JOINT DEPLOYMENT OF QUAY CRANES AND YARD CRANES IN CONTAINER TERMINALS AT A TACTICAL LEVEL

Lingxiao Wu
Research Scholar
Department of Logistics and Maritime Studies,
The Hong Kong Polytechnic University,
Hung Hom, Hong Kong.
Tel: +852-5980 4665 Email: lingxiaowu513@gmail.com

Shuaian Wang
Associate Professor
The Hong Kong Polytechnic University Shenzhen Research Institute,
Nanshan District, Shenzhen, China, 518057;
and Department of Logistics and Maritime Studies,
The Hong Kong Polytechnic University,
Hung Hom, Hong Kong.
Tel: +852-2766 7419 Email: wangshuaian@gmail.com

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Corresponding author: Shuaian Wang

ABSTRACT

This paper discusses tactical joint quay crane (QC) and yard crane (YC) deployment in container terminals. The deployments of quay cranes and yard cranes are critical for the efficiency of container terminals. Although they are closely intertwined, the deployments of QCs and YCs are usually sequential. This paper proposes a mixed-integer programming model for the joint deployment of QCs and YCs in container terminals. The objective of the model is to minimize the weighted vessel turnaround time and the weighted delayed workload for external truck (EX) service in yard blocks, both of great importance for a container terminal but rarely considered together in the literature. This paper proves that the studied problem is NP-hard in the strong sense. Case studies demonstrate that the proposed model can obtain better solutions than the sequential method. This paper also investigates the most effective combinations of QCs and YCs for a container terminal at various demand levels.

Keywords: Container Terminals, Quay Crane, Yard Crane, Joint Deployment, Tactical Level Port Planning

INTRODUCTION

Containerized maritime transportation has grown steadily in the past few decades. According to the UN Conference on Trade and Development (UNCTAD) (*1*), the total containerized trade in 2015 was estimated to be 175 million 20ft equivalent units (TEUs). Container terminals are the heart of international container transportation, providing the linkage between inland and seaborne transportation. With the annual volume of containerized trade growing rapidly, container terminals are confronting an immense increase in service demand. Besides, competition among container terminals, especially geographically close ones, has become fiercer than ever. To deal with the increased demand and win competitive advantages, port managers are working to improve terminal efficiency.

The efficiency of container terminals is largely determined by the use of port resources. The most important equipment used in port operations includes quay cranes (QCs) and yard cranes (YCs), which are both gantry cranes. QCs are located on the quay side and are used to load and discharge containers into and from vessels. YCs are used on the yard side to stack and retrieve containers stored in the blocks. Both the QCs and the YCs play very important roles in container terminals, and more often than not they are bottlenecks in container handling. In addition, because both QCs and YCs are very expensive and require high maintenance costs, it is impossible to get all the berths and blocks fully equipped with QCs and YCs. By fully equipped, we mean a berth has the maximum number of QCs that can work simultaneously for a vessel, or a block has the largest allowable number of YCs. Moreover, the workload in berths and blocks varies from time to time. To make better use of limited resources and to ensure the smooth flow of containers, container terminals should deploy QCs and YCs in an efficient way.

The QC deployment problem (QCDP) and the YC deployment problem (YCDP) are intertwined. On the one hand, the efficiency of QCs directly determines the workload of YCs in each block in the yard. On the other hand, the work of YCs in the yard has a significant impact on the speed of ship handling in berths. To make overall improvements in terminal efficiency, port managers should work out an integrated scheme that handles the QCDP and the YCDP simultaneously. Failure to handle either of these two problems efficiently may result in (i) congestion in the yard, (ii) idleness of QCs or YCs, and (iii) delays of vessels and external container trucks.

Terminal operation planning can be divided into operational and tactical levels (*2*). Tactical planning of container terminals considers the periodic calling of liner vessels. For terminal operators, tactical planning works as the foundation for operational planning. This paper studies the tactical joint QC and YC deployment problem (TJCDP). Studies in the literature have focused on the tactical planning of container terminals, with most focused on the tactical berth allocation problem (TBAP) (*3*), the tactical yard template problem (TYTP) (*4*), or the integration of the two (*5-6*). These studies are closely related to our considered problem. While the TBAP and the TYAP provide input data for the TJCDP, the TJCDP gives an implemental guarantee for the two problems.

To address the needs of container terminals, the TJCDP is considered in our paper, whose contribution is threefold. First, we propose a mixed-integer programming model for the TJCDP and we further linearize the model. Second, we prove that the considered problem is NP-hard in the strong sense. Third, a series of application cases are solved by the model and discussed in detail, and the results demonstrate the validity of our model.

Literature Review

Efforts have been devoted to operational problems in container terminals. Most discussed are (i) the berth allocation problem (BAP) (*7-8*), (ii) the QC assignment problem (QCAP) and the QC scheduling problem (QCSP) (*9-10*), (iii) the yard template problem (YTP) (*4*), and (iv) the YC deployment problem (YCDP) and the YC scheduling problem (YCSP) (*11-12*). Our research can be seen as an extension of these studies, especially the studies on QCAP and YCDP.

The QCAP usually has been considered together with the BAP. Meisel and Bierwirth (*13*) were among the first to integrate the BAP and the QCAP. They developed a heuristic algorithm to solve the integrated problem. Blazewicz et al. (*14*) formulated an integrated problem of berth allocation and QC assignment as a moldable task scheduling problem and solved the problem by a heuristic algorithm. Some studies have considered the QCAP and the QCSP simultaneously. Daganzo (*9*) studied the QC assignment and scheduling problems for multiple container ships with the objective of minimizing the delay of vessels. An exact method and a heuristic method were developed to solve small-size instances and large ones, respectively.

Studies that focused on the YCDP have included Zhang et al. (*11*) who studied a dynamic YCDP, aiming to minimize the delay of the workload in all blocks. They proposed a mixed-integer programming model for the problem, and the model was then solved by Lagrangian relaxation. Cheung et al. (*15*) proposed a mixed-integer programming model for the YCDP. A Lagrangian decomposition solution procedure and a successive piecewise-linear approximation method were developed to solve instances with various sizes. The YCDP was investigated by Linn and Zhang (*16*) who proposed a heuristic method that they proved could deliver near-optimal solutions. Jin et al. (*17*) integrated the space allocation problem and the YCDP. They proposed an integer programming model with the objective of minimizing the YC operating cost and the YC interblock movement cost. An efficient heuristic algorithm was developed to solve the problem.

Although the QCAP and the YCDP have received great attention in the literature for more than 20 years, to the best of our knowledge there have been no studies considering the joint deployment problem of QCs and YCs.

PROBLEM DESCRIPTION

A typical container terminal, as shown in Figure 1, is composed of two parts—the quay and the yard. The quay of a container terminal is divided into independent segments called berths where containers are loaded onto or discharged from vessels. The yard of a container terminal is an area reserved for storing containers. A large-scale yard may be divided into blocks, which are separated rectangular places where containers are stacked side by side and one on top of another.

Three types of containers are used in a container terminal. An inbound container is first discharged from a vessel by a QC, then transported by an internal truck (IT) from the berth to the designated block, then stacked by a YC into the block and stored in the block until it is picked up by a YC to load onto an external truck (XT) that carries the container to an inland consignee. An outbound container undergoes a reverse flow. An XT first brings a container into the designated block. A YC then discharges the container from the XT and places it into the block. When the vessel for transporting the container arrives, the container is carried to the relevant berth by an IT and then loaded by a QC onto the vessel. The third kind of container is the transshipment container, which is transshipped from one vessel to another. After a vessel calls at the berth, a transshipment container is first discharged and then transported to its designated block, using the same procedure as with an inbound container. When the vessel for transshipment arrives, the container is then transported from the block to the relevant berth and loaded onto the vessel, using the same procedure as with an outbound container.

In terminal operation planning, a working day is normally divided into several 4- or 6-hour work shifts. At the tactical level, operation plans are normally made by taking a work shift as a planning unit.

<Insert Figure 1 here>

Tactical Quay Crane Deployment

Vessels visit container terminals on a fixed, periodic schedule. Before handling starts, a container ship should first moor in a particular berth. In practice, port managers work out a tactical berth allocation plan to designate the berth and the time window for each vessel in a planning horizon for loading and discharging containers. Such a plan is settled according to the estimated time of arrival, the draft of the vessel, and the blocks assigned to the vessel.

It is of great importance to ensure that the handling of each vessel can be completed in time, because the delay in the handling of a vessel may not only cause extra expenses of the port (like demurrage), but also can lead to delays of subsequent vessels that call at the same berth. In addition, shorter turnaround time is always encouraged by liner companies and helps improve customer satisfaction. The workload of vessel handling is defined in QC work shifts, i.e., the number of work shifts needed by one QC to finish handling a vessel. Therefore, for a certain vessel, more assigned QCs mean less turnaround time.

It is a common rule in container terminals that QCs should remain in the same berth within one shift and can only be moved between berths between two shifts. Therefore, given the berth plan of all ships in a planning horizon, container terminals should decide how to deploy QCs dynamically to fully exploit their productivity and to ensure ships are handled within the allowable time.

Tactical Yard Crane Deployment

The yard serves as a buffer zone between sea transportation and inland transportation. A tactical yard management plan assigns for vessels in a planning horizon the blocks to store containers that are to be loaded onto the vessel and the blocks to store containers that are to be discharged from the vessel. When a vessel moors in a berth, containers to be loaded onto or discharged from the vessel are transported by ITs between the corresponding blocks and the berth. Besides, in negotiation with the liner companies or the consignees, port managers also need to work out a plan that schedules the timetable of certain XTs to arrive at designated blocks for discharging or loading containers.

YCs play a critical role in yard operations, serving both the ITs and the XTs. The workload (measured by YC work shifts) for YCs in a block within a work shift is composed of two parts: (i) the workload related to moving containers in the block that are to be loaded or discharged by QCs onto or from vessels in the work shift and (ii) the workload related to moving containers from or onto XTs in the work shift.

Most YCs are tire mounted and can move freely in the yard, and as shown in Figure 2, we have identified two types of YC movements between blocks. One is the movement between two blocks in the same row (e.g., movement from block 11 to block 12). Another is the movement between two blocks in separate rows (e.g., movement from block 11 to block 21). Because any movement of YCs can cause a loss in YC productivity and possible congestion in the yard, such movements should not happen frequently. A common rule is that intra-row movements of YCs are implemented only between work shifts; and inter-row movements, which cause more traffic congestion and productivity losses, can only be implemented once in a day (usually during the midnight work shift) (*11*). Moreover, to reduce interference between two working YCs, the number of YCs in a block is limited to no more than two at any time.

Given the projected workload in each block in each work shift, port managers should decide the number of YCs in each block in each work shift with the aim of ensuring smooth flows of ITs and XTs, which are critical for reducing vessel turnaround time and XT delays and improving customer satisfaction.

<Insert Figure 2 here>

Tactical Joint Quay Crane and Yard Crane Deployment Problem

Operations of QCs and YCs are closely interconnected. Ship handling processes involve inbound and outbound flows of containers and therefore call for cooperation between YCs and QCs. Hence, for a balanced use of the QCs and YCs and to ensure a smooth flow of containers, it is of great necessity for the port managers to deploy QCs and YCs in an integrated manner.

Considering vessels are heterogeneous in size, load capacity, and lateness, when making decisions, terminal managers assign a particular weight for each vessel. Similarly, the weight for delayed XT service is also assigned according to its relative importance.

The TJCDP in container terminals can be formally stated as follows: Given the berth allocation plan, the yard management plan, and the XT transportation plan for a planning horizon, the port managers make the following three decisions jointly:

- The number of QCs deployed in each berth in each work shift.
- The number of YCs deployed in each row of blocks in each working day.
- The number of YCs in each block in each work shift.

The considered objective is to minimize the summation of the weighed turnaround time of each vessel and the weighed delayed workload for XT service in blocks in a planning horizon.

Further, according to industrial practices, we make the following assumptions for the TJCDP:

A1. All the YCs are identical in terms of working efficiency.

A2. Movements of YCs between two rows happen at most once in the midnight shift of a working day.

A3. At most two YCs can work in parallel in a block at any time.

A4. Containers are not allowed to move among blocks.

A5. All the QCs are identical in terms of working efficiency.

A6. There may be lower and upper bounds for the number of QCs that can work simultaneously on a vessel. The minimum number of cranes to be assigned for handling a vessel can be specified in the contract between the corresponding terminal authority and shipping company. Further, for safety and efficiency considerations, the number of QCs that can work simultaneously on a vessel should be less than an upper bound.

A7. Efficiency losses of individual QCs and YCs due to crane interference are not considered. This assumption implies that the handling efficiencies of QCs and YCs working on the same vessel or block are proportional to the number of cranes assigned. This assumption may not be true in practice, but it is used to simplify the analysis.

A8. Deployments of QCs and YCs should remain unchanged within one work shift.

A9. Each berth can handle at most one vessel, and each handling vessel occupies only one berth at any time.

A10. Sufficient ITs carry containers between the berths and the blocks, and no crane idleness may be caused by delayed arrivals of ITs.

A11. Direct transshipments between two berthing ships in the terminal are not allowed.

MODEL FORMULATION

In this section, we present two mixed-integer programming models for the TJCDP. Notations used in this model are introduced by Table 1.

<Insert Table 1 here>

A Nonlinear Model

The first mathematical model (M1) for the TJCDP is presented as follows:

[**M1**]
$$\min F = \sum_{k \in V} \omega_1^k (c_k - s_k) + \omega_2 \sum_{t \in T} \sum_{j \in J} u_j^t \tag{1}$$

subject to:

$$\sum_{i \in I} x_i^t \le H, \ \forall t \in T \tag{2}$$

$$\sum_{k \in V_i^I \cap V_t^T} \alpha_k^t \le 1, \ \forall i \in I, \forall t \in T \tag{3}$$

$$x_i^t - q_k^{\min} \ge (\alpha_k^t - 1)M, \ \forall i \in I, \forall k \in V_i^I, \forall t \in T_k^V \tag{4}$$

$$x_i^t - q_k^{\max} \le (1 - \alpha_k^t)M, \ \forall i \in I, \forall k \in V_i^I, \forall t \in T_k^V \tag{5}$$

$$\sum_{t \in T_k^V} \alpha_k^t x_{i=b_k}^t \ge w_k^Q, \ \forall k \in V \tag{6}$$

$$y_j^t \le 2, \ \forall t \in T, \forall j \in J \tag{7}$$

$$\sum_{j \in J_r^R} y_j^t \le z_r^d, \ \forall r \in R, \forall d \in D, \forall t \in T_d^D \tag{8}$$

$$\sum_{r \in R} z_r^d \le G, \ \forall d \in D \tag{9}$$

$$y_j^t \ge \sum_{k \in V_t^T \cap V_j^J} \frac{w_{jk}^Y}{w_k^Q} \alpha_k^t x_{i=b_k}^t, \ \forall t \in T, \forall j \in J \tag{10}$$

$$c_k \ge t\alpha_k^t, \ \forall k \in V, \forall t \in T_k^V \tag{11}$$

$$u_j^t \ge u_j^{t-1} + w_{jt}^E + \sum_{k \in V_t^T \cap V_j^J} \frac{w_{jk}^Y}{w_k^Q} \alpha_k^t x_{i=b_k}^t - y_j^t, \ \forall t \in T, \forall j \in J \tag{12}$$

$$\alpha_k^t \in \{0,1\}, \ \forall k \in V, \forall t \in T_k^V \tag{13}$$

$$x_i^t \in Z_+, \ \forall t \in T, \forall i \in I \tag{14}$$

$$y_j^t \in Z_+, \ \forall t \in T, \forall j \in J \tag{15}$$

$$z_r^d \in Z_+, \forall d \in D, \forall r \in R \tag{16}$$

$$u_j^t \ge 0, \ \forall t \in T, \forall j \in J \tag{17}$$

The objective function (1) minimizes the summation of the weighted turnaround time of each ship and the weighted delayed workload of YCs for serving XTs. In the objective function, $c_k - s_k$ calculates the actual handling time of vessel k, and $\sum_{t \in T} \sum_{j \in J} u_j^t$ calculates the total delayed yard crane workload for XT services. By minimizing the objective function, we seek to find a deployment strategy that encourages shorter vessel turnaround time and less delayed XT services in blocks. This objective function also gives terminal managers the freedom to assign different priorities to different vessels and the delayed yard crane workload for XT services. Constraint (2) ensures the total number of QCs located at all berths does not exceed the total available quantity in each work shift. Constraint (3) ensures that at any time, at most one vessel can be handled in a berth. Constraints (4) and (5) define the lower bound and upper bound of the number of QCs that can work simultaneously for each vessel in each work shift. Constraint (6) enforces that workload for QCs to handle a vessel can be completed within the allowable time window.

Constraint (7) ensures that the number of YCs in a block does not exceed two in each work shift. Constraint (8) ensures that the total number of YCs located in all blocks in a row does not exceed the number of YCs assigned to the row in any work shift within a working day. Constraint (9) defines the upper bound of the total number of YCs in all blocks. Constraint (10) ensures that in each work shift, workload related to vessel handling in each block can be completed. In the right part of the constraint, $\frac{w^Y_{jk}}{w^Q_k}\alpha^t_k x^t_{i=b_k}$ calculates the proportion of workload for QCs to handle the vessel k in work shift t. Accordingly, the overall workload in block j for handling vessels in shift t is $\sum_{k\in V^T_t \cap V^J_j}\frac{w^Y_{jk}}{w^Q_k}\alpha^t_k x^t_{i=b_k}$. We assume YCs in a yard must be able to complete workload related to vessel handling, since otherwise, QCs deployed to handle certain vessels may be left idle if they have to wait for the corresponding yard work to get ready. Constraint (11) calculates the completion time of vessel handling. Constraint (12) calculates delayed workload of YCs in each block for serving XTs, which is obtained by adding the delayed workload of the block in the current work shift to the delayed workload of the block in the last work shift. Since we give priorities to the workload related to vessel handling, YC capacity in each block available for XT services in the current work shift are obtained by deducting the workload related to vessel handling $\sum_{k\in V^T_t \cap V^J_j}\frac{w^Y_{jk}}{w^Q_k}\alpha^t_k x^t_{i=b_k}$ from the total YC capacity y^t_j. Thereby, delayed workload in the current work shift can be calculated by $w^E_{jt} + \sum_{k\in V^T_t \cap V^J_j}\frac{w^Y_{jk}}{w^Q_k}\alpha^t_k x^t_{i=b_k} - y^t_j$. Constraint (13) defines binary variables. Constraints (14)-(16) ensure the corresponding variables to be non-negative integers. Constraint (17) ensures u^t_j to be non-negative.

A Strengthened Linear Model

Constraints (6), (10) and (12) in M1 involve multiplication among decision variables, making the proposed model non-linear in nature. However, popular optimization software like LINGO and CPLEX are unable to solve or can only obtain less satisfactory solutions for non-linear models. In view of this, we provide a linear model (M2) for the studied problem where $\alpha^t_k x^t_i$ in M1 is replaced by v^t_k. In addition, constraints (4) and (5) in M1 are strengthened by constraints (19) and (20) in M2 by assigning tighter bounds.

The second model (M2) is presented as follows:

[**M2**]
$$\min F = \sum_{k\in V}\omega^k_1 (c_k - s_k) + \omega_2 \sum_{t\in T}\sum_{j\in J} u^t_j \tag{1}$$

subject to:

$$\sum_{k\in V^T_t} v^t_k \le H, \ \ \forall t \in T \tag{18}$$

$$v^t_k - q^{\min}_k \ge (\alpha^t_k - 1)M^1_k, \ \ \forall k\in V, \forall t \in T^V_k \tag{19}$$

$$v^t_k - q^{\max}_k \le (1-\alpha^t_k)M^2_k, \ \ \forall k\in V, \forall t \in T^V_k \tag{20}$$

$$v^t_k \le \alpha^t_k M^2_k, \ \ \forall k\in V, \forall t \in T^V_k \tag{21}$$

$$\sum_{t\in T^V_k} v^t_k \ge w^Q_k, \ \ \forall k \in V \tag{22}$$

$$y^t_j \ge \sum_{i\in I}\sum_{k\in V^T_t \cap V^I_i \cap V^J_j}\frac{w^Y_{jk}}{w^Q_k} v^t_k, \ \ \forall t\in T, \forall j \in J \tag{23}$$

$$u_j^t \geq u_j^{t-1} + w_{jt}^E + \sum_{i \in I} \sum_{k \in V_t^T \cap V_i^I \cap V_j^J} \frac{w_{jk}^Y}{w_k^Q} v_k^t - y_j^t, \ \ \forall t \in T, \forall j \in J \tag{24}$$

$$v_k^t \in Z_+, \ \ \forall k \in V, \forall t \in T_k^V \tag{25}$$

and constraints (3), (7)-(9), (11), (13) and (15)-(17).

Constraint (18) imposes a capacity limitation for available QCs that can work simultaneously. Constraints (19) and (20) ensure that when a vessel is being handled, the number of QCs assigned for it should be limited in an allowable range. Constraint (21) enforces v_k^t to be 0 when vessel k is not being handled. Constraint (22) ensures each vessel can be handled within the allowable time window. Constraints (23) and (24) have the same functions like constraint (10) and (12) in M1. Constraint (25) defines v_k^t to be non-negative and integral.

HARDNESS OF THE PROBLEM

This section demonstrates that the TJCDP is NP-hard in the strong sense. To do this, we show that the decision version of the TJCDP is strongly NP-hard. That is, given the berth allocation plan and the yard management plan, it cannot be determined in polynomial time or even in pseudo-polynomial time whether the objective value F is no greater than a given constant λ unless P=NP.

We prove the NP-hardness of the studied problem by reducing a well-known strongly NP-hard problem--the Multi-dimensional Bin Packing Problem (MDBPP) into a decision version of the TJCDP. The decision version of the MDBPP can be stated as follows:

There is a set V of items, each of which has volumes in J dimensions, and for item k, the jth dimensional volume is w_{kj}. There is also a set T of J-dimensional bins, each of which has the capacity of σ_j in the jth dimension. Let V_t denote the subset of items packed in bin t, then the MDBPP is to decide whether there is a packing strategy such that the following two conditions hold:

$$\sum_{k \in V_t} w_{kj} \leq \sigma_j, \ \ \forall t \in T, \forall j \in \{1, 2, ..., J\} \tag{26}$$

$$\bigcup_{t \in T} V_t = V \tag{27}$$

Theorem 1: The decision version of the TJCDP is strongly NP-hard.

Proof: We only need to prove that an MDBPP can be transformed into a TJCDP in polynomial time. The transformation can be done as follows. A TJCDP instance corresponding to an arbitrary MDBPP instance has a set V of vessels to be handled in a container terminal within a planning horizon $[1, T]$, where $T = |T|$. Suppose the terminal has J blocks and there are infinite numbers of QCs, YCs and berths in the terminal. The handling time windows T_k^V for each vessel k is set equal to the planning horizon $[1, T]$, and the allowable handling QC number is set as $q_k^{\min} = q_k^{\max} = 2$. The workload of QC for handling vessel k (w_k^Q) is set equal to 2 QC work shifts, and workload of YC in block related to handling vessel k (w_{jk}^Y) is set as $\frac{2w_{kj}}{\Delta}$ YC work shifts, where $\Delta = \max_{j \in J}\{\sigma_j\}$. In addition, the workload for handling XT containers w_{jt}^E is assumed to be $2 - \frac{2\sigma_j}{\Delta}$ YC work shifts for block j in each work shift. Moreover, we set the weights for all vessels, ω_1^k s to be 0, the weight for the delayed workload of YCs for serving XTs ω_2 to be 1 and the constant value λ is set to be 0. Therefore, the decision version of the TJCDP is to determine whether there exists a deployment plan such that all the involved vessels can complete handling within the planning horizon and no delay happens for the YC workload related to XT services.

Let V_t denote the subset of vessels handled in work shift t. Observe that:

(i) The above transformation can be done in polynomial time.

(ii) All vessels require exactly one work shift to handle due to the QC assignment and QC workload settings.

(iii) Since the number of YCs is assumed to be infinite, the handling capacity of all blocks in any work shift reaches the maximum of two YC-work shifts.

(iv) A workload of $\frac{2w_{kj}}{\Delta}$ for YC is required by vessel k in block j in the work shift when it is being handled, and therefore, a total of $2-\sum_{k\in V_t}\frac{2w_{kj}}{\Delta}$ capacity is left for handling XT services in block j in work shift t.

(v) If $F \le \lambda$ then there must exist a handling plan such that the following conditions hold:

$$2-\sum_{k\in V_t}\frac{2w_{kj}}{\Delta} \ge 2-\frac{2\sigma_j}{\Delta},\ \forall t\in T, \forall j\in J \tag{28}$$

$$\bigcup_{t\in T} V_t = V \tag{29}$$

Obviously, equation (28) is equivalent to equation (26).

Therefore, it follows easily now that a TJCDP will generate an objective value of λ or less if and only if the MDBPP has a feasible solution. Thus, an MDBPP can be solved by solving a TJCDP problem. □

It is worth noting that even a specialized sub-problem of MDBPP, the Bin Packing Problem (BPP) which involves only one dimension is already NP-Hard in the strong sense. It can be naturally obtained that even when only one block is involved in the TJCDP, the problem is already strongly NP-hard. Moreover, we can also prove in a similar way that the TJCDP is strongly NP-hard by assuming the block handling capacity to be infinite but the number of available QCs to be limited.

MODEL APPLICATION

In this section, we make a series of case studies to investigate how the integration of QC and YC deployments can affect the overall performance of a container terminal. These cases are designed based on practical situations. To understand the effectiveness of the joint deployment policy, these cases are solved by model M2 and the obtained results are compared with the results delivered by the method that deploys QCs and YCs in a sequential way.

For a better observation, we need to examine optimal solutions. Thereby, in view of the complexity of the problem, we have deliberately chosen to use small test cases which can be well solved by some state-of-the-art mixed-integer software to optimum.

Experimental Settings

In the studied cases, there are 6 to 15 vessels to be handled at a container terminal within one week's time. A typical working day is divided into 6 work shifts, each containing 4 hours. There are 3 berths and 8 blocks located in 2 rows. The numbers of QCs and YCs are 6 and 8 respectively. Three classes of vessels call at the terminal periodically and detailed information of them is given in Table 2. We assume that these vessels arrive at the terminal randomly, and for each vessel, the designated blocks are also generated in a random way. Based on these data, a berth allocation plan is then generated accordingly. In addition, we assume external trucks arrive at the terminal randomly. Hence, the required YC workload for XT services (in YC-work shifts) for each block in each work shift is randomly generated within the range [0, 0.9]. Besides, the weight of delayed workload for XT services is set as 1.

Moreover, we divide the objective function F in M2 into two parts, namely $F1 = \sum_{k \in V} \omega_1^k (c_k - s_k)$ representing vessel handling cost and $F2 = \omega_2 \sum_{t \in T} \sum_{j \in J} u_j^t$ representing the cost for delayed YC workload for XT services.

<Insert Table 2 here>

We analyze the effectiveness of the joint deployment strategy by studying the differences between results obtained by M2 and those obtained by the sequential deployment policy. In order to capture the procedures of a sequential deployment, we propose two new models (M3 and M4). Model M3 is used to optimize QC deployment, and model M4 is formulated to deploy YCs based on the QC deployment pattern obtained in M3.

Before presenting the two models, we introduce several new notations. First, since in M3, deployment pattern of YCs is fixed and required as an input, a parameter b_j representing the number of deployed YC in block j is introduced. In all cases, the b_j s are set in a way to ensure a most balanced distribution of a given number of YCs among all blocks. Moreover, considering that there is a possibility that some vessels cannot finish handling within the given time window under certain YC deployment patterns, we relax the handling time window of each vessel by extending the end of the time window to T ($T \ge |T|$) to ensure feasibility of M3. The extended handling time window for vessel k is denoted as T_k^V. It is worth noting that in practice, such infeasibility caused by fixed YC deployment can be solved by rescheduling the berth allocation plan, which is, however, out of the scope of our paper.

The QC deployment model (M3) is given as follows:

[**M3**]
$$\min F_1 \tag{30}$$

subject to:

$$\sum_{t \in \tilde{T}_k^V} v_k^t \ge w_k^Q, \quad \forall k \in V \tag{31}$$

$$\sum_{k \in V_t^T \cap V_j^J} \frac{w_{jk}^Y}{w_k^Q} v_k^t \le b_j, \quad \forall t \in T, \forall j \in J \tag{32}$$

and constraints (3),(11), (13), (18)-(21), (25).

Constraint (31) ensures enough QC capacity is scheduled for vessel handling. Constraint (32) enforces yard handling capacity limitation.

The YC deployment problem is formulated in model M4:

[**M4**]
$$\min F_2 \tag{33}$$

subject to: constraints (7)-(9), (15)-(17), (23) and (24).

Note that in constraint (23) and (24), variable v_k^t is fixed and obtained by solving model M3.

Computational Results

We solve the 10 cases by M2, M3, and M4 and the results are shown in table 3.

<Insert Table 3 here>

As shown in Table 3, for all of the 10 cases, M2 delivers better solutions than M3 and M4. The minimum optimality gap between the two deployment methods is 13.68% for case 3 and the average gap for all cases goes greater than 29%. To be more specific, as for the vessel handling related cost *F1*, M2 generates results that are much better than M3 for all cases. This implies that the joint deployment policy applied in M2 which enables dynamic handling capacity adjustments among blocks can greatly reduce the time needed by a terminal to handle vessels. Further, although in M2 the costs caused by delayed YC workload for XT services *F*2 are higher than those in M4 for half of the cases, the differences are relatively small. In addition, it should be noted that the improvements of *F*2 in M4 are achieved by lessening the handling-related workload for blocks in each work shift which is realized at the expenses of much longer handling times for vessels obtained in M3. To sum up, the joint deployment policy enables container terminals to better utilize the limited resources like QCs and YCs and can greatly improve their efficiency.

As for the computational time, we can see from the above table that CPLEX is able to obtain optimal solutions for these instances within very short time (less than 200 seconds). However, additional experimental data show that when the scale of instances grows, the solution time of CPLEX increases drastically (CPLEX fails to solve instances with 5 berths, 12 blocks and 25 vessels in reasonable time). Therefore, it would be favorable to develop some heuristic algorithms to solve instances with large scales.

Sensitivity Analysis

There are a number of parameters involved in the TJCDP, and some of these parameters like number of berths or handling time windows for vessels are relatively fixed. In this part, we make sensitivity analyses upon two most “manipulable” parameters for a terminal operator, namely the number of QCs (*H*) and the number of YCs (*G*). The sensitivities of these parameters are investigated from two dimensions. We analyze when the settings of these parameters change, how the optimal objectives of the TJCDP change and how the differences between the joint deployment policy and the sequential deployment policy change. In this section, four cases with various demand levels (Cases 1, 4, 7 and 10) are solved under settings where the numbers of QCs and YCs vary from 4 to 12 and 8 to 16 respectively.

Sensitivity analysis for the optimal value

We first investigate the impact of QC and YC numbers on the optimal values of different cases. Figure 3 shows the optimal solutions of M2 of the four cases with different QC and YC numbers. Note that for a better illustration, we rotate the direction of the “VALUE” axes in this figure.

As shown in this figure, there are diminishing returns to additional QCs or YCs if all other resources are fixed. We can further identify the most effective combinations of the two resources (QCs and YCs) for different cases. The most effective combination for Case 1 is 7 QCs plus 12 YCs. For Case 4, the best combination is 7 QCs plus 12 YCs. The most effective combinations for Case 7 and 10 are 6 QCs plus 13 YCs and 6 QCs plus12 YCs respectively.

<Insert Figure 3 here>

Sensitivity analysis for the optimality gap

We further study how optimality gaps between the joint deployment policy and the sequential deployment policy in these cases change when the settings of QCs and YCs change. Figure 4 shows optimality gaps of these cases under different settings of resources. Note that the “YC” axes are rotated in this figure to give a better illustration. It is easy to see that the changes of optimality gaps in the four cases share a similar trend with regard to changing numbers of QCs and YCs.

First, when the numbers of YCs are fixed, we look at how the optimality gaps change with changing QCs numbers. The optimality gaps first sharply increase with increasing numbers of QCs before the QC numbers reach certain values (7, 8, 6 and 6 in Case 1, 4, 7 and 10 respectively) and then remain relatively stable. Next, we investigate the relationship between the optimality gaps and YC numbers, by fixing the numbers of QCs. The optimality gaps increase greatly when the numbers of YCs increase from 8 to certain values (11, 10, 12 and 11 in Case 1, 4, 7 and 10 respectively) and then experience sharp drops. It is obvious that the joint deployment policy gains the greatest superiority when the numbers of QCs and YCs are in a moderate range.

<Insert Figure 4 here>

CONCLUSIONS

The deployments of QCs and YCs are closely intertwined and are of critical importance to the efficiency of container terminals. This paper studied the TJCDP. A nonlinear mixed-integer programming model was proposed for the considered problem and then linearized to make it easy to solve. We proved that the problem is NP-hard in the strong sense. A series of case applications were solved by the model and discussed in detail. The computational results demonstrated the effectiveness of our model. We analyzed the impacts of the numbers of QCs and YCs upon the performance of the proposed model and identified the most effective combinations of QCs and YCs for a container terminal at various demand levels. We also found that the joint deployment policy gained the greatest superiority against its sequential counterpart when the numbers of QCs and YCs are modest.

For future studies, we find two promising directions. First, in view of the complexity of the problem, it would be interesting to develop efficient heuristic algorithms for solving the TJCDP on a large scale. Second, sea transportation faces great uncertainties, and future research can study the problem with uncertain berth allocation or XT transportation plans.

ACKNOWLEDGMENTS

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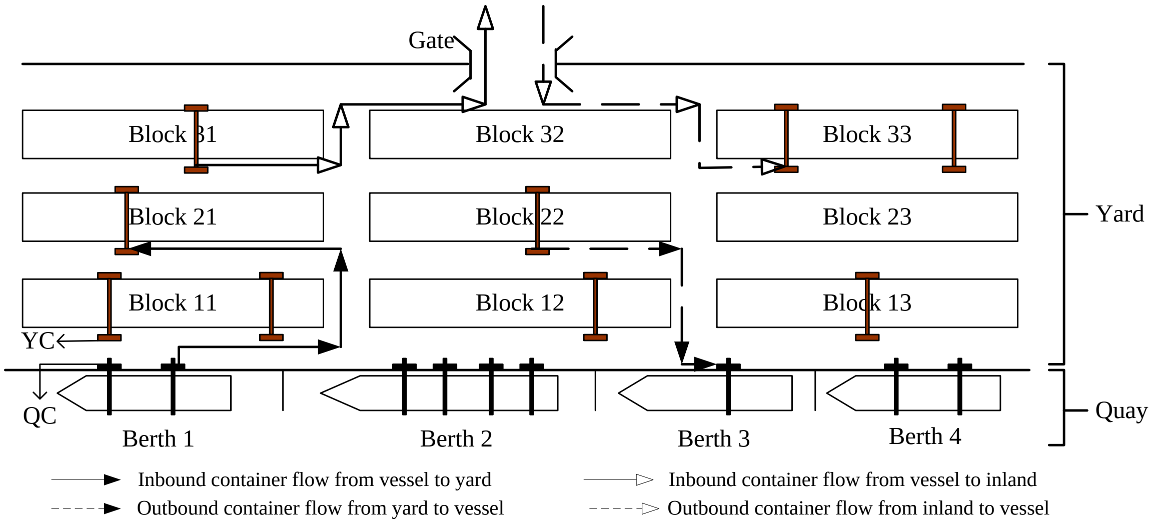

FIGURE 1 Container terminal layout.
(black and white in printed version)

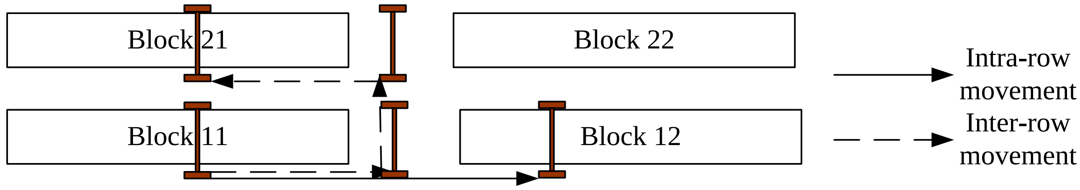

FIGURE 2 Yard crane movements.
(black and white in printed version)

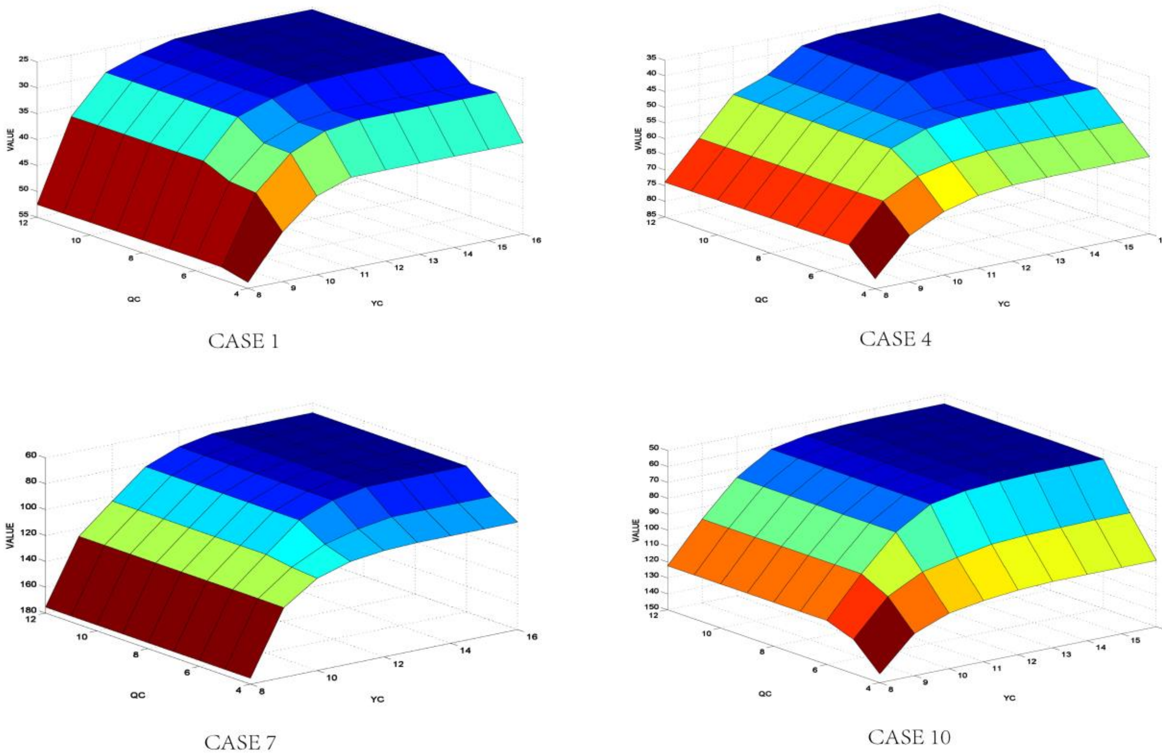

FIGURE 3 Results of M2 with changing QC and YC numbers.
(black and white in printed version)

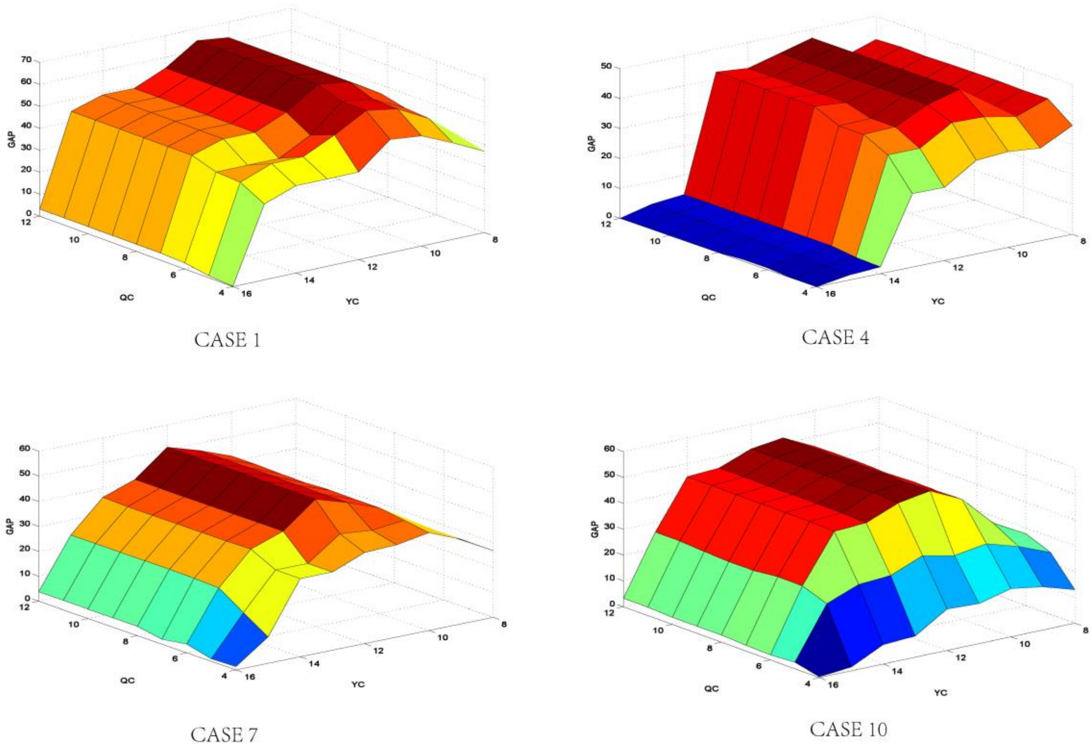

FIGURE 4 Optimality gaps with changing QC and YC numbers.
(black and white in printed version)

TABLE 1 Notations used in formulations

Indices:	
k	Index for vessels in a planning horizon, arranging in an alphabetical order
i	Index for berths, arranging in an alphabetical order
j	Index for blocks, arranging in an alphabetical order
r	Index for block rows, arranging in an alphabetical order
d	Index for working days, arranging in an alphabetical order
t	Index for work shifts, arranging in an alphabetical order
Set:	
V	Set of vessels
V_i^I	Set of vessels that moor in berth i
V_t^T	Set of vessels whose handling time windows cover work shift t
V_j^J	Set of vessels to which block j is assigned to hold containers that are to be loaded onto or discharged from them
I	Set of berths
J	Set of blocks
R	Set of rows for blocks
J_r^R	Set of blocks in row r
D	Set of work days
T	Set of work shifts
T_d^D	Set of work shifts in day d
T_k^V	Set of work shifts within the time window set for handling vessel k
Z_+	Set of non-negative integers
Parameters:	
s_k	Berthing time (work shift) of vessel k ; s_k is the smallest element in T_k
b_k	Berth assigned to vessel k
$q_k^{\min}$	Minimum number of QCs agreed to serve vessel k at any time
$q_k^{\max}$	Maximum number of QCs allowed to serve vessel k at any time
w_k^Q	Quay crane workload (in QC work shifts) for handling vessel k
w_{jk}^Y	Yard crane workload (in YC work shifts) in block j for handling vessel k
w_{jt}^E	Yard crane workload (in YC work shifts) in block j for serving XTs in work shift t
H	Number of QCs
G	Number of YCs
ω_1^k	Weight assigned to the turnaround time of vessel k in the objective
ω_2	Weight assigned to the delayed workload of YCs for serving XTs in the objective
M	A constant large enough
M_k^1	A constant set equal to $\min\{q_k^{\min}, H\}$
M_k^2	A constant set equal to $\min\{q_k^{\max}, H\}$
Decision Variables:	
α_k^t	1 if vessel k is being handled in work shift t and 0, otherwise
c_k	Completion time (work shift) of handling vessel k
x_i^t	Number of QCs assigned to berth i in work shift t
y_j^t	Number of YCs assigned to block j in work shift t
z_r^d	Number of YCs assigned to row r in working day d
v_k^t	Number of QCs assigned to handle vessel k in work shift t
u_j^t	Delayed workload (in YC work shifts) of YCs in block j for serving XTs in work shift t where the

	initial delayed workload u_j^0 is defined as 0 for each j

TABLE 2 Parameters for Vessels Used in Test Cases

Class	**Ratio**	**Handling time**	**Used QC**	**QC workload**	**YC workload**	**Dedicated blocks**	**Weight in objective**
Feeder	1/3	2-4	1-3	2-5	2-10	2-6	1-3
Medium	1/3	3-5	2-4	6-14	6-28	3-8	4-6
Jumbo	1/3	4-6	3-6	15-20	15-40	4-8	7-9

TABLE 3 Result of the Cases

Case	Vessel	M2			M3+M4			Gap (%)
		$F1^{a1}$	$F2^{a2}$	Time(s)	$F1^{b1}$	$F2^{b2}$	Time(s)	
1	6	31	21.53	2.70	76	10.35	0.44	39.17
2	7	47	29.66	147.89	68	21.59	0.43	14.43
3	8	83	34.55	3.80	112	24.18	0.68	13.68
4	9	53	20.93	3.14	110	18.25	0.43	42.36
5	10	88	33.86	12.44	169	31.94	1.28	39.36
6	11	124	32.26	21.70	172	30.99	1.75	23.02
7	12	108	67.54	37.16	193	45.90	1.31	26.52
8	13	127	67.79	193.84	213	73.72	2.76	32.06
9	14	115	54.36	120.84	241	42.11	5.91	40.18
10	15	89	33.37	16.88	140	29.45	1.33	27.78
Average		86.50	39.58	56.04	149.40	32.85	1.63	29.86

Note: 1. Gap is calculated by $100 \bullet (b1+b2-a1-a2)/(b1+b2)$.

2. The “Time” columns report the computational times of CPLEX.