Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: K Urbanowicz *et al* 2021 *J. Phys.: Conf. Ser.* **1736** 012038

Alternative numerical solution of transient flow in viscoelastic pipes

K Urbanowicz[1], **A Bergant**[2,3], **HF Duan**[4], **U Karadzić**[5] **and D Sobków**[1]

[1] West Pomeranian University of Technology, Faculty of Mechatronics and Mechanical Engineering, Szczecin, Poland
[2] Litostroj Power d.o.o., Ljubljana, Slovenia
[3] University of Ljubljana, Faculty of Mechanical Engineering, Slovenia
[4] The Hong Kong Polytechnic University, Department of Civil and Environment Engineering, China
[5] University of Montenegro, Faculty of Mechanical Engineering, Podgorica, Montenegro

kamil.urbanowicz@zut.edu.pl

Abstract. A new numerical solution of transient flow simulations in viscoelastic pipes is presented, examined, and compared with traditional solutions and experimental results. Here the viscoelastic behaviour of the pipe wall is modelled explicitly and the wall shear stress implicitly. The new solution will be able to facilitate a better understanding of the unsteady flow behaviour in a plastic pipe in water supply systems. It will also contribute to lowering the computational costs of such systems and improving the accuracy of viscoelastic pipe diagnosis such as leak detection in the future.

1. Introduction
Unsteady states are common in practice. In liquid flows, they are the results of changes in basic flow parameters. A change in the flow velocity (or pressure) can be intentional (flow closing/opening, pump or turbine load decrease/increase, etc.) or accidental (power failure, damage to system components, etc.). The pressure change (increase or decrease) depends on the intensity of this phenomenon commonly referred to as water hammer or fluid transients. In a simple reservoir-pipe-valve system, the pressure reaches maximum values when the valve closing occurs in a fraction of half of the water hammer period ($t_c < 2L/c$) and additionally, there is pressure wave interference [1]. Among the waves, a distinction is made between primary waves, resulting from the rapid closing of a flow cut-off element, and secondary ones, which can be the result of the remerging of emerging vapour areas (cavitation). Most pressure systems have several functions (transmission, cooling – e.g. in nuclear power plants, water supply, power generation, etc.); therefore, expensive devices are currently used to protect them from the discussed conditions. In order to be able to minimise costs in the future by reducing additional safety devices, it is necessary to have a thorough knowledge and practical verification of the available methods for modelling these states [2]. The unsteady flow of liquids in plastic pipes has never before been the subject of such a large amount of research [3 - 5]. However, given that in most newly designed water supply systems around the world, pipes (as well more often other devices [6]) are made of one of the commonly known synthetic polymers: PE (polyethylene), PP (polypropylene), PVC (polyvinyl chloride), ABS (acrolonityl-butadiene-styrene) and PB (polybutylene), the increased interest should not

be surprising. The water hammer phenomenon is accompanied by continuous energy dissipation as a result of liquid friction against the walls and as a result of retarded strains occurring in the walls of these pipes. Zielke [7] presented an analytical unsteady friction model characterised by high model compatibility and relative implementation simplicity. In the beginning, calculations of wall shear stress using this method required a tedious iterative procedure used to calculate the instantaneous values of the convolutional integral (of local fluid acceleration and the weighting function) solution. Fortunately, Trikha [8] came up with the idea of solving this integral in a much simpler and effective way. Over time, Trikha's method was improved by Kagawa et al. [9] and Schohl [10]. Thereafter, Vardy and Brown [11] noticed an error in the original inefficient solution according to Zielke, which was then corrected by them in the same work. The correct effective solution that will be used in this work and excludes the mentioned error was presented in Urbanowicz's work [12]. The second factor attributed to the main source of mechanical energy dissipation during the flow in plastic pipes is retarded strain. To correctly model the pipe wall deformation, it is necessary to effectively solve the second convolution integral contained in the equation of continuity, which is convoluted from the product of local pressure derivative and the weighting function of creep for the flow duration. In this work, the solution of this integral will be based on the effective Schohl procedure [10], which was implemented and discussed in [13]. The numerical procedure itself, based on the method of characteristic, will be described in this paper. Some alternative numerical solutions of the well-known system of equations (continuity and motion) describing transient flow in plastic pipes will be presented. The hydraulic resistance will be calculated implicitly, while the effect of retarded strain will be calculated using the explicit method. This is the opposite of the commonly used approach in the literature [14, 15]. Comparative studies will allow assessing what kind of impact the introduced model approach will have on the final results.

2. Alternative numerical solution

Water supply pipes are currently made of plastics due to the convenience of construction and economy of price. Their behaviour is different from elastic (metal) pipes. The internal diameter of pipes may change dynamically according to the pressure surges. When a wave of increased pressure (i.e. positive wave) is passing by, the increased diameter in this type of pipes lasts for a longer time. This behaviour is the result of the viscoelastic properties of this material. The basic system of equations describing unsteady laminar flow in this type of pipes (continuity and motion, respectively) is determined as follows [13, 16]:

$$\begin{cases} \frac{1}{\rho c^2}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} + 2\frac{\partial \varepsilon_r}{\partial t} = 0 \\ \rho\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{2\tau}{R} = 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{\rho c^2}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} + \Xi\int_0^t \frac{\partial p_{(u)}}{\partial t}\cdot w_{J(t-u)}\,du = 0 \\ \frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{32\nu}{D^2}v + \frac{16\nu}{D^2}\int_0^t \frac{\partial v_{(u)}}{\partial t}\cdot w_{(t-u)}\,du = 0 \end{cases} \tag{1}$$

where: p – pressure [Pa], t – time [s], v – mean velocity in pipe cross section [m/s], x – axial coordinate [m], ρ – density of the fluid [kg/m^3], Ξ – enhanced pipe constraint parameter [-], c – pressure wave speed [m/s], D – inner pipe diameter [m], μ – dynamic viscosity of fluid [Pa·s], ε_r – retarded strain [-], $w_{J(t-u)}$ – weighting function of creep [Pa^{-1}s^{-1}], $w_{(t-u)}$ – weighting function of friction [-].

As we may see, the partial derivative concerning the time of retarded strain ε_r is a convolution integral of the local derivative of pressure and a creep weighting function $w_{J(t-u)}$ similarly, the wall shear stress can be written as a sum of quasi-steady and unsteady wall shear stress (another convolution). With the help of the method of characteristics, the above system of partial differential equations (1) can be reduced to the system of two ordinary differential equations:

$$\pm\frac{1}{c\rho}\frac{dp}{dt} + \frac{dv}{dt} + \frac{2\tau}{\rho R} \pm 2c\frac{\partial \varepsilon}{\partial t} = 0, \text{ for } \frac{dx}{dt} = \pm c \tag{2}$$

In a typical simplified 1-D numerical solution, used for years [14, 15, 17], it is assumed that the convolutional integral from the continuity equation (models the viscoelastic nature of deformation of the pipe walls) is calculated implicitly, while the convolutional integral in the equation of motion (models unsteady hydraulic resistance) explicitly. Using the finite difference method based on a rectangular grid of characteristics (Figure 1), the following equations were introduced [15]:

$$\begin{cases} \frac{1}{c\rho}(p_P - p_A) + (v_P - v_A) + \frac{2\Delta t}{\rho R}\tau_A + 2c\Delta t\frac{\partial \varepsilon_P}{\partial t} = 0 \\ -\frac{1}{c\rho}(p_P - p_B) + (v_P - v_B) + \frac{2\Delta t}{\rho R}\tau_B - 2c\Delta t\frac{\partial \varepsilon_P}{\partial t} = 0 \end{cases} \tag{3}$$

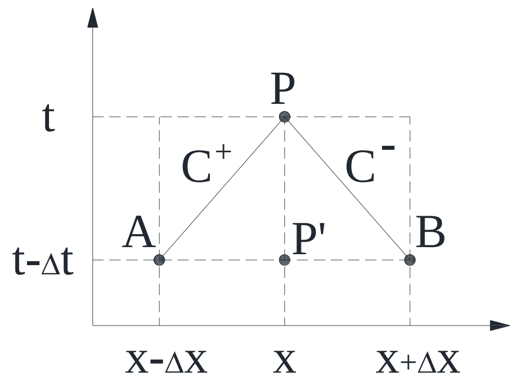

Figure 1. Grid of the method of characteristics.

In this work, an alternative solution is proposed to solve the system of ordinary equations (3). The convolution integral in the continuity equation which model the retarded strain will be calculated explicitly and the wall shear stress is modelled implicitly:

$$\begin{cases} \frac{1}{c\rho}(p_P - p_A) + (v_P - v_A) + \frac{2\Delta t}{\rho R}\tau_P + 2c\Delta t\frac{\partial \varepsilon_A}{\partial t} = 0 \\ -\frac{1}{c\rho}(p_P - p_B) + (v_P - v_B) + \frac{2\Delta t}{\rho R}\tau_P - 2c\Delta t\frac{\partial \varepsilon_B}{\partial t} = 0 \end{cases} \tag{4}$$

Let us bring the above system to the following form:

$$\begin{cases} \frac{p_P}{c\rho} + v_P + \frac{2\Delta t}{\rho R}\tau_P = C_A \\ -\frac{p_P}{c\rho} + v_P + \frac{2\Delta t}{\rho R}\tau_P = C_B \end{cases} \quad where: \begin{cases} C_A = \frac{p_A}{c\rho} + v_A - 2c\Delta t\frac{\partial \varepsilon_A}{\partial t} \\ C_B = -\frac{p_B}{c\rho} + v_B + 2c\Delta t\frac{\partial \varepsilon_B}{\partial t} \end{cases} \tag{5}$$

As can be seen from the formula (5), the values of C_A and C_B coefficients are calculated in the iterative procedure from formulas based on the knowns from previous numerical time step with values of pressure, velocity, and retarded strain which were calculated along the analysed hydraulic pipe. In this regard, the retarded strain is computed in an explicit way, rather than its instantaneous convolution.

By subtracting the second from the first equation in the above set (5), we obtain the sought formula for pressure at the internal nodes:

$$\boldsymbol{p_P = \frac{c\rho}{2}(C_A - C_B) = \frac{p_A + p_B}{2} + \frac{\rho c(v_A - v_B)}{2} - \rho c^2 \Delta t\left(\frac{\partial \varepsilon_A}{\partial t} + \frac{\partial \varepsilon_B}{\partial t}\right)} \tag{6}$$

but if we add a second equation to the first equation then we get:

$$2v_P + \frac{4\Delta t}{\rho R}\tau_P = C_A + C_B \tag{7}$$

The wall shear stress τ_P can be calculated using the corrected effective solution [11]:

$$\tau_P = \underbrace{\frac{\rho f_P v_P |v_P|}{8}}_{\tau_{q,P}} + \underbrace{\frac{2\mu}{R}\sum_{i=1}^{3}\left[\underbrace{A_i y_{i(t-\Delta t)} + \eta B_i[v_P - v_{P'}] + (1-\eta)C_i[v_{P'} - v_{P''}]}_{y_{i(t)}}\right]}_{\tau_{u,P}} \quad (8)$$

The coefficients A_i, B_i, and C_i are the friction constants and η is the match factor of the efficient weighting function – for more details see [12].

In laminar flow $f_P = \frac{64}{Re_P} = \frac{64\nu}{|v_P|D} = \frac{32\nu}{|v_P|R}$, therefore

$$\tau_P = \underbrace{\frac{4\mu}{R}v_P}_{\tau_{q,P}} + \underbrace{\frac{2\mu}{R}\sum_{i=1}^{3}\left[\underbrace{A_{i(L)} y_{i(t-\Delta t)} + \eta B_{i(L)}[v_P - v_{P'}] + (1-\eta)C_{i(L)}[v_{P'} - v_{P''}]}_{y_{i(t)}}\right]}_{\tau_{u,P}} \quad (9)$$

$$\tau_P = \frac{4\mu}{R}v_P + \frac{2\mu}{R}\sum_{i=1}^{3}\eta B_{i(L)}v_P + \frac{2\mu}{R}\sum_{i=1}^{3}\left[A_{i(L)}y_{i(t-\Delta t)} + (1-\eta)C_{i(L)}[v_{P'} - v_{P''}] - \eta B_{i(L)}v_{P'}\right] \quad (10)$$

$$\tau_{P(L)} = v_P\left(\frac{4\mu}{R} + \frac{2\eta\mu}{R}\sum_{i=1}^{3}B_{i(L)}\right) + N_{(t-\Delta t)} = v_P M_{(L)} + N_{(L),(t-\Delta t)} \quad (11)$$

Unfortunately, for turbulent flow, a solution similar to the above (11) cannot be obtained. The reason for this is the non-linear expression for quasi-steady shear stress. Below, another alternative solution, which as shown by numerical studies (not discussed in this work) ensures the stability of the numerical solution without affecting model accuracy, is presented.

Let us assume the smooth pipe walls, then the Darcy-Weisbach friction factor while a turbulent flow occurs in point P' is calculated $f_{P'} = \frac{0.3164}{\sqrt[4]{Re_{P'}}}$, while the quasi-steady shear stress in this flow will be calculated as follows:

$$\tau_{q,P} = \frac{\rho f_{P'} v_P |v_{P'}|}{8} = \frac{0.3164}{\sqrt[4]{|v_{P'}|\frac{D}{\nu}}}\frac{\rho|v_{P'}|}{8}v_P = O_{(t-\Delta t)}v_P \quad (12)$$

Then total wall shear stress in turbulent flow can be represented in an equivalent linearised way:

$$\tau_{P(T)} = v_P\left(O_{(t-\Delta t)} + \frac{2\eta\mu}{R}\sum_{i=1}^{3}B_{i(T)}\right) + N_{(t-\Delta t)} = v_P M_{(T)} + N_{(T),(t-\Delta t)} \quad (13)$$

Such linearised representation of wall shear stress equations (11) and (13) leads to the following formula for flow velocity:

$$2v_P + \frac{4\Delta t}{\rho R}\left(v_P M + N_{(t-\Delta t)}\right) = C_A + C_B \quad (14)$$

$$\boldsymbol{v_P} = \frac{\boldsymbol{C_A + C_B - \frac{4\Delta t}{\rho R}N_{(t-\Delta t)}}}{\boldsymbol{2 + \frac{4\Delta t}{\rho R}M}} = \frac{\boldsymbol{v_A + v_B + \frac{p_A - p_B}{\rho c} + 2c\Delta t\left(\frac{\partial\varepsilon_B}{\partial t} - \frac{\partial\varepsilon_A}{\partial t}\right) - \frac{4\Delta t N_{(t-\Delta t)}}{\rho R}}}{\boldsymbol{\left(2 + \frac{4\Delta t M}{\rho R}\right)}} \quad (15)$$

As one may see the received final formulas for pressure (6) and velocity (15) are different than those presented in earlier work [15]. The new solution will be examined in detail in the next chapter. In this work, the retarded strain is calculated by using the Schohl efficient convolution integral solution [13].

3. Examination of modified model

The solution derived with help of the method of characteristics in section 2 was used to write a new own in-house program in the Matlab programming language. Numerical results from the old classical [14 - 17] and the new alternative model (section 2) are compared with results from Covas measurements [14, 17]. The measurements have been performed at Imperial College, London on a HDPE pipe rig with water as a working liquid, which is a typical simple reservoir-pipe-valve system example (Figure 2).

The experimental pipeline length was L = 271.7 [m], the internal diameter D = 0.0506 [m], pressure wave speed c = 395 [m/s], water density ρ = 998.2 [kg/m^3], temperature T_W = 20 [ºC], Poisson ratio ν_P = 0.46 [-] and kinematic viscosity ν = $1\cdot10^{-6}$ [m^2/s]. Knowing Poisson ratio, the pipe-wall constraint coefficient was determined $\xi = 1.065\,[-]$ and its enhanced value $\Xi = \frac{D}{e}\xi = 8.55\,[-]$.

A quick closing globe valve (T_c = 0.13 [s]) was used for the initiation of water hammer event. The plastic pipe retarded strain creep compliance functions coefficients are: J_0 = $0.674\cdot10^{-9}$ [Pa^{-1}], J_1 = $0.1394\cdot10^{-9}$ [Pa^{-1}], J_2 = $0.0062\cdot10^{-9}$ [Pa^{-1}], J_3 = $0.1148\cdot10^{-9}$ [Pa^{-1}], J_4 = $0.3425\cdot10^{-9}$ [Pa^{-1}], J_5 = $0.0928\cdot10^{-9}$ [Pa^{-1}] with retardation times T_0 = 0 [s], T_1 = 0.05 [s], T_2 = 0.5 [s], T_3 = 1.5 [s], T_4 = 5 [s], T_5 = 10 [s]. In all numerical simulations the pipeline was divided in 32 reaches (N = 32 [-], Δx = 8.49 [m]); the resulting time step was Δt = 0.0215 [s] (dimensionless time step $\Delta\hat{t}$ = $3.36\cdot10^{-5}$ [-]). The unsteady friction weighting function coefficients are calculated using the procedure described by Urbanowicz in [18]. The old and newly presented model will be used to simulate two types of flows: laminar and turbulent. Initial system conditions are collected in Table 1 wherein $p_{v,t=0}$ – initial valve pressure, Δp_L – pressure drop along the pipe length, Δp_r – is the pressure rise inside the reservoir during the transient state.

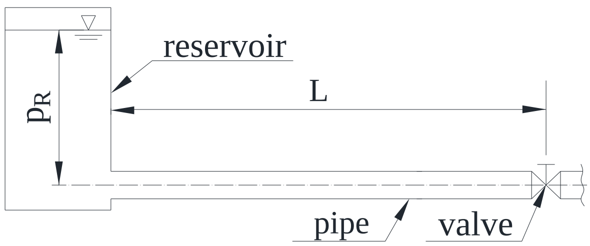

Figure 2. Reservoir-pipe-valve system.

Table 1. The initial conditions.

Case	v_0 [m/s]	$p_{v,t=0}$	Δp_L	Δp_r	p_{Final}	Re_0
C01	0.028	4.732e5	0.01e4	0.14e4	4.749e5	1417
C02	0.995	3.425e5	5.6e4	3.15e4	4.3e5	50347
$p_{Final} = p_{v,t=0} + \Delta p_L + \Delta p_r$						

The simulation results using a classical approach (dotted red line ‘old’) and the new solution presented in this paper (dashed blue line ‘new’) are compared with the experimental results (solid black line ‘exp’) for both analysed cases. With a green dash-dotted line the reservoir pressure p_R is represented, which in the experiment raised during the transient state which was probably the effect of problematic synchronization of the nearly instantaneous valve closure and stopping the running pump on the suction line. The results of comparisons for the laminar flow case are presented in Figure 3, while the turbulent flow case results are presented in Figure 4.

Comparisons for the laminar flow case (Figure 3) yield the following conclusions:

1) The tested modified model overestimates the pressure peak values at the first amplitude (Figure 3b). The value of pressure increase modelled with its help, although consistent with the increase calculated from the Joukowsky formula is not experimentally noticeable. However, it is too early to write with certainty about the ineffectiveness of the analysed modified model. Covas in her dissertation [17] discussed that the value of the increase on this first amplitude, occurring just after the rapid closing of the valve, results from several factors:

a) valve closing time – the faster, the greater the increase of pressure was recorded experimentally.

b) the type of valve used and its restraint.

An analysis of the theoretical formula analogous to the Joukowsky formula presented in [19] indicates that also in the case of plastic pipes, when an exceedingly small time step $\Delta t \rightarrow 0$ is assumed, this formula is reduced to the Joukowsky formula. This means that when the valve closes nearly instantaneously, also in the case of flow in plastic pipes, the pressure increase should be close at this initial time to the one calculated from Joukowsky's formula.

2) The simulation differences, which are most visible in the initial phase of the water hammer in the cross-section at the shut-off valve, have got no significant effect on the modelled subsequent pressure amplitudes. This overestimated value at the first amplitude in the further flow phase (on subsequent amplitudes) results only in a slight phase shift and slightly increased peak pressure values. This difference at the initial stage can be mainly attributed to the multi-dimensional effect of incident wave operation (e.g. 2D wave) such as valve closure, which has not been represented by the 1D model here [20]. Thereafter, this multi-dimensional wave effect disappears quickly (e.g. within the duration of couples of D/c), so that the 1D plane wave model is valid for simulating the flow process [21].

3) Simulations made for other cross-sections (see results at the midpoint – Figure 3c) indicate that no significant model differences are noticed. Also, in these cases, the simulated pressure with the modified model was a bit higher during the amplitude peak peaks (Figure 3d)

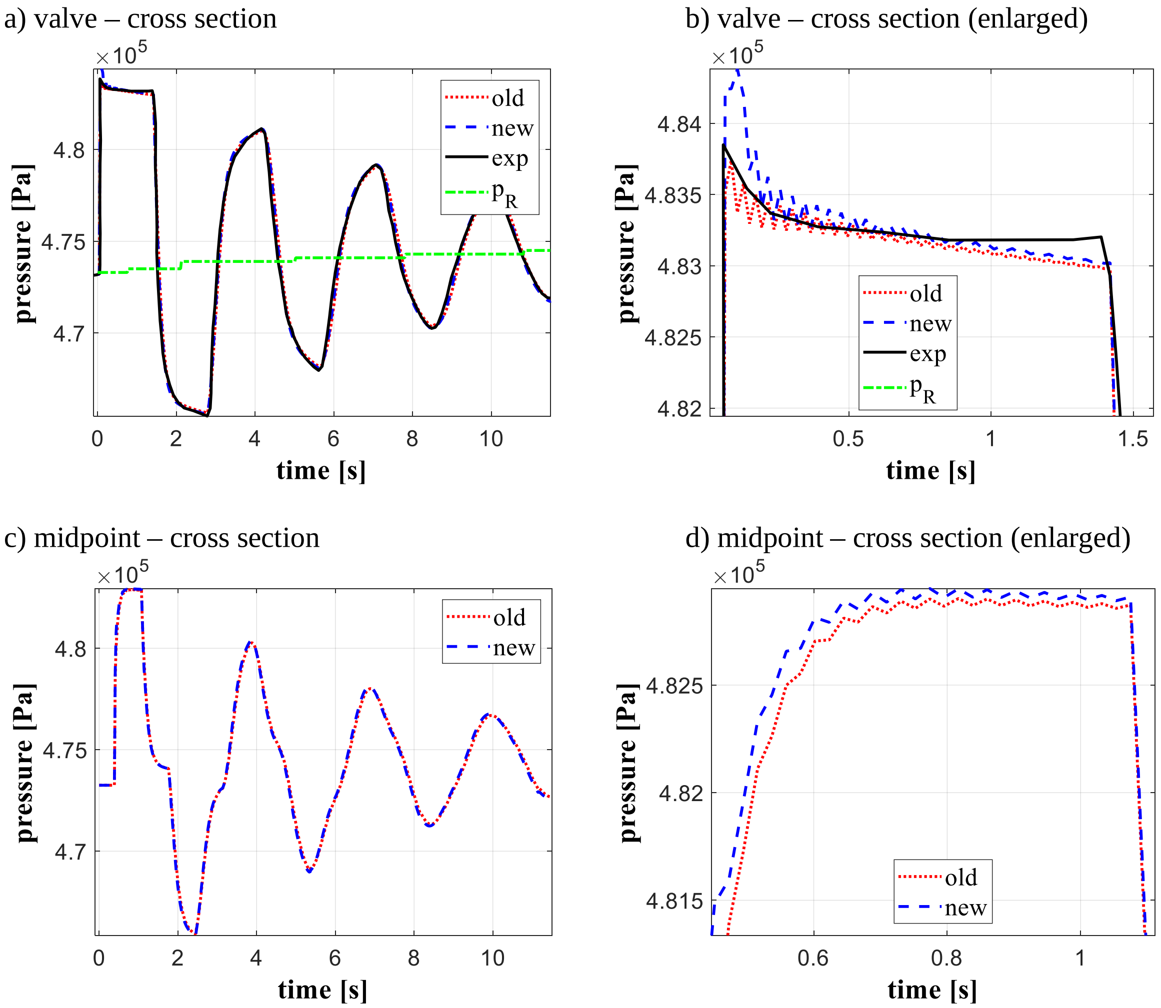

Figure 3. Laminar flow case results.

The conclusions noticed from comparisons made for the turbulent flow case (Figure 4) are remarkably similar to those observed for the laminar flow case. The significant difference that is noted between the two flow cases relates to the modelled as well as to the experimentally noticeable pressure increase at the first amplitude (Figure 4b). The phenomenon of "line packing" [22], which was correctly simulated with the use of both analysed numerical models, is becoming more and more important in a turbulent flow. This phenomenon was not visible in the laminar flow case, which is influenced by a significant difference in pressure drop along the length of the conduit in both analysed flow cases: for the laminar flow case the pressure loss along the pipe length in a pre-transient state was $\Delta p_L = 0.01 \cdot 10^4$ $[Pa]$, while for turbulent flow case it was $\Delta p_L = 5.6 \cdot 10^4$ $[Pa]$.

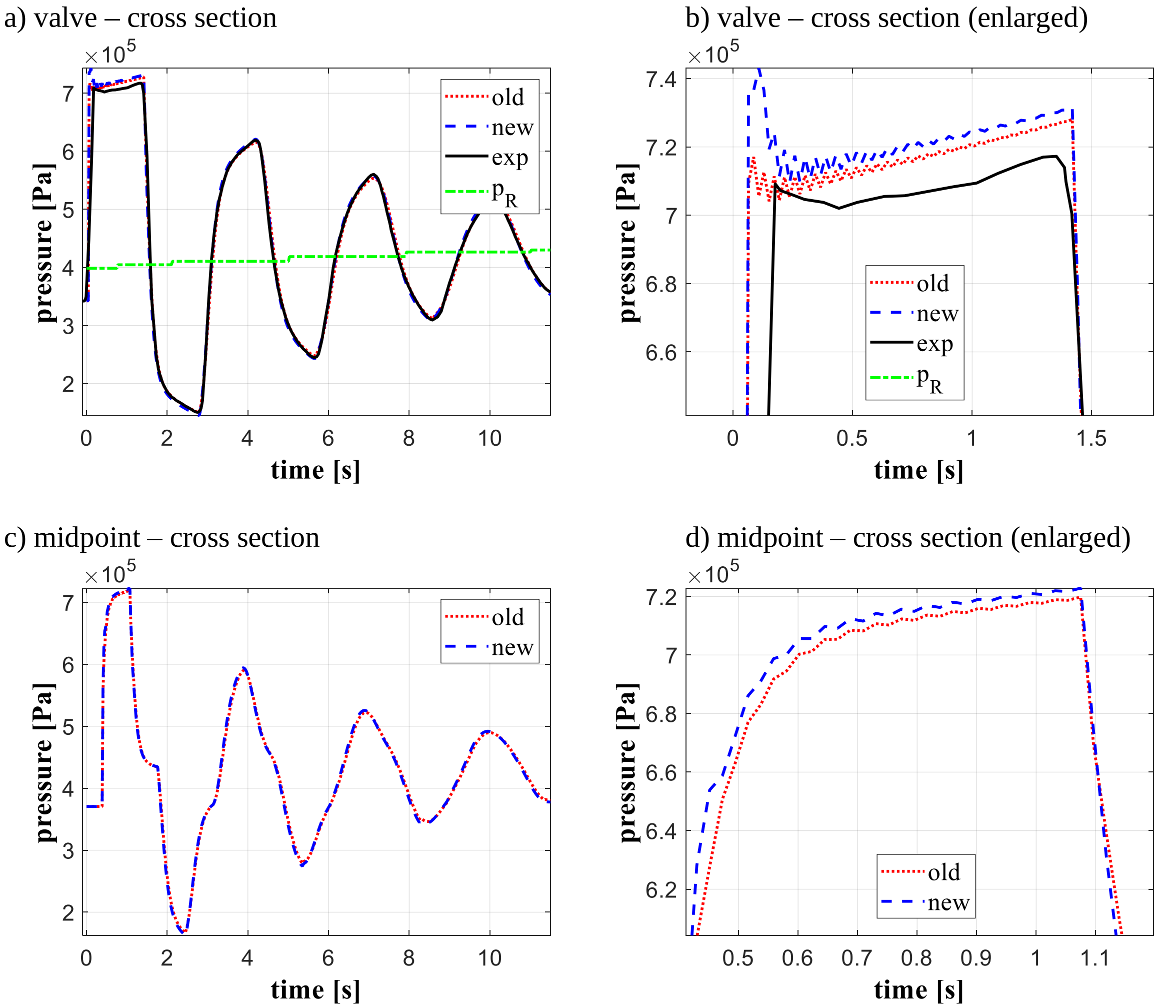

Figure 4. Turbulent flow case results.

4. Conclusions

An alternative solution to the system of continuity and motion equations describing the transient flow of liquids in a plastic pipe was presented and examined in this paper. In this alternative solution, the friction effect was calculated implicitly, while the retarded strain was estimated explicitly (from previous numerical time step). The comparison of the results indicated that the proposed alternative model may provide overall acceptable results of transient pressure with other solutions, but a relatively clear difference has been observed for the first amplitude of the transient flows. Through the result analysis in this study, such difference may be largely attributed to the multi-dimensional effect of initial wave generation (such as valve closure here), which could not be well represented by the current 1D model.

It is suggested that a 2D or 3D model will be required in the future to solve this problem as today's CFD applications seem to guarantee overlap with theoretical results [23].

From this current research, it is indicated that different simulation ways of transient flow in plastic pipes can be considered as follows: a) both the friction and retarded strain are modelled in an explicit way, b) both friction and retarded strain are modelled implicitly; c) either of these two factors is simulated explicitly, while the other in an implicit way. As a result, four different solution schemes emerge for the simulation of transient flows in plastic pipes. Consequently, more investigations and tests will be required to systematically examine the validity range and the corresponding accuracy of these four numerical schemes for the viscoelastic transient pipe flows in future work.

Acknowledgements
Bergant gratefully acknowledges the support of the Slovenian Research Agency conducted through the research project L2-1825 and the programme P2-0162.F.

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