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# A Pricing Versus Slots Game in Airport Networks[1]

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**Abstract**

This paper considers networks with two or three complementary airports. In each case, two airports independently choose between slot and pricing policies, where slot policies involve grandfather rules. We show that equilibrium policies involve slots when airport profits do not matter and pricing policies when airport profits matter. We further show that the equilibrium slot policies reach the first-best passenger quantities when congestion effects are absent. Otherwise, equilibrium slot policies will lead to excessive and equilibrium pricing policies to too low passenger quantities relative to the first best. Numerical examples indicate that slot policies can be beneficial relative to pricing policies when time valuations are low and vice versa when time valuations are high. The analysis formally distinguishes the sources for the different outcomes under slot and pricing policies by distinguishing between a *variable effect* and a *distribution effect*. The variable effect captures that decision variables are quantities in the case of slot policies and prices in the case of pricing policies. The distribution effect captures that airport slot allocation is based on grandfather rules.

*Keywords:* Airports; slots; pricing; local governments; variable effect; distribution effect

*JEL:* L93; R41; R48

# 1 Introduction

Air transport markets exhibit a couple of defining features. Airports are part of a network industry because airlines connect different airports with each other. The majority of the airports are under government ownership and, especially, local government ownership. For instance, only one percent of airports in North America exhibit private sector participation (Airports Council International, 2017).[1] While air traffic is growing fast and passenger demand is expected to double in the next 20 years (International Air Transport Association (IATA), 2016), airport capacity is growing at a slower rate in many countries. Hong Kong and London areas can serve as examples. Hong Kong International Airport and Heathrow Airport (London) both operate a two-runway system at full capacity. Although capacity expansion is on the way, the completion will take many years from now. Therefore, the capacity problems faced today are likely to become more severe in the future.[2]

The way governments deal with congestion problems caused by the excessive use of scarce capacity in the various parts of the world differs substantially. On the one hand, capacity allocation in the US is largely based on the first-come-first-serve principle. On the other hand, airports outside the US make heavy use of so-called airport slots. An airport slot (or simply "slot") is "a permission given by a coordinator for a planned operation to use the full range of airport infrastructure necessary to arrive or depart at a Level 3 airport on a specific date and time" (IATA, 2017). Essentially, slots are used to restrict airline access to airports and consequently control congestion effects by ensuring that the scheduled air traffic per time unit does not exceed the "declared airport capacity" per time unit (for example, one hour). The latter depends on, for example, the number and layout of runways, meteorological conditions, traffic mix characteristics, and the regulatory environment (Gillen et al., 2016). Only three airports in the US—John F. Kennedy (JFK), LaGuardia (LGA) and Newark Liberty (EWR)—currently make use of slots (Gillen et al., 2016), but there are around 175 airports using slots worldwide (IATA, 2017).

Another measure to mitigate congestion problems at airports is to increase airport prices in a way that reaches desirable levels of passenger demands (for example, Daniel, 1995; Brueckner, 2002; Pels and Verhoef, 2004; Zhang and Zhang, 2006; Basso, 2008; Czerny and Zhang, 2011, 2014a and 2014b). However, the proposed pricing policies can hardly be found in practice. One of the reasons presumably

[1] The share reaches between 11 and 75 percent outside the US (Airport Council International, 2017). But, airports in the US outsource many of their activities to private companies, while, for example, European airports produce many services such as ground handling in-house (Czerny et al., 2016a).

[2] Ball et al. (2010) estimated the cost of air transportation delay in the US to be $31.2 billion in 2007. FlightStats' record of global hub departure performance in April 2017 indicates that the average delay time for all delayed flights across all the international hubs is 57 minutes and only 75 percent of flights are on time. Specifically, the highest average delay time is 91 minutes at ATL, Atlanta, US and the lowest percentage of on-time flights is 42.27 percent at PVG, Shanghai, China. There also appears to be a cluster of international hubs in Asia that have low percentage of on-time flights and most of them locate in China.

is related to the possible distributional effects of such policies arising from higher payments of airlines and passengers to airports.

The present study develops and analyzes a basic model and various model extensions that integrate all the features of airport markets mentioned above in a single framework. The basic model involves a network of two complementary and uncongested airports, where each airport independently and simultaneously chooses between slot and pricing policies and their respective slot quantity and pricing levels. The present study assumes that efficient rationing is guaranteed in the sense that slots are allocated only to passengers with the highest willingness to pay. This feature implies a conservative assessment of pricing policies because the efficient allocation of slots cannot be guaranteed in reality.[3]

Two different sets of airport objectives are considered to capture that airports are owned by the (local) governments and that local governments may wish to limit payments from airlines or passengers to airports and, thus, have limited interest in airport profits. First, governments can pursue the objective of local consumer surplus maximization, which is equivalent to a situation where airport profits do not matter. Second, airports may pursue the objective of local welfare maximization, which is equivalent to a situation where airport profits matter.

The basic model and the model extensions are used to show that slots and pricing policies can be weakly dominant strategies depending on whether governments attach a zero or a unit weight to airport profits, respectively. Policy makers prefer slot policies if airport profits do not matter to them, whereas they prefer pricing policies if airport profits matter. This illustrates that equilibrium policy choices in the form of slot or pricing policies can be explained by the corresponding distribution effects.

It is well-known that price and quantity controls are equally effective in managing negative externalities in a deterministic environment (for example, Weitzman, 1974). The present analysis shows that this result on the equivalence between price and quantity controls also depends on the presence of a centralized regulator. For instance, the analysis of the two-airport network shows that the unique equilibrium in terms of pricing policies does not achieve the set of first-best passenger quantities that maximize the total welfare of both airport regions. It further shows that a set of equilibrium constellations exist in the case of slot policies. Furthermore, this set of equilibrium constellations contains an equilibrium constellation where first-best passenger quantities are reached.

To concentrate on scenarios where unique equilibria in slot and pricing strategies exist, a model extension consisting of three uncongested, complementary and asymmetric airports is considered. The three-airport network involves two active airports with local populations and one inactive dummy airport without local population. This follows a common approach in the literature where the presence of such a dummy airport is implicitly assumed (while the present study applies a more transparent approach where the presence of a dummy airport is an explicit part of the set of modeling assumptions). The presence of the dummy airport allows for the consideration of a more realistic airport network

[3] One way to achieve efficient rationing under slots can be to allow carriers to trade their slots (Brueckner, 2009).

where only a subset of the airports may be slot controlled, the consideration of unique best responses in slot quantities, and the description of a unique equilibrium in slot quantities.

Two effects that are involved in a move from pricing to slot policies are distinguished to transparently identify the causes and intuitions for the differences in local welfares implied by equilibrium policies. First, a so-called *variable effect*, which captures that such a policy change leads to a change in variables in the sense that quantities, not prices, are the decision variables. Second, a so-called *distribution effect*, which captures that such a policy change involves a change in airport revenues. The distribution effect is used to analyze the role of grandfather rules, which make it difficult for airports to internalize slot values. The goal is to find out how variable and distribution effects affect equilibrium policies relative to the first-best policies that maximize the total welfare of all airport regions.

To separate variable and distribution effects, the concept of the *slot price* is developed. In this context, the slot price represents the airport charge that would have to be (but is not) implemented to ensure that airport passenger demand equals the desired slot quantity. This slot price can also have the interpretation of a shadow price. This allows for the consideration of three scenarios. First, a scenario where airports choose slot quantities directly. This represents a scenario where quantities are the decision variables and airport revenues are zero. Second, a scenario where airports choose slot prices. This represents a scenario where prices are the decision variables and airport revenues are still zero. Third, a scenario where airports choose airport charges. This represents a scenario where prices are the decision variables and airport revenues can be strictly positive. Scenarios one and two are used to identify the variable effect of a move from pricing to slot policies. Scenarios two and three are used to identify the distribution effect of a move from pricing to slot policies.

The analysis of the three-airport network in Section 5 abstracts away from congestion effects and reveals a variable effect of zero and a strictly positive distribution effect. The variable effect is zero because the equilibrium airport behaviors are independent of whether slot quantities or slot prices are the decision variables. The distribution effect is positive because airport charges are used to exploit non-local passengers. This exploitation leads to excessive airport charges and possibly a prisoner's dilemma situation. The incentives to exploit non-local passengers are eliminated by slot policies because slots are provided for free by construction to capture the notion of grandfather rules for airport slots. As a result, equilibrium slot policies lead to the first-best result that maximizes the total welfare of all airport regions. This result is conditional on the absence of congestion, however.

Section 6 concentrates on *symmetric* (active) airports, where symmetry is assumed for tractability, and extends the three-airport network scenario by adding congestion effects. The presence of congestion does not change the result that equilibrium policies involve slot policies when airport profits do not matter and pricing policies when airport profits matter. We show that the equilibrium prices in the case of pricing policies can be written as the sum of the equilibrium slot price plus a weighted markup depending on the price elasticities of passenger demands, where this markup is higher when equilibrium prices are compared with slot quantities as decision variables than with slot price as decision variables. In this extended model version, neither slot nor pricing policies will ever reach the first-best solution in equilibrium. Airport charges will still be excessive in the case of pricing policies because of the distortions caused by the distribution effects. Such distribution effects are absent in the case of slots; but, slot quantity choices are distorted in a different way because local policy makers do not take into account how their slot choices will affect non-local passengers, which leads to too loose slot quantities relative to the first-best passenger quantities. Numerical instances are used to derive an understanding of how time valuations affect the relative performance of slot and pricing policies. These instances show that there is a critical time valuation where local welfares are independent of whether slot or pricing policies are used. They further show that local welfares are higher under equilibrium slot policies than under equilibrium pricing policies if time valuations are low relative to this critical value, while local welfares are higher under equilibrium pricing policies than under equilibrium slot policies if time valuations are high relative to this critical value. The numerical instances are further used to illustrate that the distribution effect is positive for low and negative for high time valuations, while the variable effect is always negative.

Other extensions are considered in Section 7. These extensions show that the consideration of a vertically integrated airport is under certain conditions equivalent to a situation with atomistic airlines. More specifically, these conditions involve two assumptions. The first is that local airlines exclusively serve local passengers. The second is that airports attach a unit weight to both local consumer surplus and the profits of their local airlines. The extensions are further used to show that equilibrium policy choices are robust with respect to changes in the timing of airport decisions.

The present study contributes to various strands of the literature. It contributes to the literature on slots versus pricing policies. Brenck and Czerny (2002), Czerny (2008, 2010) and De Palma and Lindsey (2018) considered an environment with uncertain demand and congestion costs. Based on Weitzman's (1974) seminal study, Czerny (2010) found that pricing is beneficial relative to slot policies from the social viewpoint when demands take linear forms because (i) demand functions will be steeper than marginal external congestion cost functions under these conditions and (ii) uncertainty in the congestion costs can lead to a negative correlation between demands and marginal external congestion costs. Slots, however, can be beneficial in the case of quadratic marginal external congestion costs. Czerny (2010) also considered a three-airport network and found that the existence of airport networks can improve the benefits of pricing policies relative to slots. In contrast to the present study, he considered a centralized airport system in the sense that airports are assumed to fully coordinate their congestion policies. De Palma and Lindsey (2018) extended this analysis by also capturing scenarios where the slot constraint may not be binding amongst other things. Brueckner (2009) considered an environment with small and large airlines and showed that slot trading or auctions can lead to an efficient outcome that would only be achievable with pricing if prices would be differentiated across carriers. Daniel (2014) highlighted the challenges of combinatorial auctions if the slots of multiple airports would be auctioned simultaneously. Basso and Zhang (2010) found that slot auctions are preferred by the airports and lead to higher total traffic relative to pricing when the social weight attached to airport profits exceeds the unit weight. Czerny (2007) and Basso, Figueroa, and Vásquez (2017) considered price versus quantity controls in the case of monopoly regulation. The main contribution here is to cover the possibility of decentralized decisions upon slot and pricing policies and to capture the possibility that the weight attached to airport profits can be lower than the unit weight, which captures scenarios where governments are reluctant to impose additional payments on airlines and passengers.

The study further contributes to the strands of the literature on local objectives and the tolling of passengers or customers from other legislations. De Borger, Proost, and Van Dender (2005) considered tax competition in the presence of congestion and rival networks. They showed that transit tolls can lead to large welfare gains from the viewpoint of the local economy. De Borger, Dunkerley, and Proost (2007) concentrated on complementary transport networks and showed that welfare can be increased in the absence of tolling. Mantin (2012) considered endogenous, local public and private ownership structures in a complementary airport network. He found that countries may be caught in a prisoner's dilemma situation with private airports and too high prices from the viewpoint of the aggregate economy because privatization is used to exploit non-local passengers. Czerny, Höffler and Mun (2014) considered endogenous, local ownership structures in a network with rival seaports. They found that countries may be caught in a prisoner's dilemma situation because they keep seaports under public ownership in order to protect local customers from excessive port prices, while private seaports and higher port prices would be helpful to exploit non-local users and increase local welfares. Wan and Zhang (2013) considered port competition as part of the rivalry between alternative intermodal transportation chains. They focused on quantity-competition model and found that an increase in road capacity by an intermodal chain will likely benefit its port while negatively influence the rival port. Wan, Basso and Zhang (2016) investigated the strategic investment decisions of local governments on regional landside accessibility in the context of seaport competition. They found that when ports are public, an increase in accessibility investment reduces port regions' welfare but improves inland regions' welfare. When ports are private and captive shipper's utility is high enough, an increase in accessibility benefits the rival port region while it harms the inland. Studies typically concentrate on local welfare maximization as the only objective of policy makers, while our study extends this view and also captures the possibility that some policy makers may attach a unit weight to consumer surplus and a zero weight to infrastructure profits.[4]

[4] See Wan, Zhang and Li (2018) for an excellent survey of papers concerned with port competition and the congestion of landside transport facilities.

The present study finally contributes to the growing literature on congestion management in airport networks. Pels and Verhoef (2004) is the study that is most closely related to the present one. It concentrated on a network of two complementary airports, and used numerical examples to illustrate that equilibrium pricing strategies differ from the first-best solution. Benoot, Brueckner and Proost (2012) analyzed the pricing of congested rival airports that serve both domestic and intercontinental passengers. They found that the presence of intercontinental passengers can distort prices in a way that leads to a welfare loss. Silva, Verhoef, and van den Berg (2014) considered congested airports and endogenous airline networks. They found that airport pricing policies cannot ensure socially efficient airline network formations. Lin (2016) investigated an airport network with one congested hub airport and several spoke airports that belong to different countries. He found that discriminatory prices for non-stop and transfer passengers can be used to maximize global welfare. Lin and Zhang (2017) considered a hub-and-spoke network and found that per-flight prices and discriminatory passenger prices are required to achieve the welfare-maximizing quantities of local and transfer passengers.

The contribution of the present study to these strands of the literature is to analyze the endogenous choice of slot and pricing policies by local policy makers with varying policy objectives in a unifying airport network. It further contributes by formally distinguishing between the variable and distribution effects that are involved in a move from pricing to slot policies.

## 2 Basic Model

Passengers travel between two airports, $A$ and $B$. Airport $i$'s passenger quantities are denoted by $q_{ij}$ for those who originate from $i$ with $i = A, B$ and $j \neq i$ (if $i$ and $j$ appear together, $j \neq i$ is assumed to hold true).[5] Passenger quantities are strictly positive on both routes, that is, $q_{ij} > 0$. The benefits of passengers with origin airport $i$ are denoted by $B_i(q_{ij})$. Benefits are strictly concave in the sense that $B_i''(q_{ij}) < 0$ by assumption.

Airports $A$ and $B$ can choose between two policy measures, which are slots, denoted by $S$, and pricing, denoted by $P$. Let $Q_i$ with $Q_i = q_{AB} + q_{BA}$ denote the total traffic at airport $i$, where the complementarity between airports implies $Q_A = Q_B$. There is an upper limit on the number of passengers at each airport, which is denoted by $\overline{Q}_i$. Together with perfect airport complementarity, this implies $Q_i \leq \min\{\overline{Q}_A, \overline{Q}_B\}$. Let $\phi_i$ with $\phi_i = S, P$ denote the policy variable. The upper limit $\overline{Q}_i$ is a function of the policy strategy, that is, $\overline{Q}_i = \overline{Q}_i(\phi_i)$. The upper limit $\overline{Q}_i(\phi_i)$ is finite in the case of slots, $\phi_i = S$, while it is infinite in the case of pricing, $\phi_i = P$.

An integrated airport that offers both infrastructure and flight services is considered in this basic model to concentrate on *airport networks* by avoiding the presence of airline profits, the need to

[5] Passengers only travel between $i$ and $j$. The notation could therefore be economized by choosing $q_i$ instead of $q_{ij}$. The more complex notation is, however, maintained because it will be useful in the extended scenarios considered in Section 5 where a third airport $C$ will be involved.

distinguish between local and non-local airline profits, and the need to distinguish the potentially differing weights policy makers attach to airline and airport profits.[6] The airport charges a non-negative ticket price $R_i$ to its passengers. Despite the assumption that air services are carried out by airports, we capture the distributional effects of slot policies based on grandfather rules by assuming that $R_i = 0$ in the case of slot policies. Thus, airports cannot earn from selling slots (as is the case under the IATA's Worldwide Scheduling Guidelines) and, thus, have zero airport revenues in the case of slot policies.[7] The airports' costs are all normalized to zero, which together with the assumption of non-negative ticket prices ensures that we can abstract away from problems of airport cost recovery under slot and pricing policies.[8]

Slots impose an upper limit on an airport's total traffic and non-negative (shadow) prices, denoted by $r_i(\phi_i)$, are used to indicate slot prices in the case of slot policies. Using the shadow prices $r_i(\phi_i)$, the airport charges can be written as $R_i = R_i(r_i(\phi_i), \phi_i)$. For $\phi_i = S$, $r_i(S) \geq 0$ is the shadow price of slots, simply called *slot price* hereafter, under efficient rationing where slots are allocated to the passengers with the highest willingness to pay. To capture that airport revenues from slot supply are zero under grandfather rules, $R_i(r_i(S), S) = 0$ is considered. For $\phi_i = P$, $r_i(P) = R_i(r_i(P), P) \geq 0$, which means that airport usage can involve strictly positive airport revenues in the case of pricing policies.

Efficient rationing where passengers with the highest willingness to pay are served first can be guaranteed under pricing policies and is assumed to be present also in the case of slot policies. However, efficient rationing cannot be guaranteed under slots and the IATA rules based on grandfathering. Altogether, this provides a conservative assessment of pricing policies relative to slot policies.

We consider a one-shot game, where airports simultaneously choose between slot and pricing policies as well as slot quantities and prices, respectively. Two sets of objectives are considered: first, local consumer surplus maximization, where local airports attach a zero weight to their airport profits (airport profits do not matter); second, local welfare maximization where local airports attach a unit weight to airport profits (airport profits matter). The airport objectives are considered as given and equal for the two airports, thus, no asymmetries regarding airport objectives are considered.

[6] We will higlight in the extensions section that the present scenario resembles a scenario where airline markets are atomistic, local passengers use their local airlines only, slots are efficiently allocated to airlines by auctions, and policy makers attach a unit weight to airline profits.

[7] That airport charges can be far from market clearing levels in reality is indicated by the high market prices for airport slots. For instance, one pair of take-off and landing slots at London Heathrow was sold for USD75 million in 2016.

[8] The consideration of integrated airport services also helps to concentrate on airport cost recovery because airport subsidies may be required to implement the first-best outcome when airline markets are oligopolistic (for example, Pels and Verhoef, 2004).

# 3 Equilibrium Policies

It is well known that policy makers are indifferent between price and quantity controls if demand and cost functions are deterministic, firms are considered in isolation and the objective is welfare maximization (for example, Weitzman, 1974; Stavins, 1996; Czerny, 2010; De Palma and Lindsey, 2018). In this section we analyze the role of airport networks and airport objectives for the choices of pricing and slot policies.

Since ticket prices are equal to airport charges, local consumer surplus, denoted by $CS_i$, can be written as

$$CS_i(q_{ij}, r_A(\phi_A), r_B(\phi_B), \phi_A, \phi_B)$$
$$= B_i(q_{ij}) - q_{ij} \cdot (R_A(r_A(\phi_A), \phi_A) + R_B(r_B(\phi_B), \phi_B)) \,. \tag{1}$$

The first term on the right-hand side is the benefit of passengers originating at airport $i$ depending on the passenger quantities $q_{ij}$. The second term is the total payment of passengers with origin $i$ who travel between $A$ and $B$ to both airports.

If airport profits do not matter, then the airports' objectives are to maximize local consumer surplus. We first show that airport $i$'s local consumer surplus under slots is higher than under pricing for any given passenger quantities and prices where $r_i(S) = r_i(P)$ and any given instances of $\phi_j$ and $r_j(\phi_j)$. If $r_i(S) = r_i(P)$ and given $R_i(r_i(S), S) = 0$ and $R_i(r_i(P), P) = r_i(P)$, local consumer surplus under slots can be written as

$$CS_i(q_{ij}, r_i(S), r_j(\phi_j), S, \phi_j) = CS_i(q_{ij}, r_i(P), r_j(\phi_j), P, \phi_j) + q_{ij} \cdot r_i(P). \tag{2}$$

The right-hand side is the consumer surplus under pricing plus the airport profit under pricing for any $r_i(P) \geq 0$. This shows that local consumer surplus can be increased by the use of slots relative to pricing because local passengers save payments $q_{ij} \cdot r_i(P)$ under slots when $r_i(S) = r_i(P)$, where the equality of prices $r_i(S)$ and $r_i(P)$ implies that passenger quantities would remain the same despite the policy change. Consider the case where $r_i(P)$ is chosen to maximize local consumer surplus. Since slots are preferred for any $r_i(P) \geq 0$, slots are also preferred if $r_i(P)$ is chosen to maximize local consumer surplus. Slots are weakly preferred if $r_i(P) = 0$, while slots are strictly preferred if $r_i(P) > 0$.

Local airport profit, denoted by $\pi_i$, can be written as

$$\pi_i(Q_i, r_i(\phi_i), \phi_i) = Q_i \cdot R_i(r_i(\phi_i), \phi_i), \tag{3}$$

and local welfare, denoted by $W_i$, can be written as the sum of local consumer surplus and local airport profit:

$$W_i(q_{ij}, q_{ji}, r_A(\phi_A), r_B(\phi_B), \phi_A, \phi_B) = CS_i(q_{ij}, r_i(S), r_j(\phi_j), S, \phi_j) + \pi_i(Q_i, r_i(\phi_i), \phi_i). \tag{4}$$

If airport profits matter, the airports' objectives are to maximize local welfare. We first show that airport $i$'s local welfare under pricing is higher than under slots for any given passenger quantities and prices where $r_i(P) = r_i(S)$ and any given instances of $\phi_j$ and $r_j(\phi_j)$. If $r_i(P) = r_i(S)$ and given $R_i(r_i(S), S) = 0$ and $R_i(r_i(P), P) = r_i(P)$, local welfare under pricing can be written as

$$W_i(q_{ij}, q_{ji}, r_i(P), r_j(\phi_j), P, \phi_j) = W_i(q_{ij}, q_{ji}, r_i(S), r_j(\phi_j), S, \phi_j) + q_{ji} \cdot r_i(P), \tag{5}$$

where the right-hand side is the welfare under slots plus the airport profit under pricing. This shows that local welfare can be increased by the use of pricing relative to slots because airport profits are increased by the revenues earned from non-local passengers, $q_{ji} \cdot r_i(P)$, under pricing. Consider the case where $r_i(S)$ is chosen to maximize local welfare. Since pricing is preferred for any $r_i(S) \geq 0$, pricing is also preferred if $r_i(S)$ is chosen to maximize local welfare. Pricing is weakly preferred if $r_i(P) = 0$, while pricing is strictly preferred if $r_i(P) > 0$.

Equilibrium policies in terms of policy variables $\phi_i$ can be summarized as:

**Proposition 1** *If airport profits do not matter, slot policies $\phi_A = \phi_B = S$ form an equilibrium in (weakly) dominant strategies, while pricing policies $\phi_A = \phi_B = P$ form an equilibrium in (weakly) dominant strategies if airport profits matter. Whether the equilibrium policy choices are determined by dominant or weakly dominant strategies depends on whether $r_i(P) > 0$ or $r_i(P) = 0$, respectively.*

This proposition highlights that equilibrium policies depend on whether airport profits matter or do not matter and it can also be used to highlight the role of airport networks for equilibrium policies. If airport profits do not matter, slot policies are the airports' preferred choices. Relevant to this study, which concentrates on the role of airport networks on policy choices, is that this is independent of whether airports are in a network or isolated. This is because the consumer surplus of own passengers is independent of the number of passengers originating from airport $j$, $q_{ji}$, in the basic model as demonstrated by the consumer surplus expression in (2). Thus, airport networks have no effect on policy choices in terms of $\phi_i$ when airport profits do not matter. The picture changes if airport profits matter. In this case, pricing policies *are* the airports' preferred choices when they are in a network, while airports are *indifferent* between slot and pricing policies when airports are isolated. This is because the local welfares depend on the number of passengers originating from airport $j$, $q_{ji}$, through the revenue they generate at the own airport as shown by the welfare expression in (5) when airports operate in a network. Thus, airport networks are relevant to policy choices in terms of $\phi_i$ when airport profits matter. Airport networks therefore eliminate the equivalence between quantity and pricing based policies that prevails in the case of isolated airports and perfect information.

Proposition 1 demonstrates that slots and pricing schemes can both occur as equilibrium solutions in airport networks. This justifies the investigation of both policies in the following section.

# 4 Equilibrium Slot Quantities Versus Equilibrium Prices

Given efficient rationing under both pricing and slot policies, the passenger demands for flights between $i$ and $j$, denoted by $D_{ij}(r_A, r_B)$, are determined by the equilibrium conditions

$$B_i'(q_{ij}) - (r_A + r_B) = 0 \tag{6}$$

for $\phi_i = S, P$, where we concentrate on those cases where passenger demands, $D_{ij}(r_A, r_B)$, are strictly positive. In equilibrium, the passengers' marginal benefit of flying between $A$ and $B$ is equal to the sum of $r_A$ and $r_B$ because every passenger flies between $A$ and $B$ and therefore will be charged twice. The strict concavity of the benefit function, $B_i(q_{ij})$, ensures the existence of a unique set of demands $D_{AB}(r_A, r_B)$ and $D_{BA}(r_A, r_B)$ with $\partial D_{ij}/\partial r_A = \partial D_{ij}/\partial r_B = 1/B_i''(q_{ij})$, where the first equality appears because it is the sum of the prices that matters to passengers.[9]

Denote the equilibrium local welfares under pricing by $W_i(P)$ and the equilibrium local welfares under slots by $W_i(S)$. A policy change from pricing to slots has two effects which we call *distribution effect* and the decision variable effect, simply called *variable effect.* Recall equation (4) which shows that, for given passenger quantities, local welfares under slots are lower because the revenues from non-local passengers are zero. Thus, a change from pricing to slots can affect equilibrium passenger quantities because it involves zero revenues from non-local passengers by construction. Furthermore, a change from pricing to slots means that airports choose quantities rather than prices as decision variables. Whether quantities or prices are the decision variables makes a difference. This is because a change of the own slot quantity may keep the total passenger quantity at the other airport unchanged if the own slot quantity is at least as high as the other airport's slot quantity, while a change in the own price will always change the own and the other airport's passenger quantity when both airports adopt pricing policies.

To formally separate variable and distribution effects, a third policy regime indicated by $\phi_i = SP$ is introduced. This regime involves the choice of slot prices when airport revenues are zero by assumption. The total welfare effect of a move from pricing to slot policies is given by the difference $W_i(S) - W_i(P)$. Adding and deducting welfares when airports use slot prices as the decision variables given by $W_i(SP)$ leads to

$$W_i(S) - W_i(P) = \underbrace{W_i(S) - W_i(SP)}_{= \text{variable effect}} + \underbrace{W_i(SP) - W_i(P)}_{= \text{distribution effect}}, \tag{7}$$

which shows that the total change in equilibrium welfare can be written as the sum of the variable and distribution effects. Variable and distribution effects are defined in terms of local welfares, while slots can be equilibrium policies only if the airport profits do not matter. This is a consistent approach because local welfares are equal to local consumer surpluses in the case of slot policies.

[9] Here and hereafter it is assumed that passenger demands are smooth functions of airport charges.

Since airport costs are normalized to zero, the sum of welfares, $W_A + W_B$, is given by the sum of benefit functions $B_A(q_{AB}) + B_B(q_{BA})$. The first-best passenger quantities are determined by the first-order conditions $B_i'(q_{ij}) = 0$ for $i = A, B$ and are denoted by $q_{ij}^*$, while the first-best total passenger quantity is denoted by $Q_i^*$ with $Q_i^* = q_{AB}^* + q_{BA}^*$. The equilibrium conditions in (6) imply the first-best prices $r_A = r_B = 0$, which reflects the zero airport costs. Congestion effects are absent in the present scenario. Thus, it is intuitive that zero prices maximize local welfares, as well as the total welfare. The following derives equilibrium passenger quantities under slots, slot prices, and prices as decision variables relative to the first-best passenger quantities implied by zero prices.

### 4.1 Equilibrium slot quantities

To analyze equilibrium slot quantities, it is useful to understand how changes in total passenger quantities, $Q_i$, translate into changes in local passenger quantities, $q_{ij}$ (the proof is regulated to Appendix A.1):

**Lemma 1** *The effect of an increase in the total passenger quantities $Q_A$ or $Q_B$ on local passenger quantities $q_{ij}$ can be written as:*

$$\frac{dq_{ij}}{dQ_i} = \frac{dq_{ij}}{dQ_j} = \frac{B_j''}{B_A'' + B_B''} > 0. \tag{8}$$

The first equality appears because, regardless of airports' slot strategies, $Q_i$ is always equal to $Q_j$. The inequality shows that an increase in the total passenger quantities will always be associated with both an increase in $q_{AB}$ and an increase in $q_{BA}$.

To derive the best responses in terms of slot quantities, it is useful to write the local passenger quantities as a function of the total passenger quantities, that is, $q_{ij} = q_{ij}(Q_i)$. The objective function can then be rewritten as $W_i(q_{ij}(Q_i))$. Consider the cases (i) $Q_i = \overline{Q}_i$, (ii) $Q_i = \overline{Q}_j$ and (iii) $Q_i < \overline{Q}_A, \overline{Q}_B$.

Part (i) implies that the total passenger quantity is equal to the own slot quantity. Local welfare can then be written as $W_i(q_{ij}(\overline{Q}_i))$ and best responses are given by the first-order condition $W_i'(q_{ij})q_{ij}'(\overline{Q}_i) = B'(q_{ij})q_{ij}'(\overline{Q}_i) = 0$, which means that slots are optimal if the slot price is equal to zero. Part (ii) implies that the total passenger quantity is equal to the other airport's slot quantity. Local welfare can then be written as $W_i(q_{ij}(\overline{Q}_j))$, which means that an increase in the own slot quantity keeps local welfare unchanged. Part (iii) implies that the total passenger quantity is lower than the own and the other airport's slot quantity. Local welfare can then be written as $W_i(q_{ij}(Q_i))$, which means that an increase in the own as well as the other airport's slot quantity keeps local welfare unchanged. Altogether, this implies:

**Proposition 2** *If there are two airports and airport profits do not matter, the full set of equilibrium slot quantities is given by*

$$(\overline{Q}_A, \overline{Q}_B) \in \{\overline{Q}_A, \overline{Q}_B : \overline{Q}_A = \overline{Q}_B < Q^*, \overline{Q}_A, \overline{Q}_B \geq Q^*\}. \tag{9}$$

**Proof.** The (Nash) equilibrium slot quantities are defined by the airports' best responses where airports have no incentive to either increase or decrease slot quantities when their objectives are to maximize local consumer surplus. First, consider $\overline{Q}_A = \overline{Q}_B < Q^*$. In this case, airports have no incentive to individually increase slot quantities because the own total passenger quantity would be limited and determined by the other airport's slot quantity and, thus, remains unchanged. Airports also have no incentive to decrease slot quantities because this would further reduce their already too low local passenger quantities. Therefore, a scenario where both airports choose the same slot quantities and these slot quantities imply total passenger quantities that are smaller than the first-best total passenger quantity, constitutes an equilibrium solution. Second, consider $\overline{Q}_A, \overline{Q}_B \geq Q^*$. In this case, the first-best passenger quantities are implemented and slot prices are equal to zero. This means that an increase in slot quantities cannot further reduce slot prices and, thus, leaves local welfares unchanged. Airports also have no incentive to decrease slot quantities beyond the first-best passenger quantity because this would reduce local welfares. This shows that both cases $\overline{Q}_A = \overline{Q}_B < Q^*$ and $\overline{Q}_A, \overline{Q}_B \geq Q^*$ constitute equilibrium constellations.

To show that these constellations determine the full set of equilibrium cases, consider $\overline{Q}_i < \overline{Q}_j < Q^*$. In this case, airport $i$ could increase slot quantity all the way to $\overline{Q}_j$, which would increase local welfare because $\overline{Q}_j < Q^*$. Thus, this is not an equilibrium constellation. Finally, consider $\overline{Q}_i < Q^* \leq \overline{Q}_j$. In this case, airport $i$ could increase its slot quantity all the way to first-best passenger quantity $Q^*$, or even beyond first-best total passenger quantity, to increase and maximize local welfare. Thus, this cannot be an equilibrium constellation, which completes the proof. ■

The full set of equilibrium slot quantities given by Proposition 2 implies that there are infinitely many sets of equilibrium slot quantities. Therefore, the set of equilibrium slot quantities is not uniquely defined. However, the set of equilibrium slot quantities where $\overline{Q}_A = \overline{Q}_B = Q^*$ might be considered as a focal point. This equilibrium set implies a scenario where both airports happen to choose slot quantities that are also the first-best passenger quantities as their best responses in equilibrium.

### 4.2 Equilibrium slot prices

In the previous subsection, slot quantity $\overline{Q}_i$ was the decision variable and airport profits were considered to be zero. In this subsection, the price is used as the decision variable and, consistently, airport profits are considered to be zero, which altogether constitutes the policy regime $\phi_i = SP$. Both regimes capture the grandfather rights but involve different variables. And this enables the comparison of the outcomes under two regimes by identifying the variable effect.

Zero airport profits mean that local consumer surplus is identical to local welfare. Local welfare can be written as

$$W_i(D_{ij}(r_A, r_B)) = CS_i(D_{ij}(r_A, r_B)) = B_i(D_{ij}(r_A, r_B)). \tag{10}$$

This equation implies that local welfare is identical to the local passengers' benefit from travelling. Assume that the best responses in terms of slot prices are determined by the first-order conditions, $\partial W_i/\partial r_i = 0$ and that the map of best responses in terms of the slot price is a contraction, which are maintained assumptions here and hereafter.[10] Using equilibrium demand conditions in (6), the first-order conditions for the best responses in terms of the slot prices can be written as

$$(r_A + r_B) \cdot \frac{\partial D_{ij}(r_A, r_B)}{\partial r_i} = 0. \tag{11}$$

The left-hand side shows how an increase in own and the other airport's slot price and the corresponding reduction in slot quantities affects own passengers' benefits from travelling. Because the demand of local passengers is strictly decreasing in the own airport charge (that is, $\partial D_{ij}(r_A, r_B)/\partial r_i < 0$), the optimality of best response requires that the sum of slot prices, $r_A + r_B$, is equal to zero. Since slot prices are non-negative, given any $r_j$, the best response for the own airport is to always choose $r_i = 0$. In other words, any positive slot price $r_i$ will decrease local welfare relative to the zero price. This is because local welfare is increasing in local passenger quantity $q_{ij}$, and the local passenger quantity is decreasing in the sum of slot prices, which is decreasing in the own slot price. Together with Proposition 2, this implies that:[11]

**Proposition 3** *If there are two airports and airport profits do not matter, zero slot prices establish a unique equilibrium in dominant strategies.*

Since zero slot prices yield the first-best passenger quantities, this further implies:

**Proposition 4** *If there are two airports and airport profits do not matter, the unique pair of equilibrium slot prices yields the first-best passenger quantities.*

Since the set of equilibrium slot quantities is not uniquely defined, this further implies:

**Corollary 1** *If there are two airports and airport profits do not matter, the variable effect is not uniquely defined.*

Consider the focal point where $\overline{Q}_A = \overline{Q}_B = Q^*$. In this special case, both airports' equilibrium slot quantities are exactly equal to the first-best passenger quantities, which implies a variable effect equal to zero by Proposition 4.

[10] Best responses are not uniquely defined if the other airport charges such an extremely high price that local passenger quantity is zero. However, we concentrate on strictly positive passenger quantities and rule out this case accordingly.

[11] See Vives (1999) for an excellent discussion of the role of the contraction condition for uniqueness of equilibrium solutions.

### 4.3 Equilibrium prices

By Proposition 1, pricing policies are relevant strategies if airport profits matter. In this case, local welfare can be written as

$$W_i(r_A, r_B) = B_i(r_A, r_B) - r_j \cdot D_{ij}(r_A, r_B) + r_i \cdot D_{ji}(r_A, r_B). \tag{12}$$

Best responses in terms of prices are assumed to be determined by the first-order conditions $\partial W_i / \partial r_i = 0$. Using the equilibrium demand conditions (6), these first-order conditions can be written as

$$r_i \cdot \frac{\partial D_i}{\partial r_i} + D_{ji} = 0. \tag{13}$$

Consider zero prices $r_i = 0$. This instance violates the first-order condition (13) because the first term on the left-hand side becomes zero, while the second term, $D_{ji}$, is strictly positive by assumption. The second-order conditions for best responses, $\partial^2 W_i / \partial r_i^2 < 0$, imply that best responses in terms of prices are strictly positive. Rearranging the first-order conditions in (13) yields

$$r_i = \frac{D_{ji}}{D_i} \cdot \left| \frac{D_i}{\partial D_i / \partial r_i} \right|. \tag{14}$$

The right-hand side is the inverse semi-price elasticity of own total demand with respect to the own price weighted by the share of non-local passengers. It shows that equilibrium prices tend to be higher if the share of non-local passengers is relatively high and if the own total demand is relatively inelastic. If airport profits matter, the equilibrium airport charge is strictly positive because it balances the local welfares of isolated airports with the profits from non-local passengers that exist in airport networks.

Given that the contraction condition for best responses in terms of prices is satisfied, this altogether leads to:

**Proposition 5** *If there are two airports and airport profits matter, the unique pricing equilibrium implies that airport charges strictly exceed first-best prices.*

By Corollary 1, aggregate welfare may or may not be maximized in equilibrium when airport profits do not matter although the equilibrium scenario where aggregate welfare is maximized could be considered as a focal point. Yet, if airport profits matter and they directly pursue local welfare maximization, it is clear that they end up with excessive airport charges, fewer total passengers and a reduction in aggregate welfare relative to the first-best solution by Proposition 5.

Airports can even be caught in a prisoner's dilemma situation in the case of pricing. For instance, if airport profits matter and airports are symmetric, they choose prices to exploit non-local passengers. But, in equilibrium, both airports choose strictly positive prices and therefore the local welfare gain from charging positive prices on non-local passengers is exactly equal to the local welfare loss that arises because the other airport charges the same on own passengers. In this situation, the gains and losses from positive airport prices cancel each other out, while aggregate welfare is reduced and, because of symmetry, also local welfares are reduced relative to the case where airport profits do not matter and both airports choose slot policies.

A prisoner's dilemma situation may or may not occur under the circumstances determined by the basic model. For instance, consider the extreme asymmetric case where the number of passengers originating from the own airport approaches zero, while the number of non-local passengers is relatively large. In this scenario, the own airport charges a strictly positive price when airport profits matter, while the other airport will charge a price that approaches zero. A prisoners' dilemma situation will not occur because the own airport mostly gains from charging a strictly positive price and derives revenue from other passengers. Nevertheless, aggregate welfare is not maximized in this scenario either.

Strictly positive airport charges mean that there are too few passengers relative to the first-best solution, which further implies:

**Corollary 2** *If there are two airports and airport profits matter, the distribution effect is strictly positive, that is,* $W_i(SP) - W_i(P) > 0$.

Altogether, Corollaries 1 and 2 show that the choice of policy variables matters to the outcomes not only because it matters whether quantities or prices are the decision variables but also because of the grandfather rules associated with slot policies and the resulting positive distribution effect. The latter is positive because slots and grandfather rules can help to avoid the incentives to exploit non-local passengers.

## 5 Three-airport Network

The variable effect is not uniquely defined in the above case of a two-airport network. To derive more conclusive insights about the variable effect, this section extends the airport network by adding a third airport, airport $C$, to the network. Airport $C$ is different from airports $A$ and $B$ because local passengers are absent at this airport and the airport charge and slot price are normalized to zero, where the latter implies that binding slot constraints are absent at airport $C$. The present study further concentrates on origin-destination passengers by assuming that the number of passengers traveling between airports $A$ and $B$ via $C$, or between $A$ and $C$ via $B$ is zero.

This three-airport network unifies two common airport network structures considered in the literature. First, if the number of passengers from $A$ and $B$ traveling to airport $C$ approaches zero, this framework reduces to the two-airport network considered above and, for example, by Pels and Verhoef (2004) and Mantin (2012). Second, if the number of passengers traveling between airports $A$ and $B$ approaches zero, the two-airport network further reduces to the single-airport (or isolated-airport as we call it in the Introduction Section) framework (to be more precise, two single-airport cases are considered in this situation because airports $A$ and $B$ each represent a single-airport scenario) which is considered in the vast majority of airport studies (for example, Brueckner, 2002; Czerny, 2006; Zhang and Zhang, 2006; Basso and Zhang, 2010). These single-airport studies implicitly assume the presence of an airport $C$ or, perhaps, multiple airports of the $C$-type as travelling destinations, without explicitly mentioning it. In this study, the presence of an airport of type $C$ is made explicit and its important role is analyzed in full detail.

Let $q_{iC} > 0$ denote the quantity of passengers who travel between airports $i$ and $C$. The total passenger quantity at airports $A$ and $B$ is given by $Q_i = q_{ij} + q_{ji} + q_{iC}$. The benefits of passengers with origin airport $i$ are denoted by $B_i(q_{ij}, q_{iC})$. Benefits are strictly concave in the sense that $\partial^2 B_i/\partial q_{ij}^2$, $\partial^2 B_i/\partial q_{iC}^2 < 0$ and $\partial^2 B_i/\partial q_{ij}\partial q_{iC} = 0$ by assumption. The latter, $\partial^2 B_i/\partial q_{ij}\partial q_{iC} = 0$, implies that air services are neither substitutes nor complements, which is for simplicity.

The characterization of equilibrium policies in Proposition 1 is independent of the presence of the third airport. This is because local consumer surplus and local welfare under slots or pricing follow the same structures as the ones given by (2) and (5), respectively. There are two differences between a two-airport and three-airport network in terms of local consumer surplus. The first difference is that the benefit of travelling in the presence of three airports is given by $B_i(q_{ij}, q_{iC})$, which is determined not only by the own passengers travelling to the other airport but also by the own passengers travelling to the third airport. The second difference involves the extra payments $q_{iC} \cdot r_i(P)$ of own passengers to the own airport in the case of pricing policies. There is only one difference between a two-airport and three-airport network in terms of local welfare, which is that the benefit of travelling in the presence of three airports is given by $B_i(q_{ij}, q_{iC})$ (payments $q_{iC} \cdot r_i(P)$ are cancelled out in the calculation of welfare). Observe that when the other airport's strategy and local passenger quantity are given, local consumer surplus is always decreased if the own airport moves from slots to pricing because local passengers will have to pay under pricing, while local welfare is always increased if the own airport moves from slots to pricing because there is an extra revenue from charging non-local passengers. The proof of Proposition 1 and the corresponding results can therefore be extended to the case with a three-airport network, which leads to:

**Proposition 6** *Equilibrium choices of policy variables $\phi_i$ are unaffected by the presence of airport $C$.*

Thus, slots and pricing policies can both be equilibrium solutions depending on whether airport profits do not matter or matter, respectively, independent of whether a third airport is absent or present. This justifies the consideration of both slot and pricing policies in the presence of a three-airport network.

### 5.1 Demand functions

The passenger demands for trips between $i$ and $j$, denoted by $D_{ij}(r_A, r_B)$, and $i$ and $C$, denoted by $D_{iC}(r_i)$ are determined by the equilibrium conditions

$$\frac{\partial B_i(q_{ij}, q_{iC})}{\partial q_{ij}} - (r_A + r_B) = 0 \text{ and } \frac{\partial B_i(q_{ij}, q_{iC})}{\partial q_{iC}} - r_i = 0 \tag{15}$$

for $\phi_i = S, P$ because efficient rationing is ensured under both slots and pricing policies by assumption. In equilibrium, the passengers' marginal benefit of travelling between $A$ and $B$ is equal to the sum of $r_A$ and $r_B$, while it is equal to $r_i$ for passengers travelling between $i$ and $C$. The strict concavity of the benefit function, $B_i(q_{ij}, q_{iC})$, ensures the existence of a unique set of demands. Cramer's rule can be applied to derive the relationship between demands and prices (the proof is relegated to Appendix A.2):

**Lemma 2** *If there are three airports, demands are decreasing in airport prices $r_i$ in the sense that*

$$(i)\ \frac{\partial D_{ij}(r_A, r_B)}{\partial r_i} = \frac{\partial D_{ij}(r_A, r_B)}{\partial r_j} < 0 \textit{ and } (ii)\ \frac{\partial D_{iC}(r_i)}{\partial r_i} < \frac{\partial D_{jC}(r_j)}{\partial r_i} = 0. \tag{16}$$

Part (i) shows that passengers who travel between airports $A$ and $B$ are indifferent between an increase in $r_A$ or $r_B$ by the same amount in the sense that passengers only care about the sum of the prices $r_A + r_B$. Part (ii) shows that passengers who travel between airports $j$ and $C$ are not affected by price changes at airport $i$. The total demand of airport $i$ is denoted by

$$D_i(r_A, r_B) = D_{AB}(r_A, r_B) + D_{BA}(r_A, r_B) + D_{iC}(r_i), \tag{17}$$

and it is decreasing in $r_i$ by Lemma 2.

### 5.2 Equilibrium slot quantities, equilibrium slot prices and equilibrium prices

A feature of the two-airport network is that the other airport's slot quantity imposes an upper limit on the own number of passengers. This is a strong assumption, which hardly reflects reality. Around 10,000 civil airports exist in the world (IATA, 2018) and around 175 airports are slot coordinated (IATA, 2017). Moreover, not all the 175 slot coordinated airports operate at full capacity during all days and operating hours. Altogether, this illustrates that it is useful to capture a scenario where the airports' passenger demands are determined by the airports themselves and not by their complementary counterparts. More specifically, with the third airport, a scenario where $Q_i$ can exceed $\overline{Q}_j$ can be considered because the number of passengers traveling between airports $i$ and $C$ is independent of airport $j$'s slot restriction.

In the three-airport network, the slot price $r_i(S)$ is implicitly determined by

$$\overline{Q}_i - D_i(r_A(S), r_B(S)) = 0.^{12} \tag{18}$$

[12] In the case of congested airports, it is assumed that slot constraints are always binding. Gillen et al. (2016) pointed out that meteorological and other stochastic factors may imply that airports may still not always operate at full capacity despite the presence of slot controls.

Applying Cramer's rule to the system of equations in (18) yields:

**Lemma 3** *If there are three airports, (i) there is a unique pair of slot prices matched with each pair of slot quantities, and (ii) slot prices are decreasing in own slot quantities* $\overline{Q}_i$ *and increasing in the other airport's slot quantities:*

$$\frac{\partial r_i(S)}{\partial \overline{Q}_i} = \frac{\partial D_j/\partial r_j}{\Phi(r_A, r_B)} < 0 < \frac{\partial r_j(S)}{\partial \overline{Q}_i} = -\frac{\partial D_j/\partial r_i}{\Phi(r_A, r_B)} < \left|\frac{\partial r_i(S)}{\partial \overline{Q}_i}\right| \tag{19}$$

*with*

$$\Phi(r_A, r_B) = \frac{\partial D_A}{\partial r_A}\frac{\partial D_B}{\partial r_B} - \frac{\partial D_B}{\partial r_A}\frac{\partial D_A}{\partial r_B} > 0. \tag{20}$$

An increase in the own slot quantity means that the passenger demand, $D_i$, at the own airport is increased, which is only possible if this increase in the own slot quantity is associated with a reduction of the own slot price. An increase in the own slot quantity increases the passenger throughput at the own airport, to avoid that this passenger increase increases the passenger throughput at the other airport, the other airport's slot price must be increased. This increase in the other airport's slot price is smaller in absolute value than the reduction of the own airport's slot price. This means that the sum of ticket prices for flights between airports $A$ and $B$ is reduced by an increase in the own slot quantity.

By Proposition 6, slots are the relevant strategies if airport profits do not matter. Thus, local consumer surplus maximization is the relevant objective in this case, where local consumer surplus can be written as $CS_i(\overline{Q}_A, \overline{Q}_B) = W_i(\overline{Q}_A, \overline{Q}_B) = B_i(\overline{Q}_A, \overline{Q}_B)$. Assume that best responses in terms of slot quantities are determined by the first-order conditions, $\partial CS_i/\partial \overline{Q}_i = 0$ (given the presence of non-negativity constraints for slot prices, this may or may not be guaranteed in general) and that the map of best responses in terms of slot quantities is a contraction, which are maintained assumptions here and hereafter. Using equilibrium demand conditions in (15), the first-order conditions in terms of slot quantities can be written as

$$\frac{\partial CS_i}{\partial r_i} \cdot \frac{\partial r_i}{\partial \overline{Q}_i} = \left( r_j \cdot \frac{\partial D_{ij}}{\partial r_i} + r_i \cdot \frac{\partial \left(D_{ij} + D_{iC}\right)}{\partial r_i} \right) \cdot \frac{\partial r_i}{\partial \overline{Q}_i} = 0. \tag{21}$$

The left-hand side shows the product of two terms where the second term, $\partial r_i/\partial \overline{Q}_i$, is strictly negative by Lemma 3. The other term is shown in the middle of the equation in more detail in parentheses. If $r_A(S) = r_B(S) = 0$, this term is zero for both airports (non-negativity is just ensured in this case), which shows that there is a unique equilibrium where slot prices are zero when slot quantities are the decision variables.

Best responses in terms of slot prices are determined by the first-order conditions, $\partial CS_i/\partial r_i = 0$, which is also true in the case of slot quantities. Since demand functions are independent of whether $r_i(S)$ or $r_i(SP)$ is considered, which implies that $D_{ij}(\overline{Q}_A, \overline{Q}_B) = D_{ij}(r_A(SP), r_B(SP))$ for $r_i(S) = r_i(SP)$, equilibrium results are independent of whether slot quantities or slot prices are the decision variables. Given that the contraction condition is satisfied for both best responses in terms of slot quantities and best responses in terms of slot prices, this altogether leads to:

**Proposition 7** *If there are three airports, equilibrium slot prices are equal to the first-best prices independent of whether slot quantities or slot prices are considered as decision variables.*

By Proposition 6, pricing policies are the relevant strategies if airport profits matter. Assume that best responses in terms of prices are determined by the first-order conditions $\partial W_i/\partial r_i = 0$. Using the equilibrium demand conditions in (15), these first-order conditions lead to a characterization of equilibrium prices that is equal to the one obtained by using the two-airport network in (14). This implies that in the presence of both two- and three-airport networks, equilibrium prices strictly exceed first-best prices.

Altogether, this implies:

**Corollary 3** *If there are three airports and airport profits do not matter, the variable effect is zero, that is,* $W_i(S) - W_i(SP) = 0$*, while the distribution effect is strictly positive when profits matter, that is,* $W_i(SP) - W_i(P) > 0$*.*

Corollary 3 shows that the variable effect is clearly identified and equal to zero in the presence of a three-airport network. The difference to a two-airport network is that the presence of a third airport allows for an interior solution for the best responses in terms of slot quantities, which ultimately and usefully leads to a unique equilibrium in slot quantities.

The discussion in this section firstly shows that a third airport has a significant effect on the slot strategy in the sense that (i) the own equilibrium demand is determined by the own slot quantity and independent of the other airport's slot quantity in equilibrium, and (ii) the variable effect is uniquely defined and equal to zero. These results show that slots in combination with grandfather rules can serve as a tool to overcome excessive airport charges caused by the incentives to exploit non-local passengers, which exist in the case of pricing policies.

# 6 Congestion Effects

This section extends the basic model with a three-airport network in order to capture congestion effects. A major factor for the cause of congestion is the ratio of traffic quantity to capacity (for example, Zhang and Czerny, 2012; Gillen et al., 2016). Traffic may be measured by passenger or flight numbers. Capacity can refer to runway capacity, air space capacity, and terminal capacity, where congestion will be positively related to flight numbers in the case of runway and air space capacity, and positively related to passenger quantities in the case of terminal capacity. Assuming given aircraft sizes and fixed load factors, as is common in the literature, the framework can represent both capacity limitations related to flights and passenger quantities because, in this case, an increase in passenger quantities translates into a fixed increase in flight numbers.[13] For tractability reasons, we concentrate on symmetric airports in the following sections.

Let $C_i(Q_i)$ with $C_i' > 0$ denote the average delays at airport $i$ and $v$ denote the passengers' time valuations. The total congestion costs, denoted by $TC_i(q_{ij}, q_{iC}, Q_A, Q_B)$, of local passengers can be written as $TC_i(q_{ij}, q_{iC}, Q_A, Q_B) = v\,(q_{ij} \cdot (C_A(Q_A) + C_B(Q_B)) + q_{iC}C_i(Q_i))$. Local consumer surplus can be rewritten as

$$CS_i(q_{ij}, q_{iC}, r_A(\phi_A), r_B(\phi_B), \phi_A, \phi_B)$$
$$= B_i(q_{ij}, q_{iC}) - q_{ij} \cdot (R_A(r_A(\phi_A), \phi_A) + R_B(r_B(\phi_B), \phi_B)) - q_{iC}R_i(r_i(\phi_i), \phi_i) - TC_i \tag{22}$$

and local welfare can be rewritten as

$$W_i(q_{ij}, q_{iC}, q_{ji}, r_A(\phi_A), r_B(\phi_B), \phi_A, \phi_B) = B_i(q_{ij}, q_{iC}) + q_{ji}R_i(r_i(\phi_i), \phi_i) - q_{ij}R_j(r_j(\phi_j), \phi_j) - TC_i. \tag{23}$$

The only difference between local consumer surpluses in (1) and (22), and welfares in (4) and (23), are the last terms on the right-hand sides of (22) and (23), which represent the total congestion costs of local passengers. Observe that congestion costs are independent of the policy variables when passenger quantities are given. The proof of Proposition 1 and the corresponding results can therefore directly be extended to the case with congestion, which leads to:

**Proposition 8** *Equilibrium policies in terms of policy variables $\phi_i$ are unaffected by the presence of airport congestion.*

Thus, slots and pricing policies can both be equilibrium solutions depending on whether airport profits do not matter or matter, respectively, independent of whether congestion effects are absent or present.

Passengers consider delays $C_i$ as given. With congestion, demands $D_{ij}(r_A, r_B)$ and $D_{iC}(r_A, r_B)$ are determined by the equilibrium conditions

$$\frac{\partial B_i(q_{ij}, q_{iC})}{\partial q_{ij}} - (r_A + r_B + v\,(C_A + C_B)) = 0 \text{ and } \frac{\partial B_i(q_{ij}, q_{iC})}{\partial q_{iC}} - (r_i + vC_i) = 0. \tag{24}$$

Passengers will travel as long as their marginal benefit from travelling is at least as high as the sum of the corresponding prices and average congestion costs. Applying Cramer's rule to the system of equations in (24) and using symmetry yields:

**Lemma 4** *In the presence of congestion and under symmetry, a marginal increase in price $r_i$ changes demands as follows:*

$$\frac{\partial D_{ji}(r_A, r_B)}{\partial r_i} = \frac{\partial D_{ij}(r_A, r_B)}{\partial r_i} < 0 \tag{25}$$

[13] See Czerny et al. (2016b) for a modelling framework with endogenous aircraft sizes and load factors.

*and*

$$\frac{\partial D_{iC}(r_A, r_B)}{\partial r_i} < 0 < \frac{\partial D_{jC}(r_A, r_B)}{\partial r_i} < \left| \frac{\partial D_{iC}(r_A, r_B)}{\partial r_i} \right|. \tag{26}$$

The result described in (25) is similar to the one described in the first part in Lemma 2 and shows that an increase in one airport's price changes the demands for trips between airports $A$ and $B$ by the same amount independent of whether local or non-local passengers are considered. This is because generalized prices are composed of the sum of airport charges and congestion costs at airports $A$ and $B$, and this sum is independent of the airport of origin. The results in (26) show how the interdependencies caused by congestion affect demand properties. Because an increase in airport $i$'s charge reduces passenger demands for flights between airports $A$ and $B$ and, thus, congestion at airport $j$, this increases the passenger demand for flights between airports $j$ and $C$. However, the increase in $D_{jC}$ is smaller than the reduction in $D_{iC}$ in absolute values, which means that an increase in the own price $r_i$ reduces the passenger demand at the own airport, $D_i$, *and* the passenger demand at airports $A$ and $B$, $D_A + D_B$.

The sum of local welfares, $W_A + W_B$, is given by the difference between the sum of local benefits, $B_A(q_{AB}, q_{AC}) + B_B(q_{BA}, q_{BC})$, and the sum of local congestion costs, $TC_A + TC_B$. The first-best passenger quantities are determined by the first-order conditions $\partial B_i/\partial q_{ij} - \partial(TC_A + TC_B)/\partial q_{ij} = \partial B_i/\partial q_{iC} - \partial(TC_A + TC_B)/\partial q_{iC} = 0$ for $i = A, B$. This together with the equilibrium conditions in (24) implies the first-best prices $r_A = D_A v C'_A$, $r_B = D_B v C'_B$. The first-best prices $D_i v C'_i$ represent the marginal external congestion costs of passengers traveling from $i$ to $C$. The sum of the first-best prices, $D_A v C'_A + D_B v C'_B$, represents the marginal external congestion costs of passengers traveling between airports $A$ and $B$.

### 6.1 Equilibrium slot quantities

The relationships between slot quantities and slot prices described in Lemma 3 is extended to the case of networks of congested airports. To see this, note that the results in parts (i) and (ii) of Lemma 3 depend on how $\partial D_i/\partial r_i$ relates to $\partial D_j/\partial r_i$. Furthermore, Lemmas 2 and 4 imply that $|\partial D_i/\partial r_i| > |\partial D_j/\partial r_i|$ independent of whether networks of uncongested or congested and symmetric airports are considered. This implies:

**Lemma 5** *The existence of a unique pair of slot prices matched with each pair of slot quantities, and the effect of changes in slot quantities $\overline{Q}_i$ on slot prices are unaffected by the presence of airport congestion in the sense that Lemma 3 extends to the present case of a three-airport network with two symmetric congested airports.*

By Proposition 8, it is still true that slots are relevant strategies if airport profits do not matter even though networks of congested airports are considered. Thus, local consumer surplus maximization is the relevant objective in this case, where local consumer surplus is the difference between the local passengers' benefits and the local passengers' congestion costs, which can be written as

$$CS_i(\overline{Q}_A, \overline{Q}_B) = B_i(\overline{Q}_A, \overline{Q}_B) - TC_i(\overline{Q}_A, \overline{Q}_B). \tag{27}$$

Best responses in terms of slot quantities are determined by the first-order conditions $\partial CS_i/\partial \overline{Q}_i = 0$. Using equilibrium demand conditions in (24), these first-order conditions can be written as

$$r_i(S)\left(\frac{\partial D_i}{\partial r_i}\frac{\partial r_i}{\partial \overline{Q}_i} + \frac{\partial D_i}{\partial r_j}\frac{\partial r_j}{\partial \overline{Q}_i}\right) - (D_{ij} + D_{iC})\, vC_i' = 0. \tag{28}$$

The first term on the left-hand side is the product of the equilibrium slot price and a term in parentheses for which, by Lemma 3, the following is true:

**Lemma 6** *An increase in the own slot quantity $\overline{Q}_i$ increases the own airport's passenger throughput $D_i$ by the same amount, that is*

$$\frac{\partial D_i}{\partial r_i}\frac{\partial r_i}{\partial \overline{Q}_i} + \frac{\partial D_i}{\partial r_j}\frac{\partial r_j}{\partial \overline{Q}_i} = 1. \tag{29}$$

Using this lemma, the equilibrium slot price can be written as

$$r_i(S) = (D_{ij} + D_{iC})\, vC_i', \tag{30}$$

where the right-hand side can be described as the marginal external congestion costs of local passengers. However, to reach the first-best solution that maximizes the welfare of the aggregate economy, a price equals to $D_i vC_i'$ with $D_i vC_i' > (D_{ij} + D_{iC})\, vC_i'$ would be required at each airport, that is, not only local but also non-local passengers should be taken into account. More specifically, from the first-best viewpoint, equilibrium slot quantities in (30) are too high, which means that the sum of local consumer surplus and welfare could be increased by reducing slot quantities relative to the equilibrium solution.

Given that the contraction condition for best responses in terms of slot quantities is satisfied, this leads to:

**Proposition 9** *If airport profits do not matter and in the case of a three-airport network with two symmetric congested airports, the unique equilibrium in slot quantities implies that airports imperfectly internalize the marginal external congestion costs, $D_i vC_i'$, relative to the first-best prices.*

This shows that equilibrium slot quantities are too loose to maximize the total welfare of all airport regions when airport profits do not matter and airports only care about local passengers.

### 6.2 Equilibrium slot prices

Consider $\phi_i = SP$ and assume that best responses in terms of slot prices are determined by the first-order conditions $\partial CS_i/\partial r_i = 0$ for $i = A, B$. Using equilibrium demand conditions in (24), these first-order conditions can be written as

$$r_i(SP)\frac{\partial\left(D_{ij} + D_{iC}\right)}{\partial r_i} + r_j(SP)\frac{\partial D_{ij}}{\partial r_i} - (D_{ij} + D_{iC})\, vC_i'\frac{\partial D_i}{\partial r_i} - D_{ji} vC_j'\frac{\partial D_j}{\partial r_i} = 0. \tag{31}$$

The first term and the second term on the left-hand side shows how an increase in slot prices and the corresponding reduction in slot quantities affects own passengers' benefits from travelling. The third term shows the reduction in congestion cost for own passengers at the own airport, while the fourth term on the left-hand side shows the reduction in congestion costs for own passengers at the other airport.

Using symmetry and solving the first-order conditions in (31) yields the equilibrium slot prices, which are strictly positive and can be written as

$$r_i(SP) = \left( \frac{\partial \left( D_A + D_B \right) / \partial r_i}{\partial D_i / \partial r_i} D_{ij} + D_{iC} \right) vC_i'. \tag{32}$$

The right-hand side of (32) shows that the congestion externalities imposed on non-local passengers are only partly taken into account because Lemma 4 implies $\left( \partial \left( D_A + D_B \right) / \partial r_i \right) / \left( \partial D_i / \partial r_i \right) < 2$. In this sense, equilibrium slot prices are too low relative to the first-best prices for both $\phi_i = S$ and $\phi_i = SP$.

Given that the contraction condition is satisfied for both best responses in terms of slot quantities and best responses in terms of slot prices, this altogether leads to:

**Proposition 10** *If airport profits do not matter and in the case of a three-airport network with two symmetric congested airports, the unique equilibrium in slot prices implies that airports imperfectly internalize the marginal external congestion costs, $D_i vC_i'$, relative to first-best prices although internalization is stronger than in the case where slot quantities are considered as decision variables.*

The intuition is that an increase in the own airport charge does not only improve the situation of local passengers by reducing passenger demand at the own airport but also by reducing passenger demand at the other airport because $\partial D_j / \partial r_i < 0$. This, therefore, provides stronger incentives to increase slot prices relative to the case where slot quantities are considered as decision variables and the other airport's total demand is independent of the own slot quantity.

The equilibrium slot prices in (32) show that airports charge a markup on slot prices in (30) where slot quantities are considered as decision variables so that it brings the equilibrium slot prices closer to the first-best prices when slot prices are considered as decision variables. This indicates that the variable effect, $W_i(S) - W_i(SP)$, is negative in sign. Considering that slot prices can be strategic substitutes, an increase in one airport's slot price could be associated with a reduction of the other airport's slot price. The total effect of a change in regimes from slots to slot prices as decision variables is therefore difficult to predict because there can be forces at play that work into opposite directions. Example 1 below will present a numerical example based on quadratic passenger benefit functions where the variable effect is indeed clear-cut and negative in sign.

### 6.3 Equilibrium prices

The local welfare function with congestion is given by (23). Using the equilibrium demand conditions in (24), the first-order conditions for the best responses in terms of the local welfare-maximizing prices, $\partial W_i/\partial r_i = 0$, can be written as

$$r_i(P)\frac{\partial\left(D_{ij}+D_{iC}\right)}{\partial r_i}+r_j(\phi_j)\frac{\partial D_{ij}}{\partial r_i}-\left(D_{ij}+D_{iC}\right)vC_i'\frac{\partial D_i}{\partial r_i}-D_{ji}vC_j'\frac{\partial D_j}{\partial r_i}$$
$$-R_j(r_j(\phi_j),\phi_j)\frac{\partial D_{ij}}{\partial r_i}+r_i(P)\frac{\partial D_{ji}}{\partial r_i}+D_{ji}=0. \quad (33)$$

Using symmetry, which implies $R_j(r_j(\phi_j),\phi_j)=r_i(P)$, as well as Lemma 4, the last three terms on the left-hand side reduce to $D_{ji}$. By the second-order conditions, this implies that best price responses, $r_i(P)$, exceed best responses in terms of slot prices, $r_i(SP)$. Solving the first-order condition (33) yields the equilibrium prices, $r_i(P)$, which can be written as the sum of the equilibrium slot prices plus a weighted markup depending on the price elasticities of passenger demands:

$$r_i(P)=r_i(SP)+\frac{D_{ji}}{D_i}\cdot\left|\frac{D_i}{\partial D_i/\partial r_i}\right|. \quad (34)$$

Thus, the markup is the product of the semi-price elasticity of demand, $D_i$, with respect to the own price and the share of other passengers at the own airport. The semi-price elasticity, $|D_i/\partial D_i/\partial r_i|$, represents the optimal price in the case of airport profit maximization. The notion of profit maximization only applies to other passengers when the airport maximizes local welfare, which is why the elasticity measure is weighted by the share of other passengers at the own airport. The markup also implies that regardless of whether it is a two-airport or three-airport network, or whether there are congestion effects or not, airports always have the incentives to charge strictly positive prices to exploit non-local passengers if airport profits matter, which again highlights the importance of considering the airport network effect.

While equilibrium slot prices are too low relative to the first-best prices, the opposite is true for equilibrium prices in the case of local welfare maximization (the proof is relegated to Appendix A.5):

**Proposition 11** *If airport profits matter and in the case of a three-airport network with two symmetric congested airports, the unique equilibrium in prices overinternalizes the marginal external congestion costs, $D_i vC_i'$, relative to first-best prices.*

This shows that equilibrium prices are used to exploit non-local passengers when airport profits matter and this leads to excessive prices in equilibrium also in the case of congested airport networks.

Table 1 summarizes the main results obtained in the present and the previous sections on how policy choices affect equilibrium passenger quantities relative to the first-best passenger quantities depending on whether uncongested or congested airports are considered and whether slot or pricing policies are considered in a three-airport network. The table shows that equilibrium slot quantities

| | Uncongested (asymmetric) | Congested (symmetric) |
|---|---|---|
| Slots | $Q_i = Q_i^*$ | $Q_i > Q_i^*$ |
| Pricing | $Q_i < Q_i^*$ | |

Table 1: Summary of the equilibrium outcomes

can reach the first-best solution when airports are uncongested, while equilibrium slot quantities imply excessive passenger quantities relative to the first-best passenger quantities when airports are congested. The table further highlights that equilibrium passenger quantities will always be too low relative to the first-best passenger quantities when pricing policies are applied.

The following example illustrates how time valuations affect the relative performance of slot and pricing policies relative to the first-best solution and the role of variable and distribution effects in the presence of congestion.

**Example 1** The benefit of travelling is given by

$$B_i(q_{ij}, q_{iC}) = \alpha_{1i} q_{ij} + \alpha_2 q_{iC} - \frac{1}{2}\left(\beta_1 q_{ij}^2 + \beta_2 q_{iC}^2\right) \tag{35}$$

for airports $A$ and $B$. To illustrate the relative performance of slot and pricing policies, the following notations are used. Using symmetry, the aggregate welfare is denoted by $W$ with $W = W(\phi_i) = 2W_i(\phi_i)$. Let $W^*$ denote the aggregate welfare under first-best prices $r_i = D_i v C_i'$. The relative welfare loss of slot policies, denoted by $\Delta(S)$, is given by $\Delta(S) = (W^* - W(S))/W^*$, while the corresponding value for pricing policies, denoted by $\Delta(P)$, is given by $\Delta(P) = (W^* - W(P))/W^*$.

Figure 1 illustrates the welfare losses of slot and pricing policies depending on time valuations in percent for parameters $\alpha_{1A} = \alpha_{1B} \in \{6/5, 1\}$, $\alpha_2 = 3/5$, $\beta_1 = 2$, $\beta_2 = 4$, where $\alpha_{1i} = 6/5$ represents a scenario with large network effects, while $\alpha_{1i} = 1$ represents a scenario where network effects are relatively less important. The solid lines ($\alpha_{1i} = 6/5$) and dashed lines ($\alpha_{1i} = 1$) represent the welfare losses when airport profits matter, $\Delta(P)$, and do not matter, $\Delta(S)$.

The figure illustrates that pricing performs particularly badly relative to slots when time valuations are low and, thus, congestion effects are of low importance to passengers. This is because in this case the distributional distortions and prisoner's dilemma situations occur under pricing. However, as time valuations increase, the too loose equilibrium slot policies with excessive levels of congestion become more problematic and, eventually, for sufficiently high time valuations, slots actually perform worse than pricing. This indicates that time valuations are crucial for the social evaluation of airport systems that rely on slots or pricing policies.

Figure 2 illustrates how time valuations affect the variable effect, $W_i(S) - W_i(SP)$, and the distribution effect, $W_i(SP) - W_i(P)$, in the presence of congestion for parameters $\alpha_{1i} = 6/5$ (solid lines) and $\alpha_{1i} = 1$ (dashed lines). When time valuations are zero, the variable effect is strictly equal to zero,

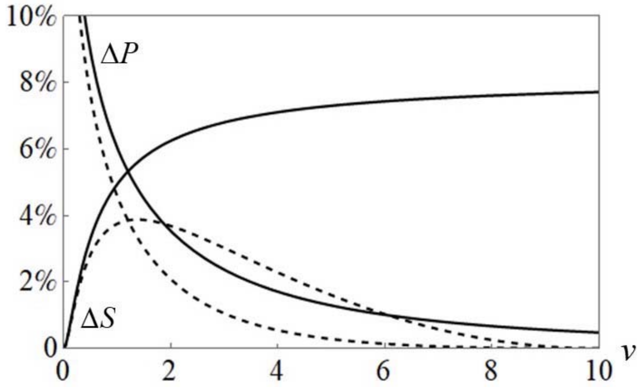


Figure 1: Welfare losses under slots, $\Delta(S)$, and pricing, $\Delta(P)$, relative to first-best in percent depending on time valuations. Parameters: $\alpha_2 = 3/5$, $\beta_1 = 2$, $\beta_2 = 4$, and $\alpha_{1i} = 6/5$ (solid lines) as well as $\alpha_{1i} = 1$ (dashed lines)

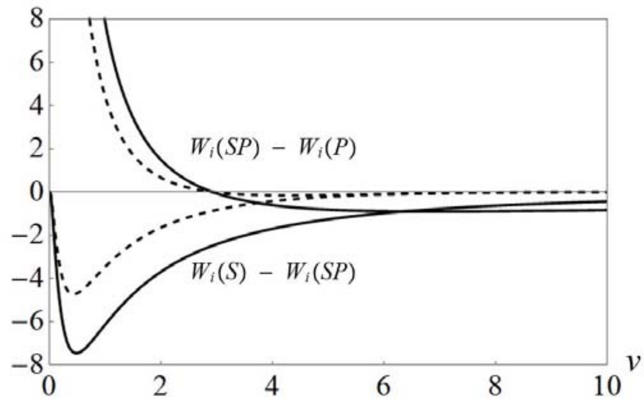


Figure 2: The variable effect, $W_i(S) - W_i(SP)$, and distribution effect, $W_i(SP) - W_i(P)$, depending on time valuations (values are multiplied by 1,000 for scaling reasons). Parameters: $\alpha_2 = 3/5$, $\beta_1 = 2$, $\beta_2 = 4$, and $\alpha_{1i} = 6/5$ (solid lines) as well as $\alpha_{1i} = 1$ (dashed lines)

while the distribution effect is strictly positive as shown in Corollary 3. The variable effect is always negative as anticipated, while the distribution effect is positive for sufficiently low time valuations and negative for sufficiently high time valuations. This shows that for large enough time valuations both the variable and the distribution effects are negative and explains the superiority of pricing policies relative to slot policies from the welfare viewpoint. ■

# 7 Other Extensions

This section considers two extensions of the three-airport network model. The first extension captures the presence of atomistic airlines, while the second extension involves a two-stage game, where airports decide whether to apply slot or pricing policies in the first stage and where they choose the specific slot quantities or airport charges, respectively, in the second stage.

### 7.1 (Atomistic) Airlines

In the previous sections, we considered integrated airports which provide both infrastructure and air services. In this sense, we abstracted away from airline companies and especially airline profits to avoid complications caused by the consideration and especially the evaluation of airline profits. To illustrate the potential complications induced by the presence of airline profits, this extension considers the presence of atomistic airlines when the airlines' costs other than the airport charges are normalized to zero.

Consider the case of uncongested airports. Airline ticket prices are equal to the sum $r_A+r_B$, where $r_i$ can represent airport charges or slot prices, for passengers flying between airports $A$ and $B$. This is true independent of whether passengers use local or non-local airlines. Thus, passengers who only care about ticket prices are indifferent between the use of local or non-local airlines when they fly between airports $A$ and $B$. Airline ticket prices are equal to $r_i$ for passengers flying between airports $i$ and $C$. Demands $D_{ij}$ and $D_{iC}$ are implicitly determined by the equilibrium conditions in (15). But, while demands $D_{ij}$ are informative with respect to the local passenger demands for flights between airports $A$ and $B$, they do not define the total passenger demands for local airlines, where the latter have a lower limit of zero and an upper limit of $D_{AB} + D_{BA}$. Let $D_i^{AB}$ denote the total passenger demand for local airlines with $D_i^{AB} \leq D_{AB} + D_{BA}$. In this case, the local airlines' total profits, denoted by $\Pi_i$, depending on whether slot or pricing policies are used, can be written as

$$\Pi_i = D_i^{AB} \cdot (r_A(\phi_A) + r_B(\phi_B) - R_A(\phi_A) - R_B(\phi_B)) + D_{iC} \cdot (r_i(\phi_i) - R_i(\phi_i)) . \tag{36}$$

The first term on the right-hand side is the profit from local passengers travelling between airports $A$ and $B$ (independent of the share of local or non-local passengers served). The second term is the profit from local passengers travelling between airports $i$ and $C$.

If airport profits matter, $\phi_A = \phi_B = P$ and $r_i(P) = R_i(P)$ in equilibrium; thus, airlines have zero profits (that is, $\Pi_i = 0$). This implies that the presence of atomistic airlines leaves the local welfare function unchanged, which leads to:

**Proposition 12** *If airport profits matter, equilibrium airport behaviors are independent of whether atomistic airlines or a vertically integrated airport is considered.*

Consider a scenario where airports auction slots to atomistic airlines so that $r_i$ reflects the auction prices for slots and where auction revenues accrue to airports. This reflects a scenario where $r_i = R_i$, which shows that Proposition 12 extends to such scenarios.

If airport profits do not matter, $\phi_A = \phi_B = S$ and airport charges are equal to $R_i(S) = 0$, while the slot prices can be positive, that is, $r_i(S) \geq 0$. This implies that airlines have positive profits if the slot quantities are small enough to ensure positive slot prices with $r_i(S) > 0$. Local consumer surplus can be written as

$$CS_i = B_i - D_{ij} \cdot (r_A + r_B) - D_{iC} \cdot r_i, \tag{37}$$

where the presence of atomistic airlines implies that passenger payments for flights are positive even when slot policies are considered. More specifically, equation (37) implies that regardless of the airports' slot or pricing strategies, local consumer surplus will always be equal to the difference between the benefits and the ticket payments of local passengers. Previously, with integrated airports, ticket prices were zero under slot strategies and, therefore, local welfare, local consumer surplus were both equal to the local passengers' benefits. The difference between vertically integrated airports and airports with atomistic airlines is that airports' slot strategies now enable atomistic airlines to gain positive profits, while the consumer surplus of local passengers is reduced.

Assume that the airport attaches a weight $\theta \in [0, 1]$ to airline profits so that the local welfare function under slots takes the form

$$W_i = CS_i + \theta \Pi_i.^{14} \tag{38}$$

If the airport attaches a unit weight to the local airline's profits and the local airlines exclusively serve local passengers, the right-hand side is equal to the benefits of local passengers, $B_i$, as it is in the case of a vertically integrated airport when airport profits do not matter. This implies:

**Proposition 13** *If airport profits do not matter, airports attach a unit weight to airline profits, and airlines exclusively serve local passengers, equilibrium slot strategies are independent of whether atomistic airlines or a vertically integrated airport is considered.*

This means that the scenario with a vertically integrated airport is equivalent to a scenario with atomistic airline markets when the airports attach unit weights to consumer surplus and airline profits and, additionally, airlines exclusively serve local passengers. But, it further means that deviations from these conditions could potentially change the results derived for the case of a vertically integrated airport, which illustrates the complications induced by the presence of atomistic airlines relative to the presence of vertically integrated airports. The following discussion and examples illustrate how these complications can affect the results derived based on the consideration of a vertically integrated airport.

Consider the first partial derivative of the local welfare functions with respect to the local slot quantities, which can be written as

$$\frac{\partial W_i}{\partial r_i}\frac{\partial r_i}{\partial \overline{Q}_i} = \frac{\partial CS_i}{\partial r_i}\frac{\partial r_i}{\partial \overline{Q}_i} + \theta \frac{\partial \Pi_i}{\partial r_i}\frac{\partial r_i}{\partial \overline{Q}_i}. \tag{39}$$

The first term on the right-hand side is strictly positive because $\partial CS_i / \partial r_i < 0$, while the second term is strictly negative when slot prices are equal to zero. If airports attach a unit weight to their local

[14] Czerny and Forsyth (2008) considered a scenario with airport slots where a unit weight is attached to consumer surplus and a weight lower than the unit weight is attached to airport and airline profits.

airlines' profits and the local airlines exclusively serve local passengers, the right-hand side is exactly equal to zero when evaluated at zero slot prices by Proposition 7. If, however, the second term on the right-hand side is smaller in absolute values because $\theta < 1$ or $D_i^{AB} \leq D_{ij}$, the non-negativity constraints for airport charges become strictly binding, and do not change the equilibrium results derived for the case with a vertically integrated airport in the sense that equilibrium slot policies still imply zero slot prices.[15] However, also positive equilibrium slot prices can occur when, for example, $D_i^{AB} > D_{ij}$.

Consider the interesting alternative scenario where airport profits do not matter, airports attach a unit weight to their local airlines' profits, the local airlines exclusively serve local passengers, and a fraction of slots are auctioned by airports to airlines as proposed by Daniel (2014). In this scenario, airline profits are reduced relative to a scenario where all slots are freely allocated to them based on the grandfather rule. To emulate such a scenario (which cannot be an equilibrium policy scenario when the auctioned fraction of slots is strictly positive and airport profits do not matter), let $\theta$ represent the discount on airline profits rather than the weight attached to airline profits where an increase in the fraction of auctioned slots is associated with a reduced value of $\theta$. This interpretation, illustrates that the above scenario is rich enough to also cover such cases and that the results derived for a vertically integrated airport extend to such cases as well.

Consider the case of congested airports. Pricing policies lead to zero profits of atomistic airlines, which is independent of whether networks of uncongested or congested airports are considered. If airline markets are atomistic, the equilibrium pricing strategies derived for the case of vertically integrated airports are therefore unaffected by the presence of both atomistic airlines and congestion.

The picture changes again in the case of slot policies. Airline profits are positive in this case and the relative weight attached to local consumer surpluses increases when the weight, $\theta$, attached to airline profits decreases (or local airlines serve few passengers relative to the local passengers' total demand). Since local consumer surpluses are decreasing functions of ticket prices, this tends to increase equilibrium slot quantities and reduce equilibrium slot prices, thus, equilibrium ticket prices. The following example numerically illustrates that the presence of atomistic airlines in combination with a less than unit weight attached to their profits loosens equilibrium slot strategies relative to the case of vertically integrated airports when congestion is involved.

**Example 2** The benefits of local passengers are given by (35) with parameters $\alpha_{1A} = \alpha_{1B} = 6/5$, $\alpha_2 = 3/5$, $\beta_1 = 2$ and $\beta_2 = 4$. Figure 3 displays the relative local welfare losses under slots relative to

[15] If airport subsidy payments from airports to airlines would be allowed, the first-order conditions $\partial W_i/\partial \overline{Q}_i = 0$ would imply negative slot prices under these conditions. The role of airline subsidies at airports with oligopolistic airlines has been highlighted by Pels and Verhoef's (2004). The present study highlights the role of airport subsidies arising from the social weights attached to airline profits and the distribution of local and non-local passengers to local and non-local airlines.

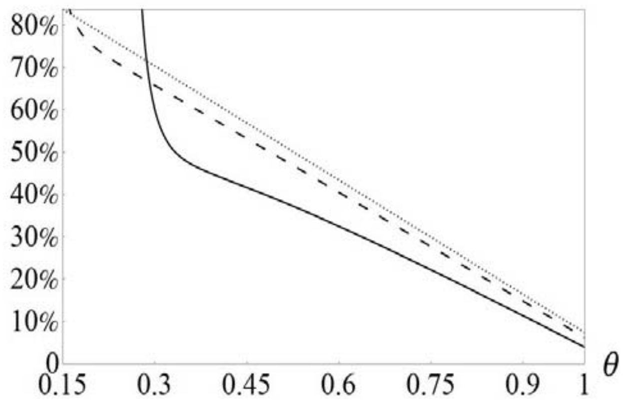


Figure 3: Relative local welfare losses under equilibrium slot policies relative to first-best policies depending on the weight attached to airline profits when time valuations are $v = 1/3$ (solid line), $v = 1$ (dashed line) and $v = 3$ (dotted line)

first-best policies depending on the weight $\theta$ attached to airline profits when airlines exclusively serve local passengers, where time valuations $v = 1/3$ (solid line), $v = 1$ (dashed line) and $v = 3$ (dotted line) are considered. The weight $\theta$ starts at $3/20$ to ensure that passenger quantities are non-negative in all markets. The figure indicates that the difference between local welfares in the case of first-best policies and local welfares in the case of equilibrium slot strategies are decreasing in the weight $\theta$. The intuition is that if airports attach lower than unit weights to airline profits, then they loosen slot quantities in equilibrium to let more local passengers travel, which increases welfare losses. ■

### 7.2 Two-stage Game

Until now, the analyses assumed one-shot games where airports simultaneously choose policy variables and prices or quantities. This seems a strong assumption because it seems plausible that airports can easily change slot quantities or prices, while it may be more difficult for them to switch between slot and pricing policies. To better capture the timing of airport decisions and test the robustness of the above-derived results, this section considers airports that choose the policy variable, $\phi_i$, in the first stage and slot quantities or prices in the second stage.

The sequential structure adds the following complication to the analysis. The simultaneous game structure involves symmetric policy choices in the sense that both airports will either choose slot policies or pricing policies depending on whether airport profits do not matter or matter, respectively. The derivation of the subgame-perfect equilibrium in policy choices requires consideration of policy constellations where one airport chooses the pricing policy, while the other chooses the slot policy, which is a scenario that could be omitted in the case of a one-shot game.

Consider uncongested airports. Furthermore, consider the case where airport profits do not matter. In this case, all airport prices (that is, prices $r_i$ and $R_i$) implied by best responses are zero independent of whether slot or pricing policies are considered. A switch between policies has no impact on passenger quantities in this case. This implies that if there are three airports, airports are uncongested and airport profits do not matter, the equilibrium results derived for the one-shot game carry over to the two-stage game structure.

Consider the case where airport profits matter. In this case, all airport prices implied by the best responses are zero for the airport that uses slot policies. However, equilibrium airport charges are strictly positive for airports that make use of pricing policies. Since airports $A$ and $B$ are complements, one would expect that airport prices are strategic substitutes. Thus, the equilibrium price of the airport that chooses pricing policies should be higher if the other airport chooses slot policies relative to a scenario where both airports choose slot policies. Furthermore, the sum of equilibrium prices should be lower if airports choose different policies relative to the scenario where both choose pricing policies. The overall effect of a unilateral move from pricing to slot policies on local welfares in the two-stage game can therefore be positive or negative. The following example illustrates the likely negative effect of a unilateral move from pricing to slot policies on local welfares when airport profits matter.

**Example 3** The benefit of travelling is given by (35) with parameters $\alpha_{1B} = 6/5$, $\alpha_2 = 3/5$, $\beta_1 = 5$ and $\beta_2 = 4$. Parameter $\alpha_{1A}$ remains undetermined to analyze asymmetric market sizes. Airport profits are assumed to matter for airports.

Figure 4 displays the equilibrium welfare of airport $A$ under slot policies (solid line) and pricing policies (dashed line) depending on the market size measured by the maximum reservation price $\alpha_{1A}$ and given that airport $B$ is engaged in pricing policies when airport profits matter. Parameter $\alpha_{1A}$ ends at 8/5 to ensure that passenger quantities are non-negative. The figure illustrates that airport $A$ has no reason to deviate from pricing policies in a sequential game structure under these conditions. This is true independent of whether it is smaller or larger than airport $B$.

Figure 5 displays the local welfare of airport $A$ under slot policies (solid line) and pricing policies (dashed line) depending on the market size measured by the maximum reservation price $\alpha_{1A}$ and given that airport $B$ chooses slot policies. Parameter $\alpha_{1A}$ starts at 2/5 to ensure that passenger quantities are non-negative. The figure illustrates that airport $A$ has no reason to deviate from pricing policies in a sequential game structure also under these conditions. This is true independent of whether it is smaller or larger than airport $B$. ■

Altogether, this indicates that the equilibrium airport policies derived above for the cases of one-shot games are robust against changes in the timing of airport decisions. While congestion effects have been abstracted away in this subsection, numerical simulations indicated that this robustness is also given in the presence of congested airports.

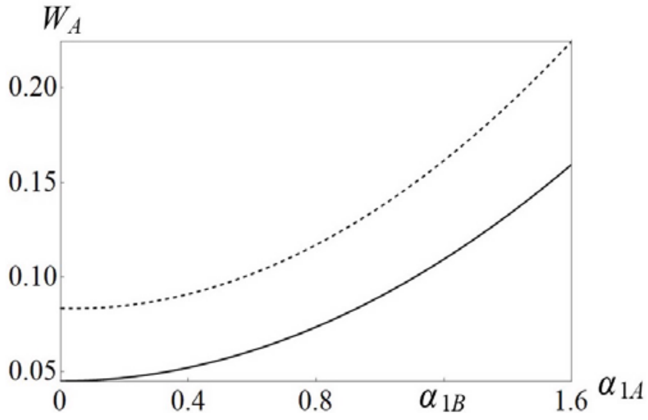


Figure 4: Equilibrium welfare of airport $A$ under slot policies (solid line) and pricing policies (dashed line) depending on the maximum reservation price $\alpha_{1A}$ when airport $B$ chooses pricing policies and airport profits matter. Parameters: $\alpha_{1B} = 6/5$, $\alpha_2 = 3/5$, $\beta_1 = 5$ and $\beta_2 = 4$

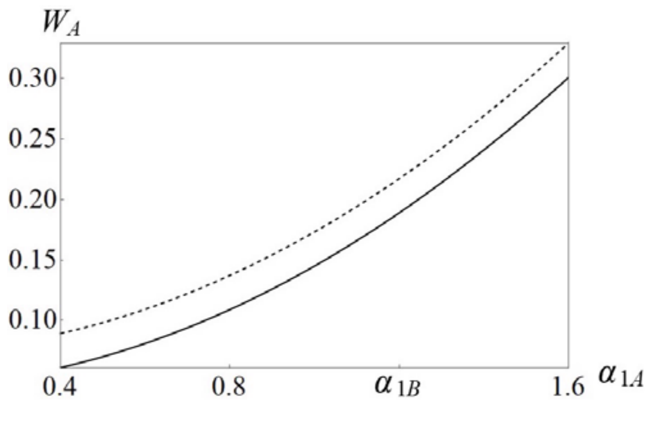


Figure 5: Equilibrium welfare of airport $A$ under slot policies (solid line) and pricing policies (dashed line) depending on the maximum reservation price $\alpha_{1A}$ when airport $B$ chooses slot policies and airport profits matter. Parameters: $\alpha_{1B} = 6/5$, $\alpha_2 = 3/5$, $\beta_1 = 5$ and $\beta_2 = 4$

## 8 Conclusions

The present study started with a consideration of a two-airport network. The analysis showed that the equivalence between price- and quantity-based airport policies that exists in the case of a single airport (and deterministic demands) breaks down when airports individually maximize their local objective functions. The latter involves scenarios where airport profits do not matter or where airport profits matter. One way to see this, is to observe the existence of a unique equilibrium in the case of pricing policies (which does not achieve the set of first-best passenger quantities), and the absence of a unique equilibrium in slot quantities (where the set of equilibrium constellations includes the set of first-best passenger quantities).

To concentrate on scenarios where unique equilibria in slot and pricing strategies exist, a three-airport network with two active airports and one inactive dummy airport was considered. This follows a common approach in the literature where the presence of such a dummy airport is often implicitly assumed (while the present study applied a more transparent approach where the presence of a dummy airport is an explicit part of the set of modeling assumptions). The presence of the dummy airport allowed for the consideration of a more realistic airport network where only a subset of the airports may be slot controlled and the consideration of unique best responses in slot quantities and the description of a unique equilibrium in slot quantities.

Networks of uncongested and congested airports were considered. The comparison between passenger quantities implied by equilibrium pricing policies lead to too low passenger quantities relative to the first-best passenger quantities independent of whether uncongested or congested airports are considered. This is because airports raise airport charges to exploit non-local passengers when airport profits matter. By contrast, equilibrium slot quantities reproduce the first-best passenger quantities in the case of uncongested airports, while they lead to excessive passenger quantities in the case of congested airports. This is because airports ignore the delay reductions for non-local passengers in their choices of slot quantities. Numerical examples were used to show that slot policies can be beneficial relative to pricing policies when time valuations are low enough, while pricing policies can be beneficial relative to slots when time valuations are high enough.

The analysis formally captures that a move from pricing to slot policies can involve two effects: first, a so called *variable effect* that arises from the change in variables in the sense that quantities not prices are the decision variables; second, a so called *distribution effect* that arises from the change in airport revenues, which captures the notion of grandfather rules that make it difficult for the airports to internalize the slot values as measured by their slot prices (shadow prices). To formally separate variable and distribution effects, a third policy regime was introduced that involves prices as decision variables and given airport profits of zero. Numerical examples were used to illustrate how time valuations affect the variable and distribution effects and how an increase in time valuations increases the total welfare achieved under equilibrium pricing policies relative to the total welfare achieved under equilibrium slot policies.

Model extensions captured the presence of atomistic airline markets and a sequential game structure where airports decide upon slot and pricing policies in the first stage and upon the specific slot quantities and prices in the second stage. The analysis of these extensions showed that appropriate assumptions can ensure that the main results are independent of whether vertically integrated or vertically separated airport and (atomistic) airline markets are considered. But, they also show that vertical separation adds complexity to the analysis.

An avenue for future research could be based on variations of the considered airport network structures. More specifically, while the present study concentrates on passenger demands that are interdependent only in the presence of congestion, there are many real-world cases where routes could be substitutes or complements. For instance, passengers may choose among different tourist destinations depending on ticket prices, meaning that there exist demand interdependencies. Another possibility is that rival hub airports compete for transfer passengers (for example, International Civil Aviation Organization, 2013). A framework with airport networks that involve rival connecting flights could be analyzed to derive a better understanding of how airport competition for transfer passengers affects equilibrium congestion policies. Finally, there is also the need to investigate the role of oligopolistic airlines for the choices of slot and pricing policies.

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# Appendix

# A Proofs

## A.1 Proof of Lemma 1

Let $x$ and $y$ represent the equilibrium conditions:

$$x = Q_i - q_{ij} - q_{ji} = 0, \tag{40}$$

$$y = B_i' - B_j' = 0. \tag{41}$$

Totally differentiating leads to

$$dx = dQ_i - dq_{ij} - dq_{ji} = 0, \tag{42}$$

$$dy = B_i'' dq_{ij} - B_j'' dq_{ji} = 0. \tag{43}$$

After rearranging, this system of equations can be written in matrix form as

$$\begin{pmatrix} -1 & -1 \\ B_i'' & -B_j'' \end{pmatrix} \begin{pmatrix} dq_{ij} \\ dq_{ji} \end{pmatrix} = dQ_i \begin{pmatrix} -1 \\ 0 \end{pmatrix}. \tag{44}$$

Cramer's Rule can be applied to derive how local passenger quantities change in total passenger quantities:

$$\frac{dq_{ij}}{dQ_i} = \frac{dq_{ij}}{dQ_j} = \frac{B_j''}{B_A'' + B_B''} > 0.$$

## A.2 Proof of Lemma 2

Let $x_{AB}$, $x_{AC}$, $x_{BA}$, and $x_{BC}$ represent the equilibrium conditions:

$$x_{AB} = \frac{\partial B_A}{\partial q_{AB}} - (r_A + r_B) = 0, \tag{45}$$

$$x_{AC} = \frac{\partial B_A}{\partial q_{AC}} - r_A = 0, \tag{46}$$

$$x_{BA} = \frac{\partial B_B}{\partial q_{BA}} - (r_A + r_B) = 0, \tag{47}$$

$$x_{BC} = \frac{\partial B_B}{\partial q_{BC}} - r_B = 0. \tag{48}$$

Totally differentiating leads to

$$dx_{AB} = \frac{\partial x_{AB}}{\partial q_{AB}} dq_{AB} + \frac{\partial x_{AB}}{\partial q_{AC}} dq_{AC} + \frac{\partial x_{AB}}{\partial q_{BA}} dq_{BA} + \frac{\partial x_{AB}}{\partial q_{BC}} dq_{BC} + \frac{\partial x_{AB}}{\partial r_A} dr_A = 0, \tag{49}$$

$$dx_{AC} = \frac{\partial x_{AC}}{\partial q_{AB}} dq_{AB} + \frac{\partial x_{AC}}{\partial q_{AC}} dq_{AC} + \frac{\partial x_{AC}}{\partial q_{BA}} dq_{BA} + \frac{\partial x_{AC}}{\partial q_{BC}} dq_{BC} + \frac{\partial x_{AC}}{\partial r_A} dr_A = 0, \tag{50}$$

$$dx_{BA} = \frac{\partial x_{BA}}{\partial q_{AB}} dq_{AB} + \frac{\partial x_{BA}}{\partial q_{AC}} dq_{AC} + \frac{\partial x_{BA}}{\partial q_{BA}} dq_{BA} + \frac{\partial x_{BA}}{\partial q_{BC}} dq_{BC} + \frac{\partial x_{BA}}{\partial r_A} dr_A = 0, \tag{51}$$

$$dx_{BC} = \frac{\partial x_{BC}}{\partial q_{AB}} dq_{AB} + \frac{\partial x_{BC}}{\partial q_{AC}} dq_{AC} + \frac{\partial x_{BC}}{\partial q_{BA}} dq_{BA} + \frac{\partial x_{BC}}{\partial q_{BC}} dq_{BC} + \frac{\partial x_{BC}}{\partial r_A} dr_A = 0. \tag{52}$$

After rearranging, this system of equations can be written in matrix form as

$$\begin{pmatrix} \frac{\partial x_{AB}}{\partial q_{AB}} & \frac{\partial x_{AB}}{\partial q_{AC}} & \frac{\partial x_{AB}}{\partial q_{BA}} & \frac{\partial x_{AB}}{\partial q_{BC}} \\ \frac{\partial x_{AC}}{\partial q_{AB}} & \frac{\partial x_{AC}}{\partial q_{AC}} & \frac{\partial x_{AC}}{\partial q_{BA}} & \frac{\partial x_{AC}}{\partial q_{BC}} \\ \frac{\partial x_{BA}}{\partial q_{AB}} & \frac{\partial x_{BA}}{\partial q_{AC}} & \frac{\partial x_{BA}}{\partial q_{BA}} & \frac{\partial x_{BA}}{\partial q_{BC}} \\ \frac{\partial x_{BC}}{\partial q_{AB}} & \frac{\partial x_{BC}}{\partial q_{AC}} & \frac{\partial x_{BC}}{\partial q_{BA}} & \frac{\partial x_{BC}}{\partial q_{BC}} \end{pmatrix} \begin{pmatrix} dq_{AB} \\ dq_{AC} \\ dq_{BA} \\ dq_{BC} \end{pmatrix} = dr_A \begin{pmatrix} -\frac{\partial x_{AB}}{\partial r_A} \\ -\frac{\partial x_{AC}}{\partial r_A} \\ -\frac{\partial x_{BA}}{\partial r_A} \\ -\frac{\partial x_{BC}}{\partial r_A} \end{pmatrix} = dr_A \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}. \tag{53}$$

Let $\Psi$ denote the determinant of the Jacobian on the left-hand side, which can be written as

$$\begin{aligned} \Psi &= \det \begin{pmatrix} \frac{\partial x_{AB}}{\partial q_{AB}} & \frac{\partial x_{AB}}{\partial q_{AC}} & \frac{\partial x_{AB}}{\partial q_{BA}} & \frac{\partial x_{AB}}{\partial q_{BC}} \\ \frac{\partial x_{AC}}{\partial q_{AB}} & \frac{\partial x_{AC}}{\partial q_{AC}} & \frac{\partial x_{AC}}{\partial q_{BA}} & \frac{\partial x_{AC}}{\partial q_{BC}} \\ \frac{\partial x_{BA}}{\partial q_{AB}} & \frac{\partial x_{BA}}{\partial q_{AC}} & \frac{\partial x_{BA}}{\partial q_{BA}} & \frac{\partial x_{BA}}{\partial q_{BC}} \\ \frac{\partial x_{BC}}{\partial q_{AB}} & \frac{\partial x_{BC}}{\partial q_{AC}} & \frac{\partial x_{BC}}{\partial q_{BA}} & \frac{\partial x_{BC}}{\partial q_{BC}} \end{pmatrix} = \det \begin{pmatrix} \frac{\partial^2 B_A}{\partial q_{AB}^2} & 0 & 0 & 0 \\ 0 & \frac{\partial^2 B_A}{\partial q_{AC}^2} & 0 & 0 \\ 0 & 0 & \frac{\partial^2 B_B}{\partial q_{BA}^2} & 0 \\ 0 & 0 & 0 & \frac{\partial^2 B_B}{\partial q_{BC}^2} \end{pmatrix} \\ &= \frac{\partial^2 B_A}{\partial q_{AB}^2} \frac{\partial^2 B_A}{\partial q_{AC}^2} \frac{\partial^2 B_B}{\partial q_{BA}^2} \frac{\partial^2 B_B}{\partial q_{BC}^2} > 0. \end{aligned} \tag{54}$$

To derive how demands change in airport price $r_A$, Cramer's Rule can be applied:

$$\begin{aligned} \frac{dq_{AB}}{dr_A} &= \frac{1}{\Psi} \det \begin{pmatrix} -\frac{\partial x_{AB}}{\partial r_A} & \frac{\partial x_{AB}}{\partial q_{AC}} & \frac{\partial x_{AB}}{\partial q_{BA}} & \frac{\partial x_{AB}}{\partial q_{BC}} \\ -\frac{\partial x_{AC}}{\partial r_A} & \frac{\partial x_{AC}}{\partial q_{AC}} & \frac{\partial x_{AC}}{\partial q_{BA}} & \frac{\partial x_{AC}}{\partial q_{BC}} \\ -\frac{\partial x_{BA}}{\partial r_A} & \frac{\partial x_{BA}}{\partial q_{AC}} & \frac{\partial x_{BA}}{\partial q_{BA}} & \frac{\partial x_{BA}}{\partial q_{BC}} \\ -\frac{\partial x_{BC}}{\partial r_A} & \frac{\partial x_{BC}}{\partial q_{AC}} & \frac{\partial x_{BC}}{\partial q_{BA}} & \frac{\partial x_{BC}}{\partial q_{BC}} \end{pmatrix} = \frac{1}{\Psi} \det \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{\partial^2 B_A}{\partial q_{AC}^2} & 0 & 0 \\ 1 & 0 & \frac{\partial^2 B_B}{\partial q_{BA}^2} & 0 \\ 0 & 0 & 0 & \frac{\partial^2 B_B}{\partial q_{BC}^2} \end{pmatrix} \\ &= \frac{1}{\Psi} \frac{\partial^2 B_A}{\partial q_{AC}^2} \frac{\partial^2 B_B}{\partial q_{BA}^2} \frac{\partial^2 B_B}{\partial q_{BC}^2} = 1 / \frac{\partial^2 B_A}{\partial q_{AB}^2} < 0. \end{aligned} \tag{55}$$

Similarly,

$$\frac{dq_{AC}}{dr_A} = 1 / \frac{\partial^2 B_A}{\partial q_{AC}^2} < 0, \tag{56}$$

$$\frac{dq_{BA}}{dr_A} = 1 / \frac{\partial^2 B_B}{\partial q_{BA}^2} < 0, \tag{57}$$

$$\frac{dq_{BC}}{dr_A} = 0. \tag{58}$$

Analogous results hold for how demands change in $r_B$.

### A.3 Proof of Lemma 3

Totally differentiating the slot conditions (18) yields

$$\begin{aligned} d\left(\overline{Q}_A - D_A\right) &= \frac{\partial\left(\overline{Q}_A - D_A\right)}{\partial r_A} dr_A + \frac{\partial\left(\overline{Q}_A - D_A\right)}{\partial r_B} dr_B + \frac{\partial\left(\overline{Q}_A - D_A\right)}{\partial \overline{Q}_A} d\overline{Q}_A \\ &= -\frac{\partial D_A}{\partial r_A} dr_A - \frac{\partial D_A}{\partial r_B} dr_B + d\overline{Q}_A = 0, \end{aligned} \tag{59}$$

$$\begin{aligned} d\left(\overline{Q}_B - D_B\right) &= \frac{\partial\left(\overline{Q}_B - D_B\right)}{\partial r_A} dr_A + \frac{\partial\left(\overline{Q}_B - D_B\right)}{\partial r_B} dr_B + \frac{\partial\left(\overline{Q}_B - D_B\right)}{\partial \overline{Q}_A} d\overline{Q}_A \\ &= -\frac{\partial D_B}{\partial r_A} dr_A - \frac{\partial D_B}{\partial r_B} dr_B = 0. \end{aligned} \tag{60}$$

In matrix form, this can be rewritten as

$$\begin{pmatrix} -\frac{\partial D_A}{\partial r_A} & -\frac{\partial D_A}{\partial r_B} \\ -\frac{\partial D_B}{\partial r_A} & -\frac{\partial D_B}{\partial r_B} \end{pmatrix} \begin{pmatrix} dr_A \\ dr_B \end{pmatrix} = - \begin{pmatrix} 1 \\ 0 \end{pmatrix} d\overline{Q}_A. \tag{61}$$

The $2 \times 2$ matrix on the left-hand side is positive definite, which ensures that each pair of slots is mapped with a unique pair of slot prices by the Gale-Nikaido Theorem (Gale and Nikaido, 1965). This proves part (i).

Applying Cramer's rule shows that an increase in the own airport's slot quantity reduces the own slot price and increases the other airport's slot price:

$$\frac{dr_i}{d\overline{Q}_i} = \frac{\frac{\partial D_j}{\partial r_j}}{\frac{\partial D_A}{\partial r_A}\frac{\partial D_B}{\partial r_B} - \frac{\partial D_B}{\partial r_A}\frac{\partial D_A}{\partial r_B}} < 0 \text{ and } \frac{dr_j}{d\overline{Q}_i} = \frac{-\frac{\partial D_j}{\partial r_i}}{\frac{\partial D_A}{\partial r_A}\frac{\partial D_B}{\partial r_B} - \frac{\partial D_B}{\partial r_A}\frac{\partial D_A}{\partial r_B}} > 0, \tag{62}$$

where the right-hand of the first equation is greater than the right-hand side of the second equation in absolute values by Lemma 2.

## A.4 Proof of Lemma 4

Let $y_{AB}$, $y_{AC}$, $y_{BA}$, and $y_{BC}$ represent the equilibrium conditions:

$$y_{AB} = \frac{\partial B_A}{\partial q_{AB}} - (r_A + r_B + v(C_A + C_B)) = 0, \tag{63}$$

$$y_{AC} = \frac{\partial B_A}{\partial q_{AC}} - (r_A + vC_A) = 0, \tag{64}$$

$$y_{BA} = \frac{\partial B_B}{\partial q_{BA}} - (r_A + r_B + v(C_A + C_B)) = 0, \tag{65}$$

$$y_{BC} = \frac{\partial B_B}{\partial q_{BC}} - (r_B + vC_B) = 0. \tag{66}$$

Totally differentiating, and after rearranging, this leads to a system of equations in matrix form as

$$\begin{pmatrix} \frac{\partial y_{AB}}{\partial q_{AB}} & \frac{\partial y_{AB}}{\partial q_{AC}} & \frac{\partial y_{AB}}{\partial q_{BA}} & \frac{\partial y_{AB}}{\partial q_{BC}} \\ \frac{\partial y_{AC}}{\partial q_{AB}} & \frac{\partial y_{AC}}{\partial q_{AC}} & \frac{\partial y_{AC}}{\partial q_{BA}} & \frac{\partial y_{AC}}{\partial q_{BC}} \\ \frac{\partial y_{BA}}{\partial q_{AB}} & \frac{\partial y_{BA}}{\partial q_{AC}} & \frac{\partial y_{BA}}{\partial q_{BA}} & \frac{\partial y_{BA}}{\partial q_{BC}} \\ \frac{\partial y_{BC}}{\partial q_{AB}} & \frac{\partial y_{BC}}{\partial q_{AC}} & \frac{\partial y_{BC}}{\partial q_{BA}} & \frac{\partial y_{BC}}{\partial q_{BC}} \end{pmatrix} \begin{pmatrix} dq_{AB} \\ dq_{AC} \\ dq_{BA} \\ dq_{BC} \end{pmatrix} = dr_A \begin{pmatrix} -\frac{\partial y_{AB}}{\partial r_A} \\ -\frac{\partial y_{AC}}{\partial r_A} \\ -\frac{\partial y_{BA}}{\partial r_A} \\ -\frac{\partial y_{BC}}{\partial r_A} \end{pmatrix} = dr_A \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}. \tag{67}$$

Using symmetry, the Jacobian on the left-hand side can be rewritten as

$$\begin{pmatrix} \frac{\partial y_{AB}}{\partial q_{AB}} & \frac{\partial y_{AB}}{\partial q_{AC}} & \frac{\partial y_{AB}}{\partial q_{BA}} & \frac{\partial y_{AB}}{\partial q_{BC}} \\ \frac{\partial y_{AC}}{\partial q_{AB}} & \frac{\partial y_{AC}}{\partial q_{AC}} & \frac{\partial y_{AC}}{\partial q_{BA}} & \frac{\partial y_{AC}}{\partial q_{BC}} \\ \frac{\partial y_{BA}}{\partial q_{AB}} & \frac{\partial y_{BA}}{\partial q_{AC}} & \frac{\partial y_{BA}}{\partial q_{BA}} & \frac{\partial y_{BA}}{\partial q_{BC}} \\ \frac{\partial y_{BC}}{\partial q_{AB}} & \frac{\partial y_{BC}}{\partial q_{AC}} & \frac{\partial y_{BC}}{\partial q_{BA}} & \frac{\partial y_{BC}}{\partial q_{BC}} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 B_A}{\partial q_{AB}^2} - 2vC'_A & -vC'_A & -2vC'_A & -vC'_A \\ -vC'_A & \frac{\partial^2 B_A}{\partial q_{AC}^2} - vC'_A & -vC'_A & 0 \\ -2vC'_A & -vC'_A & \frac{\partial^2 B_A}{\partial q_{AB}^2} - 2vC'_A & -vC'_A \\ -vC'_A & 0 & -vC'_A & \frac{\partial^2 B_A}{\partial q_{AC}^2} - vC'_A \end{pmatrix}. \tag{68}$$

The determinant of the Jacobian can be written as

$$\det \begin{pmatrix} \frac{\partial y_{AB}}{\partial q_{AB}} & \frac{\partial y_{AB}}{\partial q_{AC}} & \frac{\partial y_{AB}}{\partial q_{BA}} & \frac{\partial y_{AB}}{\partial q_{BC}} \\ \frac{\partial y_{AC}}{\partial q_{AB}} & \frac{\partial y_{AC}}{\partial q_{AC}} & \frac{\partial y_{AC}}{\partial q_{BA}} & \frac{\partial y_{AC}}{\partial q_{BC}} \\ \frac{\partial y_{BA}}{\partial q_{AB}} & \frac{\partial y_{BA}}{\partial q_{AC}} & \frac{\partial y_{BA}}{\partial q_{BA}} & \frac{\partial y_{BA}}{\partial q_{BC}} \\ \frac{\partial y_{BC}}{\partial q_{AB}} & \frac{\partial y_{BC}}{\partial q_{AC}} & \frac{\partial y_{BC}}{\partial q_{BA}} & \frac{\partial y_{BC}}{\partial q_{BC}} \end{pmatrix}$$

$$= \left( vC'_A - \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) \left( \left( \frac{\partial^2 B_A}{\partial q_{AB}^2} + 4 \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) vC'_A - \frac{\partial^2 B_A}{\partial q_{AB}^2} \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) \frac{\partial^2 B_A}{\partial q_{AB}^2} \tag{69}$$

where the right-hand side is strictly positive. Applying Cramer's rule yields:

$$\frac{dq_{AB}}{dr_A} = \frac{dq_{BA}}{dr_A} = -\frac{1}{vC'_A \left( \frac{\partial^2 B_A}{\partial q_{AB}^2} / \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) + 4vC'_A - \frac{\partial^2 B_A}{\partial q_{AB}^2}} < 0, \tag{70}$$

$$\frac{dq_{AC}}{dr_A} = -\frac{\left( \frac{\partial^2 B_A}{\partial q_{AB}^2} + 2 \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) vC'_A - \frac{\partial^2 B_A}{\partial q_{AB}^2} \frac{\partial^2 B_A}{\partial q_{AC}^2}}{\left( vC'_A - \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) \left( \left( \frac{\partial^2 B_A}{\partial q_{AB}^2} + 4 \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) vC'_A - \frac{\partial^2 B_A}{\partial q_{AB}^2} \frac{\partial^2 B_A}{\partial q_{AC}^2} \right)} < 0, \tag{71}$$

$$\frac{dq_{BC}}{dr_A} = \frac{2vC'_A}{\left( vC'_A - \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) \left( vC'_A \left( \frac{\partial^2 B_A}{\partial q_{AB}^2} / \frac{\partial^2 B_A}{\partial q_{AC}^2} \right) + 4vC'_A - \frac{\partial^2 B_A}{\partial q_{AB}^2} \right)} > 0. \tag{72}$$

## A.5 Proof of Proposition 11

Consider the equilibrium prices in (34) and the first-best prices $r_i = D_i v C'_i$ for $i = A, B$. These prices can be used to show that in the presence of congestion and symmetry, and if airports pursue local welfare objectives, the equilibrium prices exceed the first-best price if

$$\left( D_{ij} \frac{\partial (D_A + D_B)}{\partial r_i} / \frac{\partial D_i}{\partial r_i} + D_{iC} \right) vC'_i - D_{ji} / \frac{\partial D_i}{\partial r_i} > D_i v C'_i. \tag{73}$$

This condition is equivalent to the condition

$$\left( \frac{\partial D_{jC}}{\partial r_i} - \frac{\partial D_{iC}}{\partial r_i} \right) vC'_i < 1. \tag{74}$$

Using (71) and (72), it can be shown that

$$\left( \frac{dD_{jC}}{dr_i} - \frac{dD_{iC}}{dr_i} \right) vC'_i = \frac{vC'_i}{vC'_i - \frac{\partial^2 B_i}{\partial D_{iC}^2}}, \tag{75}$$

where the right-hand side is indeed strictly smaller than 1.