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**Analysis of mobilized stress ratio of gap-graded granular materials in direct shear state considering coarse fraction effect**

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## 25 Abstract

26 Weathered rockfill materials, characterized by a mixture of soil matrix and rock aggregates,  
27 are widely distributed in mountainous areas. These soils are frequently used for subgrade or  
28 riprap in engineering practice, and the mobilized shear strength is crucial for analyzing the  
29 displacement and stability of these geo-structures. A series of direct shear tests are performed  
30 on a gap-graded soil with a full range of coarse fraction. The behavior of gap-graded soils is  
31 analyzed, and a simple model is proposed for the evolution of mobilized stress ratio during  
32 direct shearing process based on mixture theory. The change of inter-aggregate configuration  
33 is incorporated by introducing a structure variable which increases with coarse fraction and  
34 decreases approximately linearly with the overall horizontal shear strain in double logarithmic  
35 plot. It reasonably reflects a gradually transformation from a matrix-sustained structure into  
36 an aggregate-sustained one with the increase of coarse fraction. The model has four  
37 parameters, and at least two direct shear tests need to be done for the calibration. Validation  
38 of the model is done by using the test data in this work and those from literature.

40 **Keywords:** Direct shear tests; Mobilized stress ratio; Gap-graded soils; mixture theory;  
41 Volume average scheme

## 1. Introduction

Weathered rockfill materials are distributed worldwide, especially in mountainous areas (Yang and Juo, 2001; Zhao *et al.*, 2007; Peters and Berney, 2010; Zhou *et al.*, 2016; Qin and Chian, 2017; Wei *et al.*, 2018). For example, Chongqing soil from western China (Chen *et al.*, 2020b) is a typical weathered gap-graded soil. The particle size between 0.425 mm and 2.0 mm is missing, and it can be characterized by a mixture of fines and rock aggregates. The fine matrix can be either originated from a weather induced disintegration or a tectonic deformation of weak rock (Chandler, 2000; Ruggeri *et al.*, 2016; Peng *et al.*, 2018; Shi *et al.*, 2019; Guo and Cui, 2020). Gap-graded soils are frequently used for subgrade or riprap in engineering practice, since they can be obtained locally (Vallejo, 2001; Chang *et al.*, 2014; Chen *et al.*, 2020a; Meng *et al.*, 2019; Tan *et al.*, 2019). The shear strength of gap-graded soils is crucial for analyzing the stability of these geo-structures.

The shear strength of gap-graded soils has been investigated by many researchers (e.g., Kumar and Wood, 1999; Yin, 1999b; Vallejo and Mawby, 2000; Simoni and Houlsby, 2006; Shin and Santamarina, 2012; Ruggeri *et al.*, 2016). They reported that the fraction of aggregates has a considerable influence on the shear strength, which is related to the internal structure of gap-graded soils. Thevanayagametal and his colleagues (Thevanayagam and Mohan, 2000; Thevanayagametal *et al.*, 2002) presented a schematic interpretation for the structural evolution of fine-coarse mixtures, and it provides a useful reference for analyzing the compressibility and shear behavior of gap-graded granular soils. Micromechanics based models (Chang and Yin, 2011; Yin *et al.*, 2014; Chang *et al.*, 2017), equivalent void concept (Thevanayagam *et al.*, 2002; Thevanayagam and Martin, 2002; Yin *et al.*, 2016), and mixture theory (Shi *et al.*, 2019; Shi *et al.*, 2020) are used to evaluate and capture the coarse fraction effect of gap-graded soils.

The ultimate shear strength of the gap-graded soils was fully analyzed based on direct shear

75 tests by previous researchers (e.g., Vallejo and Mawby, 2000; Yagiz, 2001; Simoni and  
76 Houlsby, 2006; Ruggeri et al., 2016; Zhou *et al.*, 2016). However, design approaches based  
77 on ultimate shear strength fail to reflect realistic transition of soils from the initial state to the  
78 final failure state (Bolton *et al.*, 1990a and 1990b). To this end, Bolton *et al.* (1990a, 1990b,  
79 2008) and Osman and Bolton (2004, 2006) suggested that mobilized shear strength should be  
80 used to assess the safety and to compute deformations in vicinity of retaining systems.

81 Mobilized shear strength relies on the mobilized stress ratio defined as the ratio of the shear  
82 stress to the vertical stress in direct shear tests. In this work, we present several points which  
83 were not reported by others: (1) gap-graded soils with a prescribed void ratio of matrix are  
84 tested, so that the coarse fraction effect can be assessed; (2) a new homogenization law is  
85 proposed for the stress ratio of gap-graded mixtures, which couples the overall stress ratio  
86 with that of the matrix; (3) a simple model is proposed for the evolution of the stress ratio  
87 with only two state variables. Experimental data from literature is also adopted for a further  
88 validation.

89

## 90 2. Materials and test scheme

### 91 *Materials*

92 Chongqing soil consists of two different size of components, termed as coarse aggregates and  
93 fine matrix. Most of the soil particles with a size between 0.425mm and 2.0mm were missing,  
94 indicating that the Chongqing soil is a typical gap-graded soil. The fine soil matrix is obtained  
95 by sieving Chongqing soil through a mesh opening of 0.425 mm. The particle size distribution  
96 curve of the fine soil does not follow the fractal limiting grading curve:

$$97 \quad P(d) = 100 \times \left( \frac{d}{d_{\max}} \right)^n \quad (1)$$

98 where  $P(d)$  is the cumulative percentage passing the sieve,  $d_{\max}$  is the maximum size of  
99 particles, and  $n$  is a constant parameter. The particle size distribution of the fine matrix in  
100 Chongqing soil is not consistent with Eq. (1), since the fines are originated from weathering  
101 factors (e.g., seasonal temperate and humidity oscillations). The density of fine soil particles  
102 is  $2.73 \text{ Mg/m}^3$ . The maximum and minimum void ratios of the fine matrix are 1.323 and 0.406,  
103 respectively. The optimum moisture content is 12%, and the corresponding maximum dry  
104 density is  $1.94 \text{ Mg/m}^3$ .

105 The typical sizes distinguishing the aggregates and the matrix vary according to the degrees  
106 of rock mass weathering. E.g., for the Chongqing soils, the typical sizes are 0.425mm and  
107 2mm. Particles between these two limit values are missing, and soil particles larger than 2.0  
108 mm are categorized as coarse aggregates. Since the test data may be not reliable for very  
109 coarse granular materials in direct shear tests, a coarse sand material from Hong Kong  
110 (denoted as HK sand) is used to replace the rock aggregates in Chongqing soil. The density of  
111 sand particle is  $2.63 \text{ Mg/m}^3$ . To distinguish the coarse fraction effect, only the coarse particles  
112 (HK sand) with a size between 1.13 mm and 2.0 mm are used as inclusions. The ratio of  
113 representative particle size (e.g.,  $D_{50}$ ) between aggregates and fines is ca. 8. This type of  
114 mixtures shows a distinct coarse fraction effect, and it can be categorized as a typical gap-  
115 graded soils (Ueda et al., 2011). Most of the coarse particles in HK sand have a subangular to  
116 angular shape. The basic physical properties of the matrix and aggregates are given in Table 1  
117 (BS 1377). The minimum void ratio  $e_{\min,a}$  and maximum void ratio  $e_{\max,a}$  of HK sand are 0.60  
118 and 0.95, respectively.

#### 119 *Test scheme*

120 The test apparatus utilized in this work is a modified direct shear system (Borana *et al.* 2016).  
121 The stainless-steel shear box has a diameter of 10 cm and a height of 4 cm. It is well

122 recognized that the overall behavior relies on both coarse fraction and behavior of the soil  
123 matrix. The objective of this study is to evaluate and model the coarse fraction effect. To this  
124 end, the state of soil matrix of mixtures should be the same. E.g., the moisture content and the  
125 initial void ratio of the soil matrix. In this study, an optimum water content (12%) is adopted  
126 for the matrix. Six different coarse fractions are considered in this study (0%, 20%, 40%, 55%,  
127 80%, 100%, The “coarse fraction” denotes the mass fraction of coarse aggregates, which  
128 differs from the “volume fraction of coarse aggregates”).

129 To produce gap-graded soils, the fine soil matrix and the coarse aggregates were mixed  
130 homogeneously for a visual homogeneity at room conditions. Then, the mixtures are matured  
131 in an airtight container for several hours for a uniform distribution of moisture content. After  
132 finishing the direct shear test, the mixtures from three different parts of a specimen are  
133 sampled (top, bottom and the middle in vicinity of the shear plane). The coarse fraction and  
134 moist content are measured, and most of the data are approximately close to the desired  
135 values, with a relative error less than 4%. The particles size distribution curves of the  
136 mixtures with various coarse fractions are shown in Fig. 1.

137 It is fully recognized that the initial state of soils has a significant influence on the shear  
138 strength (e.g., Chandler, 2000; Yao *et al.*, 2004; Yao *et al.*, 2009; Hong *et al.*, 2012; Shi and  
139 Yin, 2018; Zhou *et al.*, 2016). Therefore, the initial void ratio of the fine matrix in mixture  
140 samples (coarse fraction below 73%) should be the same, otherwise, one cannot distinguish  
141 the effect of coarse fraction and the initial void ratio of the matrix. The desired dry density of  
142 the fine matrix is  $1.61 \text{ Mg/m}^3$  in this study. If it is assumed that the fine matrix and the coarse  
143 aggregates are fully mixed, the overall void ratio of the mixture can be computed, as shown in  
144 Fig. 2. (‘fully mixed’ means an ideal mixing process: when the fraction of fines is higher than  
145 the porosity of the coarse aggregates, the coarse grains float in a fine matrix. Otherwise, all  
146 fines are confined within the inter-aggregate space.) The overall void ratio decreases as the

147 coarse fraction increases, and it is formulated as a function of the void ratio of fine matrix  $e_m$

148 and the coarse fraction  $\psi_a$  :

$$149 \quad e = e_m(1 - \psi_a) \quad (2)$$

150 Note that Eq. (2) is only suitable for matrix-sustained structure as shown in Fig. 3, i.e.,

151 most of the coarse aggregates float within a fine matrix. The overall void ratio decreases until

152 it approaches the transitional point (the lowest point in Fig. 2), ( $e = \tilde{e}$ , and  $\psi_a = \tilde{\psi}_a$ , see

153 Appendix-1 for details). This point corresponds to the transitional structure in Fig. 3. With a

154 further increase of coarse fraction, the overall void ratio increases, and aggregate-sustained

155 structure forms. The “ideal compaction” line can be expressed as

$$156 \quad e = (1 + e_a)\psi_a - 1 \quad (3)$$

157 where  $e_a$  is the void ratio of coarse aggregates. The overall void ratio of the mixture after

158 compaction is shown in Fig. 2 (solid diamond points). It is seen that the “ideal compaction”

159 state can be achieved in case of lower coarse fractions (0%, 20%, 40% and 55%). However,

160 the data point of the mixture with 80% coarse fraction is located above the ideal state, this

161 corresponds to the loose structure beyond the “transitional coarse fraction” in Fig. 3. A

162 possible interpretation is that fine particles of matrix are accommodated between coarse

163 aggregates during preparation, denoted as the “wedging effect”. As described in Fig. 3, the

164 fine particles separate adjacent aggregates from each other and participate in force chains. As

165 a result, the coarse aggregates are prevented from forming an ideal dense structure. This

166 phenomenon has been observed by Goudarzy et al. (2016) using micro-CT scanning.

167 All the specimens are prepared using a moist tamping technique. Different compaction

168 methods were utilized to prepare the specimen according to the coarse fraction: for a high

169 coarse fraction (80% and 100%), the specimen cannot hold itself after being extruded from a

170 mold. Therefore, it was directly compacted inside the shear box in 4 layers using a vibrating

hammer (each layer has a height of 1.0 cm after compaction). For a lower coarse fraction (0%, 20%, 40% and 55%), the volume of matrix is high enough to sustain the coarse aggregates. The specimen was slightly compacted inside a square mold using a vibrating hammer in 4 layers. The top surface of the layers was scratched after compaction to guarantee the homogeneity of the soil specimen. Finally, the specimen was extruded from the mold to the shear box for a further consolidation and shearing.

An initial stress of 4.7 kPa was applied to the specimens. It was increase to 157-451 kPa by gradually adding weights to the loading hanger, following the steps of 19.6, 39.3, 78.5, 157, 314, and 451 kPa. Finally, the specimens were sheared under a constant vertical stress (157 kPa, 314 kPa, and 451kPa) at a displacement rate of 0.02 mm/min. The shear tests were stopped if the horizontal shear strain reaches 15% or the asymptotic state is approached.

### 3. Test results and discussions

The fraction of coarse aggregates is independent of state of mixtures; therefore, it is not a good choice for analyzing the mechanical behavior of gap-graded mixtures. As an alternative, the volume fraction of the coarse aggregates  $\phi_a$  is adopted for the compressibility of lumpy soils and sand-clay mixtures (Shi and Yin, 2017; Shi et al., 2020). It varies due to different compressibility of the two constituents. It can be approximated by the overall void ratio and the coarse fraction (see Appendix-1):

$$\phi_a = \frac{\psi_a}{1+e} \quad (4)$$

where  $e$  is the overall void ratio,  $\psi_a$  is the coarse fraction of gap-graded soils. Eq. (2) and Eq. (4) incorporate the assumptions that (1) the coarse aggregates are incompressible compared with the porous soft matrix during loading, and (2) the particle density of fines is the same as that of coarse aggregates. Since the fines originate from disintegration of parent rock aggregates, this assumption is reasonable.



The volume fraction of coarse aggregates during oedometer loading can be computed by Eq. (4), and its evolution is shown in Fig. 4. As the void ratio decreases, the volume fraction of aggregates increases with increasing stress level according to Eq. (4). This is consistent with of change of the volume fraction of aggregates in Fig. 4. Note that, the change of volume fraction of aggregates is lower than 0.02, which is negligible within the tested stress levels. In the sequel, the volume fraction is treated as a constant during the compression and shearing process. Note that the change of volume fraction can not be neglected if the matrix undergoes a large deformation, e.g., at extremely high stress level, or the matrix is a soft clay slurry. However, if the volume fraction of phases is incorporated into the sequel analysis, it is also applicable to gap-graded soils with a considerable deformation.

The overall mobilized stress ratio of the gap-graded soils  $\mu_\sigma$  is defined as

$$\mu_\sigma = \frac{\tau}{\sigma'} \quad (5)$$

where  $\sigma'$  and  $\tau$  are the overall vertical stress and overall shear stress of the gap-graded soils, respectively. Note that the stresses are calculated using the initial cross area according to ASTM-3080. Therefore, the vertical stress is marked as a constant. Since the vertical stress and the shear stress are calculated using the same cross area, the stress ratio is irrelevant to the cross area. The stress field is nonuniform, as the stress in aggregates is higher than that of in fine matrix (Lielens et al., 1998; de Boer, 2006; Shi et al., 2020). The stress variables in Eq. (5) are defined as volume average values.

The relationship between overall mobilized stress ratio and overall horizontal shear strain for the gap-graded soils with various coarse fraction is shown in Fig. 5. The overall stress ratio of mixtures shows a relative smooth change at low coarse fractions (below 55%). For the samples with a high coarse fraction, there is an oscillation at a large displacement when the sample shows a constant volume state. This is consistent with the results after previous researches (Li et al., 2013; Wei et al., 2018). The mobilized stress ratio of coarse-fine

220 mixtures relies on the roughness of the shear surface. DEM simulations reveal that the coarse-  
221 fine mixtures exhibits a relative uniform distribution of contact force chains at low coarse  
222 fractions. At high coarse fractions, the contact force chains become nonuniform and  
223 concentrated between the coarse aggregates (Xu et al., 2019). This may lead to collapse and  
224 rearrangement of aggregates in vicinity of the shear plane (Wei et al., 2018). As a result, the  
225 overall shear stress shows an oscillation, especially at large strains. The curve shows strain-  
226 hardening behavior for the samples with a lower coarse fraction (0%, 20% and 40%) and  
227 strain softening behavior for pure aggregates (or specimens with a high coarse fraction at low  
228 stress levels). For a given overall shear strain within the prefailure range, the mobilized stress  
229 ratio increases with coarse fraction, except for the one with a coarse fraction of 80%. Similar  
230 phenomenon was also reported by previous researchers that the mobilized stress ratio of gap-  
231 graded specimens with a coarse fraction of ca. 80% is lower than that with smaller coarse  
232 fractions (e.g., Vallejo and Mawby, 2000; Elkady *et al.*, 2015; Wei *et al.*, 2018). This can be  
233 interpreted by the macro-pores within the inter-aggregate apace. As shown in Fig. 3, the black  
234 aggregate may move towards the macro-voids due to loss of constraints from the surrounding  
235 fines, this suggests an increase of the degree of freedom of coarse aggregates.

236 It is well recognized that the strain along the vertical profile of specimens is nonuniform.  
237 DEM simulations (Zhang *et al.*, 2018) show that the strain is concentrated in vicinity of the  
238 shear plane. For a complementary qualitative analysis, the vertical strain is calculated from  
239 the displacement of the loading cap. The vertical deformation of the gap-graded soils with  
240 various coarse fraction is plotted in Fig. 5 in terms of the overall vertical strain and overall  
241 horizontal shear strain. The maximum dilation angle is also computed and summarized in  
242 Table. 2. It is seen that the pure fine matrix and the gap-graded soil with 20% coarse fraction  
243 show volume contraction during the shear process. The shear dilatancy become more  
244 significant as the coarse fraction increases, which coincides with the evolution of mobilized

stress ratio. As reported by Monkul and Ozden (2007), the overall behavior is mainly controlled by the fine matrix, until it is reduced to the “transition fines content” ( $\phi_m = 1 - \phi_a = 0.49$ ). The coarse fraction is below “transition coarse content” for the specimens with coarse fractions of 73%. However, it still shows granular-like behavior with a distinctive shear dilatancy. This phenomenon can be interpreted by the densified clay layer between coarse aggregates. The stiffer layer acts like a bridge between adjacent coarse aggregates, which contributes to formation of a granular-like structure (Jafari and Shafiee 2004; Shi et al., 2019). With a further increase of the coarse fraction, volume contraction is suppressed, and shear-induced contraction prevails.

#### 4. A simple model for the mobilized stress ratio

If the gap-graded soils are simplified as a mixture of fines and coarse aggregates, the overall behavior of mixtures reinforced by stiff inclusions can be analyzed within mixture theory. The overall mobilized stress ratio relies on both the inter-aggregate skeleton and that of the soil matrix. Correspondingly, the following is addressed for modelling mobilized stress ratio of gap-graded soils: (1) the change of mobilized stress ratio of the fine matrix, and (2) the evolution of inter-granular structure during shearing process.

The coarse fraction  $\psi_a$  is constant during mechanical loading. Therefore, it is not suitable for describing the change of inter-aggregate structure, especially for very soft matrix e.g., sand-bentonite mixtures which shows a large deformation. As reported by Tandon and Weng (1988), the volume average concept provides satisfactory descriptions of the behavior of gap-graded geomaterials. The volume fraction  $\phi_a$  is a state variable, relying on the void ratio. Therefore, it is used for homogenizing state variables of binary gap-graded soils, and the relationship between the overall strain and the value of matrix is

$$\varepsilon_h = (1 - \phi_a) \varepsilon_{h,m} \quad (6)$$

where  $\varepsilon_h$  is overall horizontal shear strain of gap-graded soils, and  $\varepsilon_{h,m}$  is the horizontal shear strain of the matrix. They are correlated using the volume fraction of sand aggregate. It should be noted that the fines cannot fully fill the inter-aggregate space for an extremely high fraction of aggregates (e.g., 80% in this study), leading to macro-pores in the gap-graded soils. In this case, Eq. (4) is not applicable.

The mobilized stress ratio of fine matrix  $\mu_{\sigma,m}$  is defined as

$$\mu_{\sigma,m} = \frac{\tau_m}{\sigma'_m} \quad (7)$$

where  $\sigma'_m$  and  $\tau_m$  are the vertical stress and shear stress of the matrix, respectively. Both are defined as volume average values. However, the mobilized stress ratio of fine matrix cannot be computed from its definition, since the local stresses cannot be measured in direct shear tests. The change of mobilized stress ratio of pure sand is essentially dependent of vertical stress levels. A hyperbolic function is adopted to correlate the mobilized stress ratio and the horizontal shear strain of the fine matrix:

$$\mu_{\sigma,m} = \frac{\varepsilon_{h,m}}{a + b\varepsilon_{h,m}} \quad (8)$$

where  $a$  and  $b$  are model parameters for the fine matrix. The vertical stress does not show significant change during the shearing process. Therefore, the slope of tangent in Fig. 5 resembles the tangent stiffness of the mixtures. The hyperbolic function is successful in simulating decrease of stiffness and asymptotic state of various geomaterials (Duncan and Chang, 1970; Yin, 1999a; Chong and Santamarina, 2016; Nagula et al., 2018). The test data and regression curve are presented in Fig. 6 in terms of the relationship between  $\varepsilon_{h,m}/\mu_{\sigma,m}$  and  $\varepsilon_{h,m}$ , revealing an excellent regression with a correlation coefficient of 0.99.

To model the mobilized stress ratio within mixture theory, it is assumed that the inter-

aggregate space is fully filled with the fine matrix. This corresponds to the weathered soils due to intense disintegration of weak rocks. As shown in Fig. 2, macro-pores prevail between the aggregates when the coarse fraction is higher than 73%. Therefore, only the specimens with a coarse fraction less than 73% (0%, 20%, 40%, 55%) can be treated based on the homogenization theory. The initial porosity of the fine matrix is the same as that of the pure fine soil, therefore, it is reasonable to assume that the change of mobilized stress ratio of the matrix in gap-graded soils follows the one of the pure fine soil (Eq. 8). Substitution of Eq. (6) into Eq. (8) gives:

$$\mu_{\sigma,m} = \frac{\varepsilon_h}{a(1-\phi_a) + b\varepsilon_h} \quad (9)$$

The mobilized stress ratio of the matrix can be estimated from the volume fraction of aggregates and the overall horizontal strain. To create a bridge between the mobilized stress ratio of matrix and the overall value, a structure variable  $\eta$  is introduced. It is defined as the ratio of the overall mobilized stress ratio and the mobilized stress ratio of the fine matrix:

$$\eta = \frac{\mu_{\sigma}}{\mu_{\sigma,m}} \quad (10)$$

Fig. 7 illustrates the change of structure variable  $\eta$  against the overall horizontal shear strain. The structure variable decreases with the overall horizontal shear strain during the shearing process, which becomes approximately constant beyond 9.0% of the overall shear strain. The structure variable appears to be rather independent of the vertical stress level, although there is some scatter at initial shearing stage. To reduce the oscillation of test data, average value of the structure variable at three different stress levels is taken. The results of gap-graded soils with three different coarse fractions are summarized in Fig. 8. It is seen that the structure variable is affected by both the coarse fraction and the horizontal shear strain. The data points move upwards as the coarse fraction increases. After being replotted in double logarithmic plot,  $\ln \eta$  changes almost linearly with  $\ln \varepsilon_h$ :

$$\eta = \exp(\beta - \alpha \ln \varepsilon_h) \quad (11)$$

where  $\alpha$  and  $\beta$  are structure variables which seems independent of overall horizontal shear strain during shearing process. At a given overall shear strain, substitution of Eqs. (9) and (11) into Eq. (10) gives

$$\mu_\sigma = \frac{\exp(\beta - \alpha \ln \varepsilon_h) \varepsilon_h}{a(1 - \phi_a) + b\varepsilon_h} \quad (12)$$

The structure variables for the gap-graded soils with various coarse fractions are calibrated using regression analysis as given in Table 3. It is seen that both two variables increase nonlinearly with the increase of coarse fraction. There are two limit cases for the structure variables ( $\alpha$  and  $\beta$ ) of gap-graded soils. The overall mobilized stress ratio should equal the one of the matrix in case of a negligible coarse fraction ( $\phi_a \approx 0$ ), i.e.,  $\eta=1$ , and  $\alpha = \beta=0$ . When the fines fraction equals the minimum porosity of the pure coarse aggregates, the overall void ratio approaches the minimum value (the lowest point in the ideal compaction line in Fig. 2). The corresponding volume fraction of the aggregates  $\bar{\phi}_a$  is given as

$$\bar{\phi}_a = \frac{1}{1 + e_{\min,a}} \quad (13)$$

An inter-aggregate structure forms at the densest packing state of gap-graded soils, which overtakes additional external loading. Note that the stiffness of the inter-granular structure in gap-graded soils should be much higher than the one with pure aggregates: For granular specimens with pure aggregates, fracturing and collapse of load-bearing aggregates induces a rearrangement of the structure (Pestana and Whittle, 1995; McDowell 2002; McDowell and Harireche 2002; Mesri and Vardhanabhuti, 2009). This induces a remarkable decrease of the stiffness. On the contrary, the inter-aggregate space in gap-graded soils is filled with a matrix (0%, 20%, 40% and 55% coarse fraction in this study). The stress in coarse aggregates is relatively uniform due to the confining stress provided by the surrounding matrix. This

decreases the possibility of splitting and fracturing of coarse aggregates and prevents further rearrangement of inter-aggregate structure (Zhang and Baudet, 2013). Consequently, the intergranular structure for gap-graded soils is much more stable than the one with pure aggregates, and the structure variables ( $\alpha$  and  $\beta$ ) should be extremely high at the densest packing state.

Based on the above discussions, the structure variables can be expressed as follows:

$$\alpha = \left( \frac{\bar{\phi}_a}{\bar{\phi}_a - \phi_a} \right)^{\xi_1} - 1 \quad (14a)$$

$$\beta = \left( \frac{\bar{\phi}_a}{\bar{\phi}_a - \phi_a} \right)^{\xi_2} - 1 \quad (14b)$$

where  $\xi_1$  and  $\xi_2$  are structure parameters which controls the sensitivity of structure variables ( $\alpha$  and  $\beta$ ) on the coarse fraction. Since the structure parameters rely on the evolution of inter-aggregate skeleton, they depend on shape and size distribution of coarse particles (Herle and Gudehus, 1999).

The basic structure of gap-graded mixture is characterized by the size ratio between large and small particles, and the coarse fraction. Both the effect of coarse fraction and particle size ratio (between coarse and fine particles) is considered in the proposed model. The coarse fraction effect is explicitly incorporated into Eq. (14). According to mixture theory, no matter what the particle size ratio is, the mechanism governing the overall behavior of gap-graded soils is the same (Ueda et al., 2011; Zhou et al., 2016). The effect of particle size ratio (between coarse and fine particles) on the deformation behavior of gap-graded soils is implicitly incorporated into the model by means of the structure variables in Eq. 14. The structure variables,  $\alpha$  and  $\beta$ , decrease as the particle size ratio falls, and they reduce to 0 if the particle size of the fines and aggregates is the same.

## 5. Validation of the proposed model

The proposed model has four parameters:  $a$ ,  $b$ ,  $\xi_1$  and  $\xi_2$ .  $a$  and  $b$  are for the mobilized stress ratio of the fine matrix,  $\xi_1$  and  $\xi_2$  are structure parameters correlated with the change of inter-aggregate structure with increasing coarse fraction. Only two direct shear tests are required for calibrating the mentioned model parameters: one on pure fine matrix, and the other on gap-graded soil with a predefined coarse fraction.  $a$  and  $b$  can be calibrated from the relationship between mobilized stress ratio and horizontal shear strain of fine matrix. The structure parameters  $\alpha$  and  $\beta$  can be calibrated by trial and error procedure through a direct shear test on a gap-graded specimen. Experimental data of two gap-graded soils are used for validating the model: soil-sand mixtures in this work, and the sand-gravel mixtures from literature (Vallejo *et al.*, 2014).

### *Gap-graded soils in this work*

The gap-graded soils in this work is a mixture of coarse sand material and fine sandy soils, with fine sandy soils being the matrix and the coarse material being the inclusions. The proposed model is evaluated in this section by comparing simulations with the test data of the mixtures. Calibration of the model parameters are shown in Figs. 6, 9 and 10, and the value of parameters are listed in Table 4. To distinguish the data between different stress levels, the results are presented in terms of overall shear stress and overall horizontal shear strain. The simulated curves for the gap-graded soils with various fractions are shown in Fig. 11, together with the experimental results ( $\psi_a=0\%$ , 20%, 40%, 55%, and 80%).

In this work, we focus on gap-graded soils with matrix-sustained structure. As shown in Figs. 11 and 12, the model can well capture the change of stress ratio in direct shear test with coarse fraction below the transitional value. However, the model predictions deviate from the test data with a coarse fraction beyond 73% for the Chongqing soils. This is due to prevailing



macro-voids within inter-aggregated space beyond the transitional coarse fraction (Fig. 3), and the emerging macro-voids reduce the stiffness and shear strength of the gap-graded soils. As a result, the model based on mixture theory is not applicable anymore. As been reported by other researchers, the coarse fraction is usually beyond the transitional value for sandy marine deposits and gap-graded soils in intensely weathered areas (Zhao et al., 2007; Cui et al., 2017; Chen et al., 2020c). Therefore, gap-graded granular soils usually have a matrix-sustained structure, and their behavior can be well reproduced using the proposed model.

392

### 393 *Gap-graded soils from literature*

Two granular materials are used for producing gap-graded soils by Vallejo *et al.* (2014): gravel is used as coarse inclusions, and Ottawa sand was used as the matrix. The coefficient of uniformity of Ottawa sand is 1.3, with an average diameter of 0.59 mm. The maximum and minimum void ratios are 0.88 and 0.48, respectively. The average diameter of coarse inclusions is 5 mm. The minimum void ratio of the inclusion is assumed to be 0.60 for the calculation of  $\bar{\phi}_a$  in Eq. (13). The particle densities of gravel inclusions and sand matrix are 2.40 Mg/m<sup>3</sup> and 2.65 Mg/m<sup>3</sup>, respectively. Gap-graded soils with seven different coarse fractions (from 0% to 30%) were tested by the authors. The initial void ratio of the matrix in the gap-graded specimens are 0.8 regardless of the coarse fractions. Three various fractions, 0%, 15%, and 30%, are used for validation of the model, and the corresponding volume fractions of the inclusions are 0%, 8.9%, and 19.6%, respectively. The specimens were compressed to two desired vertical stresses (52 kPa and 103 kPa) followed by a direct shear process with a displacement rate of 0.02 mm/min. Fig. 12 presents a comparison of the experimental data with model predictions of the sand-gravel mixtures, suggesting that the model can represent the coarse fraction effect on mobilized shear strength in direct shear tests.

409

## 410 6. Conclusions

411 A simple model has been proposed for describing the evolution of mobilized stress ratio, and  
412 its validation is presented using the experimental data in this study and data from literature.

413 Conclusions are summarized as follows:

414 (1) The initial overall void ratio of the binary soils initially decreases approximately linearly  
415 with the coarse fraction. With a further increase of the coarse fraction, the initial overall  
416 porosity increases, and the mobilized stress ratio decreases due to arising macro-pores within  
417 the inter-aggregates space.

418 (2) The change of inter-aggregates skeleton is considered by introducing a structure variable.  
419 It increases with volume fraction of coarse aggregates and decreases approximately linearly  
420 with the overall horizontal shear strain in double logarithmic plot.

421 (3) The model has four parameters which can be easily calibrated from two direct shear tests  
422 on pure matrix and gap-graded soils, respectively. The model reproduces well the evolution of  
423 mobilized stress ratio of the gap-graded soils in this work and those from the literature.

## 425 Appendix-1

426 There are two types of structure according to the coarse fraction, matrix-sustained structure  
427 and aggregate-sustained structure. In this study, we deal with the matrix-sustained structure,  
428 and the structure of the fine matrix is relatively uniform without macro-voids (Fig. 3).  
429 Therefore, volume of voids in mixtures can be well represented by the volume of voids in fine  
430 matrix, and the overall deformation of gap-graded mixtures relies on the decrease of volume  
431 of voids in fines. The mixture can be divided into three parts: (1) the solid phase of fine  
432 matrix (denoted as  $V_{sm}$ ), (2) the volume of voids in fine matrix (denoted as  $V_{vm}$ ), and (3) the

433 volume of coarse aggregates (denoted as  $V_a$ ). Suppose that the volume of solid phase of fine

434 matrix is one unit (i.e.,  $V_{sm}=1$ ). The volume of voids in fine matrix is

$$435 \quad V_{vm} = e_m \quad (15)$$

436 Where  $e_m$  is the void ratio of fine matrix. From the definition of the void ratio of aggregates  $e_a$ ,

437

$$438 \quad V_a = \frac{V_{sm} + V_{vm}}{e_a} = \frac{1 + e_m}{e_a} \quad (16)$$

439 Hence, the coarse fraction and overall void ratio at the transitional point can be derived

440 according to its definition:

$$441 \quad \tilde{\psi}_a = \frac{V_a}{V_{sm} + V_a} = \frac{1 + e_m}{1 + e_m + e_{\min,a}} \quad (17)$$

$$442 \quad \tilde{e} = \frac{V_{vm}}{V_{sm} + V_a} = \frac{e_m e_{\min,a}}{1 + e_m + e_{\min,a}} \quad (18)$$

443 The overall void ratio can be derived according to its definition:

$$444 \quad e = \frac{V_{vm}}{V_{sm} + V_a} = \frac{e_m}{1 + V_a} \quad (19)$$

445 The dry mass of the fines  $m_s$  and aggregates  $m_a$  are

$$446 \quad m_s = \rho_s V_{sm}; \quad m_a = \rho_a V_a \quad (20)$$

447 Suppose that the density of fines and aggregates are the same, the coarse fraction is derived as:

$$448 \quad \psi_a = \frac{m_a}{m_s + m_a} = \frac{\rho_a V_a}{\rho_s V_{sm} + \rho_a V_a} = \frac{V_a}{1 + V_a} \quad (21)$$

449 The following equations can be derived from Eq. (19) and Eq. (21):

$$450 \quad e_m = \frac{e}{1 - \psi_a}; \quad V_a = \frac{\psi_a}{1 - \psi_a} \quad (22)$$

451 The volume fraction of aggregates is defined as:

$$452 \quad \phi_a = \frac{V_a}{V_{sm} + V_{vm} + V_a} = \frac{V_a}{1 + e_m + V_a} \quad (23)$$

453 Substitution of Eq. (22) into Eq. (23), it gives:

$$454 \quad \phi_a = \frac{\psi_a}{1+e} \quad (24)$$

455

## 456 List of symbols

$a, b$	Model parameters for the fine matrix
$C_u$	Coefficient of uniformity
$C_c$	Coefficient of curvature
$D_{50}$	Diameters at 50 percentiles in PSD curve
$e$	Overall void ratio
$e_a$	Void ratio of coarse aggregates.
$e_m$	Void ratio of fines
$e_{\min}$	Overall minimum void ratio
$e_{\min,a}$	Minimum void ratio of coarse aggregates
$e_{\max,a}$	Maximum void ratio of coarse aggregates
$m_s$	Dry mass of fine matrix
$m_a$	Dry mass of coarse aggregates
$V_a$	Volume of coarse aggregates
$V_{sm}$	Volume of solid phase in fine matrix
$V_{vm}$	Volume of void phase in fine matrix
$\alpha, \beta$	Structure variables
$\varepsilon_h$	Overall horizontal shear strain
$\varepsilon_{h,m}$	Horizontal shear strain of matrix
$\varepsilon_v$	Overall vertical strain
$\eta$	Structure variable
$\mu_\sigma$	Overall mobilized stress ratio

$\mu_{\sigma,m}$	Mobilized stress ratio of matrix
$\xi_1, \xi_2$	Structure parameters
$\rho_a$	Density of aggregates
$\rho_s$	Density of fine particles
$\sigma'$	Overall vertical stress
$\sigma'_m$	Vertical stress of matrix
$\tau$	Overall shear stress
$\tau_m$	Shear stress of matrix
$\phi_a$	Volume fraction of coarse aggregates
$\bar{\phi}_a$	The maximum packing density of coarse aggregates
$\psi_a$	Coarse fraction

457

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Table 1: Basic physical properties of the fine matrix and coarse aggregates

Materials	Density of soil particles (Mg/m <sup>3</sup> )	Sand (%)	Silts (%)	Clay (%)	Particle size distribution		
					D <sub>50</sub>	C <sub>c</sub>	C <sub>u</sub>
Fine matrix	2.73	89	9	2	0.19	2.41	5.31
Coarse aggregates	2.63	100	0	0	1.49	0.97	1.32

Table 2: Maximum dilation angle for gap-graded soils with various stress levels and coarse fractions

Stress level (kPa)	$\psi_a=0\%$	$\psi_a=20\%$	$\psi_a=40\%$	$\psi_a=55\%$	$\psi_a=80\%$	$\psi_a=100\%$
157	-1.6°	0.0°	8.3°	26.9°	21.0°	43.2°
314	-3.4°	0.0°	6.7°	23.7°	11.7°	32.8°
451	-3.5°	-1.5°	3.1°	22.5°	7.7°	29.7°

Table 3: Values of structure variables for the gap-graded soils with various coarse fractions

Parameters	$\psi_a=0\%$	$\psi_a=20\%$	$\psi_a=40\%$	$\psi_a=55\%$
$\alpha$	0	0.065	0.138	0.336
$\beta$	0	0.128	0.362	0.985



Table 4: Parameters of the proposed model for the gap-graded soils

Sources	$a$	$b$	$\zeta_1$	$\zeta_2$
	(%)	---	---	---
This study	5.25	1.04	0.24	0.57
Vallejo <i>et al.</i> , 2014	1.06	0.30	0.45	0.90

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Figure 12. Comparison between experimental data and model simulation (test data from Vallejo *et al.*, 2014): (a)  $\psi_a = 0\%$ , (b)  $\psi_a = 15\%$ , (c)  $\psi_a = 30\%$

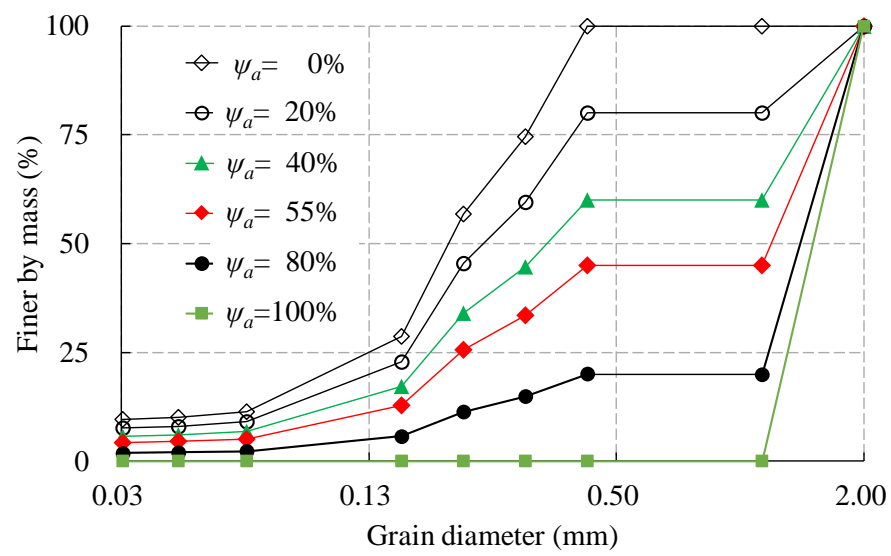


Figure 1: Particle size distribution of the tested gap-graded soil with different coarse fraction

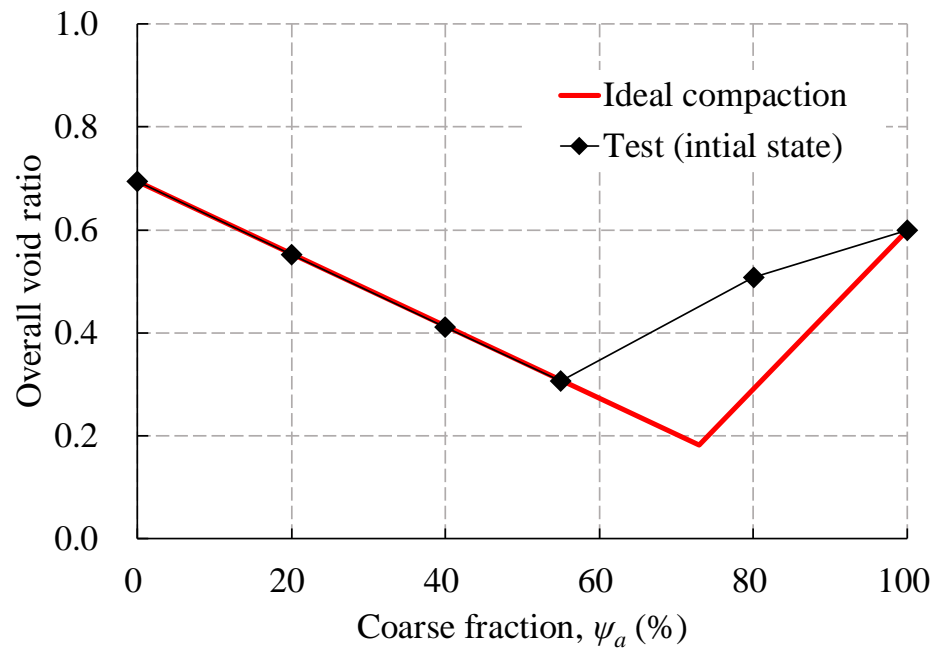


Figure 2: Change of initial void ratio with mass fraction of coarse aggregates

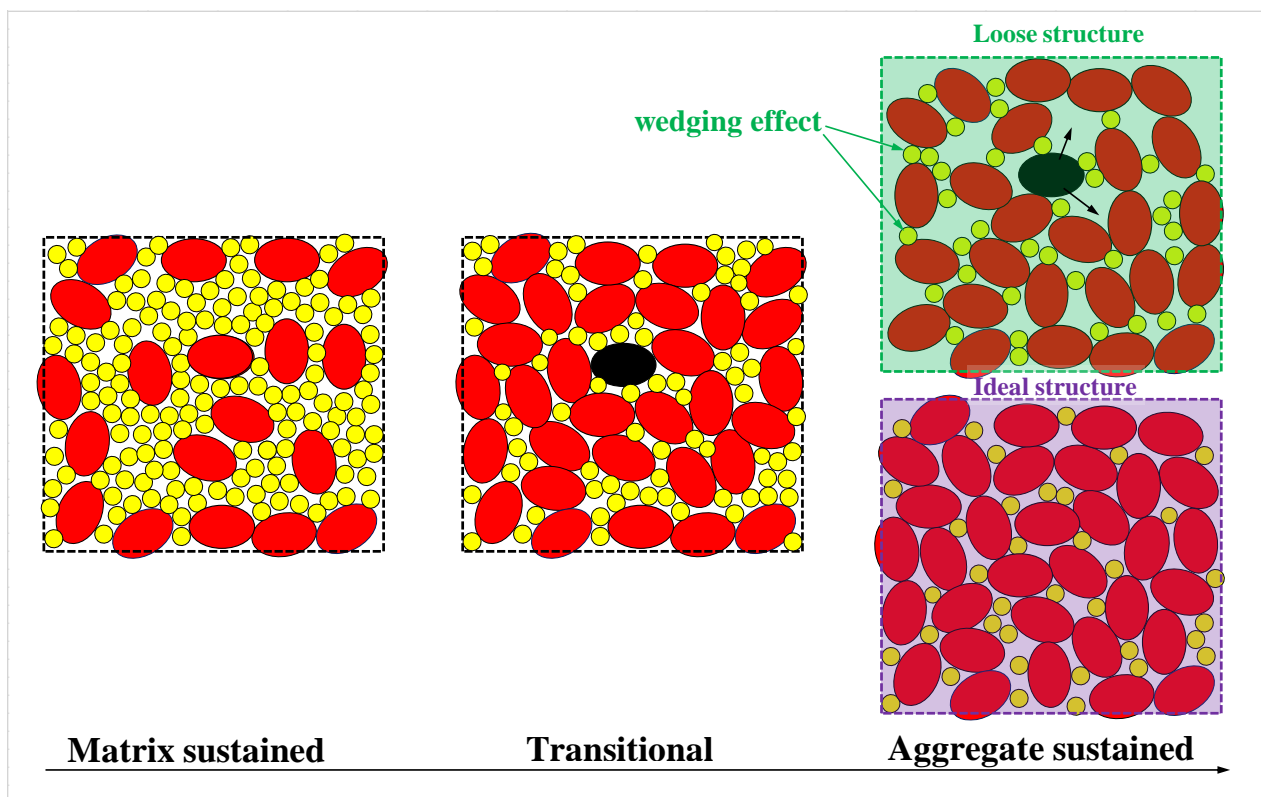


Figure 3: Soil structure with various coarse fractions considering the wedging effect (Goudarzy *et al.*, 2016)

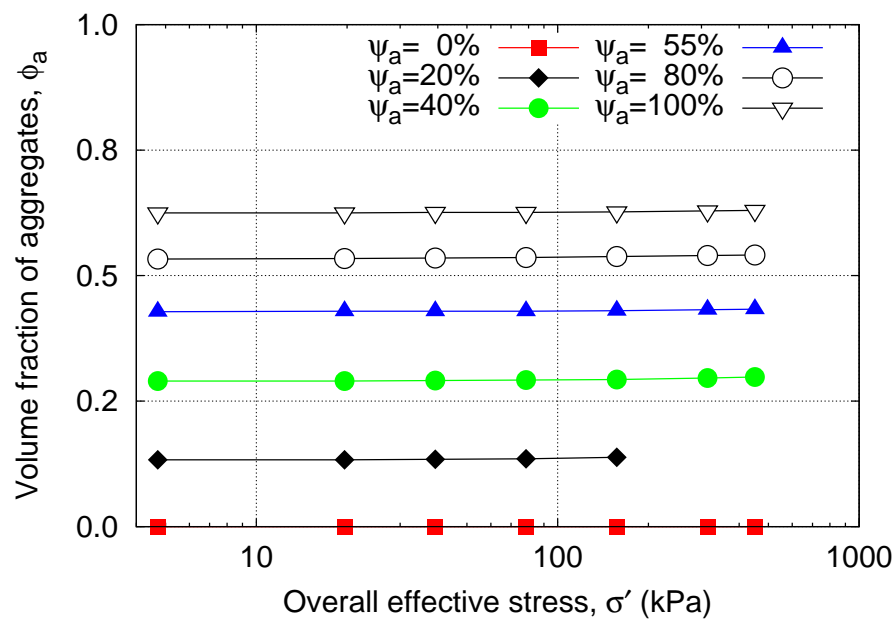
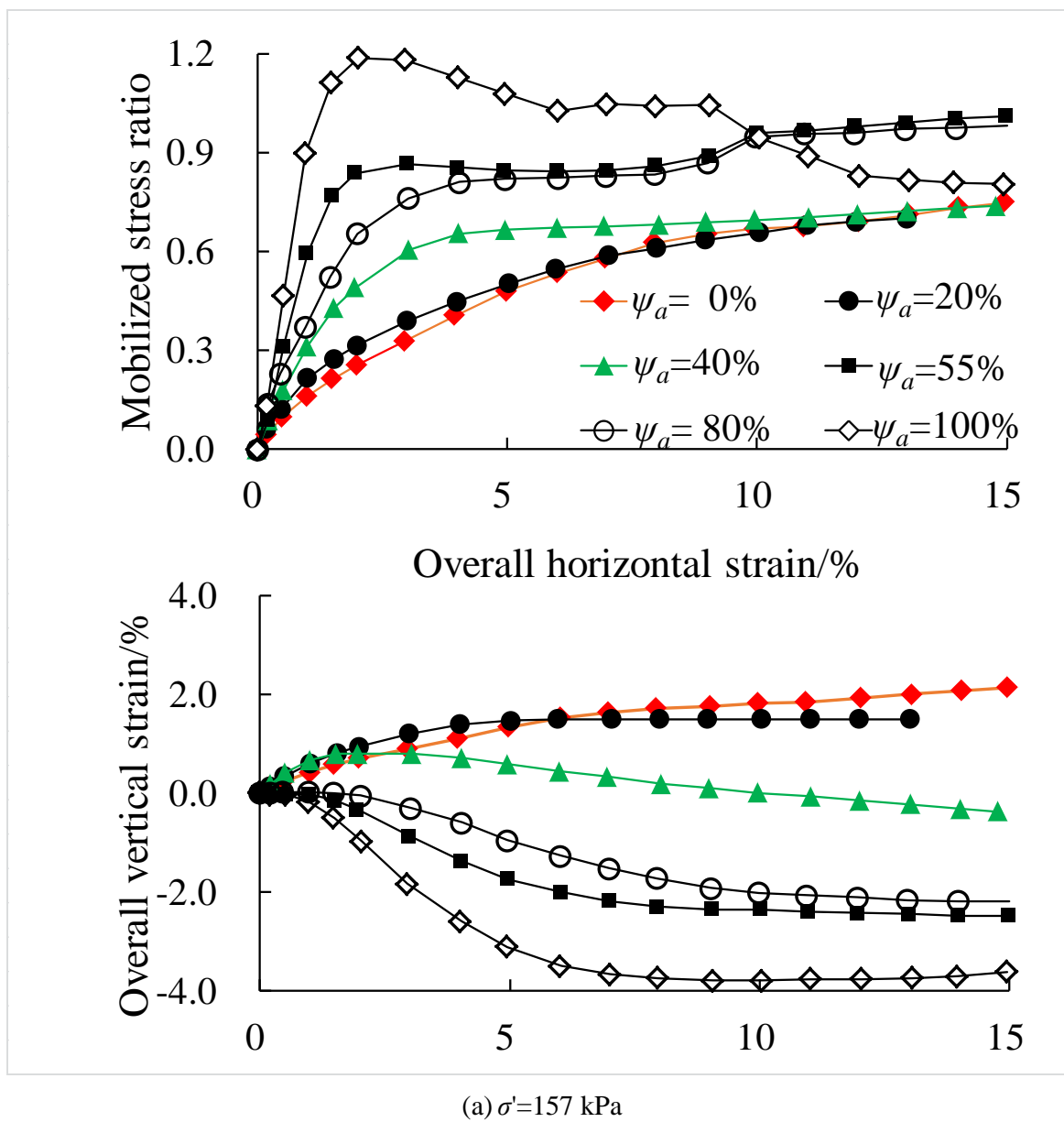
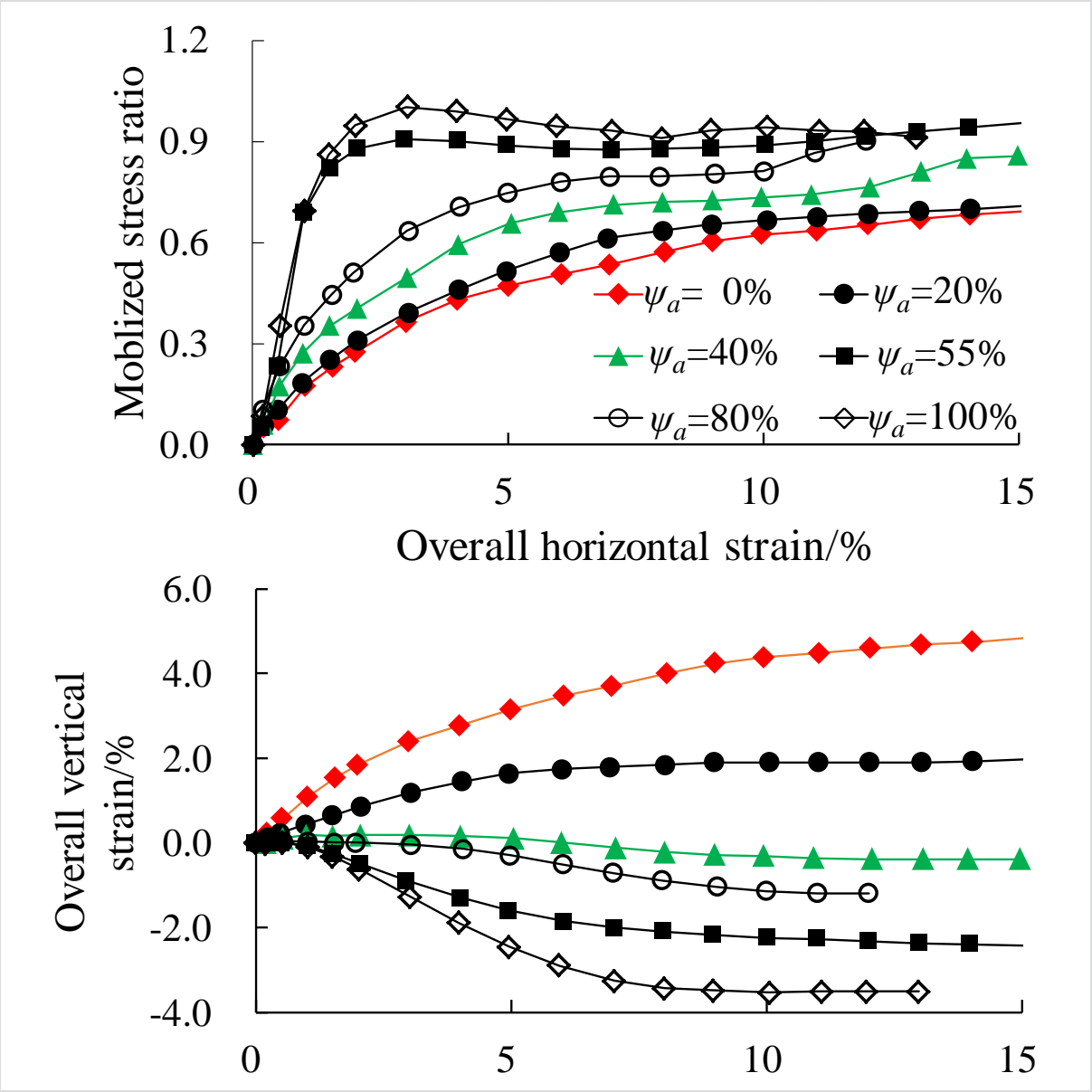


Figure 4: Change of volume fraction of coarse aggregates in the gap-graded soils





(b)  $\sigma' = 314$  kPa



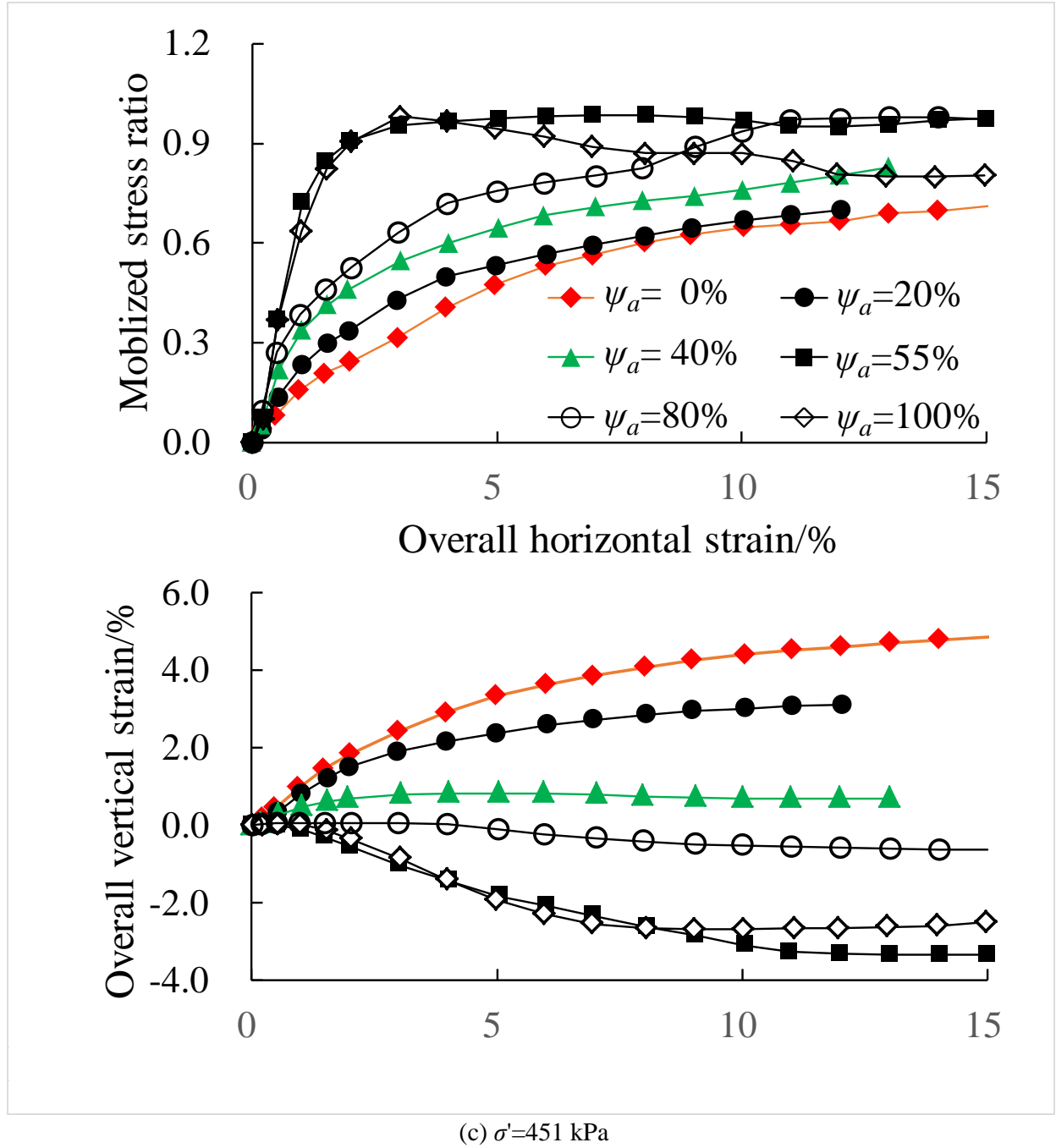


Figure 5: Change of mobilized stress ratio and overall vertical strain for the gap graded soils with various coarse fractions

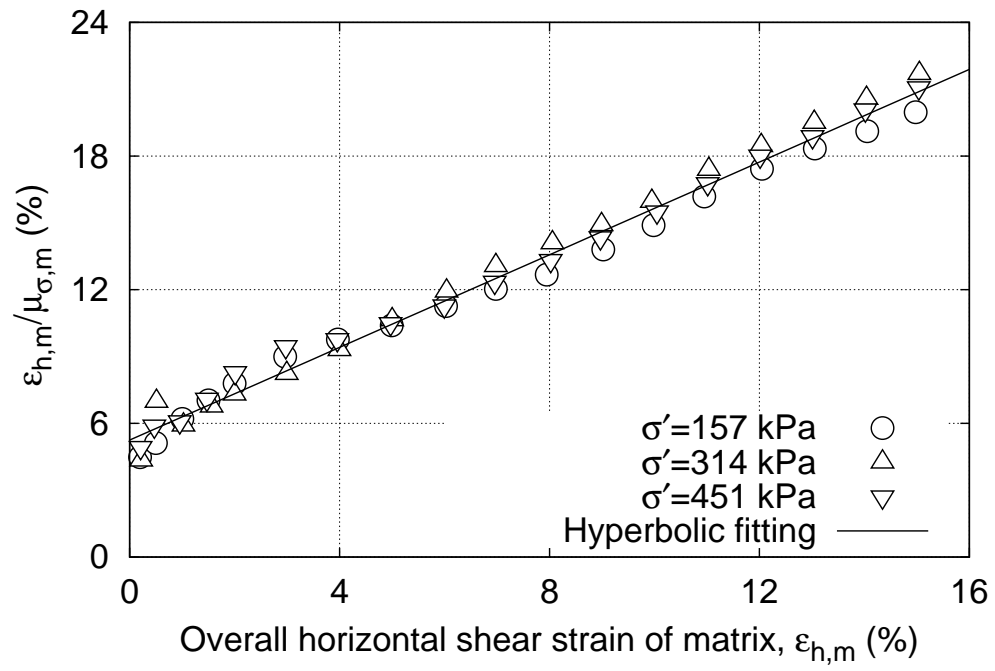
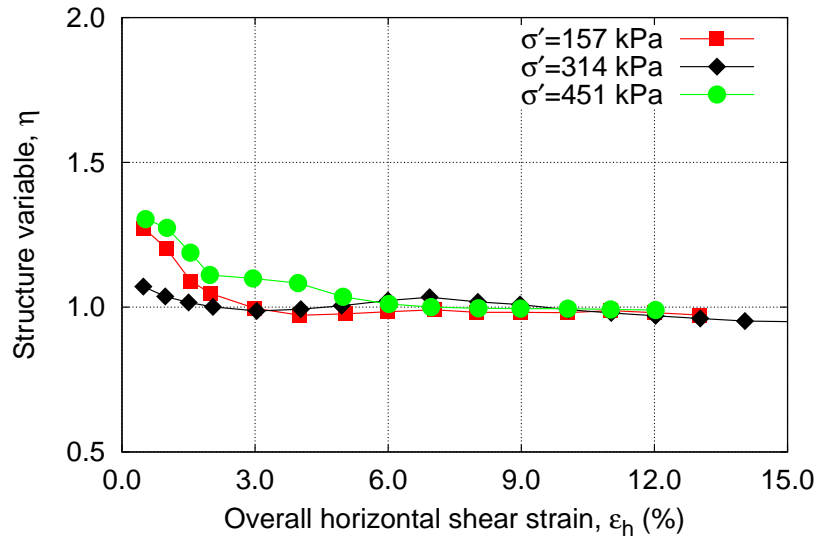
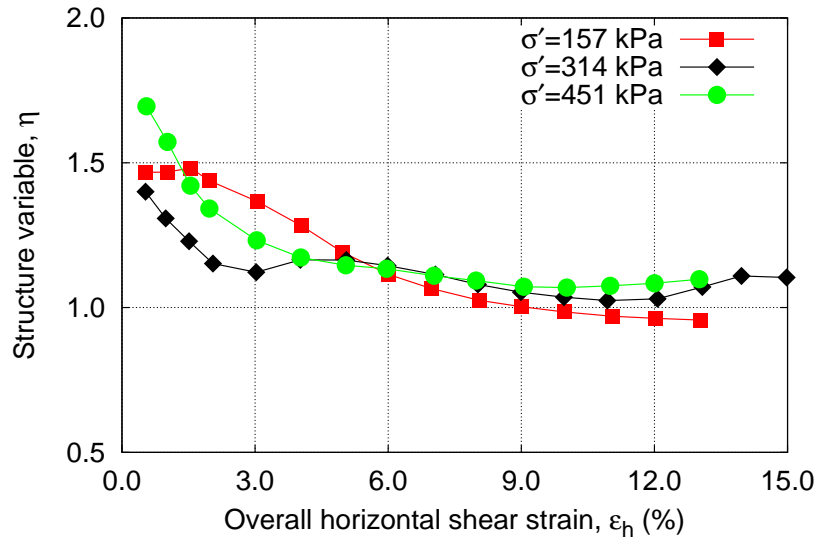


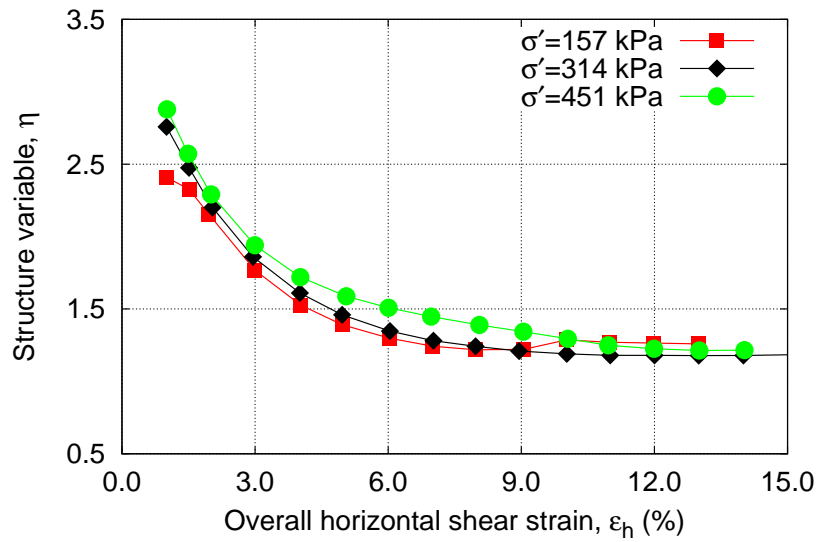
Figure 6: Correlation between the mobilized stress ratio and the overall horizontal shear strain for pure fine matrix



(a)  $\psi_a = 20\%$



(b)  $\psi_a = 40\%$



(c)  $\psi_a = 55\%$

Figure 7: Evolution of the structure variable  $\eta$  with overall horizontal shear strain

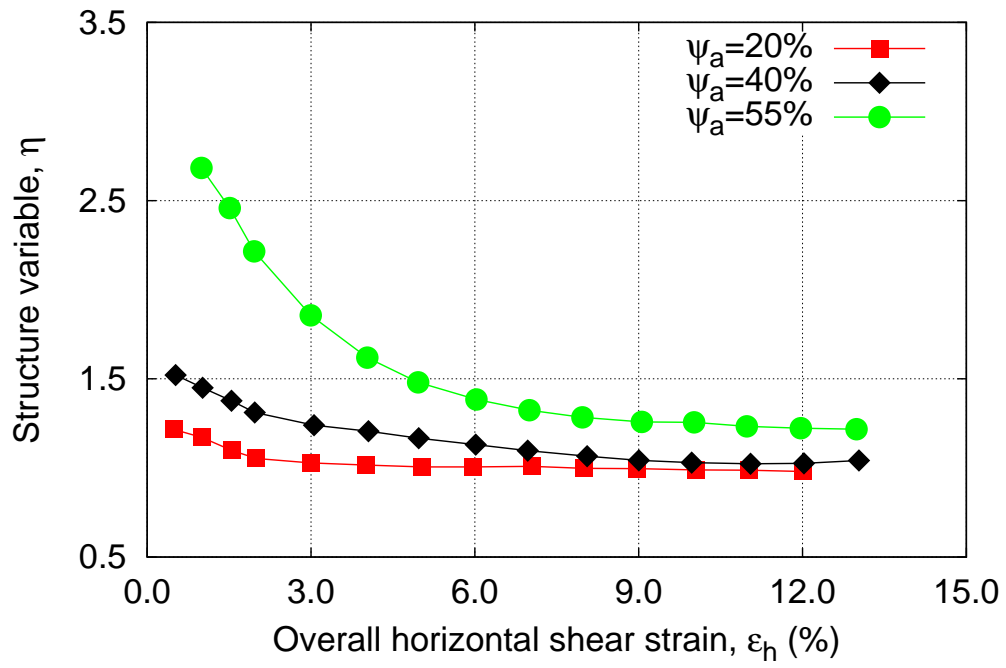


Figure 8: Change of average value of the structure variable  $\eta$  with overall horizontal shear strain

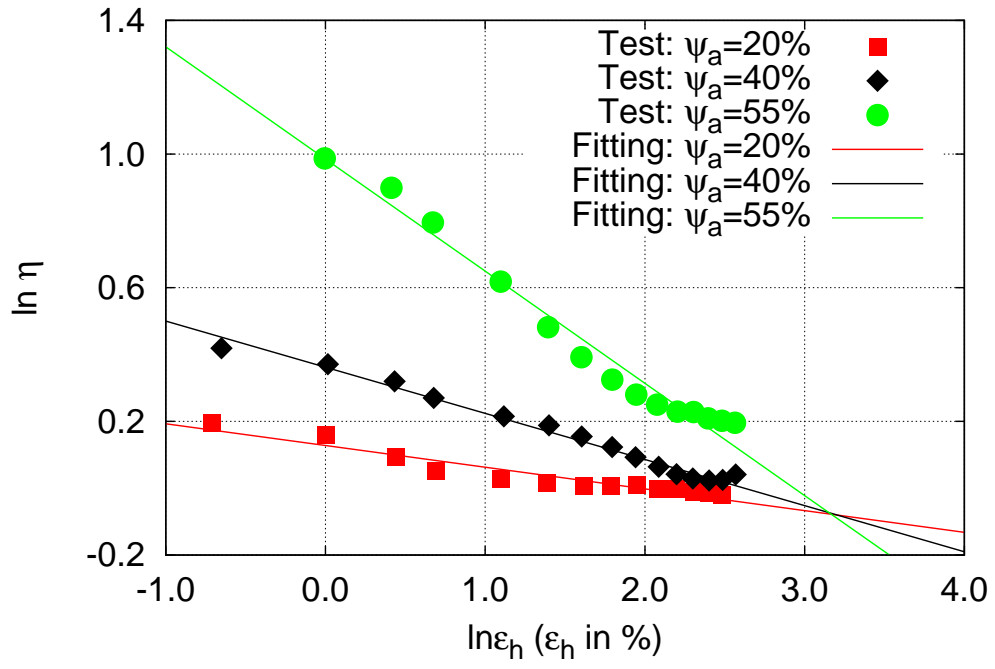


Figure 9: Relationship between logarithms of the structure variable  $\eta$  and overall horizontal shear strain

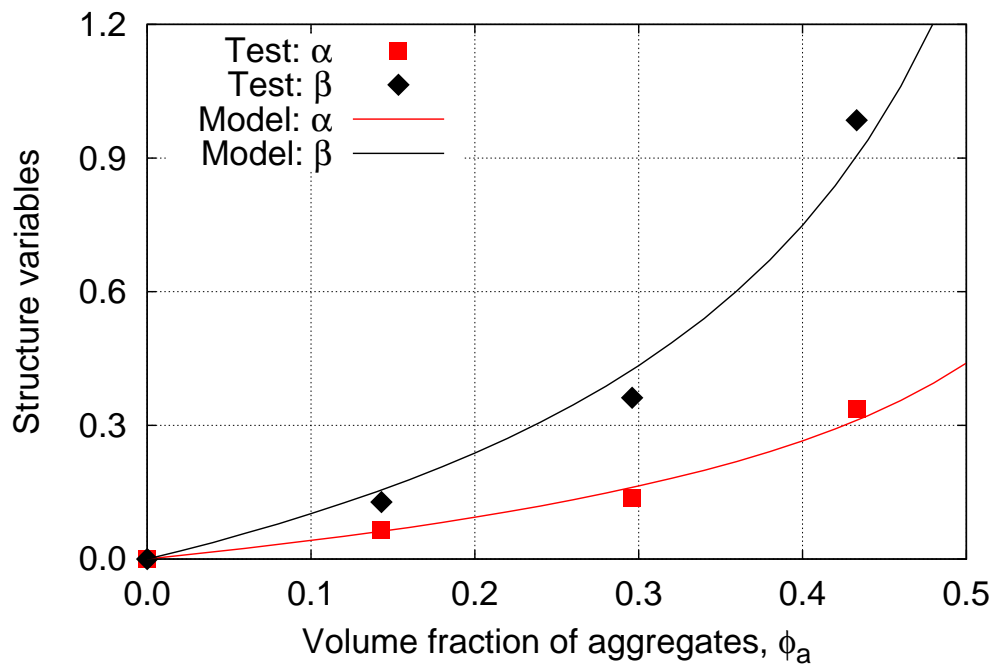
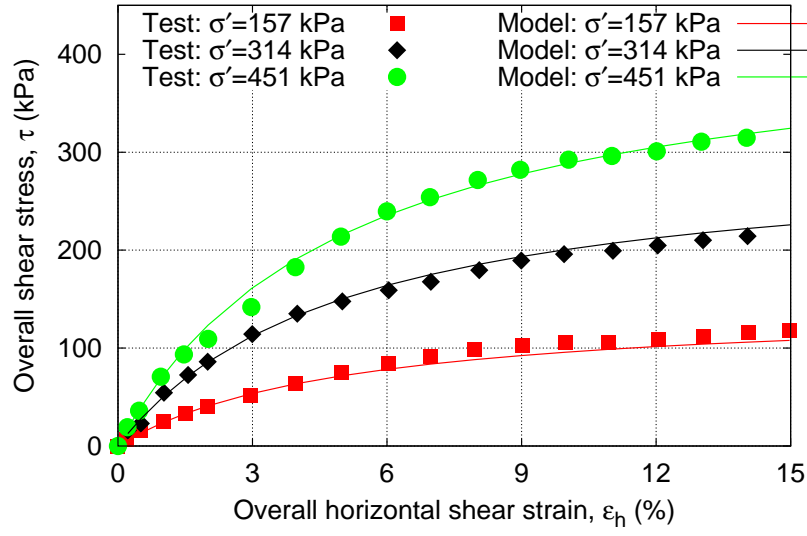
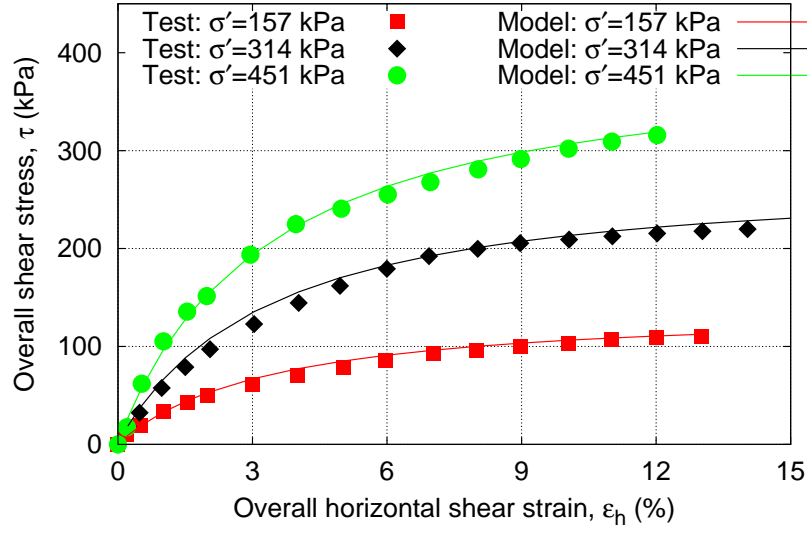


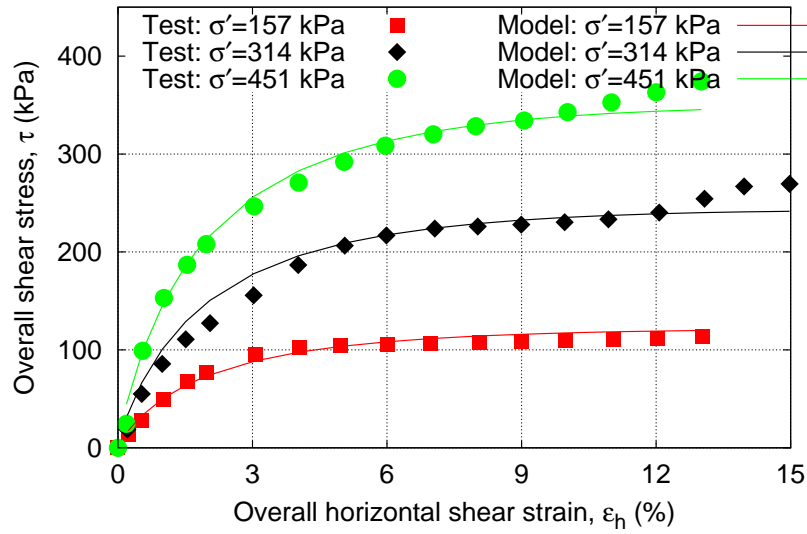
Figure 10: Change of structure variables with volume fraction of coarse aggregates



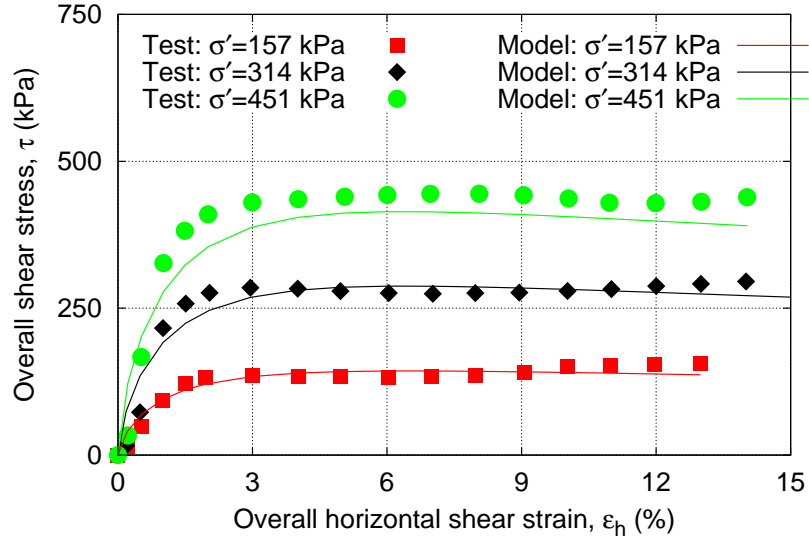
(a)  $\psi_a = 0 \%$



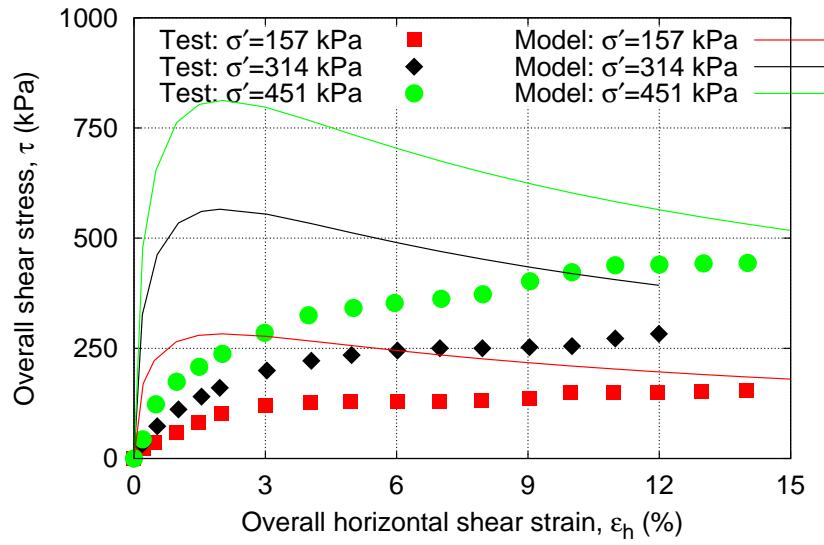
(b)  $\psi_a = 20 \%$



(c)  $\psi_a = 40 \%$



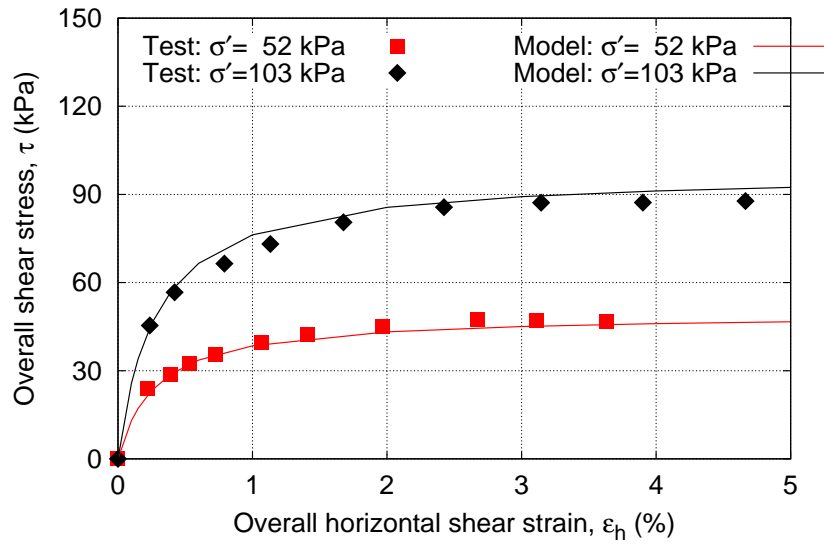
(d)  $\psi_a = 55 \%$



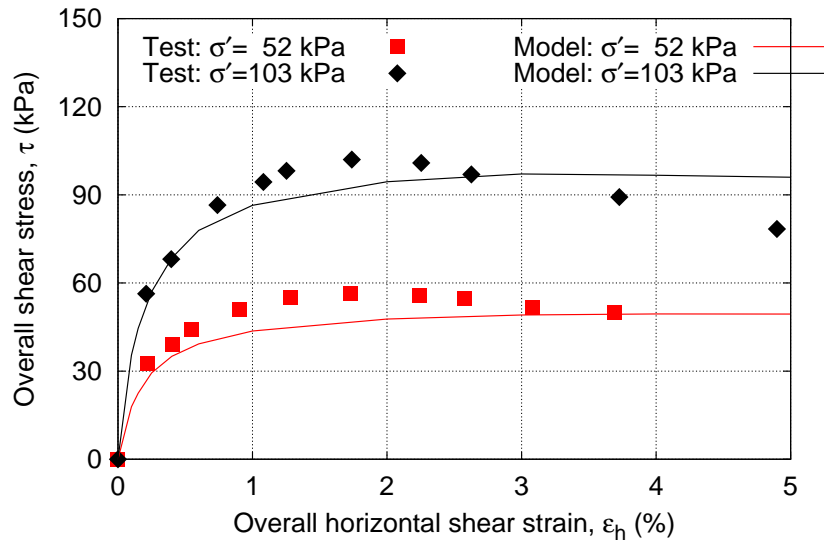
(e)  $\psi_a = 80 \%$

Figure 11: Comparison between experimental data and model simulation

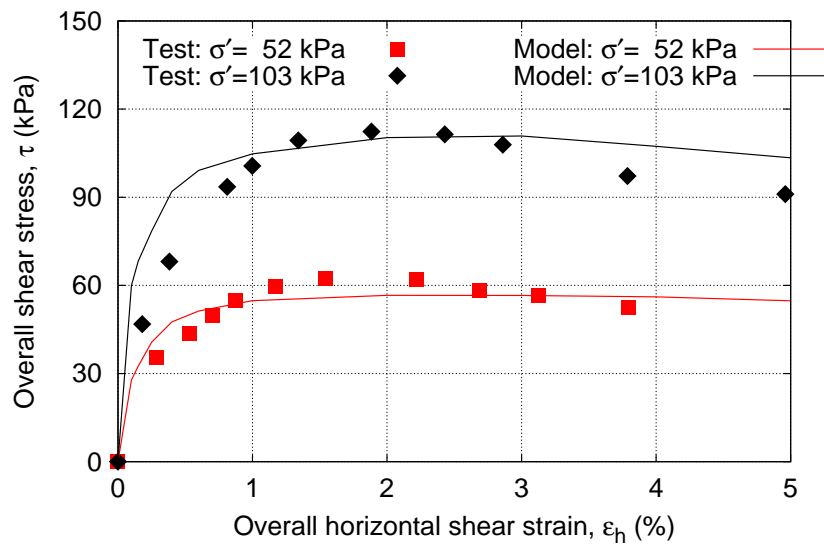




(a)  $\psi_a = 0$  %



(b)  $\psi_a = 15$  %



(c)  $\psi_a = 30$  %

Figure 12: Comparison between experimental data and model simulation (test data from Vallejo *et al.*, 2014)