing a trigger ratio between total cost and total revenue. Numerical examples were given and parameters were arbitrarily imposed. Dixit and Pindyck (1994) synthesized several of the past ideas and provided a complete framework on the issue of investment under uncertainty. They considered firm investment decision not only as a monopoly, but also in a competitive environment, using the ROA. Therefore they concentrated more on theoretical development, and empirical examples were very limited. Dixit and

Pindyck's theoretical models have since been applied to various areas, such as energy saving investment (Lin & Huang, 2011), urban development (Bar-Ilan & Strange, 1996), and technological innovations (Grenadier & Weiss, 1997).

The most common assumption of the ROA is that cash flow follows the Geometric Brownian Motion (GBM) process, as suggested in the early works by Pindyck (1982; 1988), Abel (1983), Brennan and Schwartz (1985), McDonald and Siegel (1986) and Dixit (1989). However, many economic variables exhibit the tendency of reverting to their long-term average. In such cases, the OU process is a more appropriate assumption. In comparison with the GBM process, the OU process is not often applied, because of its complexity. A discussion on its applicability can be found in the works by Bhattacharya (1978), Metcalf and Hassett (1995), Schwartz (1997), Sarkar (2003) and Tsekrekos (2010). In fact, the OU specification in the above works is known as a geometric OU process (Metcalf & Hassett, 1995; Schwartz, 1997) or a modified OU process, in which diffusion is proportional to output price (Bhattacharya, 1978; Sarkar, 2003; Tsekrekos, 2010). There has been no empirical test on the validity of these two types of the OU process. The classical arithmetic OU process is only found in the work by Sødal *et al*. (2008), in which the freight rate differential between the dry and the wet bulk is assumed to follow the classical arithmetic OU process with the aim of analyzing when to switch between the dry and the wet bulk market for a combination carrier, and used the Augmented Dickey Fuller (ADF) test to check this assumption.

The ROA has been successfully applied to other fields, but only sparsely used in maritime studies. Bendall and Stent (2003; 2005; 2007) published a series of papers using the ROA to carry out case studies. Bendall and Stent (2003) described the investment strategy in a declining and competitive market. The 2005 paper analyzed different ways of allocating container ships on the Singapore-Klang-Penang route. In 2007, they studied the investment in a new container vessel serving from the east coast of Australia to New Zealand. All these papers feature the ROA to ship investment projects under uncertainty, but do not provide any theoretical demonstration. As of yet, there is still a lack of theoretical understanding of the application of the ROA on ship investment decisions.

As for the stochastic process of freight rates, the predominant view in the existing shipping literature is that it is the result of market supply and demand (Zannetos, 1966; Hawdon, 1978; Beenstock, 1985; Stopford, 2009), and that it should, therefore, follow the traditional OU process. The formal assumption of this process first appeared in Bjerksund and Ekern (1995), its lognormal process in Tvedt (1997) and its non-linear process in Adland and Cullinane (2006). However, a body of empirical evidence shows that freight rates contain a unit root (Veenstra & Franses, 1997; Kavussanos & Alizadeh, 2002b; Alizadeh & Nomikos, 2007), indicating that it is not an OU process. Koekebakker *et al*. (2006) summarized the unit root test results from the past studies, and concluded that, except for spot rates and BFI (Tvedt, 2003), all the results show that freight rates are a non-stationary process. Spot rates and BFI (Tvedt, 2003) appear stationary only when US dollars are converted to Japanese yen, and Tvedt (2003) argued that it is the direct result of a Japanese-dominated market. However, the time charter rates in these studies were found to be non-stationary even when being measured by Japanese yen. Note that Koekebakker *et al*. (2006) used data samples upto year 2000. Freight rates in bulk shipping registered a dramatic increase in 2007 and a fast decrease in 2008, and only until recently had freight rates seemingly reverted to their pre-2003 levels. Therefore, it is likely that the stationarity of freight rates in different periods may be different, which requires further investigation.

In summary, although the ROA has been proven an efficient tool in analysing investment projects under uncertainty, it has not been sufficiently utilized in ship investment decisions, and therefore an analytical and quantitative evaluation of the critical freight rate for ship investment decisions taking into account the option of delay is still lacking.

3. Theoretical Model

This section formulates the theoretical conditions for ship investment by comparing two evaluation methods: first NPV, then ROA. In NPV, an individual shipping company has to decide, at the current time (t=0), whether to order a new ship. If ordered, it can have the ship in θ years due to the construction lag, and use the ship for N years. Denote R_t as the time-charter rate at time t, which is the net earnings of the ship without taking into account the ship investment cost, i.e., the ship price. For simplicity, the ship is assumed not to be traded in the second-hand market, and as having no salvage value at the end of its lifespan. Then, the value of the project, i.e., the present value of the total future earnings if the shipowner orders a new ship now (t=0) can be written as:

$$V_0(R_t) = E\left\{\int_{t=\theta}^{t=N+\theta} R_t e^{-rt} dt\right\} \quad (1)$$

where r is the discount rate, and E denotes an expectation operator.

From Eq. (1), the future earnings depend on the time-charter rate R_t, which is assumed to be a random variable that follows the OU process:

$$dR_t = u(m - R_t)dt + \sigma dz_t, \quad (2)$$

where m is the long-run equilibrium time-charter rate, u the speed for R_t reverting to m, σ>0 the standard deviation measuring the volatility of this process, and dz_t the increment of a Wiener process with $dz_t = \varepsilon_t\sqrt{dt}$ where ε_t~N(0,1). Then the expectation and variance of dz_t are $E(dz_t) = 0$ and $var(dz_t) = dt$.

The solution to the OU process is

$$R_t = R_0 e^{-ut} + m(1 - e^{-ut}) + \int_0^t \sigma e^{u(s-t)} dz_s, \quad (3)$$

where the expectation of R_t is

$$E(R_t) = m + (R_0 - m)e^{-ut}. \tag{4}$$

This shows that R_t tends to converge to m. Substituting Eq. (4) into Eq. (1), we get:

$$V_0(R_t) = mK_r + (R_0 - m)K_\rho \tag{5}$$

where ρ=u+r, K_r and K_ρ are the annuity present value factors and $K_\rho = \left[e^{-\rho\theta} - e^{-\rho(N+\theta)}\right]/\rho$, $K_r = \left[e^{-r\theta} - e^{-r(N+\theta)}\right]/r$. K_r and K_ρ are both positive and decreasing with r and ρ. Eq. (5) shows a positive linear relationship between the project value and freight rate under the OU assumption. If u=0, the future freight rate is a random walk, and the project value will be determined by the current freight rate R_0. If u is very big, whenever the freight rate is away from the mean, it will converge to the mean right away, so that the starting point (R_0) is not important, and the project value will be determined by the mean.

If the NPV method is used, then the project value $F_0(R_t)$ can be written as:

$$F_0(R_t) = V_0(R_t) - P_0 = R_0K_\rho + m\left(K_r - K_\rho\right) - P_0 \tag{6}$$

where P_0 is the price of the new ship at time t=0. Define the trigger rate R^*_{NPV} as the time-charter rate that makes Eq. (6) equal to zero, i.e.

$$R^*_{\text{NPV}} = \frac{P_0 - m\left(K_r - K_\rho\right)}{K_\rho}, \tag{7}$$

then whenever the time-charter rate $R_0 \geq R^*_{\text{NPV}}$, invest immediately is a good decision according to the NPV rule.

Using ROA, if the company delays the investment to a short time dt, the project value at dt can be written as $F_{dt}(R_t, P_t) = V_{dt}(R_t) - P_{dt}$. Since the main objective is to study the market driven investment behavior, small changes in ship price in dt time can be overlooked, i.e., P_{dt}=P_0. If the present value of the project value, $w_0(R_t) = E_0[F_{dt}(R_t)]e^{-rdt}$, is larger than $F_0(R_t)$, it is better to delay the project. Define the trigger rate using ROA, R^*_{ROA}, as the time-charter rate that makes the investor at least not worse off than ordering now, i.e. $w_0(R^*_{\text{ROA}}) \geq F_0(R^*_{\text{ROA}})$. The derivation of the trigger rate is shown in the Appendix. Due to the complexity of the problem, a closed form solution is not obtainable. The mathematical form of the two project values are shown below:

$$\begin{cases} F_0(R_t) = R_0K_\rho + m\left(K_r - K_\rho\right) - P_0 & R_0 \geq R^*_{\text{ROA}} \text{ (immediate investment)} \\ w_0(R_t) = \dfrac{K_\rho X(R_0)}{X'(R_0)} & R_0 < R^*_{\text{ROA}} \text{ (postponed investment)} \end{cases} \tag{8}$$

where $X(R_0)$ is an auxiliary function defined in the Appendix.

Next we analyze decisions based on the properties of the project value function. From Eq.(5), it is straightforward that:

$$\frac{dV_0(R_t)}{dR_0} = K_\rho, \tag{9}$$

which shows that the project value of investing now is a linear function of the current time-charter rate. Differentiating the project value $w_0(R_t)$ *w.r.t.* R_0, and considering the relationship of R_t with R_0 from Eq. (3), it is possible to get:

$$\frac{dw_0(R_t)}{dR_0} = K_\rho e^{-rdt}\frac{dR_{dt}}{dR_0} = K_\rho e^{-(r+u)dt} = K_\rho e^{-\rho dt}. \tag{10}$$

It is clear that when R increases, the project value of investing later increases slower than that of investing now. Their relationship can be illustrated in Figure 1.

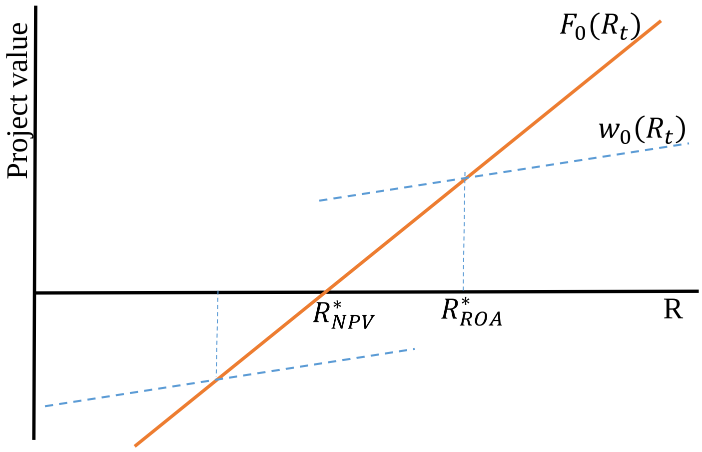


Figure 1: Relationship between $F_0(R_t)$ and $w_0(R_t)$

The intersection of $F_0(R_t)$ with the horizontal line is the trigger rate from the NPV method. The trigger rate using ROA is the intersection between $F_0(R_t)$ and $w_0(R_t)$. When the real rate R_0 is less than R^*_{ROA}, $w_0(R_t)>F_0(R_t)$, indicating that investing later is better; if R_0 is higher than R^*_{ROA}, this indicates the opposite case. Theoretically, the relationship between the current freight rate (R_0), R^*_{ROA} and R^*_{NPV}, and their implications on investment decisions, can be summarized as follows:

$R_0 \geq \max\{R^*_{\text{NPV}}, R^*_{\text{ROA}}\}$: Invest immediately;

$R^*_{\text{ROA}} > R_0 > R^*_{\text{NPV}}$: Delay the investment;

$R^*_{\text{NPV}} > R_0 > R^*_{\text{ROA}}$: No investment by the NPV method; and

$R_0 < \min\{R^*_{\text{ROA}}, R^*_{\text{NPV}}\}$: No investment by either method.

From this, it can be seen that if the freight rate does not pass the NPV criteria, no investment should happen. On the other hand, even if it does pass the NPV criteria, it may still be optimal to delay the investment. Therefore, the trigger rate using the NPV can be considered as a necessary condition for investing immediately, but not the sufficient condition.

4. Empirical Analysis of the trigger rates

This section presents an empirical analysis of the trigger rates for ship investment, using monthly data from Clarkson Research Services Limited 2010 (CRS) on new-building prices (*P*) and 1-year time-charter rates (*R*) of three ship categories — Capesize, Panamax and Handysize. Originally, the ship price was in million dollars and the time-charter rate was in dollars/day. For consistency, the time-charter rate is converted to monthly rate. The sample period is from January 1976 to July 2014, except for Capesize prices, whose sample period spans from October 1983 to July 2014. Figure 2(a) depicts the time-charter rates for the whole sample, and Figure 2(b) depicts the new-building prices for three ships and the contracting number for new-building Handysize ships.

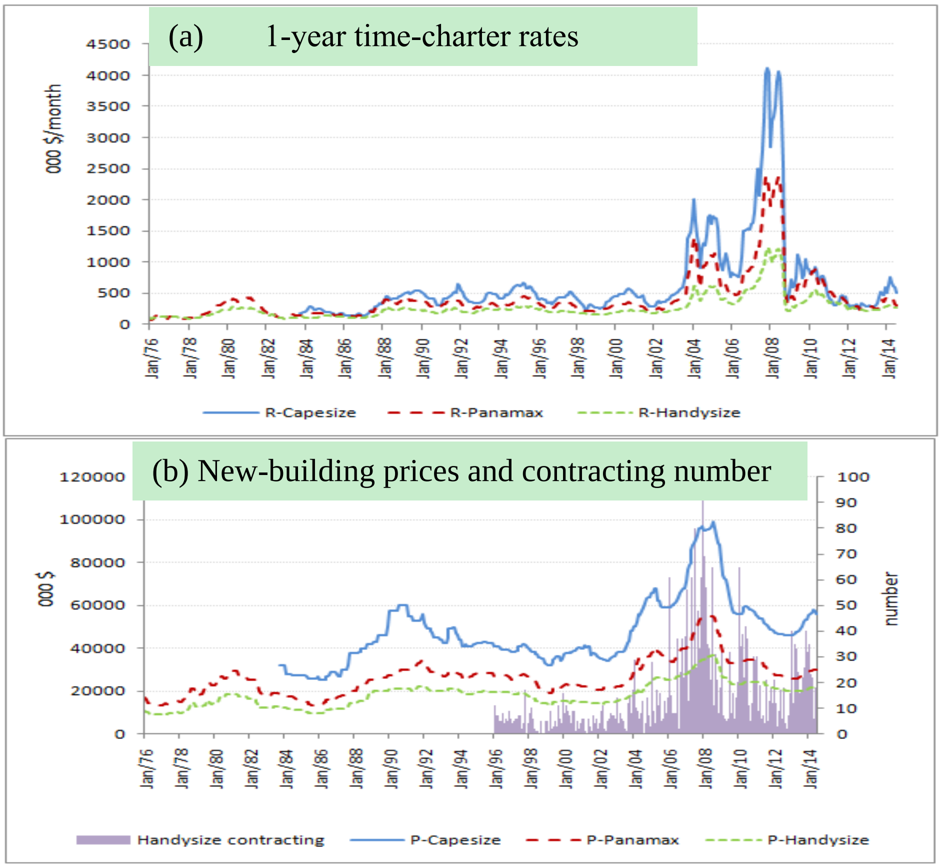


Figure 2: Time-charter rate and new-building prices over the study period

A stationarity test is applied to examine whether the time-charter rate follows the OU process. The results using the Augmented Dickey-Fuller (ADF) test are shown in Table 1. The null hypothesis is that the original series is non-stationary (with a unit root). The lag length is chosen on the basis of the Schwarz Information Criterion (SIC) (Schwarz, 1978). Different from most of the previous studies (Veenstra & Franses, 1997; Kavussanos & Alizadeh, 2002b; Alizadeh & Nomikos, 2007), the ADF test rejects the null hypothesis that *R* is non-stationary, implying that the time-charter rate *R* follows the OU process.

Table 1: ADF test of R for the whole sample

Ship types	Sample	Lags	ADF t-Statistic (SIC)	(p-value)	1% level	5% level
Capesize	1983.10-2014.07	1	-3.3989	(0.0116*)	-3.4479	-2.8692
Panamax	1976.01-2014.07	2	-3.4568	(0.0096**)	-3.4443	-2.8676
Handysize	1976.01-2014.07	1	-3.7410	(0.0038**)	-3.4443	-2.8676

Notes: * denotes rejection of the null hypothesis at the 5% significance level, while ** denotes rejection of the null hypothesis at the 1% significance level. Maximum lag is set to be 4, and lag length is automatically selected by Schwarz Information Criterion (SIC).

Next, we first present the trigger rates for the whole sample. Taking into account that there are structure breaks in the whole sample period (Kou & Luo, 2015a), we also calculate the trigger rates for each sub-sample. After that, we carry out a simple empirical test to examine whether the whole sample or the sub-sample result is closer to the real investment behaviour. Finally, new ship investment over the next two years (2014.08-2016.07) will be predicted.

4.1. Trigger rates for the whole sample

To calculate the trigger rates R^*_{NPV} and R^*_{ROA} over the whole sample, we use data from before Jan 1988 to generate the first group of parameters, then iteratively extend the sample data to generate the parameter series. The median values of each estimated parameter are used, and sensitivity analysis of these parameters on the trigger rates are provided.

4.1.1 Estimating the parameters for the whole sample

To calculate the trigger rates R^*_{NPV} and R^*_{ROA}, the first step is to estimate the parameters in the OU process, i.e. u, m and σ. Following the method in Dixit and Pindyck (1994), the discrete-time counterpart of the OU process for R_t can be written as:

$$R_t = a + bR_{t-1} + \varepsilon_t \tag{11}$$

where a and b are coefficients to be estimated and $\varepsilon_t \sim N(0,S^2)$ and S the standard deviation. After obtaining the coefficients, the value of parameters u, m, σ can be calculated by:

$$u = -\frac{\ln\hat{b}}{\Delta t},\ m = \frac{\hat{a}}{1-\hat{b}} \text{ and } \sigma = S\sqrt{\frac{2\ln\hat{b}}{(\hat{b}^2-1)\Delta t}} \tag{12}$$

where Δt is the time interval between the observations, and $\hat{a}$, $\hat{b}$ are the estimates for a and b.

From Eqs. (11) and (12), it can be seen that only one set of parameters u, m, σ can be obtained using the whole sample. However, people's perceptions about the market change over time. To study the dynamics of these parameters, we treat the pre-Jan 1988 data sample as the base sample, because Jan 1988 is the break point (Kou & Luo, 2015a). From the base sample, the first group of parameters (u_1, m_1 and σ_1) is estimated. By iteratively extending the sample data by one more observation, three time series of such parameters, u_t, m_t and σ_t, can be obtained. The descriptive statistics of these series are given in Table 2, and their detailed trends are plotted in Figure 3. In general, the parameters of smaller ships are smaller than those of larger ships. From the figure, it can be seen that m and u exhibit abnormal sudden changes mainly during the period 2003-2008, which corresponds to the time when the shipping market was extremely volatile.

Table 2: Descriptive statistics of u_t, m_t and σ_t

	Mean	Median	Max	Min	S.D.	Jarque-Bera	(Prob.)
				Capesize			
u_t	0.0171	0.0261	0.0315	-0.0947	0.02	895.37	(0.0000*)
m_t	606.06	451.2361	32011	-2733.7	1937.1	$6.39*10^5$	(0.0000*)
σ_t	71.13	29.0954	184.78	18.4	62.29	58.21	(0.0000*)
				Panamax			
u_t	0.0154	0.0202	0.0273	-0.0813	0.0139	5348.7	(0.0000*)
m_t	378.47	315.43	11182	-4504.5	787.1	$2.15*10^5$	(0.0000*)
σ_t	36.05	18.1692	79.50	17.43	24.76	53.23	(0.0000*)
				Handysize			
u_t	0.0105	0.0150	0.0181	-0.0509	0.0114	1523.8	(0.0000*)
m_t	267.26	224.18	4308.1	-679.90	335.85	$1.18*10^5$	(0.0000*)
σ_t	16.81	9.6303	36.92	8.83	10.85	66.11	(0.0000*)

Notes: Prob. is the test statistics for the series following a normal distribution; * denotes rejection of the null hypothesis at the 5% significance level.

Therefore, we set the base parameters of u, m and σ equal to the median value of their respective series for the whole sample analysis in order to capture the most common phenomena in the shipping market, and then carry out the sensitivity analysis to allow for a range of changes. The following parameter values are chosen for the base case: the construction lag and lifespan of the new ship are 2 years and 25 years respectively, this being equivalent to θ=24 months and N=300 months based on the monthly data.

The discount rate r is estimated based on the theoretical ship price-freight rate relationship. According to Kavussanos and Alizadeh (2002a), the theoretical ship price at time t should be equal to the present value of expected future earnings plus the present value of the expected resale price. Since we assume no re-selling occurs during the ship's lifespan, the theoretical relationship between R and P can be derived from Eq. (6) when $F_0(R_t)$=0 (P_0 is re-written as P_t as this could happen for all t):

$$P_t = K_\rho R_t + C + \varepsilon_t \tag{13}$$

where $C=m(K_r-K_\rho)$. From Eq. (13), the value of K_ρ and C can be estimated, denoting them as $\widehat{K_\rho}$ and $\hat{C}$. These numbers can be used to estimate the value of K_r, which in turn can be used to estimate r.

All the base parameters are shown in Table 3. The trigger rates R^*_{NPV} and R^*_{ROA} are generated using ship prices from 1988 to 2014. The relationship between R_0 and the trigger rates R^*_{NPV} and R^*_{ROA} for three ship types, as well as the project values for investing immediately or later, are plotted in Figure 4. To illustrate the ship investment decision, we also list six examples on R_0, R^*_{NPV} and R^*_{ROA} in Table

3. For example, the trigger rate from the NPV is $156,730/month in Jan 1998, while that of the ROA is $51,160/month for Capesize vessels. The real time-charter rate for such vessels is $432,000/month, which is higher than both trigger rates. Thus, investing immediately is better.

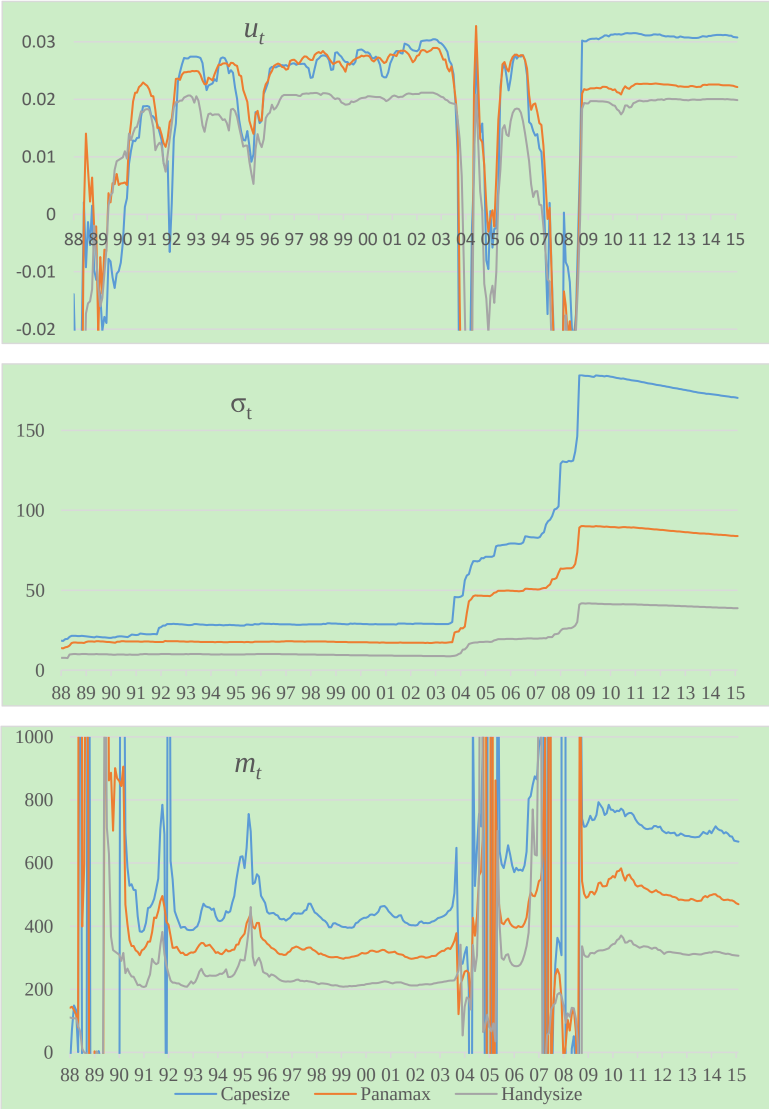


Figure 3: Parameters of u_t, m_t and σ_t evolving with time

Table 3: Base parameters and calculated trigger rates

Base Parameters				Examples			Trigger Rates(×1000)		Expected Return
u (%)	m	σ	r	Date	P(×1000)	R_0(×1000)	R^*_{NPV}	R^*_{ROA}	$F_0(R_0)$
				Capesize					
2.61	451.2361	29.0954	0.0077	1998/01	$40000	$432	$156.73	$51.16	$3625.42
				2004/08	$59000	$1312.5	$1599.37	$1892.7	$2393.44
				Panamax					
2.02	315.4324	18.1692	0.0096	1998/06	$24500	$222	$307.46	$294.65	-$838.36
				2004/08	$33000	$941.25	$824.56	$958.50	$2148.16
				Handysize					
1.50	224.1759	9.6303	0.0098	1998/05	$16500	$184.5	$196.08	$167.4	-$257.43
				2004/08	$20500	$481.89	$375.94	$422.82	$2356.16

Notes: For $F_0(R_0)$, an underlined number denotes that the NPV from immediate investment is larger than that from postponement, while the number without an underline means the opposite.

In Table 3, Capesize vessels show $R_0 \geq R^*_{\mathrm{NPV}} > R^*_{\mathrm{ROA}}$ in Jan 1998, and Handysize vessels show $R_0 \geq R^*_{\mathrm{ROA}} > R^*_{\mathrm{NPV}}$ in Aug 2004. These two cases suggest that investing immediately is a better decision. On the other hand, Capesize appears as $R_0 < R^*_{\mathrm{NPV}} < R^*_{\mathrm{ROA}}$ in Aug 2004, while Panamax shows $R_0 < R^*_{\mathrm{ROA}} < R^*_{\mathrm{NPV}}$ in Jun 1998. Therefore, it is better to invest later. For Panamax in Aug 2004, $R^*_{\mathrm{NPV}} < R_0 < R^*_{\mathrm{ROA}}$, so it is better to invest later. For Handysize in May 1998, $R^*_{\mathrm{ROA}} < R_0 < R^*_{\mathrm{NPV}}$. In this case, although the project value for immediate investment is larger than for investment later, the investment is not recommended. The empirical results here are in line with our theoretical results. When NPV>0, the project may not pass the ROA criterion. On the other hand, when NPV<0, it is a sufficient condition for not investing.

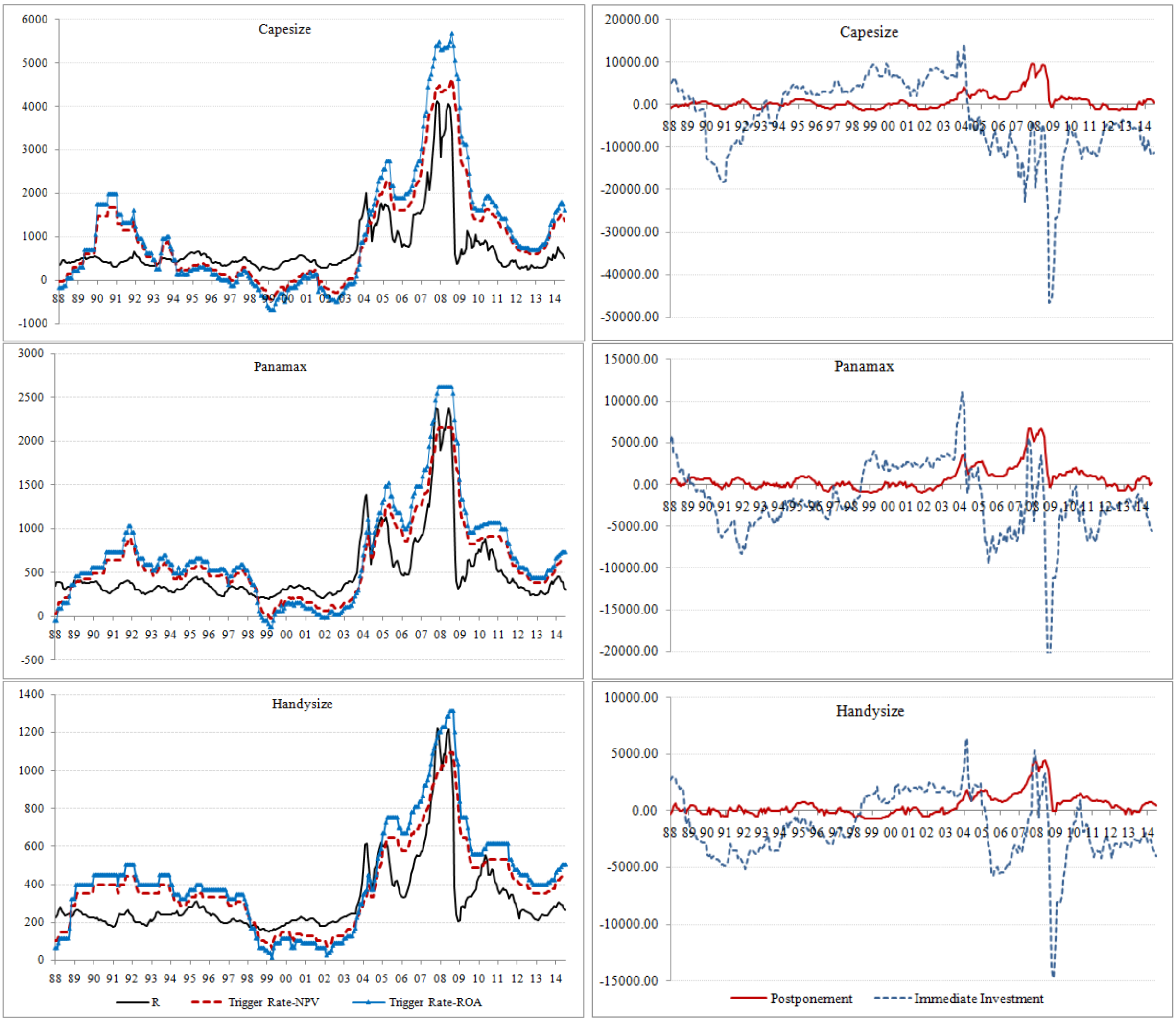
Figure 4-b plots the project values from immediate investment (dotted line) and investing later (solid line). Comparing them with the results of the trigger rates in Figure 4-a, it can be seen that when $R_0 > R^*_{\mathrm{ROA}}$, the revenues from immediate investment are higher than the revenues from postponement; when $R_0 < R^*_{\mathrm{NPV}}$, the revenues from postponement are larger. Note that since the calculation is based on the whole sample, the figures may not reflect the exact ship investment behavior in the past. Investment decisions during different shipping cycles will be provided in Section 4.2.

4.1.2. Sensitivity analysis of basic results

Since the above analysis is calculated using the median for the estimated parameters, a sensitivity analysis on the parameter values will be provided here, to help investors anticipate the possible changes in trigger rates. From Table 2, the range of parameters are: $u \in [0.006, 0.032]$, $m \in [200, 500]$, $\sigma \in [5, 150]$, $r \in [0.006, 0.0128]$, $\theta \in [12, 28]$ and $N \in [18, 35]$. Panamax vessels are used as an example, and the base ship price is assumed to be the same as Aug 2004 (i.e. $P_{\mathrm{pan}}=33000$). The results of the sensitivity analysis are plotted in Figure 5. In order to compare the sensitivity, the ranges of the left vertical axis are all in 0-1800, and the ranges of the right vertical axis are in 0-140 when the changing rate is positive and in -100-0 when the changing rate is negative.

As an indicator for market volatility, the reverting speed u has a positive effect on R^*_{ROA}, indicating that it is better to invest later, when the market is more volatile. For every 0.001 increase in the range of [0.006, 0.032], the increment of trigger rate speeds up, and the growth rate is around 3-4%.

As a market indicator, a larger m indicates a better market. R^*_{ROA} decreases with m, implying that when the market becomes better, it is better to invest now. Increasing m by every 10 units (000$/month), R^*_{ROA} decreases by around 40-50 units, except when m is around 410-430. It can be seen that this is the point where R^*_{ROA} becomes lower than R^*_{NPV}. When the market is good enough, investing immediately is always better.



(a) Trigger rates (b) Project values

Figure 4: Dynamic trigger rates and project values from 1988 to 2014

Another indicator for volatility, R^*_{ROA}, increases with σ. When the market risk increases, investment should be postponed unless the true real charter rate is very high. It seems that R^*_{ROA} is not very sensitive to a change of σ. By increasing its value by 5, the changing rate of R^*_{ROA} is less than 15 units.

The discount rate r has a positive effect on R^*_{ROA}, as a high discount rate can reduce the present value of return on investment, and therefore more investment will be postponed. In general, the increment of R^*_{ROA} is around 50 units by a 0.0002 increase in r. However, the changing rate exhibits very sensitive changes when r is in the range of 0.007 to 0.0074, which is also the point where R^*_{ROA} increases to higher than R^*_{NPV}. When the discount rate is low enough, investing immediately is always better.

The construction lag θ has a positive effect on R^*_{ROA}, indicating that immediate investment is preferred when the new-building lag is shorter. The changing rate of R^*_{ROA} increases with θ, implying that R^*_{ROA} increases faster when the construction time of a new-building ship is longer. The growth rate of the trigger rate is close to constant, which is around 3-4%.

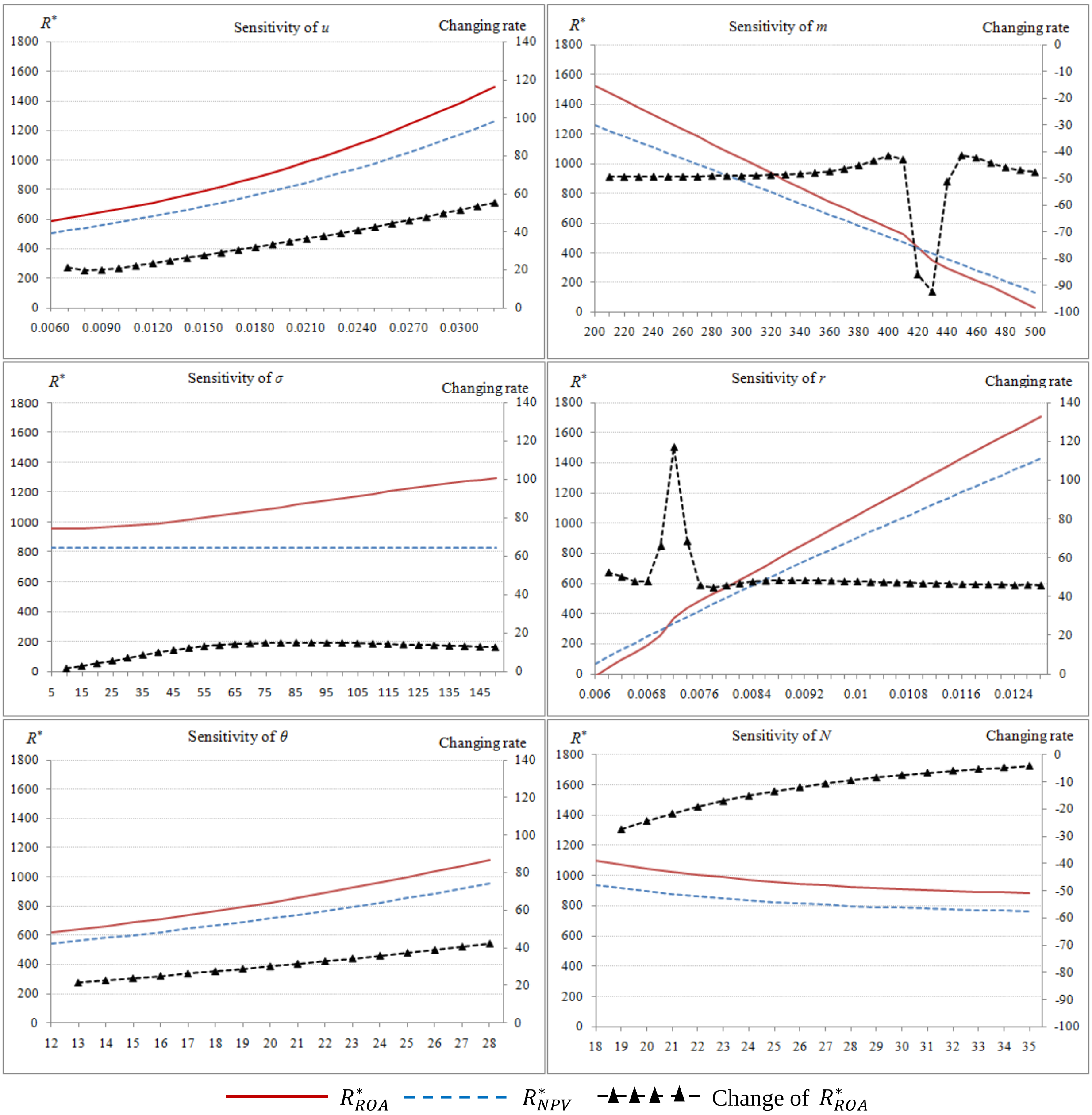


Figure 5: Sensitivity analysis of base parameters in Table 3 for Panamax carrier

The lifespan N has a negative impact on R^*_{ROA}. The longer the ship can sail, the more likely immediate investment is recommended. This result is reasonable, because a ship that has a longer lifespan has a longer service time to earn profit. This shows that the shorter the lifespan of a ship, the greater the decrement of the trigger rate.

In summary, parameters such as reverting speed, market volatility, discounted rate and construction lag have a positive effect on the trigger rate, while the long-term mean of the freight rate and the ship's lifespan have a negative impact. Among all the parameters, the trigger rate is more sensitive to changes in the long-term mean m and the discount rate r. Special attention is required when a change in m and r causes a change in the relative positions of R^*_{ROA} and R^*_{NPV}, because this will lead to systematic changes in investment strategy.

4.2. Trigger rates with structural changes

According to the ADF test in Table 1, R follows the OU process during the period 1976-2014. This contradicts most of the empirical evidence in the past (Veenstra & Franses, 1997; Kavussanos & Alizadeh, 2002b; Alizadeh & Nomikos, 2007). The main reason for this could be the different sample periods in different studies. The freight rate appears to have different properties in different periods separated by break points. This subsection presents the estimated trigger rates, taking into account the structural changes.

Kou and Luo (2015a) identified six breakpoints for Panamax and Handysize ships, and four breakpoints for Capesize vessels, over the period 1976.01-2012.07. Since we treat the pre-1988 data as the base sample in this paper (subsection 4.1.1), we only consider the shipping cycles after Jan 1988. Although the sample is extended to Jul 2014 in this study, the sub-sample size from the financial crisis in 2008 to Jul 2014 is small and no dramatic changes occurred. Therefore, we extend the last sub-period to Jul 2014 directly. Table 4 summarizes the duration of each sub-period.

Table 4: Duration of sub-period for the three ship types

Sub-periods	I	II	III	IV
Capesize	1988.01~2003.08	2003.09~2007.03	2007.04~2008.09	2008.10~2014.07
Panamax	1988.01~2002.03	2002.04~2006.12	2007.01~2008.08	2008.09~2014.07
Handysize	1988.01~2001.07	2001.08~2006.12	2007.01~2008.09	2008.10~2014.07

For each sub-period, the ADF test results are shown in Table 5. The freight rates in sub-periods I and IV can generally be described as the OU process, but not those in sub-periods II and III. For the latter two sub-periods, it is not appropriate to use Eqs. (11) and (12) to estimate the parameters for R_t, so a different method is needed for them.

Next we explain the method for estimating the parameters for R in these periods. First, for each sub-period, the changing mean m_t is generated following Kou and Luo (2015a), which allows the charter rate R_t to fluctuate around m_t:

$$R_t=m_t+\varepsilon_t \tag{14}$$

where $m_t=\alpha+\beta t$ in each sub-period. The period mean, named as m^p (p=I, II, III, IV), is then calculated according to

$$m^p=\frac{1}{n^p}\sum_{i=1}^{n^p} m_t, \tag{15}$$

where n^p is the number of observations in the corresponding sub-period. Since m_t is a liner series in each sub-period, m^p is also equal to the median value of m_t.

The period discount rate r^p is estimated using Eq. (13). Based on $\widehat{K_\rho}$ from Eq. (13), parameter ρ^p is obtained. Then the period reverting speed, u^p is generated. The period volatility σ^p is obtained using Eq. (12) with the new period u^p:

Table 5: ADF test results of the freight rate process in each sub-period

	Sub-periods	Lags	ADF t-Statistic (SIC)	(p-value)	1% level	5% level
			Capesize			
I	1988.01-2003.08	1	-2.6764	(0.0800*)	-3.4652	-2.8768
II	2003.09-2007.03	1	-2.5445	(0.1124)	-3.5925	-2.9314
III	2007.04-2008.09	0	-2.0049	(0.2822)	-3.8574	-3.0404
IV	2008.10-2014.07	1	-8.1024	(0.0000***)	-3.5270	-2.9036
			Panamax			
I	1988.01-2002.03	2	-3.5570	(0.0076***)	-3.4687	-2.8783
II	2002.04-2006.12	2	-1.7822	(0.3855)	-3.5504	-2.9135
III	2007.01-2008.08	1	-1.8874	(0.3309)	-3.8085	-3.0207
IV	2008.09-2014.07	1	-6.9637	(0.0000***)	-3.5256	-2.9030
			Handysize			
I	1988.01-2001.07	1	-2.6974	(0.0766*)	-3.4707	-2.8792
II	2001.08-2006.12	1	-1.9169	(0.3227)	-3.5349	-2.9069
III	2007.01-2008.09	1	-1.8398	(0.3522)	-3.7880	-3.0124
IV	2008.10-2014.07	1	-6.1054	(0.0000***)	-3.5270	-2.9036

Notes: * denotes rejection of the null hypothesis at the 10% significance level, while *** denotes rejection of the null hypothesis at the 1% significance level. Maximum lag is set to be 4, and lag length is automatically selected by Schwarz Information Criterion (SIC).

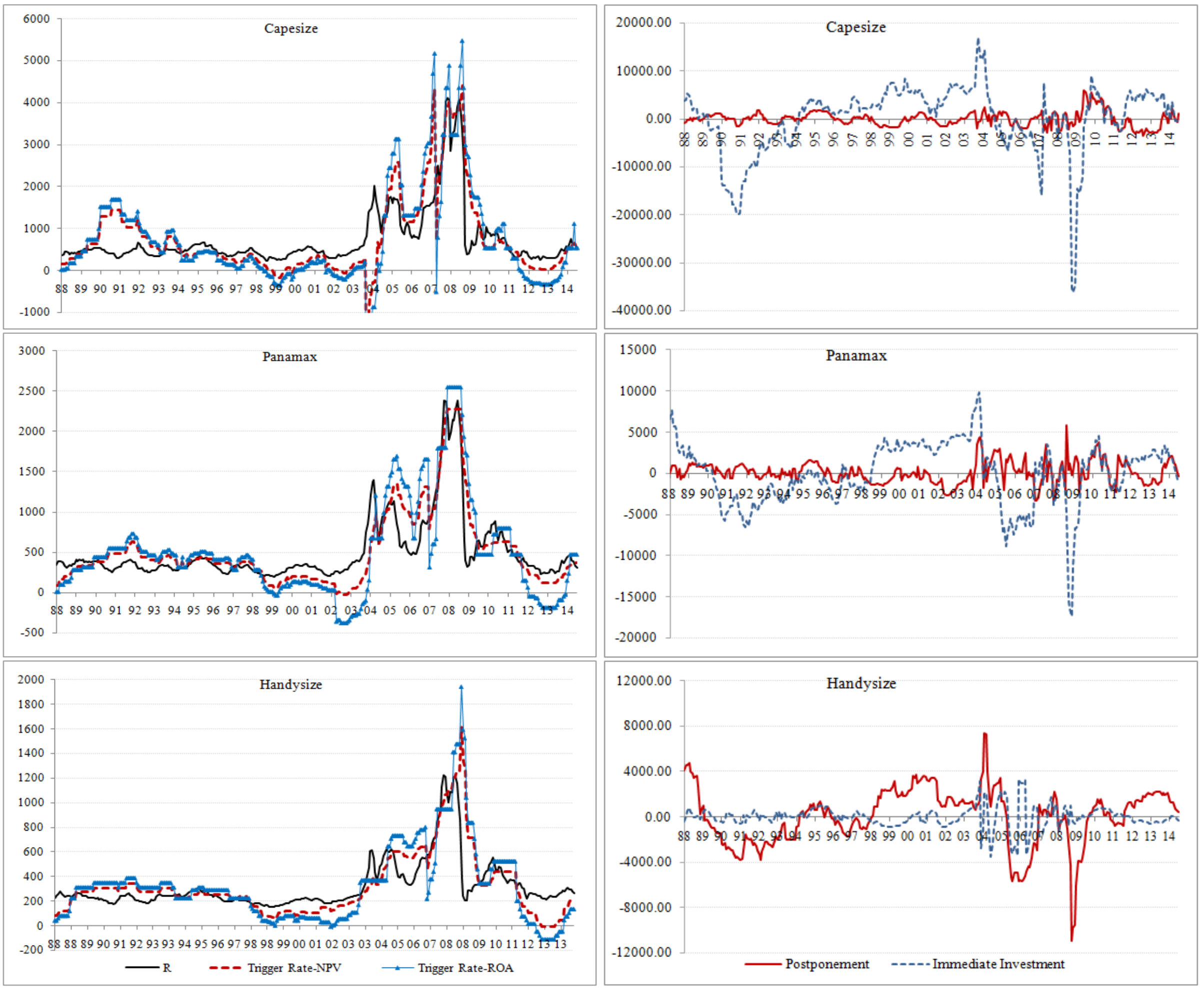
$$\sigma^p = S\sqrt{\frac{-2u^p}{e^{-2u^p\Delta t}-1}}. \tag{16}$$

The estimated parameters in all sub-periods are shown in Table 6. It can be seen that the parameters in period I are close to our base setting in Table 3, while the parameters in periods II, III and IV are generally higher than the base parameters.

Table 6: Parameters in each sub-period

Sub-periods		m^p	r^p	u^p	σ^p
			Capesize		
I	1988.01-2003.08	438.37	0.0077	0.0211	31.6918
II	2003.09-2007.03	1316.75	0.0151	0.0339	198.3069
III	2007.04-2008.09	3236.61	0.0210	0.0326	475.4158
IV	2008.10-2014.07	541.2823	0.0072	0.0200	97.8604
			Panamax		
I	1988.01-2002.03	316.1546	0.0092	0.0134	17.4441
II	2002.04-2006.12	673.2489	0.0150	0.0174	106.2119
III	2007.01~2008.08	1796.310	0.0214	0.0175	166.7076
IV	2008.09-2014.07	472.0027	0.0110	0.0182	98.0540
			Handysize		
I	1988.01~2001.07	221.1213	0.0090	0.0119	8.9463
II	2001.08~2006.12	367.6805	0.0130	0.0093	37.6735
III	2007.01~2008.09	945.5200	0.0183	0.0191	86.0221
IV	2008.10~2014.07	320.4099	0.0104	0.0310	29.3152

The dynamic trigger rates are then generated based on the parameters in Table 6, which are plotted in Figure 6 together with the project values for investing now or later. It can be seen that results when considering structural changes are different to results from the whole sample (Figure 4). In particular, trigger rates during sub-period II and III exhibit extremely dramatic changes, and trigger rates during sub-period IV are much lower than the results from the whole sample. A comparison between the result from the whole sample and that with structural breaks is provided in next section.



(a) Dynamic trigger rates (b) Project Values

Figure 6: Dynamic trigger rates and projected values with structural breaks

4.3. Comparing results from the whole sample with those from sub-samples

This section compares the "immediate investment" suggested by the whole sample with that of sub-periods, using Capesize as an example. As shown in Figure 7, the vertical lines indicate that immediate investment is recommended, while white space indicates immediate investment is not recommended. Clearly, the main differences between the results from the whole sample and from sub-periods are in sub-period III and IV. Compared with those of the whole sample, the results from sub-periods III and IV encourage immediate investment. It indicates the R^*_{ROA} from the sub-periods decreases in these sub-periods. It is known from Table 6 that four parameters in these sub-periods have been changed in comparison to the base settings in the whole sample, i.e. m, u, σ, r. We concluded in Section 4.1 that, except for m, all the other parameters (i.e. u, σ, r) have positive effects on the trigger rate, and that among them, m and r are more sensitive than the others. For sub-period III, the levels of m increase much faster than the other parameters for all three ship types. It is not surprising that R^*_{ROA} decreases in this sub-period. For sub-period IV, different ship types show different changes of the parameters. For the Capesize carrier, the increase of m and the decrease of r and u have a negative impact on the trigger rates, resulting in significant decreases.

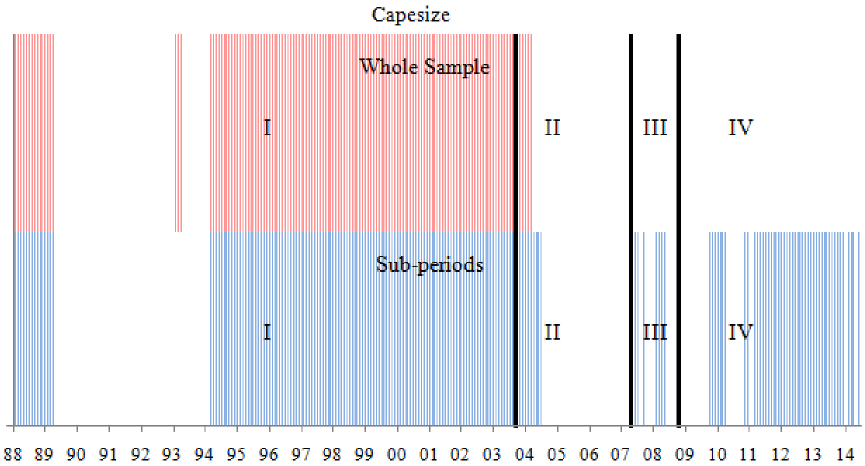


Figure 7: Immediate investment suggested by the whole sample and by sub-periods

4.4. Statistical test of the model

Finally, we carry out a simple empirical test to verify whether the actual investment behavior in the past confirms our results. We use the total deadweight ton of new orders at month t as the dependent variable $Order_t$. The charter rate R_t, the new-building ship price P_t and the estimated trigger rate R^*_t are used as explanatory variables. Regression results are summarized in Table 7. It shows that the trigger rate R^*_t generally has a significantly negative impact on the new orders, especially when in sub-periods. New ordering is more motivated by a favorable freight market, since the time charter rate has the most significant impact on $Order_t$. The sub-periods result shows that new ship ordering is encouraged when the ship price is high, which indicates that ship price and freight rate have a close connection, and that a low investment cost is not the main concern for ship investment. Instead, the freight market plays a more important role on the investment decision. From the result of R^2, it is clear that results that take account of shipping cycles are closer to actual investment behavior in the past. It is worth reiterating that our model analyzes market-driven ship investment. Other investment behaviors, such as speculations and strategic behavior in ship investment, are not modeled in this research. Therefore, the R^2 in our model is less than 50%.

4.5. Investment forecasting

This section demonstrates the use of this model to predict ship investment behavior over the coming two years (2014.08-2016.07). The first step is to forecast future movements of the freight rate process. Following Kou and Luo (2015a), the changing means m_t can be viewed as the long-run trend of the freight rate movement. Extending m_t in the last sub-period to 2016.07, we can get the new period mean m^p using Eq. (15). From Section 4.2, a new set of period parameters can be generated. Results are summarized in Table 8. Based on period parameters u^p, m^p and σ^p in Table 8, the freight rate can be estimated using Eq. (2).

Table 7: Regression of factors impacting on new-ship contracts

Regression model: $Order_t = \varphi_0 + \varphi_1 R_t + \varphi_2 P_t + \varphi_3 R^*_t + \varepsilon_t$

	φ_0	φ_1	φ_2	φ_3	R^2
			Capesize		
Whole sample	39169449	1301.4***	-995.6**	10792.3**	0.4997
Sub-period	-2397458	1279.8***	63***	-432.8***	0.5081
			Panamax		
Whole sample	2018762	1024.8***	-83.8	1215.8	0.2562
Sub-period	-1708211	1055.0***	87.3***	-1081.7***	0.3248
			Handysize		
Whole sample	798790	899.5***	-64.6	1513.8	0.4132
Sub-period	-925927	840.7***	60.5***	-663.7***	0.4788

Notes: ** and *** denote rejection of the null hypothesis at the 5% and 1% significance levels respectively.

Table 8: New parameters in the forecasting period

Time period	m^p	K_p	ρ	K_r	r^p	u^p	σ^p
			Capesize				
2008.10-2016.07	472.3467	19.1262	0.0272	115.9396	0.0063	0.0209	97.4196
			Panamax				
2008.09-2016.07	397.0564	17.0178	0.0292	77.0350	0.0097	0.0195	98.1173
			Handysize				
2008.10~2016.07	296.0594	8.9215	0.0414	76.5914	0.0098	0.0316	29.3239

The second step is to forecast the ship prices. Based on the freight rate values obtained from the first step, using the ship price-freight rate theoretical relationship in Eq. (13), ship prices in the future can be estimated.

Finally, using the parameters obtained above, the trigger rates in ROA can be calculated using Eq. (A.18) in the Appendix. Figure 8 plots the extended m_t, estimated future freight rates, and the trigger rates from ROA, together with the real contracting number of new ships up to Feb 2015. Forecasting results show that the freight market will remain stagnant during the coming two years, especially for the Capesize carriers. For Panamax and Handysize carriers, the recovery may start in 2016. For ship investment, although the predicted freight rates are sometimes higher than the trigger rates, the project values from immediate investment are all negative for both Capesize and Handysize carriers. Therefore, ship investment in these two sectors is not recommended. For Panamax carriers, immediate investment is mainly recommended in the second half of this year. From actual ship investments during the period 2014.08-2015.02, we can see that new orders have decreased, although they still exist, no matter how big the market overcapacity is. This may be explained by strategic behavior in capacity investment in a competitive market where the market share is represented by a firm's capacity (Kou & Luo, 2015b).

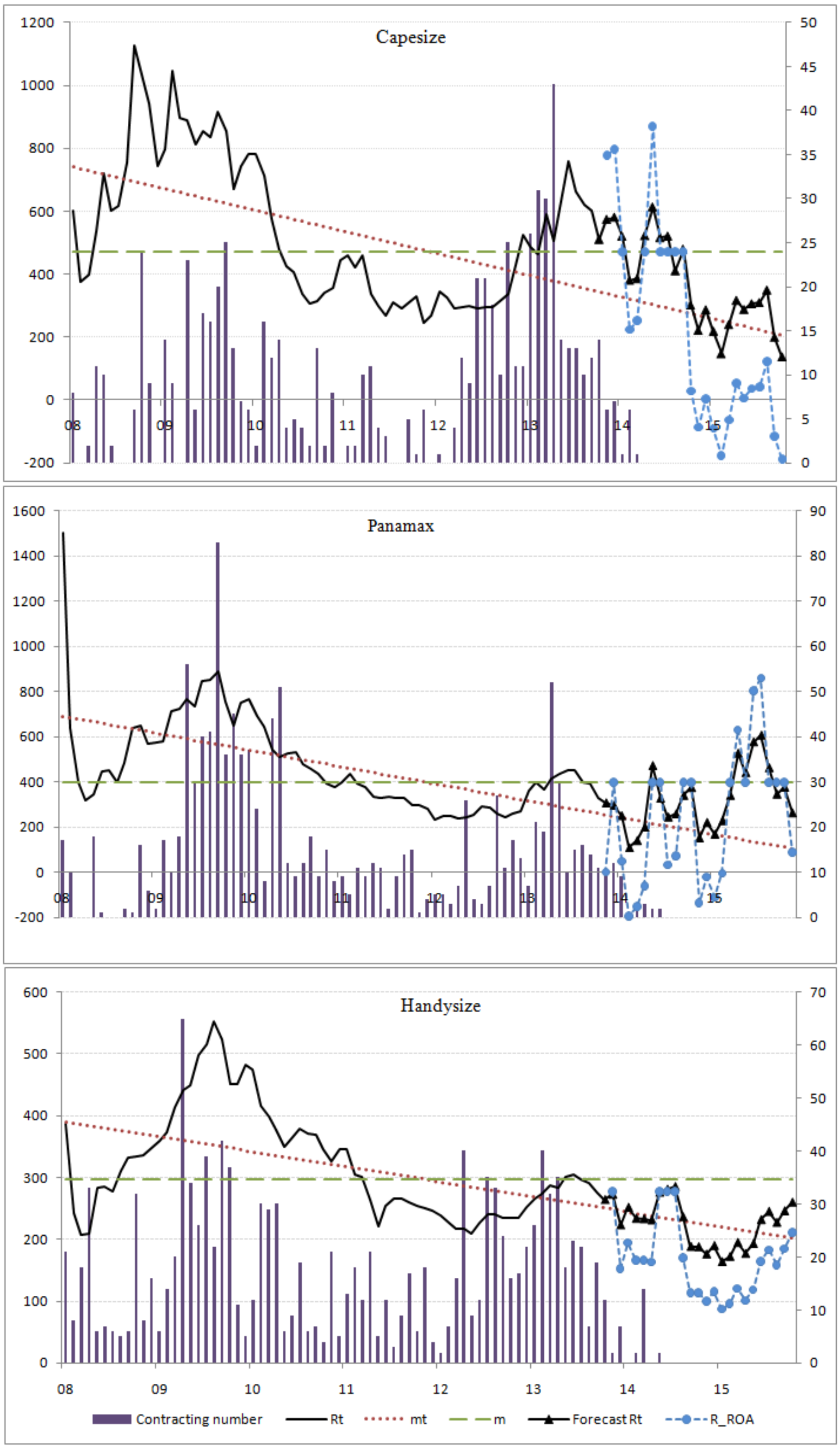


Figure 8: Freight rate and trigger rate forecasting (2014.08-2016.07)

In summary, the forecasting results show that prospects for the shipping market are not optimistic, especially for the year 2015. New ordering of ships could delay market recovery. The market rebound will not appear in the near future. Thus, small shipping companies need to be cautious when considering following the ship investment decisions of the large companies.

5. Conclusions

This paper theoretically develops market driven ship investment rules taking into account uncertainties in the cyclical shipping market using ROA. A stochastic process—the OU process—was applied to model the freight rate movement. Based on this assumption, the project values for investing now and later are defined, and the theoretical trigger rates at which investors can invest immediately are formulated.

Empirically, the OU process is accepted for the whole data sample. However, in sub-periods separated by structural breaks, the OU process is only accepted in part of the sub-periods. Parameters that determine the freight rate process are estimated dynamically, which can reflect general trends in the dynamic process. There are, however, some abnormalities in these parameters, which indicate sudden structural changes in the period studied. Sensitivity analysis finds that the mean reverting speed, market uncertainty, discount rate and construction time have a positive impact on the trigger rate, i.e., making immediate investment less likely; while the long-term mean level and the length of a ship's lifespan have a negative impact on the trigger rate, i.e., making it easier to invest. Among all the factors considered, the long-term mean level is the most significant, followed by the discount rate. Compared with trigger rates generated from the whole sample, trigger rates that have taken into account structural changes have made immediate ship investment easier over recent years. It is also found that results from sub-periods are closer to the observed new-ship contracts.

In summary, this paper provides for some flexible thinking on ship investment decisions. Unlike the traditional NPV rule, our method incorporates the cyclical nature and uncertainties of future shipping market conditions, and thus produces more accurate results. This study contributes to the application of the ROA in shipping economics, and makes the first attempt at suggesting clear rules to determine the best time to invest in new ships. The proposed fundamental option model has other applications too, such as when making decisions on buying second-hand ships, as well as on laying-up or scrapping ships. In practical terms, this study generates simple criteria for ship investment based on the current market conditions, which can help those in the industry when making ship investment decisions. Also, our prediction reveals that market conditions over the coming two years will not be sufficient to support many new orders.

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Appendix A: Trigger rate under the real option rule

To derive the trigger rate from ROA, R^{*}_{ROA}, from the analysis in Section 3, suppose the NPV from immediate investment equals to the NPV from postponement:

$$F_0(R_t) = e^{-rdt}E_0[F_{dt}(R_t)]. \quad \text{(A. 1)}$$

Since $(1+x)^{\frac{1}{x}} \approx e$ when $x \to 0$, assume that $x=rdt$. With $dt \to 0$, we can get:

$$e^{-rdt} \approx \frac{1}{1+rdt}. \quad \text{(A. 2)}$$

Substituting Eq. (A. 2) into Eq. (A. 1), we have:

$$F_0(R_t) = \frac{1}{1+rdt}E_0[F_{dt}(R_t)]. \quad \text{(A. 3)}$$

Eq. (A. 3) can be simplified as:

$$rF_0(R_t)dt = E_0[dF(R_t)] \quad \text{(A. 4)}$$

where $dF(R_t)=F_{dt}(R_t)-F_0(R_t)$.

Using Ito's Lemma to expand $dF(R_t)$ in Eq. (A. 4) and ignore the terms of order higher than two in dt, we can get:

$$dF(R_t) = F_R dR + \frac{1}{2}F_{RR}(dR)^2 \quad \text{(A. 5)}$$

where $F_R=dF/dR$, $F_{RR}=d^2F/dR^2$. Since dR follows the OU process, substitute Eq. (2) into Eq. (A. 5):

$$dF(R_t) = \left(u(m-R)F_R + \frac{1}{2}\sigma^2 F_{RR}\right)dt + \sigma F_R dz. \quad \text{(A. 6)}$$

Substituting Eq. (A. 4) into Eq. (A. 6), we can get:

$$\frac{1}{2}\sigma^2 F_{RR} + u(m-R)F_R - rF = 0. \quad \text{(A. 7)}$$

To transfer Eq. (A. 7) into a Kummer equation, following the method by Sødal *et al*. (2008), we define a variable $y = \frac{u(m-R)^2}{\sigma^2}$. Then,

$$y_R = -\frac{2u(m-R)}{\sigma^2} \quad \text{(A. 8a)}$$

$$y_{RR} = \frac{2u}{\sigma^2} \quad \text{(A. 8b)}$$

where $y_R=dy/dR$, $y_{RR}=d^2y/dR^2$.

Let $F(R)=f(y)$. It has:

$$F_R = f_y y_R = -\frac{2u(m-R)}{\sigma^2}f_y \quad \text{(A. 9a)}$$

$$F_{RR} = \frac{4u}{\sigma^2}yf_{yy} + \frac{2u}{\sigma^2}f_y \quad \text{(A. 9b)}$$

where $f_y=df/dy$, $f_{yy}=d^2f/dy^2$.

Insert Eq. (A. 9) into Eq. (A. 7) and then divide $2u$ on both sides:

$$y f_{yy} + (b-y) f_y - \gamma f = 0 \quad \text{(A. 10)}$$

where b=1/2 and γ=r/2μ. Eq. (A. 10) is the Kummer equation whose solution is:

$$f(y) = A_0 H(\gamma, b, y) + B_0 y^{1-b} H(\gamma - b + 1, 2 - b, y) \quad \text{(A. 11)}$$

where A_0 and B_0 are constants, and $H(\cdot)$ is the confluent hypergeometric function or Kummer function, given by the following series representation (Slater, 1960):

$$H(\gamma, b, y) = 1 + \frac{\gamma}{b} y + \frac{\gamma(\gamma+1)}{b(b+1)} y^2 + \frac{\gamma(\gamma+1)(\gamma+2)}{b(b+1)(b+2)} y^3 + \cdots$$
$$\lim_{y \to \infty} H(\gamma, b, y) = \frac{\Gamma(b)}{\Gamma(\gamma)} e^y y^{\gamma - b} \quad \text{(A. 12)}$$

where $\Gamma(\cdot)$ is the Gamma function.

The two constants must be related in a way which forces $f \to 0$ as $R \to -\infty$, implying

$$B_0 = -K A_0 \quad \text{(A. 13)}$$

where $K = \frac{\Gamma(b)\Gamma(\gamma-b+1)}{\Gamma(2-b)\Gamma(\gamma)}$.

Then Eq. (A. 11) can be simplified as:

$$F(R) = f(y) = A_0 (H_1 - K y^{1-b} H_2) \quad \text{(A. 14)}$$

where $H_1 = H(\gamma, b, y)$ and $H_2 = H(\gamma - b + 1, 2 - b, y)$. Let $X(R) = H_1 - K y^{1-b} H_2$, then we have:

$$X(R) = H\left(\frac{r}{2u}, \frac{1}{2}, \frac{u(m-R)^2}{\sigma^2}\right) - \frac{\Gamma(b)\Gamma(\gamma-b+1)}{\Gamma(2-b)\Gamma(\gamma)} \frac{u^{1-b}(m-R)^{2-2b}}{\sigma^{2-2b}} \left(\frac{r}{2u} + \frac{1}{2}, \frac{3}{2}, \frac{u(m-R)^2}{\sigma^2}\right) \quad \text{(A. 15)}$$

The boundary condition for the trigger rate is that investment will be made as soon as the charter rate reaches the trigger rate R^*_{ROA}:

$$A_0 X(R^*_{\mathrm{ROA}}) = R^*_{\mathrm{ROA}} K_\rho + m(K_r - K_\rho) - P_0. \quad \text{(A. 16)}$$

Another boundary condition is called the first-order "smooth pasting" condition of Eq. (A. 16):

$$A_0 X'(R^*_{\mathrm{ROA}}) = K_\rho. \quad \text{(A. 17)}$$

Eqs. (A. 16) and (A. 17) must be solved simultaneously for the two unknowns A_0 and R^*_{ROA}. Eliminating A_0, the trigger rate R^*_{ROA} satisfies:

$$K_\rho X(R^*_{\mathrm{ROA}}) - [R^*_{\mathrm{ROA}} K_\rho + m(K_r - K_\rho) - P_0] X'(R^*_{\mathrm{ROA}}) = 0. \quad \text{(A. 18)}$$

Eq. (A. 18) is a complicated non-linear equation whose closed-form solution is not available. Thus, R^*_{ROA} can only be found numerically.

Using Eq. (A. 17) and (A. 14), the net present value from immediate ordering $F_0(R_t)$ and from postponement $w_0(R_t)$ can be derived:

$$\begin{cases} F_0(R_t) = R_0 K_\rho + m(K_r - K_\rho) - P_0 & R_0 \geq R^*_{\mathrm{ROA}} \text{ (immediate invest} \\ w_0(R_t) = \dfrac{K_\rho X(R_0)}{X'(R_0)} & R_0 < R^*_{\mathrm{ROA}} \text{ (postponed invest} \end{cases} \quad \text{(A. 19)}$$