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2 **A New Simplified Method for Calculating Short-term and Long-term**
3 **Consolidation Settlements of Multi-layered Soils Considering Creep Limit**
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44 **Abstract** The long-term consolidation settlement of soft soils under infrastructures is seriously
45 concerned in coastal areas. In practice, soft soil ground is usually composed of multiple layers
46 with different properties, and the effect of soil creep is nonnegligible. However, in conventional
47 consolidation analysis, creep is seldom included during the “primary” consolidation. With
48 conventional creep model, creep strain will reach infinity with time, which is unreasonable in the
49 long-term view. In this study, a new simplified method based on Hypothesis B is presented to
50 calculate both short-term and long-term consolidation settlements of multi-layered soils
51 exhibiting non-linear creep. There are three main characteristics for this new simplified method:
52 (a) the consolidation analysis for multi-layered soil is conducted using spectral method; (b) a
53 nonlinear creep function with creep limit is incorporated to calculate both short-term and long-
54 term creep settlements; (c) the “primary” consolidation and creep are calculated independently
55 and combined with an empirical parameter α together with the consolidation degree U for
56 correcting the creep settlement. Finite element analysis with a self-developed Elastic Visco-
57 Plastic (EVP) constitutive model considering creep limit was also carried out. Verifications by
58 field cases and finite element simulation demonstrate the efficiency and accuracy of the new
59 simplified method in calculating the long-term settlements of soft soil ground under varied
60 engineering conditions. Influences of major parameters selection in the calculation of the new
61 simplified method are also discussed.

62 **Keywords:** settlements, nonlinear creep function, multi-layered soft soils, vertical drains, new
63 simplified method

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66 **1 Introduction**

67 Constructions of infrastructures and embankments on soft soils have played an important
68 role during the urbanizations around the world in the past decades in many coastal regions, such
69 as Hong Kong, Sweden, Singapore, *etc.* The soft soil ground may be formed by long-term
70 natural sedimentation or reclamation, and consolidation is needed to increase the bearing
71 capacity and stiffness of the soil. Consolidation settlement of soft soil under overlying load is a
72 significant issue, which needs to be predicted accurately and controlled strictly. Conventional
73 simplified calculations are normally based on Terzaghi's 1D consolidation equations, which
74 ignore the inter-layer differences and the creep effect during and after the consolidation.
75 However, natural soft soils usually consist of more than one layers with different values of
76 compressibility, over-consolidation ratio, permeability, *etc.*, which calls for practical analysis
77 method for these complicated situations.

78 Time-dependent settlements are contributed by the following two process: (1) the
79 increase of effective stress due to dissipation of excess pore water pressure under the applied
80 loading (*i.e.* "primary" consolidation); (2) creep of the soft soil, mainly due to the viscous
81 behaviours of the soil skeleton. Both parts are significant for soft soils and should be considered
82 in the engineering design especially for long-term service. In the earlier years, creep was used to
83 be referred to as "secondary" consolidation, occurring after the end of "primary" consolidation,
84 which is named "Hypothesis A" later. In this hypothesis, the calculations of "primary"
85 consolidation settlements and creep deformation are conducted separately. However, serious
86 concerns have been raised about Hypothesis A, as it fails to satisfy the developing theories in
87 viscous behaviours of clayey soils and usually underestimates the long-term settlements [1–3]. In
88 contrary, Hypothesis B advocates that creep happens during the "primary" consolidation [4], as

89 revealed in Fig. 1. Settlement calculations based on Hypothesis B can be conducted through fully
90 coupled finite element (FE) or finite difference analysis, which may suffer from non-
91 convergence problems. The efficiency and accuracy of fully coupled numerical simulations rely
92 on the proper selection of time steps, numerical algorithm, *etc.* The determination of parameters
93 for 2D and 3D FE analysis is also complicated. Therefore, simplified handy methods for
94 calculating settlements for soft soils based on analytical and empirical calculations are still
95 widely adopted, including the new simplified Hypothesis B methods recently developed for
96 calculating the settlement of single/double layers of soils exhibiting creep [1,5]. Without
97 necessity of iterations on small time-steps, simplified methods consider the total settlements
98 contributed by two parts: the “primary” consolidation settlements and the creep settlements from
99 the time of beginning. A reduction factor on creep settlements was introduced to consider the
100 coupling effect of excess pore pressure dissipation. One of the most important issues in this
101 method is the proper determination of correction factor varied with time and depth. The
102 consolidation analysis for multi-layered soils and modelling of creep behaviors are also
103 necessary.

104 To predict the time-dependent settlements of soft soils, the first phase of Hypothesis B
105 method is to calculate the degree of consolidation with time under different engineering
106 conditions. Terzaghi’s 1D consolidation equations are developed for a single layer of
107 homogeneous clayey soil under vertical stress. However, Terzaghi’s solutions cannot directly
108 consider horizontal drainage, multi-stage and ramp loading, as well as multi-layered soils with
109 depth-dependent properties. For soft soils with more than one layers, a number of solutions have
110 been developed using different analytical and numerical method for both double-layer [6,7] and
111 general stratified systems [8,9]. These solutions tend to be lengthy and complicated which limits

112 their applications in engineering practice. In another way, Walker et al. [10,11] applied the
113 spectral method to solve consolidation problems for multi-layered soils with high efficiency and
114 accuracy.

115 The second phase of Hypothesis B method is the calculation of creep deformation with
116 time. The calculation can be accomplished by using an elastic visco-plastic (EVP) constitutive
117 model for clays, which usually considers a correlation between strain and the logarithm of time.
118 Based on 1D straining condition, Bjerrum [12] proposed the “delayed” compression concept,
119 which is later developed into the famous “time line” model. Yin and Graham [13,14] developed
120 this model by introducing the concept of “equivalent time line” and indicated that there exists a
121 unique relationship between the stress-strain state and visco-plastic strain rate. Yin [15] later
122 improved this model with consideration of a creep limit (as shown in Fig. 2) to avoid the logical
123 error in long-term view, and proposed a nonlinear creep function. This model was later
124 developed to a 3D model and implemented in numerical analysis [16–18].

125 This paper introduces a new Hypothesis B method for calculating the short-term and
126 long-term settlements of multi-layered soft soils under ramp loading. A calculation template
127 embedded with automatically executed VBA program is developed. For the creep strain part, the
128 nonlinear creep function is adopted in the new simplified method. The proposed method was
129 verified by both in-situ measured data and numerical simulations in Plaxis 2D (2015 version). To
130 reveal the influences of permeability and creep parameters selection, sensitivity parametric
131 studies are carried out and discussed.

132

133 **2 Theoretical frameworks of the new simplified B method for multi-layered soils**

134 ***2.1 One-dimensional EVP model with creep limit***

135 Fig. 2 shows a schematic diagram for the 1D EVP model [15,17]. For simplicity, the
 136 symbols of stresses and strains in Fig. 2 and subsequent figures are in vertical direction by
 137 default (*e.g.* $\sigma' = \sigma'_z$) under 1D straining condition. The reference time line (λ -line) and instant
 138 time line (κ -line) are determined by conventional laboratory oedometer tests within t_0 period
 139 (t_0 could be 24h or the time at end of “primary” consolidation). The visco-plastic strain rate is
 140 independent of stress history but uniquely related to the current stress-strain state and can be
 141 expressed with equivalent time t_e , which is particular useful in calculating creep settlements for
 142 both over-consolidation and normally consolidation soil state. The vertical visco-plastic strain
 143 $\Delta\varepsilon^{vp}$ starting from the reference time line is calculated through Eq. (1):

$$\Delta\varepsilon^{vp} = \frac{\frac{\psi_0}{V} \ln\left(\frac{t_0 + t_e}{t_0}\right)}{1 + \frac{\psi_0}{V\Delta\varepsilon_L} \ln\left(\frac{t_0 + t_e}{t_0}\right)} \quad (1)$$

144 where t_e is the equivalent time, $\frac{\psi_0}{V}$ is the creep coefficient and t_0 is the reference time as the
 145 starting point of creep calculation and $\Delta\varepsilon_L$ is the creep strain limit. $\Delta\varepsilon^{vp}$ is equal to $\Delta\varepsilon_L$ when
 146 time is infinite. If $\Delta\varepsilon_L$ is taken as infinity, the creep strain calculation is equivalent to the linear
 147 logarithmic expression in the EVP model proposed by Yin and Graham [13]:

$$\Delta\varepsilon^{vp} = \frac{\psi_0}{V} \ln\left(\frac{t_0 + t_e}{t_0}\right) \quad (2)$$

148 Fig. 3 shows the schematic diagram of the EVP model for calculating 1D creep
 149 settlements. The dashed curves represent the actual stress-strain paths during the consolidation
 150 process. $(\sigma'_p, \varepsilon_p)$ is the pre-consolidation pressure point on the reference time line, which could

151 be calculated with initial OCR if only one single loading step is included. $(\sigma'_f, \varepsilon_f)$ represents the
 152 “final” stress-strain state after consolidation in laboratory under the targeted stress level σ'_f
 153 without considering creep. Soil at $(\sigma'_f, \varepsilon_f)$ could be normally consolidated or over-consolidated,
 154 as shown in Figs. 3(a) and 3(b) respectively. t_e can be calculated by Eq. (3):

$$\begin{aligned}
 t_e = t_{e0} + t &= \exp \left[\frac{\Delta \varepsilon_f^{vp}}{\frac{\psi_0}{V} \left(1 - \frac{\Delta \varepsilon_f^{vp}}{\Delta \varepsilon_L} \right)} \right] t_0 - t_0 + t \\
 &= \exp \left\{ \frac{\left[(\varepsilon_f - \varepsilon_p) - \frac{\lambda}{V} \log \left(\frac{\sigma'_f}{\sigma'_p} \right) \right]}{\frac{\psi_0}{V} \left(1 - \frac{(\varepsilon_f - \varepsilon_p) - \frac{\kappa}{V} \log \left(\frac{\sigma'_f}{\sigma'_p} \right)}{\Delta \varepsilon_L} \right)} \right\} t_0 - t_0 + t, \quad \text{for } \Delta \varepsilon_f^{vp} < \Delta \varepsilon_L
 \end{aligned} \tag{3}$$

155 where $\Delta \varepsilon_f^{vp}$ is the difference between the targeted strain ε_f and the reference strain ε_f^r on the
 156 reference time line.

157 Referring to Figs. 2 and 3, we can see that the value of t_e is related to the OCR. For
 158 $(\sigma'_f, \varepsilon_f)$ at normally consolidation state, $\sigma'_f \geq \sigma'_p$ and $\Delta \varepsilon_f^{vp} = 0$, as shown in Fig.3 (a), and
 159 therefore $t_e = t$. For $(\sigma'_f, \varepsilon_f)$ at over-consolidation state, $\Delta \varepsilon_f^{vp}$ is larger than zero, as shown in
 160 Fig.3 (b), and $t_e > t$. If $\Delta \varepsilon_f^{vp} \geq \Delta \varepsilon_L$, the creep strain rate is zero and there is no creep
 161 settlement after loading applied.

162

163 **2.2 One-dimensional consolidation analysis for multi-layered clayey soils subjected to ramp**
 164 **loading**

165 In Walker's solution[11], the governing equation for consolidation is presented as Eq.
 166 (4):

$$\frac{m_v}{\bar{m}_v} \frac{\partial \bar{u}}{\partial t} = - \left[dT_h \frac{\eta}{\bar{\eta}} \bar{u} - dT_v \left(\frac{\partial}{\partial Z} \left(\frac{k_v}{\bar{k}_v} \right) \frac{\partial \bar{u}}{\partial Z} + \frac{k_v}{\bar{k}_v} \frac{\partial^2 \bar{u}}{\partial Z^2} \right) \right] + \frac{m_v}{\bar{m}_v} \frac{\partial \bar{\sigma}}{\partial t} + dT_h \frac{\eta}{\bar{\eta}} w \quad (4)$$

167 where \bar{u} is the average pore water pressure at certain depth z , $\bar{\sigma}$ is the average total stress at
 168 certain depth z , η is a lumped parameter for consideration of horizontal consolidation, w is the
 169 water pressure applied on the vertical drains. The volume compressibility m_v is calculated using
 170 the total incremental strains resulted from "primary" consolidation. k_v , m_v and η are considered
 171 as depth-dependent in a piecewise linear way, which are normalized by a reference layer
 172 (with \bar{k}_v , \bar{m}_v and $\bar{\eta}$) chosen from all layers. The depth variable $Z = \frac{z}{H}$ is a normalized
 173 parameter and H is the total soil thickness. In spectral method, $\bar{u}(Z, t)$ is calculated by
 174 integrating over the whole soil depth with a uniform solution expression.

175 After solving Eq.(4) with spectral method, excess pore water pressure $\bar{u}(Z, t)$ was finally
 176 expressed with a series of matrices in Eq.(5):

$$\bar{u}(Z, t) \approx u_0 \mathbf{\Phi} \mathbf{v} \mathbf{E} (\mathbf{\Gamma} \mathbf{v})^{-1} \mathbf{\theta} \quad (5)$$

177 where u_0 is the initial reference excess pore pressure, $\mathbf{\Phi} = [\phi_1(Z) \ \phi_2(Z) \ \cdots \ \phi_N(Z)]$ is
 178 formed by a series of linearly independent sinusoidal basis functions $\phi_j(Z)$, \mathbf{E} is a diagonal
 179 matrix associated with the eigenvalues of $\mathbf{\Gamma}^{-1} \mathbf{\Psi} \cdot \mathbf{\Gamma}$, $\mathbf{\Psi}$ and $\mathbf{\theta}$ are matrices or vector relevant to
 180 soil parameters at any depth and each column of matrix \mathbf{v} is formed by eigenvector associated

181 with each eigenvalue of $\Gamma^{-1}\Psi$. N is the number of terms in Φ , which is normally taken as 30. A
 182 larger N produces the more precise results, but requires longer calculation time. Details of the
 183 derivation could be found in [10,11].

184 Walker and Indraratna [19] developed an Excel spreadsheet implemented with VBA
 185 program named SPECCON to enable convenience adoption of this method. In the program,
 186 consolidation problem of soils with up to 20 layers can be easily calculated. Details of the
 187 program can be found in [19].

188 Geometry parameters of drains, soil layers, vertical and horizontal drainage conditions,
 189 permeability, volume compressibility m_v and vertical ramp loading $\Delta\sigma'_i(t)$ are to be input. The
 190 output is the average excess pore pressure \bar{u}_i for each layer- i , from which degree of

191 consolidation for layer- i is calculated as $U_i(t) = 1 - \frac{\bar{u}_i(t)}{\Delta\sigma'_i(t)}$. Therefore, the “primary”

192 consolidation settlements under loading is obtained by:

$$S_{primary,i}(t) = U_i(t)S_{f,i} \quad (6)$$

193 where $S_{f,i}$ is the final deformation of layer- i under incremental stress without coupling of excess
 194 pore pressure. According to Fig. 3, S_f of a soil element is calculated as:

$$S_f = \begin{cases} H \frac{\kappa}{V} \ln \frac{\sigma'_f}{\sigma'_0} & \text{(for over-consolidation state)} \\ H \frac{\kappa}{V} \ln \frac{\sigma'_p}{\sigma'_0} + H \frac{\lambda}{V} \ln \frac{\sigma'_f}{\sigma'_p} & \text{(for normal-consolidation state)} \end{cases} \quad (7)$$

195 where H is the thickness of the element. It should be noted that for soils under ramp loading
 196 $\Delta\sigma'(t)$, both $\sigma'_f(t)$ and $S_f(t)$ are time-dependent within the construction time.

197 Considering the nonlinearity of soil behavior, compression strain under a stress increment
 198 varies with different depths due to different initial effective stress. Therefore, $S_{f,i}$ for each layer
 199 should be calculated by precise integration over the layer thickness H_i or by approximation as
 200 the sum of S_f in many smaller sub-layers.

201 In this study, each soil layer is divided into sub-layers with thickness of 0.5m at most, if
 202 $H_i \geq 0.5m$. For each sub-layer, the difference of initial effective stress between the upper and
 203 lower boundary is around 2.5 kPa for a clay with density of 15 kPa/m, which should be small
 204 enough since the in-situ vertical stress will be much larger for thick layers of soils. The thickness
 205 of 0.5m for sub-layers have also been adopted and examined in [1,5,20]. Thus $S_{f,i}$ is calculated
 206 as:

$$S_{f,i} = \sum_{k=1}^{\Pi\left(\frac{H_i}{0.5m}\right)} S_{f_i,k} \quad (8)$$

207 where k denotes the number of sub-layer. With $S_{f,i}$, $m_{v,i}$ is computed as $m_{v,i} = \frac{S_{f,i}}{H_i \Delta \sigma}$ before
 208 consolidation analysis.

209

210 **2.3 Calculation of total time dependent settlements with Hypothesis B**

211 According to Hypothesis B, creep starts before end of “primary” consolidation, which
 212 results the complexity of stress-strain state determination. The total settlements for each layer
 213 include two parts: the settlement resulted from excess pore pressure dissipation and the other
 214 from creep deformation, as shown in Eq. (9):

$$S(t) = S_{primary}(t) + S_{creep}(t) \quad (9)$$

215 where $S_{primary}(t)$ is the incremental settlements caused by “primary” consolidation, as calculated
 216 by Eq. (6) in the last section. S_{creep} is the settlements caused by soft soil creep. The calculation of
 217 $S_{creep}(t)$ is shown in Eq. (10):

$$S_{creep}(t) = \begin{cases} \alpha US_{creepf}(t) & t \leq t_{EOP} \\ \alpha US_{creepf}(t) + (1 - \alpha U) S_{creepd}(t) & t > t_{EOP} \end{cases} \quad (10)$$

218 In Eq. (10), S_{creep} after EOP (when the degree of “primary” consolidation reaches 98%)
 219 contains two terms: S_{creepf} and $S_{creepd} \cdot S_{creepf}$ corresponds to the assumption that creep occurs
 220 immediately after application of loading within reference time t_0 , without considering excess
 221 pore pressure dissipation. S_{creepd} corresponds to Hypothesis A that creep only occurs after EOP.
 222 As the effective stress increment is delayed by excess pore pressure dissipation, S_{creepf} will
 223 apparently overestimate the actual creep settlements before EOP. Therefore, S_{creepf} is multiplied
 224 with an empirical correction coefficient α and average consolidation degree U , both ranging
 225 between 0 and 1. When $\alpha = 1$, Eq. (10) become equivalent to Hypothesis A method. In some
 226 previous studies [5,21], $\alpha = 0.8$ was frequently adopted and validated for different clayey soils
 227 under different conditions. However, U was not included yet, resulting in overestimation at the
 228 earlier stages for the thick soil layers.

229 According to the one-dimensional EVP model, the calculation formula for S_{creepf} and
 230 S_{creepd} is shown in Eqs. (11) and (12):

$$S_{creepf}(t) = H \left[\frac{\frac{\psi_0}{V} \ln\left(\frac{t_0 + t_e}{t_0}\right)}{1 + \frac{\psi_0}{V \Delta \varepsilon_L} \ln\left(\frac{t_0 + t_e}{t_0}\right)} - \Delta \varepsilon_f^{vp} \right] \quad (\text{for } t \geq t_0) \quad (11)$$

$$S_{creepd}(t) = H \left[\frac{\frac{\psi_0 \ln\left(\frac{t_0 + t_e}{t_{EOP}}\right)}{V} - \Delta\varepsilon_f^{vp}}{1 + \frac{\psi_0 \ln\left(\frac{t_0 + t_e}{t_{EOP}}\right)}{V\Delta\varepsilon_L}} \right] \quad (\text{for } t \geq t_{EOP}) \quad (12)$$

231 where t_{EOP} is the elapsed time at end of “primary” consolidation. Other symbols have been
 232 explained in previous paragraphs. It should be noted that for soils under ramp loading $\Delta\sigma'(t)$,
 233 $\Delta\varepsilon_f^{vp}(t)$ could be time-dependent during the construction time.

234 The average consolidation degree for each layer is $U_i(t) = 1 - \frac{\bar{u}_i(t)}{\Delta\sigma'_i(t)}$. If there are more than
 235 one loading stages, αU_i for creep calculation in Eq. (10) should be replaced by $\alpha U_{multi,i,j}$ in for
 236 layer- i at stage- j with the following equation:

$$U_{multi,i,j}(t) = 1 - \frac{\sum_{k=1}^j \bar{u}_{i,k}(t)}{\sigma_{i,j}(t) - \sigma_{i0}} \quad (13)$$

237 where $\bar{u}_{i,k}(t)$ is the average excess pore water pressure of layer- i at stage- j , $\sigma_{i,j}(t)$ is the loading
 238 stage after the j -th loading and σ_{i0} is the initial value of j -th loading.

239 The calculation process of the proposed simplified method has been developed into
 240 automatically executed Excel spreadsheets using VBA programming language, in which
 241 stratified soil ground with maximum of 20 layers under ramp loading can be directly modeled
 242 and analyzed. The flow chart of the program is presented in Fig. 4.

243

244 3 Programming of EVP model for fully coupled finite element analysis

245 Based on Yin (1999)’s theory, a 3D EVP models [16,17] were developed based on the
 246 overstress theory [22,23]. The model had been validated by numbers of element tests in previous

247 studies, while simulations on large-scale field cases are still lacking. In this study, a 3D EVP
 248 constitutive model based on the abovementioned model was encoded in the user-defined
 249 modulus of Plaxis 2015 to enable finite element analysis as comparisons.

250 **3.1 Framework of the 3D EVP model**

251 In 3D stress-strain space, the strain rate of soil is divided into elastic and visco-plastic
 252 parts, as shown in Eqs. (14a-14c):

$$\dot{\boldsymbol{\varepsilon}}_{ij} = \dot{\boldsymbol{\varepsilon}}_{ij}^{vp} + \dot{\boldsymbol{\varepsilon}}_{ij}^e \quad (14a)$$

$$\dot{\boldsymbol{\varepsilon}}_{ij}^e = C_{ijkl} \dot{\boldsymbol{\sigma}}_{ij}^e \quad k=1,2,3 \text{ and } l=1,2,3 \quad (14b)$$

$$\dot{\boldsymbol{\varepsilon}}_{ij}^{vp} = \gamma \langle \phi(F) \rangle \frac{\partial f}{\partial \boldsymbol{\sigma}_{ij}'} = S \frac{\partial f}{\partial \boldsymbol{\sigma}_{ij}'} \quad (14c)$$

253 where subscript i and j represent the generalized stress-strain condition, $\dot{\boldsymbol{\varepsilon}}_{ij}^e$ is the elastic strain
 254 rate, $\dot{\boldsymbol{\varepsilon}}_{ij}^{vp}$ is the visco-plastic strain rate, C_{ijkl} is the elastic compliance tensor, f is the load
 255 potential function and $f=0$ is the yielding surface in Cam-Clay model. $\gamma \langle \phi(F) \rangle$ is a function
 256 dependent on the position of the yielding surface, in which F describes the difference between
 257 the current yielding surface and a reference yielding surface. $\gamma \langle \phi(F) \rangle$ can be replaced by a
 258 scaling function S , as proposed in [16]. Eq. (14a) can thus be transferred to Eq. (15):

$$\dot{\boldsymbol{\varepsilon}}_{ij} = \left(\frac{1}{2G^e} \dot{s}_{ij} + \frac{p'}{3K^e} \dot{\delta}_{ij} \right) + S \frac{\partial f}{\partial \boldsymbol{\sigma}_{ij}'} \quad (15)$$

259 where $K^e = \frac{Vp'}{\kappa}$ is the bulk modulus and $G^e = \frac{2(1-2\nu)K^e}{2(1+\nu)}$ is the shear modulus, $V = 1 + e_0$ is

260 the initial specific volume, $p' = \frac{tr(\sigma'_{ij})}{3} = \frac{\sigma'_{11} + \sigma'_{22} + \sigma'_{33}}{3}$ is the mean effective stress,

261 $s_{ij} = \sigma'_{ij} - \delta_{ij}p'$ is the deviatoric stress tensor, and $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$ is a Kronecker delta.

262 The flow direction of the visco-plastic strain is controlled by the load potential function

263 in Eq. (16):

$$f = \frac{q^2}{M^2 p'} + p' - p'_m = \frac{3s_{ij} : s_{ij}}{2M^2 p'} + p' - p'_m \quad (16)$$

264 where M is the slope of critical state line in $p' - q$ space and p'_m describes the position and size
265 of current yielding surface, as shown in Fig. 5.

266 According to Eq. (14c), determination of the value of S is important for calculation of
267 visco-plastic strain rate in each time step. Based on Eq. (14c), it can be derived that:

$$S = \frac{\dot{\epsilon}_v^{vp}}{\left| \frac{\partial f}{\partial p'} \right|} \quad (17)$$

268 where $\dot{\epsilon}_v^{vp}$ is the visco-plastic volumetric strain rate. In Fig. 5, the visco-plastic volumetric strain
269 rate on the same yielding surface is the same, and therefore $\dot{\epsilon}_v^{vp}$ at point (p', q) is the same
270 as $\dot{\epsilon}_{vm}^{vp}$, resulting in Eq. (18):

$$\dot{\boldsymbol{\varepsilon}}_v^{vp} = \dot{\boldsymbol{\varepsilon}}_{vm}^{vp} = \frac{\psi_0}{Vt_0} \left(1 + \frac{(\boldsymbol{\varepsilon}_{vm}^r - \boldsymbol{\varepsilon}_{vm})^2}{\Delta \boldsymbol{\varepsilon}_L} \right) \exp \left\{ \frac{V(\boldsymbol{\varepsilon}_{vm}^r - \boldsymbol{\varepsilon}_{vm})}{\psi_0 \left[1 + \frac{(\boldsymbol{\varepsilon}_{vm}^r - \boldsymbol{\varepsilon}_{vm})}{\Delta \boldsymbol{\varepsilon}_L} \right]} \right\} \quad (18)$$

271 where $\boldsymbol{\varepsilon}_{vm}$ and $\boldsymbol{\varepsilon}_{vm}^r$ locate at the current equivalent time line and referent time line respectively in
 272 the $\boldsymbol{\varepsilon}_{vm} - \ln p_m$ space, which can be calculated by Eqs. (19a) and (19b):

$$\boldsymbol{\varepsilon}_{vm} = \boldsymbol{\varepsilon}_v + \frac{\kappa}{V} \ln \frac{p_m}{p} \quad (19a)$$

$$\boldsymbol{\varepsilon}_{vm}^r = \boldsymbol{\varepsilon}_{vm0}^r + \frac{\lambda}{V} \ln \frac{p_m}{p_{mr0}} \quad (19b)$$

273 where p_{vmr0} and $\boldsymbol{\varepsilon}_{vm0}^r$ are fixed points on the reference time line.

274 **3.2 Algorithm in the finite element analysis**

275 The constitutive model was programed with Fortran language using the “user-defined soil
 276 model” module in Plaxis (2015 version). In a calculation step, the software passes the computed
 277 stresses, strains and other state parameters to the kernel with constitutive model. The new stress-
 278 strain state will be returned by the constitutive model after iterations.

279 In the initial state, the value of p_{m0} is calculated using POP (pre-over consolidation
 280 pressure, $POP = \sigma_{zp}' - \sigma_{z0}'$) or OCR with modification from K_0 -consolidation to isotropic
 281 consolidation, as shown in Fig.5. During the consolidation analysis, with the strain and time
 282 increment produced through global iterations in the software, the visco-plastic strain increment
 283 (in vector form) was calculated with Euler time integration scheme [24] as shown in Eq.(20):

$$\Delta \boldsymbol{\varepsilon}^{vp,n} = \Delta t \cdot \left[(1 - \theta) \cdot \Delta \dot{\boldsymbol{\varepsilon}}^{vp,n} + \theta \cdot \Delta \dot{\boldsymbol{\varepsilon}}^{vp,n+1} \right] \quad (20)$$

284 in which Δt is the time increment from n to $n+1$ stage, $\theta \in [0,1]$ is used to adjust the Euler
 285 integration scheme from fully explicit to fully implicit integral. In this study, θ was set as 0.5,
 286 which involves the advantages of both explicit and implicit methods with fairly efficiency and
 287 sufficient accuracy in FE analysis [25]. The increment of effective stress is determined by:

$$\Delta \boldsymbol{\sigma} = \mathbf{D} : \Delta \boldsymbol{\varepsilon}^e = \mathbf{D} : (\Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^{vp}) \quad (21)$$

288 where \mathbf{D} is the elastic stiffness vector. To solve Eqs. (20)-(21), the Newton-Raphson iteration
 289 scheme is conducted with Taylor series, as shown in Eq. (22):

$$\begin{cases} \boldsymbol{\sigma}^{n+1} = \boldsymbol{\sigma}^i + d\boldsymbol{\sigma}^i \\ \dot{\boldsymbol{\varepsilon}}^{vp,n+1} = \dot{\boldsymbol{\varepsilon}}^{vp,i} + \frac{\partial \dot{\boldsymbol{\varepsilon}}^{vp,i}}{\partial \boldsymbol{\sigma}} d\boldsymbol{\sigma} \end{cases} \quad (22)$$

290 where $\boldsymbol{\sigma}^i$ is the new stress vector and the iteration is finished once the value of $|d\boldsymbol{\sigma}^i|$ is small
 291 enough.

292

293 **4 Verification of the new simplified Hypothesis B method with in-situ measured data and** 294 **finite element simulations**

295 Computations on one field case were conducted using the proposed new simplified
 296 method. The test field is an embankment on a natural soft soil ground in Sweden without vertical
 297 drains and has been monitored for more than 50 years from 1947. The calculation results from
 298 the proposed new simplified method are presented in this section and compared with measured
 299 data. Numerical simulations using FE program Plaxis are also presented and compared.

300 **4.1 Description of the Väsby embankment**

301 In 1945, in order to select a suitable construction site for the new airfield, a field test was
 302 conducted by Swedish Geotechnical Institute (SGI) at the farm of Lilla Mellösa near Upplands

303 Väsby, Sweden. The field ground contains thick layers of soft soils, with high water content and
304 volume compressibility. Three test fills with and without vertical drains were constructed from
305 1945 to 1947, for which monitoring works on the settlements and pore pressure were continued
306 to recent years. Continuous settlements during the 60 years have been noticed and discussed by a
307 number of researchers [26,27], especially the “undrained” fill (*i.e.*, the one without installation of
308 vertical drains). The settlements are probably due to slow “primary” consolidation and long-term
309 creep deformation. In this study, the new simplified method is used to calculate the settlements
310 for one of the “undrained” fills. The profiles section of the test embankment is shown in Fig. 6.
311 According to the results of ground investigation [28][26], the ground in Väsby consists of at least
312 four types of soft clay, but without distinct boundaries. Under the soft clay there exist a thin layer
313 of medium grey sand and therefore the field was considered as a two-way drainage system in the
314 calculations. Due to the large dimensions of embankment, the total vertical settlement at the
315 center of the embankment could be considered as a one-dimensional problem.

316 Construction was started in Nov. 1947, with a 2.5m-high fill of gravels with unit weight
317 of around 16.2 kN/m^3 , constituting a vertical loading of around 40.6 kPa and was finished in 25
318 days. The loading could be considered as a ramp-increased total vertical stress uniformly
319 distributed on the top surface of the soil. In-situ measuring on both the pore pressure and the
320 settlements was started from the year of construction and has continued to recent years. A series
321 of settlement markers and piezometers were placed in different layers to monitor the settlements
322 and pore pressure.

323 ***4.2 Soil parameters and numerical model***

324 Borehole samples at different depths were taken from the site and tested in laboratories to
325 provide the permeability parameters, water content, compression curves, etc. In this study, the

326 whole soils with thickness of 14 m in total are divided into 15 different layers. During
327 calculations of S_f and m_v , each layer is divided into more than one sub-layers as mentioned in
328 2.2. Most of the parameters for each layer, including the compression indices and permeability,
329 were obtained according to the published report by Chang [28] and Larsson and Mattson [26], as
330 listed in Table 2. κ and λ were fitted using the data at the time of end of “primary”
331 consolidation (EOP). For creep behaviours, there are three major parameters: creep coefficient
332 ψ_0 , creep strain limit $\Delta\varepsilon_L$ and reference time t_0 , which are not available for each layer in the
333 original reports. In this study, the parameters were fitted with the original data from an
334 oedometer test in [28] using the method by Yin [15]. The loading step for the sample was from
335 40 to 80 kPa, and the strain-time curve in 24 hours was provided in [28], as shown in Fig. 7(a).
336 Since the vertical surcharge was 40.6 kPa in the field while the initial vertical stress was from 0
337 to around 80 kPa along the soil depth, it is reasonable to use the data from this test to estimate
338 the creep parameters. With these data, ψ_0 and $\Delta\varepsilon_L$ can be fitted using Eq. (23) transferred from
339 Eq. (4):

$$\ln\left(\frac{t_0+t_e}{t_0}\right) / \Delta\varepsilon^{vp} = \frac{V}{\psi_0} + \frac{1}{\Delta\varepsilon_L} \ln\left(\frac{t_0+t_e}{t_0}\right) \quad (23)$$

340 where t_0 was chosen as t_{EOP} in the oedometer test, which was 130 min. As the sample was
341 normally consolidated under 40 to 80 kPa, t_0+t_e equal to t_0+t , where t is the elapsed time in

342 the test. After plotting $\ln\left(\frac{t_0+t}{t_0}\right) / \Delta\varepsilon^{vp}$ versus $\ln\left(\frac{t_0+t}{t_0}\right)$ in Fig. 7(b), $\frac{V}{\psi_0}$ is the intercept and

343 $\frac{1}{\Delta\varepsilon_L}$ is the slope, so $\frac{\psi_0}{V}$ and $\Delta\varepsilon_L$ can be determined as 0.014 and 0.22 respectively. The fitted

344 strain-ln(time) curve by the EVP model is shown in Fig. 7(c), which is highly identical to the test

345 data. Therefore, $\psi_0 = \frac{\psi_0}{V} (1 + e_0) = 0.014 \times 4.15 = 0.058$ and $\Delta e_L = \Delta \varepsilon_L (1 + e_0)$
346 $= 0.22 \times 4.15 = 0.898$ could be obtained. Δe_L is the limit of change in void ratio under creep
347 condition. For different soil layers, both ψ_0 and Δe_L were assumed the same for simplicity.

348 In this paper, both the new simplified B method and FE simulation in Plaxis is conducted
349 and compared with measured data. The FE simulation was conducted in Plaxis with the
350 nonlinear EVP model in Section 3. The soil ground was built up as a plane strain model,
351 however, with assumption that the center of the embankment deformed under 1D straining
352 condition. The width of the soil ground was selected to be 1m. The numerical model after
353 meshing is shown in Fig. 8.

354 ***4.3 Comparisons of calculation results by simplified B method with FE simulation and*** 355 ***measured data***

356 In the new simplified B method, $\alpha = 0.8$ was used as the correction parameter. Since
357 $\alpha = 0.8$ has been adopted and examined in many studies, it would be meaningful to adopt the
358 same value for the new method and new case in this study. The calculation results are presented
359 in Figs. 9(a)-(d). According to the figures, the computed settlement curves at different depths by
360 the new simplified B method were highly reliable during the whole process compared with the
361 results by FE simulation and in-situ measured data, only with minor differences. At the earlier
362 stages, , the settlements at the lower positions by new simplified B method were a bit larger than
363 the measured data. The less precise results for the beginning period might be due to the delayed
364 consolidation by visco-plastic strain, which cannot be considered in the new simplified method,
365 as it is not fully coupled analysis. At the final stages, the three sets of curves become highly
366 consistent.

367 The comparisons of excess pore pressure distribution are presented in Fig. 10. For the
368 earlier stages, FE simulation gives relatively more accurate pore pressure distribution with
369 measured ones. However, the measured excess pore pressure tends to be lower than those
370 calculated in 1968 and 1979 while higher in the latest stage in 2002, possibly due to the changes
371 of permeability with time. It is also revealed that the new simplified method generally
372 underestimates the excess pore pressure even in the earlier stages, which is related to the lack of
373 fully coupled analysis for consolidation and creep. From the results of the two cases, it is
374 indicated that compared to settlement prediction, the new simplified method is less reliable in
375 prediction of excess pore pressure dissipation.

376 The settlement components $S_{primary,i}$ and $S_{creep,i}$ (consisting of $S_{creepf,i}$ and $S_{creepd,i}$) at
377 different depths by the simplified B method were plotted against time in Fig.11 and Fig.12.
378 Compared with $S_{primary,i}$, the distribution of $S_{creep,i}$ was more uniform with soil depth. Although
379 the primary compression of soil layers below 7.5m only contributed 1/3 among all soil layers,
380 their creep settlement contributed more than 50%, since their POPs and creep parameters were
381 similar to the higher layers. Fig.13 shows the evolution of $S_{primary}$, S_{creepf} and S_{creepd} for the whole
382 soil layers with time, which indicates that creep played an important role in the total settlement
383 of the embankment. S_{creepd} was much smaller than S_{creepf} and occurred at a very late stage, since
384 the “primary” consolidation process for most of the layers cost a long period.

385

386 **5 Parameter sensitivity analysis of the new simplified B method**

387 In the past section, the new simplified method performs well in settlements predictions
388 for the real embankment. It should be noted that the analysis results can be influenced by
389 selection of parameters, especially the correction parameter α and soil properties. In this study,

390 the effect of these parameters will be investigated, and the their selection principles and
391 techniques will be discussed.

392 **5.1 The effect of correction parameter α**

393 Fig. 14 compares the total settlement curves from the new simplified method using
394 different values of α , with FE simulation as well as measured data. According to Fig. 14, the
395 choice of α has significant influence on the prediction. The use of $\alpha = 0$, which is
396 corresponding to Hypothesis A, results in severe underestimation on the settlements during the
397 whole process. Using $\alpha = 0.6$ also underestimate the settlements while using $\alpha = 1$
398 overestimates the settlements. If a single parameter $\alpha = 0.8$ is used without multiplying U in Eq.
399 (10), which is the original method by Yin and Feng [5], the settlement curve is also inaccurate
400 before the final state. These results again demonstrate the necessity of adopting suitable value of
401 α before U . For this case, it is shown that $\alpha = 0.8$ is the optimal value, which has also been
402 examined for different cases in previous studies [1,5].

403 **5.2 The effect of creep parameters $\Delta\varepsilon_L$ and ψ_0**

404 According to Yin (1999)'s model, the nonlinear creep parameters $\Delta\varepsilon_L$ and ψ_0 are inter-
405 dependent and could be fitted simultaneously, as shown in Section 4. However, very few existing
406 projects paid attention to this effect and the corresponding parameters are frequently difficult to
407 determine due to lack of long-term oedometer test data. To demonstrate the influence of creep
408 parameters on settlement calculations, different values of $\Delta\varepsilon_L$ and ψ_0 will be used in the
409 calculations for comparisons.

410 The values of $\Delta\varepsilon_L$ and ψ_0 were adjusted based on the oedometer test data in the previous
411 section. From an aspect of physics, the value of $\Delta\varepsilon_L$ might range within $\left(0, \frac{e}{1+e_0}\right]$, where e is the

412 void ratio at the reference time line under a certain stress state. By adopting different values of
 413 $\Delta\varepsilon_L$ and ψ_0 can be fitted with test data through the following equation:

$$\frac{\Delta\varepsilon^{vp}V}{\left(1 - \frac{\Delta\varepsilon^{vp}}{\Delta\varepsilon_L}\right)} = \psi_0 \ln\left(\frac{t_0 + t_e}{t_0}\right) \quad (24)$$

414 By plotting the left side in Eq. (24) against $\ln\left(\frac{t_0 + t_e}{t_0}\right)$, ψ_0 was fitted as the slope of the
 415 curve. Eq.(24) could also be used for fitting conventional creep coefficient ψ in Yin and
 416 Graham's EVP model [29], with $\Delta\varepsilon_L = +\infty$.

417 A total of four sets of $\Delta\varepsilon_L$, Δe_L and ψ_0 for the oedometer sample were obtained through
 418 in this method, as listed in Table 2. In settlement calculations, Δe_L and ψ_0 were assumed the
 419 same for all layers. For case III, $\Delta\varepsilon_L = 10000$ was adopted for the new simplified method and
 420 the nonlinear EVP model in Plaxis.

421 Figs. 15(a) to (c) show the calculation results with different creep parameters by both the
 422 new simplified B method and FE analysis. It is indicated that although the parameters were fitted
 423 with the same set of laboratory test data, the settlements came out to be highly different in the
 424 field scale. In Fig. 15(a), the calculation by both the new simplified method and FE analysis
 425 using $\Delta\varepsilon_L = 0.1$ caused significant underestimation on the total settlement, especially in the
 426 long-term view. In Fig. 15(b) with $\Delta\varepsilon_L = \frac{e}{1+e_0}$, the calculated settlement curves were larger
 427 than the measured ones. In Fig. 15(c), using $\Delta\varepsilon_L = +\infty$ caused the highest overestimation.

428 The Soft Soil Creep (SSC) model in Plaxis [30] was also used in the calculation of Case
 429 III, in which no creep limit was considered. It can be found that FE analysis results by the self-

430 developed EVP model were larger than the results by SSC model. The main reason is the
431 different values of t_0 , which is 130 min in EVP model and 1 day in SSC. A larger t_0 will result
432 in smaller value of $\ln\left(\frac{t_e+t_0}{t_0}\right)$ with elapsed time and reduce the creep strain. After changing
433 t_0 into 1 day manually, the simulated curves by EVP model was highly close to the one by SSC
434 model. Therefore, t_0 has a significant influence on the calculations. The use of t_0 should be kept
435 in consistency with the position of reference time line, the compression parameters for “primary”
436 consolidation and creep parameters, as revealed in Fig. 2.

437 Therefore, accurate determination of ψ_0 , $\Delta\varepsilon_L$ and t_0 according to laboratory tests is
438 highly recommended for engineering constructions especially for the long-term design.

439 **5.3 The effect of permeability k_v**

440 In consolidation analysis, permeability values of k_v vary with void ratio of soft clays.
441 For engineering design, it is convenient to adopt constant reasonable permeability parameters
442 during the consolidation process, and the selection of k_v therefore becomes an important
443 technical issue. As indicated the last section, averaged values of k_v before and after the loading
444 is appropriate for predictions in both cases. In this sub-section, settlement calculations with
445 different permeability parameters before and after loading were conducted. The permeability
446 before and after loading in Väsby Embankment were provided by Larsson and Mattsson [26]
447 through laboratory tests, as listed in Table 3.

448 Figs. 16(a) to (b) show the results of settlement predictions with different values of
449 permeability. For both cases, the results by the new simplified method and FE analysis were still
450 fairly close. In addition, the choice of permeability parameters has significant effect on the

451 prediction curves for both cases. Using initial permeability value, the settlements tend to develop
452 faster at the earlier stage but get close to the measured results at the end of consolidation.
453 Comparatively, the settlements develop very slowly when using the final permeability value,
454 which will cause underestimation for most of the time.

455 Therefore, average permeability parameters are recommended for settlement analysis.
456 For other cases where permeability after the consolidation is unavailable, empirical correlations
457 might be used to calculate the change of k_v with estimated void ratio [31].

458 ***5.4 Verifications of the new simplified method for embankments subjected to more than one*** 459 ***loading stages***

460 In Väsby Embankment, only one loading stage is involved. As presented in Fig. 4, the
461 proposed simplified method can be applied in multi-staged loading conditions. In this study,
462 additional loading stages will be added in the case. In Stage 1, vertical stress of 40.6 kPa was
463 applied in 25 days. After 100 days of consolidation, the vertical load was increased to 100 kPa in
464 25 days in Stage 2, lasting for 20000 days. In Stage 3, the vertical load was reduced to 40.6 kPa
465 in 25 days and kept for 20000 days.

466 Both FE analysis and simplified Hypothesis B method are used for the calculation. The
467 looping method in flow chart of Fig. 4 was used to calculate the consolidation settlements at
468 three stages. Despite the absence of measured data, the fully coupled FE analysis can be used as
469 verification for the proposed simple method. The calculated settlements at different depths are
470 shown in Figs 17 (a)-(d). According to these figures, the settlement curves by simplified
471 Hypothesis B method are highly close to those by FE simulations under three stages of loading
472 and unloading.

473

474 5 Conclusions

475 This paper proposed a new simplified method based on Hypothesis B to calculate time-
476 dependent settlements for multi-layered soft soils with a nonlinear creep function considering
477 creep limit. This method can be conveniently operated using Excel spreadsheet with high
478 efficiency and stability. Walker's solution with spectral method is adopted in the "primary"
479 consolidation analysis for the multi-layered system. Yin's nonlinear creep function with a creep
480 limit is used for creep analysis. The consolidation and creep settlements are combined by
481 involving a correction factor αU in the formulation. FE analysis was also carried out in Plaxis
482 with a self-encoded 3D EVP model based on Yin's nonlinear creep function. The calculation
483 results of the new simplified method are verified using in-situ measured data and compared with
484 FE simulations for a real case in Sweden. Contributions of different components of settlements
485 subjected to primary consolidation and creep are clearly demonstrated by the proposed method.
486 A series of parametric studies have been carried out to investigate the effect of parameters
487 determination on the settlement calculations. Several important conclusions can be drawn from
488 this study:

- 489 1) The proposed simplified B method is able to predict the settlements with high accuracy.
490 Compared with FE simulations, the new simplified B method can be used to calculate similar
491 results with much higher stability and efficiency without any convergence difficulties.
- 492 2) The excess pore pressure calculated in the new simplified method was lower compared with
493 in-situ measurement and numerical simulations, which is reasonable since the consolidation
494 analysis and creep analysis were decoupled.
- 495 3) Compared with other values, the use of $\alpha = 0.8$ in the new simplified B method is the
496 optimal.

- 497 4) Different adoptions of creep parameters ψ_0 and $\Delta\varepsilon_L$ will result in different results in the
498 calculations, especially for the long-term prediction. It is highly recommended for
499 engineering designers to use proper values of ψ_0 and $\Delta\varepsilon_L$ according to the long-term
500 laboratory oedometer tests.
- 501 5) The selection of permeability has significant influences on the settlement calculations. The
502 use of averaged permeability before and after loading tests performs well in the predictions.
- 503 6) With comparisons to FE simulations, the proposed method performs well in settlement
504 calculations for multi-layered soils under multi-staged loading and unloading conditions.

505 Due to considerations of convenience in practice, complicated soil conditions such as
506 lateral drainage, horizontal deformation, soil anisotropy and spatial variations of soil properties
507 were yet not included in the proposed method. Further improvements based on these issues are
508 worth of study to widen the range of applications of the simplified Hypothesis B method without
509 much hurting of the convenience.

510

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521 **References**

- 522 [1] Feng WQ, Yin JH. A new simplified Hypothesis B method for calculating consolidation
523 settlements of double soil layers exhibiting creep. *Int J Numer Anal Methods Geomech*
524 2017;41:899–917. <https://doi.org/10.1002/nag.2635>.
- 525 [2] Le TM, Fatahi B, Khabbaz H. Viscous Behaviour of Soft Clay and Inducing Factors.
526 *Geotech Geol Eng* 2012;30:1069–83. <https://doi.org/10.1007/s10706-012-9535-0>.
- 527 [3] Kabbaj M, Tavenas F, Leroueil S. In situ and laboratory stress-strain relationships.
528 *Géotechnique* 1988;38:83–100. <https://doi.org/10.1680/geot.1988.38.1.83>.
- 529 [4] Ladd C, Foott R, Ishihara K, Schlosser F, Poulos H. Stress-Deformation and Strength
530 Characteristics. State Art Report, Proceeding 9th ISMFE, vol. 2, Tokyo: 1977, p. 421–494.
- 531 [5] Yin J-H, Feng W-Q. A new simplified method and its verification for calculation of
532 consolidation settlement of a clayey soil with creep. *Can Geotech J* 2017;54:333–47.
533 <https://doi.org/10.1139/cgj-2015-0290>.
- 534 [6] Xie K, Xie X, Jiang W. Study on Non Linear Consolidation Theory on Double Layered
535 Soils. *Comput Geotech* 2002;29:151–68.
- 536 [7] Zhu G, Yin J-H. Solution charts for the consolidation of double soil layers. *Can Geotech J*
537 2005;42:949–56. <https://doi.org/10.1139/t05-001>.
- 538 [8] Nogami T, Li M. Consolidation of clay with a system of vertical and horizontal drains. *J*
539 *Geotech Geoenvironmental Eng* 2003. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2003\)129:9\(838\)](https://doi.org/10.1061/(ASCE)1090-0241(2003)129:9(838)).
- 541 [9] Chen RP, Zhou WH, Wang HZ, Chen YM. One-dimensional nonlinear consolidation of
542 multi-layered soil by differential quadrature method. *Comput Geotech* 2005;32:358–69.
543 <https://doi.org/10.1016/j.compgeo.2005.05.003>.
- 544 [10] Walker R, Indraratna B. Consolidation analysis of a stratified soil with vertical and
545 horizontal drainage using the spectral method. *Géotechnique* 2009;59:439–49.
546 <https://doi.org/10.1680/geot.2007.00019>.
- 547 [11] Walker R, Indraratna B, Sivakugan N. Vertical and Radial Consolidation Analysis of
548 Multilayered Soil Using the Spectral Method. *J Geotech Geoenvironmental Eng*
549 2009;135:657–63. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0000075](https://doi.org/10.1061/(asce)gt.1943-5606.0000075).
- 550 [12] Bjerrum L. Engineering geology of Norwegian normally-consolidated marine clays as
551 related to settlements of buildings. *Géotechnique* 1967:81–118.
552 <https://doi.org/10.1680/geot.1967.17.2.83>.
- 553 [13] Yin JH, Graham J. Viscous-elastic-plastic modelling of one-dimensional time-dependent
554 behaviour of clays. *Can Geotech J* 1989. <https://doi.org/10.1139/t89-029>.
- 555 [14] Yin J-H, Graham J. Equivalent times and one-dimensional elastic viscoplastic modelling
556 of time-dependent stress-strain behaviour of clays. *Can Geotech J* 1994.
557 <https://doi.org/10.1139/t94-005>.
- 558 [15] Yin J-H. Non-linear creep of soils in oedometer tests. *Géotechnique* 1999;49:699–707.
559 <https://doi.org/10.1680/geot.1999.49.5.699>.
- 560 [16] Yin J-H, Graham J. Elastic viscoplastic modelling of the time-dependent stress–strain
561 behaviour of soils. *Can Geotech J* 1999:736–45.
- 562 [17] Yin JH, Zhu JG, Graham J. A new elastic viscoplastic model for time-dependent
563 behaviour of normally and overconsolidated clays: Theory and verification. *Can Geotech J*

564 2002;39:157–73. <https://doi.org/10.1139/t01-074>.

565 [18] Feng W. Experimental Study and Constitutive Modeling of the Time-dependent Stress-
566 Strain Behavior of Soils. 2016.

567 [19] Walker R, Indraratna B. A Microsoft Excel spreadsheet program for vertical and radial
568 consolidation analysis of multi layered soil using the spectral method. <[http://](http://www.uow.edu.au/eng/research/geotechnical/software/index.html)
569 www.uow.edu.au/eng/research/geotechnical/software/index.html> 2008.

570 [20] Yin J-H, Zhu G. Consolidation Analyses of Soils. 1st ed. CRC Press; 2020.

571 [21] Feng WQ, Lalit B, Yin ZY, Yin JH. Long-term Non-linear creep and swelling behavior of
572 Hong Kong marine deposits in oedometer condition. *Comput Geotech* 2017;84:1–15.
573 <https://doi.org/10.1016/j.compgeo.2016.11.009>.

574 [22] Perzyna P. Fundamental Problems in Viscoplasticity. *Adv Appl Mech* 1966.
575 [https://doi.org/10.1016/S0065-2156\(08\)70009-7](https://doi.org/10.1016/S0065-2156(08)70009-7).

576 [23] Perzyna P. The constitutive equations for rate sensitive plastic materials. *Q Appl Math*
577 1963;20:321–32. <https://doi.org/10.1090/qam/144536>.

578 [24] Katona MG. Evaluation of viscoplastic cap model. *J Geotech Eng* 1984.
579 [https://doi.org/10.1061/\(ASCE\)0733-9410\(1984\)110:8\(1106\)](https://doi.org/10.1061/(ASCE)0733-9410(1984)110:8(1106)).

580 [25] Yin Z-Y, Li J, Jin Y-F, Liu F-Y. Estimation of Robustness of Time Integration Algorithms
581 for Elasto-Viscoplastic Modeling of Soils. *Int J Geomech* 2019;19:04018197.
582 [https://doi.org/10.1061/\(asce\)gm.1943-5622.0001351](https://doi.org/10.1061/(asce)gm.1943-5622.0001351).

583 [26] Larsson R, Mattsson H. Settlements and shear strength increase below embankments
584 2003:1–98. <https://doi.org/10.1021/JM100506Y>.

585 [27] Le TM. Analysing Consolidation Data to Optimise Elastic Visco – plastic Model
586 Parameters for Soft Clay. University of Technology, Sydney (UTS), 2015.

587 [28] Chang YCE. Long term consolidation beneath the test fills at Väsby. 1981.

588 [29] Yin J-H, Graham J. Visco-elastic-plastic modelling of one-dimensional time-dependent
589 behaviour of clays. *Can Geotech J* 1989;26:199–208.

590 [30] Vermeer PA, Neher HP. A soft soil model that accounts for creep. *Beyond 2000 Comput*
591 *Geotech* 2019:249–61. <https://doi.org/10.1201/9781315138206-24>.

592 [31] Tavenas F, Jean P, Leblond P, Leroueil S. The permeability of natural soft clays. Part II:
593 permeability characteristics. *Can Geotech J* 1983;20:645–60. [https://doi.org/10.1139/t83-](https://doi.org/10.1139/t83-073)
594 073.

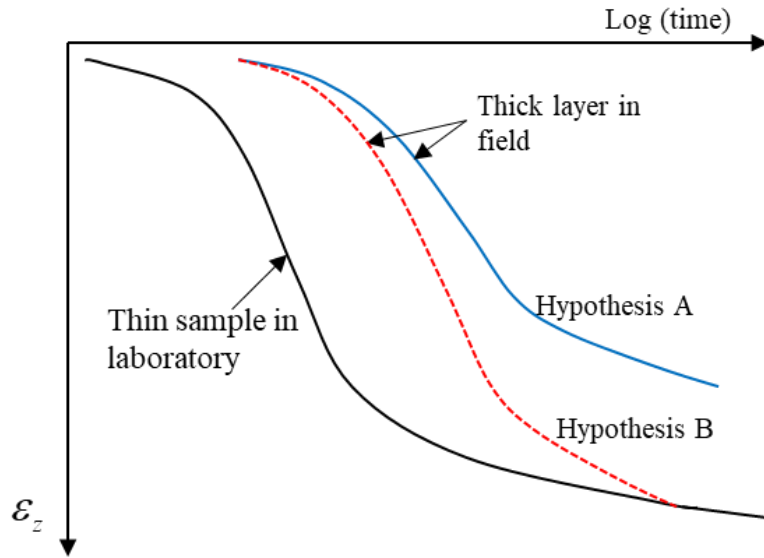
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Captions of Figures

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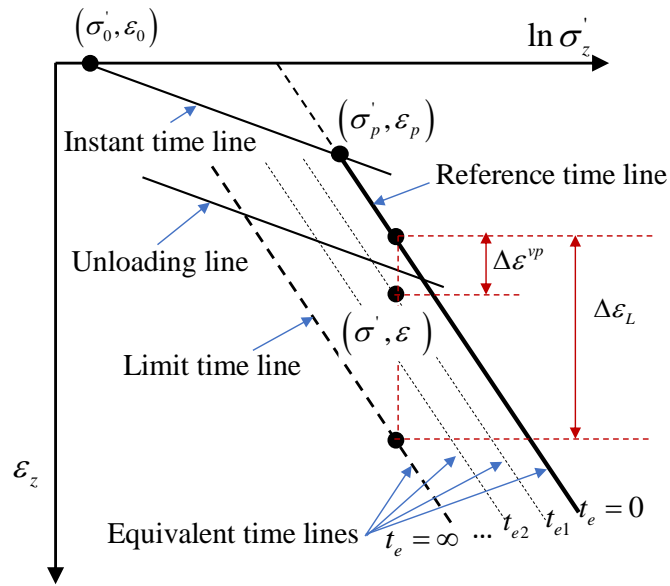


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625 **Fig. 1** The schematic diagram of the difference between Hypothesis A and Hypothesis B

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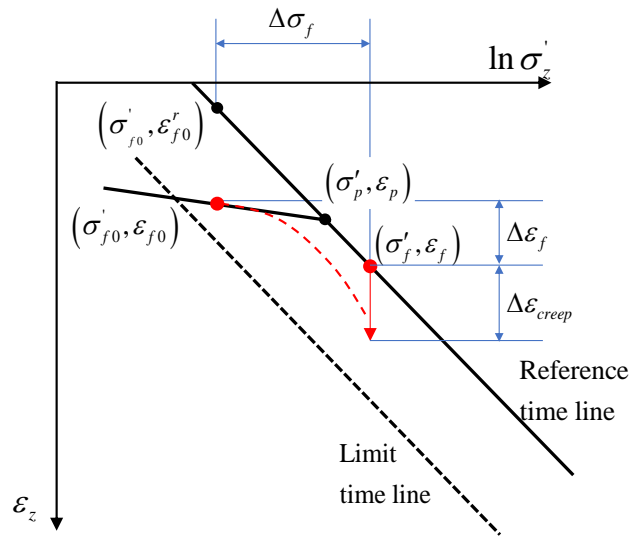


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629 **Fig.2** The schematic diagram of the 1-D nonlinear EVP model

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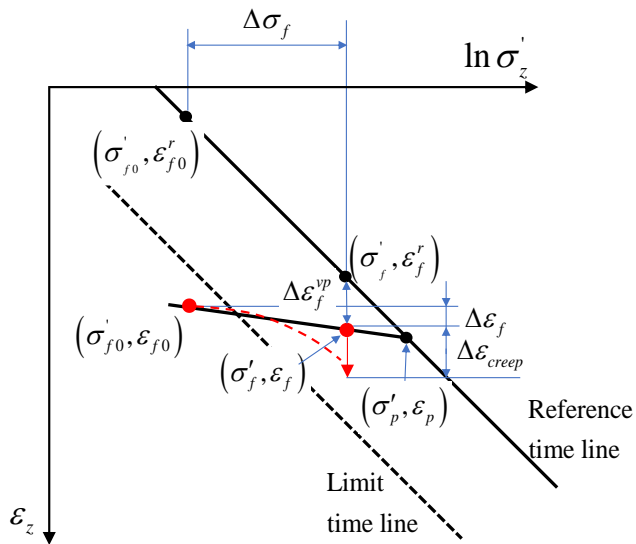
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(a)



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(b)

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637 **Fig.3** The schematic diagram of creep calculation when the final stress-strain of soil $(\sigma'_f, \varepsilon_f)$ is

638 at: (a) normally consolidation state and (b) over-consolidation state

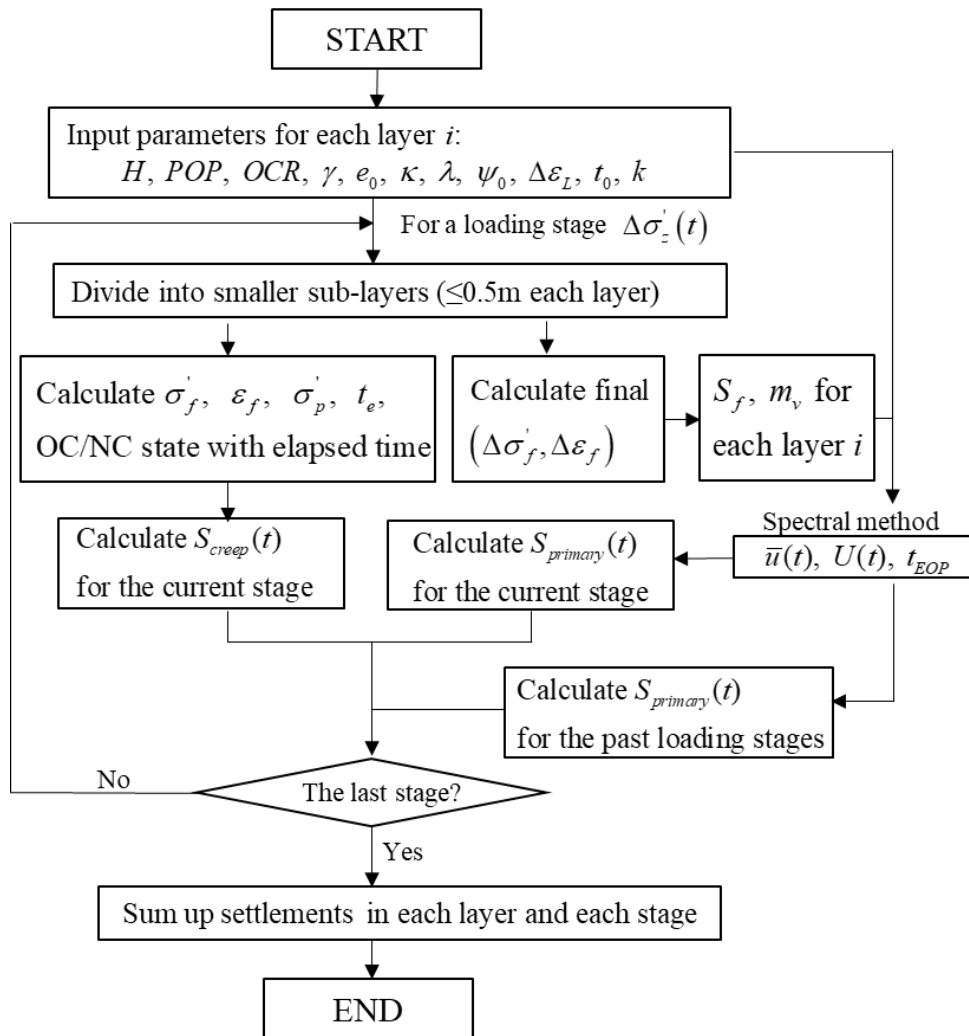
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645 **Fig.4** The calculation flow chart of the new simplified Hypothesis B method

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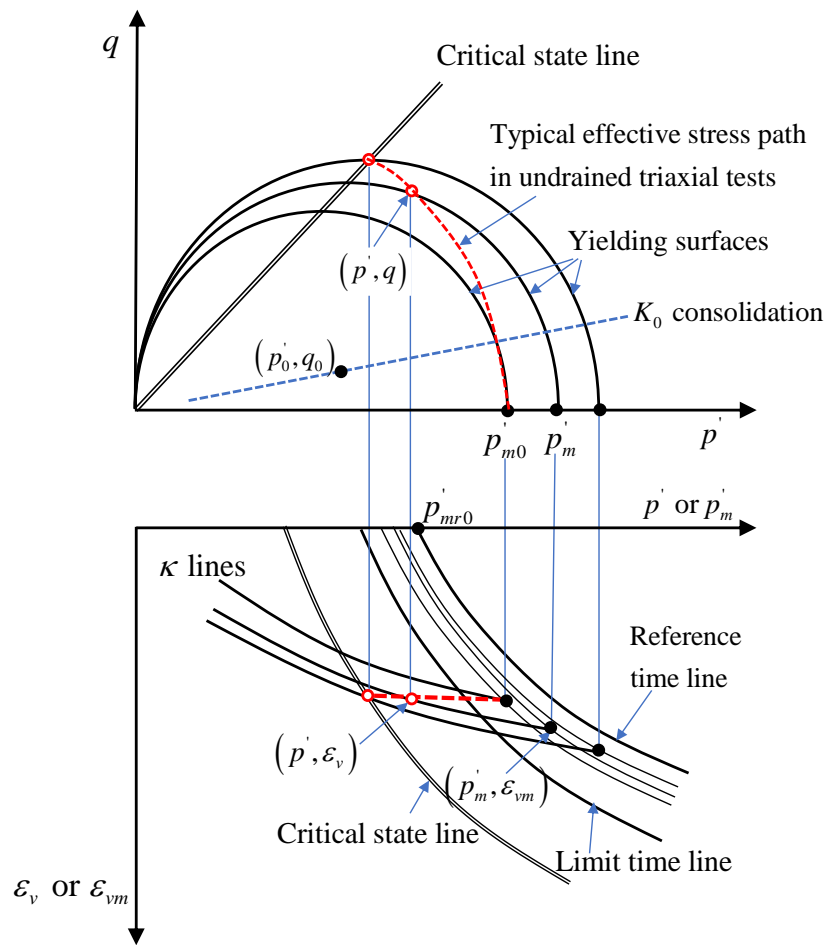
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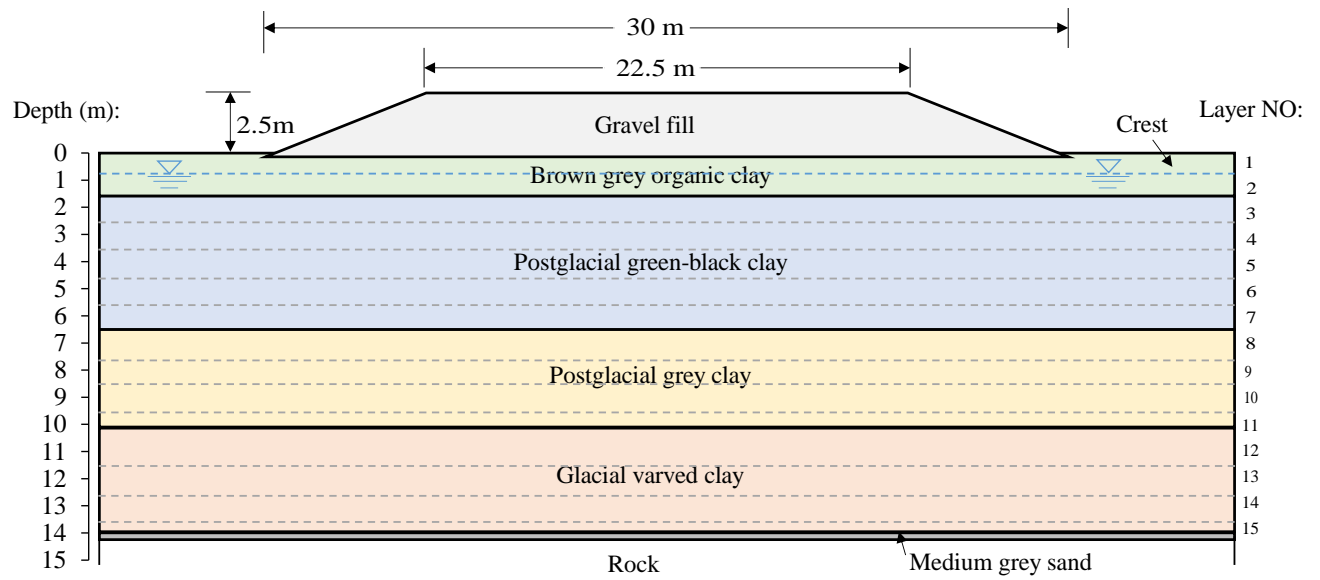
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654 **Fig.5** The schematic diagram of the 3-D nonlinear EVP model

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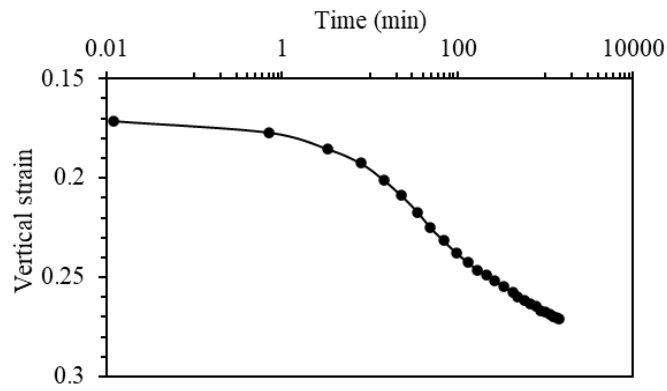
660 **Fig.6** The ground profile of the Väsby embankment

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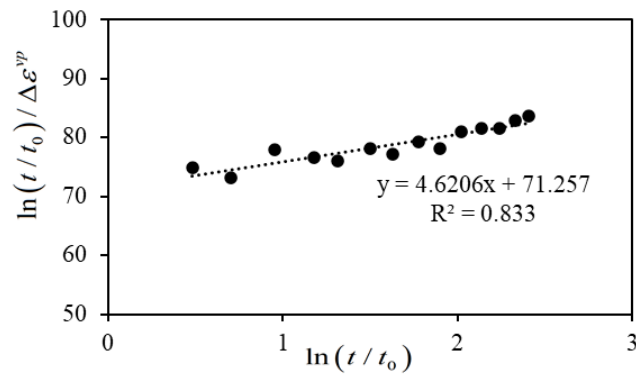
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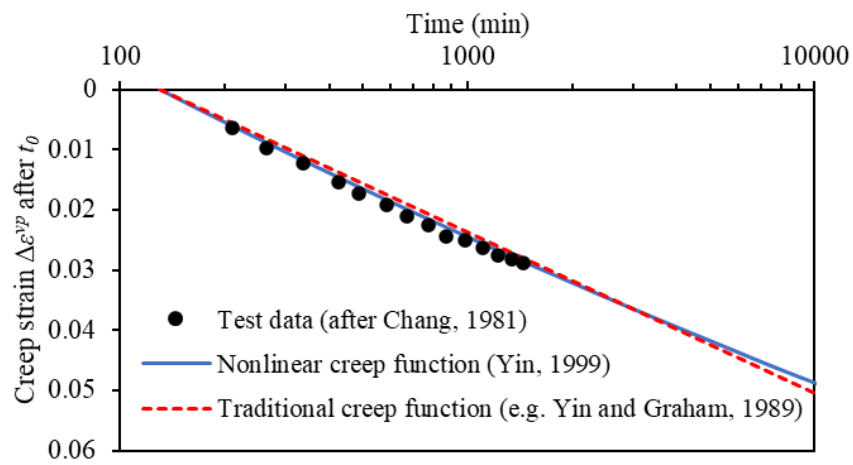
(a)



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(b)

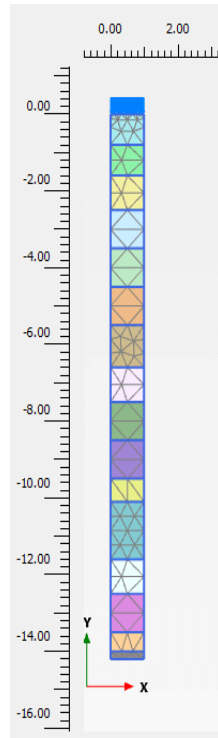


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(c)

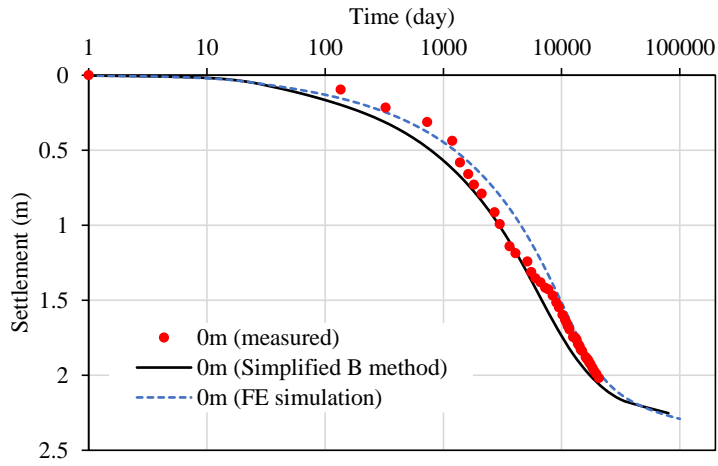
671 **Fig.7** Fitting of creep parameters with the oedometer test results: (a) the original strain-
672 log(time) curve; (b) plotting of $\ln\left(\frac{t_0+t}{t_0}\right) / \Delta\varepsilon^{vp}$ v.s. $\ln\left(\frac{t_0+t}{t_0}\right)$; (c) comparison between fitting
673 results and test data
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676 **Fig. 8** Numerical model in Plaxis with mesh

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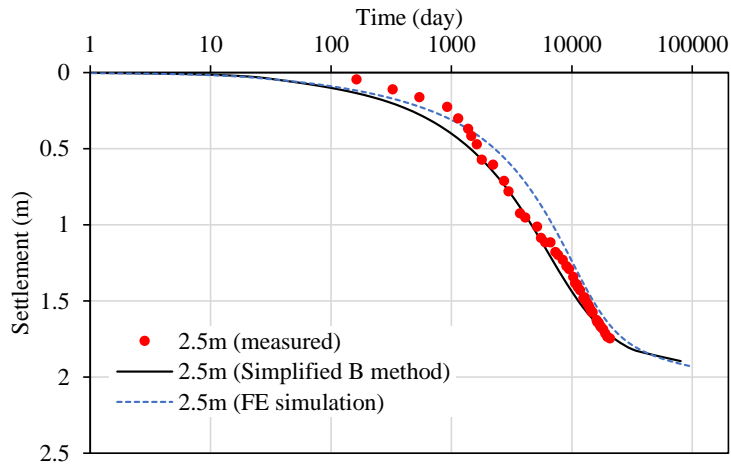
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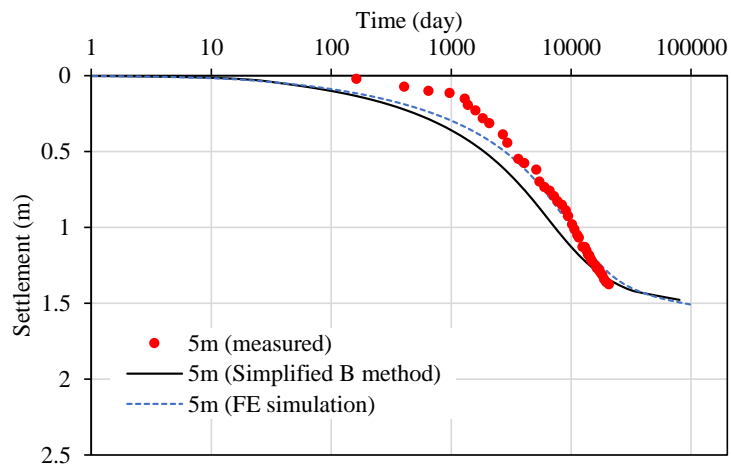
(a)



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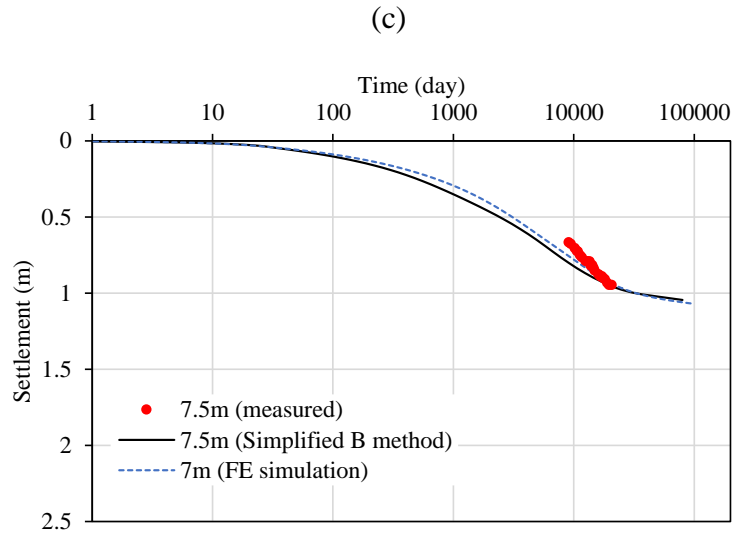
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(b)



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(d)

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690 **Fig. 9** Comparisons of settlement curves by simplified Hypothesis B method, FE simulations
691 and measurement at different depths: (a) 0m; (b) 2.5m; (3) 5m and (4) 7.5m

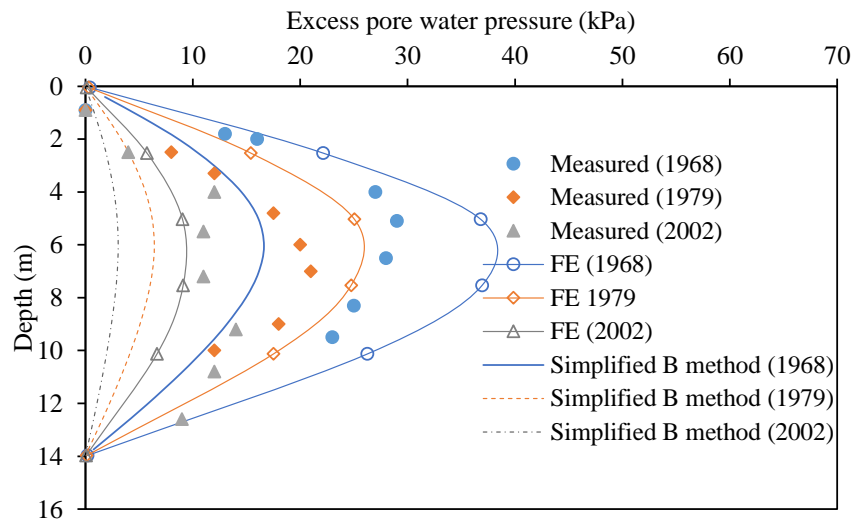
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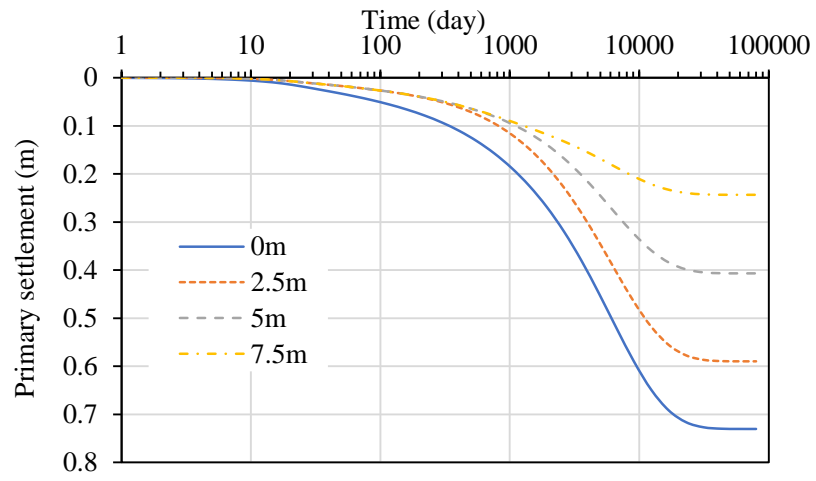
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Fig. 10 Comparisons of excess pore pressure distributions in Years 1968, 1979 and 2002 by the new simplified method, FE simulation and in-situ measurement

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Fig. 11 Computed “primary” consolidation settlement $S_{primary}$ at different depths by the new simplified Hypothesis B method

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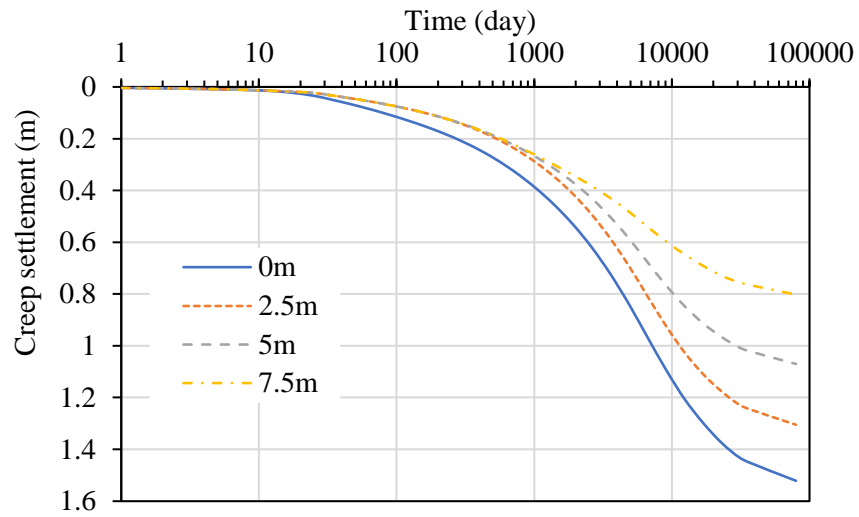
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712 **Fig. 12** Computed creep settlement S_{creep} at different depths by the new simplified Hypothesis

713 B method

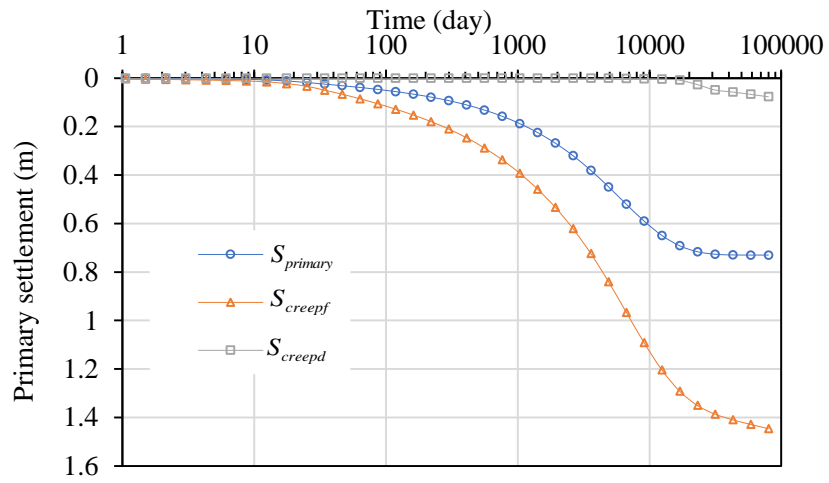
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Fig. 13 Computed $S_{primary}$, S_{creepf} and S_{creepd} at the surface by the new simplified B method

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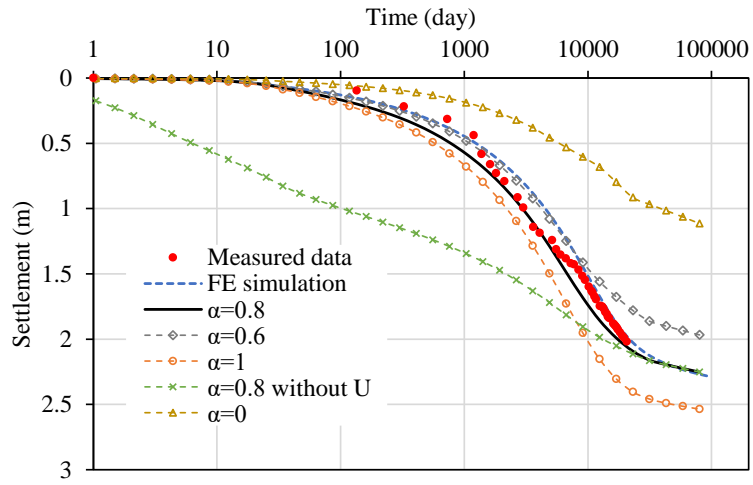
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730 **Fig. 14** Comparisons of total settlement by the new simplified B method with different α , FE
731 simulation, and measurement

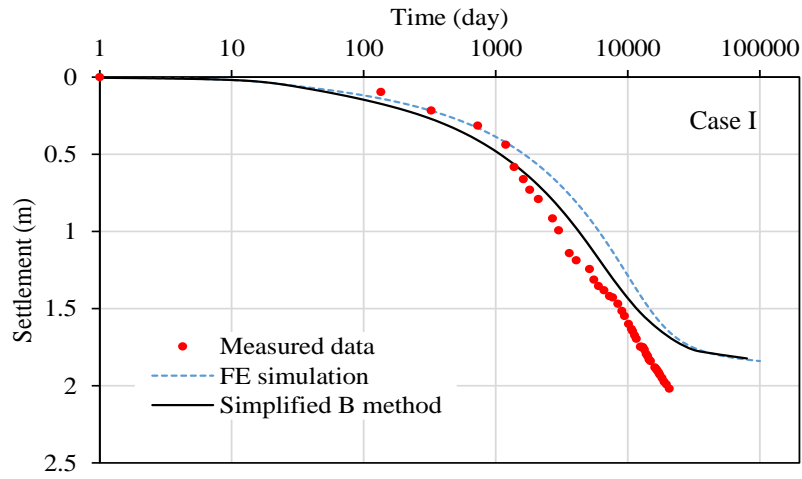
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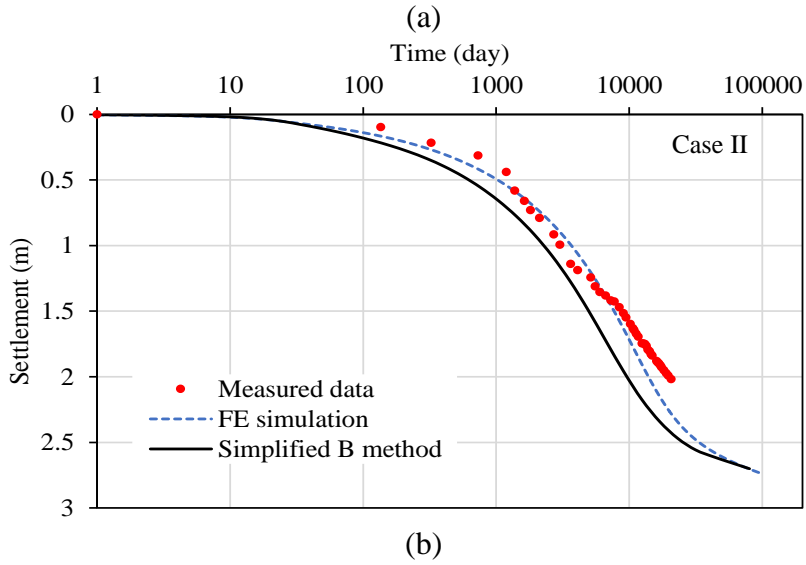
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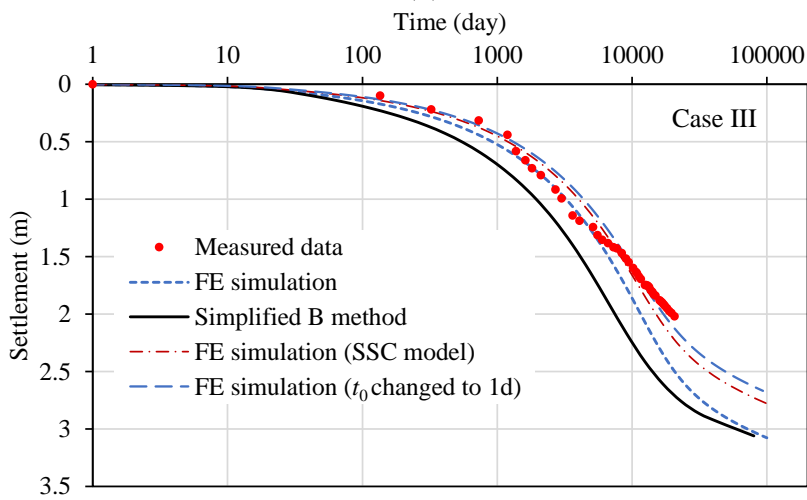
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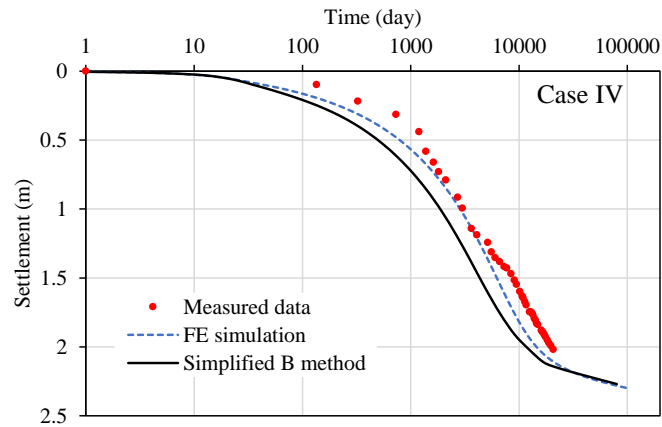
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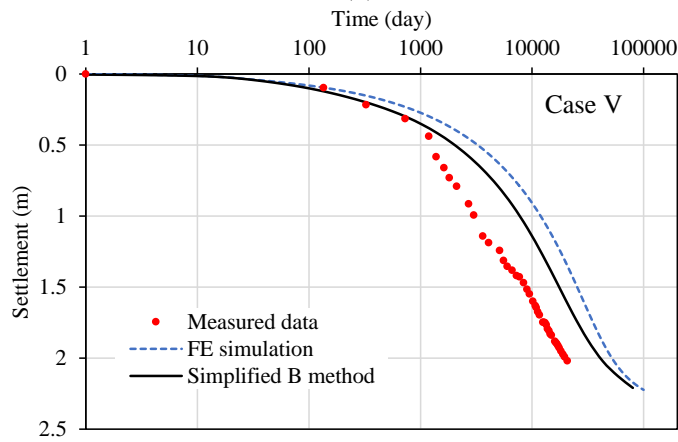
741
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(c)

744 **Fig. 15** Calculation results of total settlements with different creep parameters: (a) Case I; (b)
745 Case II; (c) Case III



(a)

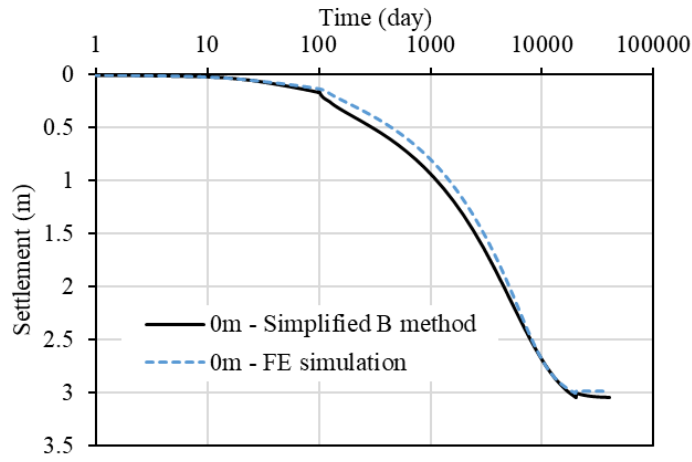


(b)

750 **Fig. 16** Calculation results of total settlements with different permeability: (a) Case IV; (b)
751 Case V

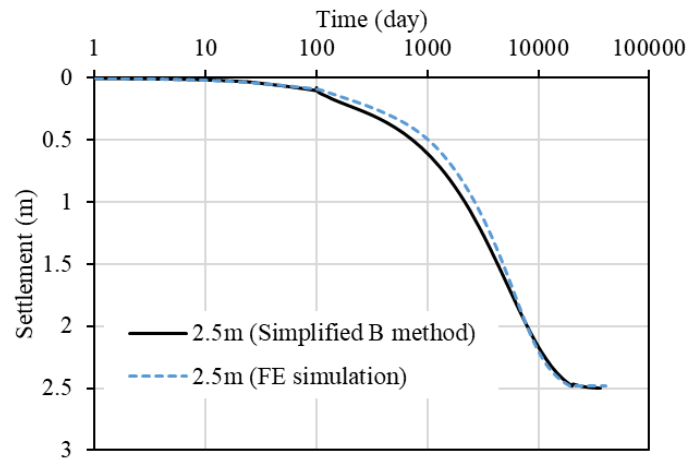
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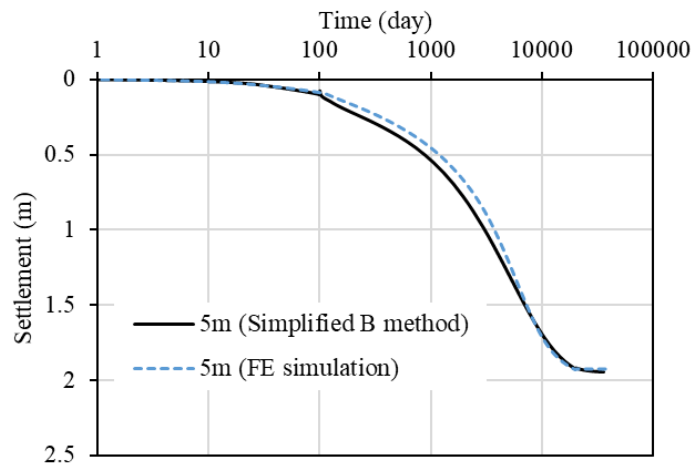
(a)

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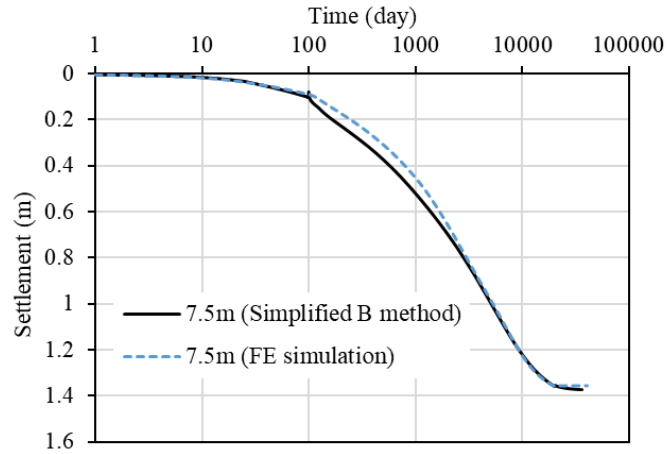
(b)

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(c)

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(d)

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Fig. 17 Calculation results of total settlements under multi-staged loadings

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Table 1 Soil parameters for Väsby embankment

Soil type	Layer No.	H (m)	POP (kPa)	γ (kN/m ³)	κ	λ	ψ_0	$\Delta\varepsilon_L$	t_0 (min)	$1+e_0$	k_v (10 ⁻⁵ m/d)
Brown grey organic clay	1	0.8	44.9	13*	0.0548	0.369	0.058	0.23	130	3.9	4.23
	2	0.8	36.3	19	0.0548	0.369	0.058	0.23	130	3.9	4.23
Post glacial green black clay	3	0.9	4.3	13.3	0.047	0.536	0.058	0.21	130	4.26	4.67
	4	1	6.0	13.7	0.0317	0.401	0.058	0.23	130	3.9	4.54
	5	1	7.2	14.0	0.0343	0.494	0.058	0.24	130	3.79	4.45
	6	1	10.1	14.3	0.0109	0.505	0.058	0.26	130	3.48	4.58
	7	1.1	5.5	14.7	0.0109	0.505	0.058	0.26	130	3.48	4.54
Post glacial grey clay	8	0.9	10.8	15.0	0.0043	0.365	0.058	0.30	130	3.01	4.75
	9	1	16.0	15.3	0.0043	0.365	0.058	0.30	130	3.01	5.27
	10	1	10.6	15.7	0.0119	0.355	0.058	0.30	130	2.99	5.27
	11	0.6	16.0	16.0	0.0119	0.355	0.058	0.30	130	2.99	5.36
Glacial varved clay	12	1.5	19.6	16.3	0.0151	0.592	0.058	0.28	130	3.17	5.27
	13	0.9	13.0	16.7	0.0079	0.460	0.058	0.31	130	2.88	5.23
	14	1	8.6	17.0	0.0007	0.328	0.058	0.35	130	2.6	5.18
	15	0.5	4.5	17.0	0.0005	0.261	0.058	0.38	130	2.38	5.18

778 (H is the thickness. γ is unit weight . γ with * is unit weight for unsaturated soils in the crust
779 above ground water level. $POP = \sigma'_{zp} - \sigma'_{z0}$ is called pre-over-consolidation pressure in history. k_v
780 is the vertical permeability.)

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784 **Table 2** Creep parameters for parametric study

	Case I	Case II	Case III	Original case
$\Delta \varepsilon_L$	0.1	$\frac{e}{1+e_0} = 0.651$	$+\infty$ or 10000	0.22
ψ_0	0.0747	0.0512	0.0482	0.0582
Δe_L	0.415	2.7	$+\infty$ or 10000	0.898

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788 **Table 3** Values of permeability for parametric study

Case name Soil type	Layer No.	Case IV	Case V	Original case
		Initial k_v (10^{-5} m/d)	Final k_v (10^{-5} m/d)	Average k_v (10^{-5} m/d)
Brown grey organic clay	1	6.91	1.56	4.23
	2	6.91	1.56	4.23
Post glacial green black clay	3	7.78	1.56	4.67
	4	7.34	1.73	4.54
	5	6.91	1.99	4.45
	6	6.48	2.68	4.58
	7	5.62	3.46	4.54
Post glacial grey clay	8	6.74	2.76	4.75
	9	7.78	2.76	5.27
	10	8.21	2.33	5.27
	11	8.64	2.07	5.36
Glacial varved clay	12	8.64	1.90	5.27
	13	8.64	1.81	5.23
	14	8.64	1.73	5.18
	15	8.64	1.73	5.18

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