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Column generation for low carbon berth allocation under uncertainty

Abstract: This paper investigates a low carbon-oriented berth allocation and quay crane assignment problem considering vessels' uncertain arrival time and loading/unloading workload for vessels. A two-stage stochastic integer programming model is formulated based on a set of scenarios. The first stage designs a baseline schedule and the second stage adjusts the schedule in each scenario. A novel tailored solution method is developed by using column generation techniques. Different from the traditional column generation-based algorithm that normally solves a deterministic problem, our proposed solution method evokes the column generation procedures at different steps to solve a two-stage stochastic problem. Numerical experiments are conducted to validate the efficiency of our column generation-based solution method. The proposed method has the potential to act as a generic methodology for solving a wide range of similarly structured two-stage models in other application fields.
Keywords: Column generation; berth allocation; quay crane assignment; uncertainty; carbon taxation.

1. Introduction

According to UNCTAD (2018), the maritime transportation activity will generate about 1.4 billion tons of carbon dioxide in 2020, which occupies 6% of the total global carbon emissions; this percentage value may increase to 18% in 2050 if ports do not take coping approaches, thus the concept of "green ports" has become increasingly popular nowadays.

Port operators have proposed some strategies to reduce the carbon emissions in their port areas, i.e., slow steaming to reduce vessels' emissions, improving port operation efficiency and allowing more environmentally friendly utilizations of resources. Among these strategies, the berth allocation problem (BAP) is one of the key decisions that impact the carbon emissions in a port's area. (1) The BAP decides vessels' mooring time, which affects their voyage speed in a port's channel and further affects their emissions. (2) The BAP also decides the assignment and usage of quay cranes (QCs) for vessels; as the most important equipment in ports, the QCs' emissions are also considerable. Thus, a reasonable berth allocation plan is critical for realizing the low carbon emission goal for ports.

Moreover, uncertain factors are included in the BAP decisions. For example, the vessels' arrival times may be different from their expected arrival times; the number of containers required for handling may also be different from the estimation. These uncertain factors complicate the traditional deterministic BAP decisions. In this study, we investigate a low carbon emission-oriented berth allocation and QC assignment problem (BAQCAP) under uncertain vessels' arrival times and uncertain workloads for loading/unloading containers.

This paper conducts an explorative study on BAQCAP. *A two-stage stochastic integer programming model* is proposed with the second-stage formulated as an (mixed) integer programming. We design a novel column generation (CG) based solution method, for solving the proposed two-stage programming model with integer resources. Numerical experiments have been conducted to validate its efficiency. This method is different from the bender decomposition method, which is commonly seen for solving such two-stage stochastic programming model, but its second stage is usually a linear programming with continuous resources. In this study, we provide an alternative choice for efficiently dealing with similarly structured two-stage models, where the second stage is generalized to an integer programming.

The remainder of this paper is organized as follows. Section 2 reviews the related works. Section 3 proposes the two-stage stochastic integer programming model, followed by a CG-based solution method elaborated in Section 4. Numerical experiment results are reported in Section 5. The closing remarks are outlined in the last section.

2. Literature review

For a comprehensive overview on container terminal operations, the review works are given by Vis and de Koster (2003), Steenken et al. (2004), Stahlbock and Voß (2008), Fransoo and Lee (2013) and Carlo et al. (2014). This study is related to the BAP, which is critical in port management and is normally the first phase to make service planning for shipping liners. The BAP has attracted significant attention among researchers (Bierwirth and Meisel, 2010 and 2015). This study is mainly oriented to the decision model and algorithm design for the green port operations under uncertainty. Thus, the related works are mainly reviewed through the following three streams: (1) the BAP considering the emission or other green issues; (2) the BAP under uncertainty; and (3) the advanced algorithms for solving variants of BAPs.

Among the recent BAP related studies, "green issue" is one of the important directions. Some proposed BAP models took account of the emissions during the berthing activities. Golias et al. (2009) present a berth scheduling policy to reduce fuel consumption and emissions produced by vessels; a genetic algorithm (GA) based heuristic was used to solve the problem. Du et al.

(2011) proposed a mixed integer second-order cone programming (SOCP) model on BAP considering fuel consumption in mooring periods; it is well known that vessels' fuel consumption is directly related to their emission during the berthing activities. Hu et al. (2014) considered fuel consumption and emissions as two objectives and designed a SOCP model for a BAQCAP, in which vessels' arrival time is defined as decision variables. He (2016) developed a mixed-integer programming (MIP) model for BAQCAP to minimize the total handling energy consumption of all vessels by QCs; a simulation optimization methodology was utilized to solve the problem. Zhen et al. (2016) studied a terminal allocation problem, which is a higher-level problem than BAP and also involves BAQCAP in each terminal. A MIP model is designed to minimize the bunker consumption cost and the inter-terminal transportation cost on a terminal allocation problem; a local branching-based method and particle swarm optimization (PSO) based method was implemented to solve the model. Wang et al. (2018) investigated a BAQCAP in the deterministic environment and proposed a branch and bound (B&B) solution method to solve the problem. Their study considered two types of carbon emission taxation policies and conducted numerical experiments to compare these policies.

The fluctuation of maritime transportation market brings plenty of uncertainties for port operations planning. Many BAP related studies are oriented to uncertain environments. One of the first studies that handled the uncertainties in port operations is Moorthy and Teo (2006), which proposed a robust berth template and used a sequence pair based simulated annealing (SA) algorithm to solve BAP considering uncertain arrival time of vessels. Han et al. (2010) further took account of both uncertain arrival time and operation time of vessels in BAQCAP and developed a proactive robust model as well as a simulation-based GA procedure to solve the problem. Hendriks et al. (2010) employed the robust optimization methodology in a BAP considering arrival time window and the periodicity of plans. Their study also validated the effectiveness of their model in the reduction of QCs. Zhen et al. (2011b) combined proactive and reactive strategies and proposed a two-stage decision model for BAP under uncertain arrival time and operation time of vessels. Zhen and Chang (2012) presented a bi-objective optimization model for robust BAP under uncertainty; a heuristic was developed to obtain the Pareto front for the bi-objective problem. Golias et al. (2014) proposed a bi-objective model for the BAP under uncertain arrival time and handling time with the aim of minimizing the average and the range of the total service times for all vessels; a GA-based heuristic was developed to solve the model. Zhen (2015) used both stochastic programming and robust optimization methodologies to build periodical BAP models considering uncertain operation time of vessels; some novel parameters are defined for connecting the models built by the two different methodologies. Ursavas and Zhu (2016) handled the BAP under stochastic arrival and handling times for different types of vessels by using a dynamic programing approach. Xi et al. (2017) formulated a bi-objective robust BAP model considering economic performance and customer satisfaction; an adaptive grey wolf optimizer algorithm (AGWO) was designed to solve the model. Iris and Lam (2019) defined recoverable robustness and applied in the model formulation for BAQCAP with uncertainties; an adaptive large neighborhood algorithm (ALNS) was developed to solve the model.

Table 1: Brief summary of algorithms for BAP variants in recent years

Algorithms		**Problems**	**Other issues**	**Related works**
Exact solutions	B&P	BAQCAP	——	Vacca et al. (2013)
		BAP	bulk port, yard assignment	Robenek et al. (2014)
	B&B	BAP	continues BAP	Xu and Lee (2018)
	cutting plane algorithm	BAQCAP	——	Türkoğullarıet al. et al. (2014 and 2016)
Heuristics	CG	BAP	yard space assignment	Jin et al. (2015)
		BAQCAP	—— tides and channel flow yard assignment	Iris et al. (2015) Zhen et al. (2017) Wang et al. (2018)
	GA	BAP	—— uncertainty	Lalla-Ruiz et al. (2014) Golias et al. (2014)
	PSO	BAQCAP	fuel consumption, terminal allocation	Zhen et al. (2016)
	AGWO	BAP	uncertainty	Xi et al. (2017)
	ALNS	BAP	——	Ribeiro et al. (2016)
		BAQCAP	QCs productivity uncertainty	Iris et al. (2017) Iris and Lam (2019)
	B&B based heuristic	BAQCAP	carbon emission	Wang et al. (2018)
	SO	BAQCAP	fuel consumption	He (2016)

Notes: In the first column, 'B&P': Branch-and-Price; 'B&B': Branch and Bound; 'CG': Column Generation; 'GA': Genetic Algorithm; 'PSO': Particle Swarm Optimization; 'AGWO': Adaptive Grey Wolf Optimizer; 'ALNS': Adaptive Large Neighborhood Search; 'SO': Simulation Optimization.

Another contribution of this study is to design a new algorithmic framework based on CG for the new BAP variant proposed. The algorithmic strategies are also hot topics in the fields of the BAPs; Table 1 summarizes some representative types of the algorithms used in the BAP. As the methodology used in this study, the CG was used for solving some BAP variants with various special features. For example, Iris et al. (2015) used the CG to solve the BAQCAP in a deterministic environment; Zhen et al. (2017) proposed a CG-based method for the BAQCAP in a tidal port; Jin et al. (2015) used the CG to solve a berth and yard template planning problem; Wang et al. (2018) further considered the QC assignment in the berth and yard template planning and also used the CG as well as some acceleration techniques to solve the problem. Although the BAP variants are usually very complex and it is challenging to design some exact solutions for them, some scholars designed and implemented exact solution methods in recent years. For example, the branch-and-price (B&P) method, which is also based on the CG, was used by Vacca et al. (2013) to solve the BAQCAP with up to 20 ships and five berths; Robenek et al. (2014) also proposed a B&P method to solve a BAP with yard space assignment for a bulk port. Xu and Lee (2018) designed a branch-and-bound (B&B) based method for continuous BAP, where a new lower bound was also proposed for accelerating the solving process. In addition, Türkoğulları et al. (2014 and 2016) designed cutting plane algorithms to solve two BAQCAPs optimally considering the time-invariant QC assignment and the time-variant QC assignment, respectively. Besides the above exact solutions, plenty of tailored heuristics have been designed to solve various BAP variants with special features. Due to the limitation of the space, the heuristics listed in Table 1 are not elaborated here. As the CG has a good performance for BAPs and inherits an algorithmic framework, which belongs to some middle ground between the exact solutions and some pure heuristics, this study adopts the CG as the methodology to develop a solution method for the problem investigated in this paper.

Compared with the related literature, the contributions of this study include the following aspects. (i) An explorative BAQCAP is investigated by considering many realistic factors, i.e., the carbon emission during the berthing activity, vessels' uncertain arrival times and workloads, the periodicity of BAP plans, and berth-dependent workload for each vessel. A two-stage stochastic integer programming model is formulated for this comprehensive BAQCAP. (ii) A CG-based solution method is developed, and an exact pseudo-polynomial procedure is also embedded in the CG-based method for solving the pricing problems to optimality. Sensitivity analysis is conducted to outline some managerial implications.

3. Model formulation

A two-stage stochastic integer programming model is formulated for the problem in this section. Before the model formulation, the related parameters and decision variables are defined first.

3.1 Notation

Indices and sets:

i, j index of a vessel

V set of vessels

b index of a berth

B set of berths

p index of a QC profile

P_i set of QC profiles for vessel i, $i \in V$

t index of a time step

T set of time steps, $T \in \{1, \dots, H + E\}$, H and E are defined later

ξ index of a scenario

Φ set of scenarios

Input parameters:

$[a_i^f, b_i^f]$ feasible service time window for vessel i, $i \in V$; the vessel's turnaround time slot must be in this time window

$[a_i^e, b_i^e]$ expected service time window for vessel i, $i \in V$; if the vessel's turnaround time slot is not in this time window, penalty cost will be incurred

c_i unit penalty cost for the vessel i's earliness and tardiness beyond its expected time window when making *baseline schedule* (unit: $/time step), which penalizes the deviation of vessels' planned dwelling time slot from their expected time window. This penalty cost reflects the service level that the port operator satisfies the requirement of shipping liners

c_i' unit penalty cost for the vessel i's actual earliness and tardiness beyond its planed time slot (unit: $/time step), which penalizes the deviation of vessels' actual dwelling time slot from the time slot planned *in the baseline schedule*. This penalty cost is related to the adjustment of other port resources, whose plans had been made according to the baseline schedule of berthing. The relationship among the above two cost coefficients is: $c_i' > c_i$

$a_{i\xi}$ actual arrival time of vessel i in scenario ξ, $i \in V, \xi \in \Phi$

h_{ip} handling time (in time steps) of vessel i by using QC profile p, $i \in V, p \in P_i$

q_{ipm} number of QCs in the mth time step if vessel i is served by QC profile p, $p \in P_i, i \in V, m \in \{1, \dots, h_{ip}\}$

Q_t number of available QCs in time step t, $t \in \{1, \dots, H\}$

$w_{i,\xi}$ realized workload (in QC time steps) for vessel i in scenario ξ, $i \in V, \xi \in \Phi$

d ratio of the workload increment to the deviation of a vessel's actually moored berth from its planed berth in baseline schedule; for example, if a vessel's actually moored berth is Berth 5 and its planned berth in baseline schedule is Berth 8, the workload for serving the vessel increases by $3d$ QC time steps. Here we assume the berths are identical and are indexed by an ascending (or descending) order along the quay side

c^{qc} unit operating cost per time step

c^{tax} taxation cost coefficient of carbon emission taxation function $g(x) \coloneqq c^{tax}x$, where x represents the amount of total QC time steps used by all the vessels

H number of time steps in the planning horizon

E maximum handling time of all vessels, that is $E = max_{i \in V, p \in P_i}\{h_{ip}\}$

$\rho(\xi)$ probability of realization for scenario ξ, $\xi \in \Phi$

Decision variables:

ω_{ib} binary, set to one if berth b is planned to be allocated to vessel i in baseline schedule; and zero otherwise; $i \in V, b \in B$

$\omega_{ib\xi}$ binary, set to one if berth b is actually allocated to vessel i in scenario ξ; and zero otherwise; $i \in V, b \in B, \xi \in \Phi$

$\varpi_{ibb'\xi}$ binary, set to one if berth b' is actually allocated to vessel i in scenario ξ and berth b is planned to be allocated to vessel i in baseline schedule; and zero otherwise; $i \in V, b, b' \in B, \xi \in \Phi$

δ_{ijb} binary, set to one if vessels i and j are planned to be moored at berth b in baseline schedule, and vessel i is followed by vessel j; and zero otherwise; $i, j \in V, i \neq j, b \in B$

$\delta_{ijb\xi}$ binary, set to one if vessels i and j are actually moored at berth b, and vessel i is followed by vessel j in scenario ξ; and zero otherwise; $i, j \in V, i \neq j, b \in B, \xi \in \Phi$

$\Delta_{i\xi}$ integer, deviation of vessel i's actually moored berth index from its planned berth in baseline schedule in scenario ξ; $i \in V, \xi \in \Phi$

γ_{ip} binary, set to one if vessel i is served by QC profile p in baseline schedule; and zero otherwise; $i \in V, p \in P_i$

$\gamma_{ip\xi}$ binary, set to one if vessel i is served by QC profile p in scenario ξ; and zero otherwise; $i \in V, p \in P_i, \xi \in \Phi$

σ_t integer, the number of used QCs at time step t in baseline schedule; $t \in T$

$\sigma_{t\xi}$ integer, the number of used QCs at time step t in scenario ξ; $t \in T, \xi \in \Phi$

μ_{it} binary, set to one if vessel i begins handling at time step t in baseline schedule; and zero otherwise; $i \in V, t \in T$

$\mu_{it\xi}$ binary, set to one if vessel i begins handling at time step t in scenario ξ; and zero otherwise; $i \in V, t \in T, \xi \in \Phi$

η_{ipt} binary, set to one if vessel i is served by QC profile p and begins handling at time step t in baseline schedule; and zero otherwise; $i \in V, t \in T, p \in P_i$

$\eta_{ipt\xi}$ binary, set to one if vessel i is served by QC profile p and begins handling at time step t in scenario ξ; and zero otherwise; $i \in V, t \in T, p \in P_i, \xi \in \Phi$

α_i integer, the start time step of the handling for vessel in baseline schedule; $i \in V$

$\alpha_{i\xi}$ integer, the start time step of the handling for vessel i in scenario ξ; $i \in V, \xi \in \Phi$

β_i integer, the ending time step of the handling for vessel i in baseline schedule; $i \in V$

$\beta_{i\xi}$ integer, the ending time step of the handling for vessel i in scenario ξ; $i \in V, \xi \in \Phi$

ρ_b, ς_b integer, start and ending time step of berth b's usage in baseline schedule; $b \in B$

$\rho_{b\xi}, \varsigma_{b\xi}$ integer, start and ending time step of berth b's usage in scenario ξ; $\xi \in \phi, b \in B$

3.2 Mathematical model

Based on the above definition, a two-stage stochastic integer programming model is formulated to minimize the cost of the baseline schedule and the expected cost over scenarios.

We first develop a baseline schedule so that other resources' schedule and ships' voyages need to be designed based on it. The baseline schedule is necessary even though actual process may be different from it. Each ship has its preferred time window, and the baseline schedule is to minimize the violations from their preferred time windows. The "violation" is reflected by the earliness (i.e., $(a_i^e - \alpha_i)^+$) and the tardiness (i.e., $(\beta_i - b_i^e)^+$) beyond their preferred time window. Thus, the first part of the objective is $\sum_{i \in V} c_i \left[(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+\right]$, where c_i is the cost coefficient in the (first) stage of making baseline schedule.

In the series of random realizations, denoted by scenarios ($\xi \in \Phi$), the "violation" between a vessel's actual berthing time slot and its planned time slot (decided in the baseline schedule) also need to be minimized; here the "violation" in each scenario ξ is reflected by the earliness (i.e., $(\alpha_i - \alpha_{i\xi})^+$) and the tardiness (i.e., $(\beta_{i\xi} - \beta_i)^+$) beyond the planned time slots. In the actual realization of each scenario, the cost of used QCs (i.e., $(c^{qc} + c^{tax})\sum_{t\in T}\sigma_{t\xi}$) is also considered in the objective because the berth allocation plan also affects handling workloads for vessels as well as the total number of used QCs; here the former one "c^{qc}" is related to QCs' operation cost, the latter one "c^{tax}" is related to the carbon emission (carbon tax) by the QCs. Therefore, the second part of the objective is the expected cost in a series of random scenarios: $\sum_{\xi\in\Phi}\rho(\xi)\{\sum_{i\in V}c_i'[(\alpha_i - \alpha_{i\xi})^+ + (\beta_{i\xi} - \beta_i)^+] + (c^{qc} + c^{tax})\sum_{t\in T}\sigma_{t\xi}\}$, where $\rho(\xi)$ denotes the probability of scenario ξ.

Based on the above analysis, our two-stage stochastic integer programming model is formulated as follows.

[M1] *Min* $\sum_{i\in V}c_i\,[(a_i^e - \alpha_i)^+ + (\beta_i - b_i^e)^+] + \sum_{\xi\in\Phi}\rho(\xi)\{\sum_{i\in V}c_i'[(\alpha_i - \alpha_{i\xi})^+ + (\beta_{i\xi} - \beta_i)^+] + (c^{qc} + c^{tax})\sum_{t\in T}\sigma_{t\xi}\}$ (1)

s.t.

$\sum_{b\in B}\omega_{ib} = 1$ $\forall i \in V$ (2)

$\sum_{t\in\{1,\dots,H\}}\mu_{it} = 1$ $\forall i \in V$ (3)

$\sum_{t\in T}\mu_{it}t = \alpha_i$ $\forall i \in V$ (4)

$\alpha_i + \sum_{p\in P_i}\gamma_{ip}h_{ip} - 1 = \beta_i$ $\forall i \in V$ (5)

$\alpha_j + (1 - \delta_{ijb})M \geq \beta_i + 1$ $\forall i,j \in V, i \neq j, b \in B$ (6)

$\delta_{ijb} + \delta_{jib} \leq \omega_{ib}$ $\forall i,j \in V, i \neq j, b \in B$ (7)

$\delta_{ijb} + \delta_{jib} \geq \omega_{ib} + \omega_{jb} - 1$ $\forall i,j \in V, i \neq j, b \in B$ (8)

$\alpha_i \geq a_i^f$ $\forall i \in V$ (9)

$\beta_i \leq b_i^f$ $\forall i \in V$ (10)

$\rho_b \leq \alpha_i + (1 - \omega_{ib})M$ $\forall i \in V, b \in B$ (11)

$\varsigma_b \geq \beta_i + (\omega_{ib} - 1)M$ $\forall i \in V, b \in B$ (12)

$\varsigma_b - \rho_b \leq H - 1$ $\forall b \in B$ (13)

$\sum_{p\in P_i}\gamma_{ip} = 1$ $\forall i \in V$ (14)

$\eta_{ipt} \geq \gamma_{ip} + \mu_{it} - 1$ $\forall i \in V, p \in P_i, t \in T$ (15)

$$\eta_{ipt} \leq \gamma_{ip} \qquad \forall i \in V, p \in P_i, t \in T \qquad (16)$$

$$\eta_{ipt} \leq \mu_{it} \qquad \forall i \in V, p \in P_i, t \in T \qquad (17)$$

$$\sigma_t = \sum_{i \in V} \sum_{p \in P_i} \sum_{m=max\{1,t-h_{ip}+1\}}^{t} \eta_{ipm} q_{ip(t-m+1)} \qquad \forall t \in T \qquad (18)$$

$$\sigma_t \leq Q_t \qquad \forall t \in \{E+1, \dots, H\} \qquad (19)$$

$$\sigma_t + \sigma_{t+H} \leq Q_t \qquad \forall t \in \{1, \dots, E\} \qquad (20)$$

$$\sum_{b \in B} \omega_{ib\xi} = 1 \qquad \forall i \in V, \xi \in \Phi \qquad (21)$$

$$\sum_{t \in \{1,\dots,H\}} \mu_{it\xi} = 1 \qquad \forall i \in V, \xi \in \Phi \qquad (22)$$

$$\sum_{t \in T} \mu_{it\xi} t = \alpha_{i\xi} \qquad \forall i \in V, \xi \in \Phi \qquad (23)$$

$$\alpha_{i\xi} + \sum_{p \in P_i} \gamma_{ip\xi} h_{ip} - 1 = \beta_{i\xi} \qquad \forall i \in V, \xi \in \Phi \qquad (24)$$

$$\alpha_{j\xi} + \left(1 - \delta_{ijb\xi}\right) M \geq \beta_{i\xi} + 1 \qquad \forall i, j \in V, i \neq j, b \in B, \xi \in \Phi \qquad (25)$$

$$\delta_{ijb\xi} + \delta_{jib\xi} \leq \omega_{ib\xi} \qquad \forall i, j \in V, i \neq j, b \in B, \xi \in \Phi \qquad (26)$$

$$\delta_{ijb\xi} + \delta_{jib\xi} \geq \omega_{ib\xi} + \omega_{jb\xi} - 1 \qquad \forall i, j \in V, i \neq j, b \in B, \xi \in \Phi \qquad (27)$$

$$\alpha_{i\xi} \geq a_{i\xi} \qquad \forall i \in V, \xi \in \Phi \qquad (28)$$

$$\beta_{i\xi} \leq b_i^f \qquad \forall i \in V, \xi \in \Phi \qquad (29)$$

$$\rho_{b\xi} \leq \alpha_{i\xi} + (1 - \omega_{ib\xi}) M \qquad \forall i \in V, b \in B, \xi \in \Phi \qquad (30)$$

$$\varsigma_{b\xi} \geq \beta_{i\xi} + (\omega_{ib\xi} - 1) M \qquad \forall i \in V, b \in B, \xi \in \Phi \qquad (31)$$

$$\varsigma_{b\xi} - \rho_{b\xi} \leq H - 1 \qquad \forall b \in B, \xi \in \Phi \qquad (32)$$

$$\sum_{p \in P_i} \gamma_{ip\xi} = 1 \qquad \forall i \in V, \xi \in \Phi \qquad (33)$$

$$\eta_{ipt\xi} \geq \gamma_{ip\xi} + \mu_{it\xi} - 1 \qquad \forall i \in V, p \in P_i, t \in T, \xi \in \Phi \qquad (34)$$

$$\eta_{ipt\xi} \leq \gamma_{ip\xi} \qquad \forall i \in V, p \in P_i, t \in T, \xi \in \Phi \qquad (35)$$

$$\eta_{ipt\xi} \leq \mu_{it\xi} \qquad \forall i \in V, p \in P_i, t \in T, \xi \in \Phi \qquad (36)$$

$$\sigma_{t\xi} = \sum_{i \in V} \sum_{p \in P_i} \sum_{m=max\{1,t-h_{ip}+1\}}^{t} \eta_{ipm\xi} q_{ip(t-m+1)} \qquad \forall t \in T, \xi \in \Phi \qquad (37)$$

$$\sigma_{t\xi} \leq Q_t \qquad \forall t \in \{E+1, \dots, H\}, \xi \in \Phi \qquad (38)$$

$$\sigma_{t\xi} + \sigma_{(t+H)\xi} \leq Q_t \qquad \forall t \in \{1, \dots, E\}, \xi \in \Phi \tag{39}$$

$$\varpi_{ibb'\xi} \geq \omega_{ib} + \omega_{ib'\xi} - 1 \qquad \forall b, b' \in B, i \in V, \xi \in \Phi \tag{40}$$

$$\varpi_{ibb'\xi} \leq \omega_{ib} \qquad \forall b, b' \in B, i \in V, \xi \in \Phi \tag{41}$$

$$\varpi_{ibb'\xi} \leq \omega_{ib'\xi} \qquad \forall b, b' \in B, i \in V, \xi \in \Phi \tag{42}$$

$$\Delta_{i\xi} = \sum_{b' \in B} \sum_{b \in B} \varpi_{ibb'\xi} |b - b'| \qquad \forall i \in V, \xi \in \Phi \tag{43}$$

$$\sum_{p \in P_i} \gamma_{ip\xi} \sum_{m \in \{1, \dots, h_{ip}\}} q_{ipm} \geq (1 + d\Delta_{i\xi}) w_{i\xi} \qquad \forall i \in V, \xi \in \Phi \tag{44}$$

$$\omega_{ib}, \delta_{ijb}, \gamma_{ip}, \mu_{it}, \eta_{ipt} \in \{0,1\} \qquad \forall i, j \in V, t \in T, b \in B, p \in P_i \tag{45}$$

$$\omega_{ib\xi}, \delta_{ijb\xi}, \gamma_{ip\xi}, \mu_{it\xi}, \eta_{ipt\xi} \in \{0,1\} \qquad \forall i, j \in V, t \in T, b \in B, p \in P_i, \xi \in \Phi \tag{46}$$

$$\sigma_t, \alpha_i, \beta_i, \rho_b, \varsigma_b \geq 0 \qquad \forall i \in V, t \in T, \xi \in \Phi, b \in B \tag{47}$$

$$\sigma_{t\xi}, \alpha_{i\xi}, \beta_{i\xi}, \Delta_{i\xi}, \rho_{b\xi}, \varsigma_{b\xi} \geq 0 \qquad \forall i \in V, t \in T, \xi \in \Phi, b \in B \tag{48}$$

Constraints (2) ensure that each vessel is allocated to one berth. Constraints (3) state that each vessel begins to be served at a certain time step. Constraints (4) connect the two decision variables (i.e., μ_{it} and α_i). Constraints (5) connect the start time step and the ending time step of the handling for each vessel. Constraints (6)–(8) ensure that vessels that have been allocated in the same berth cannot have overlap between their planned dwelling time slots. Constraints (9)–(10) emphasize the condition that planned dwelling time slot for each vessel must lie within its feasible service time window. Constraints (11)–(12) define that ρ_b (or ς_b) is no later than (or no earlier than) all of the start (or ending) time step of vessels who occupy the berth b. Constraints (13) ensure that the gap between ρ_b and ς_b is less than the planning horizon. Constraints (11)–(13) guarantee the periodicity of the baseline schedule. Constraints (14) enforce only one QC profile can be selected for each vessel. Constraints (15)–(17) link three decision variables η_{ipt} , μ_{it} and γ_{ip}. Constraints (18) calculate the number of QCs used in each time step. Constraints (19)–(20) guarantee that the number of QCs used cannot exceed the capacity Q_t in each time step. Constraints (21)–(27), Constraints (29)–(39) are similar with Constraints (2)–(8), Constraints (10)–(20), which have the similar meaning as the constraints for the baseline schedule. Constraints (28) ensure that the start time step of handling $\alpha_{i\xi}$ should be later than the actual arrival time $a_{i\xi}$ in each scenario. Constraints (40)–(42) link three decision variables $\varpi_{ibb'\xi}$, ω_{ib} and $\omega_{ib'\xi}$. Constraints (43) calculate each vessel's deviation if the vessel is not moored at its planned berth in each scenario. Constraints (44) ensure that each vessel's assigned QC-profile should satisfy the requirement of its actual workload. Constraints (45)–(48) define the domains of decision variables.

3.3 Linearization for the Objective (1)

There is a nonlinear form '$(\cdot)^+$' in objective (1). To linearize the first part, that is, $\sum_{i\in V} c_i\,[(a_i^e-\alpha_i)^+ + (\beta_i-b_i^e)^+]$, we define the additional decision variables $\tau_i^{a+},\tau_i^{a-},\tau_i^{b+},\tau_i^{b-},\forall i\in V$. By adding following constraints, the first part in the objective can be reformulated as $\sum_{i\in V} c_i\left(\tau_i^{a+}+\tau_i^{b+}\right)$

$$a_i^e-\alpha_i=\tau_i^{a+}-\tau_i^{a-},\ \forall i\in V \tag{49}$$

$$\beta_i-b_i^e=\tau_i^{b+}-\tau_i^{b-},\ \forall i\in V \tag{50}$$

$$\tau_i^{a+},\tau_i^{a-},\tau_i^{b+},\tau_i^{b-}\ge 0 \tag{51}$$

The second part, i.e., $\sum_{i\in V} c_i'\left[(\alpha_i-\alpha_{i\xi})^+ + (\beta_{i\xi}-\beta_i)^+\right]$, is treated by the same way as the first part. The second part in the objective can be reformulated as $\sum_{i\in V} c_i'\left(\tau_{i\xi}^{a+}+\tau_{i\xi}^{b+}\right)$

$$\alpha_i-\alpha_{i\xi}=\tau_{i\xi}^{a+}-\tau_{i\xi}^{a-},\ \forall i\in V,\xi\in\Phi \tag{52}$$

$$\beta_{i\xi}-\beta_i=\tau_{i\xi}^{b+}-\tau_{i\xi}^{b-},\ \forall i\in V,\xi\in\Phi \tag{53}$$

$$\tau_{i\xi}^{a+},\tau_{i\xi}^{a-},\tau_{i\xi}^{b+},\tau_{i\xi}^{b-}\ge 0,\forall i\in V,\xi\in\Phi \tag{54}$$

Based on these new decision variables and constraints, the model RRBACAP can be reformulated as a MILP model.

[M2] $Min\ \sum_{i\in V} c_i\,(\tau_i^{a+}+\tau_i^{b+})+\sum_{\xi\in\Phi}\rho(\xi)[\sum_{i\in V} c_i'(\tau_{i\xi}^{a+}+\tau_{i\xi}^{b+})+(c^{qc}+c^{tax})\sum_{t\in T}\sigma_{t\xi}]$ (55)

s.t. Constraints (2)–(54)

4. A CG based solution method

The model M2 is a two-stage stochastic integer programming model based on a set of scenarios. The model contains a very large number of variables and constraints and is intractable for commercial solvers in large-scale problem instances. Therefore, this section proposes a novel solution method by using the methodology of CG. The core idea, as well as the framework of the solution method, is addressed in Section 4.1; and the details of sub-procedures are elaborated in Sections 4.2–4.4.

4.1 Framework of the solution method

This study proposes a novel method for solving the two-stage stochastic integer programming model. Besides the specific BAPs investigated in this paper, the proposed method could also be applied to similarly structured two-stage stochastic integer programming models.

For the simplicity of exposition, we define below a standard formulation for a two-stage stochastic integer programming model.

$$\begin{aligned} Min \quad & Z\left(\boldsymbol{X}, \{\boldsymbol{Y}_\xi\}_{\xi\in\Phi}\right) = \boldsymbol{AX} + \sum_{\xi\in\Phi} \rho(\xi)[\boldsymbol{BY}_\xi] \\ s.t. \quad & \boldsymbol{CX} \geq 0 \\ & \boldsymbol{DX} + \boldsymbol{EY}_\xi \geq 0 \\ & \boldsymbol{FY}_\xi \geq 0 \end{aligned}$$

In the above model, $\boldsymbol{X}$ denotes decision variables in baseline schedule and $\boldsymbol{Y}_\xi$ denotes the decision variables in adjustment schedule in each scenario ξ. $\boldsymbol{A}$ and $\boldsymbol{B}$ are cost vectors for the first stage and second stage objectives. $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$ and $\boldsymbol{F}$ represent the coefficient matrices.

Algorithm 1: The framework of the CG-based solution method

$n \leftarrow 0$ // n denotes the iteration index
Obtain an initial baseline schedule $\boldsymbol{X}^{(n)}$ // methods are elaborated in **Section 4.2**
$bool \leftarrow true$
While ($bool$)
  For all $\xi \in \Phi$ // for each scenario
    Solve the following model with $\boldsymbol{Y}_\xi$ as variables by using the procedure of generating columns contained in the CG // elaborated in **Section 4.3**

$$\begin{aligned} Min \quad & Z(\boldsymbol{Y}_\xi) = \boldsymbol{BY}_\xi \\ s.t. \quad & \boldsymbol{DX}^{(n)} + \boldsymbol{EY}_\xi \geq 0 \\ & \boldsymbol{FY}_\xi \geq 0 \end{aligned}$$

    The generated columns for $\boldsymbol{Y}_\xi$ are recorded in a set $\mathbb{Y}_\xi$ for scenario ξ
  End For
  Solve the following model with $\boldsymbol{X}$ and $\{\boldsymbol{\chi}_\xi\}_{\xi\in\Phi}$ as variables. *//$\boldsymbol{\chi}_\xi$ is a vector with binary variables, which denote which column within the set $\mathbb{Y}_\xi$ is selected for the scenario ξ*

$$\begin{aligned} Min \quad & Z\left(\boldsymbol{X}, \{\boldsymbol{\chi}_\xi\}_{\xi\in\Phi}\right) = \boldsymbol{AX} + \sum_{\xi\in\Phi} \rho(\xi)[\boldsymbol{G\chi}_\xi] \\ s.t. \quad & \boldsymbol{CX} \geq 0 \\ & \boldsymbol{DX} + \boldsymbol{H\chi}_\xi \geq 0 \\ & \boldsymbol{J\chi}_\xi \geq 0 \end{aligned}$$

  // the above model is solved by the traditional CG; details are elaborated in **Section 4.4**
  $n \leftarrow n + 1$
  Record the solution for the above model as $\boldsymbol{X}^{(n)}$
  If $\boldsymbol{X}^{(n)}$ has not been improved for a certain number of iterations
    $bool \leftarrow false$
    Output the best $\boldsymbol{X}$ obtained so far
  End If
End While

4.2 Method for obtaining an initial baseline schedule $\boldsymbol{X}^{(0)}$

For obtaining an initial baseline schedule, we can solve a simplified model with only parameters and variables that are related to the baseline schedule. All the scenario-related constraints and the objective part are removed from model M2. A model for only optimizing baseline schedule is denoted by M3 and is formulated as follows:

$$[\mathbf{M3}]\ Min\ \sum_{i\in V} c_i\left(\tau_i^{a+}+\tau_i^{b+}\right)+\sum_{\xi\in\Phi}\rho(\xi)\sum_{i\in V} c_i'(a_{i\xi}-\alpha_i)^+ \tag{56}$$

s.t. Constraints (2)–(20), (45), (47), (49)–(51)

If the above model can be solved directly by the commercial solvers such as CPLEX within a reasonable time, we use CPLEX to solve M3 and then output solution as the initial baseline schedule for the following algorithmic procedures; otherwise, we can use a heuristic based on sequential inserting to solve M3 (Zhen et al., 2011a). Due to the limitation of the space, the details of the heuristic are not elaborated here, which can be found in Zhen et al. (2011a).

4.3 Generating columns for scenario-related plans

For each scenario ξ, we solve a model (denoted by M4) with only scenario-dependent variables and parameters (our second stage model). Here the baseline schedule is given as known for each iteration; and it is the initial solution obtained in Section 4.2.

$$[\mathbf{M4}]\ Min\ \sum_{i\in V} c_i'(\tau_{i\xi}^{a+}+\tau_{i\xi}^{b+})+(c^{qc}+c^{tax})\sum_{t\in T}\sigma_{t\xi} \tag{57}$$

s.t. Constraints (21)–(39), (44), (46), (48), (52)–(54)

$$\Delta_{i\xi}=\sum_{b\in B}\omega_{ib\xi}|b-b_i| \qquad \forall i\in V, b\in B \tag{58}$$

Model M4 can be reformulated as a set covering model and we then use the procedure of "generating columns" by the traditional CG so that we can obtain a set of columns for model M4. These obtained columns are stored in a set $\mathbb{Y}_\xi$, where each column is the adjustment plan in the realization of the scenario ξ and the adjustment is based on the baseline schedule.

In this step, this study uses the traditional CG procedure of "generating columns". Due to the limitation of space, the set covering model, linear programming (LP) relaxed restricted master problem, and the pricing problem are not elaborated here. These are similar to the ones that solve our master model (model M5) presented in the next.

4.4 CG algorithm for a model transferred from M2

Based on the sets of generated adjustment plans (columns) in scenarios, the original model M2 is transformed (simplified) by replacing the scenario-related decisions (i.e., the second stage of the model M2) by the decision on selecting an adjustment plan for each scenario. The selection decision is reflected by variables $\{\chi_\xi\}_{\xi\in\Phi}$, which are mentioned in the algorithmic framework in Section 4.1 and denote which adjustment plan within the set $\mathbb{Y}_\xi$ is selected for the scenario ξ. After the above transformation, the second-stage decision only lies in χ (i.e., binary $x_{i\xi\wp}$ variables defined later) and the solution space of the second-stage part is reduced significantly. The newly transformed model is denoted by M5 and is formulated as follows.

The model M5 will be solved by the traditional CG algorithm.

Before formulating the model, some newly defined variables and parameters are defined first.

Newly defined index and sets:

$\wp$ index of a plan

$\mathcal{P}_{i\xi}$ set of feasible adjustment plans for vessel i in scenario ξ, $i \in V, \xi \in \Phi$. It should be noted that $\mathbb{Y}_{\xi} = \bigcup_{i \in V} \mathcal{P}_{i\xi}$

Newly defined variables:

$x_{i\xi\wp}$ binary, set to one if plan $\wp$ is determined for vessel i in scenario ξ; $i \in V, \xi \in \Phi, \wp \in \mathcal{P}_{i\xi}$.

Newly defined parameters:

$\alpha_{i\xi\wp}$ start time step of handling in plan $\wp$ for vessel i in scenario ξ; $i \in V, \xi \in \Phi, \wp \in \mathcal{P}_{i\xi}$

$\beta_{i\xi\wp}$ ending time step of handling in plan $\wp$ for vessel i in scenario ξ; $i \in V, \xi \in \Phi, \wp \in \mathcal{P}_{i\xi}$

$b_{i\xi\wp}$ allocated berth in plan $\wp$ for vessel i in scenario ξ; $i \in V, \xi \in \Phi, \wp \in \mathcal{P}_{i\xi}$

[**M5**] *Min* $\sum_{i \in V} c_i (\tau_i^{a+} + \tau_i^{b+}) + \sum_{\xi \in \Phi} \rho(\xi)[\sum_{i \in V} c_i' (\tau_{i\xi}^{a+} + \tau_{i\xi}^{b+}) + (c^{qc} + c^{tax}) \sum_{i \in V} (1 + d\Delta_{i\xi}) w_{i\xi}]$ (59)

s.t.: Constraints (2)–(20), (45), (47), (49)–(54)

$$\sum_{\wp \in \mathcal{P}_{i\xi}} x_{i\xi\wp} = 1 \quad \forall i \in V, \xi \in \Phi \quad (60)$$

$$\sum_{\wp \in \mathcal{P}_{i\xi}} x_{i\xi\wp} \alpha_{i\xi\wp} = \alpha_{i\xi} \quad \forall i \in V, \xi \in \Phi \quad (61)$$

$$\sum_{\wp \in \mathcal{P}_{i\xi}} x_{i\xi\wp} \beta_{i\xi\wp} = \beta_{i\xi} \quad \forall i \in V, \xi \in \Phi \quad (62)$$

$$\Delta_{i\xi} = \sum_{b \in B} \omega_{ib} |b - \sum_{\wp \in \mathcal{P}_{i\xi}} x_{i\xi\wp} b_{i\xi\wp}| \quad \forall i \in V, \xi \in \Phi \quad (63)$$

$$x_{i\xi\wp} \in \{0,1\} \quad \forall i \in V, \xi \in \Phi, \wp \in \mathcal{P}_{i\xi} \quad (64)$$

$$\Delta_{i\xi} \geq 0 \quad \forall i \in V, \xi \in \Phi \quad (65)$$

$$\alpha_{i\xi}, \beta_{i\xi}, \Delta_{i\xi} \geq 0 \quad \forall i \in V, \xi \in \Phi \quad (66)$$

Although the decision space of the above model M5 is much restricted than the original model M2, it is still hard to solve by CPLEX directly for some large-scale instances. Thus, we propose a CG-based heuristic algorithm to derive a near-optimal solution. The restricted master problem (RMP) and the pricing problem (PP) models are defined in the following subsections.

4.4.1 Column definition and RMP for M5

Define $\mathcal{P}_i$ as the set of all feasible assignment plans (columns) for vessel i. The information contained in a column includes berth allocation and QC assignment. $\mathbb{P} = \bigcup_{i \in V} \mathcal{P}_i$ as the set of all columns. In each column $p_i \in \mathcal{P}_i$, two parameters are defined as follows:

$A_{bt}^{p_i}$ equals one if berth b is allocated to vessel i at time step t in column p_i; and zero otherwise; $\forall b \in B, t \in T$

$U_t^{p_i}$ integer, number of QCs used by vessel i at time step t in column p_i; $\forall t \in T$

For each column, its related objective value is denoted by c_{p_i}, whose calculation process is elaborated in Section 4.4.2.

Based on the above definition, we formulate RMP with a subset of feasible columns $\mathbb{P}' = \bigcup_{i \in V} \mathcal{P}_i' \subseteq \mathbb{P}$. To ensure that an initial feasible solution exists in the RMP, we derive initial $\mathbb{P}'$ for the RMP by the heuristic. The decision variable in RMP is defined as λ_{p_i}, i.e., a binary variable, which equals one if the column p_i is selected. As the usual procedure of the CG algorithm, a linear programming relaxation of RMP (LR-RMP) is solved. The LR-RMP for M5 is formulated as follows, denoted by M6, in which the binary variable λ_{p_i} is relaxed to a continuous variable between zero and one. Note the scenario-related second-stage decisions will be captured in PP rather than in the RMP.

[**M6**] *Min* $\sum_{i \in V} \sum_{p_i \in \mathcal{P}_i} c_{p_i} \lambda_{p_i}$ (67)

Subject to:

$\sum_{p_i \in \mathcal{P}_i} \lambda_{p_i} = 1$ $\quad \forall i \in V$ (68)

$\sum_{i \in V} \sum_{p_i \in \mathcal{P}_i} A_{bt}^{p_i} \lambda_{p_i} \leq 1$ $\quad \forall b \in B, t \in T$ (69)

$\sum_{i \in V} \sum_{p_i \in \mathcal{P}_i} (U_t^{p_i} + U_{t+H}^{p_i}) \lambda_{p_i} \leq Q_t$ $\quad \forall t \in \{1, \dots, E\}$ (70)

$\sum_{i \in V} \sum_{p_i \in \mathcal{P}_i} U_t^{p_i} \lambda_{p_i} \leq Q_t$ $\quad \forall t \in \{E + 1, \dots, H\}$ (71)

$0 \leq \lambda_{p_i} \leq 1$ $\quad \forall i \in V, p_i \in \mathcal{P}_i$ (72)

$\rho_b, \varsigma_b \geq 0$ $\quad \forall b \in B$ (73)

Objective (67) minimizes the total cost. Constraints (68) ensure that one column is selected for each vessel. Constraints (69) guarantee that each berth is occupied by at most one vessel in each time step. Constraints (70)–(71) ensure that the number of used QCs in each time step t does not exceed the capacity limit. Constraints (72)–(73) define the decision variables.

It is noted that the M6 does not contain the constraints that ensure the periodicity of the berth allocation, i.e., constraints (74)–(76), because M6 is linear programming relaxation of the M5's RMP, which implies each vessel i may have more than one feasible plans (columns) in the model M6 and the periodicity of the berth allocation is invalid. Therefore, the constraints on periodicity are considered in the last stage of the CG, i.e., the stage for generating integer $\lambda_{\wp_i}$ variable, which is elaborated in Section 4.4.3.

$$t \cdot \sum_{i\in V}\sum_{\wp_i\in\mathcal{P}_i} A_{bt}^{\wp_i}\lambda_{\wp_i} + M\left(1-\sum_{i\in V}\sum_{\wp_i\in\mathcal{P}_i} A_{bt}^{\wp_i}\lambda_{\wp_i}\right) - \rho_b \geq 0 \qquad \forall b\in B, t\in T \qquad (74)$$

$$t \cdot \sum_{i\in V}\sum_{\wp_i\in\mathcal{P}_i} A_{bt}^{\wp_i}\lambda_{\wp_i} + M\left(\sum_{i\in V}\sum_{\wp_i\in\mathcal{P}_i} A_{bt}^{\wp_i}\lambda_{\wp_i} - 1\right) - \varsigma_b \leq 0 \qquad \forall b\in B, t\in T \qquad (75)$$

$$\varsigma_b - \rho_b \leq H - 1 \qquad \forall b\in B \qquad (76)$$

At each iteration, the dual variables of M6 are transferred to PP to generate new columns. These dual variables are defined as follows:

y_i dual variables for constraints (68), $\forall i\in V$

m_{bt} dual variables for constraints (69), $\forall b\in B, t\in T$

n_t dual variables for constraints (70) and (71), $\forall t\in T$

Based on dual variables, PP is constructed to generate new columns with negative reduced cost. These new columns will be added to LR- RMP (i.e., M6) until no column can be added.

4.4.2 PP and its solving method

PP (pricing problem) of M6 can be decomposed into $|V|$ subproblems, each of which is formulated for one vessel. The decomposed pricing subproblem for generating columns for vessel i is formulated as follows and denoted by model M7, where note that the scenario-related second-stage decisions are made here (i.e, Constraints (60)–(66)).

[**M7**] $Min\ \left\{c_{\wp_i} - \left(y_i + \sum_{b\in B}\sum_{t\in T} m_{bt}\cdot o_{bt} + \sum_{t\in T} n_t\cdot\kappa_t\right)\right\}$ (77)

Subject to:

constraints (2)–(5), (9)–(10), (14)–(17), (45), (47), (49)–(54), (60)–(66)

$$\kappa_t = \sum_{p\in P_i}\sum_{m=max\{1,t-h_{ip}+1\}}^{t} \eta_{ipm} q_{ip(t-m+1)} \qquad \forall t\in T \qquad (78)$$

$$t + M(1-\nu_t) \geq \alpha_i \qquad \forall t\in T \qquad (79)$$

$$t \leq \beta_i + M(1-\nu_t) \qquad \forall t\in T \qquad (80)$$

$$\sum_{t\in T}\nu_t = \beta_i - \alpha_i + 1 \qquad (81)$$

$$o_{bt} \geq \nu_t + \omega_{ib} - 1 \qquad \forall t\in T, b\in B \qquad (82)$$

$$o_{bt} \leq \nu_t \qquad \forall t\in T, b\in B \qquad (83)$$

$$o_{bt} \leq \omega_{ib} \qquad \forall t\in T, b\in B \qquad (84)$$

$$c_{\wp_i} = c_i\left(\tau_i^{a+} + \tau_i^{b+}\right) + \sum_{\xi\in\Phi}\rho(\xi)\left[c_i'(\tau_{i\xi}^{a+} + \tau_{i\xi}^{b+}) + (c^{qc} + c^{tax})(1 + d\Delta_{i\xi})w_{i\xi}\right] \qquad (85)$$

$$o_{bt}, \nu_t \in \{0,1\} \qquad \forall t\in T, b\in B \qquad (86)$$

$$\kappa_t, c_{\wp_i} \geq 0 \qquad \forall t\in T \qquad (87)$$

The computational efficiency of solving PP is a key factor that affects the performance of the whole CG-based algorithm. It is time-consuming to solve the PP by using CPLEX directly, especially when facing some large-scale instances. Therefore, we propose an exact algorithm (Algorithm 2) to solve the PP efficiently based on the special features of PP.

Algorithm 2: A pseudo-polynomial exact algorithm for solving PP

Input: Parameters of vessel i, a dual vector $(\mathcal{Y}_i, \mathcal{M}_{bt}, \mathcal{N}_t)$
Output: An optimal assignment plan for a berth, planned time slot and QC assignment and its minimal reduced cost $\mathcal{Z}_{b,p,\alpha_i}$
For $b \in B$
    For $p \in P_i$
        For $\alpha_i \in \left[a_i^f, b_i^f - h_{ip} + 1\right]$ //possible start time step
            $\mathcal{Z}_{b,p,\alpha_i} \leftarrow c_i\{(a_i^e - \alpha_i)^+ + [(\alpha_i + h_{ip} - 1) - b_i^e]^+\}$
            For $\xi \in \Phi$
                For $\wp \in \mathcal{P}_{i\xi}$
                    $c''_{i\xi\wp} \leftarrow c'_i\left\{\left(\alpha_i - \alpha_{i\xi\wp}\right)^+ + [\beta_{i\xi\wp} - (\alpha_i + h_{ip} - 1)]^+\right\} + (c^{qc} + c^{tax})(1 + d|b - b_{i\xi\wp}|)w_{i\xi}$
                End for
                $\mathcal{Z}_{b,p,\alpha_i} \leftarrow \mathcal{Z}_{b,p,\alpha_i} + min_{\wp \in \mathcal{P}_{i\xi}}\{c''_{i\xi\wp}\}$
            End for
            //Calculate the number of used QCs in time step t
            $\kappa_t \leftarrow q_{ip(t-\alpha_i+1)}, \forall t \in [\alpha_i, \alpha_i + h_{ip} - 1]$
            $\kappa_t \leftarrow 0, \forall t \in T \backslash [\alpha_i, \alpha_i + h_{ip} - 1]$
            $\mathcal{Z}_{b,p,\alpha_i} \leftarrow \mathcal{Z}_{b,p,\alpha_i} - (\sum_{t \in [\alpha_i, \alpha_i + h_{ip} - 1]} \mathcal{M}_{bt} + \sum_{t \in T} \mathcal{N}_t \times \kappa_t + \mathcal{Y}_i)$
        End for
    End for
End for
Return $min_{b \in B, p \in P_i, \alpha_i \in \left[a_i^f, b_i^f - h_{ip} + 1\right]}\{\mathcal{Z}_{b,p,\alpha_i}\}$ which is the minimal reduced cost of the PP, and the new optimal assignment plan for the vessel can be extracted from the values of the decision variables (i.e., o_{bt}^*, κ_t^*) in the combination (b, p, α_i)

Algorithm 2 is based on combining a possible berth, a possible QC profile and a possible start time step. We obtain each possible berth b from B, each possible QC profile p from the set P_i and each possible start time step from a range $\left[a_i^f, b_i^f - h_{ip} + 1\right]$, which could be denoted as a set $\mathcal{X}$ containing all possible start time steps in the range. Given a combination of b, p and α_i, the algorithm basically runs a process of selecting assignment plan for each scenario, which requires $O(|\Phi||\mathcal{P}_{i\xi}|)$ computation time. Thus, the overall computational complexity of Algorithm 2 is $O(|B||P_i||\mathcal{X}||\Phi||\mathcal{P}_{i\xi}|)$.

4.4.3 Heuristic for obtaining a binary solution by embedding CG

As the model M6 is a linear relaxation, the solution on $\lambda_{\wp_i}$ yielded from the CG is fractional.

Therefore, a CG-based heuristic algorithm by embedding CG is designed to construct a feasible integer solution for the model M5. The whole procedure of the CG algorithm is shown as follows. The following procedure mainly focuses on how to obtain a binary solution ($\lambda_{\mathcal{p}_i}$) by embedding CG, while the traditional process of “generating columns”, i.e., solving LR-RMP and PP as usual practice of CG, is not elaborated.

Algorithm 3: Heuristic for obtaining a binary solution by embedding CG

1 $be\_ts_{bt} \leftarrow 1,\ QC\_num_t \leftarrow Q_t,\ \Omega_1 \leftarrow V,\ \Omega_2 \leftarrow \emptyset\ \ \forall b \in B, t \in T$
// Initialization: be_ts_{bt} equals one if berth b is available at time t; otherwise zero. QC_num_t denotes the number of available QCs at time t. Ω_1 set (or Ω_2 set) contains vessels whose plans have not (or have) been determined.
2 **Do** {
3 $\Theta_0 \leftarrow \emptyset$ // Θ_0 set will be used to include columns whose corresponding λ variable is not zero, and whose corresponding plan should satisfy port resources related constraints (68)–(71).
4 **Invoke** the procedure of “generating columns” by solving LR-RMP and PP and obtain a set of columns $\mathbb{P} = \bigcup_{i \in V} \mathcal{P}_i$, indexed by $\mathcal{p}_i$.
5 **For** $\mathcal{p}_i \in \mathbb{P}$
6 $\Theta_0 \leftarrow \Theta_0 \cup \{\mathcal{p}_i | \lambda_{\mathcal{p}_i} > 0,\ i \in \Omega_1,\ \mathcal{p}_i$ satisfies M6’s constraints (68)–(71)$\}$
7 End for
8 **For** $\mathcal{p}_i \in \mathbb{P}/\Theta_0$
9 $\lambda_{\mathcal{p}_i} \leftarrow 0$
10 End for
11 $vmax_{\Theta_0} \leftarrow \max_{\mathcal{p}_i \in \Theta_0} \lambda_{\mathcal{p}_i},\ vnum_{\Theta_0} \leftarrow \left|\{\mathcal{p}_i | \lambda_{\mathcal{p}_i} = vmax_{\Theta_0}, \mathcal{p}_i \in \Theta_0\}\right|$
12 **If** $vnum_{\Theta_0} = 1$ then
13 $\mathcal{p}_i{}^* \leftarrow \arg\max_{\mathcal{p}_i \in \Theta_0} \{\lambda_{\mathcal{p}_i}\};\ \lambda_{\mathcal{p}_i{}^*} \leftarrow 1$
14 **For** $\mathcal{p}_i \in \Theta_0/\{\mathcal{p}_i{}^*\}$
15 $\lambda_{\mathcal{p}_i} \leftarrow 0$
16 End for
17 **Update** $\Omega_1 \leftarrow \Omega_1/\{i|\lambda_{\mathcal{p}_i} = 1\};\ \Omega_2 \leftarrow \Omega_2 \cup \{i|\lambda_{\mathcal{p}_i} = 1\}$
18 End If
19 **If** $vnum_{\Theta_0} > 1$ then
20 **If** $vmax_{\Theta_0} = 1$ and $vnum_{\Theta_0} = |\Theta_0|$ and $vnum_{\Theta_0} = |\Omega_1|$ then
21 **Calculate** o_b as time span occupied by all the vessels in Ω_2 and columns in Θ_0 for each $b, b \in B$.
22 **If** $o_b \le H$ for each $b, b \in B$// check whether each berth satisfies periodicity
23 **Update** $\Omega_1 \leftarrow \emptyset, \Omega_2 \leftarrow V$
24 **Go to** Line 39
25 **Else**
26 $\mathcal{p}_i{}^* \leftarrow \arg\min_{\mathcal{p}_i \in \{\mathcal{p}_i | \lambda_{\mathcal{p}_i} = 1, \mathcal{p}_i \in \Theta_0\}} \{c_{\mathcal{p}_i}\};\ \lambda_{\mathcal{p}_i{}^*} \leftarrow 1$
27 **For** $\mathcal{p}_i \in \Theta_0/\{\mathcal{p}_i{}^*\}$
28 $\lambda_{\mathcal{p}_i} \leftarrow 0$
29 End for
30 **Update** $\Omega_1 \leftarrow \Omega_1/\{i|\lambda_{\mathcal{p}_i} = 1\};\ \Omega_2 \leftarrow \Omega_2 \cup \{i|\lambda_{\mathcal{p}_i} = 1\}$
31 **Else**
32 $\mathcal{p}_i{}^* \leftarrow \arg\max_{\mathcal{p}_i \in \Theta_0} \{\lambda_{\mathcal{p}_i}\},\ \min_{\mathcal{p}_i \in \{\mathcal{p}_i | \lambda_{\mathcal{p}_i} = vmax_{\Theta_0}, \mathcal{p}_i \in \Theta_0\}} \{c_{\mathcal{p}_i}\};\ \lambda_{\mathcal{p}_i{}^*} \leftarrow 1$

For $\wp_i \in \Theta_0/\{\wp_i^*\}$
    $\lambda_{\wp_i} \leftarrow 0$
End for
Update $\Omega_1 \leftarrow \Omega_1/\{i|\lambda_{\wp_i} = 1\}$;$\Omega_2 \leftarrow \Omega_2 \cup \{i|\lambda_{\wp_i} = 1\}$
End If
Update be_ts_{bt}, QC_num_t and M6 // according to port resources occupied by vessel i removed from Ω_1 and periodicity
} **While** ($|\Omega_1| > 0$ and $|\Omega_2| < V$)

4.5 Summary on CG-based solution method

The above subsections present a CG-based solution method for solving a low carbon BAQCAP under uncertainty, which is a two-stage stochastic integer programming model based on a set of scenarios reflecting the random realizations of context in future. The flowchart of the proposed solution method is demonstrated in Figure 1.

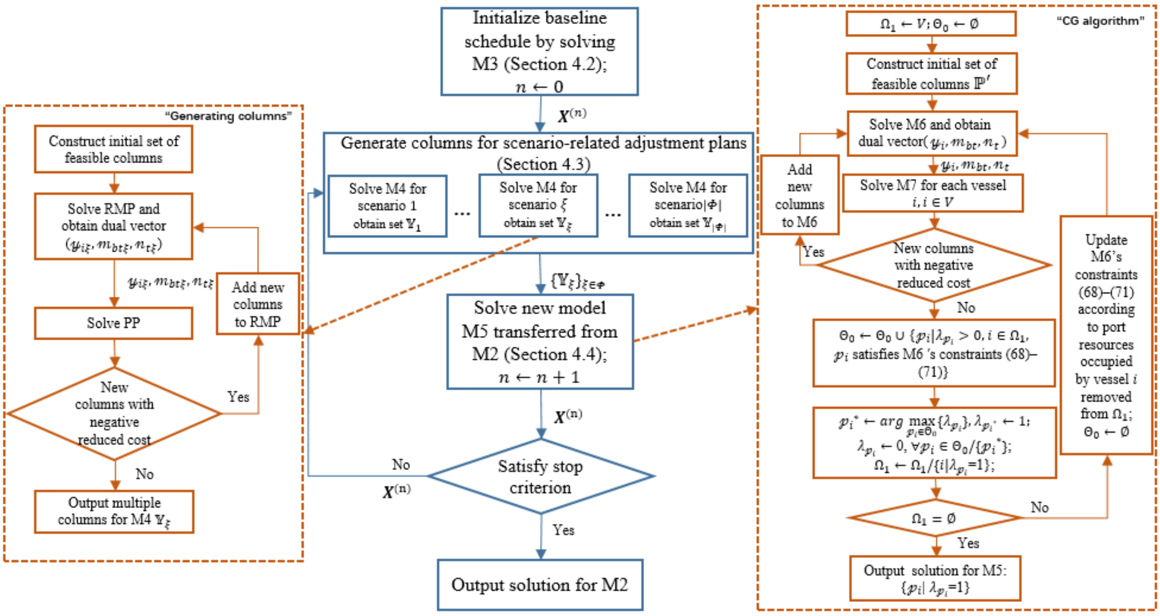

Figure 1: The whole procedure of CG based solution method

The main features of our proposed method include:

(1) The traditional CG process is used in two steps. One is at the second stage of the model for generating columns (i.e., adjustment plans) for each scenario; the other is at the first stage for solving a transformed model that includes the baseline decision and the decision on selecting a suitable adjustment plan (one of the above-generated columns) for each scenario. The usage of the CG process in the above two steps slightly differs from each other. In the formerly mentioned step, we only generate columns by using the CG procedure; while in the latter mentioned step, we use a CG-based algorithm to solve the first-stage model.

(2) Some steps embedded in the proposed method can be decomposable. For example, when solving the model M5, its PP can be decomposed into $|V|$ independent subproblems, each of which generates columns for each vessel; when generating adjustment plans for future scenarios, this step can also be decomposed into $|\Phi|$ independent sub-steps, each of which generates adjustment plans for each scenario. All the above decomposable structure makes the solution method's execution process highly efficient.

5. Computational experiments

Numerical experiments are conducted to validate the effectiveness of the proposed model and the efficiency of the algorithm. Sensitivity analysis of taxation rates and uncertain parameters are also conducted. Based on these experimental results, some managerial implications are outlined. The experiments are performed on a Lenovo Thinkstation P910 workstation equipped with two Intel Xeon E5-2680 V4 CPUs (28 cores) of 2.4 GHz processing speed and 256 GB RAM running Windows 7. All algorithms are programmed in C# (VS2015), and the RMPs are solved by CPLEX 12.6. The CPU time limit for all test instances is two hours.

5.1 Generation of the test instances

This study considers four groups of instances. The parameter settings for the four instance groups (ISGs) are listed in Table 2. Three types of ships, namely feeder vessels, medium vessels, and jumbo vessels, are involved. The vessel characteristics and the information for generating QC profiles are listed in Table 3. The planning horizon is one week (i.e., 42-time steps).

The berth utilization and QC utilization for each instance group are estimated to validate whether the above experiment setting follows the reality. The estimation method is similar to Zhen et al. (2011) and Wang et al. (2018); due to the limitation of space, it is not repeated here. Table 4 shows the results of the estimated berth utilization and QC utilization in instances; they are about 66-77%, which is close to the realistic port operations context.

Table 2: Problem scales of four instance groups

Group ID	Num. of vessels (|V|)	Num. of berths (|B|)	Num. of QCs (|Q|)	Num. of time steps (|H|)
ISG1	30	4	10	42
ISG2	36	5	13	42
ISG3	48	6	17	42
ISG4	60	8	21	42

Table 3: Information for different vessel classes

Vessel	**QC profile specifications**						
Class	Range of number of used QCs	Range of handling time (time step)	Average handling time (time step)	Range of workload (QC × time step)	Average workload (QC × time step)	Range of workload (QC × time step) in scenarios	Average workload (QC × time step) in scenarios
Feeder	1-3	2-4	3	2-5	3.5	2-7	4.5
Medium	2-4	3-5	4	6-14	10.0	6-17	11.5
Jumbo	3-5	4-6	5	15-20	17.5	13-21	17.0

Table 4: The berth and QC utilization of the instances in experiments

Group	**Berth utilization**			**QC utilization**		
	Vessel usage (|V|×4)	Port resource (|B|×|H|)	Utilization rate%	Vessel usage (|V|×10.5)	Port resource (|Q|×|H|)	Utilization rate%
ISG1	120	168	71.4%	315	420	75.0%
ISG2	144	210	68.6%	378	546	69.2%
ISG3	192	252	76.2%	504	714	70.6%
ISG4	240	336	71.4%	630	882	71.4%

5.2 Efficiency of CG-based solution method

A set of 36 randomly generated instances with varying characteristics is used for the evaluation. As shown in Table 5, CPLEX can solve the problem only for some small-scale instances, i.e., ISG1 and ISG2 groups with three scenarios. From the column "Gap1" in Table 5, we can see that the average gap between the results solved by our method and the optimal results is about 2.99%; while the computation time of our method is much shorter than the computation time by using CPLEX directly.

Table 5: Comparison with the optimal results and LBs in small-scale instances

Instance	Optimal result solved by CPLEX		CG based algorithm		LB	Gap	
ID	Obj1	t(s)	Obj2	t(s)	Obj3	Gap1	Gap2
ISG1-S3-1	30567	1065	31368	84	30560	2.62%	0.02%
ISG1-S3-2	30687	2196	31269	335	30349	1.90%	1.10%
ISG1-S3-3	31396	780	32772	67	31395	4.38%	0.00%
ISG2-S3-4	37084	1605	38880	308	37084	4.84%	0.00%
ISG2-S3-5	37084	1625	38400	186	37084	3.55%	0.00%
ISG2-S3-6	37084	1410	37320	263	37084	0.64%	0.00%
Average (%)						2.99%	0.19%

Notes: Instance ID denotes "problem scale"-"number of scenarios"-"index of instance", respectively; Gap1=(Obj2-Obj1)/Obj1; Gap2=(Obj1-Obj3)/Obj1.

By relaxing the binary variables and the QC related constraints, we propose a lower bound (LB) to evaluate the results in some large-scale problem instances, for which CPLEX cannot solve and output the optimal solutions for evaluating the performance of our proposed method. Before using the LB in the performance evaluation in large-scale instances, we should know how tight the LB is from the optimal result. The column "Gap2" shows that the average gap is 0.19%, which implies that the LB is very close to the optimal results. In this case, the LB is suitable for later evaluating the performance of our method in large-scale instances.

Table 6: Performance evaluation of our method in instances that CPLEX cannot solves

Instance	CG based algorithm		LB	Gap
ID	Obj2	t(s)	Obj3	
ISG3-S3-7	51948	325	49793	4.33%
ISG3-S3-8	52513	777	49793	5.46%
ISG3-S3-9	50988	947	49793	2.40%
ISG4-S3-10	64488	898	61854	4.26%
ISG4-S3-11	66528	816	61854	7.56%
ISG4-S3-12	63408	562	61854	2.51%
ISG1-S10-13	40405	291	39407	2.53%
ISG1-S10-14	39631	119	39407	0.57%
ISG1-S10-15	39391	280	37967	3.75%
ISG2-S10-16	49326	413	46551	5.96%
ISG2-S10-17	47886	1260	46346	3.32%
ISG2-S10-18	47646	337	46346	2.80%
ISG3-S10-19	65213	668	62188	4.86%
ISG3-S10-20	64919	1033	62188	4.39%
ISG3-S10-21	64439	543	62188	3.62%
ISG4-S10-22	82585	1641	78543	5.15%
ISG4-S10-23	80665	817	77244	4.43%
ISG4-S10-24	78167	696	77243	1.20%
ISG1-S15-25	41474	405	40458	2.51%
ISG1-S15-26	40442	278	39407	2.63%
ISG1-S15-27	41044	126	39407	4.15%
ISG2-S15-28	50563	408	47582	6.26%
ISG2-S15-29	49789	230	49329	0.93%
ISG2-S15-30	48883	333	47582	2.73%
ISG3-S15-31	66892	341	63847	4.77%
ISG3-S15-32	66580	400	63847	4.28%
ISG3-S15-33	66100	431	63846	3.53%
ISG4-S15-34	82271	828	79304	3.74%
ISG4-S15-35	82735	726	79304	4.33%
ISG4-S15-36	80231	765	79304	1.17%
Average (%)				3.67%

Notes: Gap=(Obj2-Obj3)/Obj3.

As aforementioned, we cannot compare the results solved by our method with the optimal results in some large-scale instances, thus we compare with the LBs. Table 6 shows the average gap from the LBs is about 3.67%, which may be acceptable for the studied problem such as tactical level decisions. In addition, the computation time is also not very long.

5.3 Sensitivity analysis

This study is oriented to low carbon BAQCAP under uncertainty. The feature "low carbon" is reflected by the carbon emission taxation rate considered in the objective function; while the "uncertainty" is reflected by the random arrival time of vessels and the random workload (in QC time steps) of vessels. Thus, two series of experiments are conducted on different settings on "carbon emission rate – variance of actual arrival time" and "carbon emission rate – variance of actual workload", which are shown in Figure 2 and Figure 3, respectively.

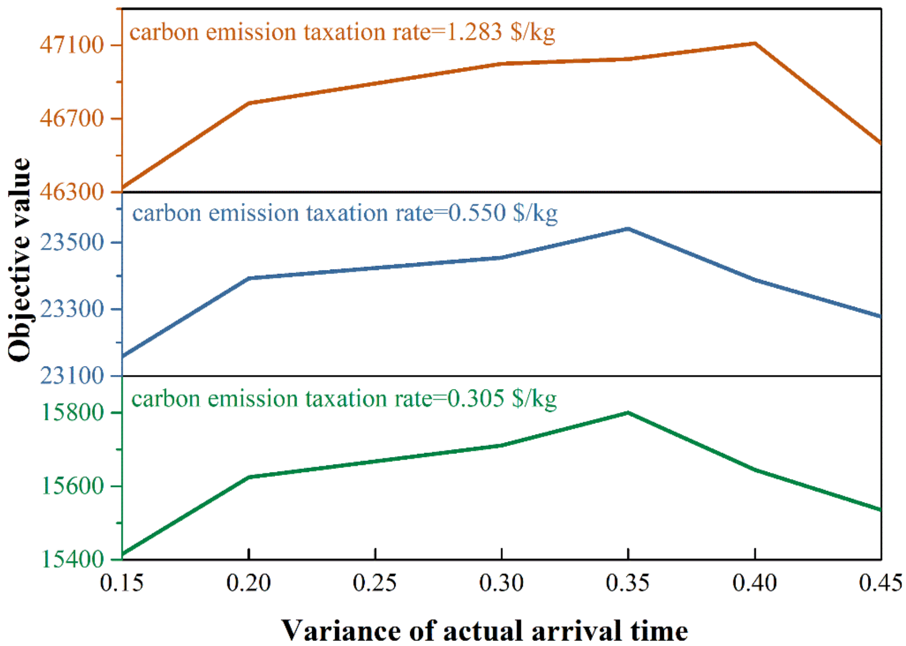

Figure 2: Sensitivity analysis on variance of actual arrival time

Figure 2 shows that the higher is the carbon emission taxation rate, the total cost is larger. Given a carbon emission taxation rate. the objective value increases first with the variance of actual arrival time growing, and then decreases when the variance is further growing. It implies that the growth of the arrival time uncertainty does not always increase the total cost. When the variance is larger than some threshold, the total cost will even decrease with the variance increases. The reason may lie in that a significant changing of the arrival time of a vessel may be "releases" a free berth-time space for other vessels' schedule to approach their expected time slot, which may cause the objective value decreases. In addition, Figure 2 also shows that there seems an increasing trend for the threshold when the carbon emission taxation rate is growing.

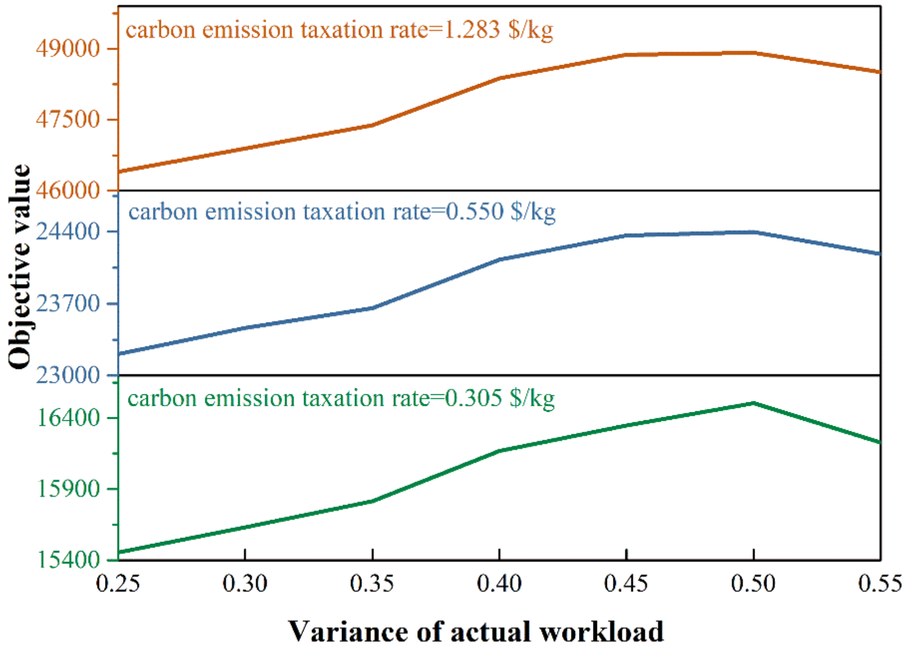

Figure 3: Sensitivity analysis on variance of actual workload

The sensitivity analysis on the variance of actual workload in Figure 3 shows the similar trends and phenomenon as Figure 2 that is related to the variance of actual arrival time. The difference is that the curves in Figure 3 are a bit higher than the curves in Figure 2 given the same carbon tax rate, which means the uncertainty of the workload brings a higher increase to the total cost than the uncertainty of the arrival time.

6. Conclusions

This paper investigates a low carbon oriented BAQCAP under uncertainty. A two-stage stochastic programming model is formulated based on a set of scenarios. A CG-based solution method is developed for solving such a scenario based two-stage model. Numerical experiments are also conducted to validate the efficiency of the proposed solution method. The contribution of this study may contain two aspects.

(1) Few studies have considered the uncertain arrival time and uncertain container loading/unloading workload of vessels in a scheduling problem on both the berth allocation and QC assignment decisions. This study formulates a two-stage stochastic integer

programming model for this problem and the model also considers the carbon emission related issues in the objective. This study may be the first one that investigates the low carbon oriented BAQCAP under uncertainty, which is potentially useful for port operators to make berthing and QC assignment plans when facing the uncertain maritime transportation market.

(2) In the fields of port operations, few studies have applied a CG based methodology to solve a two-stage stochastic integer programming model based on a set of scenarios. A tailored solution method, which evokes the CG procedure in different steps of the method, is designed and implemented for solving the two-stage model. Numerical experiments are conducted to validate the efficiency of this tailored CG-based solution method.

Besides the problem in this study, the two-stage stochastic programming is very common in the fields of decision optimization problem under uncertainties; thus, our proposed CG-based solution method may have a wide application possibility in wide contexts. Transferring the second stage's information to the first stage is a key issue for solving a two-stage model. The well-known bender decomposition method uses bender cuts; while our proposed CG based method uses columns (i.e., the extreme points in the second stage's polyhedron solution space). The CG-based method may be potentially more useful than the bender decomposition method when solving a two-stage stochastic programming model with integer resource. In the future, we will extend our proposed solution methodology to other application problems with the two-stage features and containing integer resources in the second stage, so as to further investigate the generic performance of our proposed methodology.

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