

# A New Ionosphere-free Ambiguity Resolution method for Long-range Baseline with GNSS Triple-Frequency signals

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**Abstract:** New GNSS systems (i.e. GPS modernization, BeiDou, and Galileo) will provide multiple navigation signals for reliable navigation services. The triple or even multiple frequency signals are expected to bring great benefits to the ambiguity resolution (AR) over long-range baselines, which is always regarded as a huge challenge. Another issue in the long baseline AR is the unmodeled ionospheric delay, which is one of the major errors in ranging signals. A new triple-frequency, ionosphere-free technique for ambiguity resolution of long-range baseline are developed in this study. In this technique, the optimal observation combinations are chose considering the effect of ionospheric delay. At the same time, using this technique, the double difference (DD) ionospheric delay can be nullified in the ambiguity search process. The performance of the new technique is examined using the simulated GPS triple frequency data as well as the real BDS observation. Results show that the ambiguity can be fixed in 10 min for GPS and BDS long-range baselines with this new technique.

**Key words:** triple-frequency; ionosphere-free; ambiguity resolution; long-range baseline; GPS; BeiDou

## 1 Introduction

Among new global navigation satellite systems (GNSS), both the modernized GPS and the Chinese BeiDou system (BDS) operate with triple frequency signals. Table 1 shows the frequencies/wavelengths of the Open Service signals that can be tracked from the BDS satellites, versus those of the signals that can be tracked from the satellites of the established GPS modernization (*RINEX 3.02*, 2013). From this table we can see GPS introduces the L5 signal at 1176.45MHz, in addition to the current L1 at 1575.42MHz and L2 at 1227.6MHz, while the Chinese BDS navigation satellite system would operate in three frequencies: B1 (1561.098MHz), B2 (1207.14MHz), B3 (1268.52 MHz).

**Table1** the frequency programme of GPS modernization and BDS

Frequency MHz	Wavelength cm	GPS	BDS
1575.42	19.0	L1	
1561.098	19.2		B1
1268.52	23.6		B3
1227.60	24.4	L2	
1207.14	24.8		B2
1176.45	25.5	L5	

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The triple or even multi-frequency singles can compose extra-wide lane ( $\lambda \geq 2.93$  m) and wide lane ( $0.75 \text{ m} \leq \lambda < 2.93$  m) linear observation combinations, which will bring great benefit to the ambiguity resolution. Richert (Richert, 2005) listed three motivations for using linear combinations of GNSS data. The first motivation is that linear combinations can eliminate or mitigate unwanted terms in the mathematical model that are correlated among the observations. The second motivation is to alleviate the computational burden of processing multi-frequency GNSS data. The third motivation is to reduce the communication bandwidth needed for the transmission of GNSS observations. Most research (Feng and Rizos, 2005; Feng et al., 2007; Ji et al., 2010) has shown that the initialization and the centimeter level position can be got in very short time using multi-frequency observations. And the ambiguities can be fixed more easily for long-range baseline. There are numbers of triple-frequency combination observations, but only the combinations which are beneficial to the cycle slip detection and repair, ambiguity resolution or positioning accuracy improvement are meaningful. And the selection of the optimal combinations has become a hot spot. Zhang (Zhang et al., 2003) find some triple frequency combinations of GPS and Galileo, which are ionosphere-free or the double difference noise of them are below 0.2 cycles, and that the ionosphere index, are no larger than 1.0. Ji et al. (Ji et al., 2010) defined a set of optimal combinations of Galileo inter-frequencies. Richert and El-Sheimy (Richert and El-Sheimy, 2007) defined useful combinations for the three frequency GPS and Galileo systems which eliminate or mitigate individual error sources, alleviate excessive computational burdens and reduce the communication bandwidth. Feng (Feng and Rizos, 2005; Feng et al., 2007) also defined numbers of optimal combinations of GPS, Galileo and BDS considering lower ionospheric term and total noise levels. However, most of the research above selects the optimal combinations with longer wavelength and lower noise. None of them consider the effect of different level of ionospheric delay. In this paper, we choose the optimal combinations for both GPS and BDS base on the effect of ionospheric delay. A new ionosphere-free combination is also proposed to fix the ambiguities of these optimal combinations.

Errors in ranging signals can be grouped into three parts: satellite-related, propagation-related and receiver-related errors. Satellite-related errors, such as satellite clock error and orbit error are same for receivers, and they can be removed by differential methods. Receiver related error sources include receiver clock errors, measurement noise and multipath. These errors are dependent on surroundings and the receiver hardware, and they can also be removed by differential methods and receiver Autonomous Integrity Monitoring (RAIM) methods. Propagation-related effect is introduced by troposphere and ionosphere. The tropospheric delay is small, and the effect of tropospheric decorrelation is negligible. In this study, we only focus on the ionospheric delay, the effect of which is very huge in the long-range baseline. And research has shown that the ionospheric variability in low-latitude regions is much greater than that in mid-latitude areas (Skone and Shrestha, 2002; Konno et al., 2005). During ionosphere storms, the ionospheric delay difference between the reference station and the user can reach tens of meters, and is significant although they are separated by about 10 kilometres. As a result, ionospheric delay is one of the major challenges in resolving ambiguities. Commonly, there are three methods to eliminate the ionospheric delay: the first one is to ignore it in short baseline, which is not suited for long-range baseline. The second one is to eliminate it using ionosphere-free combinations, but the combination observation noise gets much larger and the converged time would be very long. The integer feature of the ambiguity would be lost. The third way is to estimate it as a parameter in the long baseline. However, large numbers of unknown parameters would be involved, a long time should be taken to make both ionospheric delay and float ambiguity converged. A new algorithm to eliminate the ionospheric delays is proposed in this research which does not depend on the convergence of the ionospheric parameter. The substance of this algorithm is to eliminate the ionospheric delay in ambiguity search process. Simply say, once a pair of ambiguity candidates of triple frequency signals has been selected from the search space, the corresponding ionospheric delay can be removed.

A brief description to the optimal combination and the common methods to remove the ionospheric delay was given in

the first section. After that the selection and ambiguity resolution of the triple-frequency optimal combination was discussed. In the third section, the new triple-frequency, ionosphere-free technique for ambiguity resolution was proposed, and the performance of it was examined using the simulated GPS triple-frequency data and BDS real triple-frequency observation in the fourth section. In the end, conclusions were drawn.

## 2 Selection and Ambiguity Resolution of optimal combinations

### 2.1 Selection of Triple-Frequency Optimal Combinations for Extra- wide lane and wide lane

Both modernized GPS of the USA and BDS of China among new global navigation satellite systems operate with triple-frequency signals. Optimal combinations of extra-wide lane ( $\lambda \geq 2.93$  m) and wide lane ( $0.75 \text{ m} \leq \lambda < 2.93$  m) consist of triple-frequency signals significantly enhance the efficiency and reliability of the ambiguity resolution for long-range baseline. The ambiguities of them can be fixed more easily due to their large wavelength.

The selection of the optimal combinations has become a hot spot. Actually, combinations proposed by different researchers are often based on longer equivalent wavelengths or larger wavelength-to-noise ratio (Eq. 1). Generally speaking, ionospheric delay contributes most part of the noise except the measurement noise in long-range baseline (Eq. 2). However, the prior ionospheric delay which would be incorrect is employed in most research, and at the same time the improper optimal combinations are introduced. The ambiguity resolution success rate of them (especially wide lane) would be very low. In this research, optimal combinations will be selected considering the updated ionospheric delay which is estimated as a parameter in the very beginning. Table 2 and 3 show the possible optimal combinations of GPS and BDS when different ionospheric delay involved, and the measurement noise of the pseudo-range is set to 1.0m ( $\sigma_c = 1.0$ ). In Table 2 and 3, i, j and k are the combination coefficients, they should be in the integer space.  $f/f_{L_1}$ ,  $f/f_{B_1}$  are the ratio of the frequency of the optimal combinations and  $L_1$ (GPS) or  $B_1$ (BDS),  $\lambda/\lambda_{L_1}$ ,  $\lambda/\lambda_{B_1}$  are the ratio of the wavelength of the optimal combinations and  $L_1$ (GPS) or  $B_1$ (BDS) respectively. These two tables indicate that the extra-wide lane of GPS changed from 0,-1, 1 to 1,-6, 5 when the ionospheric delay ( $\sigma_{ion}$ ) increases from 0 m - 0.211 m to 0.212 m -3.0 m. For BDS, the extra-wide lane changed from 0,-1, 1 to 1, -5, 4 when ionospheric delay increases. Compared with the extra-wide lane, there are more choices for the optimal combinations of wide lane. GPS has seven wide lane combinations when ionospheric delay alters from 0 m to 3.0 m, and similar combinations are selected for BDS.

$$\text{ratio} = \lambda/\sigma \quad (1)$$

$$\sigma = \sigma_c + \sigma_{ion} \quad (2)$$

where  $\lambda$  is the wavelength of the combination,  $\sigma_c$  is the pseudo-range measurement noise,  $\sigma_{ion}$  indicates the ionospheric delay.

**Table 2** possible optimal observation combinations for GPS

	$\sigma_{ion}$ (m)	i	j	k	$f/f_{L_1}$	$\lambda/\lambda_{L_1}$	ratio
<b>extra-wide lane</b>	0.000-0.211	0	1	-1	0.03	30.77	38.84
	0.212-3.000	1	-6	5	0.06	17.11	13.46
<b>wide lane</b>	0.000-0.005	1	0	-1	0.25	3.95	75.54
	0.006-0.016	1	-1	0	0.22	4.53	48.78
	0.017-0.026	1	-2	1	0.19	5.31	28.61
	0.027-0.041	1	-3	2	0.16	6.42	22.31
	0.042-0.061	1	-4	3	0.12	8.10	18.90
	0.062-0.111	1	-5	4	0.09	11.00	16.76
	0.112-3.000	1	-7	6	0.06	17.11	15.66

**Table 3** possible optimal observation combinations for BDS

	$\sigma_{ion}$ (m)	i	j	k	$f/f_{B_1}$	$\lambda/\lambda_{B_1}$	ratio
extra-wide lane	0.000-0.851	0	1	-1	0.04	25.43	8.16
	0.852-3.000	1	-5	4	0.03	33.14	2.99
wide lane	0.000-0.011	1	0	-1	0.23	4.41	14.99
	0.012-0.101	1	-1	0	0.19	5.34	10.51
	0.102-0.171	1	-2	1	0.15	6.75	6.20
	0.172-0.286	1	-3	2	0.11	9.19	4.92
	0.287-2.316	1	-4	3	0.07	14.40	4.02
	2.317-3.000	2	-9	7	0.10	10.04	1.72

## 2.2 Ambiguity Resolution of Triple Frequency Optimal Combinations

Three/Multiple Carrier Ambiguity Resolution (TCAR/MCAR) and Cascading Integer Resolution (CIR) are the typical three/multiple-carrier ambiguity resolution methods, which are biased by the residual ionospheric delay. TCAR method is the earliest studies by Forsell et al.(Forsell et al., 1997) and Vollath et al. (Vollath et al., 1998). De Jonge et al.(De Jonge et al., 2000) and Hatch et al.(Hatch et al., 2000) proposed the CIR method. The steps of TCAR and CIR can be illustrated in the Fig1. In Fig1,  $P_{ewl}$  is the pseudo-range observation of the extra-wide lane combination,  $\hat{N}_{ewl}$ ,  $\hat{N}_{wl}$ ,  $\hat{N}_{ml}$  and  $\hat{N}_1$  are the ambiguities of the extra- wide lane, wide lane, middle line and  $L_1$ (GPS) or  $B_1$ (BDS).  $\varphi_{ewl}$ ,  $\varphi_{wl}$ ,  $\varphi_{ml}$  and  $\varphi_1$  are the corresponding carrier phase observation in cycle,  $\lambda_{ewl}$ ,  $\lambda_{wl}$ ,  $\lambda_{ml}$  and  $\lambda_1$  are the wavelengths of them. TCAR starts with the extra-wide lane signal (step 1). The ambiguity is resolved by rounding the “float” value to its nearest integer quite reliably. With this information, the wide lane combination can be resolved through rounding (step 2). With the first two ambiguity-resolved signals, the ambiguity of the third signal, e.g. narrow line or middle line is resolved with the same rounding process (step 3). The ambiguity of the  $L_1$ (GPS) or  $B_1$ (BDS) is fixed in the end (step 4).

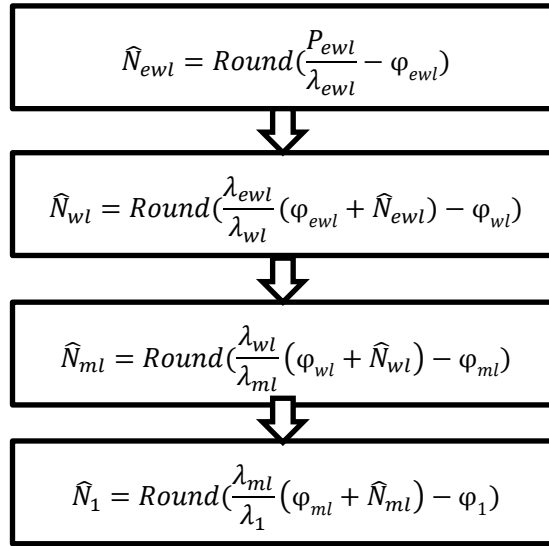


Fig 1 general steps of TCAR/CIR

In step 1,  $\hat{N}_{ewl}$  is affected by the measurement noise, while  $\hat{N}_{wl}$  in step 2 is biased by the residual ionospheric error in addition to the measurement noise. Although the wavelength of wide lane combination is very large, the double difference ionospheric error raises when the baseline length increases. So the TCAR/CIR methods will be limited to the short baseline. In this study, we propose a new ionosphere-free phase observation combined by extra-wide lane and wide lane, and fix the ambiguity of the wide line without the effect of the ionospheric effect.

We have two observation equations of extra-wide lane and wide lane:

$$P + \lambda_{wl}N_{wl} = \lambda_{wl}\varphi_{wl} + C_{wl}\delta_{ion} \quad (3)$$

$$P + \lambda_{ewl}\widehat{N}_{ewl} = \lambda_{ewl}\varphi_{ewl} + C_{ewl}\delta_{ion} \quad (4)$$

An ionosphere-free phase observation combined by extra-wide lane and wide lane:

$$C_{wl}P - C_{ewl}P + C_{wl}\lambda_{ewl}\widehat{N}_{ewl} - C_{ewl}\lambda_{wl}N_{wl} = C_{wl}\lambda_{ewl}\varphi_{ewl} - C_{ewl}\lambda_{wl}\varphi_{wl} \quad (5)$$

Then,  $\widehat{N}_{wl}$  should be:

$$\widehat{N}_{wl} = Round(((C_{wl}(P + \lambda_{ewl}\widehat{N}_{ewl}) - C_{wl}\lambda_{ewl}\varphi_{ewl} + C_{ewl}\lambda_{wl}\varphi_{wl})/C_{ewl} - P)/\lambda_{wl}) \quad (6)$$

Where,  $P$  is the distance between the satellite and the station receiver;  $\delta_{ion}$  is the ionospheric delay. The meanings of other parameters are the same as the Fig 1. Using this ionosphere-free phase observation combined by extra-wide lane and wide lane, we can fix the ambiguity of wide lane combination more easily (Eq. 6).

### 3 Theory of the new Triple-frequency, Ionosphere-free technique for Ambiguity Resolution of long-range baseline

To fix the ambiguity for the long-range baseline without the effect of the ionospheric delay, a new Triple-frequency, Ionosphere-free technique for Ambiguity Resolution is proposed in this section which does not depend on the convergence of the ionospheric parameter. The detailed flow path is below. Figure 2 shows the procedures of this new technique.

- 1) Estimate the ionospheric delay as a parameter each epoch, and then choose the optimal combinations of extra-wide lane and wide lane considering the updated ionospheric delay. Fix the ambiguity of extra-wide lane and wide lane with the methods in section 2.2.
- 2) Calculate the ambiguity search space for  $L_1, L_2, L_5$ (GPS) or  $B_1, B_2, B_3$ (BDS) by seven observation equations: two observation equations of extra-wide lane and wide lane (Eq. 7 and 8) whose ambiguities have been fixed; three phase observation equations (Eq. 9) and two functional relationships (Eq. 10 and 11) between ambiguities of  $L_1, L_2, L_5$ . Sort the ambiguity candidates according to the values of  $\nabla tPV$ .

$$AX = \lambda_{ewl}\varphi_{ewl} - \lambda_{ewl}\widehat{N}_{ewl} + \varepsilon_{pewl} \quad (7)$$

$$AX = \lambda_{wl}\varphi_{wl} - \lambda_{wl}\widehat{N}_{wl} + \varepsilon_{pwl} \quad (8)$$

$$AX + \lambda_i N_i = \lambda_i \varphi_i + \varepsilon_{p_i} \quad (9)$$

$$N_{ewl} = \kappa_1 N_1 + \kappa_2 N_2 + \kappa_3 N_5 \quad (10)$$

$$N_{wl} = \kappa_1' N_1 + \kappa_2' N_2 + \kappa_3' N_5 \quad (11)$$

- 3) Selected a pair of ambiguity candidates from the search space, and then eliminate the ionospheric delay of  $L_1, L_2, L_5$  employing the geometry-free combination (Eq. 12, 13 and 14). Once we have a new ionosphere observable, we can estimate a new float ambiguity estimate. This new float ambiguity and the corresponding variance-covariance matrix are free from the effects of the ionospheric delay. Obviously, each pair of ambiguity candidates gains their own corresponding variance-covariance matrix. Save the value of the corresponding  $\Omega_i$ (Fig 2), and find the ambiguity candidate with the minimum  $\Omega_i$ .

$$\delta_{ion12} = \left( \frac{f_2^2}{f_1^2 - f_2^2} \right) [\lambda_1 \varphi_1 - \lambda_2 \varphi_2 - (\lambda_1 \widehat{N}_1 - \lambda_2 \widehat{N}_2)] \quad (12)$$

$$\delta_{ion15} = \left( \frac{f_5^2}{f_1^2 - f_5^2} \right) [\lambda_1 \varphi_1 - \lambda_5 \varphi_5 - (\lambda_1 \widehat{N}_1 - \lambda_5 \widehat{N}_5)] \quad (13)$$

$$\delta_{ion} = (\delta_{ion12} + \delta_{ion15})/2 \quad (14)$$

- 4) Fix the ambiguities with partial solution.
- 5) Repeat steps 1-5 until there is no data left.

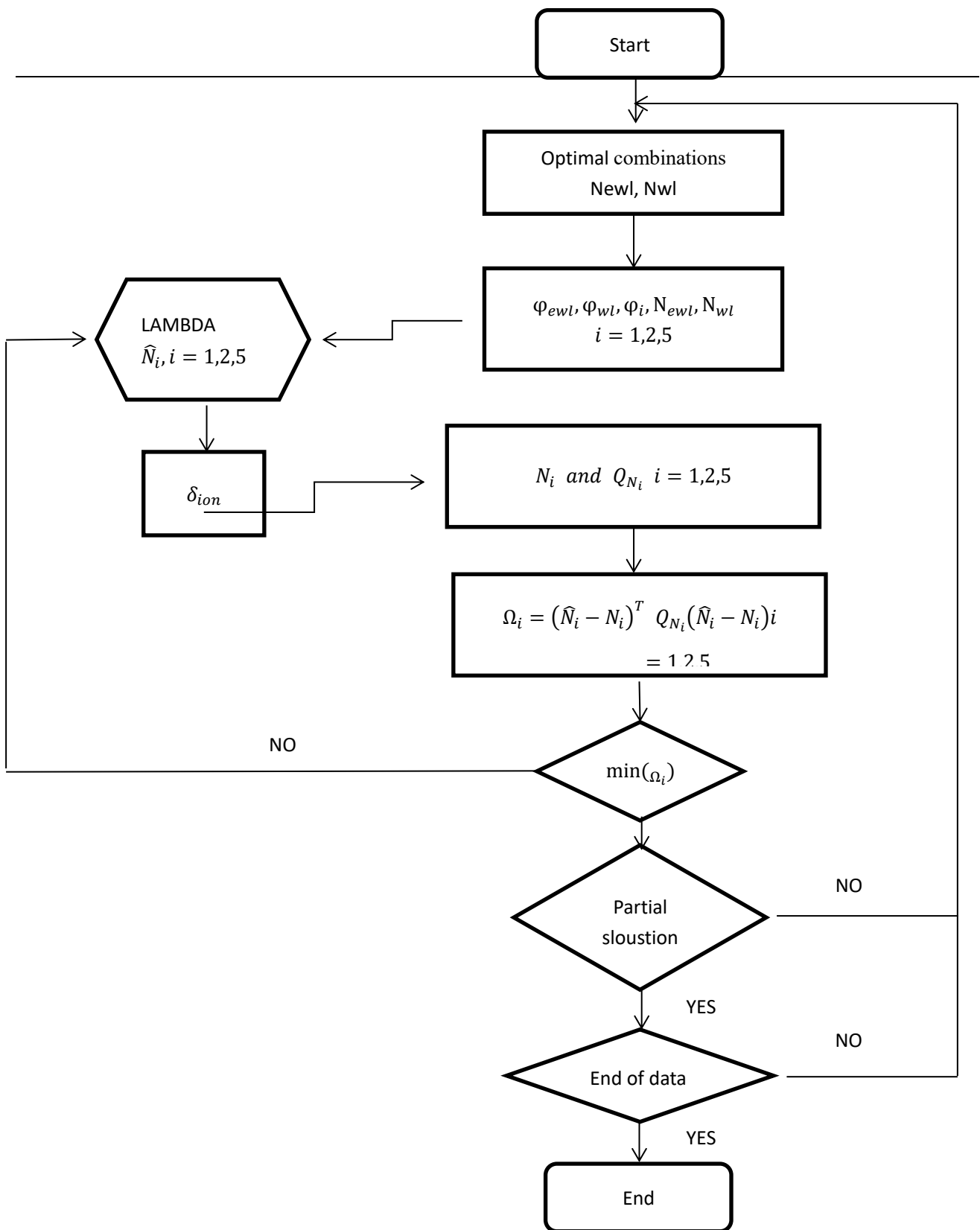


Fig 2 procedures triple-frequency ionosphere-free technique

## 4 Experiments and Analysis

### 4.1 Experiments with GPS simulated data

#### 4.1.1 Data simulation

To test the new triple-frequency ionosphere-free technique, triple-frequency GPS data of three stations (Sta1, Sta2 and Sat3) for 24 hours on December 03, 2012 were simulated. There were three baselines, and the lengths of them were 45Km,

90km and 135km. The observation interval was 30 seconds.

Both the pseudo-range and carrier phase measurements were affected by many error sources. In this simulation, the main error including the ionospheric and tropospheric delays, receiver clock error, measurement noise and multipath effect were modeled. In this simulation, only the first-order ionospheric delay was simulated using the Klobuchar Model. Then a mapping function was applied to convert it to the slant delays. In addition, the higher-order part was ignored. Hopfield model was used to simulate the troposphere delay in this research. The receiver clock was modeled as a specific offset relative to the reference time, including: systematic clock errors and thermal noise errors. White noise was used to represent the clock random errors. Both code and carrier phase observation noise were simulated according to Gaussian normal distribution model and the sizes were user configurable. The code range error and carrier phase range error due to the multipath effect were simulated too.

#### 4.1.2 Optimal combination

In this study, the initial value of ionospheric delay was modeled and estimated as a parameter, and the optimal combinations of extra-wide lane and wide lane of the three baselines in this research were defined considering the updated ionospheric delay. Table 4 and 5 show the coefficients of the optimal combinations, the required time to fix the ambiguity of them as well as the success rate of the ambiguity resolution. From Table 4 we can see that different baselines have the same extra-wide lane optimal combination but different wide lane optimal combinations due to different values of ionospheric delay. Using the method proposed in this study, both ambiguities of extra-wide lane and wide lane can be fixed in the 2.5 min and the success rates of them are high than 99%.

**Table 4** extra-wide lane optimal combination of three baselines

Length of baseline	extra-wide lane	AR Required time(min)	Success rate
45 km	0, 1,-1	0.5	100%
90 km	0, 1,-1	0.5	100%
135 km	0, 1,-1	0.5	100%

**Table 5** wide lane optimal combination of three baselines

Length of baseline	wide lane	AR Required time(min)	Success rate
45 km	1,-1,0	1.0	100%
90 km	1,-2,-1	2.0	100%
135 km	1,-6,5	2.5	99%

Table 6 indicates the observation noise of the optimal combinations which are calculated using Eq. 15. Assuming the noise of carrier phase  $\varepsilon_p$  is 0.003m, the observation noise of the extra-wide lane is 0.0997m and the noises of wide lane are 0.0172m, 0.0318m and 0.3114m separately, which is much lower than that of the pseudo-range. As a consequence, we can adopt them to provide the initial value for the rover position.

$$\varepsilon = \sqrt{\left(i \frac{\lambda}{\lambda_1}\right)^2 + \left(j \frac{\lambda}{\lambda_2}\right)^2 + \left(k \frac{\lambda}{\lambda_3}\right)^2} \varepsilon_0 \quad (15)$$

Where  $\varepsilon$  is the noise of the optimal combinations,  $\lambda$  is the wavelength of them,  $\varepsilon_0$  is the noise of GPS carrier phase observation.

**Table 6** observation noise of the optimal combinations

	i	j	k	Noise(m)
<b>Extra-wide lane</b>	0	-1	1	0.0997
<b>wide lane</b>	1	-1	0	0.0172
	1	-2	1	0.0318
	1	-6	5	0.3114

#### 4.1.3 Performance of the new triple-frequency, ionosphere-free technique

To examine the performance of the new triple-frequency, ionosphere-free technique for ambiguity resolution, three schemes are employed to fix the ambiguities of the GPS triple frequency signals for three baselines.

Scheme 1: ambiguity resolution using GPS triple-frequency pseudo-range and carrier phase observation.

Scheme 2: ambiguity resolution with ambiguity fixed extra-wide lane and wide lane combinations replacing the pseudo-range observation

Scheme 3: ambiguity resolution with the new triple-frequency, ionosphere-free technique.

**Table 7** time required to fix the ambiguity (unit: min)

	45 km	90 km	135 km
<b>Scheme 1</b>	18.5	35.0	41.0
<b>Scheme 2</b>	10.0	32.5	38.0
<b>Scheme 3</b>	4.5	7.0	9.0

Table 7 indicates the time required to fix the ambiguity for baselines with different lengths using the three schemes. From the table we can see,

- 1) About 18.5 – 41.0 minutes are needed for the ambiguity resolution with scheme 1 when the baseline is shorter than 135 km.
- 2) Compared with that of scheme 1, the speed of the ambiguity resolution using scheme 2 increases 45.9%, 7.14% and 7.32%, respectively. So using the ambiguity fixed carrier phase observation of optimal combinations to provide the initial value for the rover position is benefit to the ambiguity resolution.
- 3) Only 4.5 – 9.0 minutes are required to fix the ambiguity using the new triple-frequency, ionosphere-free technique, which is shorter than that of Scheme 2 by 55%, 78.46% and 76.32%. which means that we can fix the ambiguity within 10 minutes when the baseline is shorter than 135 km adopting the new triple-frequency, ionosphere-free technique.

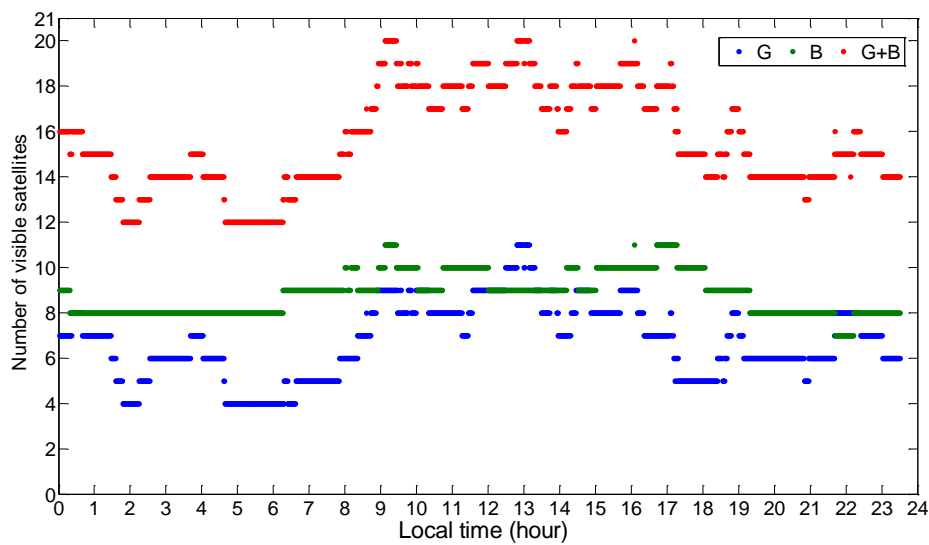
## 4.2 Experiments with BDS observation

### 4.2.1 Observation overview

Hong Kong lies between Latitude 22°08' North and 22°35' North, Longitude 113°49' East and 114°31' East. As a city located at low latitude, strong disturbances and scintillation of the ionosphere exist in Hong Kong due to the complicated interactions among solar radiation, the geomagnetic field and the atmosphere. Consequently, the ionospheric delays are significantly different, even with baselines less than 10 km (Kao et al. 2013; Chen et al. 2008; Chen et al. 2002). This situation is very similar to that of long-range baseline in high- and mid-latitude areas. So Hong Kong is a nature experimental filed for the ionospheric delay investigation for long-range baseline. Triple frequency BDS and double frequency GPS observation of three Integrity Monitoring (IM) stations from Hong Kong Satellite Positioning Reference



Station Network (SatRef) were collected on May 1, 2013 from 00:00:00 to 23:59:59 (GPS time). The three stations were HK01, HK02, HK03, and the receiver of them was Trimble NetR9. The sample interval was 15 seconds. Two baselines, including HK01-HK02 (about 25km), HK01-HK03 (about 35km) were processed in this study. Fig 3 shows the number of visible satellites of GPS, BDS and GPS+BDS. In this study, the cutoff elevation angle was set to  $15^\circ$  for GPS satellites, and which was set to  $20^\circ$  for BDS due to the large noise of BDS observation below 20 (Ji et al. 2014). For GPS, there were only dozens of epoch observation available for G17, G22 and G25. In the data processing, we just deleted them. For BDS, the observed satellites included BDS satellites from C01 to C14. But for C13 and C14, only observations of two frequency bands (B1 and B2) were available. The used BDS satellites in data processing included 5 GEOs (C01, C02, C03, C04, C05), 5 IGSOs (C06, C07, C08, C09, C10) and 2 MEOs (C11, C12). From Fig 3 we can see that there are 7 – 11 BDS visible satellites in the experimental period, while 4 – 11 GPS satellites are available. For the BDS and GPS combined system, over 12 satellites can be applied in the data processing.



**Fig 3** Number of visible satellites of GPS, BDS and GPS+BDS

#### 4.1.2 Performance of the new triple-frequency, ionosphere-free technique

In this section, the performance of the new technique was examined with BDS triple-frequency observations. Besides, the ambiguity resolution performance for combined BDS and GPS system was investigated and compared with that of BDS alone. To achieve these purposes, data was processed every hour with BDS alone for the two baselines using the three schemes below.

Scheme 1: ambiguity resolution using BDS triple-frequency observation.

Scheme 2: ambiguity resolution with new triple-frequency, ionosphere-free technique.

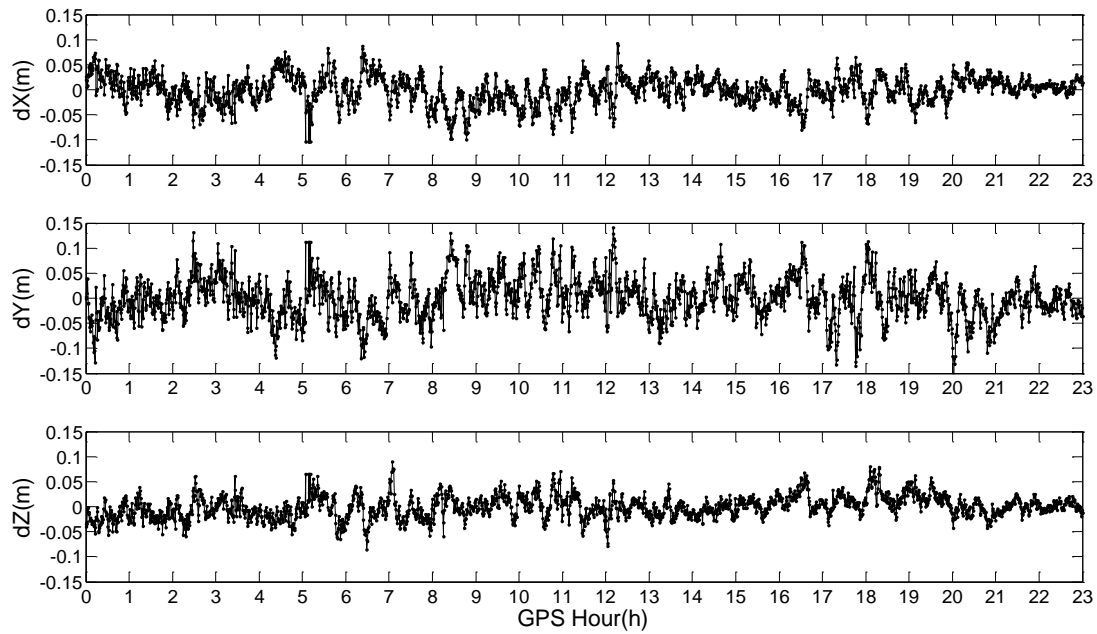
Scheme 3: ambiguity resolution using BDS triple-frequency observation and GPS dual-frequency observation with new triple-frequency, ionosphere-free technique.

The performance of the baseline HK01-HK02 (25 km) was better than that of baseline HK01-HK03 (35 km), only the latter one was shown in this paper. Table 8 indicates the time required to fix the ambiguity with these three schemes. From Table 8 we can see that the required time of ambiguity resolution with new technique is generally less than that with scheme 1, and the ambiguity resolution are speeded up by about 4.35% to 29.88%. When dual-frequency GPS observation are involved, ambiguity resolution can be achieved in about 10 minutes, which shows that GPS observations is beneficiary to fixing BDS ambiguities. The possible reason may be that GPS observations are helpful to float BDS ambiguity solution.

After ambiguities are corrected, the position errors of dynamic double difference baseline are summarized in Fig. 3 and table 9. From the figure and table we can see GPS and BDS have the similar precisions. The precision of both GPS and BDS in each direction are better than 42mm. And the combined system has the best performance.

**Table 8** time required to fix the ambiguity (min)

<b>Hour</b>	<b>Scheme 1</b>	<b>Scheme 2</b>	<b>Improvement by Scheme 2</b>	<b>Scheme 3</b>	<b>Improvement by Scheme 3</b>
<b>1</b>	7	6.5	7.14%	6	7.69%
<b>2</b>	11.5	10.5	8.70%	8.5	19.05%
<b>3</b>	10	9.5	5.00%	9	5.26%
<b>4</b>	8	7	12.50%	5.5	21.43%
<b>5</b>	12.5	11.5	8.00%	10.5	8.70%
<b>6</b>	15	13	13.33%	12	7.69%
<b>7</b>	26.8	20	25.37%	16.5	17.50%
<b>8</b>	16	15	6.25%	14.5	3.33%
<b>9</b>	16.3	13.3	18.40%	11	17.29%
<b>10</b>	4.75	4	15.79%	3.5	12.50%
<b>11</b>	7	6.5	7.14%	6	7.69%
<b>12</b>	14.5	12.5	13.79%	8	36.00%
<b>13</b>	11.5	11	4.35%	10	9.09%
<b>14</b>	13	11	15.38%	10.5	4.55%
<b>15</b>	17	16.5	2.94%	15	9.09%
<b>16</b>	24.8	20.5	17.34%	17	17.07%
<b>17</b>	6.5	6	7.69%	5.5	8.33%
<b>18</b>	7	6	14.29%	5.5	8.33%
<b>19</b>	32.8	23	29.88%	15	34.78%
<b>20</b>	10.5	8.75	16.67%	7.5	14.29%
<b>21</b>	12	11	8.33%	9	18.18%
<b>22</b>	13	11.5	11.54%	10	13.04%
<b>23</b>	9	8	11.11%	7.5	6.25%
<b>mean</b>	13.61	11.64	12.45%	9.89	13.61%



**Fig 4** double difference positioning difference

**Table 9** double difference positioning precision

<b>std</b>	<b>dx</b>	<b>dy</b>	<b>dz</b>
<b>GPS</b>	0.032	0.040	0.021
<b>BDS</b>	0.028	0.042	0.023
<b>G+B</b>	0.024	0.035	0.018

## 5 conclusions

In this research, we select the optimal combination for the triple frequency GPS and BDS signals and fix the ambiguities of them considering the effect of the ionospheric delay. A new triple-frequency, ionosphere-free technique for ambiguity resolution of long-range baseline is developed. To test the performance of the new technique, triple-frequency GPS simulated data and BDS observation were processed. From all the discussions above, we can get the conclusions below:

- 1) Ambiguity resolution can be achieved in 2.5 min for both extra- wide lane and wide line using the combination proposed in this study.
- 2) With the triple-frequency ionosphere-free technique, ambiguities can be fixed in 10 min with GPS simulated data for baselines which are short than 135Km.
- 3) With the triple-frequency ionosphere-free technique, ambiguities can be fixed in 12 min with BDS observation for baselines which are short than 35Km in low-latitude regions.
- 4) With BDS (triple-frequency) and GPS (dual-frequency) observation, ambiguities of BDS can be fixed in 10 min for baselines which are short than 35Km in low-latitude regions.
- 5) The new triple-frequency ionosphere-free technique is effective.

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