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**THE EFFECT OF GROUND ALTITUDE ON LIGHTNING STRIKING DISTANCE
BASED ON A BI-DIRECTIONAL LEADER MODEL**

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18 **ABSTRACT**

19 The effect of ground altitude on lightning striking distance has been investigated based on a
20 bi-directional leader model. The model, which is a development of that proposed originally by Mazur
21 and Ruhnke in 1998, enables the calculation of leader channel parameters, such as leader charge
22 density, leader current, leader potential, and lightning striking distance to flat ground. In the model,
23 the lightning striking distance is directly related to the critical electric field in the negative streamer
24 zone in front of the leader tip and to the leader potential. The former may vary with the ground
25 altitude above sea level and the latter may also be affected by it. Based on this thought and on the
26 bi-directional leader model, the effect of regional ground altitude on lightning striking distance was
27 investigated. The result shows that the striking distance increases significantly as the ground altitude
28 increases. This is because the critical electric field necessary for sustaining the negative streamer zone
29 decreases as the ground altitude increases. The result is useful to both physical and engineering
30 application.

31 **Keywords:** leader potential, lightning striking distance, return stroke, ground altitude

32

33 **1 INTRODUCTION**

34 In negative cloud-to-ground flashes, the first return stroke is always initiated by a stepped leader
35 that travels from the cloud to the ground. As the stepped leader descends to tens to hundreds of meters
36 above ground, the charge in the leader channel will generate a relatively large electric field in front of
37 the leader tip leading to an electric breakdown between the leader tip and the ground or objects
38 connected to it. This final breakdown is usually referred to as the lightning attachment, and the height
39 of the leader tip above the ground at which the lightning attachment starts is usually referred to as the
40 “striking distance”. Obviously, it is this lightning attachment that determines the lightning striking
41 point on an object to be struck, making the striking distance the most important parameter in lightning
42 protection issues.

43 Usually the lightning attachment process, which determines the striking distance, may involve an
44 upward connecting leader from the ground or grounded objects. Due to this, many existing striking
45 distance models have paid more attention to the criterion for upward leader inception, such as critical
46 radius concept [1-2], critical potential criterion [3-4], critical streamer length criterion [5], and leader
47 stabilisation field concept [6-8]. However, if the ground is completely flat, once the corona streamer
48 from the stepped leader tips touches the ground, the final breakdown will take place before or without

49 any initiation of upward connecting leader. In such case, the striking distance was evaluated by the
50 length of the negative streamer region when it reaches the ground. In fact, even if a positive upward
51 connecting leader (for negative cloud-to-ground flashes) is successfully incepted from the ground or a
52 structure (for ordinary ground structures), it starts slowly and moves continuously, and does not have
53 too much time to develop [9]. As a result, it has no more influence on the development of a downward
54 stepped leader and therefore has less importance for striking distance.

55 Traditionally, the striking distance has been correlated with the prospective return stroke lightning
56 peak current and further, with the geometric parameter of the grounded structure [5, 10-11]. However,
57 many studies show that the striking distance is determined mainly by the last step length of the
58 downward leader near the ground [12-14], which can be estimated with both the leader tip potential
59 just before it touches the earth and a constant critical electric field along the negative streamer region
60 in front of the leader tip. Thus, the leader tip potential is believed by those studies to be a dominant
61 factor affecting leader interaction with a grounded structure. In some studies [4, 13-16], the leader is
62 assumed to be a conducting wire extending in the ambient electric field of a thundercloud, thus, the
63 leader potential is largely controlled by the potential profile of the cloud, the point of leader initiation
64 and the shape of leader path.

65 The critical electrical field sustaining the streamer zone propagation varies with the air pressure
66 and water vapour content, therefore may vary with the regional altitude above sea level. Phelps and
67 Griffiths in [17] studied the effects of the pressure and humidity on the critical electric field for the
68 positive steamer propagation and further on the striking distance. However, the effects of regional
69 altitude on the critical electric field in the downward negative streamer in front of the downward
70 leader tip, on the potential of the leader tip, and further on the striking distance have not been
71 addressed yet.

72 In this paper, by using a triple cloud charge model and a bidirectional leader model in
73 combination with the leader potential concept of striking distance [14], the effect of the regional
74 altitude on the striking distance is mainly investigated.

75

76 **2 MODEL DESCRIPTION**

77

78 **2.1 Tripole structure of the cloud charge**

79 It has been recognised that the charge structure allowing a flash to emerge from the thundercloud
80 is the typical tripole structure: a dominant negative charge in the middle of the cloud, a positive

81 charge above it and a (usually small) positive charge below. The lower positive charge is essential for
82 the initiation of negative cloud-to-ground lightning [18-20]. Although the electrical structure of
83 thunderstorms may be more complex, the potential profile in a thunderstorm is typical and quite
84 stable, and is consistent with a simple dipole or tripole charge model [21].

85 In particular, the charge configuration used in this study is similar to the modified tripole charge
86 model in [15]. The charge model is consisted of a dominant dipole and a small high charge density
87 core enclosed at the lower part of the main negative charge region. The core corresponds to an
88 intensive updraft region, which is split in two: a small positive charge core and a small negative
89 charge core (see Fig.1). Where the parameter H stands for the altitude of boundary between the two
90 small charge cores, h the ground altitude above sea level, h' the height of the cloud base above
91 ground, and $h_1\lambda$, $h_2\lambda$ and $h_1\lambda$ the layer depth of the small positive charge core, the small negative
92 charge core, and the upper positive charge region, respectively. Among these, $h_1 = 3$ km, $h_2 = 4$ km, and
93 λ is a zoom factor of the depth of charge regions. For selected parameters H , h and λ , the profile of
94 potential U and vertical electrical field E along the axis of the model are obtained by integrating the
95 disk charge.

96 Fig.2 shows the profile of U and E for the model for $H = 7$ km, $h = 2$ km and $\lambda = 1$. There are two
97 competed maximum fields at boundaries between the two opposite charge regions. The lower one,
98 which is closely associated with the presence of the lower positive charge region, is crucial to the
99 initiation of the negative cloud-to-ground discharge.

100

101 **2.2 Bidirectional leader development**

102 For the development of lightning leader, the concept of the bidirectional lightning leader model,
103 which is widely accepted in the interpretation of a variety of physics processes of lightning in [15,
104 22-25], was employed. The essence of the concept is: lightning development in cloud occurs as a
105 bi-directional, bipolar, zero-net-charge leader and as an electrodeless discharge [26-29]. In a simple
106 one-dimension bi-directional model in [15], the leader channel was assumed to be a perfect thin
107 conductor that develops vertically downward and upward from the initiation point with constant
108 speed. The leader potential was determined by the average ambient potential along the conductor
109 length. A similar approach was used in [30] to investigate the evolution of initial leader velocities
110 during intra-cloud lightning. Instead of using the constant velocity, they considered the self-consistent
111 evolution of the leader. In principle, the leader propagation model in this study is a combination of

112 those in [14-15] and [30], however, the potential gradient in the leader channel is considered, in
 113 addition, the streamer zone in front of the leader tip is also included.

114 Studies of positive spark in long air gap show that the propagation of the leader channel involves
 115 a corona zone at its head [31-33]. The leader channel advancement acts as a metallic electrode,
 116 sustaining the field at the front of the leader head for continuous breakdown. The active ionisation at
 117 the front of the leader head supplies the current and energy input necessary to sustain the thermal
 118 transition at the leader head for its extension. There is apparent asymmetry between positive and
 119 negative leaders. The propagation of negative leader is harder to initiate, more intermittent and
 120 stepwise. However, in consideration of the evolution of negative leader within a given step, we
 121 assume the leader for positive and negative is similar, both propagating with corona at their head,
 122 which is characterised by a constant critical electric field E_s [12, 31-33]. The length of the corona can
 123 be evaluated by the geometric construction as the length between the leader tip and the point defined
 124 by the intersection of the ambient potential curve with a straight line of slope E_s in corona.

125 There is pressure dependence for the critical breakdown or breakdown field in the atmosphere [5,
 126 38-39]. Assume the same relation for the critical field for both the positive and negative streamer
 127 zone, it gives

$$128 \quad E_s = E_{s0} \exp(-z / z_0), \quad (1)$$

129 where, z_0 is a constant of 8.4 km, z the height above sea level, E_{s0} the critical field at sea level which is
 130 assumed to be 5.0 kV/cm and 7.5 kV/cm for positive polarity and negative polarity, respectively.

131 It is believed that the leader is consisted of a thin conducting core surrounded by a corona
 132 envelope of tens of meters in diameter. To calculate the electric field to find the streamer zone in front
 133 of the leader tip, it is first necessary to specify the leader charge distribution within the corona sheath
 134 along the channel, which is governed by the leader potential and the ambient potential. The leader,
 135 however, is usually with a potential gradient along its channel to maintain a current to sustain the
 136 leader extension. To estimate the potential gradient in the leader channel, the semi-empirical model
 137 proposed by Bazelyan and Raizer in [4] is adopted, in which the leader tip speed, leader tip current
 138 and the leader potential gradient are controlled by the potential difference ΔU_{tip} between the potential
 139 of the leader tip and the ambient potential at the edge of the corona zone (also see [36]). In their
 140 model, the average leader speed is given as

$$141 \quad v_L = a(\Delta U_{tip})^{1/2}, \quad a = 15mV^{-1/2}s^{-1}. \quad (2)$$

142 The average leader tip current i_L is then related to v_L and the leader tip charge density τ_L by

$$143 \quad i_L = \tau_L v_L = \tau_L a(\Delta U_{tip})^{1/2}. \quad (3)$$

144 Because of the negative current-voltage characteristic of the leader channel, the longitudinal electrical
 145 field (potential gradient) in the channel is related to the channel current by

$$146 \quad E_L = \frac{b}{i_L}, \quad b = 3 \times 10^4 \text{ VAm}^{-1} . \quad (4)$$

147 For a leader that grows steadily, the electric field in its channel core E_L must not be stronger than the
 148 undisturbed external field E_0 , $E_L \leq E_0$. Related to (4), the leader survival condition is

$$149 \quad i_L \geq \frac{b}{E_0} . \quad (5)$$

150 With the potential gradient of the leader channel E_L and the potential of the middle point of the
 151 leader channel φ_0 , the leader potential along the leader channel $\varphi(z)$ is readily obtained, then the
 152 charge density in the leader channel can be calculated.

153 In calculation of the charge density in the leader channel, instead of the method in [12], the
 154 charge simulation method in [37-38] is employed. The major reason is that the method of charge
 155 density in [12] is sensitive to the radius of the corona sheath of the leader channel assumed, while the
 156 charge simulation method in [37-38] is not. At a moment during the leader development, the channel
 157 of leader is divided into N segments; each segment is assumed a uniformly charged cylinder. Let τ_i be
 158 the charges per unit channel length of the i -th segment, the leader potential $\varphi(z_k)$ at given point z_k
 159 along the leader-streamer channel and below it is written as

$$160 \quad \sum_{i=1}^N \alpha_{ki} \tau_i + U(z_k) = \varphi(z_k) , \quad (6)$$

161 where, $U(z_k)$ is the undisturbed ambient potential due to the thundercloud for the point z_k , and

$$162 \quad \alpha_{ki} = \frac{1}{4\pi\epsilon_0 R^2} [(z_{i2} - z_k)(\sqrt{(z_{i2} - z_k)^2 + R^2} - |z_{i2} - z_k|) - (z_{i1} - z_k)(\sqrt{(z_{i1} - z_k)^2 + R^2} - |z_{i1} - z_k|) \\ + R^2 \log \frac{z_{i2} - z_k + \sqrt{(z_{i2} - z_k)^2 + R^2}}{z_{i1} - z_k + \sqrt{(z_{i1} - z_k)^2 + R^2}}] \quad (7)$$

163 is a potential coefficient of the i -th segment relating to the point z_k , z_{i2} and z_{i1} are the coordinates of the
 164 ends of the i -th segment of the leader channel, and R is the radius of the charged leader channel. As
 165 shown by Eq. (7), the α_{ki} is not sensitive to the R unless the segment z_i is very close to the point z_k
 166 concerned.

167 A summary of the leader channel radius can be found in the work by Vargas [25], in which it
 168 says the leader channel radius ranges from 0.5 m to 5 m. From photographic measurements in South

169 Africa, Schonland [39] reported luminous stepped-leader radius between 0.5 - 5 m. In [12, 14], Mazur
 170 proposed that the radius of the leader channel is related to its charges and hence to its potentials. A
 171 leader with a potential of -10 MV and -40 MV would have a channel radius of about 1 m and 5 m
 172 respectively. In [40], Cooray pointed out that the radius of the leader channel corresponding to a
 173 charge density of -0.001 C/m would be about 3 m. All these also suggest that the leader radius may
 174 change with its charges and potentials. The larger the leader potential is the larger of its radius. A
 175 leader radius smaller than its actual value may lead to underestimation of its charge on one hand, and
 176 on the other hand it may lead to the calculated E-field within the leader sheath exceeding the air
 177 breakdown electric field which is against the definition of the leader sheath radius.

178 Referring to [12, 14], as the leader potential involved in this paper is up to more than 90 MV, the
 179 corresponding leader radius should be larger than 5 m. But taking account of the upper limit of the
 180 observed leader radius by Schonland, we use 5 m as the leader radius in this paper. In fact, since the
 181 leader charge is axis-symmetrically distributed along the leader channel, different leader radius should
 182 make no more difference in the potential profile along the thin leader core and out of the leader
 183 sheath, and hence it has no more influence on the calculation of the strike distance that is based on the
 184 leader core potential. But it does little difference in the charge distribution within the leader sheath.
 185 To examine the sensitivity of the striking distance to the leader radius, for the case of H=7km, h=2km
 186 and $\lambda=1$, the evolutions of leader charge density and leader potential profile have been calculated and
 187 compared for leader radii of 3 m and 5 m. The results show that the charge density for the leader
 188 radius of 3 m is smaller than that for the leader radius of 5 m by less than 5%. Meanwhile, the
 189 potential profiles for the leader radii of 3 m and 5m are almost the same.

190 Eq. (6) is written for all selected points of the bi-polar leader channel. The set τ_i are solved with
 191 the least square error method [38] in such a way that the potential at the middle point of the leader
 192 channel φ_0 (therefore the leader channel potential $\varphi(z_k)$) is adjusted so that the total charge induced in
 193 all segments of the bipolar leader is zero,

$$194 \quad \sum_{i=1}^N \tau_i \Delta d_i = 0 \quad , \quad (8)$$

195 where Δd_i is the length of the i -th segment of the leader channel.

196 Once the line charge density along the leader channel $\tau(z)$ is obtained with the aforementioned
 197 charge simulation method, the total charge deposit in part of the leader channel is given by

$$198 \quad Q = \int_{z_1}^{z_2} \tau(z) dz \quad , \quad (9)$$

199 where z_1 and z_2 are the altitude of the lower end and upper end of the leader channel concerned.

200 Particularly, the charge deposited in the downward negative leader Q_1 can be estimated using Eq.
201 (9) by setting z_1 as the lower negative leader tip and z_2 as the center sign reversal point of the
202 bidirectional leader channel. Similarly, the charge deposited in the upward positive leader Q_2 can be
203 estimated using Eq. (9) by setting z_1 as the center sign reversal point of the leader and z_2 the upper
204 positive leader tip. As the leader channel is zero-net-charge, the charge deposited in the downward
205 negative leader Q_1 is equal to that in the upward positive leader Q_2 . The current at the center charge
206 sign reversal point of the leader channel, i_0 , is given by

$$207 \quad i_0 = \frac{dQ_2}{dt} = -\frac{dQ_1}{dt} \quad . \quad (10)$$

208 When the negative streamer zone in front of the leader tip reaches the ground, the striking distance
209 with respect to flat ground is then obtained as the streamer length [12] by

$$210 \quad S = \phi / E_{sg} , \quad (11)$$

211 where Φ is the leader tip potential just before it touches the ground and E_{sg} is the critical field for the
212 negative streamer zone at ground level.

213 In deriving the above leader model, several assumptions for simplification were made, which are
214 discussed below:

215 1) The tortuosity and branching are neglected, which may lead to over-estimation of the
216 striking distance according to [11] and [25], due to the over-estimation of the leader potential [4, 15].
217 In addition, the tortuosity and branching may be associated with the stochastic behaviour of the
218 lightning striking point. Since the effects of those factors are unknown and debatable, neglecting them
219 is justifiable.

220 2) The other atmospheric conditions, such as temperature and humidity are not considered.

221 3) Instead of the step transient speed of negative stepped leader, the step average speed is
222 introduced to describe the extension of leader within a step. In practice, the striking distance, which is
223 dominated by the leader tip potential, is also affected by the step-wise behaviour of the leader just
224 before the attachment, with the length of the last step being its upper limit. What is calculated based
225 on Eq. (11) and by using the step average speed is actually the maximum striking distance with the
226 influence of step-wise behaviour on the striking distance being ignored. To estimate leader tip
227 potential with a high accuracy, the increment of the leader channel extension (or time step) in the
228 numerical calculation should be much smaller than the step length. In this study, by numerical testing,
229 a time step of $50 \mu s$ subjecting to Eq. (2) is considered to be adequate.

230 4) The semi-empirical model (Eq. (2) to Eq. (5)) derived by Bazelyan and Raizer [4] for
231 positive leaders under switching impulse voltages are extrapolated to the negative leaders under
232 atmospheric conditions. This rough extrapolation may introduce errors in estimates of leader potential
233 gradients via the leader speeds and currents; furthermore it may affect the accuracy of the estimated
234 leader tip potential and therefore the estimated striking distance. Fortunately, the leader potential is
235 largely controlled by the ambient potential profile. The leader potential gradient is just a small factor
236 in determining the leader tip potential.

237

238 3 RESULTS AND ANALYSES

239

240 3.1 Evolution of the leader

241 It is assumed that a cloud-to-ground lightning is initiated at the altitude of H where the electric
242 field is the maximum, and the leader channel has an initial length of $L = 1$ km with an initial potential
243 gradient along the channel of 1 kV/m and grows in a $50 \mu\text{s}$ time step. For the simulation in a step (say
244 step n), with the leader channel length, the leader speed and the leader channel potential gradient
245 calculated in the previous step (step $n-1$), the charge distribution (τ) along the leader channel for the
246 step n can be estimated based on Eq. (6), (7) and (8). With the recalculated charge distribution, the
247 ambient potential profile for the step n can be recalculated. Consequently, the leader tip speed (v_L ,
248 subject to Eq. (2)), the leader tip currents (i_{bottom} and i_{top} , subject to Eq. (3)) and the leader potential
249 gradient (E_L , subject to Eq. (4)) for the step n can be estimated. The recalculated leader speed and
250 leader potential gradient for the step n can then be used for the simulation in the next step (step $n+1$).
251 The ambient potential profile below the leader tip is calculated as the undisturbed cloud-produced
252 potential plus the potential due to the leader channel charges by Eq. (6). In addition, Eq. (5) is tested
253 in each step. If Eq. (5) is fulfilled, then go to the next step, otherwise, the leader stops.

254 Fig.3 shows the evolution of potential profile along the bidirectional leader-streamer channel at a
255 6.5 ms time interval for the case $H = 7$ km, $h = 2$ km and $\lambda = 1$, based on the charge simulation
256 method with the cloud charge model shown in Fig.1. The result is similar to that in [15], but different
257 in the following two points. First, we have considered the potential gradient along the leader channel.
258 Second, we have considered the streamer zone in front of the leader tip. The potential distribution in
259 the streamer zone is characterised with a straight line of a slope of the critical electrical field E_s . The
260 length of the streamer zone is determined as the distance between the leader tip and the point defined

261 by the intersection of the ambient potential curve with a straight line of slope E_s in the streamer zone.

262 Fig.4 shows the evolution of the induced charge profile along the leader channel corresponding to
263 Fig.3, calculated based on the charge simulation method. It can be seen that the charge densities at the
264 leader tips usually show a sharp increase due to the “electrode tip” effect. For the upper positive
265 leader part, the charges are mainly concentrated at the height of $H = 7$ km, where the main negative
266 charge region of the cloud is, while the charges are mainly concentrated near the tip for the lower
267 negative leader part. This feature is consistent with the ambient potential distribution shown by the
268 dot-line in Fig. 3, as the charge on the leader channel is induced due to the ambient potential.

269 Fig.5 demonstrates the evolution of the current at the two ends of the leader channel and that at
270 the leader center corresponding to Fig.3 and Fig.4. It can be seen that the upward positive leader
271 current initially increases and then decreases to the minimum, while the current at the negative end
272 increases steadily when it approaches to the ground. This is in accord with the variation of potential
273 difference at the leader tip ΔU_{tip} (see Fig.3) and the variation of charge density along the leader
274 channel (see Fig.4). The center current is less than a hundred ampere, which agrees with the result of
275 others [4, 14].

276

277 **3.2 Striking distance variation with ground altitude**

278 In investigating the striking distance variation, three parameters, H , h and λ were used to adjust
279 the charge structure. It is believed that the initiation of cloud-to-ground lightning always occurs at the
280 same height as the boundary between the mid-level negative and lower positive charge regions, where
281 it is corresponding to the charge reversal temperature of about -10 to -20 °C [40]. Starting from this
282 basic point of view, following case studies are introduced.

283 Case 1 – Since the cloud temperature is dependent with the altitude above sea level, thus, as Case
284 1, it is assumed that the leader initiation altitude remains as a constant at $H = 7$ km with $\lambda = 1$.
285 Changing the altitude of the ground h and the cloud base height h' , the variation of the striking
286 distance with altitude of the ground h is then obtained as shown in Fig.6. It can be seen that the
287 striking distance increases as the altitude of the ground h increases. This is expected by Eq. (1) and
288 Eq. (11) as the critical field of the negative streamer at ground level exponentially decreases with the
289 increase of the ground altitude, while the leader potential changes in a narrow range of 90 to 93 MV
290 as the ground altitude varies.

291 Case 2 – In addition to the leader initiation altitude, the variation of potential profile with the
292 ground altitude also affects the striking distance. To investigate this effect, as Case 2, the height of the

293 cloud base to ground ($h' = 2$ km with $\lambda = 1$) remains as a constant while the ground altitude h changes.
294 In such case the cloud background potential remains invariant and the striking distance is only
295 affected by the critical field. Similar calculations is also repeated for $h' = 0.5$ km with $\lambda = 1$, which is
296 referred as Case 3. The result of Case 2 is similar to that of Case 1, while there is a less than 10 m
297 difference in striking distance between Case 2 and Case 3.

298 In the above cases, the variation of cloud charge structure with charge region altitudes is not
299 taken into account. However, both theoretical and experimental works show that the electric field
300 within cloud is limited by the critical field of lightning initiation. According to this, the electric field at
301 the leader initiation altitude should remain invariant regardless of the change of the ground altitude.
302 Therefore, as Case 4, all other things are the same as Case 1, except the zoom parameter λ . With the
303 increase of ground altitude h , the λ is adjusted so that the electric field at lightning initiation height
304 remains the same as the case when the ground altitude is at sea level. Fig.7 shows the comparison of
305 striking distance between Case 1 and Case 4. It can be seen that with the increase of the ground
306 altitude, the thunderstorm charge may become weaker ($\lambda < 1$), and the striking distance may decrease
307 also. However, the difference is less than 13 m between the two cases.

308

309 **3.3 Leader charges and return stroke charge transfer and striking distance**

310 The leader tip potential that determines the striking distance is highly associated with the ambient
311 potential profile. On the other hand, the charge stored on the negative leader channel is also highly
312 associated with ambient potential profile. Therefore, the striking distance may be associated with the
313 charge stored on the downward negative leader channel. Some engineers are also like to associate the
314 striking distance with the total charge induced in the channel of the return stroke just following the
315 downward leader, as the total charge induced in the return stroke channel is theoretically associated
316 with the ambient potential profile too. But the problem is that the return stroke occurs after the
317 attachment from the striking distance, while the leader potential and its charges precede the
318 attachment. So, from the point of view of physicists, only the relationship between the leader charges
319 and the striking distance makes sense to them.

320 For the bidirectional leader in this study, the total return stroke charge transfer Q is equivalent to
321 the positive charge induced in the whole bidirectional leader channel by the cloud charge after the
322 leader touches the ground, which can be easily calculated by setting the leader channel potential $\varphi(z_k)$
323 to the ground potential in Eq. (6) based on the charge simulation method. The negative charge stored
324 in the downward negative leader channel just before touching the ground is referred as Q_1 , which can

325 be estimated based on Eq.(9) given that the charge distribution along the leader channel is obtained.

326 Fig.8 shows the variation of the charge stored on the negative leader channel before touching
327 ground Q_l , and the total return stroke charge transfer Q , against the striking distance S , when the
328 ground altitude changes from 0 to 3.0 km with $H = 7$ km and $\lambda = 1$. It shows that the charge stored on
329 the negative leader channel decreases with the increase of striking distance (or the ground altitude).
330 This is understandable as the length of the leader channel decreases with the increase of ground
331 altitude, so does the charge associated with the leader channel.

332 To further investigate the effect of ground altitude on the relationship between the striking
333 distance and the charge stored on the negative leader channel, the following study is done.

334 First, fix the ground altitude h but change the parameter λ , so that both the striking distance S and
335 the charge stored on the negative leader channel Q_l are changed. A relation between the S and Q_l is
336 then obtained by curve fitting with a formula as

$$337 \quad S = kQ_l^v . \quad (12)$$

338 Secondly, repeat the above process for different ground altitudes h ranging from 0 to 2km, a
339 series of $S - Q_l$ curves are obtained as shown in Fig.9. It is found that both the parameters k and v in
340 Eq. (12) are changed with the change of ground altitude h . The k increases from 54 to 89 while the v
341 increases from 0.96 to 0.87, when the h increases from 0 to 2 km. This result can be specified as

$$342 \quad S = (54 + 17.5h)Q_l^{(0.96 - 0.045h)} , \quad (13)$$

343 where h is the ground altitude above sea level in km.

344 In engineering practice, the striking distance S is conventionally related to the peak current of the
345 first return stroke I as

$$346 \quad S = aI^b . \quad (14)$$

347 where, a and b are two constants. And in many studies [41-42], the peak current of the first return
348 stroke is associated with the total charge transferred to the ground by the return stroke excluding the
349 continuous current. However, Mazur and Ruhnke in [14] have shown that correlation between the
350 leader potential (which determines the striking distance) and the peak return stroke current is
351 extremely weak. The return stroke is affected by both, the features of the preceding leader as well as
352 by characteristics of the grounding system of the object. The firm association of the peak current to
353 the total charge transfer by the return stroke is questionable unless the waveforms of every return
354 stroke are the same for every cloud-to-ground flash.

355

356 **4 CONCLUSION**

357 In this work, a simple self-organised propagation model of bi-directional leader heading by a
358 streamer zone was employed to discuss the possible effect of the ground altitude on the lightning
359 striking distance. The model, which is a further development of that of Mazur and Ruhnke [13-15]
360 and Behnke et al. [30] with a streamer zone in front of the leader tip and with the charge simulation
361 method [37-38], enables the calculation of many parameters of the channel, such as the charge
362 distribution along the leader channel, leader current, leader potential, and the striking distance to flat
363 ground. The main parameters obtained, such as the leader charge distribution and the leader current,
364 are in agreement with previous results. Based on the leader tip potential concept of striking distance,
365 the effects of regional ground altitude on the striking distance was investigated. It shows that the
366 striking distance increases significantly with the increase of the regional ground altitude mainly due to
367 the decrease of the critical electric field necessary for sustaining the negative streamer zone.

368

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374

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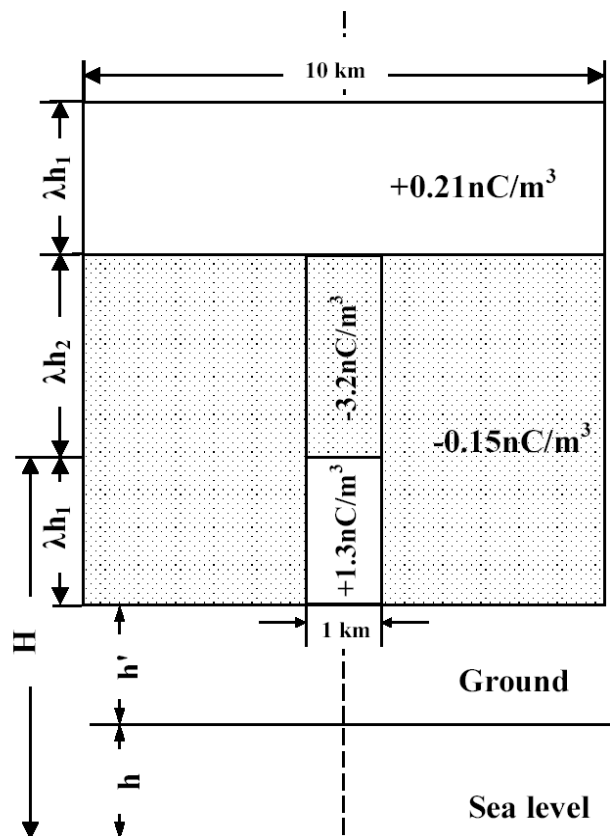
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471 FIGURES

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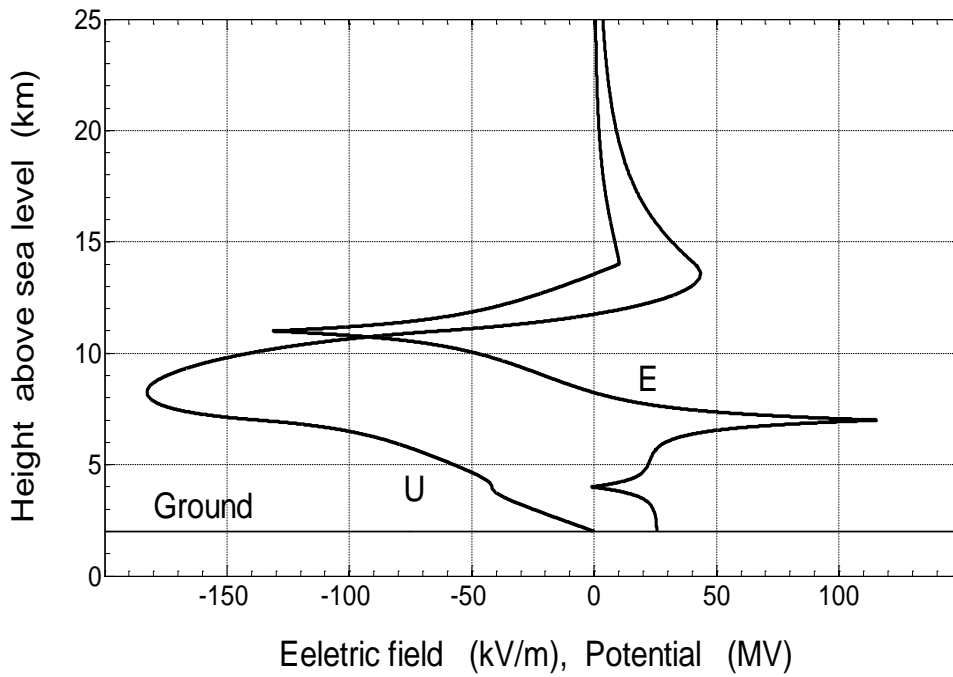
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475 **Fig.1** A modified two-dimensional axis symmetrical tripole charge model of a thunderstorm.

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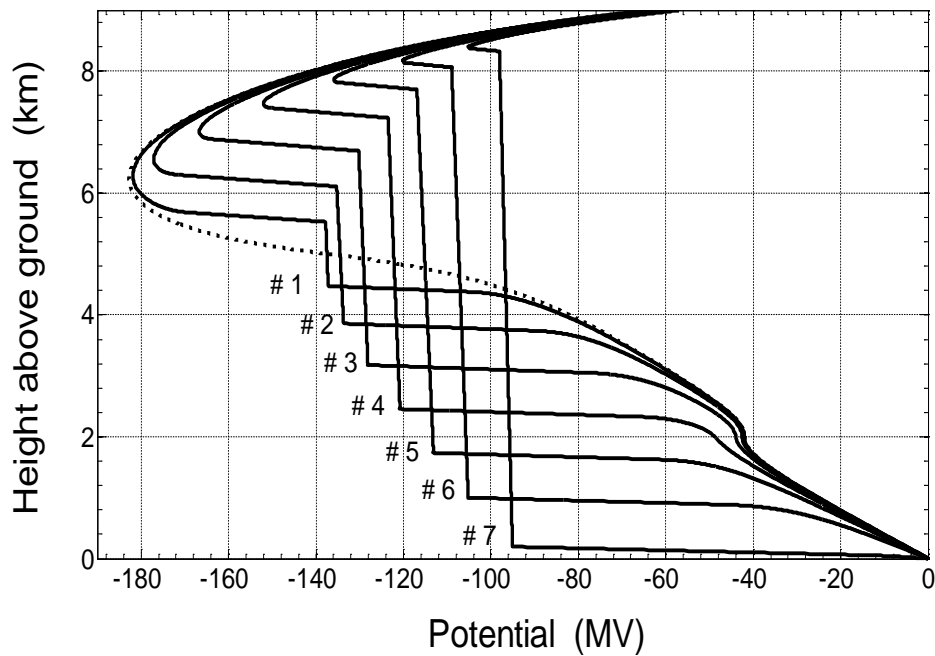


479

480 **Fig. 2** Profiles of the electric fields and potentials along the axis of the cloud charge model in Fig.

481 1 for $H = 7$ km, $h = 2$ km, $\lambda = 1$. U – potentials, E – vertical electric fields.

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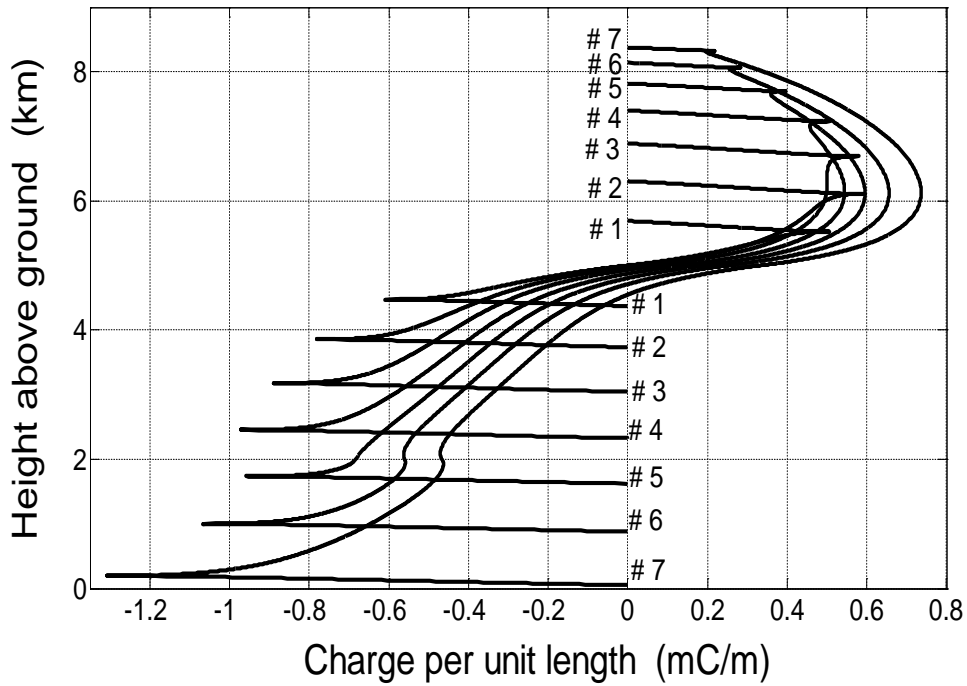
484 **Fig.3** Evolution of the potential profiles on a developing bidirectional leader-streamer system at

485 about 6.5 ms time intervals (solid-lines, indicated by #1, #2 ... #7, respectively) with the

486 cloud charge model shown in Fig.1 and the charge simulation method, for $H = 7$ km, $h = 2$

487 km and $\lambda = 1$. Dot-line: cloud ambient potential profile as in Fig.2.

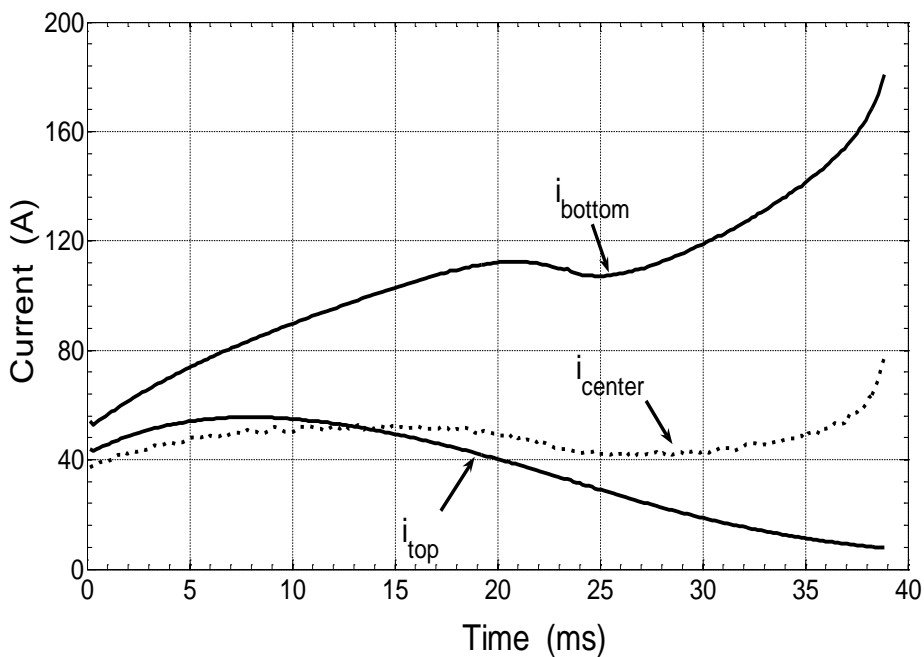
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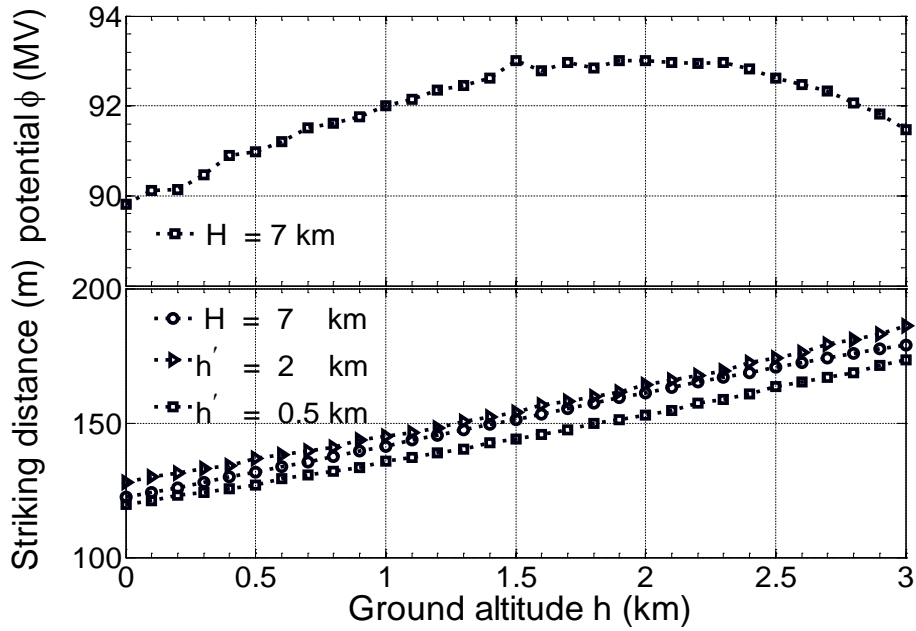
490 **Fig.4** Evolution of the induced charge profiles on a developing bidirectional leader-streamer
 491 system at about 6.5 ms intervals (indicated by #1, #2 ... #7, respectively) with the cloud
 492 charge model in Fig.1 and the charge simulation method, for $H = 7$ km, $h = 2$ km, and $\lambda = 1$.

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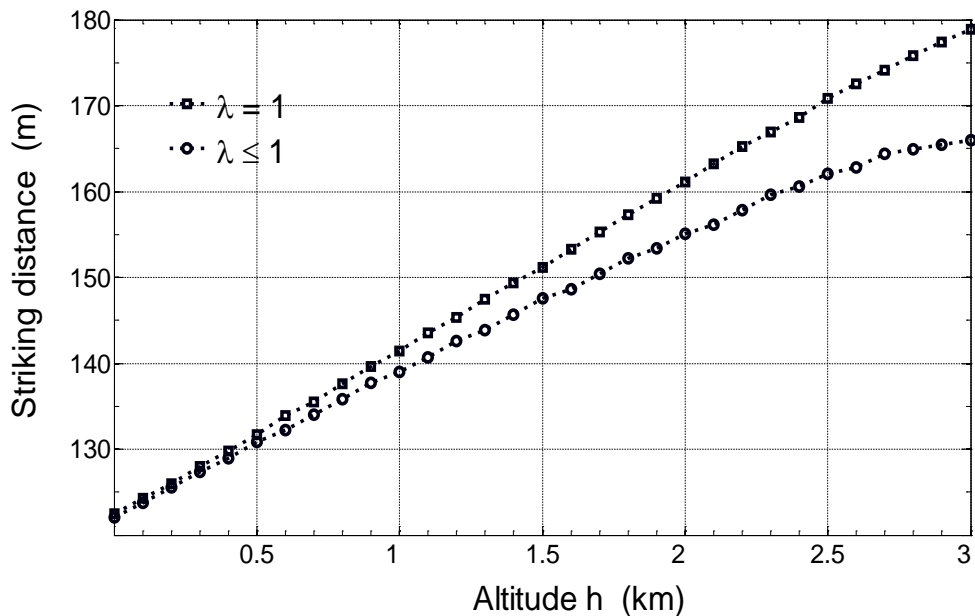
495 **Fig.5** Time variation of the currents at the upper end (i_{top}), bottom end (i_{bottom}) and the center (i_{center}) of
 496 the bidirectional leader-streamer system during its propagation as in Fig.4 ($H = 7$ km, $h = 2$
 497 km, $\lambda = 1$).



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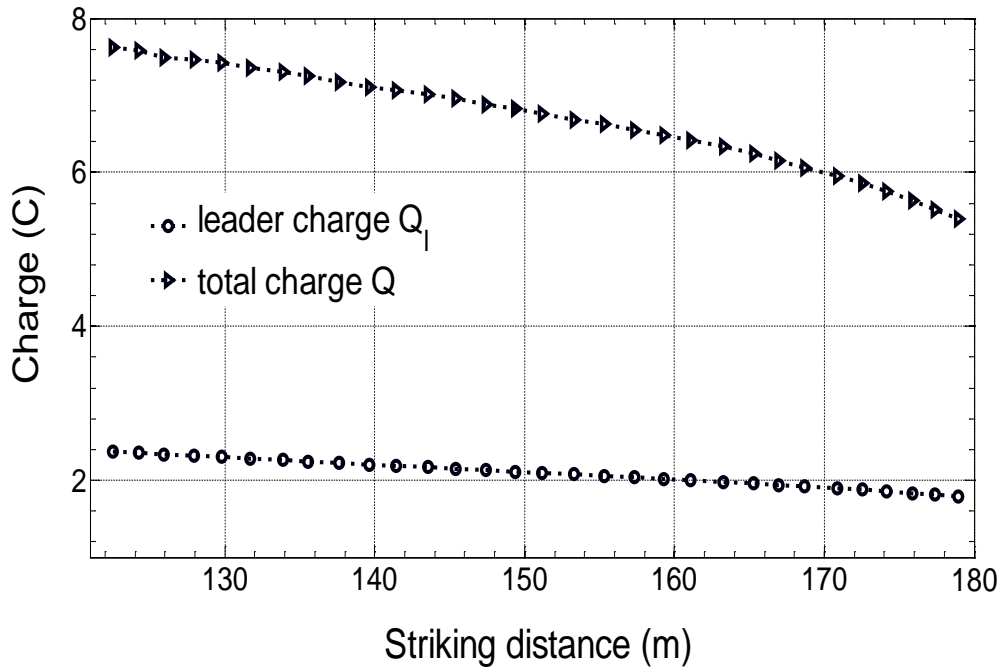
499 **Fig.6** Upper: leader potential before touching ground versus ground altitude for leader initiation
 500 height fixed at $H = 7$ km; Lower: striking distance versus ground altitude for (i) leader
 501 initiation height fixed at $H = 7$ km (circle), (ii) cloud base fixed at $h' = 2$ km (triangle) and
 502 (iii) cloud base fixed at $h' = 0.5$ km (square), with $\lambda = 1$.

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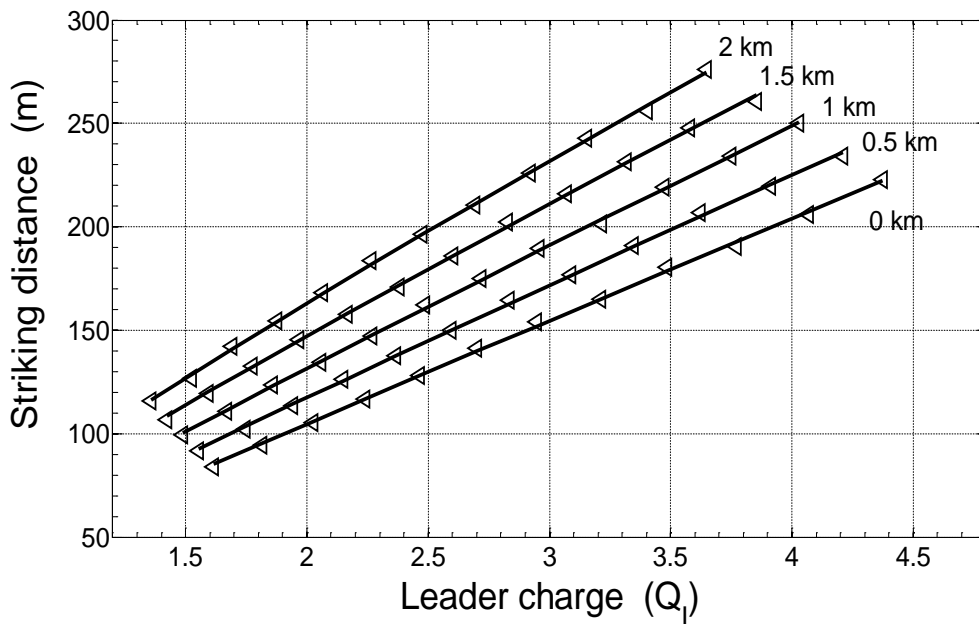
505 **Fig.7** Comparison on striking distance between Case1 (the charge structure unchanged, $\lambda = 1$)
 506 (square) and Case 4 (the electric field at the leader initiation altitude unchanged by adjusting
 507 the zoom factor, $\lambda \leq 1$) (circle).



509

510 **Fig.8** The charge deposit in the negative leader channel before touching ground (Q_l) and the total
 511 charge transfer by the return stroke (Q).

512



513

514 **Fig.9** Striking distance versus the charge (in coulomb) stored on the downward negative leader
 515 channel for different altitude of the ground h changing from 0 to 2 km.