Pricing of Shared-parking Lot: An Application of Hotelling Model

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Abstract

Shared-parking lot brings utilization improvement, but also has its disadvantage compared with traditional parking lot while they are competing for public users. In the market including both shared-parking lot and traditional parking lot, parking lot operators need to know how to deal with parking price to be competitive in the market. The Hotelling model is applied in this paper to study the product differentiation of traditional parking lot and shared-parking lot, with some equilibrium analyses to figure out equilibrium parking prices of both parking lots while considering their competition in the market. Two points of indifferent consumers exist in the competition of the traditional parking lot and the shared- parking lot. The traditional parking lot serves 2/3 of the market, and the shared-parking lot serves 1/3 of the market in the special case of equal penalty cost and equal operation cost.

1. Introduction

Sharing economy has become very popular in the last few years. Multiple sharing economy platforms are booming, such as Uber, Airbnb, Relay Rides and so on, which enable owners to rent out their durable goods while not using them. New Internet-based markets occur correspondingly, then several issues related to ownership, rental rates, quantities, prices, and surplus generated are heatedly discussed both in the industry and academic area. People claim that shared economy brings efficiency, opportunity and sociability. This opinion can be well represented in the aspect of urban logistics.

In the progress of city modernization, the number of vehicles increases as more people are attracted to the metropolis. Gradually, urban areas are saturated commercial tall buildings but lack of parking lots. One solution is public transport (Liu et al., 2013, 2016; Liu and Meng, 2014), however, private cars are attractive for various reasons. Parking of private cars has become a confusing and conflicting problem in many areas which occurs every day. Such problem cannot be solved only by constructing more parking lots to fulfill the parking requirement of vehicles, because the land resources are too limited, especially metropolis such as Hong Kong. Instead of such conventional method, some new solution is in need to improve the efficiency and capacity of existing parking lots, with considering the cost and feasibility.

'Shared-parking strategy' is a relatively new method to improve parking efficiency suggested recently by Shao (2015). This idea comes from the phenomenon that a large number of private parking spots in residential communities are empty when their owners go to work during the day, and are only occupied at night when their owners go back home. (These owners live in one location and work in another, and their travel pattern can be predicted very well daily, weekly or monthly.) If the owners are willing to offer their personal parking spots to other people, so-called public users, when the parking spots are not occupied in the daytime, the shortage of parking spaces nearby could be alleviated to a great extent and parking efficiency could also be improved, as can be seen in Figure 1.

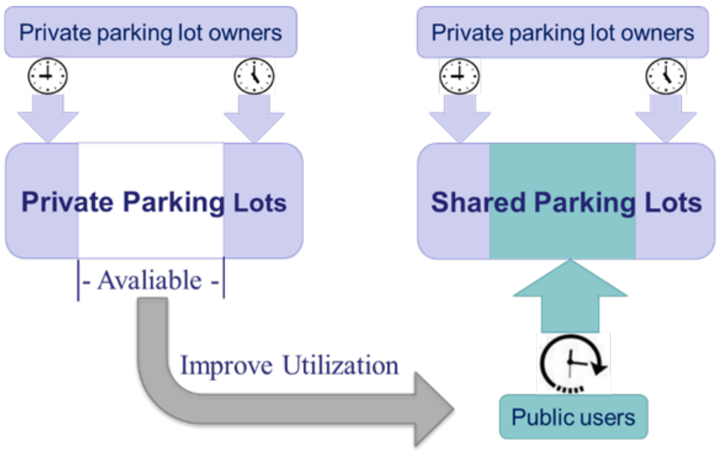

Figure 1. Basic idea of shared-parking lots

There is some existing literature on shared-parking lots, most related to parking lot allocation. These studies often assume that 'demand' from public users and 'supply' from parking spot owners are both given before allocation, and the parking fee is fixed and preset. For example, Shao et al. (2015) build binary integer linear programming models to make an optimum allocation, aiming to maximize the profit under constraints of parking space and time duration. Parking pricing, so far, has received relatively little attention in the literature. However, a better understanding of strategic parking pricing is important, not only because adjusting parking fees can regulate parking demand (Cassandras and Geng, 2013) and relieve the heavy parking pressure (Teodorovic and Lucic, 2006), but also because drivers are always searching for cheaper parking lots (Chou et al., 2008). The competition between the traditional parking lot and shared-parking lot, so far, has received little attention, and parking lot operators should know how to deal with parking price to compete in the market, especially for shared-parking operator owing to the inconvenience nature of the shared-parking lot.

Inconvenience of the shared-parking lot is that, public users have to request parking (do reservations beforehand) for integrate time intervals, which means that if a public user wants to park during 9:30-11:45 am, she will be suggested to request for three-time interval 9:00-10:00 am, 10:00-11:00 am and 11:00-12:00 am, as can be seen in Figure 2. The reason behind is that, each parking spot has different available time gaps (because the parking spot owners have different departure time and coming back time), only collecting such parking requests with integrate time intervals, the shared-parking company can allocate certain public user to specific parking spot, so that to make the shared-parking lot as fully occupied as possible to make more profit. For shared-parking lot operator to do the optimum allocation, public users have to request parking for integrate time intervals. But for the traditional parking lot, the public user can start parking at any time as preference, which is 9:30 am in this example. She does not need to do reservation which starts from 9:00 am, actually earlier than her real parking start time. She also does not need to delay the original schedule to start parking at 10:00 am. This is the main differentiation of traditional parking lots and shared-parking lots.

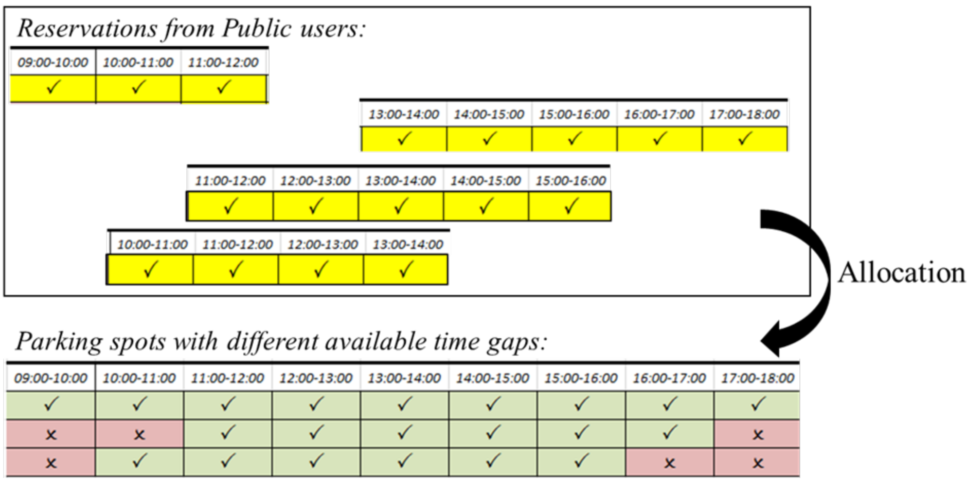

Figure 2. Allocation for shared-parking lot

For studying product differentiation in markets with multiple competitors, the Hotelling model is probably the most well-known one. Hotelling's classic paper studied two firms, which are located at the two endpoints of a linear city with length one. Firm one is located at $x=0$, firm two is located at $x=1$. Consumers are uniformly distributed with density 1 along this linear city and incur a transportation cost t per unit of distance. Each consumer has unit demand and will buy from the firm which has a cheaper generalized price (price plus transportation cost) and does not exceed surplus s. Let p_A and p_B denote the price by firm A and firm B, the demand for firm A is given by $D_A(p_A, p_B) = \tilde{x}$ where $p_A + t\tilde{x} = p_B + t(1-\tilde{x})$, and the demand for firm B is given by $D_B(p_A, p_B) = 1 - D_A(p_A, p_B)$, as can be seen in Figure 3. The customer located at $\tilde{x}$ is called an indifferent consumer who is just indifferent between firm A and firm B. Once we know the indifferent consumer and then define the demand function of firm A and B, the payoff functions of both firms can be derived, therefore the equilibrium prices can be determined.

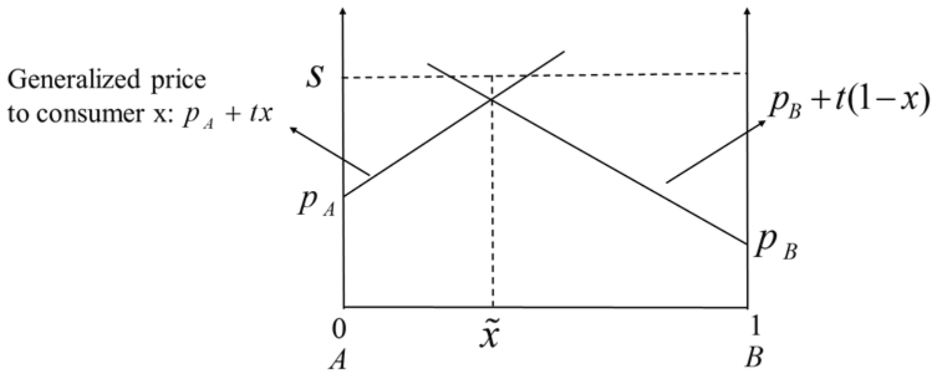

Figure 3. The equilibrium of the classic Hotelling model

Although the hoteling model originally framed in the context of location differentiation along a linear market, it has various possible interpretations. In this case of two kinds of parking lots, we may intuitively think that, the parking price of the shared-parking lot could be lower than the traditional parking lot to attract more drivers. Although shared-parking lot has more inconvenience compared with traditional parking lot, under lower parking price motivation of shared-parking lot, public user who wants to park at 10:30 am may move up her original schedule to start parking at 10:00 am or to delay the original schedule to start parking until 11:00 am so as to park in the shared parking lot. Then, Hotelling' s 'space' becomes 'parking time' for public users, and 'transport costs' between consumers and suppliers become 'early arrival cost' and 'late arrival cost', with considering the cost of being late is usually higher than the cost of being early (Small, 1982). In this paper, we propose an application of the Hotelling model that has exactly this interpretation, with some equilibrium analysis to figure out equilibrium parking prices of the traditional parking lot and the shared-parking lot while considering the competition between them.

2. Problem description and model formulation

2.1. Problem description

Indifferent public users and two kinds of parking lots, traditional parking lot A and shared-parking lot B, are considered as participants in the market. To simplify the problem, indifferent public users (parking demand) are all generated from the one-time interval. In this time interval, the arrival of public users follows a Poisson distribution, as usually considered in previous literature (Richardson, 1974; Cleveland, 1963; Blunden, 1971). Suppose each time interval has one-hour length. For example, our modeling time interval is from 9:00 to 10:00, then 9:00 indicates 0 and 10:00 indicates 1 in the Hotelling model, and public users are uniformly distributed along this interval $[0,1]$. Compared with the classic Hotelling model that a linear city has length one, here a time interval has length one, that is to say, we consider time distance instead of space distance in this problem.

Both parking lots are in the same residential area other than scattered, and if the public user wants to park in this area, she has no preference for specific parking lot in the aspect of geographic reason. The only differentiation of two parking lots is that, for the traditional parking lot A, public users can park at any time once they arrive at the parking lot; for the shared-parking lot B, public users whose prefer starting parking time is in the middle of a time interval need to decide to move up the schedule to the beginning of this time interval, or to delay the schedule until the beginning of the next time interval. Each public user has unit demand, either choosing to park in the traditional parking lot A or the shared-parking lot B.

Suppose all the public users generated in this time interval want to park for a whole daytime. This makes sense for public users who work nearby the parking lot so that they need to park for a whole daytime. What is more, the start time of their work always concentrates on one specific time interval, for example, from 8:00 to 9:00, which is to be considered as our modeling period. As the ends of the working hour in the afternoon also do not have big differences, if we charge the parking fee per hour which is the same as the practical situation, the parking revenue earning from each public user of our interests just has a small variance limited in one or two hour's revenue. Compared to the whole day profit, this small part can be neglected in order to simplify the question. So, the parking fee can be set for one unit consumption of parking spot, that is to say, traditional parking lot p_A and shared-parking lot p_B are both considered as a whole day parking fee. Both traditional parking lot A and shared-parking lot B have the marginal cost as they sell one parking spot, which is defined as c_A and c_B, being considered as the operation cost for the companies to maintain daily operation.

The inconvenience of the shared-parking lot will be captured by early arrival cost coefficient t_1 and late arrival cost coefficient t_2. Both costs are up to public users to pay, indicating the unwillingness of public users to change their schedules. For the early arrival cost coefficient t_1, although the public user can still arrive at the shared-parking lot according to her schedule, she has to pay for the integrate time interval, so that the early arrival cost can also be interpreted as the unhappy emotion (the public user may think that she pays more parking fee than she actually deserves to be charged). For the late arrival cost coefficient t_2, it represents the bad outcome for the public user if she is late, for example, to go to work after the regulated working start time, late to arrive her office. As the cost of being late is usually higher than the cost of being early (Small, 1982), the late arrival cost coefficient t_2 is larger than the early arrival cost coefficient t_1.

2.2. Model formulation

In order to derive the demands of the traditional parking lot A and the shared-parking lot B, we need to derive the public user with the preference arrival time x that is just indifferent between parking in the traditional parking lot A and the shared-parking lot B. Suppose a public user actually wants to park her car at a time duration of x from the beginning of the time interval (or $(1-x)$ to the end of the time interval). The early arrival cost is $t_1 x$ if she chooses to arrive at the shared-parking lot earlier than her original time schedule, the late arrival cost is $t_2(1-x)$ if she chooses to arrive at the shared-parking lot later than her original time schedule. So, the total cost for the public user parking in the traditional parking lot is p_A, but the total cost for the public user parking in the shared parking lot is $p_B + t_1 x$ or $p_B + t_2(1-x)$. The Nash equilibrium can be expressed by equation (1) and equation (2), then x_1 and x_2 can be derived as equation (3) and equation (4).

$$p_A = p_B + t_1 x_1 \tag{1}$$

$$p_A = p_B + t_2(1-x_2) \tag{2}$$

$$x_1 = \frac{p_A - p_B}{t_1} \tag{3}$$

$$x_2 = \frac{p_B - p_A + t_2}{t_2} = \frac{p_B - p_A}{t_2} + 1 \tag{4}$$

Figure 4 can be derived by above equations, there exist two time points defined as x_1 and x_2. For the public user who has the parking preference at the time x_1, it is indifferent for her to park at the traditional parking lot A with a higher parking fee p_A or park at the shared-parking lot B with a lower parking fee p_B but a higher inconvenience to move up her schedule. For the public user who has the parking preference at the time x_2, it is indifferent for her to park at the traditional parking lot A with a higher parking fee p_A, or park at the shared-parking lot B with a lower parking fee p_B but a higher inconvenience to delay her schedule.

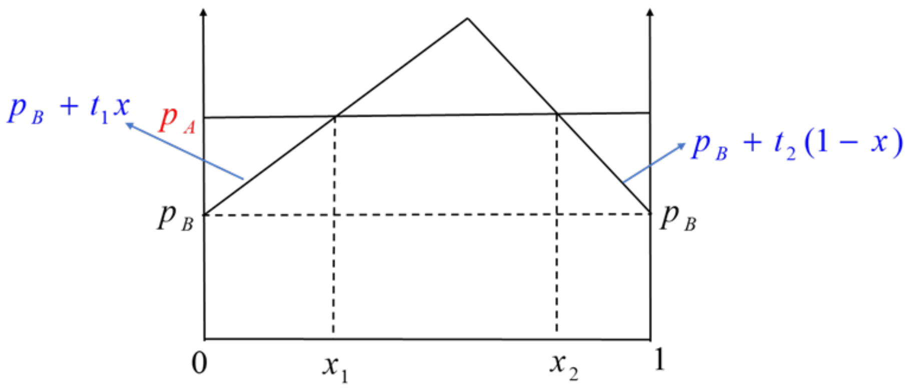

Figure 4. The equilibrium of the traditional parking lot and shared-parking lot in $[0,1]$

Once we know the indifferent public user, we may define the demand function of the traditional parking lot A by equation (5) and the demand function of the shared-parking lot B by equation (6). It can be seen in figure 4 that parts of the market are covered by the shared-parking lot B, which are the public users who have the parking preferences in the time duration of $[0, x_1]$ and $[x_2, 1]$. Residual market is covered by the traditional parking lot A, which are the public users who have the parking preferences in the time duration of $[x_1, x_2]$. This result is intuitive since, for the public users with the parking preferences in the middle of the time interval, either to move up their schedules or to delay their schedules is not a wise choice.

$$D_A(p_A, p_B) = \int_{x_1}^{x_2} 1dz = z\Big|_{x_1}^{x_2} = x_2 - x_1 = 1 + \frac{p_B - p_A}{t_2} - \frac{p_A - p_B}{t_1} \quad (5)$$

$$D_B(p_A, p_B) = \int_{0}^{x_1} 1dz + \int_{x_2}^{1} 1dz = z\Big|_{0}^{x_1} + z\Big|_{x_2}^{1} = x_1 + (1 - x_2) = \frac{p_A - p_B}{t_1} - \frac{p_B - p_A}{t_2} \quad (6)$$

The maximization problem of the traditional parking lot A is

$$\underset{p_A}{Max}\,\Pi^A(p_A, p_B) = (p_A - c_A)D_A(p_A, p_B) = (p_A - c_A)(1 + \frac{p_B - p_A}{t_2} - \frac{p_A - p_B}{t_1}) \tag{7}$$

By deriving its first-order condition $\frac{d\Pi^A}{dp_A} = 0$, we have

$$\begin{aligned} &(1 + \frac{p_B - p_A}{t_2} - \frac{p_A - p_B}{t_1}) - \frac{p_A - c_A}{t_2} - \frac{p_A - c_A}{t_1} = 0 \\ &p_A = (\frac{t_1 t_2}{t_1 + t_2} + p_B + c_A)/2 \end{aligned} \tag{8}$$

The maximization problem of the shared-parking lot B is

$$\underset{p_B}{Max}\,\Pi^B(p_A, p_B) = (p_B - c_B)D_B(p_A, p_B) = (p_B - c_B)(\frac{p_A - p_B}{t_1} - \frac{p_B - p_A}{t_2}) \tag{9}$$

By deriving its first-order condition $\frac{d\Pi^B}{dp_B} = 0$, we have

$$\begin{aligned} &\frac{p_A - p_B}{t_1} - \frac{p_B - p_A}{t_2} - \frac{1}{t_1}(p_B - c_B) - \frac{1}{t_2}(p_B - c_B) = 0 \\ &p_B = \frac{p_A + c_B}{2} \end{aligned} \tag{10}$$

Then, by equation (8) and (10), the equilibrium prices are determined as follow

$$p_A^* = (\frac{2t_1 t_2}{t_1 + t_2} + 2c_A + c_B)/3 \tag{11}$$

$$p_B^* = (\frac{t_1 t_2}{t_1 + t_2} + c_A + 2c_B)/3 \tag{12}$$

We define θ as the price differentiation between the traditional parking lot A and shared-parking lot B which is expressed by equation (13). If the marginal operating costs of two kinds of parking lots are equal (when $c_A = c_B$, θ can be simplified as θ^0 expressed by equation (14)), θ^0 is actually the benefit of the convenience of the traditional parking lot, or, the cost of the inconvenience of the shared-parking lot, which is only related with early arrival cost coefficient t_1 and late arrival cost coefficient t_2. The higher is $t_1 t_2 / (t_1 + t_2)$, the more differentiated are the two kinds of parking lots from the point of view of public users.

$$\theta = p_A^* - p_B^* = (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3 \quad (13)$$

$$\theta^0 = p_A^* - p_B^* = \frac{t_1 t_2}{3(t_1 + t_2)} \quad (14)$$

The indifferent public users can be expressed by equation (15) and (16).

$$x_1^* = \frac{p_A^* - p_B^*}{t_1} = (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3t_1 \quad (15)$$

$$x_2^* = \frac{p_B^* - p_A^* + t_2}{t_2} = 1 - (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3t_2 \quad (16)$$

The demand functions of traditional parking lot A and shared-parking lot B can be derived as equation (17) and (18).

$$\begin{aligned} D_A^*(p_A^*, p_B^*) &= \int_{x_1^*}^{x_2^*} 1dz = z\Big|_{x_1^*}^{x_2^*} = x_2^* - x_1^* \\ &= 1 - (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3t_2 - (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3t_1 \\ &= \frac{2}{3} - \frac{t_1 + t_2}{3t_1 t_2}(c_A - c_B) \end{aligned} \quad (17)$$

$$\begin{aligned} D_B^*(p_A^*, p_B^*) &= \int_0^{x_1^*} 1dz + \int_{x_2^*}^{1} 1dz = z\Big|_0^{x_1^*} + z\Big|_{x_2^*}^{1} = x_1^* + (1 - x_2^*) \\ &= (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3t_1 + (\frac{t_1 t_2}{t_1 + t_2} + c_A - c_B)/3t_2 \\ &= \frac{1}{3} + \frac{t_1 + t_2}{3t_1 t_2}(c_A - c_B) \end{aligned} \quad (18)$$

Then, the profit earned by the traditional parking lot A and the shared-parking lot B can be expressed by equation (19) and (20) as follows.

$$\Pi^{A^*} = (p_A^{\ *} - c_A)D_A^{\ *} = \left[(\frac{2t_1t_2}{t_1+t_2} - c_A + c_B)/3 \right] \left[\frac{2}{3} - \frac{t_1+t_2}{3t_1t_2}(c_A - c_B) \right]$$
$$= \frac{1}{9}\left[\frac{4t_1t_2}{t_1+t_2} + 4(c_A - c_B) + \frac{t_1+t_2}{t_1t_2}(c_A-c_B)^2 \right] \quad (19)$$

$$\Pi^{B^*} = (p_B^{\ *} - c_B)D_B^{\ *} = \left[(\frac{t_1t_2}{t_1+t_2} + c_A - c_B)/3 \right] \left[\frac{1}{3} + \frac{t_1+t_2}{3t_1t_2}(c_A - c_B) \right]$$
$$= \frac{1}{9}\left[\frac{t_1t_2}{t_1+t_2} + \frac{t_1+t_2}{t_1t_2}(c_A-c_B)^2 \right] \quad (20)$$

We can further modify the above model by changing the interval $[0,1]$ to the interval $[0,a]$, as can be seen in Figure 5. If $[0,1]$ indicates the time interval of one hour, then $[0,a]$ can indicate the time interval for several hours, for example, half an hour (when $a=0.5$) or two hours (when $a=2$).

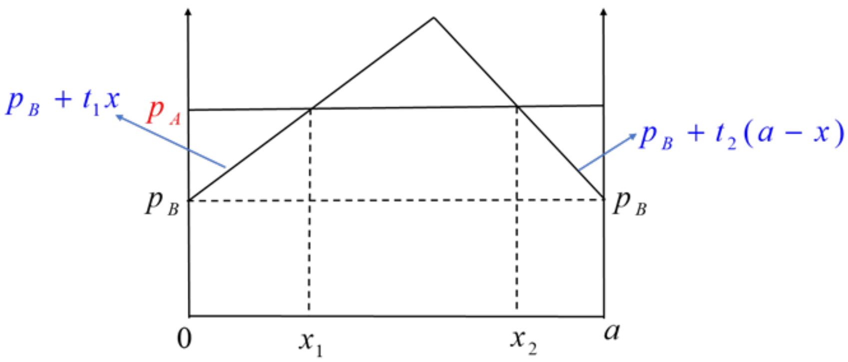

Figure 5. The equilibrium of the traditional parking lot and shared-parking lot in $[0,a]$

Follow the same logic, the equilibrium prices can be determined by equation (21) and (22),

$$p_A^* = (\frac{2at_1t_2}{t_1+t_2} + 2c_A + c_B)/3 \quad (21)$$

$$p_B^* = (\frac{at_1t_2}{t_1+t_2} + c_A + 2c_B)/3 \quad (22)$$

θ is still defined as the price differentiation between the traditional parking lot A and the shared-parking lot B which is expressed by equation (23). If the marginal operating costs of two kinds of parking lots are equal ($c_A = c_B$), θ can be simplified as θ^0 expressed by equation (24).

$$\theta = p_A^* - p_B^* = (\frac{at_1t_2}{t_1 + t_2} + c_A - c_B)/3 \tag{23}$$

$$\theta^0 = p_A^* - p_B^* = \frac{at_1t_2}{3(t_1 + t_2)} \tag{24}$$

Then, we can see that θ^0 is not only related with early arrival cost coefficient t_1 and late arrival cost coefficient t_2, it is also related with a (the length of one time interval), which further reflects the flexibility or the degree of inconvenience for public users parking at shared-parking lots. Longer one time interval is, more inconvenience is brought to public users. Then, the price differentiation of the traditional parking lot and the shared-parking lot should be larger to make the shared-parking lot more attractive, so that the shared-parking lot can compete for public users in the market.

2.3. A special case for equal penalty cost and equal operation cost

We know that in the classic Hotelling's case, the transportation cost to both firms which are located at two ends of the interval is the same, and each of the two firms serves half the market. In the classic Hotelling model, there only exists one point of the indifferent consumer, which is just located in the middle ($x = 1/2$) of the interval. However, in the case of the competition of the traditional parking lot and the shared-parking lot, there exists two points of indifferent consumers. To draw a better understanding of this particular case, we compare it with the classic Hotelling model by setting $t_1 = t_2 = t$, and $c_A = c_B = c$. Then, the equilibrium prices can be simplified as equation (25) and (26). Equation (27) shows the price differentiation.

$$p_A^* = (\frac{2t_1t_2}{t_1 + t_2} + 2c_A + c_B)/3 = \frac{t}{3} + c \tag{25}$$

$$p_B^* = (\frac{t_1t_2}{t_1 + t_2} + c_A + 2c_B)/3 = \frac{t}{6} + c \tag{26}$$

$$\theta^0 = p_A^* - p_B^* = \frac{t}{3} - \frac{t}{6} = \frac{t}{6} \tag{27}$$

Then, the indifferent public users can be expressed by equation (28) and (29), which are in $1/6$ and $5/6$ part of the time interval.

$$x_1^* = \frac{p_A^* - p_B^*}{t_1} = \frac{1}{6} \quad (28)$$

$$x_2^* = \frac{p_B^* - p_A^* + t_2}{t_2} = \frac{p_B^* - p_A^*}{t_2} + 1 = \frac{5}{6} \quad (29)$$

Therefore, the equilibrium is shown in Figure 6 as follows.

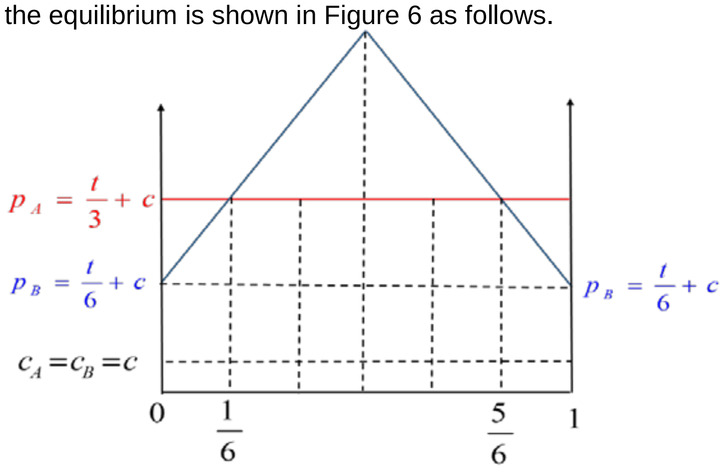

Figure 6. The equilibrium of the traditional parking lot and shared-parking lot in $[0,1]$ when $t_1 = t_2 = t$ and $c_A = c_B = c$

The demand functions of the traditional parking lot A and the shared-parking lot B can be derived from equation (30) and (31). We can see that the traditional parking lot A serves 2/3 of the market, and the shared-parking lot B serves 1/3 of the market.

$$D_A^*(p_A^*, p_B^*) = \int_{x_1^*}^{x_2^*} 1dz = z\Big|_{x_1^*}^{x_2^*} = x_2^* - x_1^* = \frac{2}{3} \quad (30)$$

$$D_B^*(p_A^*, p_B^*) = \int_0^{x_1^*} 1dz + \int_{x_2^*}^{1} 1dz = z\Big|_0^{x_1^*} + z\Big|_{x_2^*}^{1} = x_1^* + (1 - x_2^*) = \frac{1}{3} \quad (31)$$

The profit earned by the traditional parking lot A and the shared-parking lot B can be expressed by equation (32) and (33) as follows.

$$\Pi^{A^*} = (p_A^* - c)D_A^* = \frac{2t}{9} \tag{32}$$

$$\Pi^{B^*} = (p_B^* - c)D_B^* = \frac{t}{18} \tag{33}$$

3. Conclusion and future research

The classic Hotelling model originally framed in the context of location differentiation along a linear market, but in this paper another interpretation is considered that timing is a relevant measure of product differentiation. We study two competing parking lots, a shared-parking lot and a traditional parking lot. Hotelling's 'space' becomes 'parking time' for public users, and 'transport costs' of consumers become 'early arrival costs' and 'late arrival costs' of parking lot public users. A general formulation that has exactly this interpretation is proposed in this paper with equilibrium analyses, to figure out equilibrium parking prices of two kinds of parking lots. Another big difference from the classic Hotelling model is that, we have two points of indifferent consumers in the competition of the traditional parking lot and the shared- parking lot, instead of only one point of the indifferent consumer in the classic Hotelling's case.

In the future, the proposed model can be extended. In this paper, the parking fee of the traditional parking lot p_A and of the shared-parking lot p_B are set as whole day parking fees. This assumption can be relaxed, for example, parking fees are charged per hour and are different between each hour based on the parking demand in the specific time interval. Then, the distribution of parking duration of public users should also be considered (the parking duration cannot be a whole daytime as considered in this paper), that is to say, the situation of the current time interval may make influence on the situations of the following time intervals, the demand for each time interval is not independent and elastic demand or even stochastic demand (Qu et al., 2017) can be considered to make the problem more realistic.

4. Acknowledgment

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5. References

[1] Blunden, W.R., 1971. The Land-Use/Transport System. Analysis and Synthesis.

[2] Chou, S., Lin, S. Li, C., 2008. Dynamic parking negotiation and guidance using an agent-based platform. *Expert System with Application*, 35(3), 805– 817.

[3] Cleveland, D. E., 1963. Accuracy of the periodic check parking study. *Traffic Engineering*, 33(12), 14–17.

[4] Geng, Y., Cassandras, C., 2013. New "smart parking" system based on resource allocation and reservations. *IEEE transactions on Intelligent Transportation Systems*,14 (3), 1129-1139.

[5] Hotelling, H., 1929. Stability in competition. *The Economic Journal*, 39(153), 41–57.

[6] Liu, Z. and Meng, Q., 2014. Bus-based park-and-ride system: a stochastic model on multimodal network with congestion pricing schemes. *International Journal of Systems Science*, 45(5), 994–1006.

[7] Liu, Z., Wang, S., Chen, W. and Zheng, Y., 2016. Willingness to board: a novel concept for modeling queuing up passengers. *Transportation Research Part B: Methodological*, 90, 70–82.

[8] Liu, Z., Yan, Y., Qu, X. and Zhang, Y., 2013. Bus stop-skipping scheme with random travel time. *Transportation Research Part C: Emerging Technologies*, 35, 46–56.

[9] Qu, X., Zhang, J. and Wang, S., 2017. On the stochastic fundamental diagram for freeway traffic: model development, analytical properties, validation, and extensive applications. *Transportation Research Part B: Methodological*, 104, 256–271.

[10] Richardson, A.J., 1974. An improved parking duration study method. *Proceedings of the 7th Australian Road Research Board Conference*, Adelaide, Australia, 397–413.

[11] Shao, C., Yang, H., Zhang, Y. and Ke, J., 2016. A simple reservation and allocation model of shared parking lots. *Transportation Research Part C: Emerging Technologies*, *71*, 303–312.

[12] Small, K. A., 1982. The scheduling of consumer activities: work trips. *American Economic Review*, 72(3), 467–79.

[13] Teodorovic, D., Lucic, P., 2006. Intelligent parking systems. *European journal of Operational Research*, 175(3), 1666–1681.