Static Congestion Pricing Considering the Cumulative Impact of Day-to-Day Dynamics with Weibit Adjustment Process

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Abstract

Static pricing is currently the most prevalent form of congestion pricing. Most existing methods for static pricing design use equilibrium-based indices as the optimization objective function. However, in reality, the equilibrium may not be reached within the toll planning horizon, and thus equilibrium-based pricing schemes might be ineffective in alleviating traffic congestion. To address this issue, this paper proposes a new method for the optimal static congestion pricing design. Specifically, we propose a new measure called cumulative network performance (CNP) as the optimization objective function. The CNP measure explicitly considers the fluctuation in network performance over the toll planning horizon, instead of only focusing on the performance at the equilibrium state. To capture the effect of static congestion pricing on CNP, we introduce a day-to-day dynamic model with the Weibit-based route flow adjustment, which can overcome the drawbacks of the user equilibrium or Logit-based route-choice criterion that are used in most current day-to-day dynamic models. Numerical experiments are conducted to show the effectiveness and advantages of the toll scheme resulting from the proposed method.

Keywords: Static congestion pricing; Weibit model; day-to-day dynamics; cumulative network performance; network equilibrium.

1. Introduction

1.1 Literature review

Road congestion pricing has been long regarded as an efficient instrument to relieve traffic congestion. The concept of road congestion pricing can be traced back to Pigou (1920) and later researchers such as Walters (1961), Beckmann (1965), and Vickrey (1969). Since the successful implementation of congestion pricing in Singapore in 1975, many countries and cities, such as Norway, London, Stockholm, and Milan, have established and implemented their own road-congestion pricing policies.

Previous approaches to road congestion pricing can be broadly classified in terms of the degree of time differentiation into two categories: static pricing and dynamic pricing (de Palma and Lindsey, 2011). In static congestion pricing, the toll scheme is usually predetermined and fixed over a certain long time period (also known as the toll planning horizon), such as 3 months in Singapore, after which the toll pattern can be adjusted. Note that time-of-day pricing, in which the toll varies by the time of day or day of the week, is also a form of static pricing, as it follows a predetermined toll schedule. In contrast, in the dynamic pricing, the tolls are changed based on a short time period (such as on a daily basis) according to the network conditions of the previous time period. Generally, two traffic assignment models are widely adopted in congestion pricing studies: static models and day-to-day dynamic models. According to the toll type (i.e., static or dynamic pricing) and model type (static or day-to-day dynamic models), we provide a summary of the existing studies of congestion pricing focusing on three categories of research: static pricing with static traffic model, dynamic pricing with day-to-day traffic model, and static pricing with day-to-day traffic model, as shown in Table 1. The assumptions of and remarks on pricing type, model type, as well as pricing and model type are summarized in Table 2.

Table 1 Summary of representative studies on congestion pricing by model type and pricing type

Research category	Pricing type	Traffic model type	References
1	Static pricing	Static traffic model	Beckmann et al. (1956); Marchand (1968); Dafermos (1973); Smith (1979); Yang and Huang (1998, 2004, 2005); Yang (1999); Maher et al. (2005); Meng et al. (2012); Di et al. (2016)
2	Dynamic pricing	Day-to-day dynamic traffic model	Sandholm (2002); Friesz et al. (2004); Yang and Szeto (2006); Yang et al. (2007); Guo et al. (2013, 2016); Tan et al. (2015); Wang et al. (2015); Rambha and Boyles (2016); Han et al. (2017, 2021); Gehlot et al. (2020)
3	Static pricing	Day-to-day dynamic traffic model	Liu et al. (2017); Ma et al. (2021); **This paper**

Table 2 Assumptions of and remarks on pricing type, model type, as well as pricing and model type

	Type	Assumptions/ Rationale	Remarks
Pricing type	Static pricing	The toll structure is fixed and predetermined within a toll planning horizon	Easier to implement for managers, and more understandable and acceptable for road users
	Dynamic pricing	The toll structure is adjusted frequently, such as on a daily basis	Difficult to implement for managers and confusing for road users, despite being flexible
Model type	Static traffic model	Concern the final equilibrium state only	Can only obtain the network performance at the final equilibrium state
	Day-to-day dynamic model	Concern the day-to-day evolution of the network state	Can obtain day-to-day network performance fluctuation
Pricing and model type	Static pricing with static traffic model	The final equilibrium state can be reached	Easy to implement and straightforward, but might be ineffective since it assumes the final equilibrium state can be reached, which may not be true in reality
	Dynamic pricing with day-to-day dynamic model	The final equilibrium state can be reached	More effective and flexible, but might be difficult to implement for managers and confusing for road users
	Static pricing with day-to-day dynamic model	The final equilibrium state need not necessarily be reached	Easy to implement and straightforward, and is also relatively effective

The well-known method of first-best marginal pricing is a type of static pricing, which states that road users should pay the difference between the marginal social cost and the marginal private cost. With the marginal pricing, a transportation system can be driven to the system optimal (SO) with the minimum total travel time (TTT) (Beckmann et al., 1956) or the stochastic system optimal (SSO) with the minimum expected total travel time (ETTT) (Yang, 1999; Maher et al., 2005) at the equilibrium state. Static equilibrium models, including static User Equilibrium (UE) and Stochastic User Equilibrium (SUE) models, have been used. The marginal pricing has also been extended to consider multiclass, multicriteria, link flow interaction, and demand elasticity in transportation networks (Dafermos, 1973; Smith, 1979; Yang and Huang, 1998, 2004). In addition, some static second-best pricing approaches under physical and economic constraints have also been investigated, such as the cordon-based and distance-based pricing (e.g., Marchand, 1968; Yang and Huang, 2005; Meng et al., 2012; Di et al., 2016).

The abovementioned static pricing methods usually optimize an objective function based on the equilibrium state. This process is straightforward, but suffers from the following two drawbacks. First, it implicitly assumes that an equilibrium state can be reached, regardless of how long it may take. However, in reality, any new toll pattern will affect travelers' route choice decisions. Thus, network flows may take a long time to approach equilibrium, and this may be far longer than the toll planning horizon. For example, Cho and Hwang (2005) tested a small network and found that it took nearly 200 days to reach equilibrium. Clearly, the time period needed to reach equilibrium in the network of a large city would be much longer. Second, only the network performance at the final equilibrium state is optimized. However, after the implementation of a (new) toll scheme, network performance fluctuates for a long time before approaching equilibrium, due to adjustments in travelers' day-to-day route choices. Therefore, in designing static toll schemes, it is necessary to consider day-to-day flow dynamics, rather than only consider the final equilibrium states.

On the other hand, day-to-day dynamic pricing has recently attracted extensive attention in the literature (Sandholm, 2002; Friesz et al., 2004; Yang and Szeto, 2006; Yang et al., 2007; Guo et al., 2013, 2016; Tan et al., 2015; Wang et al., 2015; Rambha and Boyles, 2016; Han et al., 2017, 2021; Gehlot et al., 2020). They can overcome some drawbacks of static pricing using day-to-day dynamic models. However, daily varied pricing is difficult to implement from a manager's perspective and is also confusing from a traveler's perspective, despite the ubiquity of advanced communications technology. Thus, there has been little use of day-to-day dynamic pricing. Moreover, there are problems with the methods used in some current studies of day-to-day dynamic pricing. First, most of these studies assume that the toll scheme resulting from their methods can drive the given initial network state to a target equilibrium state or a steady state, regardless of the time required to achieve this. Second, most of these studies use either the user equilibrium (UE) or the Logit route choice criterion in their day-to-day dynamic models. It is well known that the UE model has the unrealistic perfect information assumption, while the Logit model cannot account for route overlapping and heterogeneous trip variance due to the independently and identically distributed (IID) perception error assumption.

Very few of the current studies on static pricing design consider day-to-day dynamics. In a recent example, Liu et al. (2017) proposed a robust optimization approach for static distance-based pricing design considering day-to-day dynamics. Although the approach considers day-to-day network flow fluctuations, the robust optimization only considers the objective function in the worst case (day). In other words, the network performances on the other days are not explicitly considered in the objective function. In addition, travelers' route adjustments are assumed to follow the Logit model, which suffers from the IID assumption. In another example, Ma et al. (2021) introduced a new day-to-day dynamic model that uses a bi-objective UE for pricing design. This method calculates an optimal toll pattern for each sub-time period to minimize the total system travel time within the entire investigated time period. This method also assumes that the equilibrium state can be reached, which would require thousands of days as illustrated in their example.

1.2 Motivations

The following observations and summaries can be made from the above literature review. Regarding the model type, static traffic model cannot account for network performance fluctuation while day-to-day dynamic model can capture day-to-day network performance fluctuation induced by travelers' route choice adjustment on each day. Regarding the toll type, dynamic pricing is more difficult to implement for traffic managers and more confusing for road users compared to static pricing, although it seems more flexible. Thus, static pricing scheme considering day-to-day dynamics is a more ideal practice, but receives little attention in the literature. Of the only two studies (Liu et al., 2017; Ma et al., 2021) on static pricing scheme considering day-to-day dynamics, they both have their respective limitations mentioned above. Thus, in this study we attempt to fill the gap in the literature by proposing a new method for static pricing design considering day-to-day dynamics.

1.3 Objectives and contributions

Motivated by the above observations, we propose a new method for static pricing that explicitly accounts for network performance fluctuation over the toll planning horizon. Specifically, instead of focusing only on the performance at the equilibrium state as most static pricing methods do, we propose a new measure called cumulative network performance (CNP) as the optimization objective function. Besides, we introduce a day-to-day dynamic model with the Weibit-based route flow adjustment to calculate the CNP measure. The Weibit-based route choice criterion in the proposed day-to-day dynamic model can overcome the drawbacks of the widely-used UE or Logit-based route choice criterion that are used in most existing day-to-day dynamic models. We show the difference between the route flow evolution curves under the Weibit flow adjustment process and the Logit flow adjustment process, and how such a difference further influences the toll scheme design. Numerical examples are provided to show the effectiveness of the pricing scheme developed from the proposed method and to demonstrate its advantages over some existing static pricing schemes.

The remainder of this paper is organized as follows. Section 2 presents the methodology, including the proposed day-to-day dynamic model with the Weibit-based route choice adjustment, the new network performance measure, and the bi-level programming model for deriving the optimal toll schemes as well as its solution algorithm. Section 3 uses numerical examples to show the advantages of the toll schemes derived from the proposed method over the marginal pricing scheme and the robust pricing scheme. Finally, conclusions and future research directions are summarized in Section 4.

2. Methodology

In this section, we first introduce the proposed day-to-day dynamic model. Then, we propose a new network performance measure as the optimization objective function. Finally, a bi-level model to derive the optimal static pricing scheme is established, together with its solution algorithm. The main notation used in this paper is summarized in Table 3.

Table 3 Notation and explanations

Notation	Explanation
W	The set of origin-destination (O-D) pairs
R^w	The set of paths between an O-D pair $w \in W$
Υ_r	The set of links comprising path r
A	The set of links
D	The toll planning horizon, i.e., the toll pattern is changed every D days
d	The d^{th} day after the toll implementation, $d = 1,2, \dots, D$
$f_{w,r}$	The traffic flow on path $r \in R^w$ between O-D pair $w \in W$
f	The column vector of all of the path flows, $\mathbf{f} = (f_{w,r}, r \in R^w, w \in \mathrm{W})^T$
m	The column vector of all of the target flows, $\mathbf{m} = (m_{w,r}, r \in R^w, w \in \mathrm{W})^T$
α^d	The flow adjustment ratio, which is assumed to be constant throughout D days
$g_{w,r}$	The route travel cost of route r that connects O-D pair w
τ_a	The link travel cost of link a
$H_{w,r}$	The predicted path travel cost of route r that connects O-D pair w
$h_{w,r}$	The mean of the predicted path travel cost of route r that connects O-D pair w
γ	The weighting parameter of the actual path cost on the previous day
q^w	The travel demand between $w \in \mathrm{W}$
q	The column vector for all of the travel demands, $\mathbf{q} = (q^w, w \in W)^T$
v_a	The flow on link $a \in A$
v	The column vector of all of the link flows, $\mathbf{v} = (v_a, a \in A)^T$
τ_a	The link travel cost of link a
t_a	The travel time function of link $a \in A$
t	The column vector of the link travel time functions, $\mathbf{t} = (t_a, a \in A)^T$
δ_{ra}^w	$\delta_{ra}^w = 1$ if path $r \in R^w$ contains link a, and $\delta_{ra}^w = 0$ otherwise
y	The column vector of the link toll pattern, $\mathbf{y} = (y_a, a \in A)^T$
β^w	The shape parameter of O-D pair w in the Weibit model
ω	The value of time

2.1 A discrete day-to-day dynamic model incorporating a Weibit-based flow adjustment process

As mentioned in the Introduction, it is necessary to consider the fluctuation of day-to-day network performance in static pricing design. Thus, an appropriate day-to-day dynamic model is needed to capture day-to-day traffic dynamics. As we focus here on a day-to-day dynamic model that incorporates a Weibit-based flow adjustment process, we first introduce some basics of the Weibit model.

2.1.1 Review of some basics of the Weibit model

The Weibit-based route choice probability expression is given by:

$$p_{w,r} = \frac{\left(g_{w,r}\right)^{-\beta^w}}{\sum_{l \in R^w}\left(g_{w,l}\right)^{-\beta^w}}, \quad \forall r \in R^w, \quad w \in W \tag{1}$$

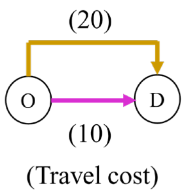
where $p_{w,r}$ is the probability of choosing route *r* connecting O-D pair *w*; $g_{w,r}$ is the travel cost of route *r* that connects O-D pair *w*; and β^w is the shape parameter in the Weibull random error distribution of O-D pair *w*, which can be calibrated following Kurauchi and Ido (2017).

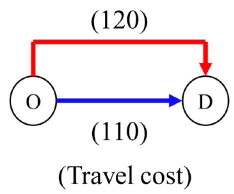
The Weibit model can capture travelers' non-identical perception errors toward travel costs. Unlike the Logit model which requires IID distributed Gumbel variates, the Weibit model uses the Weibull variates, without the need of the identically distributed assumption (Castillo et al., 2008; Kitthamkesorn and Chen, 2013, 2014). As a result, the perception variance in the Weibit model is route-specific, whereas the perception variance in the Logit model is identical for all routes. More specifically, in the Logit model the trip variance is calculated by $\left(\sigma_{w,r}\right)^2 = \pi^2/(6\varphi^2)$, where φ is the dispersion parameter in the Gumbel distribution. However, in the Weibit model, the perception variance of a route depends on its travel cost, i.e.,

$$\left(\sigma_{w,r}\right)^2 = \left[\frac{g_{w,r}}{\Gamma\left(1+\frac{1}{\beta^w}\right)}\right]^2 \left[\Gamma\left(1+\frac{2}{\beta^w}\right) - \Gamma^2\left(1+\frac{1}{\beta^w}\right)\right], \quad \forall r \in R^w, \quad w \in W \tag{2}$$

where $\left(\sigma_{w,r}\right)^2$ is the perception variance of route *r* connecting O-D pair *w*; and $\Gamma(\cdot)$ is the gamma function, i.e., $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t}\, dt$. From Eq. (2), it can be seen that routes with different costs have different perception variances, despite the shape parameter β^w is an O-D specific parameter. This heterogeneous perception variance in the Weibit model is more reasonable than the fixed and identical variance of all routes in the Logit model, because travelers on longer routes have larger perception variances than those on shorter routes in reality (Sheffi, 1985). Some recent empirical studies have verified the above advantage of the Weibit model over the Logit model in various travel choice problems (Kurauchi and Ido, 2017; Li et al., 2020; Tinessa et al., 2020).

The following example is used to illustrate the advantage of the Weibit model over the Logit model in route choice problems. Note that similar examples have also been provided in some previous studies (e.g., Kitthamkesorn and Chen, 2013, 2014; Xu et al., 2015). Consider the two-route networks shown in Fig. 1. In both networks, the route cost of the upper route is 10 units larger than that of the lower route. Nevertheless, the route cost of the upper route is two times larger than the route cost of the lower route in the short network, while the route cost of the upper route is less than 10% larger than the route cost of the lower route in the long network. As expected, the Logit model produces the same route choice probability for both the short and long networks, due to its identically distributed perception error assumption. In contrast, the Weibit model produces a smaller route choice probability for the lower route in the long network than in the short network, which is more consistent with the fact that travelers would have larger perception errors in the long network. The probability density functions (PDF) of the perceived travel cost of the four routes in the two networks are plotted in Fig. 2. It can be seen that the distributions of the four routes with different travel costs share the same variance in the Logit model. In contrast, the Weibit model leads to different variances for each of the four routes. Thus, the optimal pricing scheme to be investigated adopting the Weibit model as the route choice criterion is more plausible than those in the majority of current pricing studies using the UE or Logit model as the route choice criterion.





Logit ($\varphi = 0.1$) $\quad p_{lower} = \dfrac{e^{-1}}{e^{-1} + e^{-2}} = 0.73$ $\qquad$ Logit ($\varphi = 0.1$) $\quad p_{lower} = \dfrac{e^{-11}}{e^{-12} + e^{-11}} = 0.73$

Weibit ($\beta^w = 3.7$) $\quad p_{lower} = \dfrac{10^{-3.7}}{10^{-3.7} + 20^{-3.7}} = 0.93$ $\qquad$ Weibit ($\beta^w = 3.7$) $\quad p_{lower} = \dfrac{110^{-3.7}}{110^{-3.7} + 120^{-3.7}} = 0.58$

(a) Short network $\qquad$ (b) Long network

Fig. 1 Logit and Weibit route choice probabilities in the two-route networks

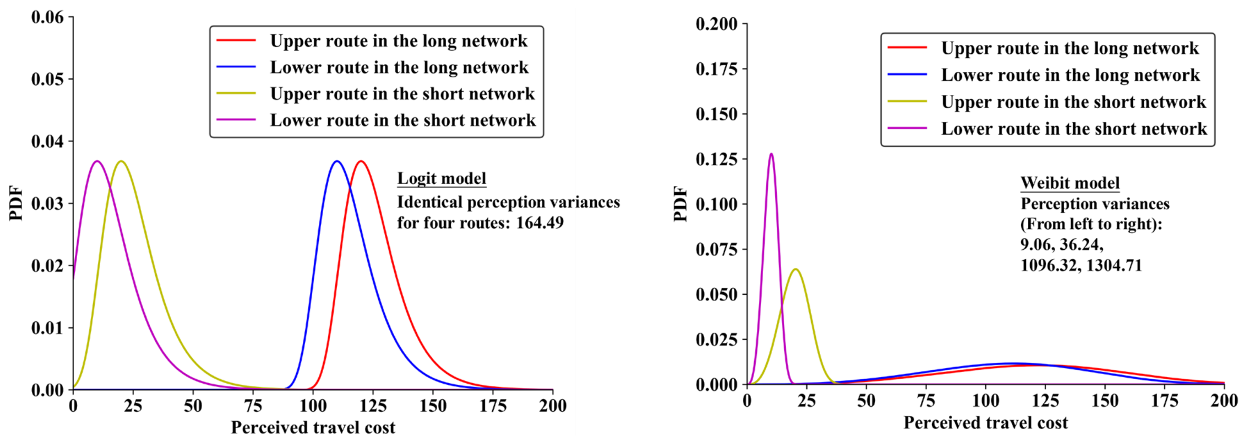


Fig. 2 Perceived travel cost distributions in the Logit and Weibit models

2.1.2 The day-to-day dynamic model with a Weibit-based flow adjustment process

In the literature, day-to-day dynamic models have received extensive attention. They can be classified according to various dimensions, including deterministic models versus stochastic models, discrete-time models versus continuous-time models, and link-based models versus path-based models. For more comprehensive reviews of day-to-day dynamic models, interested readers may refer to Cantarella and Watling (2016), Lou et al. (2017), Zhou et al. (2017), Mahmoodjanlou et al. (2019), Wang et al. (2021), and Xu et al. (2021).

Various continuous day-to-day dynamic models have been used to investigate day-to-day dynamic pricing (Friesz et al., 2004; Tan et al., 2015; Guo et al., 2016). These continuous day-to-day dynamic models are formulated as ordinary differential equations, and possess good mathematical properties. However, real-life travelers' route choices are repeated daily, and it is thus more appropriate to use discrete day-to-day dynamic models to characterize travelers' behavior (Watling and Hazelton, 2003; Rambha and Boyles, 2016; Xiao et al., 2019; Ye et al., 2021). Therefore, we use a discrete day-to-day dynamic model.

The generic flow adjustment process of a discrete day-to-day dynamic model is:

$$\mathbf{f}^{d+1} = (1 - \alpha^d)\mathbf{f}^d + \alpha^d \cdot \mathbf{m}^d \tag{3}$$

where $\mathbf{f}$ is the vector form of route flows, i.e., $\mathbf{f} = \{f_{w,r}, r \in R^w, w \in W\}$, and $f_{w,r}$ is the traffic flow on route r connecting O-D pair w; $\mathbf{f}^{d+1}$ and $\mathbf{f}^d$ are the route flow patterns of day $d + 1$ and day d; $\boldsymbol{m}^d$ is the target route flow pattern on day $d + 1$ after travelers finish their travels on day d; and α^d is the route flow adjustment ratio on day d, denoting the percentage of road users that change routes on each day. The parameter α^d ranges from 0 to 1, reflecting travelers' inertia behavior, and can be calibrated following Ye et al. (2018) and Cheng et al. (2019). On any day d, the definitional constraint between path flow and link flow is given by:

$$v_a^d = \sum_{w \in W} \sum_{r \in R^w} f_{w,r}^d \delta_{ra}^w, \qquad \forall a \in A \tag{4}$$

where v_a^d is the flow of link a on day d; and δ_{ra}^w is the link-path incidence indicator: $\delta_{ra}^w = 1$ if link a is on route r between O-D pair w, and $\delta_{ra}^w = 0$ otherwise.

Eq. (3) is a general framework of discrete day-to-day dynamic models. The differences among various discrete day-to-day dynamic models lie in their strategies of determining the target flow pattern and the adjustment ratio. The adjustment ratio is a positive constant that reflects travelers' inertia or reluctance to change. We use a fixed constant for the adjustment ratio, as has been done in many studies of discrete day-to-day dynamic models (e.g., Bifulco et al., 2016; Cantarella and Watling, 2016; Liu et al., 2017; Cheng et al., 2019). Next, we introduce the target flow pattern determination strategy that follows the Weibit-based route choice criterion.

The Weibit model has a multiplicative route cost structure, i.e., the travel cost of a route is the product of the cost of the links that comprise the route (Kitthamkesorn and Chen, 2013, 2014):

$$g_{w,r} = \prod_{a \in \Upsilon_r} \tau_a \qquad \forall r \in R^w, w \in W \tag{5}$$

where Υ_r is the set of links on path r; and τ_a is the link travel cost, which is often specified as an exponential function of the link travel time t_a, e.g., $\tau_a = e^{0.075 t_a}$ (Kitthamkesorn and Chen, 2013, 2014). The link cost function can be calibrated following Hensher and Truong (1985). Since congestion pricing is involved in this study, we include the link toll into the link travel cost function by setting $\tau_a = e^{0.075(t_a + y_a/\omega)}$, where y_a is the toll on link a, and ω is the value of time (VOT). By substituting $\tau_a = e^{0.075(t_a + y_a/\omega)}$ into Eq. (5), we have

$$g_{w,r} = e^{0.075 \sum_{a \in \Upsilon_r} (t_a + y_a/\omega)} \qquad \forall r \in R^w, w \in W \tag{6}$$

Next, we introduce a reasonable assumption about road users' travel utility.

Assumption 1. We assume that travelers' route choice decisions on any day d are affected by the predicted path travel cost $\mathbf{H}^d$. Some previous studies have also used this assumption (e.g., Cantarella and Watling, 2016; Liu et al., 2017; Cheng et al., 2019). The predicted path travel cost for the next day ($\mathbf{H}^{d+1}$) is a weighted combination of the current day's predicted and actual travel costs ($\mathbf{H}^d$ and $\mathbf{g}^d$). Instead of using the additive weighted arithmetic mean as used by Liu et al. (2017), we use the weighted geometric mean shown in Eq. (7), which is consistent with the multiplicative structure of the Weibit model.

$$\mathbf{H}^{d+1} = (\mathbf{g}^d)^\gamma \times (\mathbf{H}^d)^{1-\gamma} \tag{7}$$

where γ is a weighting parameter $0 < \gamma \leq 1$, which reflects the tradeoff travelers make between the actual path cost on the previous day and the predicted path travel cost. This parameter can be calibrated following Ye et al. (2018). Note that the sign "×" is the Hadamard product, i.e., $\mathbf{z} = \mathbf{x} \times \mathbf{y} \Leftrightarrow z_i = x_i y_i, i = 1,2, \dots, I$, where I is the length of the vectors $\mathbf{x}$ and $\mathbf{y}$.

Eq. (7) can be further recursively expanded as follows:

$$\begin{aligned} \mathbf{H}^{d+1} &= (\mathbf{g}^d)^\gamma \times (\mathbf{H}^d)^{1-\gamma} \\ &= (\mathbf{g}^d)^\gamma \times [(\mathbf{g}^{d-1})^\gamma \times (\mathbf{H}^{d-1})^{1-\gamma}]^{1-\gamma} \\ &= (\mathbf{g}^d)^\gamma \times (\mathbf{g}^{d-1})^{\gamma(1-\gamma)} \times (\mathbf{H}^{d-1})^{(1-\gamma)^2} \\ &= (\mathbf{g}^d)^\gamma \times (\mathbf{g}^{d-1})^{\gamma(1-\gamma)} \times [(\mathbf{g}^{d-2})^\gamma \times (\mathbf{H}^{d-2})^{1-\gamma}]^{(1-\gamma)^2} \\ &= \cdots\cdots \\ &= (\mathbf{g}^d)^\gamma \times \prod_{k=2}^{d-1} (\mathbf{g}^{d-k+1})^{\gamma(1-\gamma)^{k-1}} \times (\mathbf{H}^2)^{(1-\gamma)^{d-1}} \end{aligned} \tag{8}$$

Eq. (8) only applies to cases where $d \geq 3$.

If $d = 2$, from Eq. (7), we have:

$$\mathbf{H}^3 = (\mathbf{g}^2)^\gamma \times (\mathbf{H}^2)^{1-\gamma} \tag{9}$$

If $d = 1$, we assume that

$$\mathbf{H}^2 = \mathbf{g}^1 \times \boldsymbol{\varepsilon} \tag{10}$$

where $\boldsymbol{\varepsilon} = (\varepsilon_{w,r}, \forall r \in R^w, \quad w \in W)^T$ is a vector of random variables that reflect commuters' perception errors regarding the path travel cost. Thus, Eq. (8) becomes

$$\mathbf{H}^{d+1} = (\mathbf{g}^d)^\gamma \times \prod_{k=2}^{d-1} (\mathbf{g}^{d-k+1})^{\gamma(1-\gamma)^{k-1}} \times (\mathbf{g}^1)^{(1-\gamma)^{d-1}} \times \boldsymbol{\varepsilon}^{(1-\gamma)^{d-1}} \tag{11}$$

From Eqs (6) and (11), it can be seen that given the initial route flow and toll pattern, the predicted travel costs on any following day can be obtained. However, Eq. (11) implies that the predicted cost of a route on day d+1 depends on all the past travel costs of the route, which is often known as the infinite learning process (Cantarella and Watling, 2016). In reality, travelers have a finite memory length, and thus their route choice decisions are largely influenced by recent states (Cascetta, 1989; Cantarella and Watling, 2016; Liu et al., 2017). Given this, and consistent with previous studies, we further assume that:

Assumption 2. Travelers' route choice decisions are determined by the network states in the past m days, i.e.,

$$\mathbf{H}^{d+1} = (\mathbf{g}^{d})^{\gamma} \times \prod_{k=2}^{m} (\mathbf{g}^{d-k+1})^{\gamma(1-\gamma)^{k-1}} \times (\mathbf{g}^{1})^{(1-\gamma)^{d-1}} \times \boldsymbol{\varepsilon}^{(1-\gamma)^{d-1}} \tag{12}$$

Note that the exponents of $\mathbf{g}(\cdot)$ on the right-hand side of Eq. (11) sum to 1, whereas the exponents of $\mathbf{g}(\cdot)$ on the right side of Eq. (12) do not sum to 1, due to the presence of the finite memory length m. Hence, we impose a scaling factor $\frac{1}{1-(1-\gamma)^m}$ on the right side of Eq. (12) to make the exponents sum to 1. Thus, Eq. (12) becomes:

$$\mathbf{H}^{d+1} = (\mathbf{g}^{d})^{\frac{\gamma}{1-(1-\gamma)^m}} \times \prod_{k=2}^{m} (\mathbf{g}^{d-k+1})^{\frac{\gamma(1-\gamma)^{k-1}}{1-(1-\gamma)^m}} \times \boldsymbol{\varepsilon}^{\frac{(1-\gamma)^{d-1}}{1-(1-\gamma)^m}} \tag{13}$$

It is easy to verify that $\frac{\gamma}{1-(1-\gamma)^m} + \frac{\gamma}{1-(1-\gamma)^m} \cdot \sum_{k=2}^{m} (1-\gamma)^{k-1} = 1$.

From Eq. (13), the predicted path cost $\mathbf{H}$ is a vector of random variables. In this study, we assume that ε follows the Weibull distribution, thus $\varepsilon^{\frac{(1-\gamma)^{d-1}}{1-(1-\gamma)^m}}$ also follows the Weibull distribution. Then, each element of $\mathbf{H}$, which is the product of a deterministic value and a Weibull variate, also follows the Weibull distribution. We denote $\mathbf{h}^{d+1}$ as the mean of $\mathbf{H}^{d+1}$ (i.e., $\mathbf{h}^{d+1} = (\mathbf{g}^{d})^{\frac{\gamma}{1-(1-\gamma)^m}} \times \prod_{k=2}^{m} (\mathbf{g}^{d-k+1})^{\frac{\gamma(1-\gamma)^{k-1}}{1-(1-\gamma)^m}}$, which is the deterministic part of the right side of Eq. (13)). By replacing the travel cost $g_{w,r}$ in Eq. (1) with the mean of predicted travel cost $h_{w,r}$, we then have

$$p_{w,r} = \frac{\left(h_{w,r}\right)^{-\beta^w}}{\sum_{l \in R^w} \left(h_{w,l}\right)^{-\beta^w}}, \quad \forall r \in R^w, \quad w \in W \tag{14}$$

Finally, assuming that the travel demand $\mathbf{q}$ is fixed, the target route flow $\mathbf{m}$ is the multiplication of the O-D demand $\mathbf{q}$ and the route choice probability $\mathbf{p}$:

$$\mathbf{m} = \mathbf{q} \times \mathbf{p} \tag{15}$$

At this point, we are able to calculate the target flow pattern $\mathbf{m}$ used in Eq. (3).

2.1.3 Summary of the proposed day-to-day dynamic model

Fig. 3 provides a summary of the proposed day-to-day dynamic model.

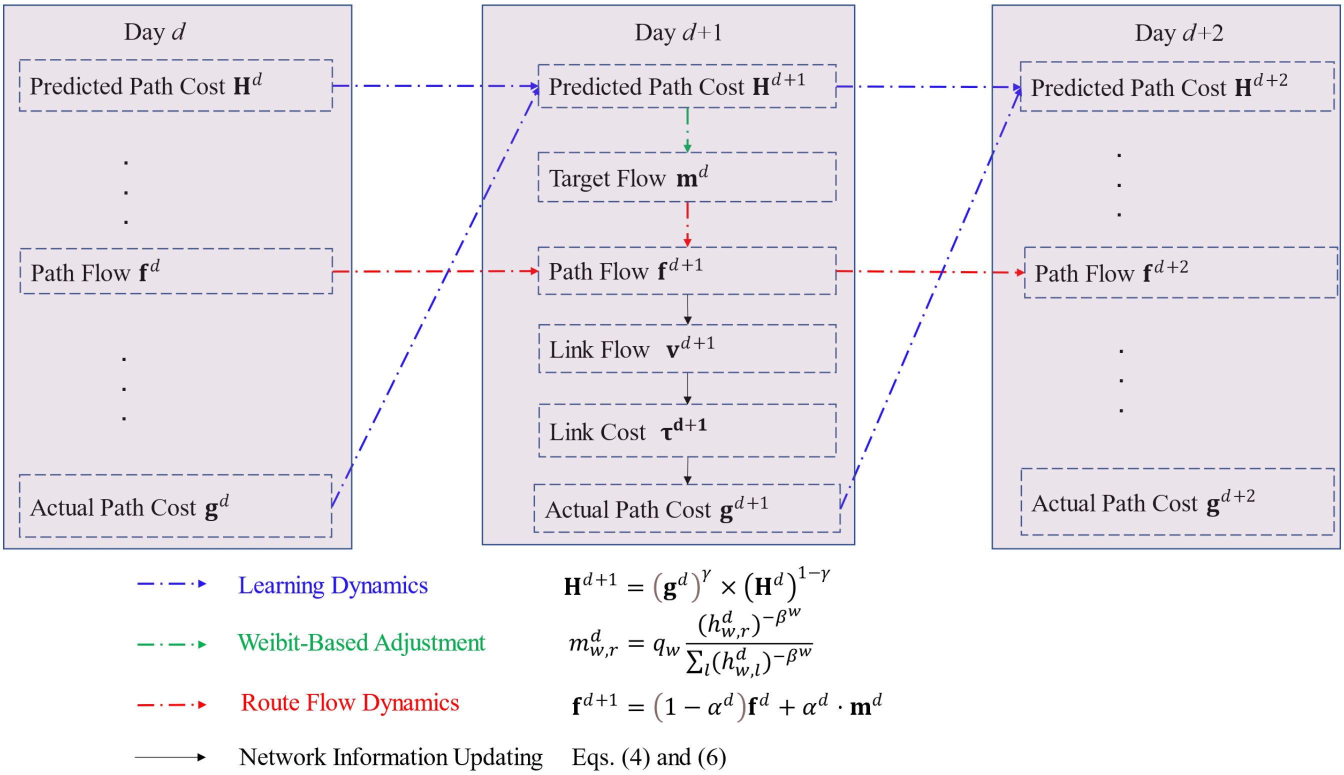


Fig. 3 Summary of the day-to-day dynamic model

According to the day-to-day dynamic model introduced in this section, given a pricing scheme and initial flow pattern on day 1, the network flow and cost on each day within the toll planning horizon can be calculated. They will be used to evaluate the effectiveness of the toll scheme based on our proposed network performance measure, as presented in Section 2.2.

2.1.4 Comparison between Weibit and Logit route adjustment criteria

Now we compare the day-to-day route flow dynamics under the Weibit and Logit route adjustment criteria. We still use the two-route network shown in Section 2.1.1 but here we consider traffic congestion. The parameters are set following Xu et al. (2021): the free-flow travel time of the upper and lower routes are set to be 10 and 5 respectively; both links are set to have the same capacity of 100; the O-D demand is set as 100; the initial flow pattern is (50, 50). The shape parameter of the Weibit model is set as 3.7; and the dispersion parameter of the Logit model is set as 0.85503 in order to keep the same coefficient of variation (CoV) with the Weibit model. Besides, we set the flow adjustment ratio $\alpha^d = 0.5$, memory length $m = 3$, and weighting parameter $\gamma = 0.4$ for both day-to-day dynamic models.

Fig. 4 demonstrates the evolution curves of the route flows under two route adjustment criteria. At the equilibrium state, the Weibit model assigns more flow to the longer route (or less flow to the shorter route) than the Logit model. This is still because the Weibit model can consider the route-specific perception variances unlike the Logit model which assumes identical trip variances for two routes, and thus there is a larger probability for road users to make the "mistake" of choosing the longer route in the Weibit model. Besides, the parameters for the two day-to-day dynamic models (i.e., α^d, m, and

γ) are set to be the same as mentioned above, which means that the two day-to-day dynamic models may take similar days to reach the equilibrium. Thus, evolving from the same initial flow pattern to their respective equilibrium points within similar days, both flow evolution curves from the Weibit model are less steep than those from the Logit model.

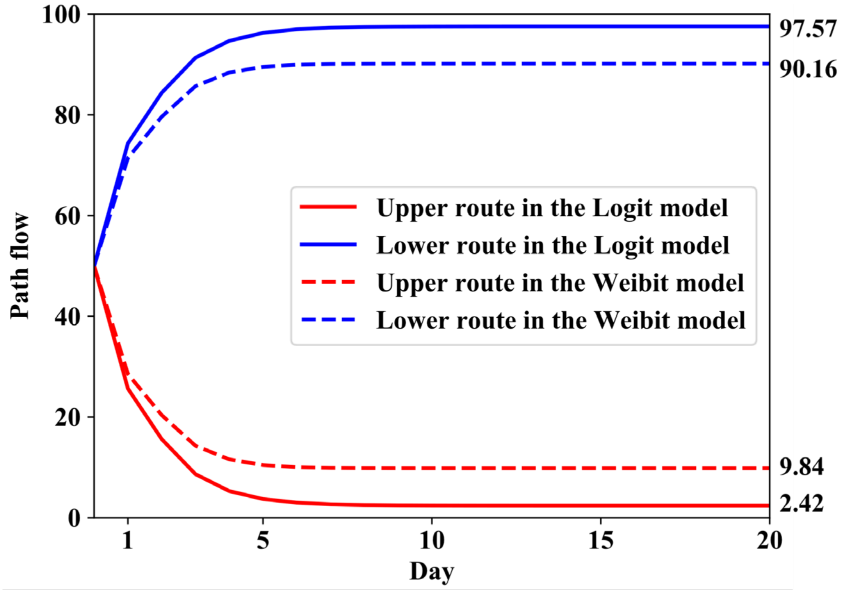


Fig. 4 Path flow evolution curves under the Logit and Weibit route adjustment processes

2.2 A new network-wide performance measure

As mentioned in the Introduction, most current static pricing methods use the indices based on the final equilibrium state as their optimization objective functions, which implicitly assume that the equilibrium state can be reached. However, in practice, the time period needed to reach the equilibrium can be much longer than the toll planning horizon. As a result, these methods may generate toll schemes that are ineffective in alleviating traffic congestion within the toll planning horizon. Therefore, it is necessary to develop a new optimization objective function for the static pricing design to explicitly account for the network performance fluctuation within the toll planning horizon, without the need to assume that the equilibrium could be reached as in the majority of existing studies on static pricing. In our study, the proposed toll scheme is based on the following assumption.

Assumption 3. We assume that policymakers are more concerned about the smooth cross-day operation of a network rather than that on a particular day. Thus, the proposed toll scheme is based on the optimization of overall network performance throughout a toll planning horizon, which is rarely explored in the existing studies.

To explicitly account for network performance fluctuations due to travelers' route choice adjustment after the toll implementation within the toll planning horizon, we propose a new metric as the optimization objective function of the static pricing design, which is denoted cumulative network performance (CNP).

Fig. 5 illustrates the proposed CNP from day 1 to day *D* (i.e., the end of the toll planning horizon). Consider that a toll scheme is implemented on day 1, then the network-wide performance (NP) would change from day to day until day *D*. CNP is calculated by the area cycled by the network performance evolution curve derived by the proposed day-to-day dynamic model and the horizontal axis from day 1 to day *D*.

Mathematically, CNP is given by the following formulation:

$$\mathrm{CNP} = \int_{1}^{D} \mathrm{NP}^{t}\, dt \tag{16}$$

where NP^t is the network-wide performance on day *t* after toll implementation, which is derived from the proposed day-to-day dynamic model. NP^t in Eq. (16) is not limited to a particular measure of network performance, and some typical measures pertaining to congestion toll-design problems can be used herein, such as total travel time (TTT), total social benefit, and expected total travel time under uncertainty.

Note that Eq. (16) is continuous with regard to time t, and thus seems more applicable to the continuous day-to-day dynamic model. However, Eq. (16) is also applicable to the discrete day-to-day dynamic model. That is, we can also obtain a continuous evolution curve by connecting the discrete points generated by the discrete day-to-day dynamic model one by one along the time dimension, and then calculate the measure according to Eq. (16). Note that in the case where a discrete day-to-day dynamic model is used, the curve in Fig. 5 is a piecewise linear function. To fit in the proposed discrete day-to-day dynamic model, Eq. (16) can also be discretized as $CNP = \sum_{t=\{1,2,\ldots,D-1\}} \frac{\mathrm{NP}^t + \mathrm{NP}^{t+1}}{2}$.

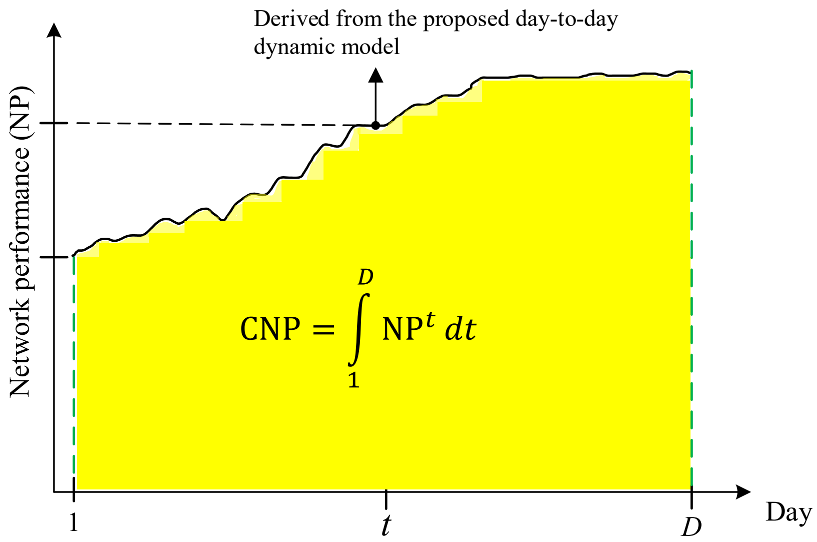


Fig. 5 Illustration of CNP

The advantage of using CNP as the optimization objective function for the optimal static pricing design is that CNP explicitly considers the network performance on each day. This is distinct from the objective function in Liu et al. (2017), which only optimizes the worst case within a toll planning horizon, or the metrics in most other static pricing methods (such as the TTT in marginal pricing), which only focus on the final equilibrium state. In addition, as the network performance on each day is explicitly incorporated into CNP, this new metric applies to both equilibrium and non-equilibrium cases, whereas most traditional static pricing metrics are only applicable to cases where the equilibrium can eventually be reached. Thus, CNP has wider real-life applications, as an equilibrium may not be reached within a given toll planning horizon, especially in large networks, as mentioned in the Introduction.

After choosing the CNP as the optimization objective function, we now present an example to show how the Weibit and Logit route adjustment processes influence the effectiveness evaluation of congestion pricing schemes. We continue to use the two-route network with the same settings in Section 2.1.4. The two toll schemes to be compared (named as Toll A and Toll B, both in time units) are (0.015, 0.965) and (0.699, 0.502), respectively. The upper (longer) route is charged more in Toll B than in Toll A, and the lower (shorter) route is charged more in Toll A than in Toll B.

Table 4 Comparison of two toll schemes from the Weibit and Logit adjustment processes

Type of adjustment process	CNP under Toll A	CNP under Toll B	Which is better?
Weibit adjustment process	7398.25	7289.72	Toll B
Logit adjustment process	11057.78	11082.96	Toll A

Note: The CNP magnitude difference between the Weibit and Logit adjustment processes lies in their different expressions of the expected total travel time.

From Table 4, the day-to-day dynamic model with the Weibit adjustment process suggests Toll B as the better toll scheme, while the day-to-day dynamic model with the Logit adjustment process suggests Toll A as the better. This is because the day-to-day dynamic model with the Weibit adjustment process assigns more flow to the longer (lower) route on each day, as can be shown in Fig. 4 in Section 2.1.4. Thus, the longer route contributes more to the CNP measure in the Weibit day-to-day dynamic model than in the Logit day-to-day dynamic model. Therefore, the Weibit day-to-day dynamic model would suggest the toll scheme with a higher toll on the longer route (Toll B) as the better, which is different from the Logit day-to-day dynamic model. From this analysis, we can readily deduce that the two route adjustment processes may lead to different optimal toll patterns: the Weibit day-to-day dynamic model leads to higher (lower) toll on the longer (shorter) route than the Logit day-to-day dynamic model.

2.3 The bi-level programming model and the solution algorithm

The optimal toll design can be obtained by minimizing the CNP:

$$\min_{\mathbf{y}} \ \mathrm{CNP} \tag{17}$$

In Eq. (17), the CNP follows Eq. (16), where the required information can be calculated according to the day-to-day dynamic model in Section 2.1, and $\mathbf{y} = \{y_a, a \in A\}$ is the link-based toll pattern. The proposed method can also be easily extended to the design of other forms of toll schemes, such as distance-based pricing schemes.

Under SUE, congestion pricing seeks to achieve an SSO (i.e., a minimum ETTT) rather than the SO (i.e., a minimum TTT) in the UE (Maher et al., 2005). Therefore, ETTT is used as the NP in Eq. (16). According to Maher et al. (2005), ETTT is the sum of the product of the travel demand and the satisfaction function of each O-D pair. Below we derive the ETTT corresponding to the Weibit model.

Following the logarithmic satisfaction function of the Weibit-SUE model in Kitthamkesorn and Chen (2014), we have

$$\begin{aligned} S^w(\ln\mathbf{g}^w) &= -\frac{1}{\beta^w}\ln\sum_{r\in R^w}\left(g_{w,r}\right)^{-\beta^w} \\ &= -\frac{1}{\beta^w}\ln\left[\frac{\left(g_{w,r}\right)^{-\beta^w}}{p_{w,r}}\right] \\ &= -\frac{1}{\beta^w}\left[\ln\left(g_{w,r}\right)^{-\beta^w} - \ln p_{w,r}\right] \\ &= -\frac{1}{\beta^w}\left[-\beta^w\ln\left(g_{w,r}\right) - \ln p_{w,r}\right] \\ &= \ln\left(g_{w,r}\right) + \frac{1}{\beta^w}\ \ln p_{w,r} \end{aligned} \tag{18}$$

Then, we have

$$S^w(\ln\mathbf{g}^w) = \sum_{r\in R^w} p_{w,r}S^w(\ln\mathbf{g}^w) = \sum_{r\in R^w} p_{w,r}(\ln g_{w,r} + \frac{1}{\beta^w}\ln p_{w,r}) \tag{19}$$

Finally, the ETTT can be derived as follows:

$$\begin{aligned} \mathrm{ETTT} &= \sum_{w\in W} q^w S(\ln\mathbf{g}^w) \\ &= \sum_{w\in W} q^w\sum_{r\in R^w} p_{w,r}\left(\ln g_{w,r} + \frac{1}{\beta^w}\ln p_{w,r}\right) \\ &= \sum_{w\in W}\sum_{r\in R^w} f_{w,r}\left(\ln g_{w,r} + \frac{1}{\beta^w}\ln p_{w,r}\right) \\ &= \sum_{w\in W}\sum_{r\in R^w} f_{w,r}\left(\ln g_{w,r} + \frac{1}{\beta^w}\ln\frac{f_{w,r}}{q^w}\right) \\ &= \sum_{w\in W}\sum_{r\in R^w} f_{w,r}\ln g_{w,r} + \sum_{w\in W}\sum_{r\in R^w}\frac{1}{\beta^w}f_{w,r}\ln f_{w,r} - \sum_{w\in W}\frac{1}{\beta^w}q^w\ln q^w \end{aligned} \tag{20}$$

By including the day index d, the expected total travel time on day d, denoted as ETTT^d, is:

$$\text{ETTT}^d = \sum_{w\in W}\sum_{r\in R^w} f_{w,r}^d \ln g_{w,r}^d + \sum_{w\in W}\sum_{r\in R^w} \frac{1}{\beta^w} f_{w,r}^d \ln f_{w,r}^d - \sum_{w\in W}\frac{1}{\beta^w} q^w \ln q^w \tag{21}$$

As the travel demand q^w is assumed to be fixed, we can eliminate the third term in Eq. (21) from the optimization model.

The above model of solving the proposed toll scheme is a bi-level programming model.

The upper-level program:

$$\min_{\mathbf{y}} \int_1^D \text{ETTT}^t \, dt$$

where

$$\text{ETTT}^d = \sum_{w\in W}\sum_{r\in R^w} f_{w,r}^d \ln g_{w,r}^d + \sum_{w\in W}\sum_{r\in R^w} \frac{1}{\beta^w} f_{w,r}^d \ln f_{w,r}^d$$

The lower-level program:

$$\mathbf{f}^{d+1} = (1-\alpha^d)\mathbf{f}^d + \alpha^d \cdot \mathbf{m}^d$$

$$g_{w,r}^d = e^{0.075\sum_{a\in\Upsilon_r}(t_a^d + y_a/\omega)}, \qquad \forall r \in R^w, w \in W$$

where the determination of $\mathbf{m}^d$ follows Eqs. (11)-(15).

The upper-level subprogram is used to find the optimal toll pattern $\mathbf{y}$, and the lower-level subprogram is used to obtain the day-to-day network flows and costs under toll pattern $\mathbf{y}$ for calculating the CNP in the upper-level subprogram. We use the Artificial Bee Colony (ABC) algorithm to solve the bi-level programming model. As an advanced evolutionary algorithm, the ABC algorithm has a better local search mechanism to improve the solution quality compared with other evolutionary algorithms. Recently, the ABC algorithm has been adopted to solve some transportation-related problems (See, e.g., Szeto et al., 2011; Szeto and Jiang, 2012, 2014; Chen et al., 2015). For more details about the ABC algorithm, interested readers may refer to Szeto et al. (2011).

3. Numerical Examples

In this section, we conduct numerical experiments in two networks. In the first illustrative network, we show the advantages of the first-best link-based pricing scheme from the proposed method over the marginal pricing scheme and the robust pricing scheme, under different flow adjustment ratios. Sensitivity analyses are also conducted to examine the effects of different parameter settings. Then, we carry out a more detailed analysis in the larger Sioux Falls network, to show the effectiveness of the second-best link-based pricing scheme generated by the proposed method. The numerical experiments are coded in Python 3.8 running on a laptop with Inter(R) Core (TM) i7-10510U CPU @ 1.80 GHz, 2.30 GHz and 12.00G RAM.

In both networks, travelers' memory length (m) is set as 3, the weighting parameter (γ) is set as 0.4, and the value of time (ω) is set to be ¥1 per time unit, following Liu et al. (2017). The currency form of the toll patterns used in the following experiments is also in ¥. The shape parameter β^w is set as 3.7. The toll planning horizon D is set as 30. The standard Bureau of Public Roads (BPR) function is adopted to capture the relationship between the link travel time and link flow, i.e.,

$$t_a = t_a^0(1 + 0.15 \times \frac{v_a^4}{C_a^4}) \tag{22}$$

where t_a^0 is the free-flow travel time of link a, and C_a is the capacity of link a.

The parameter settings in the ABC algorithm are as follows: colony size $n_c = 40$, number of employed bees $n_e = 40$, number of onlookers $n_o = 20$, maximum iteration value $i_{max} = 500$, and limit $=$ 2. Readers may refer to Szeto et al. (2011) for the meanings of these parameters.

3.1 Illustrative network

3.1.1 Experimental setting

The first numerical experiment is conducted using the network shown in Fig. 6, which is from Liu et al. (2017). This network consists of nine nodes and thirteen links. There are two O-D pairs, i.e., O-D pair 1-8 and 1-9, connected by four and seven routes, respectively. Both O-D pairs have the same travel demand of 8,000 units. We further assume that the travel demand of each O-D pair is evenly distributed onto the paths initially connecting the O-D pair. The assumption has also been adopted by previous studies such as Guo et al. (2013) and Xu et al. (2021). The link performance characteristics of this network are shown in Table 5.

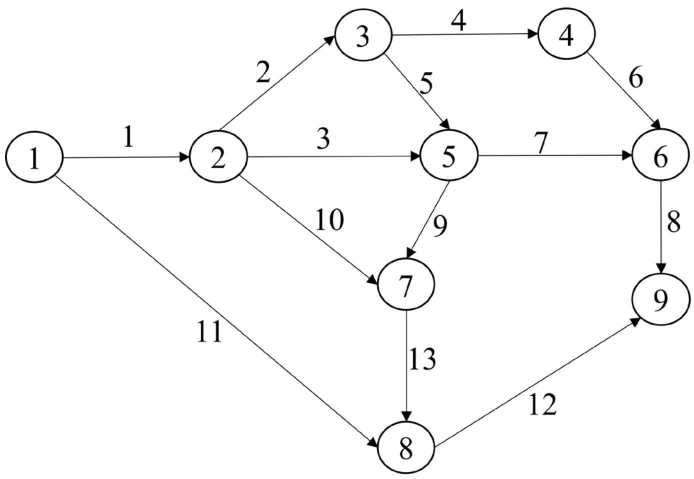


Fig. 6 Topology of the illustrative network

In this network, we impose tolls on all links. The following three pricing schemes and the baseline scheme are compared:

✧ **Scheme 1**: The first-best pricing scheme, which is generated by the proposed method. For clarity, we refer to Scheme 1 as the *proposed toll scheme* in the following text.

✧ **Scheme 2**: The robust pricing scheme. Similar to Liu et al. (2017), the robust pricing scheme to be compared is a toll pattern $\mathbf{y}$ that minimizes the maximum network performance within the given toll planning horizon. Specifically, the optimization objective function of the robust pricing scheme is given by:

$$\min_{\mathbf{y}} \max_{d} \ \text{ETTT}^d \tag{23}$$

Although the robust pricing scheme considers the day-to-day dynamics, it only optimizes the ETTT under the worst-case conditions, and the ETTTs on other days are not explicitly considered.

- ✧ **Scheme 3**: The marginal pricing scheme, which is based on the network flow from the static Weibit-SUE model.
- ✧ **Baseline scheme**: No pricing scheme.

We consider four values of α^d = 0.3, 0.4, 0.5, and 0.6, corresponding to four levels of network performance fluctuation (from slight to large) resulting from different flow adjustment ratios. Sensitivity analyses on the initial flow pattern, the length of the toll planning horizon, and the memory length are provided in Appendix A.

Table 5 Detailed network information

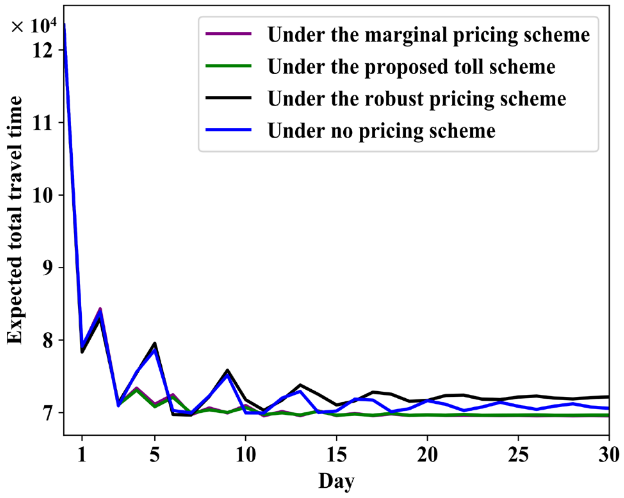
Link ID	Tail	Head	Free-flow travel time	Capacity
1	1	2	2	6,000
2	2	3	2	4,000
3	2	5	8	6,000
4	3	4	2	2,000
5	3	5	4	2,000
6	4	6	6	1,000
7	5	6	2	4,000
8	6	9	6	6,000
9	5	7	3	4,000
10	2	7	9	2,000
11	1	8	26	3,000
12	8	9	4	3,000
13	7	8	5	3,000

3.1.2 Results and analyses

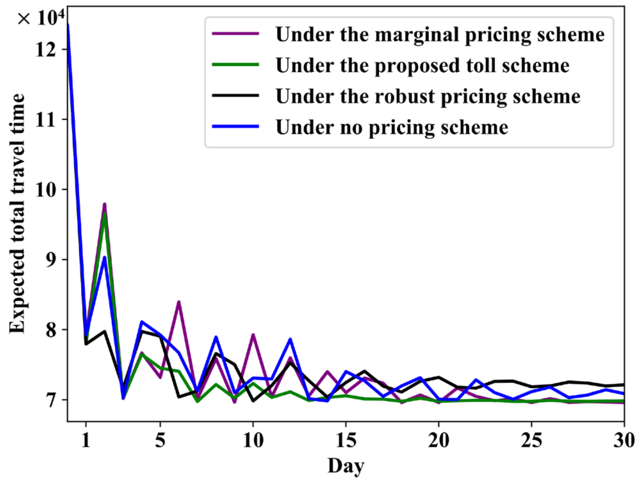
The optimal link-based pricing schemes generated by the proposed method under the four values of α^d are shown in Table 6. Fig. 7 demonstrates the evolution trajectories of ETTT under different pricing schemes and different values of α^d. As a comparison, we also calculate the marginal tolls. Specifically, by replacing the cost function t_a with $t_a + v_a \cdot \frac{dt_a}{dv_a}$, the equilibrium link flow pattern $\mathbf{v}^*$ can be obtained. Then, the marginal link tolls $\boldsymbol{y}_m$ can be calculated by $\mathbf{v}^* \cdot \frac{d\mathbf{t}}{d\mathbf{v}}$, which results in $\boldsymbol{y}_m$ = (20.78, 1.27, 5.04, 0.27, 7.73, 12.82, 6.11, 8.25, 0.40, 6.74, 38.48, 0.00, 20.62). Note that marginal pricing under SUE can lead to SSO; this corresponds to the minimum ETTT at the equilibrium state (Maher et al., 2005), which is 69,562 in this experiment.

Table 6 Optimal toll schemes generated by the proposed method under different values of α^d

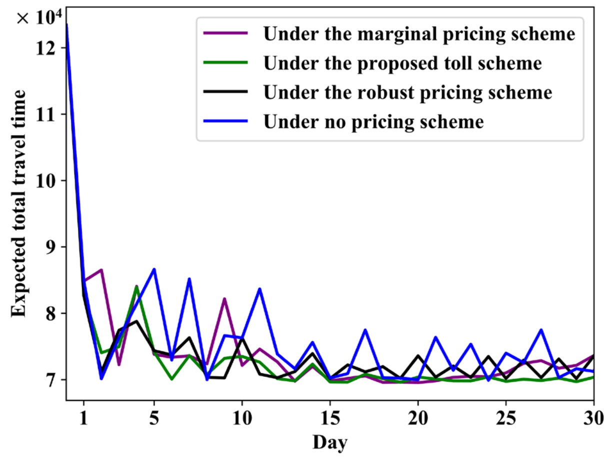
α^d \ Link	1	2	3	4	5	6	7	8	9	10	11	12	13
0.3	19.30	1.53	6.36	1.77	8.11	14.47	9.89	6.76	1.09	8.43	38.07	1.21	20.32
0.4	18.80	4.55	5.94	0.51	4.62	10.24	7.31	5.43	2.01	8.35	38.69	0.38	18.49
0.5	18.94	2.79	4.98	0.40	6.22	9.55	5.40	9.45	1.53	6.00	38.69	0.90	19.50
0.6	20.58	3.18	7.10	0.45	8.12	14.79	5.54	7.94	1.28	8.78	37.82	3.37	19.16



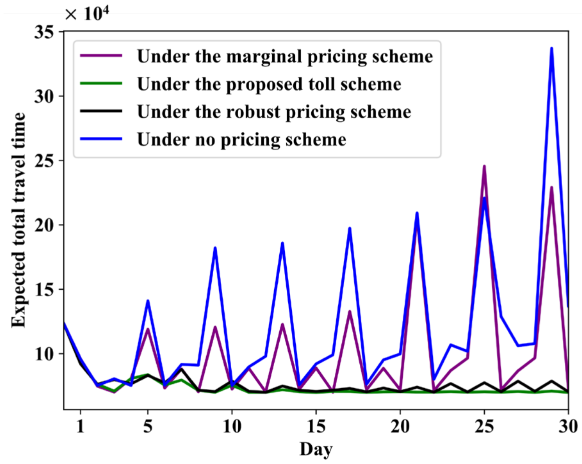
(a) $\alpha^d = 0.3$



(b) $\alpha^d = 0.4$



(c) $\alpha^d = 0.5$



(d) $\alpha^d = 0.6$

Fig. 7 Evolution trajectories of expected total travel time under different pricing schemes and different values of α^d

In the following section, we demonstrate the advantages of the proposed toll scheme by comparing its performance with those of the marginal pricing scheme and the robust pricing scheme.

(1) The proposed toll scheme versus the marginal pricing scheme.

- The case where the equilibrium can be reached within the toll planning horizon

In a network in which the travelers are reluctant to alter their travel choices, it is likely that the equilibrium can be eventually reached within the toll planning horizon, particularly in the illustrative network herein, due to its simple structure. In the four investigated cases, the equilibrium state can be reached within 30 days when $\alpha^d = 0.3$ under both the proposed toll scheme and the marginal pricing scheme, as both schemes lead to only small fluctuations in network performance. From Table 7, despite the proposed toll scheme produces a slightly larger ETTT on day 30, it has a smaller CNP than the marginal pricing scheme. In this case, both schemes are similarly effective, as they give rise to similar lower CNPs (2.152×10^6 and 2.153×10^6) and ETTTs on day 30 (6.966×10^4 and 6.956×10^4) than the no pricing case (2.192×10^6 for CNP; 7.059×10^4 for ETTT on day 30).

Table 7 Comparison between the proposed toll scheme and the marginal pricing scheme

α^d	Pricing scheme	CNP ($\times 10^6$)	ETTT on day D ($\times 10^4$)
0.3	The proposed toll scheme*	**2.152**	6.966
	Marginal pricing scheme*	2.153	**6.956**
0.4	The proposed toll scheme	**2.182**	6.983
	Marginal pricing scheme	2.220	**6.961**
0.5	The proposed toll scheme	**2.184**	**7.038**
	Marginal pricing scheme	2.224	7.365
0.6	The proposed toll scheme	**2.212**	**6.995**
	Marginal pricing scheme	3.035	7.217

Note: (1): The red bold number denotes the better performance
(2): The asterisk means that in this case the equilibrium state can be reached within the toll planning horizon.
(3): CNP = cumulative network performance; ETTT = expected total travel time.

- Cases where the equilibrium cannot be reached within the toll planning horizon

When travelers have a medium or strong willingness to adjust their route choices based on their past travel experience, the network flow state may fluctuate strongly, and thus the equilibrium may not be reached within the toll planning horizon. In this network, from Fig. 7(b), (c) and (d), the equilibrium cannot be reached under any of the three pricing schemes within D days in the cases of $\alpha^d = 0.4, 0.5$, and 0.6, which verifies that it could be unreasonable to use the ETTT at the equilibrium state as the optimization objective function for static pricing. From Table 7, it can be seen that the proposed toll scheme leads to a smaller CNP than the marginal pricing scheme under all three values of α^d. This superiority of the proposed scheme increases as α^d increases. Specifically, the differences between the CNP of the marginal pricing scheme and the proposed pricing scheme are 3.8×10^4, 4×10^4, and 8.23×10^4 when $\alpha^d = 0.4$, 0.5 and 0.6, respectively. Surprisingly, the proposed toll scheme even produces a smaller ETTT on day D than the marginal pricing scheme when $\alpha^d = 0.5$ (7.038×10^4 versus 7.365×10^4) and 0.6 (6.995×10^4 versus 7.217×10^4), despite giving a slightly worse result when $\alpha^d = 0.4$ (6.983×10^4 versus 6.961×10^4).

Besides, it is notable that the patterns of the blue and purple curves (corresponding to the no pricing and marginal pricing cases) in Fig. 7 (d) are unstable, which are different from the stable blue and purple curves in Figs. 7 (a), (b), and (c). In contrast, the flow fluctuation can be largely alleviated by the proposed pricing scheme and robust pricing scheme. Note that such instability in a day-to-day dynamical system has also been found and investigated by some previous studies, such as Cantarella and Cascetta (1995), Zhou et al. (2017), and Ye et al. (2021). The reason for this phenomenon is mainly because $\alpha^d = 0.6$ is larger than a certain threshold, leading to an unstable traffic state. As a result, without a proper travel demand management, the initial disequilibrium network flow under $\alpha^d = 0.6$ cannot be spontaneously mitigated as in Figs. 7 (a), (b), and (c), and might even get worse instead. More details about the stability of day-to-day dynamic models (i.e., how to find the threshold) can be found in Ye et al. (2021). Note that this phenomenon also emphasizes the importance of considering day-to-day dynamics in the static pricing scheme.

(2) The proposed toll scheme versus the robust pricing scheme

From Table 8, one can see that the robust pricing scheme indeed produces a smaller maximum ETTT under all four values of α^d than the other two pricing schemes. However, this advantage comes at the expense of inducing larger CNPs, as shown in Table 9. Specifically, the differences in the maximum ETTT between the robust pricing scheme and the proposed toll scheme are 0.92×10^3, 16.82×10^3, 1.31×10^3, and 0.32×10^3 as α^d increases from 0.3 to 0.6; while the differences in the CNP improvement (CNP improvement percentage) between the proposed toll scheme and the robust pricing scheme are respectively 6.1×10^4 (2.78%), 4.2×10^4 (1.88%), 2.5×10^4 (1.10%), and 6.4×10^4 (1.87%) when α^d= 0.3, 0.4, 0.5 and 0.6. The two schemes appear to be similar if only comparing the CNP improvement percentage in Table 9 (or the difference in CNP improvement percentage, indicated by the numbers in the parenthesis above), but this "small" percentage essentially corresponds to a nontrivial difference due to the large magnitude of CNP (i.e., 10^6). It is also interesting to note that the CNP improvement percentage between the two schemes gets close as α^d increases (i.e., their difference evolves from 2.78% to 1.78%). This is largely because the worst-case network performance can be so significant that it becomes the key determinant of the CNP as α^d increases. For example, when $\alpha^d = 0.6$, if no pricing is implemented the largest ETTT is 33.717×10^4, as shown in Fig. 7(d), which, however, could be reduced by more than 3 times under both the proposed toll scheme and the robust toll scheme. Hence, optimizing the worst-case performance could indeed lead to a small CNP when travelers actively make route choice alterations.

Table 8 Comparison of the maximum ETTT within D days under three pricing schemes

α^d	Pricing scheme	Maximum ETTT within the toll planning horizon ($\times 10^4$)
0.3	The proposed toll scheme*	8.388
	Robust pricing scheme	**8.296**
	Marginal pricing scheme *	8.431
0.4	The proposed toll scheme	9.656
	Robust pricing scheme	**7.974**
	Marginal pricing scheme	9.791
0.5	The proposed toll scheme	8.396
	Robust pricing scheme	**8.265**
	Marginal pricing scheme	8.652
0.6	The proposed toll scheme	9.242
	Robust pricing scheme	**9.210**
	Marginal pricing scheme	24.562

Note: (1) The red bold number denotes the best performance under the three schemes
(2) ETTT = expected total travel time

Table 9 Comparison between the proposed toll scheme and the robust pricing scheme in terms of cumulative network performance (CNP)

α^d	Pricing scheme	CNP $\times 10^6$)	CNP improvement ($\times 10^6$)*	Percentage improvement in CNP **
0.3	The proposed toll scheme	2.152	0.021	0.96%
	Robust pricing scheme	2.213	-0.04	-1.82%
0.4	The proposed toll scheme	2.182	0.05	2.24%
	Robust pricing scheme	2.224	0.008	0.36%
0.5	The proposed toll scheme	2.184	0.087	3.83%
	Robust pricing scheme	2.209	0.062	2.73%
0.6	The proposed toll scheme	2.212	1.411	38.95%
	Robust pricing scheme	2.276	1.347	37.18%

*: CNP improvement denotes the difference in CNP between the no pricing scheme and the pricing scheme in question.
**: Percentage improvement in CNP refers to the ratio of the CNP improvement in question to the CNP in the no pricing scheme

On the other hand, since the robust pricing scheme merely optimizes the worst-case performance, it may lead to worse performance on the other days, as can be seen by comparing the black curves with the other curves in Fig. 7. Furthermore, the overall performance within the toll planning horizon may also be worsened. For example, when $\alpha^d = 0.3$, the ETTT evolution trajectory (black curve) lies above all of the other curves most of the time, even the curve of the no pricing scheme case (i.e., the blue curve), as shown Fig. 7(a). As a result, the CNP under the robust pricing scheme is also larger than the CNPs under the other three schemes (2.213×10^6 vs. 2.152×10^6, 2.153×10^6, and 2.192×10^6). This means that the robust pricing scheme may worsen the overall transportation system performance within the toll planning horizon.

3.1.3 Summary

These numerical results demonstrate the necessity of using the new metric CNP as the optimization objective function for static pricing design. This is because CNP is a more comprehensive metric that explicitly incorporates the network performances on all days, and also because the equilibrium may not be reached within a toll planning horizon, such as when $\alpha^d = 0.4$, 0.5, and 0.6. In addition, regardless of whether the equilibrium could be reached or not, the proposed toll scheme not only achieves the best performance in terms of optimizing the overall network performance, but also leads to moderate performance in terms of ETTT on day *D* or the maximum ETTT within *D* days compared to the other two pricing schemes.

It is also noticeable that different flow adjustment ratios lead to different optimal toll patterns: that is, the optimal toll pattern for one case may not be effective in reducing congestion for another. Hence, for policymakers who would like to design their pricing strategies based on the proposed method, it is necessary to determine the flow adjustment ratio using real traffic data for a given transportation network before calculating the toll scheme (e.g., Ye et al., 2018; Cheng et al., 2019).

3.2 Sioux Falls network

The second numerical experiment is conducted using the Sioux Falls network. This network consists of 24 nodes, 76 links, and 550 O-D pairs. A pre-generated route set with 3,441 acyclic routes from Bekhor et al. (2007) is used, which was generated by using a combination of the link elimination method of Azevedo et al. (1993) and the penalty method of de la Barra et al. (1993) with a penalty of 5% on travel times on the shortest route links. We again assume that the travel demand of each O-D pair is evenly distributed onto the paths initially connecting the O-D pair. We consider that congestion pricing is imposed on 10 links (i.e., links 25, 26, 27, 28, 29, 30, 32, 43, 48, and 51) in the central area, as shown in Fig. 8. The flow adjustment ratio α^d is set as 0.35.

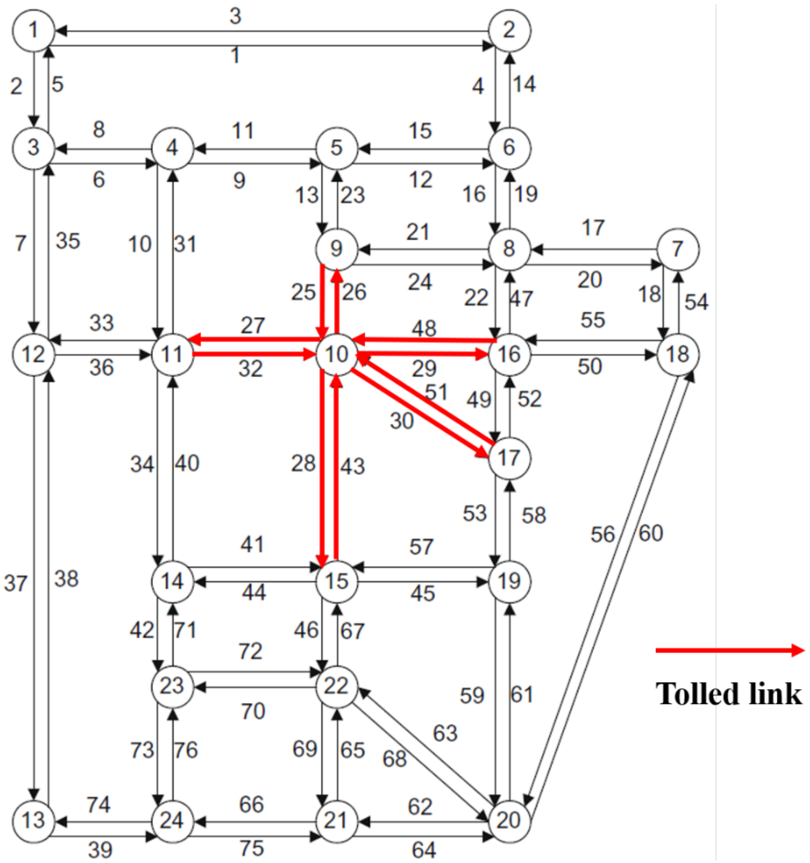


Fig. 8 Sioux Falls network and the tolled links

According to the proposed method in Section 2, the optimal toll scheme $\mathbf{y}^*$ is (1.82, 2.04, 3.34, 2.76, 8.44, 7.43, 4.60, 2.22, 9.57, 6.20). As to the computational time of the ABC algorithm, executing one iteration requires 31 seconds in the Sioux Falls network.

- Network-level performance

First, we examine the effectiveness of the proposed toll scheme in optimizing the network-level performance of this larger network. The evolution trajectories of ETTT under the proposed toll scheme and no pricing scheme are plotted in Fig. 9. It can be seen that the ETTT fluctuation within 30 days under the proposed toll scheme is smaller than that under the no pricing scheme. The CNP and ETTT on day *D* under the proposed toll scheme are 30,788 and 464, which are 8.0% and 6.6% lower respectively than those under the no pricing scheme (33,471 and 497). Moreover, under the proposed toll scheme, the network state is near equilibrium on day 30, as can be observed from the green curve in Fig. 9. The maximum rates of change of ETTT between two consecutive days in the last 10 days are 4% and 34% under the proposed pricing scheme and no pricing scheme, respectively. Thus, the effectiveness of the second-best link-based pricing scheme generated by the proposed method in optimizing the network performance over the entire toll-planning horizon is further validated.

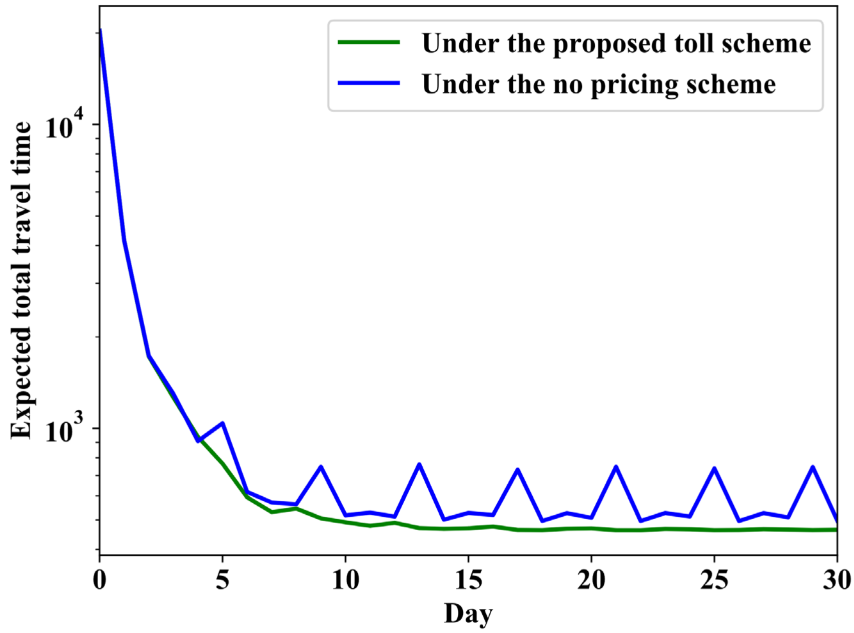


Fig. 9 Expected total travel time evolution trajectories under the proposed toll scheme and no pricing scheme

- O-D-level performance

Next, we use the O-D cost (ODC) to explore the effectiveness of the proposed toll scheme in improving users' travel experience at the O-D level. According to Kitthamkesorn and Chen (2014), the O-D cost of the O-D pair *w* on day *d* in the Weibit model is:

$$\mathrm{ODC}_w^d = -\frac{1}{\beta^w} \ln \sum_{r \in R^w} \left(g_{w,r}^d\right)^{-\beta^w} \tag{24}$$

We compare the percentage improvement in the ODC of each O-D pair on the last day of the toll planning horizon (i.e., $d = 30$), where this improvement is defined as the ratio of the difference in the ODC_w^{30} between no pricing scheme and the proposed toll scheme to the ODC_w^{30} under the no pricing scheme. The cumulative distribution function (CDF) of the percentage improvement in the ODC on day D is shown in Fig. 10. Out of the total of 550 O-D pairs, 354 (i.e., 64.4%) O-D pairs have a lower ODC_w^{30} under the proposed toll scheme than under the no pricing scheme. The largest improvement is 56.28%, and the average improvement is 6.76%. Hence, the proposed toll scheme is capable of improving the travel experience of the majority of commuters in this network. Nevertheless, similar to the traditional pricing methods, the proposed toll scheme would also lead to an increase in O-D cost for the other users. To overcome this problem, an equity constraint can be incorporated into the model to achieve a balance between network-wide efficiency and users' travel costs (Meng and Yang, 2002).

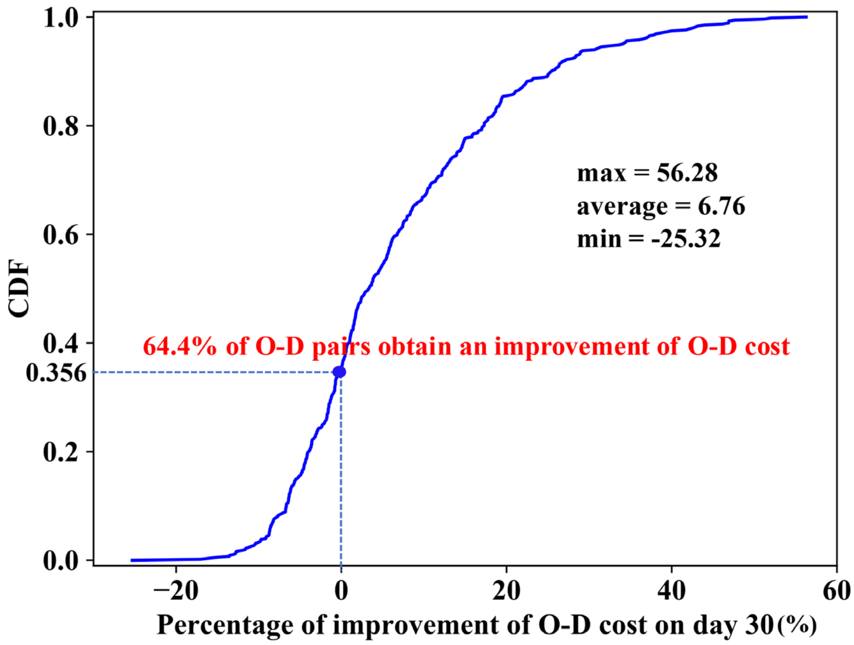
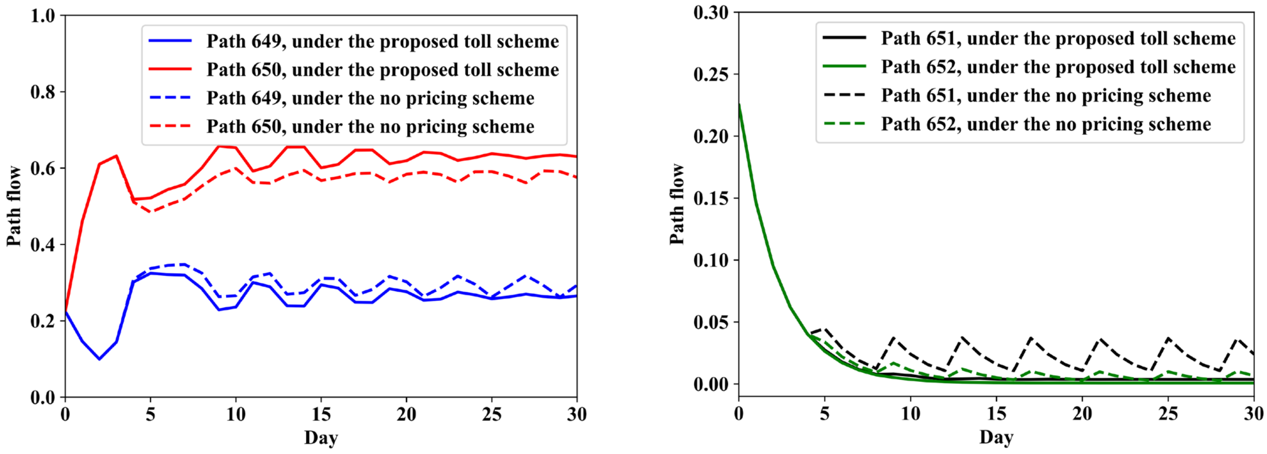


Fig. 10 Cumulative distribution function (CDF) of the percentage improvement of O-D cost on day 30

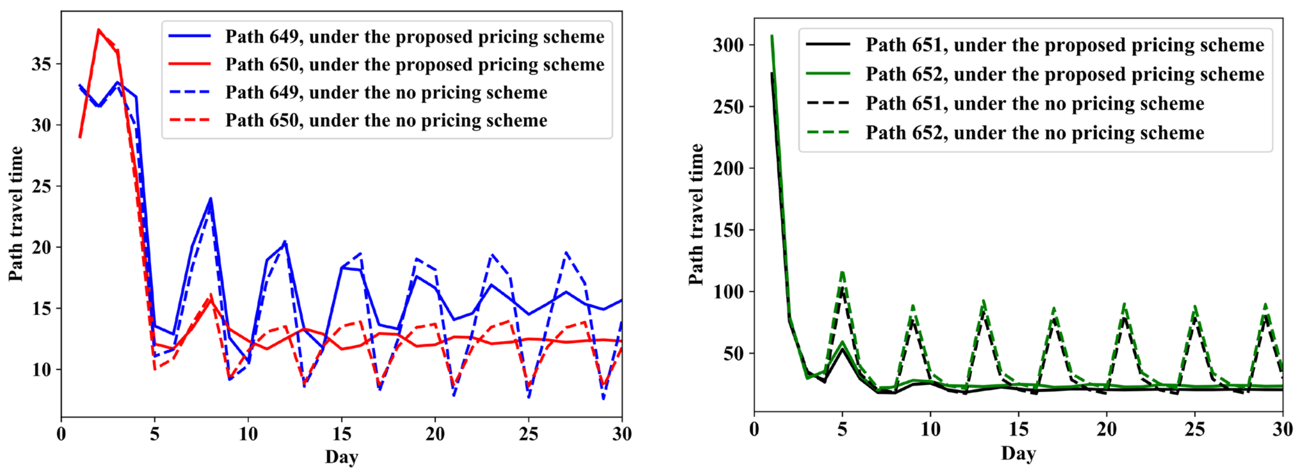
- Path-level and link-level performance

We further investigate how a pricing scheme influences the evolution of path flows. The O-D pair 5-8, which is connected by four paths, is selected for the analysis. Specifically, path 649 (links 12-16) and path 650 (links 13-24) do not traverse the tolled links, while path 651 (links 13-**25**-**29**-47) and path 652 (links 11-10-**32**-**29**-47) traverse two tolled links (indicated in bold). The flow evolution trajectories of the four paths are presented in Fig. 11. Over the toll planning horizon, both paths that traverse the tolled links (paths 651 and 652) have lower flows under the proposed pricing scheme than under the no pricing scheme, as shown in Fig. 11(b). These reduced flows are directed to path 650, which does not traverse the tolled links, and this path thus has a flow increase of 9.4% (i.e., from 0.576 to 0.630) on day 30. Fig. 12 shows the travel time evolution trajectories of the four paths. From Fig. 12 (b), it can be seen that the toll scheme leads to a significant reduction in the travel time of paths 651 and 652. Specifically, their travel times on day 30 are more than 10 times less than those on day 1. Thus, the proposed pricing scheme effectively decreases the number of users originally traveling through the tolled links, and thereby further reduces congestion (travel time) in the pricing area within the toll planning horizon.



(a) Paths 649 and 650 (not traversing tolled links) (b) Paths 651 and 652 (traversing some tolled links)

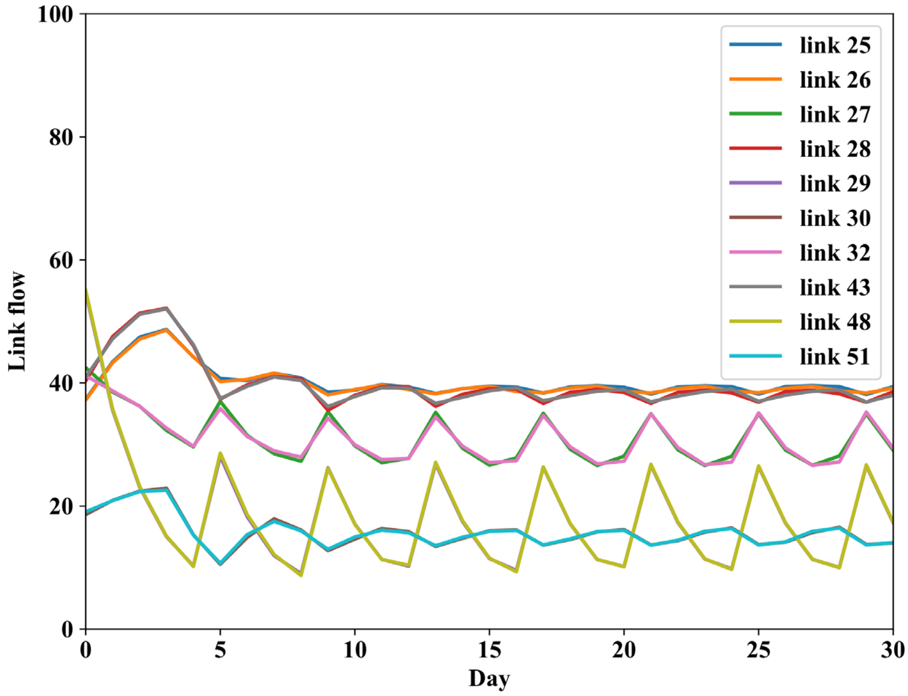
Fig. 11 Flow evolution trajectories of the four paths that connect O-D pair 5-8



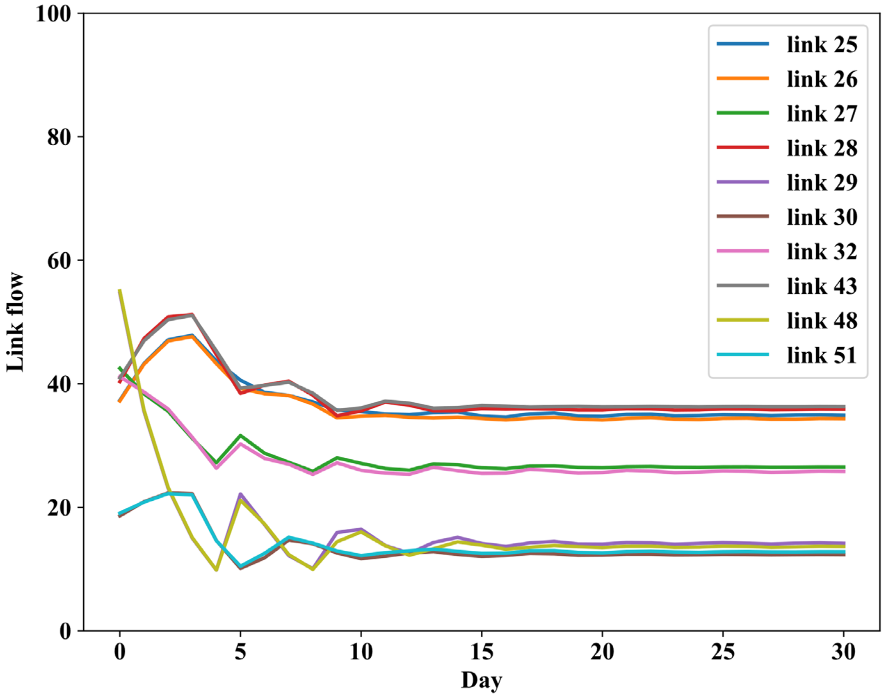
(a) Paths 649 and 650 (not traversing tolled links) (b) Paths 651 and 652 (traversing some tolled links)

Fig. 12 Travel time evolution trajectories of the four paths that connect O-D pair 5-8

Finally, we examine the effectiveness of the proposed pricing scheme in reducing the traffic flow at the 10 tolled links. The flow evolution trajectories at the tolled links under the pricing scheme and under the no pricing scheme are shown in Fig. 13. Further, the evolution trajectories of the sums of flows at the tolled links under the two schemes are plotted in Fig. 14. Fig. 13(a) shows that links 27, 32, and 48 experience large flow fluctuations under the no pricing scheme. In contrast, the flow at each of the ten links almost converges under the proposed toll scheme, as can be seen in Fig. 13(b). Moreover, Fig. 13 demonstrates that the proposed toll scheme could lead to a lower sum of flows at the tolled links than that under the no pricing scheme on every day. At the end of the toll planning horizon, each of the 10 tolled links has a lower flow than it does under the no pricing scheme, with the percentage of link flow reduction ranging from 4.5% to 18%. Hence, the proposed toll scheme can effectively alleviate traffic congestion at the tolled links.



(a) Under the no pricing scheme



(b) Under the proposed toll scheme

Fig. 13 Flow evolution trajectories at the 10 tolled links

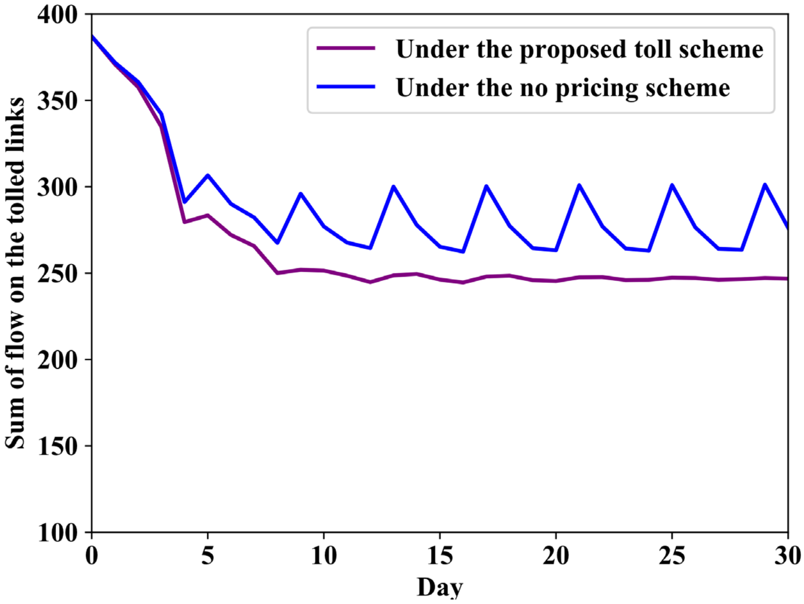


Fig. 14 Evolution trajectories of the sums of flows at the tolled links under the proposed toll scheme and under the no pricing scheme

4. Conclusions

Static pricing is a dominant form of congestion pricing in practice. Most current methods for static pricing design assume that an equilibrium can be eventually reached. However, in reality, the equilibrium may not be reached within the toll planning horizon, and thus the equilibrium-based pricing schemes may be ineffective in alleviating traffic congestion. To address this problem, this study proposes a new method for static congestion pricing design in a bi-level modeling framework. In particular, we propose a new measure called CNP as the optimization objective function explicitly accounting for the network performances on each day within the toll planning horizon, rather than only focusing on the performance at the equilibrium state, as is done by most existing static pricing methods. In addition, we introduce a day-to-day dynamic model with the Weibit-based route flow adjustment to calculate the CNP measure. The Weibit-based route choice criterion in the proposed day-to-day dynamic model can overcome the drawbacks of the widely-used UE or Logit-based route choice criterion in most existing day-to-day dynamic models. Overall, the proposed method provides a new perspective in static pricing design for decision makers by optimizing the cumulative network performance within the toll planning horizon and considering the day-to-day dynamics with the Weibit-based adjustment process.

The results from a series of numerical examples demonstrate the necessity of using the CNP measure as the optimization objective function for static pricing design, as the equilibrium state may not be reached within the toll planning horizon in some cases. The optimal toll scheme from the proposed method has a lower cumulative ETTT (network performance over the entire toll planning period) than the other two pricing schemes (i.e., marginal pricing and robust pricing) under various flow adjustment ratios. In addition, the proposed toll scheme generates moderate outcomes in terms of ETTT on day D (i.e., the network performance on the last day of the toll planning horizon) and the worst-case ETTT within the toll period. Apart from the network-level performance improvement, we also examine the effectiveness of the proposed toll scheme at the O-D level, path-level, and link-level.

When applying the proposed toll scheme design method, it is necessary to calibrate the model parameters from the real-world data in advance, including the flow adjustment ratio, the shape parameter in the Weibit model, the memory length, the weighting parameter, the link travel cost function, and the value of time. Naturally, the accuracy of the parameters can influence the effectiveness of the toll scheme. Besides, determining an optimal length of the toll planning horizon is also critical for policymakers. A too-long toll planning horizon may be ineffective while a too-short toll planning horizon might be confusing to road users and difficult to implement for traffic managers.

Finally, a few directions are worthy of further investigation. (1) Although this study investigates the link-based toll scheme, the proposed framework can be extended to the design of other types of pricing schemes, such as distance-based pricing. (2) Empirical analyses are needed to validate the proposed day-to-day dynamic model. (3) Although the Weibit model used in this study is free from the identically distributed perception error assumption, it cannot cope with the route overlapping problem due to the independent perception error assumption. We can overcome this drawback by introducing a path-size factor. (4) To be more consistent with reality, the flow adjustment ratio should be calibrated from real traffic data, such as Global Positioning System (GPS) data (Cheng et al., 2019), which may vary from one day to another (Zhou et al., 2017; Ye et al., 2018). (5) The initial path flow settings in this paper are hypothetical. To get an efficient toll scheme, it is necessary to input an accurate initial path flow pattern obtained from real-world data.

Appendix A: Sensitivity analysis in the illustrative network

In the following, we conduct sensitivity analyses on the setting of the initial flow pattern, the length of the toll planning horizon, and the memory length in the illustrative network used in Section 3.1. All the following experiments are conducted with the flow adjustment ratio $\alpha^d = 0.5$.

(1) The setting of the initial flow pattern

Below we compare a different initial path flow pattern with the previously used evenly distributed path flow pattern. The new initial path flow pattern (3034.42, 1741.97, 3034.42, 189.19, 1522.23, 2651.64, 873.87, 1522.23, 501.67, 873.87, 54.49) is obtained by performing the Weibit loading with link free-flow times. We plot the evolution curves of the expected total travel time under the no pricing scheme and under the proposed optimal toll schemes corresponding to the two initial path flow patterns.

As shown in Fig A.1, the proposed pricing schemes under both settings of initial flow pattern can effectively alleviate the network performance fluctuation and drive the network flow states to similar near-equilibrium states within 30 days. This well demonstrates the effectiveness and robustness of our proposed method.

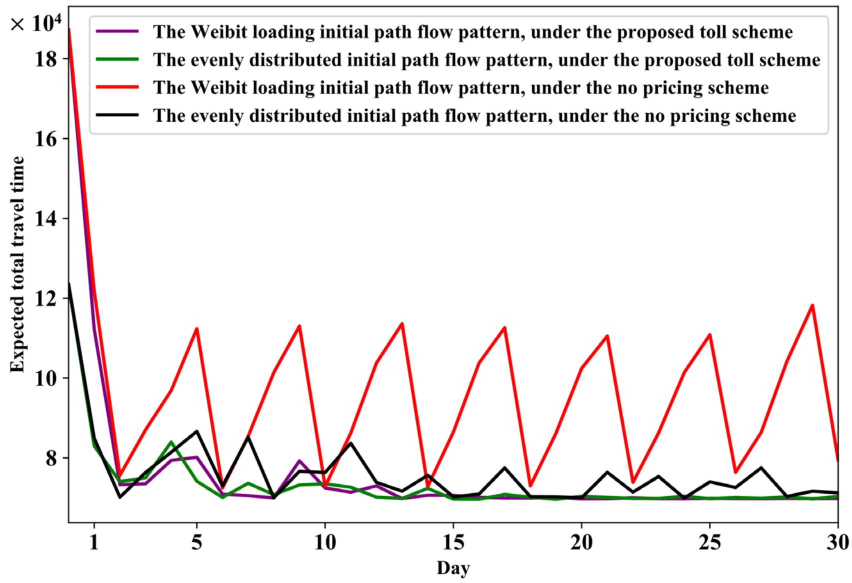


Fig A. 1 The evolution curves of expected total travel time under the no pricing scheme and under the proposed optimal toll schemes corresponding to the two initial flow patterns

(2) The length of the toll planning horizon

Below we compare four settings of toll planning horizon: D = 20, 30, 40 and 50. We plot the evolution curves of the expected total travel time under the optimal toll schemes corresponding to the four values of D in Fig A.2. Since we only focus on one toll horizon, the former three curves are truncated before 50 days. The evolution curve of the expected total travel time under the no pricing scheme within 50 days is also plotted for comparison.

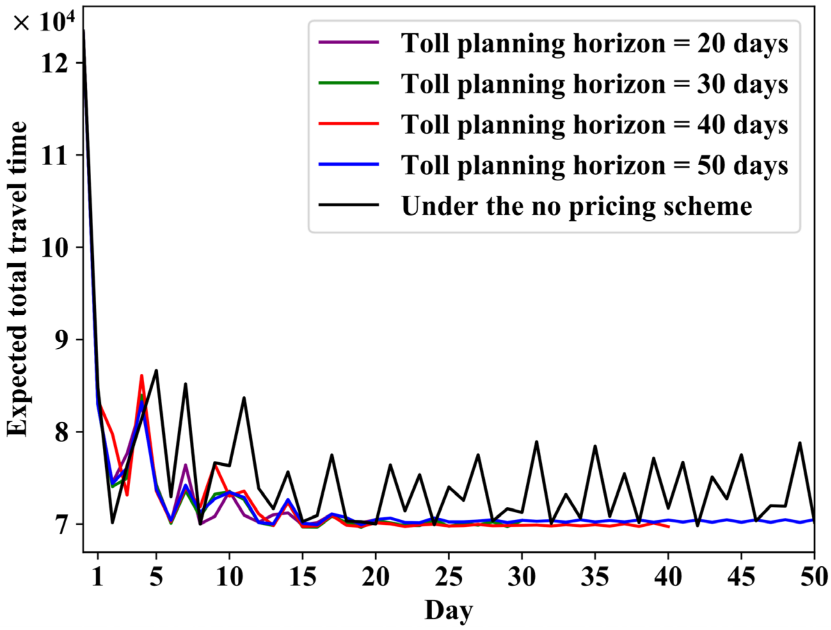


Fig A. 2 the evolution curves of expected total travel time under the optimal toll schemes corresponding to the four values of D

From Fig A.2, all the four pricing schemes can effectively mitigate the network performance fluctuation regardless of the length of toll planning horizon. Additionally, the ETTT at the end of the toll horizon tends to increase with the length of toll planning horizon. For example, the ETTT of day 50 on the blue curve is larger than that of day 40 on the red curve. This implies the benefit of updating the toll pattern more frequently (i.e., setting a shorter toll planning horizon) according to the previous network condition. The results indicate that there is a tradeoff for planners to determine the length of the toll horizon, such that not only the tolling effectiveness can be guaranteed but also the road users would not get confused due to a short toll horizon.

(3) The memory length

Below we compare the infinite memory length with the previous used finite memory length (i.e., m = 3). In the case of infinite memory length, Eqs. (7) - (10) are adopted to calculate the predicted travel cost on each day instead. The evolution trajectories of the expected total travel time under the two assumptions of memory length without pricing are plotted in Fig. A.3.

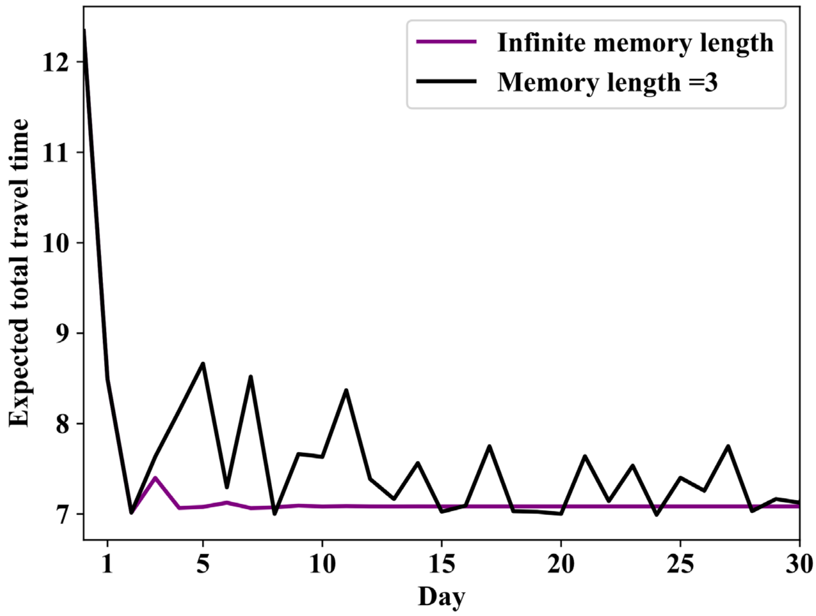


Fig A. 3 The evolution trajectories of the expected total travel time under the two assumptions of memory length without pricing

From Fig A.3, we can observe that adopting the infinite memory length assumption in the day-to-day dynamic model may underestimate the network flow fluctuation. This assumption may further lead to a less effective toll scheme since travelers' memories are finite in reality. It is thus necessary to consider travelers' memory length in modelling day-to-day dynamics as in many existing studies (e.g., Cascetta, 1989; Cantarella and Watling, 2016; Liu et al., 2017).

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