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Peer-to-Peer Energy Trading with Energy Path Conflict Management in Energy Local Area Network

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***Abstract*—The increasing penetration of distributed renewable generations has given rise to a novel energy management mechanism, peer-to-peer (P2P) energy trading. The concept of Energy Internet (EI) is proposed as a new energy system framework to facilitate P2P energy trading where all distributed electrical devices are interconnected via energy routers (ERs). A proper market clearing approach is required to coordinate decentralized decision making in P2P market for demand-supply balance and feasible end-to-end energy delivery. In this paper, a decentralized market clearing mechanism considering energy path conflict management is proposed for P2P energy trading in energy local area network (e-LAN). An adjusted conflict-based search (ACBS) algorithm is proposed to deal with the non-convexity of the social welfare maximization problem where the nonconvex problem is transformed into multiple independent convex problems. Considering the privacy protection of participants, a dual decomposition based approach is proposed to solve these convex problems in a decentralized manner where self rationality of each participant can be satisfied by maximizing its personal welfare. In addition, energy path conflicts caused by decentralized optimization is resolved by imposing penalty fee and observing the rule of energy sharing maximization. Numerical simulations are presented to verify the effectiveness of the proposed market clearing approach.**

***Index Terms*—P2P energy trading, Energy Internet, energy path conflict management.**

I. Introduction

A. Background and Motivation

SMART grid is envisioned as the next-generation power grids that enables environmentally friendly energy generation and smart energy exchange mechanism [1], [2]. In this context, end users are motivated to install renewable energy generators, which transforms them from passive consumers into active prosumers who can play both roles of energy producers and energy consumers. Considering the possible mismatch between renewable generation of a single prosumer and its power demand, a novel market mechanism, peer-to-peer (P2P) energy trading, is promoted in the emerging smart grid for energy balance [3], [4]. However, the rigid structure of the existing power grid based on centralized power generation and unidirectional power transmission cannot achieve flexible end-to-end transmission and distribution required by P2P energy trading between prosumers [5].

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In recent years, the concept of Energy Internet (EI) has been proposed as an innovative and promising framework for future smart grid [6], [7]. Under the proposed EI framework, all electrical devices including generators, loads and energy storages, are interconnected in the form of mesh network by using energy routers (ERs). Similar to the information Internet, wide-area EI is organized into small-scale energy local area networks (e-LAN) for more effective operation and management, as shown in Fig.1 [8]. ERs are perceived as the core equipment of the EI as they are responsible for routing energy across the EI in collaboration with other ERs. Within each e-LAN, prosumers are allowed to trade their energy surplus or deficiency with other prosumers for generating revenues or reducing energy cost.

As P2P energy trading in e-LAN facilitates the utilization of distributed RESs [9], a proper market clearing mechanism is required for P2P energy trading involving multiple producers and multiple consumers. The first necessary requirement for the market clearing mechanism is to ensure a normal operation of the e-LAN with P2P energy market. Unlike traditional electricity market which plans power delivery in a centralized manner by following the solution of optimal power flow problem, P2P energy market allows decentralized end-to-end power delivery. Due to the physical constraints of e-LAN, power deliveries of all P2P transactions may not be achieved simultaneously, which leads to conflicting interests. Therefore, the market clearing mechanism should be able to resolve energy path conflict in a fair fashion. Moreover, to motivate prosumers to participate in P2P energy trading, the market clearing mechanism should allow prosumers to pursue their benefits without revealing their important private information to a third party. Hence, the aim of this paper is to address the aforementioned challenges by designing a privacy-preserving and fair market clearing mechanism with energy path conflict management.

B. Related Work

In the literature, many efforts have been devoted to design suitable P2P energy trading mechanisms, which can be mainly divided into two categories, P2P energy trading with demand response (DR) and P2P energy trading without DR [10] [11]. Although the market design of P2P energy trading without DR is simpler [12]–[14], it can be observed from [15]–[18] that P2P energy trading with DR reveals significant advantages in

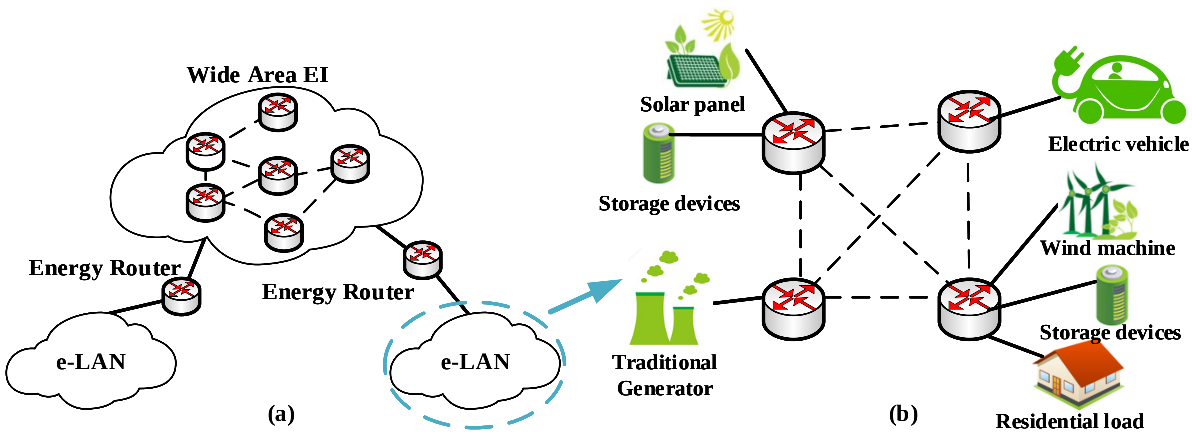

Fig. 1. (a) An example wide area EI. (b) An example e-LAN.

balancing energy supply and demand and improving system's economic efficiency compared with P2P trading without DR.

Several fully open P2P energy trading models with DR are presented in [15], [17], [19]–[21] [22]. By means of establishing direct communication between sellers and buyers, all details of each P2P transaction can be negotiated and determined without coordination of the system operator, which avoids exposing their trading information to the system operator. Although these models provides much freedom and information protection for prosumers, they are too idealistic to be implemented in practice due to several reasons. First, in the absence of the coordination of the system operator, energy delivery of transactions planned in a fully decentralized manner may violate network constraints. In other words, to ensure a normal operation of the whole e-LAN, the system operator must know transactions information in advance for feasibility check. Second, various assumptions are predefined for trading negotiations in these models to ensure P2P market converges to equilibrium status or the optimal solution. For example, [15] assumes that each buyer in P2P market will utilize all its budget for purchasing energy regardless of energy price, [17], [19] requires that each seller must sell its all available energy out every time it participates in P2P energy trading, and it is assumed in [20], [21] that each seller updates its price and quantity with the aim to maximize the social welfare instead of its personal welfare, and [22] assumes that each buyer is allowed to trade with only one seller . However, since personal welfare cannot be guaranteed maximum under these assumptions, each prosumer in real life may not follow the assumed actions so that negotiations about price and quantity may not proceed to produce the expected results of these models.

In [16], [23], [24], a centralized operator is introduced into P2P market as a coordinator for price management. By collecting power supply and demand data from all participants, the coordinator regulates P2P energy trading price so as to eliminate any mismatch between supply and demand. Each participant can autonomously adjust its supply or demand according to the updated price for pursuing its maximum welfare.A cooperative Stackelberg game model is proposed where the grid behaves as a leader to promote P2P energy sharing by regulating the grid price and users are considered as followers to pursue their benefits by participating in P2P energy trading [25]. In this regard, each participant is not required to reveal its welfare function information to the coordinator so that its private information can be protected. With the guidance of the coordinator, P2P market can operate efficiently and converge in the expected direction. However, the cost associated with transmission loss, *i.e.* transmission cost, is ignored in these models. It is notable that transmission cost in P2P energy trading has to be borne by the trading pairs instead of the utility companies. The final market outcome may thus be different if each participant must bear the transmission cost involved in P2P energy trading.

To bridge this gap, [21], [26], [27] incorporate transmission cost into P2P energy trading model by charging each participant loss compensation cost or network utilization fee. It is found that the decision of each buyer depends on the prices offered by sellers as well as the potential transmission cost. In these models, energy paths are pre-defined for transmission cost calculation and the energy path planning is ignored. However, as prosumers in P2P energy market are allowed to select their preferred trading partners, end-to-end energy path optimization is necessary for each trading pair to save their transmission cost. As a result, a decentralized Dijkstra algorithm based method is proposed in [22] [28]–[31] to minimize end-to-end power transmission cost for P2P energy trading. However, these approaches ignore possible path conflicts caused by decentralized energy transmission, in which each trading pair behaves greedily and always pursues its optimal energy path. Energy deliveries of all P2P transactions may violate the physical constraints of e-LAN and conflicting interests may occur among these P2P energy trading pairs. Any unfair conflict-resolving mechanisms may discourage disadvantaged trading pairs to participate in P2P energy trading. Moreover, the existing energy path optimization algorithms such as Dijkstra algorithm and Minimum-cost flow algorithm are for confirmed P2P transactions. As a consequence, trading decision and energy path optimization are considered separately and thus the optimality of prosumers' personal welfare cannot be guaranteed when applying these algorithms. Therefore, the P2P energy trading mechanism should be designed in such a way that each prosumer can jointly optimize its trading strategies and energy path.

C. Main Contributions and Paper Organization

In light of the shortcomings of the existing models, this paper proposes a P2P energy trading model incorporating energy path conflict management where the market clearing of P2P energy trading is formulated as a social welfare maximization problem considering transmission cost and network constraints. Compared with the previous works, the novelty and main contributions of this paper is summarized as follows:

1) An adjusted conflict-based search (ACBS) algorithm is proposed for solving the social welfare maximization problem where a constraint tree based two-step iteration is designed to address the non-convexity of power flow direction (PFD) constraints.
2) A decentralized method is proposed for market clearing in P2P energy market that preserves the privacy of participants. Based on the dual decomposition principle, the market clearing problem is decomposed into a set of personal welfare maximization problems so that each participant formulates its own trading strategy independently of other participants. In the proposed decentralized method, a dynamic penalty fee method for handling congested transmission lines is proposed to achieve congestion management in e-LAN.
3) A tripartite graph based decentralized energy path optimization method is proposed where a hybrid approach that unifies depth-first search (DFS) algorithm and Lagrange multiplier method is used to find the jointly optimal solution for trading decision and energy path.

The rest of the paper is organized as follows. In Section II, a system model of P2P energy trading is presented and the social welfare maximization problem is formulated. Section III discusses ACBS algorithm for market clearing. The effectiveness of the proposed P2P energy trading model is verified in Section IV by numerical simulations. Finally, conclusions are drawn in Section V.

II. System Model

An e-LAN can be visualized as a community-scale power network through which a small group of prosumers are interconnected. It is assumed that each prosumer in the e-LAN is equipped with some forms of energy generation and storage devices and local loads. As depicted in Fig. 1(b), each prosumer is interfaced to the e-LAN via an ER which enables energy exchanges between its energy generation and storage devices and local loads as well as to allow them to exchange energy with other prosumers in the e-LAN.

Currently, there is no standard design for ER and a typical ER design is shown in Fig. 2. It is composed of a power conversion module, multiple input/output ports, and a routing control module. Energy generation and storage devices and loads with different terminal characteristics are interfaced to a common internal dc bus via suitably designed power converters, and exchange energy with one another via the dc bus. The routing control module is responsible for establishing communication between the local devices and other ERs and managing power flows between them.

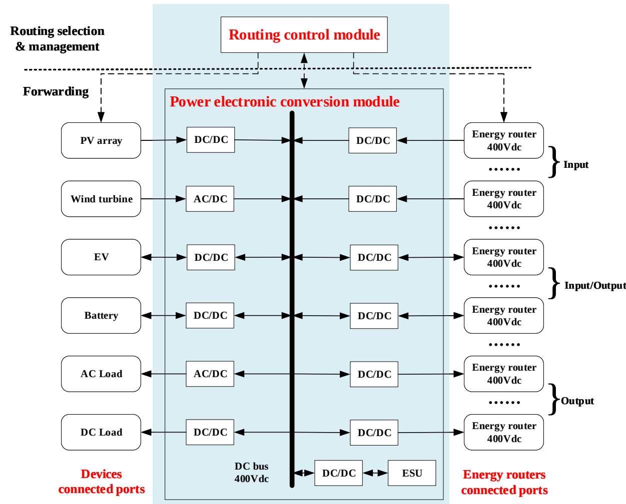

Fig. 2. An example structure of energy router.

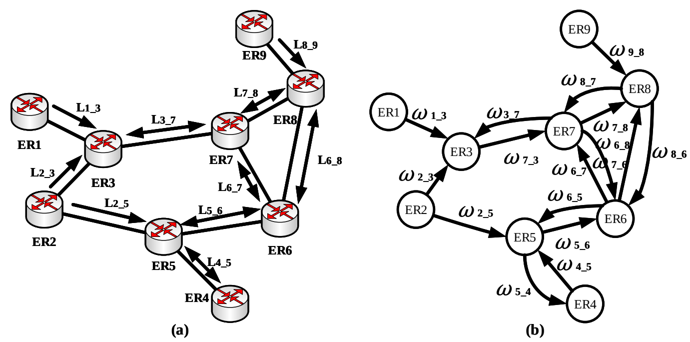

Fig. 3. (a) An example e-LAN. (b) Graph model of the e-LAN in Fig.3(a).

Due to its mesh network topology, an e-LAN is commonly modeled as a graph for analysis. Fig.3 shows an example of a 9-node e-LAN represented as a directional network graph G where each node denotes an ER and the transmission lines that interconnects adjacent ERs are depicted by the edges of the graph. The direction of an edges represents the direction of power flow on the transmission line and the edge weight W denotes the transmission cost arising from transmission loss.

Under such framework, each prosumer is allowed to trade its energy surplus or deficiency with other prosumers. To model this, we define B as the set of buyers and S as the set of sellers in a given energy trading period, where B and S are two disjoint sets. Moreover, a network system operator (NSO) is established to coordinate P2P energy trading and the corresponding energy transmission. Since the market clearing model of a single trading period can be extended to multiple trading periods by adding time-coupled constraints, this paper focuses on the market design of a single trading period to more explicitly demonstrate the performance of the proposed model.

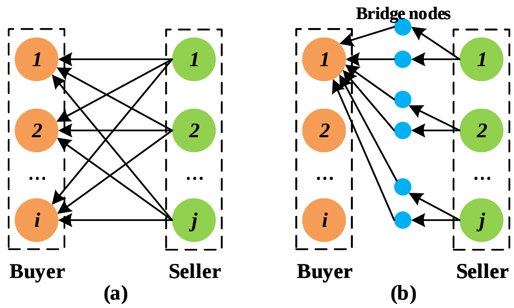

Fig. 4. (a) P2P market model proposed in [17], [20]. (b) P2P market model proposed in this paper.

A. Welfare Function on Buyer and Seller Sides

Energy transactions in traditional P2P energy trading market [17], [20] can be represented by the bipartite graph in Fig.4(a), where each buyer $i \in B$ can purchase energy from multiple sellers at the same time. When energy transmission is considered, the proposed P2P energy trading market is depicted in Fig.4(b) where bridge nodes denote transmission paths. Each bridge node represents a possible energy transaction between the two participants connected to it. Thus, each participant in the P2P energy trading market will formulate its strategy by evaluating the profit gained from each bridge node connecting to itself.

Based on the tripartite graph, the welfare function W_i of buyer i is expressed by (1), which consists of four terms: (i) u_i – satisfaction derived from consuming all purchased electricity, (ii) C_{i-p} – cost of purchasing electricity through the pth path in P2P market, (iii) T_{i-p} – transmission cost associated with the pth path, and (iv) C_{i-g} – cost of purchasing electricity from the grid. It is notable that in this paper buyers are assumed to bear the transmission cost involved in P2P energy transactions.

$$W_i = u_i - \sum\nolimits_{p=1}^{|\Omega_i|}(C_{i-p} + T_{i-p}) - C_{i-g} \tag{1}$$

where Ω_i denotes the set of all transmission paths between buyer i and all sellers in S.

Specifically, the expressions of the four terms in (1) are given by (2)-(5) respectively.

$$u_i = \beta_i(\sum\nolimits_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g}) - \frac{\theta_i}{2}(\sum\nolimits_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g})^2 \tag{2}$$

$$C_{i-p} = \pi_{i-p} P_{i-p} \tag{3}$$

$$T_{i-p} = \sum_{l_{(m,n)} \subset \ell_p} \pi_{line}(\omega_{line-(m,n)} + \omega_{ER-(m,n)}) \tag{4}$$

$$C_{i-g} = \pi_{b-g} P_{i-g} \tag{5}$$

where P_{i-p} is the amount of power transmitted through the pth path. β_i and θ_i in (2) are the preference parameters of buyer i which characterize its satisfaction derived from consuming electricity. P_{i-g} is the amount of electricity purchased from the grid. π_{i-p} is the electricity price of the seller associated with the pth path. In (4), $l_{(m,n)}$ denotes the edge or transmission line (m,n) connecting ER m and ER n on the pth path, ℓ_p denotes the set of all edges on the pth path, π_{line} is the unit price of the compensating power for the transmission loss in each transmission line, $W_{line-(m,n)}$ is the conduction loss in transmission line (m,n) given by (6), and $W_{ER-(m,n)}$ is the power conversion losses in ER m and ER n given by (7). T_{i-p} therefore denotes the total transmission cost summed over all transmission lines and ERs associated with the pth path. π_{b-g} is price of purchasing power from the grid.

$$W_{line-(m,n)} = \frac{R_{(m,n)}}{V_{(m,n)}^2}((P_{i-p} + P_{ex(m,n)})^2 - P_{ex(m,n)}^2) \tag{6}$$

$$W_{ER-(m,n)} = [(1 - \eta_{out-m}) + (1 - \eta_{in-n})]P_{i-p} \tag{7}$$

The welfare function W_j of seller j comprises two parts: (1) u_j – satisfaction derived from self-consumption or self-storage, and (2) ϕ_j – revenue generated from trading energy, which are given by (8)-(10).

$$W_j = u_j + \phi_j \tag{8}$$

$$u_j = \beta_j(P_{j-\max} - \sum\nolimits_{j=1}^{|\Psi_j|} P_{j-p} - P_{j-g}) - \frac{\theta_j}{2}(P_{j-\max} - \sum\nolimits_{j=1}^{|\Psi_j|} P_{j-p} - P_{j-g})^2 \tag{9}$$

$$\phi_j = \pi_j \sum\nolimits_{j=1}^{|\Psi_j|} P_{j-p} + \pi_{s-g} P_{j-g} \tag{10}$$

where β_j and θ_j in (9) are the preference parameters of seller j which characterize its satisfaction derived from self-consumption or self-storage. $P_{j-\max}$ in (9) denotes the total available power of seller j. Ψ_j is the set of all paths from buyers to seller j, P_{j-p} denotes the amount of power transmitted through the pth path supplied by seller j and P_{j-g} is the amount of power sold to power grid directly. π_j and π_{s-g} denotes the price of seller j when selling electricity to P2P participants and power grid, respectively.

B. Optimization Problem

The aim of the proposed P2P energy trading market is to maximize the social welfare, as given by the optimization problem (11).

$$\max \quad (\sum\nolimits_{i=1}^{|B|} \widehat{W}_i + \sum\nolimits_{j=1}^{|S|} \widehat{W}_j) \tag{11a}$$

$$s.t. \quad D_{i-\min} \le \sum_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g} \le D_{i-\max}, \forall i \in B \tag{11b}$$

$$P_{j-\max} - P_{j-p2p} - P_{j-g} \ge D_{j-\min}, \forall j \in S \tag{11c}$$

$$\sum\nolimits_{i=1}^{|B|} \sum\nolimits_{p=1}^{|\Omega_{(i,j)}|} P_{i-p} = P_{j-p2p}, \forall j \in S \tag{11d}$$

$$\begin{cases} \sum_{i \in B} P_{i-(m,n)} \le Cap_{(m,n)}, \forall l_{(m,n)} \in \Xi \\ P_{i-(m,n)} = \sum_{p=1}^{|\Omega_{i-(m,n)}|} P_{i-p}, \forall i \in B \end{cases} \tag{11e}$$

$$\begin{cases} f_{(m,n)} + f_{(n,m)} = 1, \forall l_{(m,n)} \in \Xi \\ f_{(m,n)}, f_{(n,m)} \in \{0,1\}, \forall l_{(m,n)} \in \Xi \\ P_{i-p} = P_{i-p} * \prod\limits_{(m,n)\in \ell_p} f_{(m,n)}, \forall \ell_p \in \Omega_i, \forall i \in B \\ \text{Resolving PFD conflicts by observing the energy} \\ \text{sharing maximization rule} \end{cases} \tag{11f}$$

where $\widehat{W_i} = W_i + \sum_{p=1}^{|\Omega_i|} C_{i-p}$ and $\widehat{W_j} = W_j - \pi_j \sum_{p=1}^{|\Psi_j|} P_{j-p}$. $D_{i-\min}$ and $D_{i-\max}$ in ($11b$) denote the non-flexible and total power demand of buyer i, respectively. In ($11c$), $P_{j-p2p} = \sum_{p=1}^{|\Psi_j|} P_{j-p}$ and $D_{j-\min}$ is the non-flexible power demand of seller j. $\Omega_{(i,j)}$ in ($11d$) denotes the set of all transmission paths between buyer i and seller j. In ($11e$), $P_{i-(m,n)}$ is the total transmitted power of buyer i through transmission line (m,n) and $Cap_{(m,n)}$ is available capacity of transmission line (m,n). Ξ denotes the set of all transmission lines in an e-LAN and $\Omega_{i-(m,n)}$ is the set of buyer i's all transmission paths to all sellers in S incorporating transmission line (m,n). $f_{(m,n)}$ in ($11f$) denotes power flow direction (PFD) of transmission line (m,n) and it can take 1 or 0. "1" or "0" means that power can or cannot flow from ER m to ER n respectively.

As the payment of buyers will become revenue of sellers, $\sum_{i=1}^{|B|}\sum_{p=1}^{|\Omega_i|} C_{i-p} = \sum_{j=1}^{|S|} \pi_j \sum_{p=1}^{|\Psi_j|} P_{j-p}$. Maximizing $(\sum_{i=1}^{|B|} W_i + \sum_{j=1}^{|S|} W_j)$ is equivalent to maximize (11). ($11b$) and ($11c$) are the demand constraints for buyer i and seller j, respectively. ($11d$) is to achieve demand-supply balance in P2P energy trading market. ($11e$) and ($11f$) are two physical constraints to ensure a normal operation of the e-LAN. ($11e$) is the capacity constraint for each transmission line to avoid congestion and ($11f$) is the PFD constraints of transmission lines to resolve PFD conflicts, which happens when a transmission line is required to achieve simultaneous power transmission in two opposite directions by P2P trading pairs. In the energy sharing maximization rule of ($11f$), the PFD of a transmission line achieving a better RESs sharing will be preferred when PFD conflict occurs in the transmission line. It is known that the purpose of P2P energy trading is to promote the sharing of RESs between participants and reduce their dependence on the power grid. As a consequence, NSO will give a higher priority to the PFD of a transmission line that produces less power demand on the powergrid and a larger quantity of traded energy in P2P market.

III. Market Clearing Approach

Since the value domain of $f_{(m,n)}$ is discrete, the constraint ($11f$) is a non-convex set and thus the problem (11) cannot be solved by using convex optimization methods directly. A transformation can be performed on (11) so that convex methods can be applied on it. As a matter of fact, the discrete set ($11f$) can be viewed as a union set of multiple independent point sets. For a transmission line, its PFD has two situations. The combinations for PFDs of all transmission lines in e-LAN are $2^{|\Xi|}$. Since PFDs of all transmission lines are determined in each combination, each combination is a convex point set. Consequently, constraint ($11f$) can be interpreted as an union set of the $2^{|\Xi|}$ independent convex sets. It transforms the non-convex optimization problem (11) into $2^{|\Xi|}$ child convex optimization problems where the objective function is ($11a$) and the constraints are ($11b$)-($11e$) and a child convex set of ($11f$). By solving the $2^{|\Xi|}$ child problems respectively and comparing the $2^{|\Xi|}$ solutions, the optimal solution of the non-convex problem (11) can be found. However, a major disadvantage of this method is that it may consume too much computing resources when $|\Xi|$ is large. Therefore, this paper proposes an adjusted conflict-based search (ACBS) method to deal with the non-convex problem (11). Traditional CBS algorithm is proposed for using multi-robot path finding problem and is not perfectly suitable for the problem in this paper. More details about CBS algorithm can be found in [32].

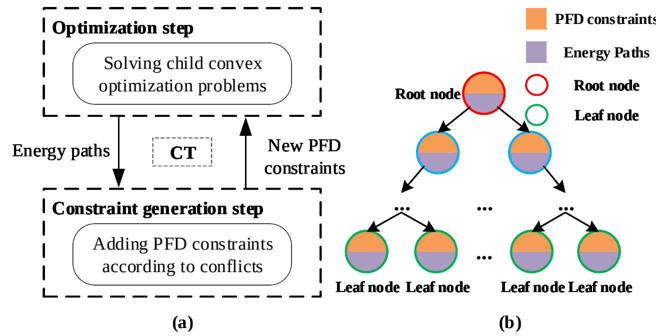

Fig. 5. (a) The proposed ACBS algorithm. (b) CT in the ACBS algorithm.

A. ACBS Algorithm

As shown in Fig.5(a), ACBS algorithm is composed of two steps, optimization step and constraints generation step. The mentioned idea of transforming (11) into multiple child convex optimization problems is adopted by ACBS algorithm, in which optimization step is established to solve these child problems. To avoid the curse of dimentionality, ACBS algorithm employs the constraints generation step to grow valid child PFD constraints according to the conflicts in the results of optimization step rather than enumerating all possible child PFD constraints as the mentioned method. During the process of iteration, energy paths and PFD constraints are two pieces of information that need to be exchanged between the two steps. In view of increasing PFD constraints and changing energy paths, a constraint tree (CT) is established as data storage for information exchange. As depicted in Fig. 5(b), the CT is a rooted binary tree where each node is used to store energy paths and PFD constraints.

B. Two-Step Iteration

The two-step iteration based on CT is shown in Fig.6. In the optimization step, each child problem can be formulated by (12). Note that the root node is also leaf node when CT has only one node. The optimization step is to find the optimal solutions for (12) under PFD constraints of each leaf node and insert the energy paths in each obtained solution to the corresponding leaf node. After that, the CT with updated leaf nodes will be sent to the constraint generation step for PFD conflict check. Since a leaf node represents an independent child set, the constraint generation step analyzes and expands

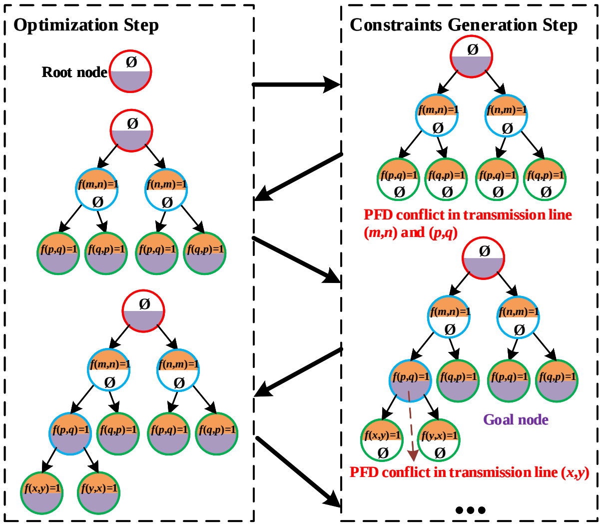

Fig. 6. Diagram of two-step optimization based on CT

each leaf node individually. For a leaf node under analysis, it is called sub-root node and the expansion for a sub-root node is as follows. When PFD conflict occurs in a transmission line, two mutually exclusive point sets denoting two PFD situations of the transmission line respectively will be produced as new PFD constraints. A new level will be added to CT by assigning two child nodes incorporating the two new PFD constraints respectively to the sub-root node. At the same time, these newly added child nodes will be updated as new leaf nodes. It is possible that there exists multiple PFD conflicts in the energy paths of a sub-root node. Every time a new PFD conflict is found, CT will be further expanded by adding a new level and each new leaf node subject to the sub-root node will be assigned two child nodes with new PFD constraints. After traversing the energy paths in all leaf nodes, the updated CT will be returned to optimization step. It is notable that each child node inherits the constraints of its parent node. To do such, each leaf node denotes a possible situation to resolve all PFD conflicts found so far. Therefore, the optimization step only needs to focus on the newly generated leaf nodes. For those leaf nodes without PFD conflicts, they are labeled as goal nodes and will not be visited in later iterations. The two-step iteration repeats until all leaf nodes in the CT become goal nodes and the optimal solution can be obtained by comparing the energy sharing in all goal nodes.

$$\max \quad (11a) \tag{12a}$$

$$s.t.(11b,c,d,e)\&\text{PFD constraints in a leaf node} \tag{12b}$$

In order to show the advantages of ACBS method over exhaustive computation, the computational costs of the two methods are compared based on the number of child problems solved. From the viewpoint of mathematics, the ACBS algorithm will incur a higher computational cost than exhaustive computation approach only if PFD conflicts occur in all transmission lines of an e-LAN. When $(|\Xi|-1)$ transmission lines in an e-LAN have PFD conflicts, the maximum computational times of ACBS is $(2^0+2^1+...+2^{|\Xi|-1}=2^{|\Xi|}-1)$, less than $2^{|\Xi|}$ of exhaustive computation. However, in electrical networks, the scenario that all transmission lines have PFD conflicts will not occur as some edges connecting to the nodes possessing only one adjacent node such as $l_{(1,3)}$, $l_{(4,5)}$ and $l_{(8,9)}$ will not be simultaneously employed by multiple trading pairs. Apparently, PFD conflicts will not occur in these transmission lines. Therefore, compared with exhaustive computation, ACBS method will consume a lower computation cost.

C. Distributed Optimization

Although the problem (12) can be solved by the network operator in a centralized fashion, each participant may suffer from privacy violation. Therefore, this paper proposes a decentralized approach to solve (12) where each participant solves its sub-problems locally to avoid exposing its welfare function information to NSO. Since (12) is a quadratic programming problem with all constraints being affine functions, the duality gap is always zero [33]. Therefore, the problem (12) holds strong duality. The technique of dual decomposition [34] can be used to decompose (12) into a series of independent sub-problems. (13) formulates the Lagrangian Γ of (12).

$$\begin{aligned}\Gamma(\mathbf{P_i},\mathbf{P_j},\boldsymbol{\pi},\boldsymbol{\gamma}) &= \sum\nolimits_{i=1}^{|B|}\widehat{W}_i+\sum\nolimits_{j=1}^{|S|}\widehat{W}_j\\ &+\sum\nolimits_{j=1}^{|S|}\pi_j(P_{j-p2p}-\sum\nolimits_{i=1}^{|B|}\sum\nolimits_{p=1}^{|\Omega_{(i,j)}|}P_{i-p})\\ &+\sum\nolimits_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}(Cap_{(m,n)}-\sum\nolimits_{i\in B}P_{i-(m,n)})\end{aligned} \tag{13}$$

where π_j is the Lagrange multiplier for the constraint $(11d)$ of seller j, and can also be viewed as the electricity price of seller j to maintain balance between its supply and demand to it from an economic viewpoint. $\gamma_{(m,n)}$ denotes the Lagrange multiplier for the capacity constraint of transmission line (m,n). Since $(11b)$ and PFD constraints are for buyers and $(11c)$ is for sellers, they are not included in (13).

The dual problem of (12) is defined as

$$\begin{aligned}&\min_{\boldsymbol{\pi},\boldsymbol{\gamma}}\sup_{\mathbf{P_i},\mathbf{P_j}}\Gamma(\mathbf{P_i},\mathbf{P_j},\boldsymbol{\pi},\boldsymbol{\gamma})\\ &=\min_{\boldsymbol{\pi},\boldsymbol{\gamma}}[\sum\nolimits_{i=1}^{|B|}\vartheta_i(\pi,\gamma)+\sum\nolimits_{j=1}^{|S|}\varsigma_j(\pi)\\ &\quad+\sum\nolimits_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}Cap_{(m,n)}]\end{aligned} \tag{14a}$$

$$s.t.\pi_j\geq 0,\forall j\in S,\gamma_{(m,n)}\geq 0,\forall l_{(m,n)}\in\Xi \tag{14b}$$

where $\vartheta_i(\boldsymbol{\pi},\boldsymbol{\gamma})$ and $\varsigma_j(\boldsymbol{\pi})$ are the sub-problems to be solved by buyer i and seller j.

Based on the sub-gradient projection method, dual problem (19) can be solved by the following a distributed update.

Buyer i updates $\mathbf{P_i}$

$$\begin{aligned}\vartheta_i(\boldsymbol{\pi}^{(k)},\boldsymbol{\gamma}^{(k)}) &\triangleq \max_{\mathbf{P_i}}[\widehat{W}_i-\sum\nolimits_{j=1}^{|S|}\pi_j^{(k)}\sum\nolimits_{p=1}^{|\Omega_{(i,j)}|}P_{i-p}\\ &-\sum\nolimits_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}^{(k)}P_{i-(m,n)}]\\ &\triangleq\max_{\mathbf{P_i}}[W_i-\sum\nolimits_{l_{(m,n)}\in\Xi}\gamma_{(m,n)}^{(k)}P_{i-(m,n)}]\end{aligned} \tag{15a}$$

$$s.t.\quad (11b)\&\text{PFD constraints} \tag{15b}$$

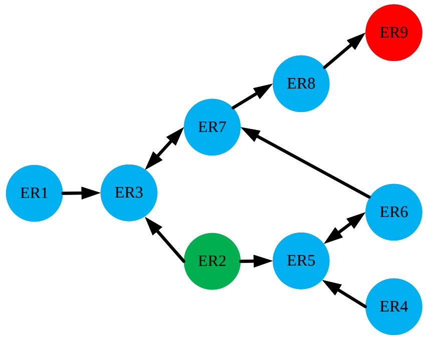

Fig. 7. DFS tree of the Energy Internet shown in Fig.3(a).

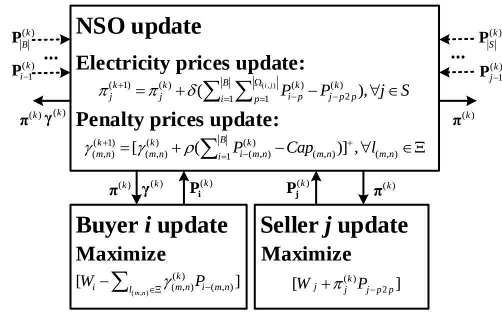

Fig. 8. Information exchange between NSO and prosumers

where $\pi_j^{(k)}$ and $\gamma_{(m,n)}^{(k)}$ denotes dual variable π_j and $\gamma_{(m,n)}$ at the kth iteration. $\gamma_{(m,n)} P_{i-(m,n)}$ can be interpreted as a penalty fee imposed on the trading pairs for using the transmission line (m, n) to avoid congestion. When $\sum_{i=1}^{|B|} P_{i-(m,n)} < Cap_{(m,n)}$, $\gamma_{(m,n)} = 0$.

Constructing the set $\Omega_{(i,j)}$ is to find all available paths between source node j and sink node i, which can be solved by using depth-first-search (DFS) based algorithm in graph theory. Fig.7 presents the DFS tree of the e-LAN shown in Fig.3(a). For example, the two paths between node 2 and node 9 can be identified as 2→3→7→8→9 and 2→5→6→7→8→9 by the DFS algorithm. More details about DFS algorithm can be found in [35].

As (15) is a convex problem, Lagrange multiplier method and Karush-Kuhn-Tucker (KKT) conditions is used to solve it, as given by (16) and (17) where $\underline{\lambda}_i$ and $\overline{\lambda}_i$ are the Lagrange multipliers.

$$\zeta(P_i, \lambda) = W_i + \underline{\lambda}_i(D_{i-\min} - \sum_{p=1}^{|\Omega_i|} P_{i-p} - P_{i-g})$$
$$+\overline{\lambda}_i(\sum_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g} - D_{i-\max}) - \sum_{l_{(m,n)}\in\Xi} \gamma_{(m,n)}^{(k)} P_{i-(m,n)} \tag{16}$$

$$\begin{cases} \frac{\partial \zeta}{\partial P_{i-p}} = 0 \\ \underline{\lambda}_i(D_{i-\min} - \sum\limits_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g}) = 0 \\ \overline{\lambda}_i(\sum\limits_{p=1}^{|\Omega_i|} P_{i-p} + P_{i-g} - D_{i-\max}) = 0 \end{cases} \tag{17}$$

The solution variables $\mathbf{P_i}$ are labeled by $\mathbf{P_i}^{(k)}$, which is used for the following update.

Seller j updates $\mathbf{P_j}$

$$\varsigma_j(\pi_j^{(k)}) \triangleq \max_{\mathbf{P_j}} \ [\widehat{W}_j + \pi_j^{(k)} P_{j-p2p}]$$
$$\triangleq \max_{\mathbf{P_j}} \ W_j \tag{18a}$$
$$s.t. \quad (11c) \tag{18b}$$

Similarly, the optimal solution of (18) can be obtained by using Lagrange multiplier method and KKT condition as follows and the solution P_{j-p2p} is expressed by $P_{j-p2p}^{(k)}$ for the following update.

$$\xi(P_j, \mu) = W_j + \mu(P_{j-\max} - P_{j-p2p} - P_{j-g} - D_{j-\min}) \tag{19}$$

$$\begin{cases} \frac{\partial \zeta}{\partial P_{j-p2p}} = 0, \frac{\partial \zeta}{\partial P_{j-g}} = 0 \\ \mu(P_{j-\max} - P_{j-p2p} - P_{j-g} - D_{j-\min}) = 0 \end{cases} \tag{20}$$

NSO updates $\boldsymbol{\pi}$, $\boldsymbol{\gamma}$

$$\pi_j^{(k+1)} = \pi_j^{(k)} + \delta(\sum\nolimits_{i=1}^{|B|} \sum\nolimits_{p=1}^{|\Omega_{(i,j)}|} P_{i-p}^{(k)} - P_{j-p2p}^{(k)}) \tag{21}$$

For $\forall l_{(m,n)} \in \Xi$,

$$\gamma_{(m,n)}^{(k+1)} = [\gamma_{(m,n)}^{(k)} + \delta(\sum_{i\in B} P_{i-(m,n)}^{(k)} - Cap_{(m,n)})]^+ \tag{22}$$

where δ is the step size. To ensure convergence of the distributed update, δ should less than $\frac{L}{2}$ to ensure convergence, in which L is the Lipschitz constant subject to [36]

$$\left\|\nabla\Theta(x) - \nabla\Theta(y)\right\| \le L \left\|x - y\right\|, \forall x, y \in (\boldsymbol{\pi}, \boldsymbol{\gamma}) \tag{23}$$

The implementation of the distributed update requires iterations between NSO and prosumers, which can be achieved by information exchange in Fig.8. After undergoing finite iterations, the dual update will converge to an optimal solution. The proposed distributed method for problem (12) is scalable in the number of participants where all participants can solve their own optimization problems in parallel.

IV. Simulation Results and Analysis

This section presents numerical simulations for performance evaluation of the proposed P2P energy trading model. In the simulation study, we consider an e-LAN modified from IEEE 14-bus system [30], as shown in Fig.9. According to [8], [28], [30], the conversion efficiency of each ER is randomly set from [0.97,0.99] and the transmission voltage between ERs is 400Vdc. For the P2P market model, it is assumed that there are four buyers and four sellers in the e-LAN and their basic information is listed in Table I [17], [21]. In Hong Kong, the price of purchasing electricity from the grid $\pi_{\max}$ is 2.0 HKD/kWh, and the price of selling electricity to the grid is

TABLE I
BUYERS AND SELLERS PARAMETERS

Buyers	β_i (HK\$/kWh)	θ_i (HK\$/kWh^2)	$D_{i-\min}$ (kW)	$D_{i-\max}$ (kW)
B_9	2.29	0.023	10.3	19.5
B_{14}	2.12	0.052	11.5	20.3
B_7	2.27	0.026	12.2	22.5
B_2	1.98	0.061	11.3	18.7
Sellers	β_j (HK\$/kWh)	θ_j (HK\$/kWh^2)	$D_{j-\min}$ (kW)	$P_{j-\max}$ (kW)
S_{10}	2.14	0.086	8.9	33.9
S_{13}	1.95	0.079	12.7	22.9
S_8	1.82	0.081	9.6	19.4
S_1	1.77	0.057	11.5	23.2

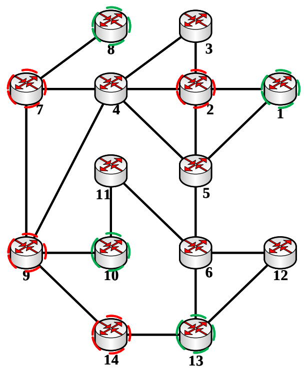

Fig. 9. An e-LAN modified from IEEE 14-bus system.

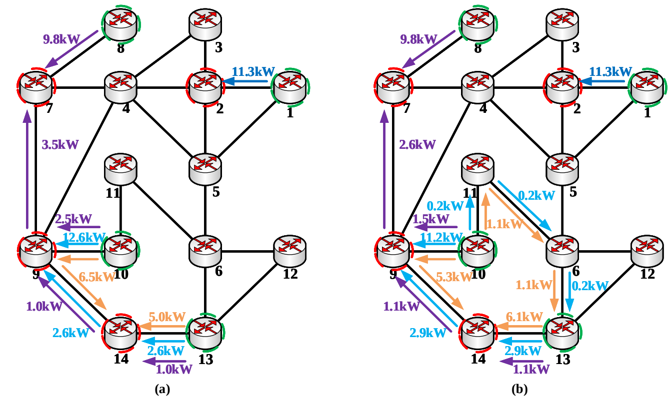

Fig. 10. Energy paths of (a)Model 2 and (b)Model 3.

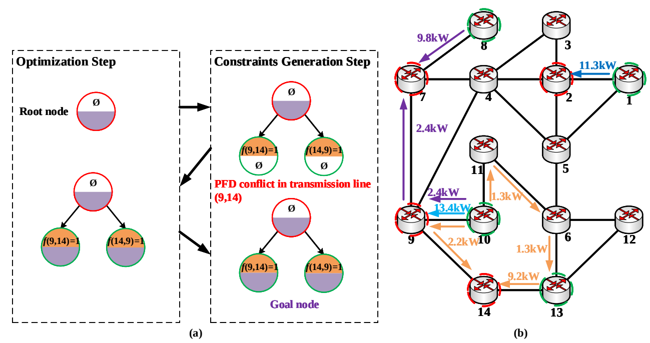

Fig. 11. (a) Growth of constraint tree. (b) Energy Paths when $f_{(9,14)} = 1$.

assumed to be 0.5 HKD/kWh. Price adjustment parameter σ_j and Lagrange multiplier $\gamma_{(m,n)}$ are both taken as 0.001.

To demonstrate the merits of the proposed P2P energy trading model, comparative simulations are performed between the following four P2P energy trading models. It is noteworthy that Model 3 is designed by this paper as a comparison object to exhibit the advantages of Model 4 (the proposed P2P energy trading model) more clearly.

- Model 1: P2P energy trading ignoring transmission cost and network constraints [16];
- Model 2: P2P energy trading considering only transmission cost [21];
- Model 3: P2P energy trading considering transmission cost and capacity constraint;
- Model 4: P2P energy trading considering transmission cost, capacity constraint and PFD constraint.

Table II shows the market outcomes of the four models. It can be observed that the participants in Model 2 exhibit different market behavior as compared with the participants in Model 1. When transmission cost is considered in P2P energy transactions, trading strategy formulation of each participant depends not only on the electricity price but also on the transmission cost. Although Model 2 reveals the impact of transmission cost on market outcomes, the energy paths produced by Model 2 violate the physical constraints of the e-LAN. As shown in Fig.10(a), B_7, B_9 and B_{14} all plan to purchase electricity from S_{10} as S_{10} possesses convenient electrical location close to the three buyers. Nevertheless, in their planned energy paths, transmitted electricity through transmission line (10,9) is 21.6kW more than its available capacity 18kW. In addition, transmission line (9,14) is required to transmit power simultaneously in both two directions. As an improved version of Model 2, Model 3 deals with capacity constraints by using a dynamic penalty fee method. Fig.10(b) shows the energy paths of Model 3 where the three buyers reduce their transmitted powers on transmission line (10,9) due to the effect of penalty fee. However, there still exists PFD conflict in transmission line (9,14). Although seller 10 is more attractive than other sellers for buyer 9, the quadratic characteristic of the transmission cost function and penalty fee proportional to its transmitted power through transmission line (9,14) cause it purchase power from multiple buyer to avoid high transmission cost. In Model 4, PFD constraint is considered by using ACBS algorithm. The growing process of the CT is shown in Fig.11(a), where two child nodes containing PFD constraints are generated to resolve the PFD conflict found in transmission line (9,14). The market outcomes for two child nodes are shown in Table III. It is found that B_{14} needs to purchase 2.9kW power from the grid to meet its demand in the situation of $f_{(14,9)} = 1$, while each buyer has no power demand from the grid when $f_{(9,14)} = 1$. Therefore, the child node containing $f_{(9,14)} = 1$ is preferred by NSO and its energy paths is shown in Fig.11(b).

In this case, the computation for child optimization problem (12) runs total $(2^0 + 2^1 = 3)$ times, which is significantly less than 2^{18} times associated with exhaustive computation (18 edges in the e-LAN). Moreover, the optimality and convergence of the proposed decentralized algorithm in the opti-

TABLE II
MARKET OUTCOMES OF MODEL 1, MODEL 2, MODEL 3 AND MODEL 4.

	Model 1						Model 2					Model 3					Model 4				
	S_1	S_8	S_{10}	S_{13}	G	T	S_1	S_8	S_{10}	S_{13}	G	S_1	S_8	S_{10}	S_{13}	G	S_1	S_8	S_{10}	S_{13}	G
π_j	1.91	1.91	1.91	1.91	—	—	1.74	1.89	1.85	1.82	—	1.74	1.92	1.66	1.84	—	1.74	1.93	1.65	1.78	0
B_2	—	—	—	—	0	16.7	11.3	0	0	0	0	11.3	0	0	0	0	11.3	0	0	0	0
B_7	—	—	—	—	0	11.5	0	9.8	2.5	1.0	0	0	9.8	1.5	1.1	0	0	9.8	2.4	0	0
B_9	—	—	—	—	0	14.0	0	0	12.6	2.6	0	0	0	11.4	2.7	0	0	0	13.4	0	0
B_{14}	—	—	—	—	0	11.3	0	0	6.5	5.0	0	0	0	6.4	5.0	0	0	0	3.5	7.9	0
G	0	0	0	0	—	—	0	0	0	0	—	0	0	0	0	—	0	0	0	0	—
T	22.2	9.6	9.8	11.7	—	—	'G' is the grid and 'T' denotes Total														

TABLE III
MARKET OUTCOMES OF TWO LEAF NODES.

	$f_{(9,14)} = 1$					$f_{(14,9)} = 1$				
	S_1	S_8	S_{10}	S_{13}	G	S_1	S_8	S_{10}	S_{13}	G
B_2	11.3	0	0	0	0	11.3	0	0	0	0
B_7	0	9.8	2.4	0	0	9.8	3.9	0	0	0
B_9	0	0	13.4	0	0	0	14.1	0	0	0
B_{14}	0	0	3.5	7.9	0	0	0	1.6	7.0	2.9
G	0	0	0	0	—	0	0	0	0	—

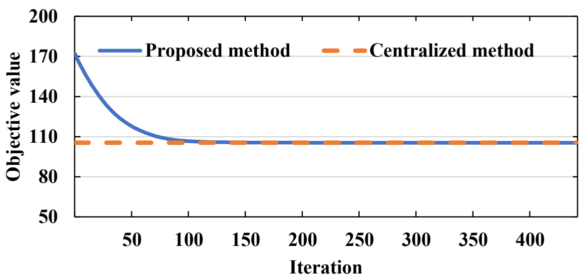

Fig. 12. Development of objective value.

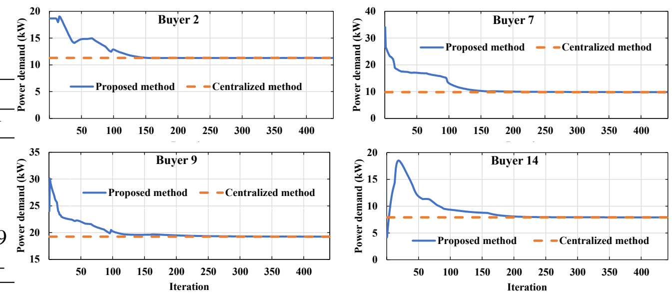

Fig. 13. Evolution of buyers' power demand.

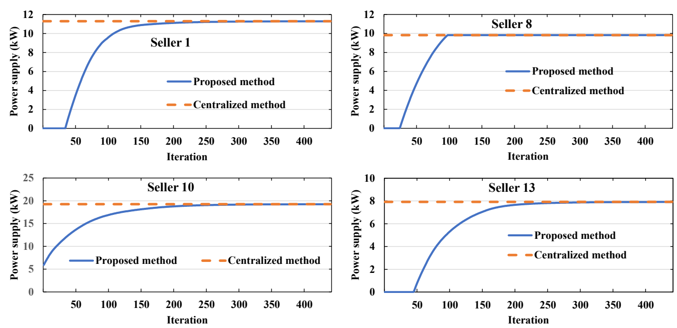

Fig. 14. Evolution of sellers' power supply.

mization step of ACBS algorithm is also verified by numerical simulations. A centralized method based on fmincon function with interior-point method in Matlab R2014a is employed as a benchmark to evaluate the performance of the proposed method. Fig.12 shows the development of the objective value obtained by the proposed decentralized method, from which it can be observed that the objective value computed by the proposed decentralized algorithm gradually converges to the optimal objective value produced by the centralized method. In addition, the evolution of buyers' power demand and sellers' power supply from the proposed decentralized algorithm are shown in Fig.13 and Fig.14, respectively. It is found that power demand/power supply of each buyer/seller matches the globally optimal demand/supply obtained by the centralized method.

V. CONCLUSION

This paper proposes an ACBS method to achieve P2P energy trading in e-LAN with energy path conflict management. The market clearing of P2P energy trading is formulated as a nonconvex social welfare maximization problem. By designing a two-step iteration based on CT, the nonconvex optimization problem is transformed into multiple independent convex problems according to PFD conflicts. A dual decomposition based method is designed to solve these convex problems in a decentralized way where the electricity prices are regulated to ensure demand-supply balance in P2P energy market and network congestion is avoided by using a dynamic penalty fee method. Numerical simulation results show that the proposed P2P energy trading model can converge to the optimal solution without violating network's physical constraints.

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