Redevelopment Strategies and Building Ages

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Abstract: An improved real option pricing model that incorporates the depreciation effect for two-stage redevelopment projects will be demonstrated to estimate the redevelopment project values in a wide range of scenarios. Based on a stochastic differential equation, numerical analyses will focus on new factors that influence the expected exercise time, that is, depreciation rate, the annual increase in average building age, building age of the targeted property, and the average building ages in the same region. The depreciation rate is the most influential factor in this model. More importantly, this study will summarize three exercise strategies that cover all combination of the parameters. In some cases, with a low capital return rate, or high depreciation rate, or both, the traditional simultaneous exercise strategy based on the optimal demolition price-to-cost ratio is not feasible. Instead, either a sequential exercise or simultaneous exercise that is based on the optimal rebuilding price-to-cost ratio is the best choice. Detailed procedures on how to adopt the best strategy will be demonstrated. The acquisition price of the old property proved to be sensitive to the depreciation rate and the capital return rate within a certain range. To apply this model properly, the feasible time interval for the traditional simultaneous exercise strategy and the depreciation rate should be estimated accurately. DOI: 10.1061/(ASCE)UP.1943-5444.0000686.

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Introduction

Redevelopment is one of the effective urban renewal measures to achieve the goals of sustainable urban development and land use efficiency (Chan and Yung 2004). As a city grows to a certain scale, limited vacant land is available for new developments. In addition, this problem is urgent in Hong Kong. Residential and commercial building stocks will become insufficient if only new developments are relied on, which threatens the city's sustainability (Ho et al. 2012). Urban renewal activities, which include redevelopment, rehabilitation, regeneration, and preservation, are major measures that ensure housing supply in highly developed cities. Among these measures, redevelopment is sometimes required to replace the old-fashioned, low-density buildings. Excluding a large proportion of old buildings, which could be repaired to extend their lives, for the remaining buildings, a problem exists: what is the optimal timing to start redevelopment when the future market price is uncertain.

The real option approach is applied to solve this problem, to determine the redevelopment option value or the optimal exercise strategy (Capozza and Sick 1991; Rosenthal and Helsley 1994; Williams 1997; Sing and Lim 2004; Hui and Ng 2008; Chen and Lai 2013). However, few pricing models have addressed the depreciation effect of building structures during the option period. The property value is composed of the building structure value and the value of the land on which the property is built. The building structure value of a new building depreciates on its completion. This depreciation effect is crucial for multiple ownership buildings, because the residents in this type of property usually have separated ownership and seldom consider redeveloping these buildings themselves. Then, the depreciation effect increases the price difference between old properties to be demolished and new ones to be rebuilt. A reason to introduce the depreciation effect is the diversification of acquisition standards in actual projects. Redevelopers tend to acquire old properties at their current value and the residents aim to sell their properties at a higher value, which includes the redevelopment potential (Yau and Chan 2008). The negotiation between redevelopers and residents becomes more complicated when the number of owners is large, or if some residents ask for increased potential appreciation due to redevelopment. This negotiation process depends on the acquisition standards for old properties.

Because the development history, population density, land ownership system, economic conditions, and building aging conditions are not the same in different cities and different countries, the acquisition (or compensation) standards implemented by different governments are diverse. The Urban Renewal Authority in Hong Kong summarized related policies in several Asian cities (Law et al. 2009). In Japan and South Korea, where land is usually freehold, land right conversion, replacement lots with discounted prices (or even for free), and monetary compensation are major compensation methods. In mainland China, where land is state-owned, monetary compensation that is based on prices adjusted from market value is a major method. In addition, Hong Kong offers a flat-to-flat compensation scheme that allows residents to purchase relocation flats that have fixed prices.

Although the policies for acquisitions diverge significantly in these examples, monetary compensation that is based on the market prices of old or new properties is the most common policy. Then, the acquisition price should be determined by the uncertain market price and predictable depreciation status. To compare the policy influence on redevelopment timing, an improved real options model with depreciation effect is required. This continuous-time model should include the concept of average building age in the surrounding region and should account for the impact of previous developments in the same region (Zhong and Hui 2020).

Another reason to include the depreciation effect is the availability of new property price index. In mainland China, the statistical department offers the newly built property price index and the second-hand property price index. The depreciation effect could be completely removed when estimating the future value of a new property. In Singapore, Private Residential Properties Price Index (released by Urban

Redevelopment Authority, 2021) includes new units that are sold by developers. The basic model by Chen and Lai (2013) could be adopted directly in these cases.

However, in some property markets, the proportion of newly built properties is very low. For example, single-family housing stock grew <20% from 2000 to 2010 in 13 out of the 20 major cities in the US, based on US Census Bureau data (S&P Dow Jones Indices 2017). The most popular property indices in the US, S&P CoreLogic Case-Shiller Home Price Indices, announced that the sample sale prices excluded new construction cases. Some other examples, such as House Price Indices in the US (published by Office of Federal Housing Enterprise Oversight), House Price Indices in the UK (published by Land Registry, Land and Property Services Northern Ireland, Office for National Statistics and Registers of Scotland), Private Domestic Price Indices in Hong Kong (published by Rating and Valuation Department 2017) and Centa-City Indices in Hong Kong (published by Centaline Property Agency Limited) include either old properties as the majority, or only include old properties. The basic model might not be used based on these indices.

One solution is to reconstruct an age-effect-adjusted property index to capture the market return rate and volatility for option pricing. However, this would require a large number of original transaction records and a strict calculation standard to guarantee the reliability, which would incur significant time and money costs for a developer. Another approach would be to improve the option pricing model to adjust the depreciation effect in an approximate way, which is the objective of this study.

The contents of this study is structured as follows. The next section, "Literature Review," is a summary of the major findings in real option models that were applied for redevelopment, and depreciation effects related to redevelopment option value. The section that follows, "Methodology," provides details of the model construction, solutions for continuous-time option pricing models with depreciation effects, and model properties on the influential factors. In "Numerical Analysis," the model properties were verified through a series of numerical analyses. Finally, the concluding section summarizes the major findings, implications for policymakers and investors, and potential improvements in future studies.

Literature Review

A real option approach and depreciation effect are major components in this study of redevelopment projects. Before the development of formal models, the redevelopment condition was proposed by Brueckner (1980) and Wheaton (1982), which is satisfied when the potential income from a rebuilt property exceeds the present income from an old property. Munneke (1996) provided evidence to support this condition. The crucial problem in the real options approach is choosing the appropriate model assumptions based on targeted scenarios.

The first important research into the real option in the real estate field was conducted by Titman (1985). Titman highlighted that the uncertainty in future revenue would reduce the incentive to construct in the current period. Then, 10 years after Titman's work, Capozza and Helsley (1990), Capozza and Sick (1991, 1994), Williams (1991), Quigg (1993), Capozza and Li (1994), Rosenthal and Helsley (1994), and Williams (1997) developed different real options models that could be applied to specific development or redevelopment situations.

The well-known continuous-time development models were developed by Williams (1991) and Quigg (1993). In their models, property price and construction costs follow geometric Brownian motions. In addition, building density was a variable. The optimal exercise strategy included the optimal price-to-cost ratio and optimal building density, to maximize project profits. This strategy was extended to a two-stage redevelopment model by Chen and Lai (2013), which investigated the impact of potential factors on the expected exercise time. The major contribution was that the optimal strategy for an infinite-time redevelopment option was to exercise demolition and rebuild options simultaneously. In addition, this study discussed the finite-time scenarios.

The mainstream real option research into real estate redevelopment was mainly based on the previous models and achieved various empirical findings (Sing and Lim 2004; Hui and Ng 2008; Ho et al. 2009; Hui et al. 2011; Downing and Wallace 2002; McMillen and O'Sullivan 2013; Krause 2015). However, these studies did not take the unit value difference between new and old properties in the same location into account, which was caused by the depreciation effect. The old property value, average market value, and new property value were assumed to change synchronously at the same rate. This assumption might indicate that redevelopment was due to the land value increase and was related to the aging of buildings.

The main reason to take the depreciation effect into account is that the depreciation of building structure and the appreciation of land value are two different subjects. When the property market expands due to an increase in housing demand, the aging building structure will cause higher maintenance cost and then a lower net cash flow from renters. Williams (1997) focused on the deterioration of building structure during redevelopment. This research discussed the multiple redevelopment cases and assumed that property quality depreciated at a constant rate after each redevelopment. The actual rent was a product of exogenous rent and property quality. The redevelopment occurred when the quality decreased to a predetermined level. Therefore, when the property value decreases to a certain percentage of the value of a new property with the same structural, location, and other factors, it will be demolished and rebuilt. The quality of building to be redeveloped and rebuilt are assumed to be variables when maximizing the project value. This model assumed that the owner made the redevelopment decision based on the residual value of the property, instead of a profit maximization standard.

Dixit and Pindyck (1994) proposed a depreciation model in corporate finance; however, this study does not assume a survival function for a property as they did for a machine. Compared with machines in a factory, the life expectancy of a property is usually much longer. A small portion of buildings must be replaced within a certain period. Except for this case, the holder of a redevelopment option seldom accounts for the survival function of a property. Without the survival function assumption, the derivation of a real option pricing model will be different.

Some theoretical and empirical studies have proved the existence of this type of depreciation (Hulten and Wykoff 1981; Baum and McElhinney 2000; Jeffrey et al. 2005; Bokhari and Geltner 2018; Francke and van de Minne 2017). Based on Rosen's (1974) basic hedonic model, new hedonic pricing models that are combined with redevelopment options or special terms related to age effects have been developed (Clapp and Giaccotto 1998; Clapp and Salavei 2010; Clapp et al. 2010, 2012a, b, 2013; Munneke and Womack 2013). Exponentially depreciated hedonic factors (Clapp and Salavei 2010; Munneke and Womack 2013), and the impact of building age on redevelopment option values (Clapp et al. 2012a, 2013), provided explanations on the downward bias that exists in the traditional hedonic pricing model when valuing an old property. In these studies, finite-time models focus on the factors that influence compound option values and infinite-time models focus on factors that influence optimal exercise timing. For the latter case, the volatility of market price and construction cost, risk-free interest rate, capitalization rate (or average rent rate), maximum density, the correlation coefficient between market price and construction cost, the depreciation rate, the price elasticity of scale, the cost elasticity of scale, and the redevelopment limitation in density or timing have demonstrated their impact on the optimal exercise timing in different models (Williams 1991; Capozza and Li 1994; Williams 1997; Cunningham 2006; Hui and Ng 2008; Bulan et al. 2009; Hui et al. 2011; Chen and Lai 2013).

However, these models did not explore the inside structure of redevelopment options. Therefore, these models only investigated the existing influences that were caused by the depreciation effect. Neither the redevelopment option value nor its expected exercise time could be predicted if some market conditions were changed in the future.

To address the knowledge gaps in previous research, this study will embed the property value depreciation term into a continuoustime real options model that is based on a stochastic differential equation. A profit maximization standard is chosen to reflect a developer's rational behaviors. An infinite-time real option is developed to discuss the potential influence of different economic factors on redevelopment decisions by changing the values of these factors. Therefore, the investigation into the depreciation effect focuses on its influence on the expected exercise time, instead of the real option value. However, it is impossible to derive the value for a permanent US option using a backward algorithm. There are two major concepts in Zhong and Hui's (2020) work: (1) constant depreciation rate that affects the demolition and rebuild option; and (2) annual increase of average building age in the same region, which is applied to develop a new continuous-time model. A general solution is generated for all possible exercise strategies. In addition, the proposed new model is used to compare the expected exercise time under different acquisition standards for old properties.

Methodology

Model Assumptions

A redevelopment project consists of two stages. According to the traditional continuous-time real option frameworks (Williams 1991; Quigg 1993; Dixit and Pindyck 1994), in both stages, the unit market price S(t) and the unit construction cost K(t) follow geometric Brownian motion.

Assume that the option maturities in both stages are infinite. This assumption suggested a suitable time, which ensured that the optimal exercise timing was not bound by the option maturities.

First, the developer acquires an old property from the original owners and then demolishes it. Then, a new building is constructed on vacant land. In addition, the demolition of the targeted old property and the rebuilding of the new one does not change the market demand. To simplify, the time to demolish old properties and to rebuild new ones was assumed to be close to zero and was ignored.

In Hong Kong, the government does not require the developer to start the rebuilding process within a predetermined period when the redevelopment project is not directed by the government. Therefore, an infinite-maturity real option could be constructed to achieve the optimal strategy in this example and could be applied to other cities with the same instructions.

Assume that S(t) is a stochastic variable, which has a constant and risk-neutral growth rate (or drift rate) (ν_S) and constant variance (σ_S). The risk-neutral growth rate is a theoretical measure. It is used to derive the option value of equilibrium between different investors, such as risk-averse investors and risk-loving investors. The risk-neutral growth rate is then an expected value of the future growth rate. S(t) follows a geometric Brownian motion. This is the most classical assumption to describe the random changes in market prices, with a fixed drift term and a random term, which follows Brownian motion. The geometric Brownian motion is written as

$$dS(t) = \nu_S S dt + \sigma_S S dZ_S \qquad (1)$$

where Z_S =a Wiener process. It satisfies $E(dZ_S)=0$ and $Var(dZ_S)= dt$.

Similarly, K(t) follows a geometric Brownian motion as:

$$dK(t) = \nu_K K dt + \sigma_K K dZ_K \qquad (2)$$

The correlation coefficient between Z_S and Z_K =ρ. $Cov(Z_S, Z_K)= \rho\sigma_S\sigma_K$

Then, define the rate of risk-adjusted capital return as r(r>0). This is constant and risk-adjusted. To prevent the option value from being unbounded, assume $\nu_S < r$ and $\nu_K < r$.

Then, define the annual increase in average building age in the same region y($0 \le y \le 1$). Redevelopments of old properties or new developments decrease the average building age. Therefore, the higher the proportion of redevelopments or new developments in the sample period [to determine the drift rates and volatilities of S(t) and K(t)], the smaller y is.

Assume the constant annual depreciation rate is $\xi(0 < e^{\xi} \le 1$, or $\xi \le 0)$. For two properties that have the same structural and locational characteristics, if they age g and g + z at time t respectively, the relationship between their S(t) is

$$\ln \frac{S(t)^{g+z}}{S(t)^{g}} = z\xi, \text{for any integer } t, \quad g > 0,\ z \geq 0 \qquad (3)$$

In the infinite time model, a quadratic depreciation term, that is, $\ln(S(t)^{g+z}/S(t)^{g})=z\xi_1+z^2\xi_2$ leads to biased estimations of market volatility. Then, assume the price information in n periods is included to derive the square of market volatility

The first term is the square of market volatility when the depreciation effect is ignored. The second term is an increasing function of n. The third term depends on the trend $S(t+i)^g/S(t+i-1)^g$. Increasing the number of sampling periods (n) cannot reduce the values of the second and the third terms. Therefore, the quadratic depreciation effect is not applied in this model.

This depreciation rate was based on the total property value instead of the building structure value. The main reason was that the

$$\frac{1}{n-1}\sum_{i=1}^{n}\left[\ln\frac{S(t+i)^{g+iz}}{S(t+i-1)^{g+(i-1)z}}-\frac{1}{n}\sum_{i=1}^{n}\ln\frac{S(t+i)^{g+iz}}{S(t+i-1)^{g+(i-1)z}}\right]^2$$

$$=\frac{1}{n-1}\sum_{i=1}^{n}\left[\ln\frac{S(t+i)^{g}}{S(t+i-1)^{g}}-\frac{1}{n}\sum_{i=1}^{n}\ln\frac{S(t+i)^{g}}{S(t+i-1)^{g}}\right]^2+\frac{n\xi_2^2 z^4}{n-1}\cdot\frac{2n(n+1)(2n+1)-n(n+1)^2}{\ldots}$$

$$+\frac{1}{n-1}\sum_{i=1}^{n}2\ln\left[\frac{S(t+i)^{g}}{S(t+i-1)^{g}}-\frac{1}{n}\sum_{i=1}^{n}\ln\frac{S(t+i)^{g}}{S(t+i-1)^{g}}\right]\xi_2 z_2(2i-1-n) \qquad (4)$$

depreciation rate must be estimated from previous transaction records. These records only contain total property values, instead of structure values and land values individually. Subjective assumptions on land leverage and the growth rate of land value must be included in the model to estimate the structure values and land values in all the transaction records. To reduce the number of subjective assumptions, the depreciation rate was based on the total property value in this study.

Option Period and Two Redevelopment Stages

The solution process assumed an infinite option period, with a sufficient observation period (in this study it was assumed to be 100 years). Although we assume a destination of the observation period (100 years), this compound option was not a finite redevelopment option. In Chen and Lai's (2013) study, the finite period model was adopted because of the length of the land lease. There are two reasons why the compound option as an infinite redevelopment option is valued.

First, the development density usually increases after the redevelopment. Some redevelopment projects include a change of land use, that is, from residential use to residential-commercial mixed use. In some urban renewal projects, the government requires the developer to embed several community facilities in the new building plan. All the previous examples required that a new land lease was signed after the demolition stage, which included a new lease period. In this case, the developer would be limited to rebuilding a new property in the second stage within a fixed period. If the redevelopment option was exercised simultaneously, the construction process must start when the demolition is finished. Then, the maximum construction period has a negligible impact on exercise decisions. If Type 2 (simultaneous) exercise was used as the major optimal decision (Chen and Lai 2013), the limitation of construction time for the land lease was not important.

Second, the land lease of the old property could be extended automatically, or after some extension fee has been paid in many countries, which include mainland China. If the land was taken over by the landowner when the lease expired, the developer could negotiate with the landowner to redevelop the old property. Therefore, the expiry date of a land lease for an old property was not an actual time restriction for redevelopment. Therefore, an infinite redevelopment option is more common.

The following model for the construction process is based on previous assumptions.

In the second stage, at a specific time, because the risk-neutral growth rate is derived from nearby properties, which depreciate over time, the new property to be built is over-depreciated. Therefore, the adjusted market unit price [S(t)] is

$$S(t)*L_2*e^{-T_{ave}\xi}*e^{-\xi yt} \qquad (5)$$

where L_2 =the plot ratio of the new property to be built. At the start of the compound option, the age difference between nearby properties and the new property to be built is T_{ave}. In addition, this difference increases by y annually. Therefore, the term $e^{-T_{ave}\xi}$ offsets the over-depreciation section before t=0 and the term $e^{-\xi yt}$ offsets the over-depreciation section from t=0 to the exercise time of the rebuild option.

If the rebuild option is exercised at time t, the option value equals

$$V_2(S(t), K(t)) = S(t)*L_2 *e^{-T_{ave}\xi} *e^{-\xi yt} - K(t)*L_2 \qquad (6)$$

Then, if $S_2(t)=S(t)*e^{-\xi yt}$

$$dS_2(t) = d[S(t)*e^{-\xi yt}]$$
$$= e^{-\xi yt}dS(t) - S(t)\xi ye^{-\xi yt}dt$$
$$= e^{-\xi yt}(\nu_S S(t)dt + \sigma_S S(t)dZ_S) - S(t)\xi ye^{-\xi yt}dt$$
$$= S_2(t)[\nu_S - \xi y]dt + S_2(t)\sigma_S dZ_S \qquad (7)$$

In addition, $S_2(t)$ follows a geometric Brownian motion, with a constant and risk-neutral growth rate $\nu_S - \xi y$, and a constant variance σ_S. The remaining part, $L_2 *e^{-T_{ave}\xi}$, is a constant. The value of this rebuild option, which is defined as $V_2(S(t), K(t))$is obtained from the following partial differential equation:

Then

$$f_2(\phi^*_2) = \frac{V_2(S_2^*, K^*)}{K^*} = L_2 *e^{-T_{ave}\xi}\phi^*_2 - L_2 \qquad (14)$$

$$\frac{1}{2}\left(\sigma_S^2 S_2^2 \frac{\partial^2 V_2}{\partial S_2^2} + 2\rho\sigma_S\sigma_K S_2 K \frac{\partial^2 V_2}{\partial S_2 \partial K} + \sigma_K^2 K^2 \frac{\partial^2 V_2}{\partial K^2}\right) + S_2[\nu_S - \xi y]\frac{\partial V_2}{\partial S_2} + K\nu_K \frac{\partial V_2}{\partial K} - rV_2 = 0 \qquad (8)$$

Assume at the optimal exercise timing the critical boundary of V_2 is $V_2(S_2^*, K^*)$then, Eq. (6) becomes

$$V_2(S_2^*, K^*) = L_2 *e^{-T_{ave}\xi}S_2^* - L_2K^* \qquad (9)$$

This is the value-matching condition. The smooth-pasting conditions are

$$\frac{\partial V_2(S_2^*, K^*)}{\partial S_2} = L_2 *e^{-T_{ave}\xi} \qquad (10)$$

$$\frac{\partial V_2}{\partial K}(K^*) = -L_2 \qquad (11)$$

To derive the real option value with price and cost uncertainty, the introduction of a stochastic variable that stands for the ratio of these two variables is necessary (McDonald and Siegel 1986; Williams 1991; Dixit and Pindyck 1994).

Then, define $\phi_2 =(S_2/K)$. Of note, the ratio of S(t) and K(t) is not chosen directly. Instead, combine the time-dependent depreciation term into the price variable as S_2. Then

$$f_2(\phi_2) = \frac{V_2(S_2, K)}{K} \qquad (12)$$

$$d\phi_2 = (\nu_S - \xi y - \nu_K + \sigma^2_K - \rho\sigma_S\sigma_K)\phi_2 dt$$
$$+ \sqrt{\sigma^2_S + \sigma^2_K - 2\rho\sigma_S\sigma_K}\,\phi_2 dZ \qquad (13)$$

and define

$$f_2(0) = 0 \qquad (15)$$

The smooth-pasting condition requires that

$$f'_2(\phi^*_2) = L_2 *e^{-T_{ave}\xi} \qquad (16)$$

$$f_2(\phi^*_2) - \phi^*_2 f'_2(\phi^*_2) = -L_2 \qquad (17)$$

and stochastic Eq. (8) becomes

$$\frac{1}{2}(\sigma^2_S - 2\rho\sigma_S\sigma_K + \sigma^2_K)\phi_2^2 f''_2(\phi_2)$$
$$+ [\nu_S - \xi y - \nu_K]\phi_2 f'_2(\phi_2) - (r - \nu_K)f_2(\phi_2) = 0 \qquad (18)$$

The general solution to this second-order differential Eq. (18) is

$$f_2(\phi_2) = C_2\phi_2^{\lambda} \tag{19}$$

Combined with the value-matching condition Eq. (14), boundary condition Eq. (15), and smooth-pasting conditions Eqs. (16) and (17), based on Øksendal's work (1998), the solution is

$$f_2(\phi_2) = \begin{cases} C_2\phi_2^{\lambda} & \text{if } \varphi_2 \le \varphi_2^* \\ L_2\phi_2 e^{-T_{ave}\xi} - L_2 & \text{if } \varphi_2 > \varphi_2^* \end{cases} \tag{20}$$

where

$$C_2 = \frac{L_2 e^{-T_{ave}\xi}\phi_2^* - L_2}{\phi_2^{*\lambda}} \tag{21}$$

The optimal rebuilding ratio is

$$\phi_2^* = \frac{\lambda}{\lambda - 1} e^{T_{ave}\xi} \tag{22}$$

where the rebuilding option elasticity (λ) is

$$\lambda = \frac{1}{2} - \frac{\nu_S - \xi y - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2} + \sqrt{\left(\frac{1}{2} - \frac{\nu_S - \xi y - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}\right)^2 + \frac{2(r - \nu_K)}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}} \tag{23}$$

Only $\lambda > 1$ is discussed, or the option would not be exercised in the second stage.

In the first stage, $f_2(\phi_2)$ and ϕ^*_2 are known parameters. Assume the age of the old property to be demolished is T_{old} when the compound option becomes valid

$$S(t)^* L_1 {}^* e^{(T_{old} - T_{ave})\xi} {}^* e^{\xi(1-y)t} \tag{24}$$

where L_1 = the plot ratio of the old property. At the start of the compound option, the age difference between nearby properties and the old property to be demolished is $T_{old} - T_{ave}$. Then, the old property depreciates faster than its estimated market price that was derived from nearby properties. Therefore, the term $e^{(T_{old}-T_{ave})\xi}$ offsets the under-depreciation section before t = 0 and $e^{\xi(1-y)t}$ offsets the under-depreciation section from t = 0 to the exercise time of demolition option.

If the demolition option is exercised at time t, the option value equals

$$V_1(S(t),K(t)) = \mathrm{Max}[V_2(S(t),K(t)) - S(t)^* L_1 {}^* e^{(T_{old}-T_{ave})\xi} {}^* e^{\xi(1-y)t} - K(t)^* L_1 {}^* DEM] \tag{25}$$

where DEM=a constant ratio of demolition cost to construction cost for the same building area in the same period.

Similar to Eq. (8), the value of $V_1(S(t), K(t))$ should satisfy the following partial differential equation:

$$\frac{1}{2}\left(\sigma_S^2 S_1^2 \frac{\partial^2 V_1}{\partial S_1^2} + 2\rho\sigma_S\sigma_K S_1 K \frac{\partial^2 V_1}{\partial S_1 \partial K} + \sigma_K^2 K^2 \frac{\partial^2 V_1}{\partial K^2}\right) + S_1[\nu_S + \xi(1-y)]\frac{\partial V_1}{\partial S_1} + K\nu_K \frac{\partial V_1}{\partial K} - rV_1 = 0 \tag{26}$$

where $S_1 = S(t)^* e^{\xi(1-y)t}$.

Assume at the optimal exercise timing, from Eq. (25) that the critical boundary (value-matching condition) of V_1 is $V_1(S_1^*, K^*)$

$V_1(S_{1*},K_*)=V_2(S_{2*},K_*)-L_1{}^*e^{(T_{old}-T_{ave})\xi}S_{1*}-L_1{}^*DEM^*K_*$ (27) Then, define $\phi_1=(S_1/K)$ and

$$f_1(\phi_1) = \frac{V_1(S_1, K)}{K} \quad (28)$$

$$d\phi_1 = (\nu_S + \xi(1-y) - \nu_K + \sigma_K^2 - \rho\sigma_S\sigma_K)\phi_1 dt + \sqrt{\sigma_S^2 + \sigma_K^2 - 2\rho\sigma_S\sigma_K}\,\phi_1 dZ \quad (29)$$

Then, the value-matching condition becomes $f_1(\phi^*_1) = \frac{V_1(S_1^*, K^*)}{K^*} = f_2(\phi_2(\phi^*_1))$

$$- L_2 - L_1{}^*e^{(T_{old}-T_{ave})\xi}\phi^*_1 - L_1{}^*DEM \quad (30)$$

and define

$$f_1(0) = 0 \quad (31)$$

Of note, although ϕ_2 is a different variable to ϕ_1, it is linearly correlated to ϕ_1 when $\phi_1(t)$ is observed with time t. For a specific t

$$\frac{\phi_1(t)}{\phi_2(t)} = \frac{S_1(t)}{S_2(t)} = \frac{e^{\xi(1-y)t}}{e^{\xi(1-\xi-y)t}} = e^{\xi t} \leq 1 \quad (32)$$

The smooth-pasting condition requires that

$$f'_1(\phi^*_1) = f'_2(\phi_2(\phi^*_1)) - L_1{}^*e^{(T_{old}-T_{ave})\xi} \quad (33)$$

$$f_1(\phi^*_1) - \phi^*_1 f'_1(\phi^*_1) = -L_1 \quad (34)$$

Stochastic Eq. (25) becomes

$$\frac{1}{2}(\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2)\phi_1^2 f''_1(\phi_1) + [\nu_S + \xi(1-y) - \nu_K]\phi_1 f'_1(\phi_1) - (r - \nu_K)f_1(\phi_1) = 0 \quad (35)$$

The general solution to this second-order differential Eq. (35) is

$$f_1(\phi_1) = C_1\phi^{\theta}_1 \quad (36)$$

Derivation of Solutions

To determine C_1 and θ, it is necessary to compare the critical value ϕ^*_2 in the second stage with ϕ^*_1 in the first stage. From the solution of $f_2(\phi_2)$, the relationship between ϕ^*_1 and ϕ^*_2 will determine the value of $f_2(\phi^*_1 e^{-\xi t})$, which is an important term to solve the value of ϕ^*_1. To avoid ignoring any possible solution for ϕ^*_1, the following section will discuss three different cases. Case 1. Assume $\phi^*_1 < \phi^*_2$, and $\phi^*_1 e^{-\xi t} < \phi^*_2$. ϕ^*_1 should satisfy

$$(\theta-\lambda)C_2(\phi^*_1 e^{-\xi t})^{\lambda} = (\theta-1)L_1{}^*e^{(T_{old}-T_{ave})\xi}\phi_{1*} + \theta L_1{}^*DEM \quad (37)$$

ϕ^*_1, as a function of t, satisfies Eq. (37) where

$$\theta = \frac{1}{2} - \frac{\nu_S + \xi(1-y) - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2}$$

$$\frac{+}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2} \quad \frac{1}{2} - \frac{\nu_S + \xi(1-y) - \nu_K}{\sigma_S^2 \pm 2\rho\sigma_S\sigma_K + \sigma^2} + 2(r - \nu_K) \quad {}^2 \tag{38}$$

K

Here, θ>1.

Case 2. Assume $\varphi_1^* < \varphi_2^* \le \varphi_1^* e^{-\xi t}$.

The optimal ratio is

ϕ*1=θ−θ 1 * L2e−Tave L2ξe+−ξLt 1−*LDEM1e(Told−Tave)ξ (39) where

$$\theta = \frac{1}{2} \quad \overline{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2} \quad -\nu_S + \xi(1-y) - \nu_K$$

$$\frac{+}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma_K^2} \quad \frac{1}{2} - \frac{\nu_S + \xi(1-y) - \nu_K}{\sigma_S^2 - 2\rho\sigma_S\sigma_K + \sigma^2} {}^2 + 2(r - \nu_K) \quad {}^2 \tag{40}$$

K

Here, θ>1.

Of note, ϕ^*_1 is a function of t, which means that for different t, the optimal exercise ratio calculated from $(S(t)^* e^{\xi(1-y)t})/K(t)$ should be different.

Case 3. Assume $\varphi_1^* \ge \varphi_2^*$.

The solution is the same as Case 2 when $\varphi_1^* < \varphi_2^* \le \varphi_1^* e^{-\xi t}$.

In the following discussions, the conditions in Case 1 will be written as Type 1 (sequential exercise), and the conditions in Cases 2 and 3 will be written as Type 2 (simultaneous exercise) for short. Of note, in Type 1 and 2 strategies, ϕ^*_1 is a function of t. This indicates that in some scenarios, the solution for ϕ^*_1 might not exist in some time intervals. Therefore, in these scenarios, despite the size of the initial price-to-cost ratio is, the optimal exercise strategies in Type 1 or 2 were only feasible in parts of the time intervals.

The appreciation of property value due to urban expansion, or increasing housing demand, or both was assumed to be measured in ν_S, in the geometric Brownian motion of S(t). In addition, ν_S indicates the assumption that the appreciation of property value is based on the total property value, that is, the building structure value and the land value. The same argument as Bokhari and Geltner (2018) was applied in this study, and the estimate of the land value appreciation rate was not determined individually, although the major component of property value appreciation should be the land value appreciation. Bokhari and Geltner (2018) claimed that the land value appraisal contained subjective assumptions, or even some type of depreciation method, which led to an unforeseeable bias in the property value estimation.

Numerical Analysis

Potential Important Factors

A Monte–Carlo simulation method was introduced to investigate the potential factors that influenced the redevelopment option. In the redevelopment option model, only S(t) and K(t) were assumed to follow geometric Brownian motion. The remaining parameters, which include the depreciation rate and price volatility based on total property values, were assumed to be constant during the pricing period. All the parameters (ν_S, ν_K, σ_S, σ_K, ρ, ξ, r, y, T_{ave}, T_{old}, L_1, and L_2) were assigned the baseline values (Table 1). Most of these baseline values refer to Chen and Lai's (2013) study for comparison. L_1, L_2 were set based on the Hong Kong government's instructions (ARCADIS 2016). In the following analysis, several groups of scenarios were designed to illustrate different market conditions or building factors on the expected waiting time. Except for the parameter to be discussed, the other parameters remained at the baseline values.

To describe the model properties more accurately, three statistics were introduced, the conditional expected exercise time, the percentage of exercised paths, and the censored expected exercise time. These statistics could provide more accurate estimations for the Monte–Carlo simulations, because the option value and exercise timing might vary significantly under different simulation paths. Because the optimal exercise strategy in a redevelopment option with depreciation effect is time-dependent, a fixed expected exercise time cannot be directly derived as in normal cases (Øksendal 1998; Chen and Lai 2013). Instead, a large set of sample paths were simulated over a long period (in this study 100 years). It was observed whether the variable ϕ_1 was larger than the optimal exercise strategy ϕ^*_1 in each minimum time interval (dt). In addition, it was confirmed whether the compound option value was positive when $\phi_1 > \phi^*_1$. Only the paths where the redevelopment option was exercised were accounted for to calculate the conditionally expected exercise time. In addition, the percentage of

Table 1. Baseline for different factors

Parameters	Baseline values
Drift rate of market price (ν_S)	4%

Drift rate of construction cost (ν_K)	2%
Volatility of market price (σ_S)	10%
Volatility of construction cost (σ_K)	10%
Correlation coefficient between S and K (ρ)	0
Depreciation rate (ξ)	Log(0.99) (1% annual depreciation rate)
Annual expected capital return rate (r)	10%
Annual increase of average building age (y)	0.5
Average building age in the same region at the beginning of the option (T_{ave}) (year)	30
Building age of the targeted property at the beginning of the option (T_{old}) (year)	50
Plot ratio of the new property to be built (L_1)	9.0[a]
Plot ratio of the old property to be demolished (L_2)	5.2[a]
Initial ratio of market price to construction cost [S(0)/K(0)]	1.5
Ratio of demolition cost to construction cost (DEM)	0.03

Source: Data from ARCADIS (2016).

Note: According to Building (Planning) Regulations in Hong Kong (Cap. 123, Laws of Hong Kong SAR), "Class B Site means a corner site that abuts on two specified streets, as defined under B(P)R, neither of which is less than 4.5 m wide."

[a] In Hong Kong, when the Class B Site building height is <30 m (but >27 m), the maximum domestic plot ratio is 5.2. When the Class B Site building higher is >61 m, the maximum domestic plot ratio is 9.0.

exercised paths was estimated to explain the coverage of this conditional expected exercise time. The censored expected exercise time included the paths where the option was not exercised within this period. In these cases, the exercise time was recorded as the maximum period plus a dt. Therefore, it was called the censored expected exercise time. The conditional expected exercise time was <the censored expected exercise time and <the actual expected exercise time.

Then, $(\theta-\lambda)C_2(\phi^*_1 e^{-\xi t})^\lambda$ were compared (ϕ^*_1 is the optimal exercise strategy in Type 2). If $(\theta-\lambda)C_2(\phi^*_1 e^{-\xi t})^\lambda < (\theta-1)\, L_1 * e^{(T_{old}-T_{ave})\xi}\phi^*_1 + \theta L_1 * DEM$ for any t, ϕ^*_1 was the unique optimal strategy for all possible cases within 100 years.

The following subsections will discuss the impacts on conditional or censored expected exercise time from different parameters. These simulations include some extreme cases that might exist in practice but were ignored in previous studies. Special phenomena were detected when some parameters had more extreme variations.

Influential Factors on Expected Exercise Time

The simulations in the subsections of "Influential Factors on Expected Exercise Time," "Feasible Time Intervals for Different Exercise Strategies", and "Sensitivity Test for the Acquired Property Value" were conducted us statistical software MATLAB R2018b. The main findings from simulations of different scenarios were summarized as follows:

1. When the market price was more volatile, the conditional and censored expected exercise time was larger (Table 2). However, when market price volatility was high enough (20%) as given in Table 2, the conditional and censored expected exercise time increased first as construction cost volatility increased but decreased later.
2. A higher capital return rate encouraged the developer to postpone their redevelopment decision, and therefore, reduced the conditional or censored expected exercise time (Table 3). However, a capital return rate that was too close to the market price drift rate impeded the exercise of the option.
3. The higher correlation coefficient indicated higher potential when the market price and construction cost moved in the same direction. The uncertainty of both parameters was partially offset, and therefore, the conditional and censored expected exercise time reduced (Table 3).

Table 2. Conditional expected exercise time (years), percentage of exercised paths (%), and censored expected exercise time (years) with various volatilities (other parameters were set as baseline values)

Parameters	σ_S=0.02	σ_S=0.05	σ_S=0.10	σ_S=0.15	σ_S=0.20
σ_K=0.02	<0.01	0.7848	5.3663	10.2758	13.3490
	100%	100%	99.85%	97.49%	90.99%
	<0.01	0.7848	5.5045	12.5281	21.1607
σ_K=0.05	0.7347	2.0353	5.9823	10.4824	13.4440

	100%	100%	99.81%	97.44%	91.52%
	0.7347	2.0353	6.1610	12.7744	20.7813
σ_K=0.10	3.9466	4.7722	7.5086	10.9764	13.3941
	99.99%	99.98%	99.63%	97.65%	92.52%
	3.9562	4.7950	7.8527	13.0687	19.8712
σ_K=0.15	6.3045	6.8667	8.6346	11.1341	13.3716
	99.92%	99.88%	99.48%	98.10%	94.15%
	6.3776	6.9748	9.1079	12.8227	18.4382
σ_K=0.20	7.6878	8.0791	9.2796	11.2258	13.1954
	99.86%	99.74%	99.43%	98.35%	95.75%
	7.8189	8.3218	9.7968	12.6907	16.8833

Note: All paths were exercised in Type 2 strategy in all scenarios. Type 2 strategy was the optimal choice for t in all scenarios.

These findings agree with previous results (McDonald and Siegel 1986; Williams 1991; Chen and Lai 2013). Different from the previous traditional factors, several new factors were investigated and discussed in detail.

4. The depreciation rate was the most influential factor in this redevelopment option model (Table 4). A high depreciation rate shortened the expected waiting time significantly. As the depreciation rate increased, the future value fluctuations of the current old property and the future new property reduced following the depreciation adjustments. Both types of predictable value change reduced the proportion of market uncertainty in the total market fluctuations. Therefore, a downward adjustment in the expected exercise time existed when the depreciation effect was embedded in the model.

To further investigate the impact of the depreciation effect, the exercise criteria was calculated in both types when $\xi=\ln(0.98)$ (2% per annum) and $\xi=\ln(0.985)$ (1.5% per annum). In some scenarios, the time interval when the option was exercised in Type 2 standard was zero. Then, the option might be exercised sequentially, or in other strategies, this was different from Types 1 or 2. Further discussions on the optimal exercise strategy in a high depreciation rate environment are presented in the subsection of "Feasible Time Intervals for Different Exercise Strategies."

The annual increase in average building age appears with the depreciation rate in pricing models. Therefore, the effect of y is influenced by the changes in depreciation rate. When the depreciation rate is higher, a larger y indicates a smaller time interval for possible optimal Type 1 and 2 strategies. The optimal exercise time could increase significantly when the optimal Type 2 strategy was not feasible (the first column when y=0.5). If both exercises were not feasible, the option was never exercised optimally in both strategies.

When the annual depreciation rate became 1%, a larger y indicated a longer waiting time to redevelop. This trend was insignificant when $\xi=\ln(0.995)$ (0.5% per annum). When the depreciation rate was close to zero, the option exercise time was rarely influenced by y. If y was close to zero, this indicated that many new properties had been completed in the sample period y (and some might have

Table 3. Conditional expected exercise time (years), percentage of exercised paths (%), and censored expected exercise time (years) with various capital return rates and correlation coefficients (other parameters were set as baseline values)

Parameters	r=0.06	r=0.08	r=0.10	r=0.12	r=0.14
ρ=−0.5	1.0821	11.8885	9.4412	7.1519	5.5680
	0.04%	97.06%	98.90%	99.33%	99.50%
	99.9724	14.4793	10.4357	7.7778	6.0421
ρ=−0.25	0.8200	11.2681	8.5146	6.2338	4.8270
	0.01%	97.79%	99.39%	99.51%	99.70%
	>100	13.2311	9.0726	6.6933	5.1088
ρ=0	>100	10.4196	7.5086	5.3828	3.9009
	0.00%	98.35%	99.63%	99.80%	99.84%
	>100[a]	11.8942	7.8527	5.5702	4.0527
ρ=0.25	>100	9.1169	6.1279	4.0710	2.6830
	0.00%	99.20%	99.84%	99.93%	99.97%

	>100[a]	9.8440	6.2819	4.1404	2.7161
ρ=0.5	>100	7.5814	4.4243	2.4177	1.1335
	0.00%	99.73%	99.99%	99.98%	100%
	>100[a]	7.8310	4.4376	2.4392	1.1354

[a] Except for these 3 unexercised scenarios, all paths were exercised in a Type 2 strategy since a Type 1 strategy was never the optimal choice for any t when r=0.06. A Type 2 strategy was the optimal choice for all t in the remaining scenarios.

Table 4. Conditional expected exercise time (years), percentage of exercised paths (%), and censored expected exercise time (years) with various depreciation rates and an annual increase of average building ages (other parameters were set as baseline values)

Parameters	ξ= ln(0.98) 2% per annum	ξ= ln(0.985) 1.5% per annum	ξ= ln(0.99) 1% per annum	ξ= ln(0.995) 0.5% per annum	ξ=ln(1) =0
y=0	<0.01	<0.01	6.6467	19.8786	36.5919
	100.00%	100.00%	99.32%	96.99%	87.94%
	<0.01	<0.01	7.2816	22.2921	44.2414
y=0.25	<0.01	<0.01	7.0696	19.8957	36.8157
	100.00%	100.00%	99.50%	97.45%	87.95%
	<0.01	<0.01	7.5380	21.9354	44.4281
y=0.5	39.8202	<0.01	7.4919	19.7384	36.7821
	99.92%	100.00%	99.62%	97.56%	87.86%
	39.8708[a]	<0.01	7.8454	21.6954	44.4554
y=0.75	53.1990	0.0243	7.9559	19.6025	36.5239
	99.91%	99.99%	99.76%	98.03%	87.93%
	53.2402[a]	0.0303	8.1408	21.1882	44.1879
y=1	>100	0.5980	8.3386	19.6489	36.6858
	0.00%	99.59%	99.82%	98.25%	87.89%
	>100	1.0017	8.5054	21.0537	44.3544

[a] These scenarios could not be exercised in a Type 2 strategy. The expected exercise time was from a Type 1 strategy, which stands for a sequential exercise. The option was never exercised when ξ=ln(0.98) and y=1. The remaining scenarios were all exercised in a Type 2 strategy, which stands for a simultaneous exercise.

replaced other old properties). This successful development and redevelopment experiences reduced the uncertainty of proposed new buildings, and therefore, the expected waiting time to start.

5. A higher building age of the old property and a higher average building age reduced the expected waiting time to redevelop (Table 5). The high average building age increased the depreciation adjustment between the new property to be built and the nearby buildings, and then decreased the uncertainty of the new property value (compared with the total depreciation-adjusted value).

Feasible Time Intervals for Different Exercise Strategies

This section will focus on a special issue in the depreciation-adjusted model: the feasible time intervals for different optimal exercise strategies.

In the original model, if the depreciation rate was assumed to be zero, $\theta=\lambda$. Then, Eq. (37) should have no solution for ϕ^*_1 (i.e., $\theta>1$). Therefore, if the depreciation effect could be ignored, or could be completely excluded by introducing a newly built property index, the compound option was never exercised optimally in a sequential strategy. The only optimal exercise strategy must be a simultaneous exercise. In addition, in the traditional model, the optimal exercise ratio in stages 1 and 2 are constants. The relationship between ϕ^*_1 and ϕ^*_2 is independent of t. If $\phi^*_1 > \phi^*_2$ when t = 0 this inequality holds for all t.

However, as mentioned in the "Introduction," the depreciation adjusted model is a superior choice in some markets. The developer must consider the depreciation adjustments if no newly built property market index is available. This adjustment has two consequences. First, the Type 1 strategy becomes possible in some scenarios. Second, the relationship between $\phi^*_1e^{-\xi t}$ and ϕ^*_2 depends on t. Therefore, Type 1 or 2 strategies might only feasible for some time intervals.

Table 5. Conditional expected exercise time (years), percentage of exercised paths (%), and censored expected exercise time (years) with various average building ages and building ages of old property (other parameters were set as baseline values)

Parameters	T_{old} =40	T_{old} =50	T_{old} =60
T_{ave} =25	10.6527	9.1307	7.7151
	99.43%	99.55%	99.56%
	11.1656	9.5360	8.1231
T_{ave} =30	9.2232	7.4919	6.0920
	99.54%	99.62%	99.68%
	9.6391	7.8454	6.3907
T_{ave} =40	6.0660	4.3103	2.7885
	99.77%	99.81%	99.89%
	6.2783	4.4959	2.8993

Note: All paths were exercised in a Type 2 strategy in all scenarios. A Type 2 strategy was the optimal choice for t in all these scenarios.

In the discussions of the previous potential factors, the Type 2 strategy (traditional optimal simultaneous exercise) was feasible and the previous choice for all t in most scenarios. As mentioned in the subsection of "Influential Factors on Expected Exercise Time", three parameters, that is, capital return rate, depreciation rate, and an annual increase of average building age influenced the feasible time intervals for Type 1 and 2 strategies. To further illustrate their impacts on the ranges of feasible time intervals, the feasible time intervals in 45 scenarios (5×3×3) were calculated (Table 6).

The term $\phi^*_1e^{-\xi t}$ was a strictly decreasing function of t for Type 2 strategy. However, for Type 1 strategy, it was a decreasing function of t first, and then an increasing function of t. The turning point is usually large, which means that $\phi^*_1e^{-\xi t}$ decreased over a long period.

Several trends in the ranges of feasible time intervals are available in Table 6.

1. When the capital return rate and an annual increase of average building age were fixed, a smaller depreciation rate leads to a higher lower-bound of feasible time intervals for Type 1 strategy, and a higher upper-bound of valid time intervals of Type 2 exercise. Therefore, a lower depreciation rate derived a smaller range of feasible time intervals for the Type 1 strategy, and a larger range of feasible time intervals for the Type 2 strategy.
2. When the capital return rate and depreciation rate were fixed, a larger annual increase of average building age resulted in a lower upper-bound (and a smaller range) of feasible time intervals for the Type 2 strategy. There was no consistent trend between the annual increase of average building age and the range of feasible time intervals for the Type 1 strategy.
3. When the depreciation rate and annual increase of average building age were fixed, a larger capital return rate caused a higher upper-bound (and a larger range) of feasible time intervals for the Type 2 strategy. The relationship between the capital return rate and the range of feasible time intervals for the Type 1 strategy was much more complicated. When the depreciation rate was 2% annually, the lower-bound of feasible time intervals increased as capital return rate increased. When the depreciation rate was 1.5% annually, no consistent trend was found.
4. Within the three parameters, the depreciation rate had the greatest impact on the range of feasible time intervals.

Table 6 reveals that a Type 2 strategy might be partially or completely unfeasible during the observation period. Instead, a Type 1 strategy becomes attainable in some scenarios. Then, the problem to determine a superior strategy over a long observation period exists. To comprehensively discuss this problem, a

Table 6. Available time intervals for 2 types of exercise from 45 selected scenarios

Capital return rate	The annual increase of average building age	ξ=ln(0.98) 2% per annum	ξ=ln(0.985) 1.5% per annum	ξ=ln(0.99) 1% per annum
r=0.06	y=0	Type 1: [0, 100]	Type 1: [18.72, 100]	Type 1: Null
		Type 2: Null[b]	Type 2: Null[b]	Type 2: [0, 25.98]
	y=0.5	Type 1: [0, 100]	Type 1: [0, 100]	Type 1: Null
		Type 2: Null[b]	Type 2: Null[b]	Type 2: [0, 0.41]
	y=1	λ<1	Type 1: [0, 100]	Type 1: [0, 100]
			Type 2: Null[b]	Type 2: Null
r=0.07	y=0	Type 1: [2.44, 100]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: [0, 4.83]	Type 2: [0, 61.83]
	y=0.5	Type 1: [0, 100]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: Null[a]	Type 2: [0, 41.05]
	y=1	Type 1: [0, 100]	Type 1: [0, 100]	Type 1: Null
		Type 2: Null[b]	Type 2: Null[b]	Type 2: [0, 19.45]
r=0.08	y=0	Type 1: [19.34, 100]	Type 1: [90.16, 100]	Type 1: Null
		Type 2: Null[b]	Type 2: [0, 21.75]	Type 2: [0, 90.32]
	y=0.5	Type 1: [6.39, 100]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: [0, 5.02]	Type 2: [0, 72.29]
	y=1	Type 1: [0, 100]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: Null[a]	Type 2: [0, 54.16]
r=0.09	y=0	Type 1: [23.62, 100]	Type 1: [79.06, 100]	Type 1: Null
		Type 2: [0, 3.59]	Type 2: [0, 35.99]	Type 2: [0, 100]
	y=0.5	Type 1: [34.36, 100]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: [0, 20.89]	Type 2: [0, 97.93]
	y=1	Type 1: [15.00, 81.61]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: [0, 6.05]	Type 2: [0, 81.90]
r=0.10	y=0	Type 1: [23.83, 100]	Type 1: [70.45, 100]	Type 1: Null
		Type 2: [0, 12.13]	Type 2: [0, 48.34]	Type 2: [0, 100]
	y=0.5	Type 1: [39.42, 100]	Type 1: Null	Type 1: Null
		Type 2: Null[b]	Type 2: [0, 34.42]	Type 2: [0, 100]
	y=1	Type 1: Null	Type 1: Null	Type 1: Null
		Type 2: Null[a]	Type 2: [0, 20.96]	Type 2: [0, 100]

Note: The periods were the available time intervals when a Type 1 and 2 strategy was feasible.

[a] In these scenarios, the option could not be exercised in Type 1 or 2 strategies.

[b] In these scenarios, the option could not be exercised in Type 2 strategy.

The option was exercised in a Type 2 strategy in the remaining scenarios.

new comparison strategy was introduced. In this comparison strategy, the compound option was exercised simultaneously if the inequality

$$\frac{S(t)}{K(t)} * e^{-\xi y t} > \phi^{*2} \qquad (41)$$

held for the first t, that is, if the optimal rebuilding ratio in Stage 2 was achieved, the compound option was exercised simultaneously. In total, 9 out of the 15 scenarios when the annual depreciation rate was 1.5% as given in Table 6 were chosen to compare the three different strategies. The numbers of exercised paths in 100,000 simulations were collected for Type 1 and 2 strategies. For comparison, the numbers of paths where the option values in the comparison strategy were higher than those in Type 1 (or Type 2) strategy were generated. The conditional expected option values for the comparison strategy, Type 1, or 2, or both were calculated. The term conditional assumed that the option value after the observation period was ignored. If a path was not exercised until 100 years, the option value of this path was zero. All the previous statistics are summarized in Table 7. To obtain a more diversified view, the initial price-to-cost ratio was 1.2 to avoid immediate exercise at t=0.

The first set provides three scenarios when a Type 2 strategy was not feasible. However, the range of feasible time intervals for a Type 1 strategy was long. Although a high percentage of paths were exercised with positive values based on the optimal Type 1 strategy, a significant proportion of paths (46%–65%) had higher values when they were exercised based on the comparison strategy. The conditional expected option value in the comparison strategy was higher than that in a Type 1 strategy in two scenarios. This suggested that even when a Type 2 strategy was not feasible, the comparison strategy, which is a type of simultaneous exercise strategy, was still usually a superior choice. When the capital return rate was very low, and the annual increase of average building age was small, a Type 1 strategy had a higher conditional expected option value.

The second set provided three scenarios when a Type 1 strategy was not feasible, and the feasible time intervals for a Type 2 strategy are small. When the feasible time intervals were <10 years, approximately ⅓ of the paths were not exercised based on a Type 2 strategy. The proportion of paths exercised in the comparison strategy was relatively small; however, the conditional expected option values of these paths were higher than those exercised in a Type 2 strategy. When a significant proportion of paths could not be exercised in a Type 2 strategy, the comparison strategy might be a superior choice.

The third set provided three scenarios when Type 1 and 2 strategies were partially feasible during the observation period. In these scenarios, a Type 2 strategy was the most valuable solution. The number of paths with higher option values and the conditional expected option value supported the choice of a Type 2 strategy. A Type 1 strategy was much worse than the comparison one.

Table 7. Numbers of exercised paths and conditional expected option value based on different strategies (with 100,000 simulations in each scenario)

	Set (I)			Set (II)			Set (III)		
Scenarios	r=0.06 y=0	r=0.06 y=0.5	r=0.07 y=1	r=0.07 y=0	r=0.09 y=0.5	r=0.09 y=1	r=0.08 y=0	r=0.09 y=0	r=0.10 y=0
Type 1 boundary	[18.72, 100]	[0, 100]	[0, 100]	Null	Null	Null	[90.16, 100]	[79.06, 100]	[70.45, 100]
# of paths exercised in Type 1	96,750	98,642	99,439	—	—	—	94,672	96,398	97,546
# of paths exercised in Type 1 (positive option value)	96,265	92,045	96,730	—	—	—	94,534	96,059	97,045
# of paths where the intrinsic option value in the comparison strategy was larger than that in Type 1	45,961	65,111	59,055	—	—	—	99,110	99,891	99,988
Conditional expected option value in Type 1 strategy	572.8155	670.3719	682.7291	—	—	—	38.8775	26.6141	19.4356
Type 2 boundary	Null	Null	Null	[0, 4.83]	[0, 20.89]	[0, 6.05]	[0, 21.75]	[0, 35.99]	[0, 48.34]
# of paths exercised in Type 2	—	—	—	62,671	93,116	61,275	92,992	97,292	98,711
# of paths exercised in Type 2 (positive option value)	—	—	—	62,671	93,116	61,275	92,992	97,292	98,711
# of paths where the intrinsic option value in the comparison strategy was larger than that in Type 2	—	—	—	36,425	6,748	38,589	6,710	2,754	1,400
Conditional expected option value in Type 2 strategy	—	—	—	370.8524	597.5184	457.3479	523.7142	503.5282	468.9839

Conditional expected option value in the comparison strategy	544.2152	731.4694	718.9531	437.6559	391.3815	479.9780	364.4314	323.0678	322.8327

Note: The depreciation rate was 1.5% per annum for all scenarios.

Based on the discussions obtained from Tables 6 and 7, a comprehensive procedure to select the optimal exercise strategy was derived:

1. Find the feasible time intervals for a Type 2 strategy. If a Type 2 strategy was completely feasible during the observation period, take it as the optimal exercise strategy. This is the most general case.
2. If a Type 2 strategy is partially feasible during the observation period, compare the conditional expected option values based on a Type 2 strategy and the comparison strategy. Take the strategy that has a higher conditional expected option value.
3. If a Type 2 strategy is completely unfeasible during the observation period, find the feasible time intervals for a Type 1 strategy. Compare the conditional expected option values based on a Type 1 strategy and the comparison strategy. Take the strategy that has a higher conditional expected option value.

In general, a Type 2 strategy was the most valuable choice in most scenarios. In a limited number of scenarios, a Type 1 strategy should be considered following a comparison with a simultaneous exercise strategy that was based on the optimal rebuilding ratio. A Type 2 strategy was no longer the only possible strategy in all scenarios, which was an extended conclusion in this depreciation-adjusted model. In this study, when a Type 2 strategy was not feasible, as given in Tables 3 and 4, the optimal strategy was the comparison strategy (based on the optimal rebuilding ratio).

A higher depreciation rate, a lower capital return rate, and a higher annual increase in the average building age all caused a smaller range of feasible time intervals for a Type 2 strategy, and then there was a lower probability to choose a Type 2 strategy as the optimal strategy. These factors have the same type of impacts on the option pricing process, that is, to increase the newly built property value in future. If the depreciation rate is taken as representative then, the optimal ratio of the compound option is shown in Eqs. (39) and (40).

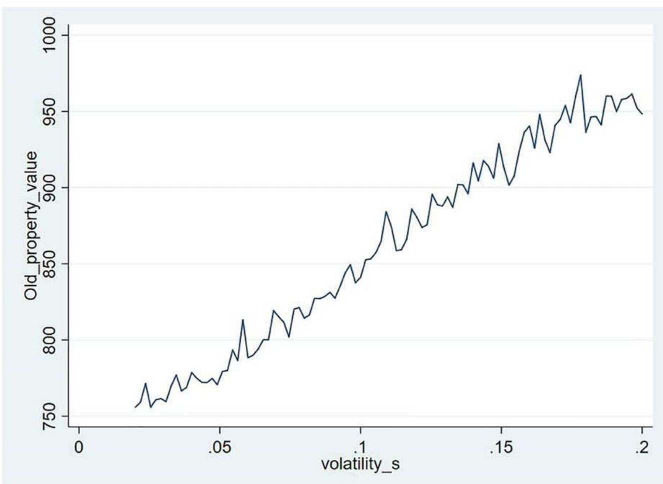
The optimal ratio of the rebuilding option is shown in Eqs. (22) and (23).

As the depreciation rate increased, the value of $\phi^*_1(t)$ decreased much faster than that of ϕ^*_2. Therefore, the optimal ratio to demolish the old property decreased much faster than the optimal ratio to rebuild a new one. However, based on the derivation of a Type 2 strategy, $\varphi_2^* \le \varphi_1^*(t)e^{-\xi_t}$, this becomes a contradiction. Therefore, a high depreciation rate induced a smaller range of feasible time intervals for a Type 2 strategy. Similarly, as the capital return rate decreased, or as the annual increase in average building age increased, the value of ϕ^*_2 increased much faster than that of $\phi^*_1(t)$. When the building age of the old property was high, the optimal demolition ratio might be lower than the optimal rebuilding ratio. The other parameters, such as the volatilities, the correlation coefficient, and the average building age did not have similar impacts. If the condition of $\varphi_2^* \le \varphi_1^*(t)e^{-\xi_t}$ became false, using the optimal rebuilding ratio φ_2^* as the trigger for the whole compound option was a better choice.

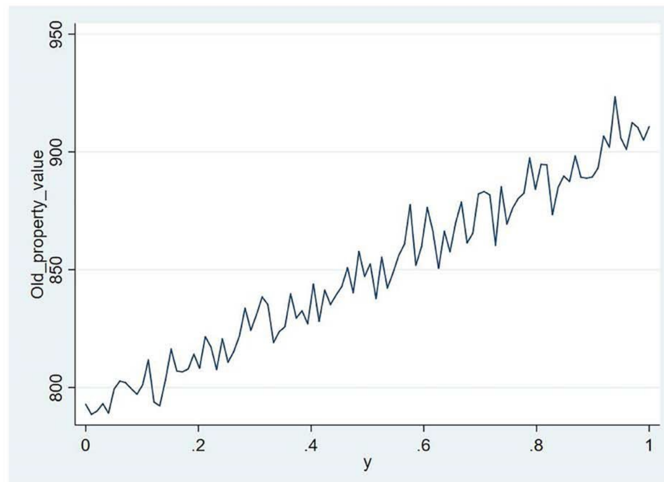
Sensitivity Test for the Acquired Property Value

In addition to the average exercise time and compound option value, the owners of old properties that are acquired will be concerned about the sensitivity of old property values and other parameters. The discussion of this acquisition value is a good example that demonstrates the scope of application for this compound option. Fig. 1 shows the relationships between old property values and the main parameters in this compound option. As shown in each graph in this figure, only one target parameter changes and the rest remain the baseline values given in Table 1. The range of the target parameter (x-axis) was divided into 100 equal parts. Then, the compound option was exercised in the optimal strategy, which contained the acquisition value of the old property during the first stage. The y-axis is the present value of the acquisition price that is discounted by the annual expected capital using return rate (r). In addition, simulations were first conducted using MATLAB R2018b. Then, the curves were generated by Stata 15. The compound option was exercised in all scenarios shown in Fig. 1.

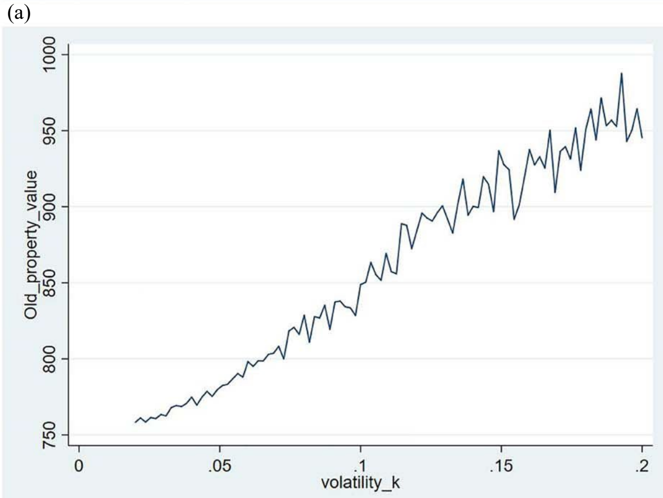
As shown in Figs. 1(a–d), the optimal strategy was the Type 2 strategy for the whole range. As shown in Fig. 1(e), the option could not be an exercise in a Type 2 strategy for the first two data points, where the comparison strategy was adopted to fill the graph. Fig. 1(f) shows the most complicated case, although the option could be exercised in a Type 2 strategy for the first 14 data points, the option values were smaller than those if the option was exercised during the comparison strategy. Therefore, the acquisition values of old properties under two different strategies are provided, as shown in Fig. 1(f). The upper curve represents the acquisition value in the comparison strategy.


Old_property_value
750
800
850
900
950
1000
0
.05
.1
.15
.2
volatility_s

(a)


Old_property_value
800
850
900
950
0
.2
.4
.6
.8
1
y

(d)


Old_property_value
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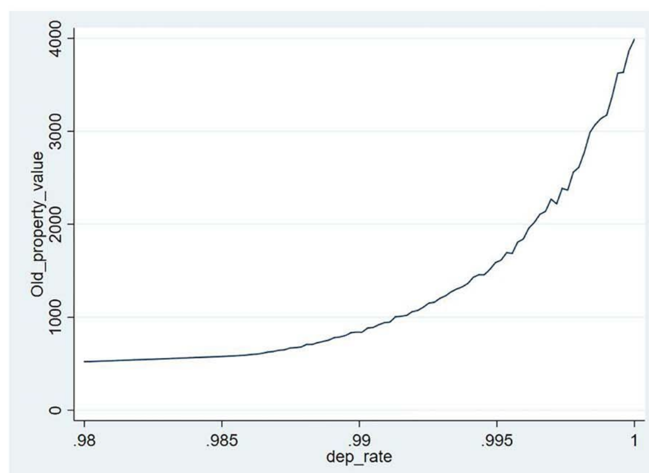
800
850
900
950
1000
0
.05
.1
.15
.2
volatility_k

(b)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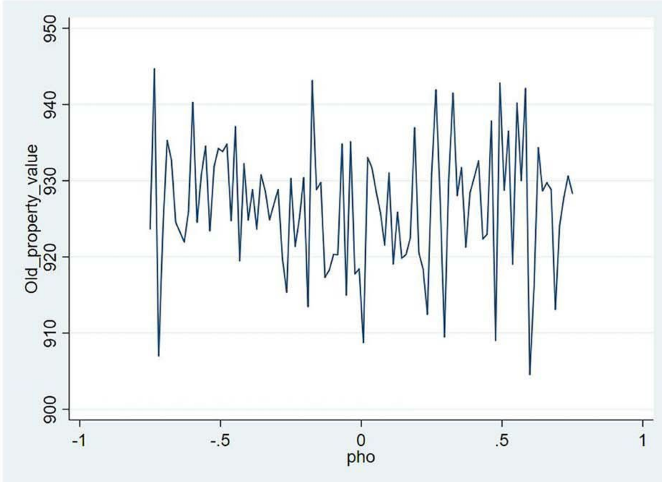
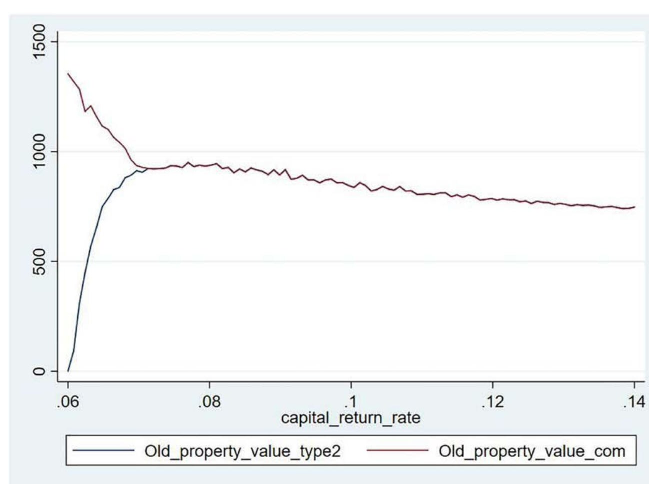
2000
3000
4000
.98
.985
.99
.995
1
dep_rate

(e)


Old_property_value
900
910
920
930
940
950
-1
-.5
0
.5
1
pho


0
500
1000
1500
.06
.08
.1
.12
.14
capital_return_rate
Old_property_value_type2
Old_property_value_com

(c)

(f)

Fig. 1. The sensitivity test for the acquisition price of old property at the current value: (a) volatility of market price; (b) volatility of construction cost; (c) correlation coefficient between market price and construction cost; (d) annual increase of average building age; (e) depreciation rate; and (f) annual capital return rate. The upper curve stands for the present value of acquisition price in the comparison strategy; and the lower curve stands for the present value in the Type 2 strategy.

The acquisition value was not sensitive to the correlation coefficient between market price and construction cost, and slightly sensitive to the annual increase in average building age (Fig. 1(c) and 1(d)). As the compound option value, the acquisition value was sensitive to volatilities of market price and construction cost (Figs. 1(a) and 1(b)). The most interesting cases are presented in the Figs. 1(e) and 1(f). In Fig. 1(e), the acquisition value was not sensitive to the depreciation rate when the depreciation rate was >1.5%. However, this sensitivity increased rapidly as the depreciation rate decreased. The acquisition value when the depreciation rate is zero becomes as large as that when the depreciation rate is 1%. As shown in Fig. 1(f), the sensitivity level was median when the capital return rate was >8% per annum. However, the sensitivity increased dramatically, because the comparison strategy became the optimal choice.

In addition to the volatilities of market price and construction cost, the acquisition price of old properties was sensitive to the depreciation rate and the capital return rate. When the depreciation rate was low, or the capital return rate was low, or both this sensitivity increased dramatically. These results correspond with the findings in the subsection of "Influential Factors on Expected Exercise Time." The difference of estimated values between a Type 2 and comparison strategies was presented in this sensitivity test, which provided visible evidence to explain the importance of adopting the correct exercise strategy.

Advantages and Limitations of This Model

This model is suitable for cities that do not announce restrictions on the maximum rebuilding period, such as Hong Kong. Two main advantages of this new model were summarized from the previous data analysis process. First, this new model is a further extension to Chen and Lai's (2013) model, to release the restriction that no depreciation effect was embedded in the existing two-stage redevelopment option. This covered most redevelopment cases when only two participants (developer and owner) were involved. By adjusting the depreciation rate and the annual increase in average building age, this new model could be adapted for a wide range of scenarios.

Second, the solution of this new model has high operability by the provision of detailed procedures that adopt the optimal exercise strategy in the subsection of "Feasible Time Intervals for Different Exercise Strategies." These strategies covered the cases when some parameters had extreme values, such as a low capital return rate and a high depreciation rate. This solution could help the developer and the owner to estimate the sensitivity of the acquisition price for the old property.

However, two limitations should be highlighted when this new model is applied in practical use. First, the optimal exercise strategy was more complicated than the existing one or two-stage redevelopment models. A simple analytical solution for the exercise standard for all scenarios does not exist. Instead, the detailed procedures suggested that the user could initially calculate the feasible time interval for a Type 2 strategy. If this time interval covered the overall option period, the simple analytical solution would hold, which is the most common case. This major limitation could occur in some extreme scenarios. Second, the estimation of depreciation rate must be accurate. The sensitivity test in the subsection of "Sensitivity Test for the Acquired Property Value" emphasized that the option value and acquisition price were sensitive to the depreciation rate that was based on the total property value. Then, the developer needs to determine an accurate estimation of the depreciation rate based on the transaction records in this region. This is the most important new parameter compared with existing redevelopment models. The developer should build a comprehensive estimation model that excludes the potential disturbances from other housing factors. Although the annual increase in average building age is another new parameter, it could be generated accurately from the building ages of nearby properties with less effort.

Conclusions

This study provided an extension to the traditional redevelopment option model with a permanent exercise period, by the introduction of the depreciation rate and annual increase in average building age. Its main contribution was to help estimate the optimal exercise time and the value of a certain redevelopment option more accurately, which does not ignore the upward bias from the lack of depreciation effect.

This study investigated in detail the influential factors on optimal exercise timing in the new model. Three new statistics were applied to describe the impact of potential factors on redevelopment decisions: (1) the conditional expected exercise time within a certain period; (2) the percentage of exercised options in all simulation paths; and (3) the censored expected exercise time. Higher price and cost volatilities, lower capital return rate, and lower correlation coefficient were related to a longer waiting time to exercise the redevelopment option. In addition, older targeted property, older buildings in the same region, and larger depreciation rates contributed to the shorter waiting time to redevelop. The depreciation rate had the largest influence. Because the property value depreciation indicated a time-varying critical value to exercise the option, many new characteristics were observed.

The traditional simultaneous exercise strategy, which was to demolish the old property and then rebuild a new property as soon as possible, was re-examined under the new conditions. A high depreciation rate, low capital return rate, and a large annual increase of the average building age, or a high building age of the old property, or both might lead to the phenomenon that the optimal demolition ratio was lower than the optimal rebuilding ratio. This led to the contradiction in the condition for the traditional optimal simultaneous exercise strategy (defined as a Type 2 strategy in this study), which assumed that the trigger ratio should be no less than the optimal rebuilding ratio. A solution could be to adopt the optimal rebuilding ratio as the trigger of the whole compound option. In some extreme scenarios (e.g., depreciation rate is 2% per annum and capital return rate is 7% per annum), the optimal sequential exercise strategy (defined as a Type 1 strategy) had a higher expected option value than the previous two optimal simultaneous exercise strategies. When a Type 2 strategy was not feasible, it is necessary to compare the remaining strategies.

In addition, this study summarized the sensitivity analysis between the acquisition value of old properties and model parameters. The acquisition value was sensitive to the depreciation rate and the capital return rate.

This new model could be applied to estimate the optimal exercise timing more comprehensively and more accurately than the previous two-stage redevelopment option model under all the different scenarios. To achieve reliable results, the depreciation rate should be accurately estimated. This model could help the government to review the compensation standard for future redevelopment projects. A satisfactory acquisition standard should be accepted by the affected residents to reduce potential social conflicts. It should help developers maintain their incentive to operate redevelopment projects. In addition, it could lead to a reasonable expected exercise time to redevelop, and a proper market price for the new housing units. Further studies should be conducted to improve this model in two ways. First, introduce a new assumption to predict the depreciation rate in property markets, which would reduce the complexity of model solutions. Second, suggest a more appropriate method for the estimation of regional depreciation rates.

Data Availability Statement

Some or all data, models, or codes generated or used during the study are available in a repository online following funder data retention policies, including plot ratio information used in the data analysis, which is open data from the ARCADIS' Annual Construction Cost Handbook. Downloaded from: https://www.arcadis .com/en/asia/our-perspectives/research-and-publications/arcadis -construction-cost-handbook/.

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request, including the program code for this continuous-time redevelopment option model.

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