



**Keywords:** Arching effect; Discrete element method; Fill height; Relative density.

**1. Introduction**

 The arching effect is a common phenomenon existing in many geotechnical applications (Terzaghi, 1943; Wang and Chen, 2019; Chen et al., 2020a), triggered by the relative movement between the stationary and mobilized portions of soils. Due to the arching effect, part of the load transfers from the mobilized portion to the stationary portion through the shear stress at the interface of these two portions. The trapdoor test is an efficient method to investigate the arching effect in granular materials, firstly developed by Terzaghi (1943). To date, such testing apparatus has been utilized in many laboratory tests to characterize the settlement field and load transfer mechanism which are related to the arching effect (Iglesia et al., 1999; Dewoolkar et al., 2007; Chevalier et al., 2009; Costa et al., 2009; Han et al., 2017; Liang et al., 2020).

 The load transfer induced by the arching effect is highly affected by the soil properties (Jenck et al., 2009; Eskişar et al., 2012). Among the soil properties, the 50 relative density  $D_r$  is an important parameter that characterizes the corresponding overall behavior of granular materials (Oda et al., 1980). During shearing, granular materials show the softening and the hardening mechanical behavior in dense and loose states, respectively. Such different mechanical behaviors affect the arching effect, induced by the shearing of granular materials. Thereby, the influence of the relative density should be considered in the arching effect. However, relevant studies remain scarce. Through a series of centrifuge tests, Dewoolkar et al. (2007) found that the minimum stress on the trapdoor of a dense sample is slightly smaller than that of the



zone is more stable than that passing through the loosen zone (Lai et al., 2018).

 Therefore, the partial arching can be defined as that the loosen zone propagates to the surface and only the unstable "arch force bridge" forms. By contrast, the full arching is 81 that the loosen zone forms inside the fill with the formation of stable "arch force bridge" above the loosen zone. Consequently, the arching effect can be classified into two patterns including the partial arching and the full arching. However, the influence of the relative density on different patterns of the arching effect is still not clear. Although Badakhshan et al. (2020) investigated the influence of the relative density on load transfer and porosity changes with different arrangements of piles, only the full arching formed in the high embankment was considered in their simulations.

 In the past decades, many theoretical models have been proposed to calculate the 89 load transfer induced by the arching effect (Terzaghi, 1943; Even, 1983; **BSI 2010;** 90 EBGEO (DGGT, 2010); van Eekelen et al., 2013). In these theoretical models, the fill height is one of the predominant parameters. However, none of these models considers the relative density (or the dilation angle) of granular materials, because these models can only calculate the load transfer induced by the arching effect at the critical state 94 (i.e., soils have reached the ultimate state). Nevertheless, the arching effect does not necessarily reach the critical state in some practical cases. As a result, it is important to understand the intermediate state of the arching effect (i.e., the evolution of load transfer with the movement of the trapdoor). Recently, some theoretical models have been proposed to reveal the progressive development of the arching effect with the 99 movement of the trapdoor (Iglesia et al., 1999; Han et al., 2017; Rui et al., 2018). The



114 the formulae should be further clarified.

 Numerical modelling is a useful tool to simulate the mechanical characteristics of the practical engineering cases (Han and Gabr, 2002; Wang et al., 2020). Compared with the finite element method (FEM), a particle is considered as a basic element in the discrete element method (DEM). Due to the advantages of showing the distribution of the interparticle contact force, particle displacement, and fabric, etc., the DEM has made significant contributions to studying granular materials over the last decades



 from macro to micro perspectives, including the arching ratio, the particle motion, the local porosity, the contact force, the coordination number, and the normal force fabric. 

#### **2. DEM modelling**

#### *2.1 Development of the DEM model*

 Compared with the three-dimensional (3D) DEM simulation, the two-dimensional (2D) DEM simulation provides high computational efficiency and maintains the key evolutions of the granular structure during the formation of the arching effect (Jenck et al., 2009; Lai et al., 2018). Also, the simulation results of 2D DEM can be directly compared with those of purely two-dimensional trapdoor tests (using the Taylor- Schneebeli soil analogues with perfect disk cross-sections as backfill). Therefore, in this study, DEM modelling is based on the purely two-dimensional trapdoor test by Xu et al. (2019). The testing setup is shown in Fig. 1 (a). The trapdoor system consists of seven blocks (B1-B7; width of each block: 128 mm), which can be moved upward and downward independently. The middle block (B4) is used as the trapdoor to move downward in Xu et al. (2019). To maintain a pure 2D condition, the backfill used in the experiment is an analogical soil of aluminium rods with different diameters (3, 4, and 5 mm), mixed at a mass ratio of 1:1:1. This kind of backfill has a uniform disk cross- section which is consistent with the basic elements (circular particle) in the 2D DEM modelling. To ensure uniform force transfer, five layers of aluminium rods with a diameter of 5 mm are laid on the blocks as a cushion. Then, the aluminium rods with  different diameters are placed by layers (thickness for each layer: 20 mm) with an equal mass ratio.

 Accordingly, Fig. 1 (b) shows the scheme of DEM modelling. The length of the model and the width of the trapdoor are 896 mm and 128 mm, respectively. The model height is dependent on the fill height *H .* Two vertical walls with a height of 25.6 mm are set on both edges of the trapdoor to prevent particles from escaping. The procedures of sample preparation in DEM modelling are listed as follows: (1) creating a testing box according to Fig. 1 (b); (2) generating the cushion layer by application of circular particles with a diameter of 5 mm in the hexagonal arrangement and fixing the velocity of these particles; (3) using the Improved Multi-layer Compaction method (Lai et al., 2014) to generate the particles by layer with approximately 20 mm thickness until the target fill height is reached; (4) setting free the velocity of cushion particles. It should be mentioned that gravity is zero during the first three steps, whereas it is restored after 176 the target fill height is reached. The importance of employing the Improved Multi-layer 177 Compaction method is that it is more suitable for generating homogeneous samples with different porosities and reasonable initial stress field under gravity, especially for 179 the loose sample (Jiang et al., 2003; Lai et al., 2014).



10



# 207 simulation is similar to that in the experiment. Accordingly, the inverse modelling of

- 208 trapdoor tests was conducted to validate the micromechanical parameters.
- 
- 209 Table 1. Micro-mechanical parameters used in DEM modelling



Note:  $n_{\text{max}}$  and  $n_{\text{min}}$  are determined from the method proposed by Wood and Maeda 210

211 (2007).



216 Figs. 2 (c) and (d) show a good consistency for numerical and experimental results 217 of the trapdoor tests, suggesting that the selected micromechanical parameters are 218 reliable to capture the macroscopic behavior of the aluminium rods used in the 219 **laboratory tests.** Table 2 shows modelling samples in this study, including 12 cases with 220 various fill heights and relative densities. The ratio between the fill height and the 221 trapdoor width ranges from 0.5 to 4. Three relative densities are selected, revealing the

222 differences of the arching effect in granular materials from the loose to the dense sample. 223 According to the maximum and the minimum porosity listed in Table 1, the relative 224 density  $D_r$  of the sample in DEM simulation can be calculated as (Wood and Maeda, 225 2007):

226 
$$
D_r = \frac{(n_{\text{max}} - n)(1 - n_{\text{min}})}{(n_{\text{max}} - n_{\text{min}})(1 - n)} \times 100\%
$$
(1)

227 where  $n$  is the porosity of the sample. Using this equation, the target porosities are 228 0.191, 0.178, and 0.164, respectively, when the relative densities are 30%, 60%, and 229 90%. It can be seen from Table 2 that the difference between the initial and target 230 porosity is less than 0.001, indicating the high accuracy of the Improved Multi-layer 231 Compaction method. In addition, the dilation angle increases with the increase of the 232 relative density at a given fill height. However, the dilation angle varies insignificantly 233 with the increase of the fill height at a given relative density, which can be attributed to 234 the minor difference of the mean effective stress (the minimum and the maximum

### 235 values are  $1.40$  kPa and  $11.56$  kPa).



# Table 2. Modelling samples





## 253 **3. Numerical Results**

254 *3.1 Arching ratio and contact force*

255 To quantitively evaluate the arching effect, the arching ratio  $\rho$  is defined as (Han 256 and Gabr, 2002): *v* <u> Andrewski predstavanju i postava u objavljanju i postava u objavljanju i postava u objavljanju i objavljanju </u>



258 where  $\gamma$  is the unit weight of granular materials; H is the height of the granular  $m$  materials;  $q_0$  is the surcharge on the surface; The displacement of the trapdoor is 259 260 expresented by normalized displacement N<sub>D</sub> expressed as: 261 **b**  $N_{\rm p} = \frac{v_{\rm td}}{B} \times 100\%$  (3) 262 where  $B$  is the width of the trapdoor  $(128 \text{ mm in this study}).$ 263 Fig. 3 depicts the variations of the arching ratio with normalized displacement at 264 different fill heights. In this figure, the hollow points and the curves represent 265 simulation data and fitting results, respectively. The fitting function is the continuous 266 ground reaction curve (GRC) proposed by  $\overline{\text{Lin et al.} (2021)}$  as:  $\rho = 1 - b - (aN_{\rm p} - b)e^{-mN_{\rm p}^c}$  (4) 267 268 where *<sup>a</sup>* , *<sup>b</sup>* , *<sup>c</sup> ,* and *m* are fitting parameters, explained in detail as follows. 269 According to Equation (4), the following characteristics can be deduced: 270 (1) For the case of  $N_{\text{D}}=0$ , the arching ratio  $\rho$  is 1, indicating that the fitting curve 271 passes through (0,1) in the figure of the arching ratio versus the normalized 272 displacement; (2) For the case of  $N_{\rm p} \rightarrow 0$ , the arching ratio  $\rho$  is  $(1 - aN_{\rm p})$ , meaning that a 273 274 represents the slope of the straight line at the beginning of the fitting curve, 275 corresponding to the development of the arching effect at or close to the 276 geostatic state (this slope can also be called the modulus of arching (Iglesia et 277 al., 1999)).  $\delta$ 

278 (3) For the case of  $N_{\rm p} \rightarrow \infty$ , the arching ratio  $\rho$  is approaching the ultimate value

 $\int$  of  $(1-b)$ .

280 (4) According to the previous parametric analysis conducted by  $\text{Lin}$  et al. (2021), *m* reflects the normalized displacement corresponding to the maximum arching effect (i.e., the minimum arching ratio). The larger the value of *m* is, the smaller the normalized displacement is needed to reach the maximum arching effect. The strength of load recovery (i.e., the slope of the line from the minimum to the ultimate arching ratio) is more dependent on *c* . A larger *c* 286 corresponds to the higher strength of load recovery.

 It can be seen from Fig. 3 that the fitting curves show a good agreement with the simulation data. All coefficients of determination  $R^2$  are larger than 0.70 and increase as the fill height increases. For the cases with low fill height, fewer particles participate in the shearing. Each particle around the shear band bears a large percentage of contact forces. However, because of the movement of these particles, force chains build and collapse. An abrupt fluctuation of trapdoor load occurs, resulting in low coefficients of determination. Nevertheless, the fitting curve well predicts the variation trend of the arching ratio with normalized displacement, which is sufficient for qualitative analysis. For all cases, the arching ratio decreases to the minimum value and then increases to the ultimate value with the increase of normalized displacement, with the observation namely as the stick- slip behavior (Rui et al., 2019). The stick-slip behavior is more obvious when  $H$  < 2.0 B. At a given fill height, denser samples exhibit more obvious stick-slip 300 behavior than looser samples.



301

302 Fig. 3. Variations of the arching ratio with normalized displacement at different fill 303 heights: (a) *H*= 0.5 *B*; (b) *H*= 1.0 *B*; (c) *H*= 2.0 *B*; (d) *H* = 4.0 *B*

 To have a better understanding of the differences of the arching effect at different conditions, Fig. 4 presents the variation of fitting parameters with different fill heights 306 at different relative densities. Fig.  $4$  (a) shows that  $a$  (the modulus of arching) increases with the increase of the fill height or the relative density, indicating that the arching effect develops rapidly in the large stress field or the dense sample. However, the modulus of arching of the dense sample reaches the maximum value and then remains stable with the increase of the fill height, while that of the loose sample shows continuous growth. This is because, for the loose sample, the increase of fill height will gradually compact particles close to the trapdoor at the initial state due to the large stress field induced by gravity. Therefore, the modulus of arching of the loose sample

 increases gradually to that of the dense sample as the fill height increases. Such compaction is more difficult for dense samples, leading to the convergence of the modulus of arching as the fill height increases. The empirical value of the modulus of arching (Iglesia et al., 1999) can be regarded as a lower boundary but is unsuitable for 318 the loose sample with relatively low  $H/B$ .









344 Fig. 4. Variations of fitting parameters of the arching ratio curve with different fill 345 heights at different relative densities

 $346$  Figs. 4 (c) and 4 (d) show that both m and c increase with the decrease of the fill 347 height or the increase of the relative density. The certain fluctuation occurred on both 348 *m* and *c* in the cases with  $H=0.5B$  is attributed to the fitting fluctuation mentioned  previously. The stick-slip behavior is more obvious in lower fill height or the higher relative density. The normalized displacement corresponding to the maximum arching decreases as the fill height decreases or the relative density increases.

352 Figs. 4 (e) and 4 (f) represent the minimum arching ratio  $\rho_{\text{min}}$  and corresponding normalized displacement  $d_{\min}$ . It can be observed that the minimum arching ratio decreases with the increase of the fill height or the relative density. However, the 355 influence of the relative density is negligible for the cases with  $H < 2.0B$ . The theoretical models proposed by Evens (1983) and Iglesia et al. (1999) show a good agreement with the minimum arching ratio of the dense samples in all ranges of the fill height, with better performance of the method proposed by Evens (1983). Because their theoretical models were proposed based on their trapdoor experiments with the dense sample, both theoretical models underestimate the minimum arching ratio in the loose and middle dense samples. In addition, the normalized displacement  $d_{\min}$  corresponding to the minimum arching ratio shows an anti-correlation with the fitting parameter of *m* . The empirical range proposed by Iglesia et al. (1999) is reasonable but overestimates  $d_{\min}$  for dense samples. 

 From a micro perspective, contact forces are the main factor controlling the stress. Figs. 5 (a) and 5 (b) show the variation of contact force at different normalized displacements, with the magnitudes of the contact force shown in the same rainbow 368 legend. The white lines in  $N<sub>D</sub> = d<sub>min</sub>$  are the lowest continuous force chain connecting the edges of the stationary region at the maximum arching state. It can be observed from  Fig. 5 (a) that, the contact force is almost invisible in all normalized displacements for 0.5B90. The contact force propagates from both edges of the trapdoor at the maximum arching state for 1.0B90. However, the entire "bridge" is not formed by contact forces, corresponding to the characteristics of the partial arching. By contrast, contact forces propagate from both edges of the trapdoor and form a close "bridge" for 2.0B90. 375 According to Fig. 5 (b), the "bridge" is more obvious when  $H=4.0B$ . There is a region in which contact forces are relatively small below the "arch force bridge" at the maximum arching state (the region beneath the white line), resulting in a distinct release of stress in the trapdoor (i.e., a small arching ratio). The proportion of the height of this region to the fill height increases as the relative density increases or the fill height decreases. However, as the normalized displacement increases, contact forces in this region gradually recover, and force chains act directly on the trapdoor. The stick-slip behavior of the arching effect can be explained from micro perspectives as follows. The free contact force region temporarily generates beneath the "arch force bridge" at the maximum arching state because of the interlocking and the frictional resistance of particles. A further movement of the trapdoor destroys this free contact region. The greater the ratio of the height of the free contact force region to the fill height leads to the more obvious the stick-slip behavior.



#### *3.2 Particle motion*

 Fig. 6 shows the final displacement field at different fill heights and relative densities. 399 Because the cases with  $H=0.5B$  show a similar trend as  $H=1.0B$ , the figures for  $H = 0.5B$  are not shown herein. The magnitude of the displacement is represented by the colour of particles. At a given relative density, the displacement region enlarges with the increase of the fill height. On the other hand, for a constant fill height, the displacement region enlarges with the decrease of the relative density. A larger displacement region means that more particles are involved in the load transfer. Furthermore, as shown in Fig. 6 (a), the relative density only shows a slight influence 406 on the displacement field in the cases with low fill heights, in which the loosened zone (blue part of particles) propagates from the trapdoor to the surface. Therefore, stable "arch force bridge" cannot be generated, indicating the partial arching. On the contrary, Figs. 6 (b) and 6 (c) show that the relative density has a significant effect on the 410 displacement field in in the cases with high fill heights. For the cases with  $H = 2.0B$ , the loosened zone becomes smaller with the increase of the relative density. The loosened zone of 2.0B30 is close to the surface, bringing challenges for the formation of stable "arch force bridge". However, the loosened zone of 2.0B90 is much smaller than that of 2.0B30, indicating a suitable condition for the formation of stable "arch 415 force bridge". This result confirms that  $H = 2.0B$  is the characteristic height in which the partial arching develops to the full arching with the increase of the relative density. Accordingly, all cases with  $H=4.0B$  show the full arching at different relative densities  because the loosened zone is far from the surface. Nevertheless, the boundary of the displacement and the stationary region (i.e., the border of yellow and red) is governed by the relative density. It becomes more vertical with the increase of the relative density. This result indicates that the failure changes from diffusion to localization due to the relative density, verified by the observation from the synchrotron X-ray computed tomography test (King et al., 2019).



425 Fig. 6. The final displacement field of (a)  $H = 1.0 B$ , (b)  $H = 2.0 B$  and (c)  $H = 4.0 B$ 426 at different relative densities

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 The surface settlement is another concerning issue for designers to evaluate the reliability of underground structures in geotechnical engineering. The variation of the surface settlement trough with the relative density at different fill heights is shown in Fig. 7. The simply modified Gaussian curve fitting method can be defined as (Peck, 431 1969):

432 
$$
S = S_0 + S_{\text{max}} \exp(-\frac{x^2}{2i^2})
$$
 (5)

where  $S$  is the settlement of surface;  $S_0$  means the **unified** displacement of surface 433 434 settlement trough, which is a modified parameter compared with Peck (1969) to eliminate the influence of boundary effect;  $S_{\text{max}}$  represents the maximum vertical 435 436 surface settlements; *i* is the horizontal distance from the centre to the inflexion point 437 of the settlement trough.



438

439 Fig. 7. Surface settlement trough of different relative densities at 440 (a) H = 0.5 B; (b) H = 1.0 B; (c) H = 2.0 B and (d) H = 4.0 B

Accordingly, the coefficients of determination  $R^2$  higher than 0.85 except for 441 442 4.0B60. This exception can be attributed to the asymmetry initial defects of fabric close 443 to the surface, but the trend of the simulation data and the fitting curve is consistent. The solid points and lines represent the simulation data and fitting results at  $N_{\rm p}$ 444

( $\delta_{\rm td}$  = 5.12 mm), respectively, while hollow points and dash lines are those at 445  $N_{\rm p} = 20\%$  ( $\delta_{\rm td} = 25.6$  mm). It can be seen that, the relative density shows an 446 447 insignificant influence on the development of the surface settlement when  $H < 2.0 B$ , 448 while it causes a remarkable effect when  $H \ge 2.0 B$ . As illustrated by Figs. 7 (a) and 449 7(b), the maximum surface settlement is larger than 85% of the trapdoor displacement 450 for the cases with low fill heights for both dense and loose samples. According to Figs. 451 7 (c) and 7 (d), the surface settlement decreases as the relative density increases for the 452 cases with high fill heights. In these cases, the loosened zone cannot entirely propagate 453 to the surface. To clarify the influence of the fill height and the relative density on the surface settlement, Fig. 8 shows the variations of fitting parameters (*i* and  $S_{\text{max}}$ ) for the 454 surface settlement trough at the final state ( $N<sub>D</sub> = 20%$ ). *i* and  $S<sub>max</sub>$  are normalized by 455 the width *B* of trapdoor and trapdoor displacement  $\delta_{\rm td}$ , respectively. It can be observed 456 that *i / B* increases as the fill height increases, but the variation trend of  $S_{\text{max}} / \delta_{\text{td}}$  is 457 458 opposite. This result demonstrates that the displacement region gradually widens as the 459 fill height increases, while the maximum settlement of the surface decreases. In addition, *i* / *B* and  $S_{\text{max}}$  /  $\delta_{\text{td}}$  are rarely affected by the relative density when *H* < 2.0 *B*. A 460 461 significant influence of the relative density occurs when  $H \ge 2.0 B$ . Both parameters 462 decrease as the increase of the relative density for a constant fill height, indicating that 463 the displacement region narrows and the maximum settlement decreases.



465 Fig. 8. Variations of fitting parameters of surface settlement trough with fill height at 466 different relative densities

467 The interparticle rolling is another particle motion in controlling the strength of 468 granular materials (Jiang et al., 2005). The high gradient of particle rotation corresponds 469 to the boundary of the shear band (Oda and Kazama, 1998). Fig. 9 demonstrates the 470 rotation field at the final state with different fill heights and relative densities. The high 471 gradient of particle rotation is depicted by the blue dash line. In general, the area where  $472$  the rotation of the particles is greater than  $10^{\circ}$  enlarges as the fill height increases at a 473 given relative density. Besides, it changes from diffusion to localization as the relative 474 density increases for a constant fill height, which is similar to the result of the 475 displacement field. Moreover, several places present the high gradient of particle 476 rotation, which indicates that the multiple shear band develops with the movement of 477 trapdoor. These shear bands gradually develop upward and outward as the relative 478 density or the fill height increases. This result has a good agreement with the illustration 479 of the progressive development of the shear surface in the trapdoor proposed by Stone 480 and Wood (1992).



 Fig. 9. Rotation field at final state with different (a) fill heights and (b) relative densities

 Both the displacement and rotation of particles can be attributed to the change of contact including pure sliding, pure rolling, and a combined sliding and rolling. Fig. 10 depicts the evolution of the proportion of changing contact to the total contact at different fill heights and relative densities. It can be observed that a remarkable change of contact occurs in a relatively small normalized displacement. Moreover, the total proportion of changing contact increases with the increase of the fill height at a given relative density, indicating that more particles and contacts are included in the formation of the arching effect as the fill height increases. As shown in Fig. 10 (b), the total proportion of changing contact decreases with the increase of the relative density, demonstrating the differences between the diffuse and localized failure modes. From the micro perspective, higher relative density can effectively improve the stability of the granular system and reduce the change of contact. Therefore, the influencing range of the rotation and the displacement of particles after the movement of the trapdoor is reduced in the cases with higher density, showing localized characteristics.



 Fig. 10. Evolution of the proportion of changing contact for different (a) fill heights and (b) relative densities with normalized displacement

#### *3.3 Local porosity and Coordination number*

 Figs. 11 and 12 show the variation of the local porosity with different fill heights at  $D_r = 90\%$  and with different relative densities at  $H = 4.0 B$ , respectively. The local porosity is obtained by the measurement circles, with the contour acquired by the grid data method (Chen et al., 2020b). The negative and the positive values of porosity correspond to the compression and dilation, respectively. The black lines are the boundary of compression and dilation. It can be observed from Fig. 11 that the local porosity of different fill heights is similar at the initial state due to the same relative density. At  $N_{\rm p} = 7\%$ , a significant dilation happens near both edges of the trapdoor for all three fill heights, while the region of dilation propagates from the edges of the trapdoor to the surface for 0.5B90 and 1.0B90. Compression regions occur near the surface due to the formation of the surface settlement trough. That is, the U-shaped settlement trough causes the particles on both sides to squeeze inward, which increases











544 coordination number  $C_N$  (defined as the average contact number per particle) is used. Since the field of particle motion and porosity distribution are basically symmetrical with the centreline of the trapdoor, only half of the region from the edge of the trapdoor to the inflexion point of the settlement trough *i* is selected to calculate the average coordination number, as the statistical region shown in Fig. 1 (b). This statistical region can be regarded as the arching zone or displacement zone. Fig. 13 illustrates the variation of the average coordination number inside the arching zone with normalized displacement at different fill heights and relative densities. The initial coordination numbers of 0.5B90, 1.0B90, 2.0B90 and 4.0B90 are around 3.8. Nevertheless, the initial coordination number decreases with the decrease of the relative density, showing the values of 3.8, 3.63 and 3.51 for 4.0B90, 4.0B60 and 4.0B30 and, respectively. On the whole, the coordination number decreases rapidly in the beginning and then remains stable. A great fluctuation of coordination number occurs for 0.5B90 as the normalized displacement increases. There are three reasons for this phenomenon: (1) the statistical fluctuation due to the smaller numbers of balls (200) and contacts (800); (2) a more obvious compression region near the surface than other cases at  $N<sub>D</sub> = 7\%$ ; (3) free movement of particles because of the small stress field (unstable state of systems). Such 561 fluctuation disappears with the increase of fill height ( $H \ge 2.0 B$ ) due to the increase of the stress field and the formation of the full arching. In addition, as shown in Fig. 13 (b), the cases with different relative densities share the same average coordination number (around 3.5) at the critical state when the normalized displacement is larger 565 than 10%, implying that the further displacement of the trapdoor (or particles) has an 566 insignificant influence on the arching effect after reaching the critical state. Moreover,



567 the relative density has no significant effect on the critical state.

568

570 Fig. 13. Variation of coordination number inside the arching zone with normalized 571 displacement at different (a) fill heights and (b) relative densities

572 *3.4 Normal force fabric*

573 Polar histograms are frequently used to visualize the distribution of the contact force in 574 various studies (Rothenburg and Bathurst, 1989), particularly for the arching effect (Lai 575 et al., 2014) which has a close relationship to the normal force fabric (Chen et al., 576 2020b). Thereby, only the normal force fabric is presented in this study. The direction 577 and the magnitude of normal contact force are collected in the same statistical region 578 as the average coordination number and categorized at a predefined bin angle  $\Delta\theta = 10^{\circ}$ . 579 Meanwhile, the histogram of the normal force fabric is obtained by normalizing the 580 normal contact force by the average normal contact force of total contacts in the 581 statistical region. It is fitted according to a Fourier-series expansion proposed by 582 Rothenburg and Bathurst (1989), with the detailed mathematical expression as:

$$
\overline{f}_n(\theta) = \overline{f}_0[1 + a_n \cos 2(\theta - \theta_n)] \tag{6}
$$

where  $f_n(\theta)$  is the distribution of the average normal contact force density in the direction between  $[\theta - \Delta \theta, \theta]$ ;  $f_0$  represents the average contact force over all contacts; 586  $a_n$  and  $\theta_n$  are the second-order coefficient and principal direction of the normal force fabric, respectively.

 Fig. 14 depicts the evolution of the normal force fabric with different fill heights at different normalized displacements. The blue dashed lines are the fitting curves according to Equation (6). The principal direction of the normal force fabric is represented by the long axis orientation of the fitting curve (red dashed-dotted lines), and the coefficient of the average normal force anisotropy is illustrated by the size of the enclosed area by the fitting curve. The initial anisotropy of normal force increases with the fill height, while its principal direction also gradually approaches the direction of gravity (vertical direction). The exception of 0.5B90 is attributed to the low-stress state, the inclined boundary of the statistical region, and fewer contacts inside the 597 statistical region. When the arching effect develops to the maximum state ( $N_{\text{p}} = d_{\text{min}}$ ), the anisotropy of normal force increases while its principal direction inclines to the right (because the statistical region is on the left side of the trapdoor). The initial vertical anisotropy transfers to a certain direction anisotropy with the formation of the arching effect, indicating the rotation of principal stress in macro perspectives. Such transformation becomes more and more obvious with the increase of the fill height. Further trapdoor movement ( $N<sub>D</sub>$  = 10% and  $N<sub>D</sub>$  = 20%) leads to a greater fluctuation 

 of the anisotropy of the normal force fabric, especially for the case of 0.5B90. Compared with the principal direction of the anisotropy, the magnitude of the anisotropy shows more fluctuation, suggesting that the preferred direction of contact force changes slightly but the magnitude varies significantly. However, such fluctuations are likely to be the adjustment of the contact force inside the arching zone, because they mainly occur around the preferred direction of the normal force fabric for 1.0B90 and 2.0B90. The preferred direction and the magnitude of the normal force fabric show a remarkable change compared with those at the maximum arching state in 0.5B90, demonstrating destruction of the arching effect. Such destruction leads to a large soil arching ratio (close to 1) in macro perspectives.



615 Fig. 14. The normal contact force fabric inside the arching with different fill heights at (a)  $N_{\text{D}} = 0\%$ , (b)  $N_{\text{D}} = d_{\text{min}}$ , (c)  $N_{\text{D}} = 10\%$  and (d)  $N_{\text{D}} = 20\%$ 616

617 Fig. 15 demonstrates the evolution of the normal force fabric with different relative 618 densities at different normalized displacements. In general, the anisotropy of the normal 619 force fabric in the initial state aligns with the vertical axis for the cases with different





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639

638 Fig. 15. The normal contact force fabric inside the arching with different relative densities at (a)  $N_{\text{D}} = 0\%$ , (b)  $N_{\text{D}} = d_{\text{min}}$ , (c)  $N_{\text{D}} = 10\%$  and (d)  $N_{\text{D}} = 20\%$ 

640

#### **4. Discussions**

 From a practical point of view, the significance of this study is that providing an enhanced basis for the displacement-related theoretical model to calculate the load transfer in the intermediate state of the arching effect. Although the empirical ground reaction curve presented in this study can reflect the evolution of the arching ratio with the movement of the trapdoor, it is not a strict theoretical physical model. The stick-slip behavior of the arching ratio under different relative densities and patterns of the arching effect is clearly revealed in this study, so is the displacement field, which is beneficial to propose a strict displacement-related theoretical physical model in the future. Besides, a characteristic height is proposed to give a reference to engineers on when to consider the relative density of granular materials in practical cases associated with the arching effect. Special care should be taken when the fill height is around the characteristic height. The relative density directly determines the pattern of the arching effect and further affects the settlement and the load transfer associated with the arching effect.

 Meanwhile, there are some limitations of this study. A few simplifications were made for numerical models to avoid time-consuming calculation and complicated modelling. The 3D problems, including the geometry of the trapdoor system and the spital distribution of void, were converted to 2D problems. As previously mentioned, 661 the particle shape and the relative density are two independent influencing factors for the arching effect, which can be studied individually. This study takes a simpler particle 663 shape to consider the influence of the relative density on the arching effect. Otherwise, 664 the influencing parameters are not easy to be identified for the simulation results (such as the surface settlement, soil arching ratio and porosity etc.). Further studies are needed 666 to clarify the coupling effect of the particle shape and the relative density on the arching 667 effect.

 Despite the aforementioned limitations, the evolution trend of the arching ratio with the normalized displacement in this study is similar to that in laboratory tests using 670 sands (Iglesia et al., 1999; Chevalier et al., 2009; Liang et al., 2020). The minimum and the ultimate arching ratios are also comparable with the theoretical models (Terzaghi, 1943; Liang et al., 2020; Even, 1983). On the other hand, the variations of the displacement field with the relative density and the fill height show a good agreement with the laboratory tests from other relevant study (Moussaei et al., 2019). Therefore, the observations of this study are reasonable and significant qualitatively. Comprehensive analysis in this study shows the significant differences of the arching effect in different relative densities and patterns from macro to micro perspectives, allowing to gain insight into the mechanisms associated with the arching effect under different conditions.

### **5. Conclusion**

 Numerical trapdoor tests at four fill heights with three relative densities were conducted using the DEM to unravel the role of the relative density on the arching effect. An  empirical ground reaction curve coupled with the parametric analysis was presented to reflect the arching effect at different conditions. The comprehensive comparisons from the micro to the macro perspectives at different conditions were performed in this study. The main conclusions are summarized as follows:

- 1. In general, with the increase of normalized displacement, the arching ratio decreases to the minimum value and then increases to the ultimate value. The 690 empirical ground reaction curve is able to appropriately reflect the variation of the arching ratio with the normalized displacement under different conditions, which can be used to evaluate the progressive development of the arching effect. 2. For the low fill height, the diffuse failure emerges and only the partial arching forms without stable "arch force bridge". For the high fill height, the diffuse failure gradually translates to the localized failure as the relative density increases. Stable "arch force bridge" can form, corresponding to the full arching state. The partial and the full arching have corresponding critical states, respectively, while the relative density has an insignificant influence on the critical states of the arching effect.
- 3. A characteristic height is identified, where the relative density affects the 701 **pattern of the arching effect (** $H=2.0 B$  in this study). It is related to the dilation angle of granular materials. With the increase of the relative density, the arching effect evolves from the partial to the full state at the characteristic height, manifesting the formation of stable "arch force bridge" with the decreasing

705 range of the loosened zone. The characteristics of the surface settlement trough changes significantly with the relative density when the fill height is larger than 707 the characteristic value.

 4. From a micro perspective, the full arching is more stable than the partial arching according to the less fluctuation of the average coordination number on the arching zone. The average coordination number maintains constant in the arching zone after reaching the critical state, which only depends on the patterns of the arching effect and the basic characteristics of particles. The main evolution of normal force fabric occurs before the maximum arching state. This normal force fabric becomes more obvious with the increase of the relative density or the fill height.

#### **CRediT Statement**

 **Qi-wei Liu:** Software, Formal analysis, Visualization, Writing - original draft, Writing - review and editing. **Han-lin Wang:** Validation, Writing - review and editing. **Ren-peng Chen:** Conceptualization, Supervision, Funding acquisition. **Zhen-yu Yin:** Methodology, Validation, Writing - review and editing. **Xing-tao Lin**: Methodology, Validation.

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