

# Prediction of fatigue damage in ribbed steel bars under cyclic loading with a magneto-mechanical coupling model

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**Abstract:** As a ferromagnetic material, the magnetization of ribbed steel bars will change with the development of fatigue damage under cyclic loading, which can be used to evaluate the fatigue damage state of steel bars. However, the quantitative relationship between fatigue damage and the variation in magnetization is still unclear, and the existing magneto-mechanical model cannot be applied to ribbed steel bars directly. To accurately predict the magnetization and the fatigue damage state during fatigue, it is also necessary to consider the influence of stress concentration caused by ribs on stress and fatigue life. This paper proposes a magneto-mechanical model suitable for ribbed steel bars, which use the Neuber law and Coffin-Manson relationship to determine the stress range and fatigue life under stress concentration. Additionally, the magnetic induction intensity of the HRB400 ribbed steel bar in the tensile fatigue test was measured to verify the proposed model, and the mechanism of the magnetization change trend during fatigue was analyzed in detail. Through the fatigue damage formula based on the magnetic indicator, the simulation and experimental comparison results show that the proposed model can effectively describe the fatigue damage state of ribbed steel bars in the fatigue.

**Keywords:** Cyclic loads; fatigue damage; magneto-mechanical effect; ribbed steel bar.

## 23 1. Introduction

24 With the development and application of high-strength building materials, and the probability  
25 limit state design method based on reliability theory and concrete multi-axis strength theory is widely  
26 used in practical engineering design. The safety of the reinforced concrete (RC) structures under static  
27 load has been greatly guaranteed, they are difficult to destroy since they exceed their ultimate bearing  
28 capacity. More attention should be paid to the impact of long-term structural performance, one of the  
29 key points is the fatigue which causes the degradation of material mechanical properties. The  
30 structure is not only subjected to static load, but also often subjected to cyclic loads, such as  
31 earthquakes, wind loads, and vehicle loads, lower than the ultimate load during service. These loads  
32 usually cause the bars in the RC structure to develop fatigue cracks and irreversible damage to the  
33 local microstructure. When a particular critical value of damage is reached, the bars will undergo  
34 brittle fatigue fracturing, which causes RC structure failure <sup>1</sup>. Therefore, accurate evaluations of  
35 fatigue damage to steel bars are an essential means to ensuring structural safety and reliability.  
36 Currently, the research methods for assessing the fatigue of materials include S-N curves, fracture  
37 mechanics and damage mechanics. These methods are mainly based on traditional mechanical  
38 indicators to analyze the fatigue properties of materials <sup>2</sup>. However, the variation in the stress-strain  
39 hysteresis curve of steel bars during fatigue is slight and monotonous, which makes it difficult to  
40 reflect the microscopic damage and accurately evaluate the fatigue damage state. Additionally, strain  
41 measuring equipment needs to be installed inside the structure, and regular maintenance and  
42 replacement are required to ensure long-term effectiveness. These factors result in damage to the  
43 structure and high costs.

44 There are certain limitations to using traditional mechanical indicators to reflect the material  
45 fatigue damage. In recent years, spontaneous magnetic field (SMF) technology, which has been used  
46 to estimate the damage of ferromagnetic materials by detecting the change in the weak magnetic  
47 signal on the surface of the materials, is considered to be of great feasibility for applications in fatigue  
48 damage detection involving ferromagnetic materials <sup>3</sup>. As a ferromagnetic material, the magnetization

49 properties of ribbed steel bars are very sensitive to the stress loading history and fatigue microscopic  
50 damage<sup>4</sup>. Villari<sup>5</sup> first systematically studied the magnetic behavior of ferromagnetic materials under  
51 an external magnetic field and stress. The study showed that the magnetic domains within materials  
52 would move and rotate under a monotone loading, leading to changes in the surface magnetic field  
53 of the material. Erber et al.<sup>6</sup> found that the surface magnetic signals of ferromagnetic materials under  
54 cyclic loading reflect internal dislocation slippage due to the accumulated microplasticity of fatigue.  
55 Guralnick et al.<sup>7</sup> measured the magnetic induction intensity of AISI 1080 steel under tensile and  
56 compressive fatigue tests. They found that the magnetic hysteresis curve of specimens contained more  
57 information about fatigue damage, and the area of the magnetic hysteresis curve changed in three  
58 stages with the development of fatigue damage. Subsequently, Bao et al.<sup>8-11</sup> also conducted similar  
59 fatigue test research, which showed that the magnetic hysteresis curve could more sensitively reflect  
60 the microstructure damage than the corresponding mechanical hysteresis curve. In torsion and  
61 bending fatigue, the variations in magnetic signals also show regularity<sup>12</sup>. Xu et al. found that the  
62 magnetic signal in medium carbon steel in a rotational bending fatigue test showed a systematic three-  
63 stage trend: a rapid increase in the early stage, a stable development in the middle stage, and a sudden  
64 change in the final stage. To verify the feasibility of SMF application on ribbed steel bars, Zhang et  
65 al.<sup>4</sup> measured the surface magnetic induction of ribbed steel bars in tensile fatigue tests and reflected  
66 the fatigue damage state of the bars by the change rate in the magnetic induction. Current  
67 experimental studies have proven that the magnetization property and SMF of ferromagnetic  
68 materials have the potential to be used as detection indicators for fatigue damage. However, most of  
69 them are still in a state of qualitative research. To accurately evaluate the fatigue damage stage of  
70 ferromagnetic materials, a theoretical model to quantitatively describe the magnetization of  
71 ferromagnetic materials during fatigue is indispensable.

72 The magnetization and SMF of ferromagnetic materials are mainly related to the magneto-  
73 mechanical effect during fatigue. Jiles and Atherton<sup>13-15</sup> proposed a phenomenological model (J-A

74 model) for ferromagnetic materials to characterize the relationship between stress and magnetization,  
75 which has been widely used in studying the magnetic behavior of ferromagnetic materials under  
76 stress. In the actual magnetization process, dislocations and domain wall pinning will affect the  
77 magnetic properties of the material. Sablik et al. <sup>16-18</sup> introduced pinning coefficients into the J-A  
78 model and established the Jiles-Atherton-Sablik (J-A-S) hysteresis model. Then, Li et al. <sup>19</sup> took the  
79 influence of plastic deformation on magnetization into account and modified the plastic term of the  
80 J-A-S model. To facilitate calculating the fatigue, Xu et al. <sup>20</sup> proposed the magnetization local  
81 equilibrium state based on dislocation magnetization theory and established a relationship between  
82 the fatigue damage and magnetization by the modified J-A-S model. On the other hand, Zheng and  
83 Liu <sup>21</sup> established the Zheng-Liu (Z-L) model using thermodynamic relations, which can predict a  
84 magnetostrictive strain under different magnetic fields with different pre-pressures. Based on the Z-  
85 L model and combined with the approach law, Shi et al. <sup>22</sup> made the model more accurate in  
86 describing the magnetization under a few cyclic loads, but it did not consider the effect of fatigue  
87 damage on the magnetization property of the material. Recently, Huang et al. <sup>23</sup> combined the Z-L  
88 model and the modified J-A-S model based on the plastic strain obtained from the test and simulated  
89 the magnetization of smooth specimens processed by HRB400 steel bars during fatigue. However,  
90 the fatigue model and the simulation of ribbed steel bars remain to be studied.

91 In general, most of the existing magneto-mechanical fatigue models are suitable for smooth and  
92 homogeneous specimens, but the ribbed bars in RC structures are not smooth and homogeneous  
93 materials. Their ribs on the surface will significantly affect the stress distribution, and these stresses  
94 are closely related to fatigue life and magnetization. In addition, the simulation results of the existing  
95 models depend on the plastic strain measured during each cycle. If theoretical analysis or data  
96 regression can be used to make them independent of the test data, it will bring great convenience to  
97 the simulation. To accurately predict the magnetization evolution of ribbed steel bars in RC structures  
98 during fatigue under complex fatigue stress, the fatigue model of ribbed bars will provide reference

99 **and guidance.** In this paper, based on the model established by Huang et al.<sup>23</sup>, the influence of the  
100 stress concentration is considered in order to determine the simulated stress range and fatigue life,  
101 and the residual plastic strain formula of ribbed steel bars conforming to the three-stage development  
102 law is combined to establish a fatigue magneto-mechanical model applicable to ribbed steel bars. In  
103 addition, tensile fatigue tests of HRB400 ribbed steel bars are performed, and the simulation results  
104 of the modified model are compared with the test results to verify the applicability and accuracy.

## 105 **2. Magneto-mechanical theoretical framework**

### 106 *2.1. Magneto-mechanical model under ideal magnetization*

107 If the influence of the stress loading history on the magnetization of ferromagnetic materials is  
108 ignored, the materials will ideally be magnetized in the magnetic field, and their magnetization will  
109 tend to be anhysteretic. In addition, ferromagnetic materials subjected to stress will produce  
110 magnetostrictive strain, which manifests the displacement and rotation of the domain walls. In this  
111 process, part of the stress energy will be converted into magnetization energy, and the macroscopic  
112 magnetization eventually changes.

113 Because the ribbed steel bar's axial dimension is much larger than its radial dimension, it can be  
114 regarded as a one-dimensional isotropic ferromagnetic rod. Under the isothermal environment and a  
115 constant axial weak magnetic field, ignoring the loading history, the total derivative of energy per  
116 unit volume of material satisfies the following equation<sup>24</sup>:

$$117 \quad dU = \sigma d\varepsilon + TdS + \mu_0(H - N_d M)dM \quad (1)$$

118 where  $\sigma$  is the stress,  $\varepsilon$  is the strain,  $T$  is the temperature,  $S$  is the entropy density,  $\mu_0 = 4\pi \times$   
119  $10^{-7}$  A/m is the vacuum permeability,  $H$  is the magnetic field strength,  $N_d$  is the demagnetization  
120 coefficient, and  $M$  is the magnetization.

121 The Gibbs free energy of ferromagnetic materials during magnetization can be expressed as<sup>25</sup>

122 
$$G(\sigma, M, T) = U - TS - \sigma\varepsilon \quad (2)$$

123 For the isothermal case,  $SdT = 0$ , the following thermodynamic relations can be obtained.

124 
$$\varepsilon = -\frac{\partial G}{\partial \sigma}, \mu_0(H - N_d M) = \frac{\partial G}{\partial M} \quad (3)$$

125 Use independent variables to extend the Gibbs free energy function at  $G(\sigma, M) = G(0,0)$ , and  
 126 substitute the expanded Taylor series into equation (3) to obtain the polynomial expression of the  
 127 effective magnetic field  $H_e$  with stress  $\sigma$  and magnetization  $M$ . Combined with the theoretical  
 128 analysis of Jiles et al.<sup>13</sup>, the magnetostriction curve of the ferromagnetic materials is a symmetrical  
 129 even function, so that the one-dimensional magneto-mechanical constitutive relationship can be  
 130 expressed as<sup>23</sup>:

131 
$$H_e = \begin{cases} H_0 + \alpha M - N_d M + \frac{2\lambda_s M}{\mu_0 M_{ws}^2} \left( \sigma - \frac{\sigma_s}{\beta} \ln \cosh \frac{\beta \sigma}{\sigma_s} \right) - \frac{4\theta \lambda_s \sigma (M^3 - M_r^3(\sigma))}{\mu_0 M_{ws}^4}, \sigma \geq 0 \\ H_0 + \alpha M - N_d M + \frac{2\lambda_s M}{\mu_0 M_{ws}^2} \left( \sigma - \frac{\sigma_s}{4\beta} \ln \cosh \frac{2\beta \sigma}{\sigma_s} \right) - \frac{4\theta \lambda_s \sigma (M^3 - M_r^3(\sigma))}{\mu_0 M_{ws}^4}, \sigma < 0 \end{cases} \quad (4)$$

132 
$$M_r(\sigma) = \begin{cases} M_{ws}(1 - \tanh(\beta\sigma/\sigma_s)), \sigma \geq 0 \\ M_{ws}(1 - \tanh(2\beta\sigma/\sigma_s)/2), \sigma < 0 \end{cases} \quad (5)$$

133 
$$M_{an} = M_s \left( \coth \left( \frac{H_e}{a} \right) - \frac{a}{H_e} \right) \quad (6)$$

134 where  $H_e$  is the effective magnetic field with applied stress,  $H_0$  is the external magnetic field, which  
 135 can be regarded as the geomagnetic field without an applied magnetic field,  $a$  and  $\beta$  are the shape  
 136 coefficients,  $\lambda_s$  is the saturated magnetostrictive strain,  $M_{ws}$  is the magnetization as the domain wall  
 137 motion process is completed without stress,  $\sigma_s$  is the stress when the magnetostrictive strain is  
 138 saturated,  $M_r(\sigma)$  is the transition point of  $M_{ws}$  denoting the saturation magnetization associated with  
 139 the domain wall motion under a given stress  $\sigma$ , and  $\theta$  is the step function. When  $M \geq M_r(\sigma)$ ,  $\theta =$   
 140  $3/4$ , the magnetization increases monotonically, and when  $M < M_r(\sigma)$ ,  $\theta=0$ , the magnetic domain

141 rotation causes the magnetization to decrease.  $M_{an}$  is the anhysteretic magnetization,  $M_s$  is the  
142 saturation magnetization, and  $\alpha$  is the coefficient that characterizes the binding ability of the magnetic  
143 moment to the magnetization.

144 By solving Eqs. (4), (5) and (6),  $M_{an}$  of the ribbed steel bar under the stress and external magnetic  
145 field under ideal magnetization conditions can be obtained.

146

## 147 2.2. Magneto-mechanical model during fatigue

148 According to the description of the approach law proposed by Jilse<sup>14</sup>, the  $M$  of ferromagnetic  
149 material tends to be  $M_{an}$  during magnetization under cyclic loads, as shown in Eq. (7). However, due  
150 to the accumulation of fatigue damage in the material, magnetic domain wall pinning acts as an  
151 obstacle during magnetization, and the material cannot be ideally magnetized to  $M_{an}$ . From the  
152 perspective of a macro energy balance, Xu et al.<sup>20</sup> assumed that the magnetization state under an  
153 energy loss caused by pinning is the local equilibrium magnetization  $M_0$ , considered that the  $M$  in  
154 the fatigue continues to approach the  $M_0$ , and modified the J-A-S model. The local equilibrium  
155 magnetization considering the pinning effect is shown in Eq. (8), which means that  $M_0$  will  
156 continuously overcome pinning obstacles under cyclic loads and finally reach  $M_{an}$ .

$$157 \quad \frac{dM}{d\sigma} = \frac{\sigma}{\xi E} (M_{an} - M) + c \frac{dM_{an}}{d\sigma} \quad (7)$$

158 where  $E$  is the Young's modulus,  $\xi$  is a parameter related to the energy density, and  $c$  is a parameter  
159 describing the flexibility of domain wall motion.

$$160 \quad \frac{dM_0}{d\sigma} = \frac{\sigma}{AEk} (M_{an} - M_0) \quad (8)$$

161 where  $A$  is a parameter determined by the test results,  $k$  is the domain wall pinning coefficient,  $k =$   
162  $n\langle\varepsilon_\pi\rangle/(2m)$ ,  $n$  is the density of the pinning,  $\langle\varepsilon_\pi\rangle$  is the 180° domain wall pinning average energy,

163  $\langle \varepsilon_\pi \rangle \propto 2m\mu_0 H_e$ , and  $m$  is the magnetic moment per unit volume <sup>26</sup>.

164 Dislocations are defects caused by the local irregular arrangement of atoms in the material,  
165 which can hinder pinning of the motion of domain walls. The pinning point in the material that blocks  
166 the movement of the domain wall increases with an increasing dislocation. It is assumed that the  
167 pinning point density is proportional to the dislocation density, as follows <sup>27</sup>:

$$168 \quad n \propto (n_0 + l) \quad (9)$$

169 where  $n_0$  is the pinning point density caused by the initial defect of the crystal and  $l$  is the plugging  
170 dislocation density.

171 Dislocation will move inside the material under cyclic loading. According to the dislocation  
172 plugging theory, the density of dislocations per unit volume, regardless of temperature, can be  
173 expressed as:

$$174 \quad l = \rho b \bar{\lambda} \quad (10)$$

175 where  $\rho$  is the dislocation density,  $b$  is the Berkeley vector magnitude of the dislocation, and  $\bar{\lambda}$  is the  
176 average dislocation slip distance.

177 Substituting Eqs. (8) and (9) into  $k = n\langle \varepsilon_\pi \rangle / (2m)$ , the relationship between the domain wall  
178 pinning coefficient and the dislocations is as follows:

$$179 \quad k \propto \mu_0 (n_0 + b\rho\bar{\lambda}) H_e \quad (11)$$

180 Referring to the Gilman relationship <sup>28</sup>, there is a quantitative relationship between the shear  
181 plastic strain and dislocation,  $\gamma_p = \rho b \bar{\lambda}$ . Since the shear plastic strain is linearly related to the axial  
182 plastic strain, Eq. (11) is substituted into Eq. (8) in order to obtain the expression of the local  
183 equilibrium magnetization related to the plastic strain:

$$184 \quad \frac{dM_0}{d\sigma} = \frac{\sigma(M_{an} - M_0)}{A\mu_0 H_e E (n_0 + \varepsilon_p)} \quad (12)$$



185 In the fatigue,  $M$  constantly tends to  $M_0$  instead of  $M_{an}$ , modifying the approach law described  
186 in Eq. (7) proposed by Jiles, the magnetization of ferromagnetic materials satisfies the following  
187 equation.

$$188 \quad \frac{dM}{d\sigma} = \frac{\sigma}{\xi E} (M_0 - M) + c \frac{dM_0}{d\sigma} \quad (13)$$

189 Substituting  $M_{an}$  into Eq. (12), and solving Eqs. (12) and (13) simultaneously by the ODE  
190 function in the commercial software MATLAB,  $M$  can be calculated in each cycle.

### 191 3. Tensile fatigue test of HRB400 ribbed steel bars

192 Both the mechanical and magnetization properties of ribbed steel bars are affected by the  
193 composition, processing technology and geometric size. To verify the proposed model, HRB400  
194 ribbed steel bars, which are widely used in the construction of RC structures in China, were used in  
195 tensile fatigue tests. The mechanical and magnetic data of the specimens during the fatigue tests were  
196 measured and recorded in real time.

#### 197 3.1. Specimen preparation

198 The length and nominal diameter of the HRB400 ribbed steel bar used in the test are 400 mm  
199 and 14 mm, respectively, and the local geometric dimensions of the specimen are shown in Fig. 1(a).  
200 The chemical composition and mechanical properties of the material are shown in Table 1 and Table  
201 2. To avoid test failure caused by fracture at the clamping end due to the stress concentration, two  
202 protective aluminum layers with a thickness of 0.3 mm were wrapped at both ends of the specimen  
203 to reduce the stress concentration, as shown in Fig. 1(b).

204 Table 1. Chemical composition of the HRB400 ribbed steel bar. (wt%)

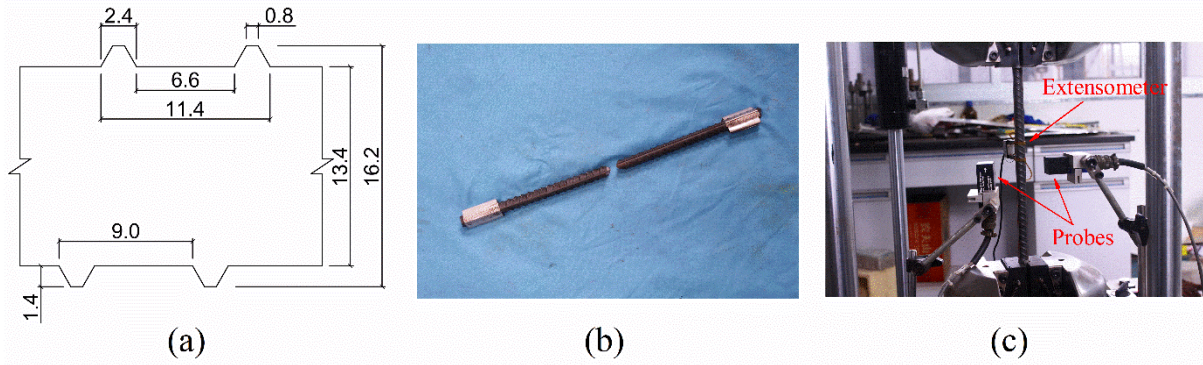
Element	C	Fe	Mn	Si	S	P	V
Wt	0.19	97.8	1.34	0.53	0.014	0.028	0.040

205

206 Table 2. Mechanical properties of the HRB400 ribbed steel bar.

Material	Young's modulus $E/\text{GPa}$	Yield Strength $\sigma_y/\text{MPa}$	Ultimate strength $\sigma_u/\text{MPa}$	Elongation $\delta/\%$
HRB400	207.29	425.17	581.20	25.77

207



209 Fig. 1. Details of the specimen and sensors: (a) geometry of the specimen (mm); (b) protective layers  
210 of the clamping end; (c) specimen and sensor installations.

211 *3.2. Loading and measurement*

212 An electrohydraulic servo-controlled fatigue testing machine performed tests with a peak  
213 capacity of 250 kN for sinusoidal wave loading at a loading frequency of 2 Hz. A CRIMS  
214 extensometer with a range of 20 mm was used to measure the strain. Two APS 428D fluxgate  
215 magnetometers were placed in the middle of the specimen, 33 mm away from the specimen's surface,  
216 to measure the normal magnetic induction and tangential magnetic induction with a range of 1G and  
217 accuracy of  $\pm 0.1\%$ , respectively. A CRONOS Compact 400-08 dynamic data acquisition instrument  
218 with a sampling frequency of 1000 Hz was used for the synchronous data acquisition of stress, strain  
219 and magnetic induction intensity. **Due to the great difference of magnetization properties and the  
220 fluctuation of environmental magnetic field, the magnetic signal of the specimens was zeroed out  
221 before the test.** The test device installations and the loading parameters are shown in Fig. 1(c) and  
222 Table 3, respectively.

223 Table 3. Parameters and results of the fatigue test.

224

Number	$S_{\min}$	$S_{\max}$	$\Delta\sigma$	$N_f$
TFB1	0.1	0.60	290.60	353586
TFB2	0.1	0.60	290.60	283553
TFB3	0.1	0.60	290.60	282566
TFB4	0.1	0.65	319.66	177239
TFB5	0.1	0.65	319.66	246789
TFB6	0.1	0.65	319.66	236897
TFB7	0.1	0.65	319.66	273019
TFB8	0.1	0.65	319.66	232575
TFB9	0.1	0.70	348.72	145077
TFB10	0.1	0.70	348.72	143913

Note:  $S_{\min} = \sigma_{\min}/\sigma_u$ ,  $S_{\max} = \sigma_{\max}/\sigma_u$ ,  $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$ , and  $\sigma_u$  is the ultimate strength.

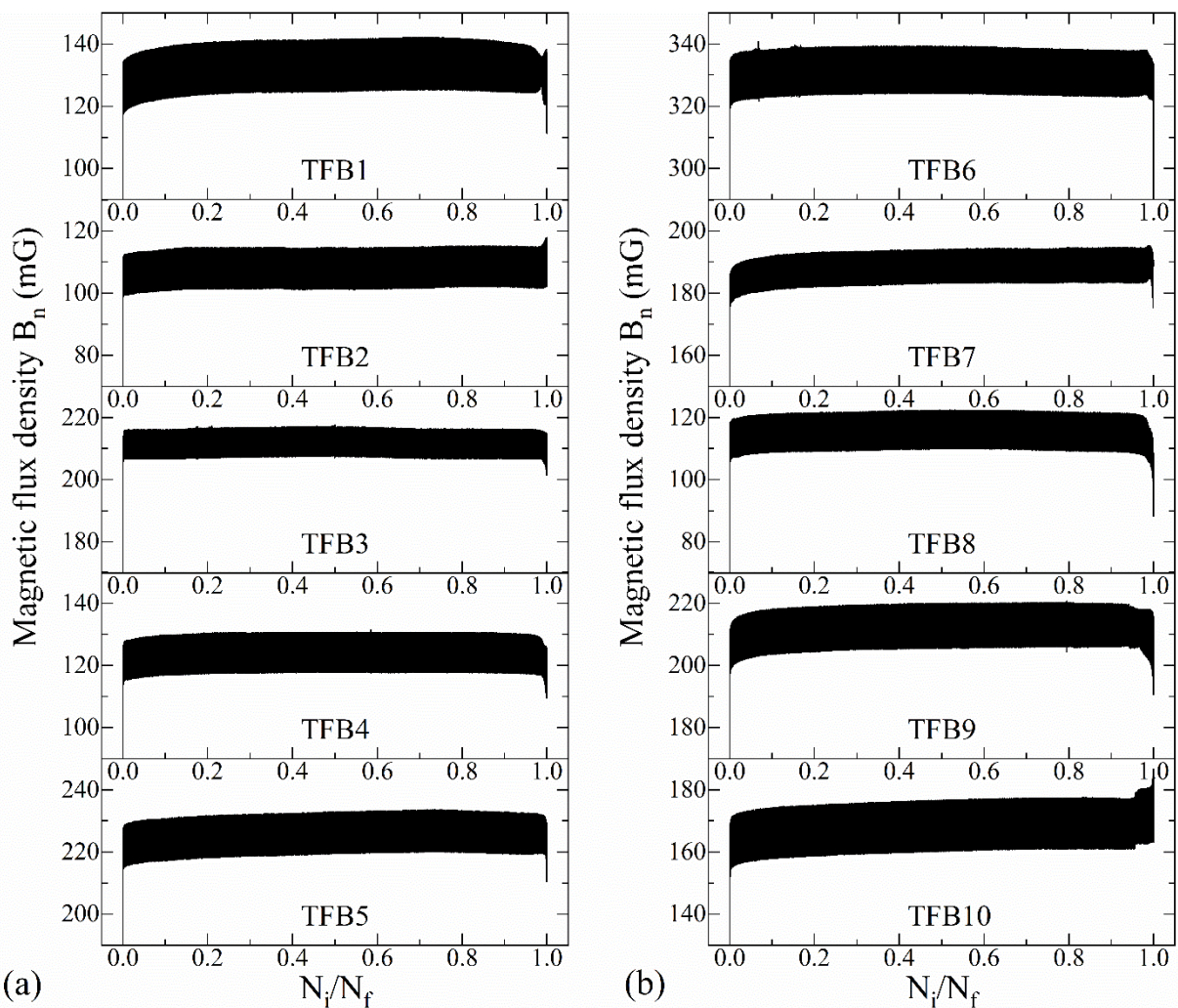
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226

#### 227 4. Analysis and discussion of the results

228 The failure mode of all specimens is fatigue fracture near the middle position. Table 3 shows the  
229 fatigue life  $N_f$  of each specimen. Due to the displacement of the specimen during the loading process,  
230 the relative position of the measuring point and the magnetic probe changed continuously, which will  
231 affect the measured magnetic induction intensity. The test results show that the disturbance amplitude  
232 of the normal magnetic induction is much smaller than that of the tangential magnetic induction, and  
233 the test results of normal magnetic induction have more obvious regularity. Therefore, the normal  
234 magnetic induction intensity  $B_n$  is mainly used for the following experimental analysis. The  $B_n$  of  
235 each specimen versus the number of loading cycles  $N$  is displayed in Fig. 2. Despite the differences  
236 in the loading ranges, the  $B_n$  of all specimens presented similar and systematic trends. At the initial  
237 stage, the  $B_n$  of each specimen rose rapidly and tended to a stable value. In the first half of the middle  
238 stage, the  $B_n$  of each specimen showed a steady increase, but the increase rate was very slow. In the  
239 second half of the middle stage, the  $B_n$  of some specimens dropped slightly, such as TFB1, 3, 5, 6,  
240 and 8. At the end of fatigue, the  $B_n$  of each specimen gradually accelerated, especially near fatigue  
241 fracture, where  $B_n$  varied dramatically. **The  $B_n$  of the specimens at this stage may increase or**

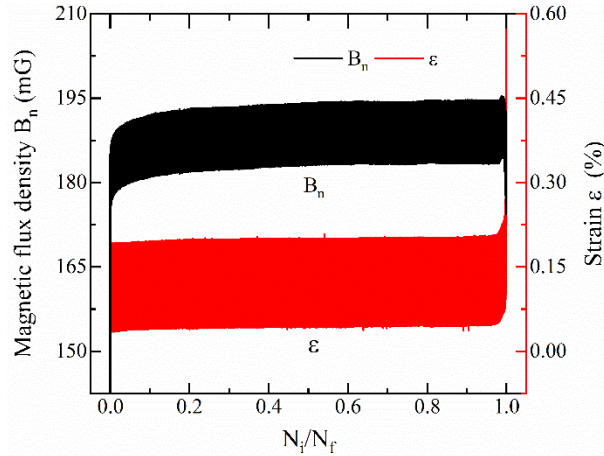
242 decrease, except for the  $B_n$  of TFB2 and TFB10, which significantly increased. The  $B_n$  of remaining  
 243 other specimens were reduced at the end of fatigue, and the reason for the difference in the variation  
 244 trend of specimens in the second half of the middle stage and last stage will be mentioned in the  
 245 following analysis. In addition, although the mechanical properties of ribbed bars are similar, their  
 246 magnetic properties are totally different, the magnitudes of the magnetic signal of these specimens  
 247 are very discrete. It is necessary to propose a reasonable evaluation method to characterize the  
 248 relationship between magnetic signal and fatigue, which will be discussed in the simulation section.



249  
 250 Fig. 2. Magnetic induction intensity curve in the fatigue test: (a) TFB1-5; (b) TFB6-10.  
 251

252 To analyze the characteristics of the magnetic signal more specifically, specimen TFB7 was  
 253 taken as a typical example. The  $B_n$  and strain  $\varepsilon$  of the specimen versus cycle  $N$  are given in Fig. 3. It

254 can be seen from the figure that  $B_n$  and  $\varepsilon$  show similar development trends. At the early stage of  
 255 fatigue, both of them rapidly reached a stable value. Subsequently, their growth velocity began to  
 256 decline but changed significantly before fatigue rupture.

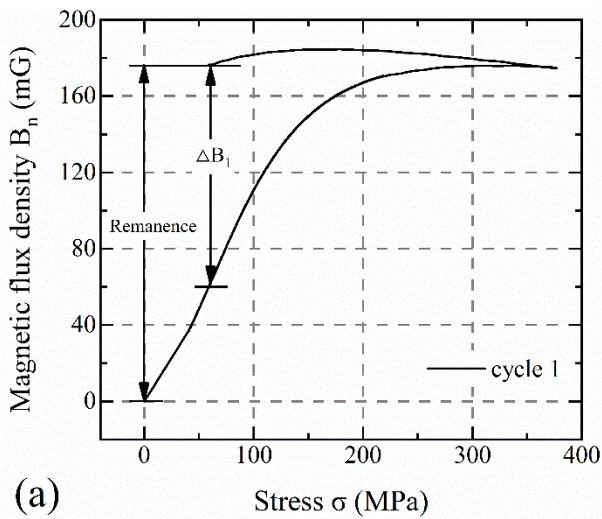


257  
 258 Fig. 3. Magnetic induction intensity curve and the strain curve of TFB7.

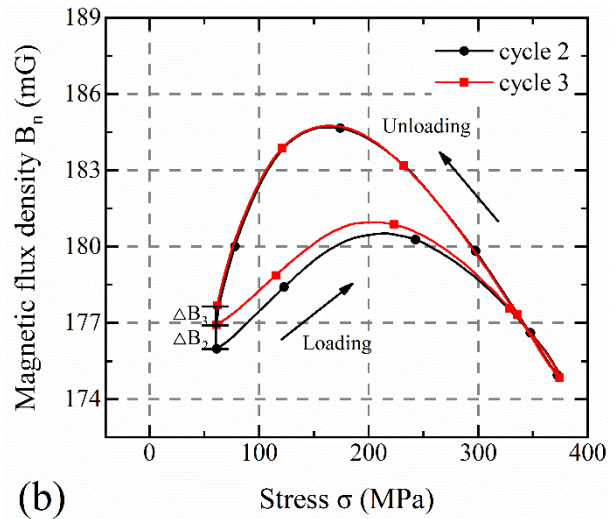
259  
 260 Fig. 4(a-b) displays the magnetic hysteresis curve of TFB7 in cycles 1-3. From Fig. 4(a), it can  
 261 be observed that the specimen left an obvious remanence with 176.0 mG when it was unloaded to  
 262  $\sigma_{min}$  for the first time, indicating that the  $B_n$  of the specimen is very sensitive to the loading history.  
 263 The magnetic hysteresis curve in cycles 2-3 is given in Fig. 4(b). Analyzing the difference value of  
 264  $B_n$  at the unloading point between adjacent cycles,  $\Delta B_1$ ,  $\Delta B_2$  and  $\Delta B_3$  are 118.8 mG, 0.9 mG and  
 265 0.4 mG, respectively, in the first three cycles, and it can be determined that the growth rate of  $B_n$   
 266 decreases as the number of cycles increases. The same trend is also applicable to the early and middle  
 267 stages of fatigue. The four characteristic hysteresis curves for cycles 10000-40000 in the early stage  
 268 of fatigue are shown in Fig. 4(c). The values of  $B_n$  at the unloading point in each cycle are 183.6 mG,  
 269 184.6 mG, 185.5 mG and 185.8 mG, and the corresponding growths are 1 mG, 0.9 mG and 0.3 mG,  
 270 respectively. Similarly, Fig. 4(d) gives the other four characteristic hysteresis curves for cycles  
 271 50000-200000 in the middle of fatigue. The values of  $B_n$  at the unloading point in each cycle are  
 272 186.1 mG, 187.0 mG, 187.7 mG and 188.1 mG, respectively. The average growth rate of  $B_n$  is



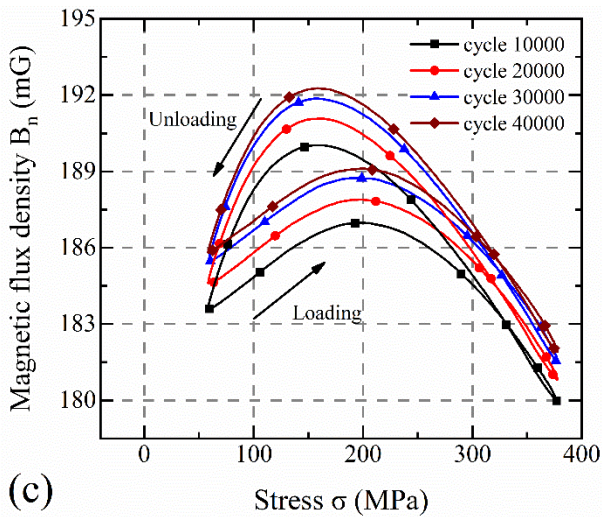
273 approximately 0.18 mG, 0.14 mG and 0.08 mG every 10000 cycles. Compared with the early stage  
 274 of fatigue, the growth rate of  $B_n$  was still declining until the last stage of fatigue. From the five  
 275 characteristic hysteresis curves for cycles 250000, 270000 and 273017-273019 at the end of fatigue  
 276 shown in Fig. 4(e), it can be observed that both the geometry and magnitude of the magnetic hysteresis  
 277 curves obviously varied, and the shape of the curves gradually changed from full to narrow and no  
 278 longer closed. At the same time, the change rate of  $B_n$  increased rapidly, and the value of  $B_n$  at the  
 279 unloading point decreased from 175.6 mG to 174.0 mG and finally decreased to 29.9 mG before  
 280 fatigue fracture.



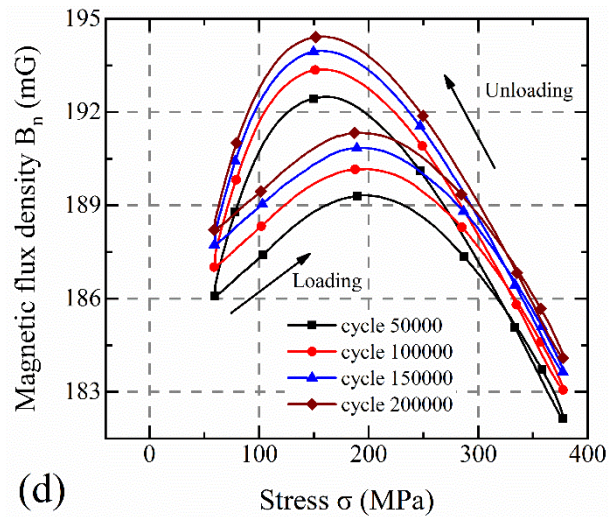
(a) Stress  $\sigma$  (MPa)



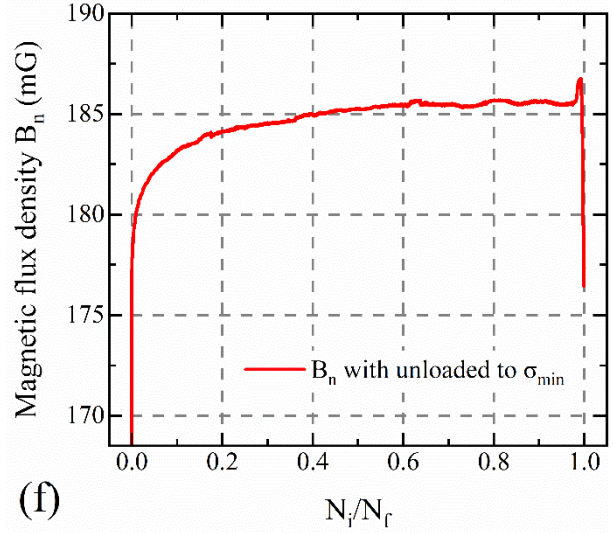
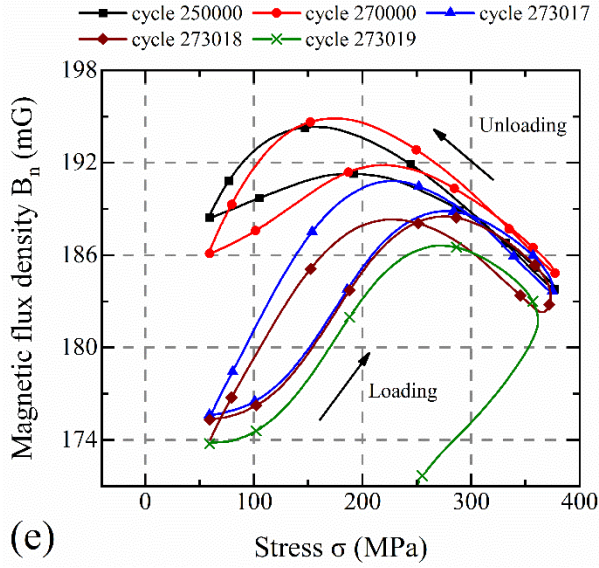
(b) Stress  $\sigma$  (MPa)



(c) Stress  $\sigma$  (MPa)



(d) Stress  $\sigma$  (MPa)



281 Fig. 4. Magnetic hysteresis loops of TFB7 during fatigue: (a) magnetic hysteresis loops for cycle 1;  
 282 (b) magnetic hysteresis loops for cycles 2-3; (c) magnetic hysteresis loops for cycles 10000, 20000,  
 283 30000 and 40000; (d) magnetic hysteresis loops for cycles 50000, 100000, 150000 and 200000; (e)  
 284 magnetic hysteresis loops for cycles 250000, 270000 and 273017-273019; (f)  $B_n$  unloaded to  $\sigma_{min}$  in  
 285 each cycle.  
 286

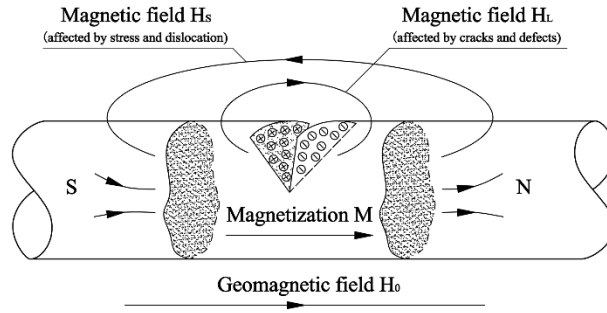
287 To facilitate the analysis of the variation trend of  $B_n$ , take  $B_n$  unloaded to  $\sigma_{min}$  in each cycle as  
 288 the characteristic point to obtain the  $B_n$  curve during fatigue, as shown in Fig. 4(f). The reasons for  
 289 the change in  $B_n$  will be analyzed in detail with the proposed model and the microscopic mechanism.  
 290 Before the specimen was loaded, the initial dislocation density was relatively low. This is because  
 291 the crystal grains of the material were intact, and the internal defects and cracks mainly came from  
 292 the production and processing processes. In the early stage of fatigue, due to the quick growth of the  
 293 plastic strain, the dislocation within the material increased rapidly to a stable state, and microcracks  
 294 began to initiate during this process<sup>29</sup>. In the middle stage of fatigue, the plastic strain grew slightly.  
 295 Since the dislocations reached a steady saturated state, the dislocation density also did not increase  
 296 obviously. However, the dislocation structure began to change significantly, and the slip band formed  
 297 and gradually stabilized<sup>30</sup>. As the fatigue progresses, accompanied by the formation and increase in  
 298 the primary and secondary slip bands, microcracks continue to develop. It can be seen from Eq. (11)

299 that the domain wall pinning and dislocations are proportional, and the dislocations of the specimens  
300 grow rapidly in the early fatigue stage. According to the magnetic domain wall pinning effect, the  
301 motion of the magnetic domains will be blocked by pinning. As a result, the growth rate of  $B_n$  dropped  
302 rapidly in the early fatigue stage. In the middle fatigue stage, the dislocations were in a stable state  
303 and changed slowly, so that the influence of domain wall pinning on the magnetization also showed  
304 a stable trend, which made  $B_n$  stable and varied slightly in the middle of the fatigue.

305 At the end of fatigue, the  $B_n$  of the specimen changed considerably, mainly affected by the  
306 leakage magnetic field at the cracks caused by fatigue damage. The direction and strength of these  
307 leakage magnetic fields are related to the location and size of the cracks. The dislocation density at  
308 this stage did not vary much, but the dislocation structure changed significantly. The microcracks  
309 grew and merged with dislocation slipping and aggregation, leading to the formation and propagation  
310 of macroscopic cracks<sup>31</sup>, which means that with the development of cracks in the last stage of fatigue,  
311 the influence of the magnetic flux leakage on  $B_n$  became more significant, and the magnetic leakage  
312 effect played a leading role in the variation in  $B_n$ . Fig. 5 gives a schematic diagram of the magnetic  
313 field composition of the specimen. The magnetic field around the specimen is mainly composed of  
314 an environmental magnetic field  $H_0$  and a magnetic field  $H_S$  caused by stress and a leakage magnetic  
315 field  $H_L$  at the crack. In the early stage and the first half of the middle stage, under the geomagnetic  
316 field and cyclic loads, the internal magnetization  $M$  of the material changed under the influence of  
317 dislocation magnetization, which caused the magnetic field  $H_S$  to change. At the end of fatigue, due  
318 to the continuous expansion of cracks caused by fatigue damage, many macroscopic cracks occurred  
319 inside and on the surface of the material. Magnetic charges accumulated at the cracks and generated  
320 leakage magnetic fields  $H_L$ . These leakage magnetic fields caused sudden changes in the local  
321 magnetic field at the cracks, which led to abrupt variations in  $B_n$  in the last fatigue stage. On the other  
322 hand, the  $B_n$  measured in the test is affected by the measurement position, magnetization properties  
323 of the material, crack position and geometric dimension. The dispersion of the magnetization



324 properties and the randomness of the crack location and size caused  $B_n$  to increase or decrease in the  
 325 last fatigue stage. In addition, due to the ribs, the specimens have stress concentrations, which  
 326 accelerate the development of fatigue cracks. Some specimens were affected by the leakage field at  
 327 cracks in the second half of the middle stage, so  $B_n$  did not increase steadily, but rather decreased  
 328 slightly.



329

330 Fig. 5. Magnetic field schematic diagram of the specimen at the end of fatigue.

331

### 332 5. Equivalent local stress and fatigue life considering stress concentration

333 Due to the stress concentration caused by ribs, not only will the fatigue life of ribbed steel bars  
 334 be reduced but the actual stress will also be changed, which affects the magnetic signal. The  
 335 equivalent local stress and fatigue life of ribbed steel bars under a nominal cyclic stress will be  
 336 determined by the Neuber law and the Coffin-Manson relation.

#### 337 5.1. Equivalent local stress and strain

338 The constitutive curve of ribbed steel bars under cyclic loading can be described by the  
 339 Ramberg-Osgood (R-O) model <sup>32</sup>.

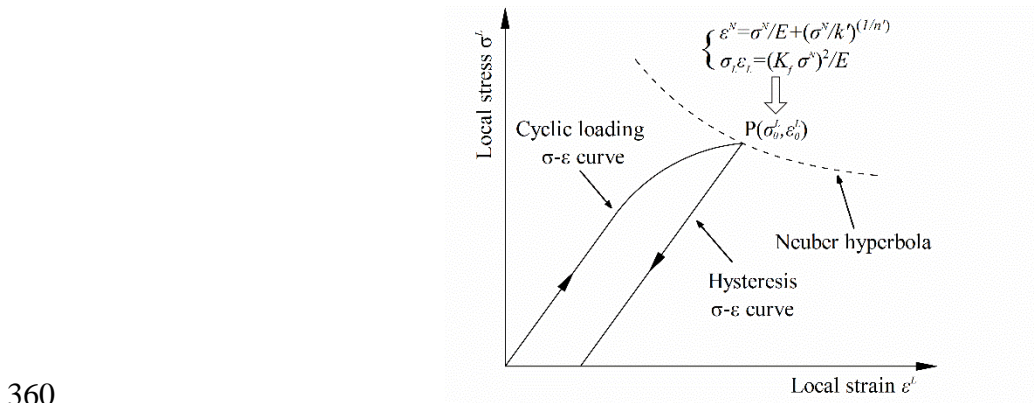
$$340 \quad \varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{k'}\right)^{\frac{1}{n'}} \quad (14)$$

341 where  $\sigma$  is the stress,  $\varepsilon$  is the strain,  $E$  is Young's modulus, and  $k'$  and  $n'$  are the cyclic strain  
 342 hardening coefficient and exponent, respectively.

343 Because the stress concentration mainly occurs in the ribs and its influence range is relatively  
 344 small, the overall constitutive relationship of ribbed steel bars still satisfies the R-O model, while the  
 345 stress and strain in the local area affected by the stress concentration are higher than the nominal  
 346 stress and strain, respectively. There are currently many methods that can be used to determine the  
 347 local stress and strain of ribbed steel bars, among which the Neuber law is most widely used, as shown  
 348 in Eq. (15)<sup>33</sup>. By introducing the stress concentration coefficient, the nominal stress and strain are  
 349 correlated with the equivalent local stress and strain. Substituting the maximum nominal stress  $\sigma_{max}^N$   
 350 and the minimum nominal stress  $\sigma_{min}^N$  into Eqs. (14) and (15), the equivalent local stress and strain  
 351 can be obtained  $(\sigma_{max}^L, \sigma_{min}^L, \varepsilon_{max}^L, \varepsilon_{min}^L)$ . Fig. 6 gives the specific relation diagram, in which the  
 352 Neuber equation can be approximated by a hyperbola, and the intersection point of the Neuber  
 353 equation and the R-O model is the equivalent local  $\sigma - \varepsilon$  point.

$$354 \quad \sigma^L \varepsilon^L = (K_f \sigma^N)^2 / E \quad (15)$$

355 where  $\sigma^L$  is the equivalent local stress,  $\varepsilon^L$  is the equivalent local strain,  $\sigma^N$  is the nominal stress, and  
 356  $K_f$  is the fatigue notch coefficient, which is affected by the notch geometry and material. Elrefai et  
 357 al.<sup>34</sup> believe that the  $K_f$  in RC beams under cyclic loading can be set at 2.0. Badawi et al.<sup>35</sup> suggested  
 358 that  $K_f$  could be set at 2.0 with a short fatigue life and 2.1 with a long fatigue life through fatigue  
 359 tests of RC beams.



360  
 361 Fig. 6. The intersection of the Neuber hyperbola and the cyclic stress-strain curve.

362 5.2. Fatigue life of ribbed steel bars

363 Ribbed steel bar fatigue fracture usually occurs in the rib, and the fatigue life is largely  
 364 influenced by the local stress and strain of the rib area. Based on the periodic  $\sigma - \varepsilon$  behavior of the  
 365 bars, the massing hypothesis can be used to determine the stable  $\sigma - \varepsilon$  hysteresis curve, and the local  
 366 stress range  $\Delta\sigma^L$  and strain range  $\Delta\varepsilon^L$  can be obtained by substituting the nominal stress range  $\Delta\sigma^N =$   
 367  $\sigma_{max}^N - \sigma_{min}^N$  into Eqs. (16) and (17).

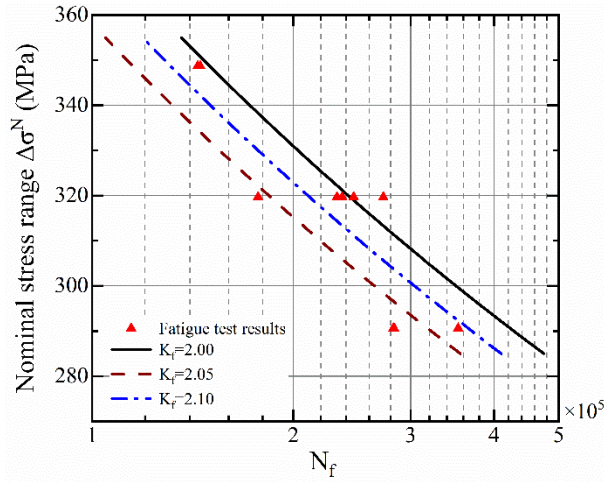
$$368 \quad \Delta\varepsilon^L = \frac{\Delta\sigma^L}{E} + 2\left(\frac{\Delta\sigma^L}{2k'}\right)^{\frac{1}{n'}} \quad (16)$$

$$369 \quad \Delta\sigma^L \Delta\varepsilon^L = (K_f \Delta\sigma^N)^2 / E \quad (17)$$

370 The Coffin-Manson relationship can be used to express the relationship between the strain range  
 371 and fatigue life, as given in Eq. (18)<sup>36, 37</sup>. The local strain range  $\Delta\sigma^L$  can be obtained by the Neuber  
 372 law mentioned above, and the coefficients  $\sigma'_f$ ,  $\varepsilon'_f$ ,  $b$  and  $c$  are usually determined by fatigue tests on  
 373 smooth specimens processed by ribbed steel bars. Jin et al.<sup>38</sup> performed a fatigue test on smooth  
 374 specimens processed with HRB400 steel bars and obtained  $\sigma'_f = 1368.01$  MPa,  $\varepsilon'_f = 0.313$ ,  $b =$   
 375  $-0.124$ , and  $c = -0.485$ . In addition, the coefficients  $k'$  and  $n'$  in the RO model can be  
 376 approximated by the parameters of the Coffin-Manson relationship<sup>39</sup>, where  $n' = b/c$ ,  $k' =$   
 377  $\sigma'_f / (\varepsilon_f'^{n'})$ . Through the Neuber law and the Coffin-Manson relationship, the fatigue life of specimens  
 378 under different nominal stress ranges  $\Delta\sigma^N$  and the fatigue notch coefficients  $K_f$  can be obtained, as  
 379 shown in Fig. 7. It can be determined that the calculation results are closer to the test results with  
 380  $K_f = 2.05$ , so the following numerical simulation analysis will take  $K_f = 2.05$ .

$$381 \quad \frac{\Delta\varepsilon^L}{2} = \frac{\sigma'_f}{E_s} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (18)$$

382 where  $\sigma'_f$  is the fatigue strength coefficient,  $\varepsilon'_f$  is the fatigue ductility coefficient,  $b$  is the fatigue  
 383 strength index,  $c$  is the fatigue ductility index, and  $N_f$  is the fatigue life.



384

385 Fig. 7. Stress range versus fatigue life of the ribbed steel bars.

## 386 6. Numerical simulation and analysis

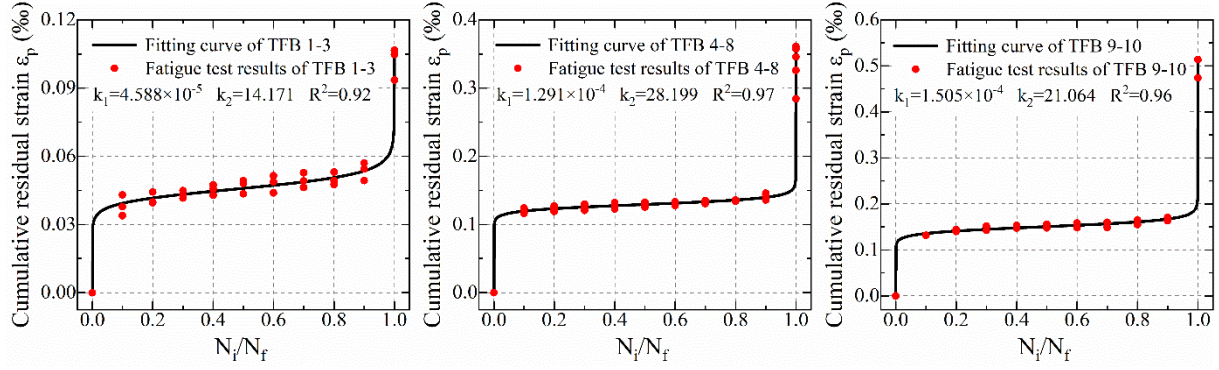
### 387 6.1. Determining the residual plastic strain of ribbed steel bars

388 According to Eqs. (11) and (12), it can be seen that the magnetization state of ribbed steel bars  
 389 is affected by domain wall pinning, which can be characterized by plastic strain. Under cyclic loads,  
 390 the domain wall pinning will vary constantly, which is difficult to simulate and analyze. To facilitate  
 391 the simulation, the domain wall pinning coefficient  $k$  of each cycle is taken as a fixed value, which  
 392 is calculated by the accumulated residual strain  $\varepsilon_p$  in each cycle. Since the variation trend of the  
 393 magnetization in the proposed model is mainly affected by the development trend of plastic strain,  
 394 the fitting formula of residual strain that satisfies the three-stage fatigue development trend given by  
 395 Ouyang et al. <sup>40</sup> in the tensile fatigue test of ribbed steel bars is used to fit the test data. The fitting  
 396 results of the plastic strain of the specimens with different loading levels are shown in Fig. 8. Since  
 397 the stress at the unloading point of the test is  $0.1\sigma_u$ , the cumulative residual strain is taken as  
 398  $\varepsilon_p(n) = \varepsilon_{min} - 0.1\sigma_u/E$ .

399

$$\varepsilon_p(n) = k_1 \left( \frac{1}{1 - \frac{n}{N_f}} - 1 \right)^{\frac{1}{k_2}} \quad (20)$$

400 where  $\varepsilon_p(n)$  is the cumulative plastic strain after  $n$  cycles and  $k_1$  and  $k_2$  are the fitting coefficients.



401

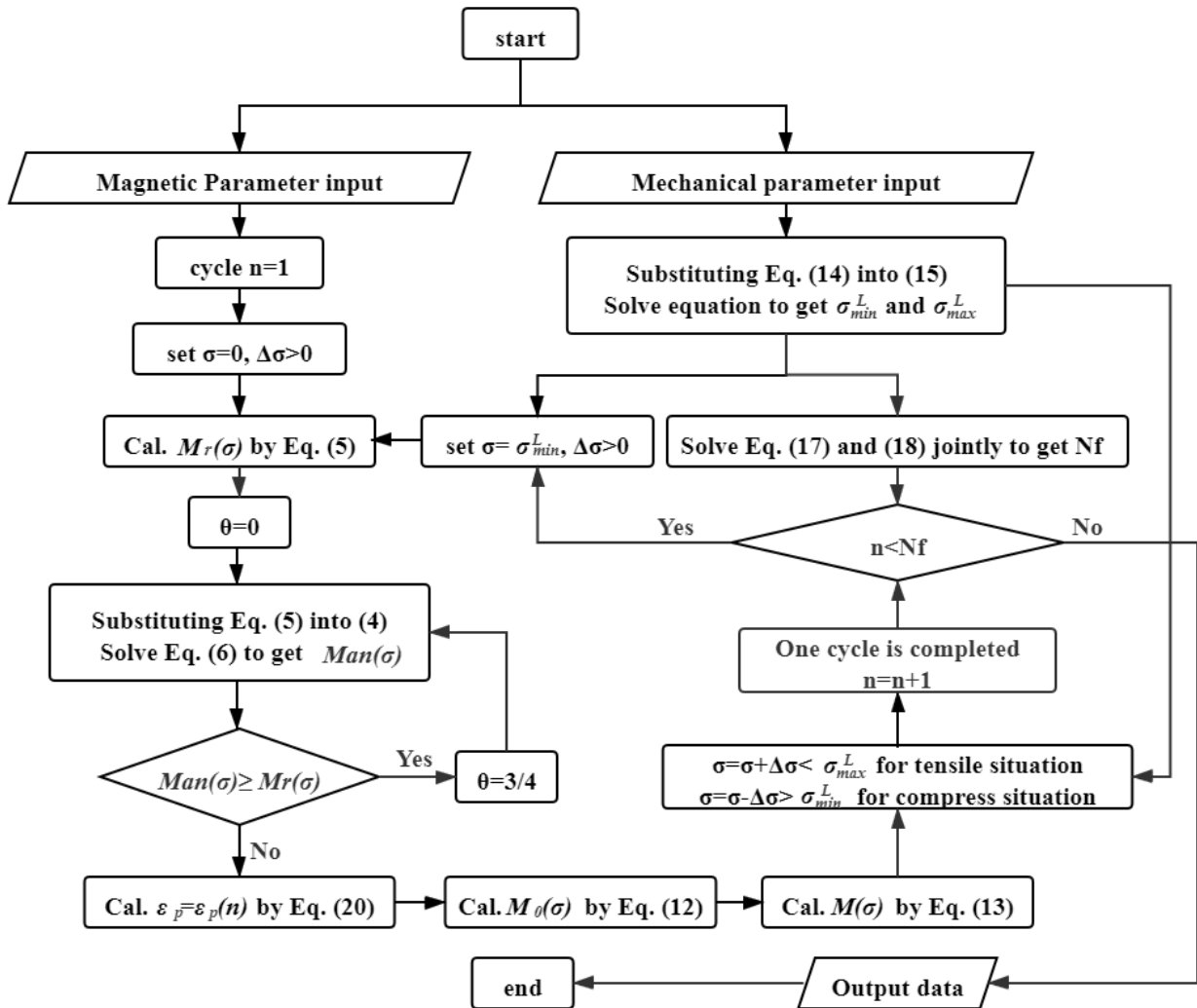
402 Fig. 8. Cumulative residual strain fitting curve of the specimens.

403

#### 404 6.2. Numerical simulation of the magnetization of ribbed steel bars

405 Because the magnetization performance of specimens is mainly affected by the material, partial  
 406 parameters of the simulation are determined by the existing references to make the simulation results  
 407 more reliable. The geomagnetic field  $H_0$  can be considered as 40 A/m. The main component of the  
 408 ribbed steel bars is mild steel, which can be regarded as a soft ferromagnetic material rod. Zhou et al.  
 409 <sup>25</sup> and Shi et al. <sup>22</sup> gave the simulation parameters of the soft ferromagnetic material rod,  $N_d = 1.0 \times$   
 410  $10^{-3}$ ,  $\lambda_s = 4.17 \times 10^{-6}$ ,  $M_s = 1.5 \times 10^6$  A/m,  $M_{ws} = 1.0 \times 10^6$  A/m, and  $\beta = 1.5$ . The simulation  
 411 parameters of mild steel given by Jiles et al. <sup>13</sup> are  $\alpha = 1.1 \times 10^{-3}$ ,  $\xi = 605$ ,  $a = 0.9 \times 10^3$ ,  $c =$   
 412  $0.1$ . The remaining parameters are taken from the simulation results of Huang et al. <sup>23</sup> on smooth  
 413 specimens processed by HRB400 steel bars,  $\sigma_s = 400$  MPa,  $A = 1.0 \times 10^{10}$ , and  $n_0 = 1.0 \times 10^{-4}$ .  
 414 Substituting the equivalent local stresses  $\sigma_{max}^L$  and  $\sigma_{min}^L$  obtained by the Neuber law, the fatigue life  
 415  $N_f$  obtained from the Coffin-Manson relationship, and the cumulative residual strain  $\varepsilon_p(n)$  of each  
 416 cycle obtained by fitting into the proposed model, the magnetization  $M$  of the specimen in each cycle

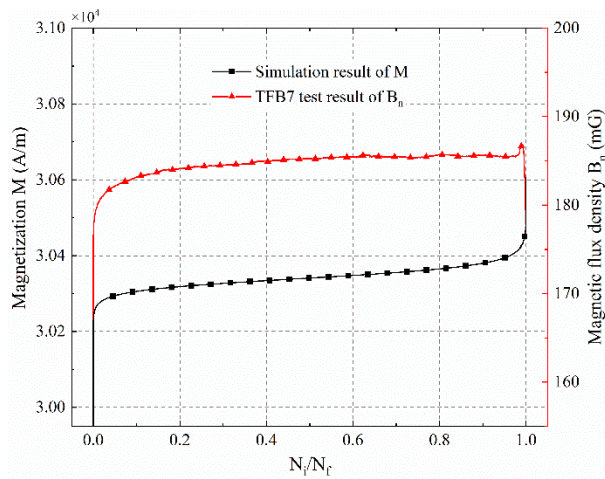
417 can be calculated. The magnetization analysis process of ribbed steel bar under cyclic loading is  
 418 shown in Fig. 9.



419  
 420 Fig. 9. The magnetization analysis process of ribbed steel bar under cyclic loading.

421 The simulation results  $M$  and the test results  $B_n$  unloaded to the minimum stress point in each  
 422 cycle are shown in Fig. 10. It can be observed that the development trends of the test curve and the  
 423 simulation curve are similar in the early and middle stages of fatigue.  $B_n$  increased rapidly in the early  
 424 stage, and the growth rate of  $B_n$  continued to decrease until fatigue failure, which satisfies the  
 425 description of the approach law. However, the change trends of the simulation curve and the test  
 426 curve are quite different. The reason is that the proposed model only considers the magneto-  
 427 mechanical effects in fatigue. At the end of fatigue, the magnetic flux leakage effect played a leading

428 role in  $B_n$ , which caused the magnetic induction intensity at the measuring point to decrease at this  
 429 stage and drop dramatically before the fracture. According to the proposed model,  $M$  will  
 430 continuously approach  $M_0$  during fatigue. It can be seen from Eq. (13) that the change in  
 431 magnetization with stress  $dM/d\sigma$  is related to the value of  $M_0 - M$ . Because the specimen was not  
 432 stressed before loading, the difference between  $M_0$  and  $M$  was large. In the early stage of fatigue, the  
 433  $M$  of the specimen quickly approached  $M_0$ . In the subsequent fatigue process,  $M$  gradually reached  
 434  $M_0$ , and the value of  $M_0 - M$  declined, resulting in a decrease in the growth rate of  $M$ . In addition,  
 435 the magnitude of  $M$  is very different from that of  $B_n$  measured by the test. The main reason is that the  
 436 simulation is the internal magnetization of the material, the magnetic induction measured by the test  
 437 is affected by the measurement position, and its magnitude will be much smaller than the internal  
 438 magnetization of the material.



439  
 440 Fig. 10. Magnetic flux density from the experiment and magnetization from the numerical simulation  
 441 of TFB7.

442

### 443 6.3. Fatigue damage formula of the magnetic indicators

444 To reduce the influence of the magnetic flux leakage effect on the magnetic induction intensity  
 445 of specimens, the following analysis focuses on the first two stages of fatigue ( $0 < N \leq 0.85N_f$ ),  
 446 where the magnetic leakage effect has a relatively small influence. At the same time, to eliminate the

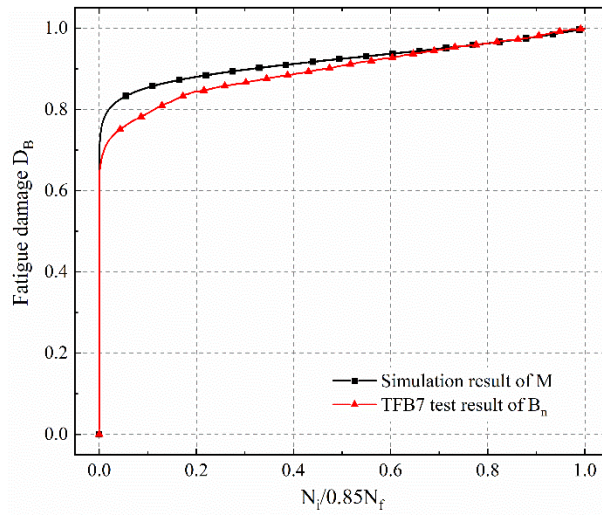
447 magnitude difference between the simulated magnetization  $M$  and the measured magnetic induction  
 448 intensity  $B_n$ , the fatigue damage formula based on the magnetic indicators<sup>4</sup> is introduced to verify  
 449 the accuracy of the proposed model. Considering that the  $B_n$  of some specimens in the middle and  
 450 final stages slightly decreased (TFB1, 3, 5, 6, and 8), the decreased part of  $B_n$  still reflected the fatigue  
 451 damage, and the absolute value of the  $B_n$  variation was taken to calculate the fatigue damage.

$$452 \quad D_B(N) = \frac{\sum_{i=1}^N |\Delta B_i|}{\sum_{i=1}^{0.85N_f} |\Delta B_i|} \quad (19)$$

453 where  $\Delta B_i = B(i) - B(i - 1)$  is the difference in  $B_n$  at the characteristic points of cycle  $i$  and cycle  
 454  $i - 1$ , and  $\sum_{i=1}^N |\Delta B_i|$  and  $\sum_{i=1}^{0.85N_f} |\Delta B_i|$  are the cumulative absolute values of the  $B_n$  variation in  
 455 cycles  $1 - N$  ( $1 \leq N \leq 0.85N_f$ ) and cycles  $1 - 0.85N_f$ , respectively.

456 The fatigue damage curve with the simulation and test data of TFB7 calculated by Eq. (19) are  
 457 shown in Fig. 11. The two curves have similar variation trends, showing nonlinear rapid growth in  
 458 the early stage of fatigue and an almost linear growth in the middle stage of fatigue. The simulated  
 459 fatigue damage curve, which is slightly higher than the test curve overall, shows that the simulated  
 460 results are more conservative than the test results. The fatigue damage comparison results of the  
 461 remaining specimens also reflect the same law. Fig. 12(a-c) shows the fatigue damage curve under  
 462 the three sets of loading levels. In addition to the above rules, for specimens TFB1, 3, 5, 6, and 8,  
 463 whose  $B_n$  decreased slightly in the second half of the middle stage, it can be determined from the  
 464 partially enlarged curve that the slight decrease in  $B_n$  caused the fatigue damage curve to produce a  
 465 slope turning point. After the turning point, the slope of the fatigue damage curve increases, and  
 466 fatigue damage develops more rapidly. It has been explained above that the decrease in  $B_n$  may be  
 467 caused by the leakage field at the fatigue crack. The accelerated development of the fatigue damage  
 468 caused by fatigue cracks will also be reflected in the measured magnetic signal, which proves the  
 469 high sensitivity of the magnetic signal to microscopic damage.

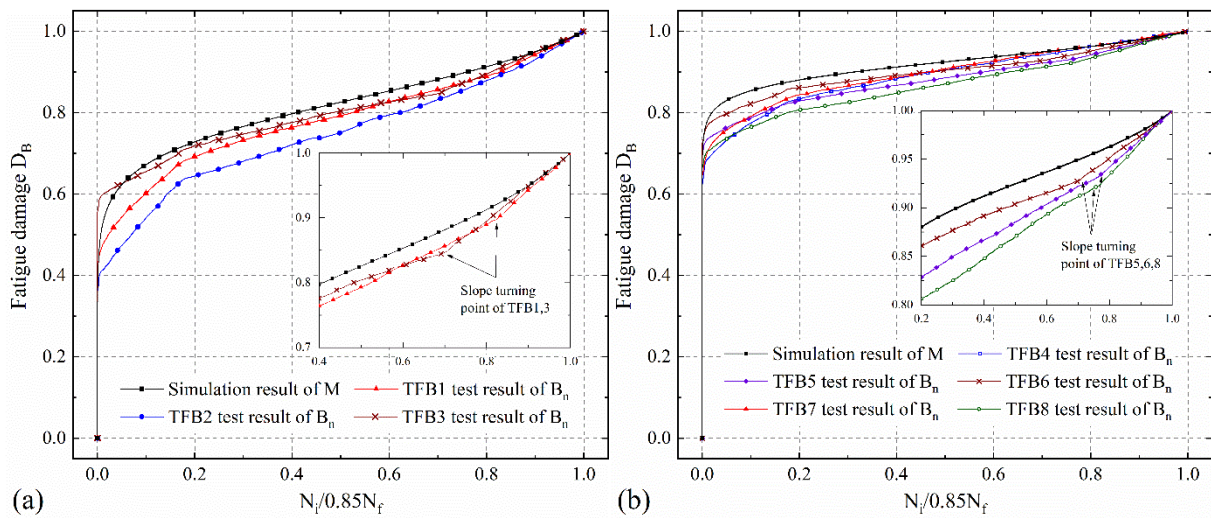




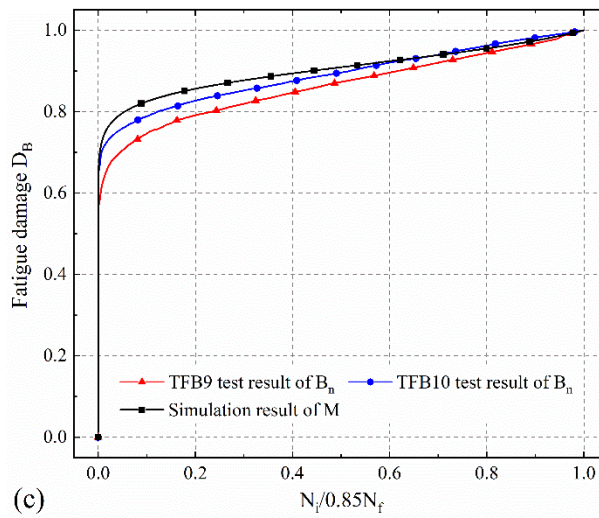
470

471 Fig. 11. Fatigue damage curves based on experimental data and simulated results of TFB7.

472



473



474

475 Fig. 12. Fatigue damage curves based on experimental data and simulated results: (a) TFB1-3; (b)  
476 TFB4-8; (c) TFB9-10.  
477

## 478 7. Conclusion

479 Considering the influence of the stress concentration at the ribs of steel bars, the equivalent stress  
480 and the fatigue life of the ribbed steel bars were determined by the Neuber law and Coffin-Manson  
481 relationship, and a magneto-mechanical model of the ribbed steel bar was established to predict the  
482 magnetization during fatigue. In addition, the cumulative plastic strain fitting formula that reflects  
483 the three-stage fatigue trend was used so the simulation parameters would no longer depend on the  
484 measured test data. According to the magnetic induction intensity measured by the tensile fatigue test  
485 of the HRB400 ribbed steel bars, the relationship between the magnetic induction intensity and the  
486 fatigue damage was discussed from the perspective of micro-mechanisms. **The magnetization of the  
487 specimen conforms to the description of the proposed model. When there is no macro crack, the  
488 growth rate of magnetization decreases continuously, which is mainly affected by pinning related to  
489 plastic deformation. And the growth trend of magnetic signal can be used to predict the macroscopic  
490 damage propagation.** Comparing the test and simulation results calculated by the fatigue damage  
491 formula based on the magnetic indicator, the results showed that the proposed model could effectively  
492 describe the fatigue damage state of ribbed steel during fatigue, which verifies the applicability and  
493 accuracy of the model. These results are very helpful in understanding the magneto-mechanical  
494 behavior of ribbed steel bars in RC beams during fatigue, which is of great significance to fatigue  
495 damage monitoring in actual engineering.

496

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