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Investigation on Groundwater Velocity Based on the Finite Line Heat Source Seepage Model

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Abstract Groundwater seepage can improve the heat transfer performance of borehole ground heat exchanger (BGHE), and the corresponding velocity is the significant parameter which shows the degree of seepage role. The paper presents the mathematical model while groundwater flows through BGHE, and the comparisons between pure conduction and the combined heat transfer including conduction and convection are made. Points are set around borehole to test the temperature response at different time and then the goal functions containing both model results and test results are established. Next, the back calculation method is employed to obtain the value and orientation of velocity and therefore the convection role can be expressed. The reasonable points' locations along both depth and radial directions are analyzed; the comparisons of points' temperature responses are made according to the variation of seepage orientation and value. The relativity between points' locations and velocity value is discussed to make the calculation result acceptable. In addition, a number of trials are made to check the validity of back calculation method. The temperature response curves of points are shown and the characteristics embodied are investigated. Accordingly, the finite line heat source seepage model is significant to realize groundwater velocity.

Keywords: groundwater seepage; borehole ground heat exchanger; velocity; finite line heat source; back calculation; partial derivative.

Nomenclature

k	thermal conductivity (W m^{-1} K^{-1})
m	substitute variable
F	sum of squared deviation
a	thermal diffusivity (m^2 s^{-1})
c_p	specific heat (J kg^{-1} K^{-1})
t_0	initial temperature (K)
t	temperature (K)
r	distance between point and borehole center (m)
Fo	Fourier number
u	value of groundwater velocity (m/s)
U	dimensionless value of groundwater velocity
x ,y, z	rectangular coordinate (m)
X,Y,Z	dimensionless rectangular coordinate
h	depth of borehole
H	dimensionless depth of borehole
q_l	heating rate per meter line heat source (W m^{-1})

Superscript

$'$	integration parameter
$\triangle$	the first order derivative

Subscripts

i	infinite line heat source
f	finite line heat source
exp	experiment
cal	calculation

Greek symbols

β	angular coordinate of points
τ	time (s)
φ	orientation of groundwater velocity
Θ	dimensionless excess temperature
θ	excess temperature (K)

1. Introduction

The groundwater seepage often exists in underground medium and therefore groundwater can flow through borehole ground heat exchanger (BGHE) while ground source heat pump (GSHP) system is employed. The heat transfer mode is converted from pure conduction to combined style including conduction and advection [1]. It is generally appreciated that the performance of the GSHP system is greatly determined by the heat transfer ability of BGHE, this is because the heat is released to underground in summer and extracted from underground in winter. However, it is difficult to obtain the accurate velocity of groundwater as the underground composition is complex, and most of the time groundwater effect is always ignored. Consequently, the traditional conduction calculation is still the main mode in terms of designing size of BGHE. There is a belief that the moving of groundwater is favorable to improve heat transfer performance, because groundwater advection makes the heat accumulation around BGHE alleviated and therefore the thermal transmission from BGHE to the surrounding becomes easier [2]. The design size of BGHE can be saved provided that the groundwater seepage is taken into account, the initial cost spent on drilling boreholes and installing thermal exchange tubes can be reduced, which means the economic efficiency is improved and this will promote the application and development of GSHP technology [3]. The diagram about groundwater seepage is shown in Fig.1 showing that groundwater flows

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through BGHE as a result of hydraulic gradient [4]. The velocity value and orientation are both determined by the local hydraulic gradient, the larger the gradient, the intenser the seepage.

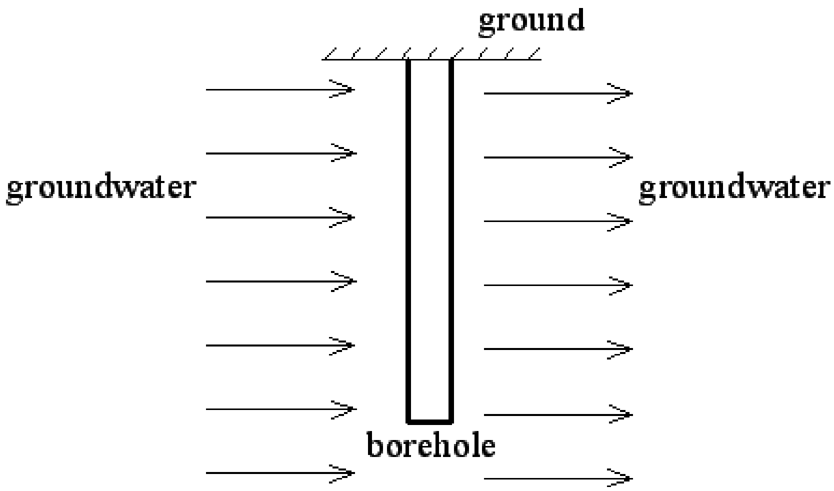

Fig.1 The schematic diagram of groundwater which flows past BGHE

The order magnitude of velocity value is always minor and thus it is not easy to comprehend the value and orientation of velocity [5], the groundwater velocity is determined by the local hydraulic gradient [6]. As a result, the calculation results of heat transfer are unsatisfactory even though the existing model is employed. Attempts have been made to develop back calculation depending on the infinite line heat source model with groundwater seepage and both the value and orientation can be acquired. The difference between infinite and finite model is whether the depth of BGHE is considered. The infinite model only takes two-dimensional heat transfer into account but the depth of any BGHE is finite rather than infinite; the calculation result of the infinite model is not accurate due to the discrepancy with actual BGHE length. The impact of ground boundary should be considered once the finite model is applied not only for pure conduction but also for combined heat transfer [7-9], the finite model is in possession of distinctive characteristics compared with the infinite case. Given that the velocity is obtained with the help of back calculation based on the finite line heat source seepage model, the result is more reasonable than that acquired from infinite case.

The underground temperature response shows different states while the value and orientation of groundwater velocity differ. Therefore, the calculation at relevant positions can be carried out to lay a firm foundation for back calculation. The analysis on groundwater seepage is significant because it can demonstrate the advection role of groundwater. The heat transfer from BGHE to the surrounding underground medium includes conduction through its solid matrix and liquid (water) in its pores as well as by convection of the moving groundwater. In addition, the detailed researches are conducted to understand groundwater velocity. The design size of BGHE can be reduced if the

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velocity is recognized, which means the economic performance will be improved. The finite line heat source seepage model is significant to achieve the groundwater velocity.

2. The mathematical model and corresponding analysis

2.1 The finite line heat source seepage model

The BGHE can be regarded as a line heat source because the ratio of length to radius is obviously large; its depth is usually between 50m and 150m, and the radius is often from 130mm to 150mm. It is evident that the radius is very small compared with the length and therefore the line heat source is feasible [10, 11], and the line source emits heat from the time τ' with the heat transfer intensity q_l. When groundwater passes BGHE, if line source is regarded as immovable then the groundwater is movable.

In general, the hydraulic gradient is also two-dimensional within a certain depth of strata though sometimes three-dimensional seepage exists; the two-dimensional flow is recommended as the precondition to simplify the difficulty of investigation. On condition that the line heat source locates at *z*-axis and emits heat from the time τ' while groundwater flows through it, the underground temperature response at any point except heat source at the time τ can be shown while the ground boundary effect is ignored, the corresponding formula is shown in Equation (1).

$$\theta_i = \frac{q_l}{4\pi k}\int_0^{\tau} d\tau' \frac{1}{(\tau-\tau')}\exp\left[-\frac{\left[x-u\cos\varphi(\tau-\tau')\right]^2+\left[y-u\sin\varphi(\tau-\tau')\right]^2}{4a(\tau-\tau')}\right] \mathrm{d}\tau' \quad (1)$$

where u and φ are respectively the value and the orientation of groundwater velocity, orientation is the intersection angle from the positive x axis to the direction of seepage, thus the parameter φ is used to depict the this parameter. $\theta_i = t - t_0$, t and t_0 are respectively transient temperature and initial temperature of any underground point except heat source, a is the thermal diffusivity of underground medium. However, the depth of actual BGHE is finite and the infinite model cannot embody the accurate heat transfer process; the existence of ground boundary should be emphasized and thereby the finite line heat source model is suggested to establish a mathematical model presenting both conduction and advection, and the detailed information is demonstrated in Equation (2).

$$\begin{cases}\left\{\frac{\partial\theta}{\partial\tau}+u\cos\varphi\frac{\partial\theta}{\partial x}+u\sin\varphi\frac{\partial\theta}{\partial y}=a\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)\right. \\ \text{for} -\infty<x<+\infty, -\infty<y<+\infty, 0\le z<+\infty \\ \tau=\tau', t=t_0 \\ \tau>\tau', \sqrt{x^2+y^2}\to\infty : t=t_0 \\ \tau>\tau', \sqrt{x^2+y^2}\to 0 : -k\frac{\partial\theta}{\partial\sqrt{x^2+y^2}}2\pi\sqrt{x^2+y^2}=q_l \\ \tau\ge\tau', z=0 : t=t_0\end{cases} \tag{2}$$

The boundary temperature is constant during the whole thermal exchange period [12], BGHE extends from the boundary to the certain depth location below ground, the initial coordinate and the termination coordinate of *z* for BGHE are respectively 0 and *h*, in such a way the analysis procedure can be relatively convenient. The problem of convection can be considered either as cases in which heat sources move through a fixed groundwater, or as cases of heat production with fixed sources past which the groundwater flows. The BGHE is regarded as motionless while groundwater flows in *x*- and *y*-direction, and the corresponding velocity along two directions are u_x and u_y respectively. Based on Equation (2), define the motionless coordinates as (*x*, *y*, *z*) and the coordinates moving together with the medium as (*ξ*, *η*, *ζ*). The conversion between the two coordinate systems are $x = \xi + u_x\tau$, $y = \eta + u_y\tau$, $z = \zeta$. The heat conduction caused by moving medium can be solved by Green function, that is, by way of integration of the solutions for instantaneous point source. If an amount of heat *ρc* is released at the point (x', y', z') at time τ', the point (*ξ*, *η*, *ζ*) of the moving medium at time *τ*, was at [$x-u_x(\tau-\tau')$, $y-u_y(\tau-\tau')$, z] at time *τ*. The velocity along x- and y-direction can be expressed as $u_x= u\cos\varphi$ and $u_y= u\sin\varphi$. Afterwards, the temperature response to the instantaneous point source emitted at (x', y', z') at *τ*' is the Green function under the condition of groundwater convection, and it can be written in Equation(3).

$$M(x,y,z,\tau;x',y',z',\tau')=\frac{1}{8\left[\pi a(\tau-\tau')\right]^{3/2}}\exp\left\{-\frac{\left[x-x'-u\cos\varphi(\tau-\tau')\right]^2+\left[y-y'-u\sin\varphi(\tau-\tau')\right]^2+(z-z')^2}{4a(\tau-\tau')}\right\} \tag{3}$$

Accordingly, the temperature response of the finite heat source model while groundwater seepage exists can be obtained by means of integration of Green function, and the corresponding expression is displayed as follows.

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$$\theta_f = \frac{q_l}{\rho c}\int_0^{\tau}\int_0^{h} M dz' d\tau' = \frac{q_l}{8\pi k}\int_0^{\tau}\frac{d\tau'}{\tau-\tau'}\times\exp\left[-\frac{[x-u\cos\varphi(\tau-\tau')]^2+[y-u\sin\varphi(\tau-\tau')]^2}{a(\tau-\tau')}\right]\times \left\{erfc\left[\frac{z-h}{2\sqrt{a(\tau-\tau')}}\right]-erfc\left[\frac{z-0}{2\sqrt{a(\tau-\tau')}}\right]-erfc\left[\frac{z+0}{2\sqrt{a(\tau-\tau')}}\right]+erfc\left[\frac{z+h}{2\sqrt{a(\tau-\tau')}}\right]\right\} \quad (4)$$

Equation (4) which a number of parameters are involved in is complex . To simplify the form the non-dimensional parameters are introduced.

$\Theta_f = k\theta_f / q_l$, $X = x/h$, $Y = y/h$, $U = uh/a$, $Fo = a\tau/h^2$, $Z = z/h$.

The dimensionless expression of Equation (4) is listed in Equation (5):

$$\Theta_f = \frac{1}{8\pi}\int_0^{Fo}\frac{dFo'}{Fo-Fo'}\times\exp\left[-\frac{\left[X-U\cos\varphi(Fo-Fo')\right]^2+\left[Y-U\sin\varphi(Fo-Fo')\right]^2}{4(Fo-Fo')}\right]\times \left\{erfc\left[\frac{Z-1}{2\sqrt{Fo-Fo'}}\right]-2*erfc\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right]+erfc\left[\frac{Z+1}{2\sqrt{Fo-Fo'}}\right]\right\} \quad (5)$$

Equation (5) clearly reveals the temperature response of the finite line heat source model with groundwater seepage, which can be tilted as the finite line heat source seepage model. The seepage role and ground boundary role are both reflected, accordingly the finite model is a significant progress because it can improve the accuracy of calculation.

2.2 The difference between having groundwater seepage and without

When there is no groundwater seepage, the heat is emitted from the BGHE to the surrounding; then the thermal response increases gradually until a stable state as a result of constant temperature of ground boundary [13,14]; conduction is the only mechanism achieving heat transmission. Groundwater flows through BGHE and then takes away a certain ratio of heat accumulated around BGHE, the temperature difference between heat source and the surrounding increases to motivate thermal transmission, and that thermal response degree is weaker than that induced by pure conduction. It is beyond question that the relief level to heat accumulation which seepage give rise to depends on seepage intensity i.e. velocity value U . With the increase of U, the contribution of advection to the whole heat transfer process becomes increasingly outstanding [15,16]; this will inevitably result in smaller and smaller temperature response. The two kinds of variation curves are both illustrated in Fig.2, the temperature responses of seepage model adopted are the mean

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responses. For one thing, pure conduction leads to larger response compared with combined heat transfer from the beginning to the end under the condition of constant velocity value *U*, this can prove that the seepage phenomenon is indeed favorable to improve heat transfer performance of BGHE. For another, if the time maintains unchanged, there is no velocity at all for pure conduction so that the temperature response holds a fixed value all the time, but the groundwater advection involved in combined heat transfer can let the response drop with the raised seepage strength [17].

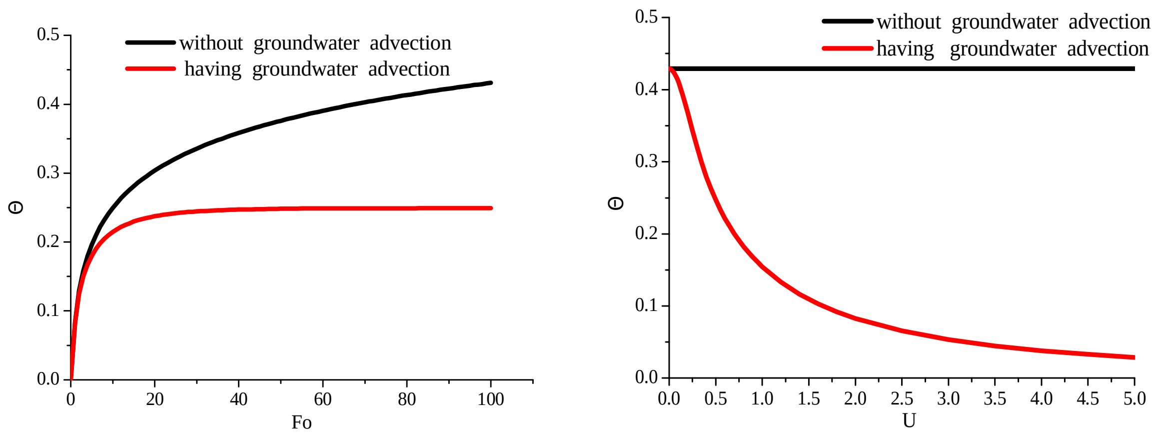

Fig.2 The comparison between pure conduction and combined heat transfer

2.3 The temperature field around BGHE

According to the analysis in section 2.2, the changes in terms of time and seepage intensity can bring about different thermal responses. BGHE is regarded as a line heat source with the constant heating rate q_l; the surrounding underground medium presents changing temperature distributions with the velocity value or the time when groundwater passes BGHE. To exhibit the temperature distribution, the isothermals are shown while the seepage angle is 0˚, which means at this time groundwater flows along positive direction of *X*-axis and some isothermals can be revealed while *F*o and *U* respectively changes. Firstly, the temperature field varies with velocity values if a certain value is given to *F*o, the isothermals are shown in Fig.3.

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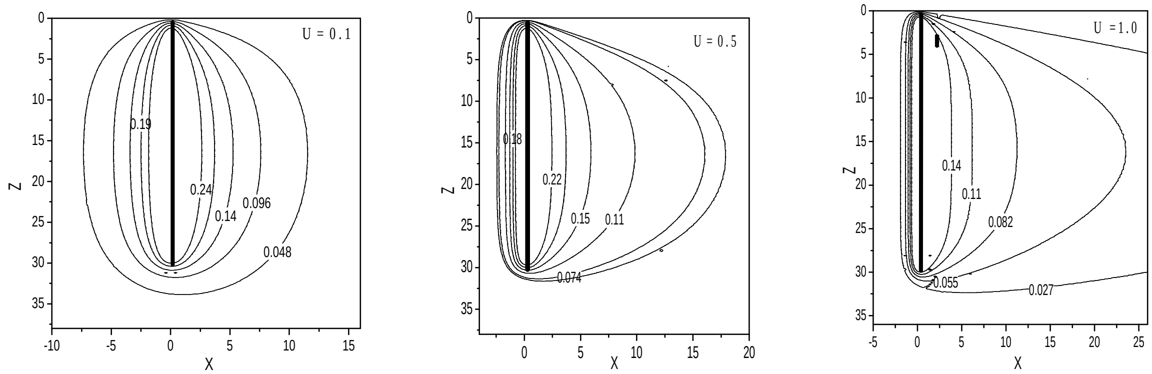

Fig.3 The isothermals with the increase of *U*

There shows delicate asymmetry of temperature distribution on both sides of *Z*-axis while *U* adopts minor value, because it seems that only pure conduction plays role in the heat exchange process. But if *U* attains a certain value, then the seepage effect is obvious. Isothermals depict the advection role and the temperature asymmetry is gradually notable; Fig.4 shows that temperature response on both sides of Z-axis presents different distribution with the time in the premise of constant *U*, which means the seepage effect is reflected by degrees.

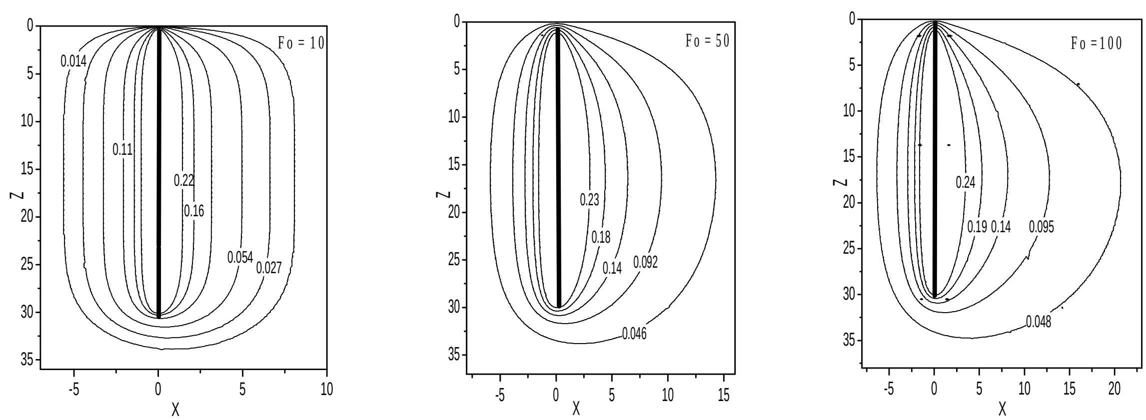

Fig.4 The isothermals with the increase of *F*o

2.4 How the seepage orientation influences mean temperature response of BGHE

It is indisputable that the hydraulic gradient direction differs in different regions or areas; the seepage orientation exerts influence on the underground temperature field. The intersection angle between positive *x*-axis and seepage direction is from -180˚ to 180˚. If the value of groundwater velocity is fixed i.e. the seepage intensity is unchangeable, different seepage directions lead to temperature variation of any underground point [18]. However, from the perspective of mean temperature response, the calculation findings are nearly the same. Equation (5) is the analytical solution of temperature response at any point except heat source. Considering that the orientation of seepage is two-dimensional, the integral average method can be utilized to acquire the mean temperature response. Another integral is added to Equation (5) and the corresponding expression is shown in Equation (6) which is a double integral.

$$\overline{\Theta_{ave,f}} = \frac{1}{8\pi}\times\frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{0}^{Fo}\frac{d\beta dFo'}{Fo-Fo'}\times\exp\left[-\frac{\left[R\cos\beta-U\cos\varphi(Fo-Fo')\right]^2+\left[R\sin\beta-U\sin\varphi(Fo-Fo')\right]^2}{4(Fo-Fo')}\right]\left\{erfc\left[\frac{Z-H}{2\sqrt{Fo-Fo'}}\right]-2*erfc\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo-Fo'}}\right]\right\} \qquad (6)$$

During the selection of intersection angles, some typical angles are chosen to discuss whether the variation of groundwater seepage can induce the change of mean temperature response surrounding BGHE. When assessing the improvement effect that groundwater seepage produces, it is significant to calculate the mean temperature response rather than the response of one location or several locations. We can obtain the mean temperature response trend of the external surface of BGHE with the time and the circumstance is unfolded in Fig.5, and the value of groundwater velocity is constant. Many angles including both negative and positive cases explain that the variation of angles exerts a little impact on the mean temperature response of BGHE.

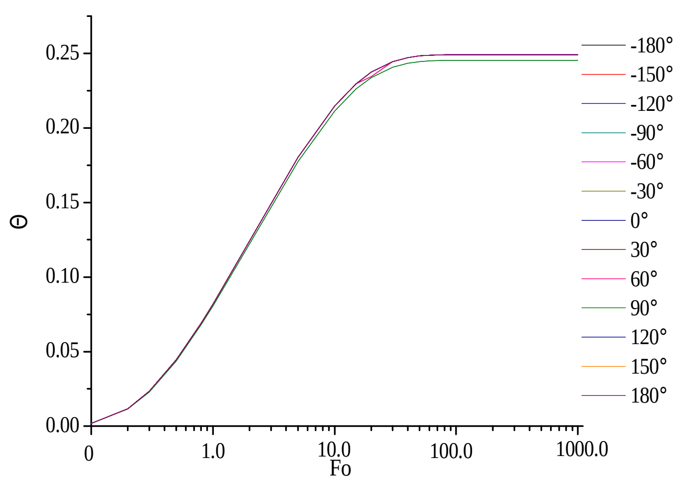

Fig.5 The mean temperature response with the time when φ adopts different angles

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Another trend is the variation curve of the mean temperature response with the increase of seepage intensity. Fig.6 shows that no matter what degree the velocity intensity is, the mean temperature responses of different seepage orientations are almost equal with each other while parameter U is the same.

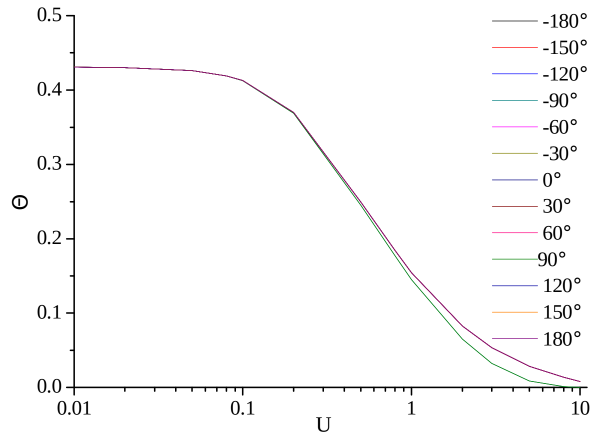

Fig.6 The mean temperature response with the velocity value when φ adopts different angles

3. The back calculation for groundwater velocity

The role of groundwater seepage mainly depends on its velocity, but from what has been stated in section 1, the difficulty in obtaining the value and orientation of velocity is a remarkable problem that engineers and scholars have to deal with. The back calculation for groundwater velocity needs to be investigated; though the infinite line heat source seepage model is simpler than the finite case, BGHE has finite depth so that the finite line heat source seepage model is more suitable for the mathematical calculation. It is worthwhile to conduct the back calculation method based on the finite line heat source seepage model. The following paragraphs expound the fundamental principles of back calculation and corresponding characteristics.

3.1 The applied measures before back calculation

The BGHE emits heat along different radial directions [19, 20], thus the temperature response of any point with the same radius to the center of BGHE should be equal with each other if there is no groundwater seepage, because the pure conduction executes with the same degree at every radial direction. At a certain depth, if some points with the same radius around BGHE are chosen, the temperature responses of these points will be different under the influence of groundwater seepage. Some thermal resistors are installed respectively at different points chosen. The accurate velocity is

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not known at first but the range is set in advance, thereby the value U and orientation φ are put into the finite line heat source seepage model in the process of back calculation,; the accurate value can be predicted while the difference between the calculation result and test data reaches the minimum or even equal. It should be admitted that the back calculation result is not accurate if only one point is applied, but the ultimate velocity can be acquired while the number of points attain a certain value, which means the temperature response of every point obtained by mathematical model simultaneously achieve the nearest approximation of test data, accordingly the U and φ can be estimated. We suggest that three points are distributed at first to verify the back calculation method; these points with the same radius are well even-distributed around BGHE, that is, the intersection angle between every two adjacent points are equal with each other, this intersection angle is 120˚. Fig.7 gives a sample of arranging three points.

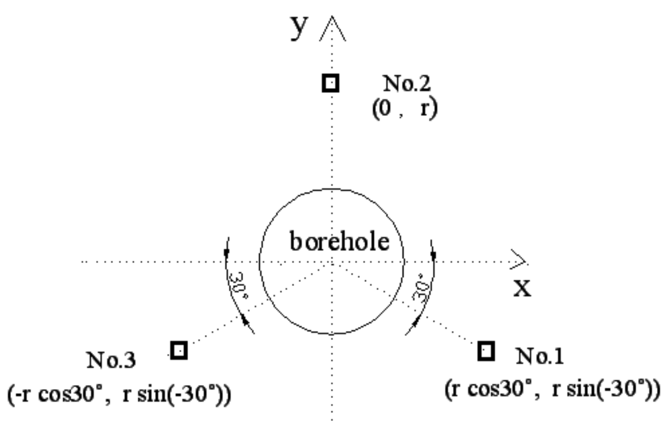

Fig.7 The schematic diagram of three even-distributed points around BGHE

Fig.7 shows one mode distributing three points; there are different modes if three points are even-distributed, such as some cases depicted in Fig.8.

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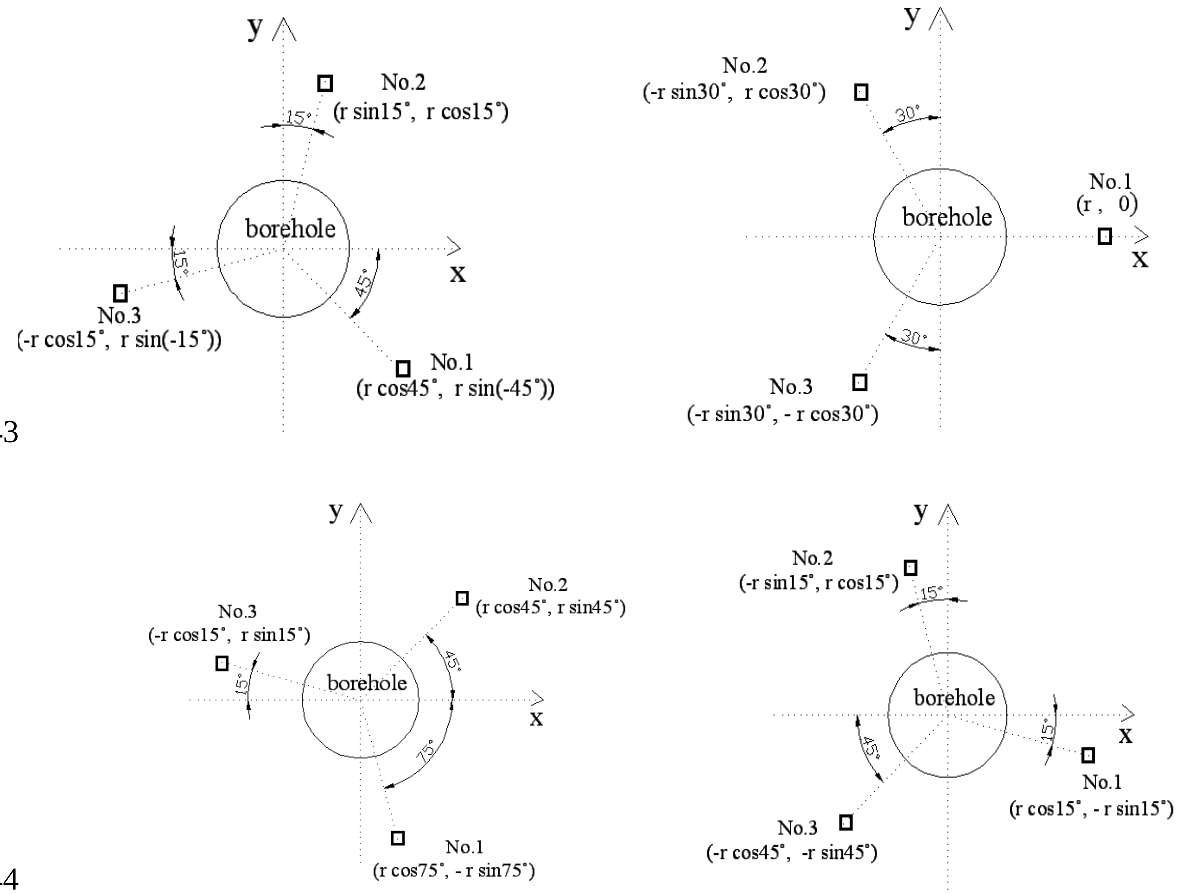

Fig.8 Some distributing modes for three points around BGHE

From the perspective of the number of points, the more the better, because added points can make the accuracy of back calculation higher and higher, these points had better be arranged even-distributed, four points, five points and six points are shown in Fig.9.

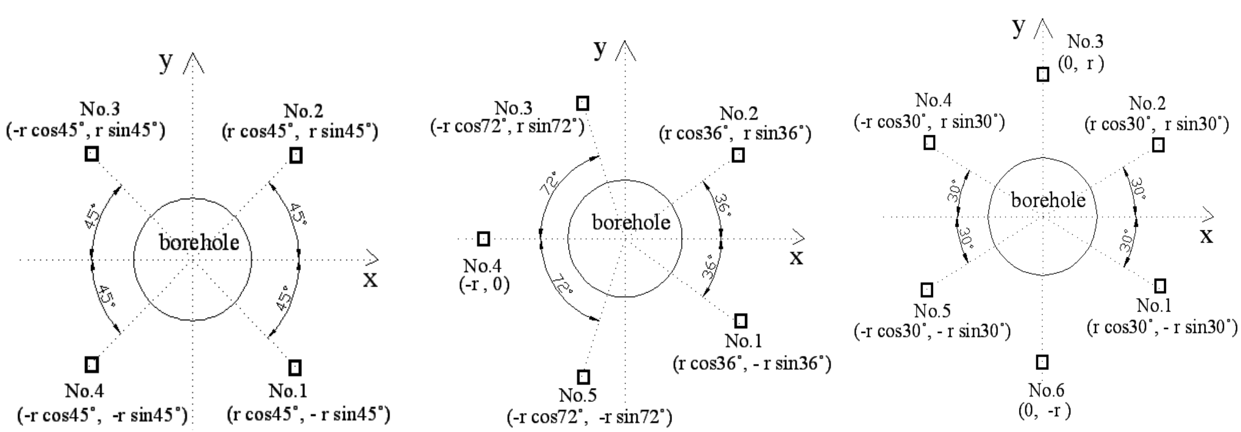

Fig.9 The distributing diagram while there are different number points

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3.2 The depth location of distributing points

As stated in section 3.1, the figures only describe the horizontal distributing information, now that the depth of any borehole is finite, how to select the depth location i.e. the value of *Z* is an important issue because this can determine the back calculation effect. The finite line heat source seepage model causes different temperature response degrees along *Z*-axis [21], and the temperature field on both sides of *Z*-axis are asymmetrical due to groundwater seepage [22,23]. The mean temperature response by means of Equation (5) can be calculated while *Z* picks different values along depth direction, and the corresponding thermal responses along *Z*-axis are listed in Fig.10 while different values are assigned to *U*. It is clear that the temperature responses of the starting location area and final position area are weaker than other areas of depth direction; the middle area shows obviously stronger temperature responses. The horizontal plane while *Z* adopts the middle point of borehole depth can be selected to set three points.

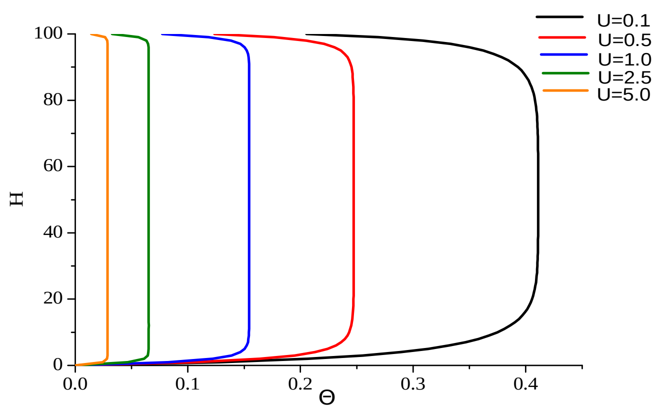

Fig.10 The mean temperature response along *Z*-axis

3.3 The relativity between points' radius to borehole center and velocity intensity

Though there are different options for selecting the number of points and the distributing modes, we still make full use of the case described in Fig.7 as the research basis of investigation. The middle depth location of borehole is decided to serve as the plane for distributing points, but how to determine the radius of point mainly rests with the seepage intensity. Three points are even-distributed no matter what the orientation is, and the difference must be generated in terms of temperature response. The next problem is to study the radius of point or the distance between points and the borehole center, because the radius should be adjusted according to the variation of *U*. In view of this, the analytical solutions of temperature response of the finite line heat source

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seepage model can be changed, if the radius from the point to the borehole center is *r*, the following non-dimensional parameters can be introduced.

$\Theta_f = k\theta_f / q_l$, $X = x/r$, $Y = y/r$, $U = ur/a$, $Fo = a\tau/r^2$, $H = h/r$, $Z = z/r$.

Because *x* = *r* cos *β* and *y* = *r* sin *β*, *β* means the angular coordinate of points distributed around borehole. Afterwards the new dimensionless temperature response is obtained as follows:

$$\Theta_f = \frac{1}{8\pi}\int_0^{Fo}\frac{dFo'}{Fo-Fo'}\times\exp\left[-\frac{\left[\cos\beta - U\cos\varphi(Fo-Fo')\right]^2+\left[\sin\beta - U\sin\varphi(Fo-Fo')\right]^2}{4\left(Fo-Fo'\right)}\right]\cdot \left\{erfc\left[\frac{Z-H}{2\sqrt{Fo-Fo'}}\right]-2erfc\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo-Fo'}}\right]\right\} \tag{7}$$

The non-dimensional parameter *U* consists of actual velocity value *u*, radius *r* of three points and thermal diffusivity *a* of underground medium, accordingly the parameter *U* can embody the relativity between *u* and *r*. Because *r* delegates the distance from those points to the borehole center, this distance should be adjusted to make the back calculation result satisfactory. Equation (7) is taken advantage of while *Z* adopts *H* / 2 to calculate the temperature responses of three points. The range of seepage orientation is [-180˚, 180˚], and a certain value from this range is chosen and then taken into Equation (7), the variation trend of temperature responses with *U* is expressed in Fig.11, it is clear that the response degrees decrease with the enhancement of seepage intensity *U*. Two factors should be considered for choosing the value of *U*, one factor is that the temperature response difference of three points should be clear as a result of *U* because notable difference is beneficial to carry out back calculation; another factor is that their temperature responses should not be too small because too small values are easy to result in calculation error.

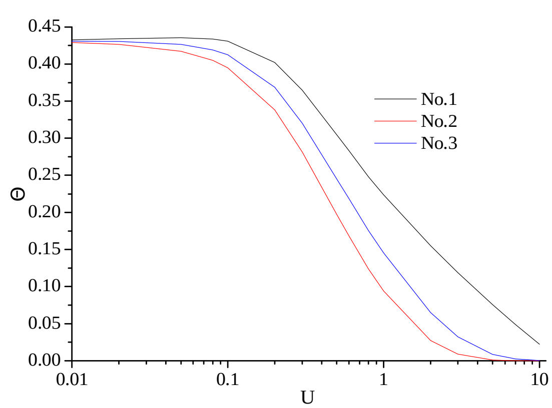

Fig.11 The temperature responses of three points with *U*

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Based on the two factors the range of U should be [0.1, 5.0], which means the relativity between points' radius r to borehole center and actual velocity intensity u can be summarized, i.e. the product of u and r lies in the range [0.1 a, 5.0a].

3.4 The back calculation principles

The data of actual temperature t of three points can be recorded at regular time interval if these points have been set at suitable locations. Non-dimensional temperature response $\Theta_{f,\text{exp}}$ at different time is $k\theta_f / q_l$ i.e. $k(t - t_0) / q_l$, the initial temperature t_0 and the thermal conductivity of underground medium can be known by relevant test equipment, and the heat transfer quantity per meter BGHE q_l can be calculated based on relevant parameters which are obtained directly by test equipments. As stated above, although the accurate velocity of groundwater cannot be known at first, the range can be set for the value U and direction φ and the corresponding interval should be small enough. Afterwards U and φ are continually taken out from the corresponding range to be put into Equation (7), the parameters except U and φ are known at different time. The corresponding $\Theta_{f,\text{cal}}$ can be achieved after finishing calculation according to Equation (7). There are a number of data being recorded at regular time interval; meanwhile, the calculation result by means of model at regular time interval can be acquired. Therefore, there exists recorded data and the corresponding calculation data at the same time. When the comparisons are made between two different kinds of data, the difference between them at different time should be calculated altogether because the total difference of the whole process should be the basis of back calculation. If the total difference reaches the minimum, it can be concluded the corresponding U and φ are respectively the actual cases. For that reason a goal function F shown in Equation (8) is the sum of squared deviation from test result to calculation result.

$$F(U,\varphi) = \sum_{i=1}^{n} (\Theta_{f,\text{cal}} - \Theta_{f,\text{exp}})^2 \tag{8}$$

From the beginning to the end, the data are recorded and calculated with the time at regular interval. If F can achieve the minimal value, the actual U and φ can be determined [24,25]. Equation (8) is a binary function with two independent variables U and φ. If the goal function F makes first order partial derivative respectively towards parameter U and φ , and the symbol F'_U and F'_φ are the corresponding first order partial derivatives. When Eq.(8) arrives at the minimum, at that time the values of F'_U and F'_φ must be zero. Considering that both U and φ are discrete variables

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rather than continuous variables, it cannot be guaranteed that the values of F'_U and F'_φ must be zero. But the minor values which is next to zero can be endowed respectively to F'_U and F'_φ, and the minor values can be adjusted according to the calculation process to limit U and φ to smaller and smaller range, the best finding is the single U and φ can be found at last.

Firstly, the formula of F'_U is demonstrated in Equation (9)

$$F_U{}^{\Delta} = 2\sum_{i=1}^{n}(\Theta_{f,\mathrm{cal}}-\Theta_{f,\exp})\times\Theta'_{f,\mathrm{cal}} = 2\sum_{i=1}^{n}\left\{\begin{array}{l}\frac{1}{8\pi}\int_0^{Fo_i}\frac{1}{(Fo_i-Fo')}\exp\left[-\frac{\left[\cos\beta-U\cos\varphi(Fo_i-Fo')\right]^2+\left[\sin\beta-U\sin\varphi(Fo_i-Fo')\right]^2}{4\left(Fo_i-Fo'\right)}\right]\times\\ \left\{erfc\left[\frac{Z-H}{2\sqrt{Fo_i-Fo'}}\right]-2*erfc\left[\frac{Z}{2\sqrt{Fo_i-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo_i-Fo'}}\right]\right\}dFo'-\Theta_{f,\exp}\end{array}\right\}$$

$$\times\frac{1}{8\pi}\int_0^{Fo_i}\frac{1}{(Fo_i-Fo')}\exp\left[-\frac{\left[\cos\beta-U\cos\varphi(Fo_i-Fo')\right]^2+\left[\sin\beta-U\sin\varphi(Fo_i-Fo')\right]^2}{4\left(Fo_i-Fo'\right)}\right]\times \tag{9}$$

$$\left\{erfc\left[\frac{Z-H}{2\sqrt{Fo_i-Fo'}}\right]-2*erfc\left[\frac{Z}{2\sqrt{Fo_i-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo_i-Fo'}}\right]\right\}\times$$

$$\frac{2\times\left[\cos\beta-U\cos\varphi(Fo_i-Fo')\right]\times\cos\varphi(Fo_i-Fo')+2\times\left[\sin\beta-U\sin\varphi(Fo_i-Fo')\right]\times\sin\varphi(Fo_i-Fo')}{4\left(Fo_i-Fo'\right)}dFo'$$

Secondly the detailed information on F'_φ is illustrated in Eq.(10).

$$F_\varphi{}^{\Delta} = 2\sum_{i=1}^{n}(\Theta_{f,\mathrm{cal}}-\Theta_{f,\exp})\times\Theta'_{f,\mathrm{cal}} = 2\sum_{i=1}^{n}\left\{\begin{array}{l}\frac{1}{8\pi}\int_0^{Fo_i}\frac{1}{(Fo_i-Fo')}\exp\left[-\frac{\left[\cos\beta-U\cos\varphi(Fo_i-Fo')\right]^2+\left[\sin\beta-U\sin\varphi(Fo_i-Fo')\right]^2}{4\left(Fo_i-Fo'\right)}\right]\times\\ \left\{erfc\left[\frac{Z-H}{2\sqrt{Fo_i-Fo'}}\right]-2*erfc\left[\frac{Z}{2\sqrt{Fo_i-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo_i-Fo'}}\right]\right\}dFo'-\Theta_{f,\exp}\end{array}\right\}$$

$$\times\frac{1}{8\pi}\int_0^{Fo_i}\frac{1}{(Fo_i-Fo')}\exp\left[-\frac{\left[X-U\cos\varphi(Fo_i-Fo')\right]^2+\left[Y-U\sin\varphi(Fo_i-Fo')\right]^2}{4\left(Fo_i-Fo'\right)}\right]\times \tag{10}$$

$$\left\{erfc\left[\frac{Z-H}{2\sqrt{Fo_i-Fo'}}\right]-2*erfc\left[\frac{Z}{2\sqrt{Fo_i-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo_i-Fo'}}\right]\right\}\times$$

$$\frac{-2\left[\cos\beta-U\cos\varphi(Fo_i-Fo')\right]\times U\sin\varphi(Fo_i-Fo')+2\left[\sin\beta-U\sin\varphi(Fo_i-Fo')\right]\times U\cos\varphi(Fo_i-Fo')}{4\left(Fo_i-Fo'\right)}dFo'$$

As there are three points and therefore three goal functions need to be established. Only one goal function may not determine the accurate velocity, but three goal functions are highly possible to

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determine the velocity. For every goal function, the values of F'_U and F'_φ are respectively zero or minor values, which means two limitations are set for every function to let function reach the minimum. Accordingly, there are six conditions while three functions simultaneously reach the minimum. There are many conditions so that velocity can be limited to a single case, and this is the reason of setting three points rather than only one point. But if the accurate velocity cannot be found in such a way, there are some or a small range of velocities meeting the six conditions, these remaining velocities will be put into Equation (8) one by one for comparing different results, then the single velocity can be discovered because this velocity let the result of Equation (8) reach the minimum.

4. The relevant characteristics and trials of back calculation

4.1 The influence that orientation exerts on the comparisons of three points

The locations of three points have been fixed as shown in Fig.7. The temperature responses of points will raise with the time when *U* and *φ* are given confirmed values, and the relative size with each other is explicit. One example of specific *U* and *φ* is shown in Fig.12, and the orientation of groundwater flow is 60˚.

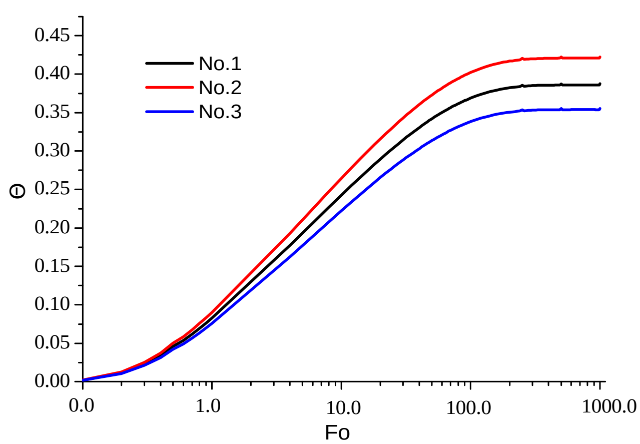

Fig.12 The temperature responses of three points with the time

Seepage role is an incitement of impacting temperature responses of points, and the influence degrees rest with the velocity [26]. All these temperature responses decrease with the increase of velocity intensity *U*, but when it comes to comparisons made for three temperature responses, the seepage orientation indeed plays a vital role. With the variation of orientation, the temperature

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response of every point shows fluctuation ceaselessly if the time and velocity intensity are constant, it means that the relative size in comparison to each other varies with the adjustment of orientation, the detailed information is observed in Fig.13.

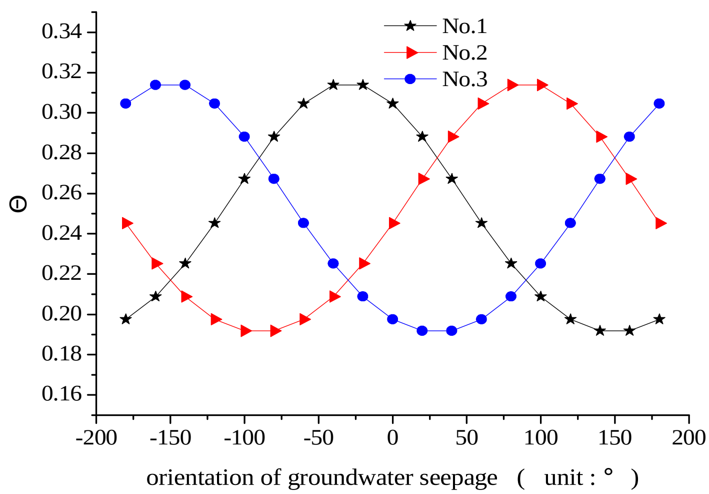

Fig.13 The temperature responses of three points with the change of orientation

4.2 The change trend of slope of temperature responses

The curves shown in Fig.12 indicate that the whole trend of temperature response is ever-increasing until a stable state. In the process of temperature increase, firstly those curves go through the stage that slopes keep continuous growth, next there is a period when slopes decrease, and at last all the curves will arrive at stable states. Equation (7) is the analytical solution of any point except heat source, thereby the slope of temperature response to time can be listed by means of the first order derivative of Θ to *F*o. The substitute variable *m* is used and $m = Fo - Fo'$ so that Equation (7) is transformed into Equation (11).

$$\Theta_f = \frac{1}{8\pi}\int_0^{Fo}\frac{dm}{m}\times\exp\left[-\frac{(\cos\beta - Um\cos\varphi)^2 + (\sin\beta - Um\sin\varphi)^2}{4m}\right]\times \left\{erfc\left[\frac{Z-H}{2\sqrt{Fo-Fo'}}\right] - 2erfc\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right] + erfc\left[\frac{Z+H}{2\sqrt{Fo-Fo'}}\right]\right\} \tag{11}$$

And the first order derivative of Θ to *F*o can be gained in Equation (12).

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$$\Theta_{Fo}{}^{\Delta}=\frac{1}{8\pi}\times\frac{1}{Fo}\times\exp\left[-\frac{(\cos\beta-U\times Fo\times\cos\varphi)^2+(\sin\beta-U\times Fo\times\sin\varphi)^2}{4Fo}\right]\times\left\{erfc\left[\frac{Z-H}{2\sqrt{Fo-Fo'}}\right]-2erfc\left[\frac{Z}{2\sqrt{Fo-Fo'}}\right]+erfc\left[\frac{Z+H}{2\sqrt{Fo-Fo'}}\right]\right\}/2 \tag{12}$$

For three points, the change trends of slopes with the time are displayed in Fig.14 while U and φ remain changeless.

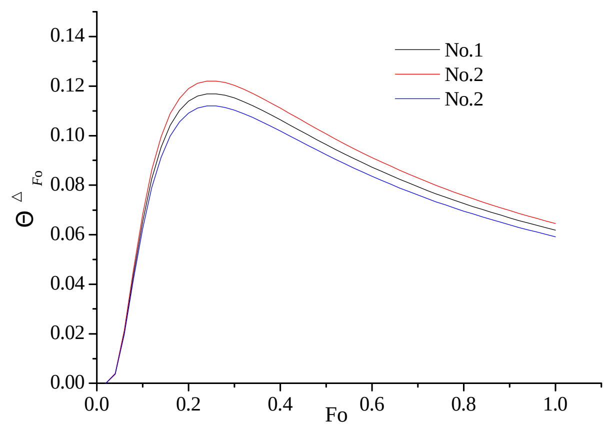

Fig.14 The variation trend of slopes of three pints while U and φ remain changeless

It can be found that the time when the slopes stop rising trend and begin to decrease are nearly the same for three points. From another perspective, because change trends of slopes of all points present the same regular pattern and the corresponding time is nearly equal. Consequently one point can be chosen to explore the slope trend under the conditions of different seepage directions, at that time the seepage intensity U is invariable. Meanwhile, the influence which velocity intensity exerts on the slope trend of one point can be studied if the seepage orientation is fixed. Fig.15 can depict corresponding conclusions with reference to these problems.

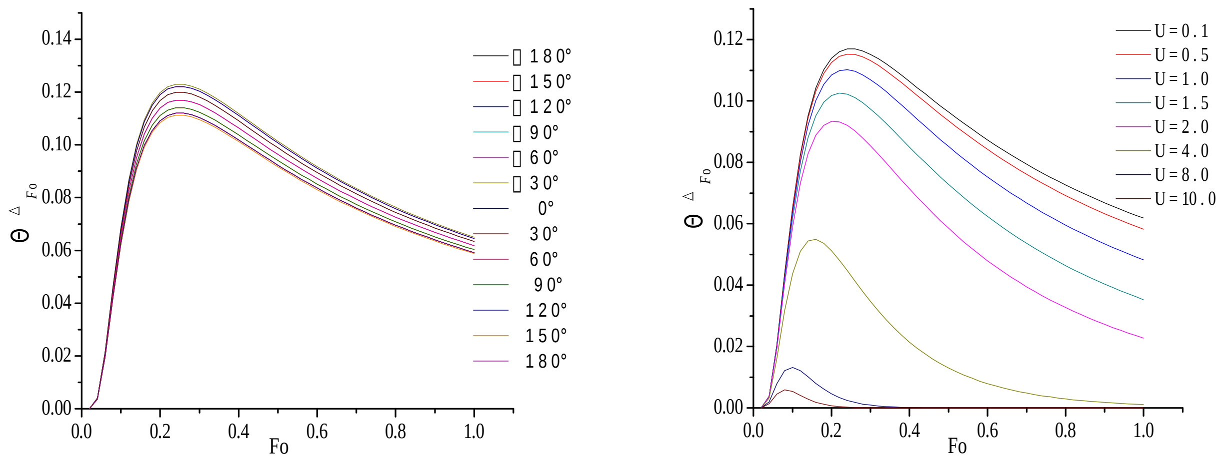

Fig.15 The variation trend of slope when orientation and velocity intensity varies respectively

The curves shown in Fig.15 explain that change of orientation can slightly alter the extent of slope but has no impact on the whole regular pattern even route, especially for the time point at which slop turns from increase to decrease, this time point does not change with the variation of orientation. In addition, the strengthening of U can affect the slope curves not only the extent but also the route. The slope extent drops with the enhancement of velocity value, and the time when the transformation between rise and fall occurred becomes shorter and shorter.

4.3 The back calculation trials

The principles of back calculation for obtaining velocity of groundwater are summarized above. Next, the actual experimental data and the theoretical results will be employed to check the rationality of back calculation. Three points are distributed around BGHE to record the experimental data at regular time interval. The range of U is known according to the local underground hydraulic data and the range of φ is [-180˚, 180˚]. The experimental data of temperature response with the time can be recorded and the corresponding non-dimensional temperature response can be obtained. The range of U and φ are set and then the iterations for them are conducted, which means U and φ are continuously picked from their ranges and then the values are put into Equation (7) to obtain the non-dimensional temperature response of theoretical model. Thus, the temperature response curves of three points can be obtained by theoretical calculation. Commonly the recorded data curves fluctuate and have obvious deviation with the model curves. By means of back calculation method, the accurate values of U and φ can be determined while the goal function achieves the minimum. The relevant signs CM and CE are respectively the calculation

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result of model and the experiment data. The theoretical model circumstances and experimental circumstances of three points are illustrated in Fig.16. Some examples were selected in the process of trials, for example, when U and φ respectively adopts 0.2 and 60˚, theoretical result of the model can be obtained and the experimental data are recorded. Having used the back calculation based on the principles introduced above, the velocity intensity U and orientation φ can be found. In addition, other examples were tried to verify the back calculation, and the effect can shows that the back calculation method is reasonable of obtaining groundwater velocity.

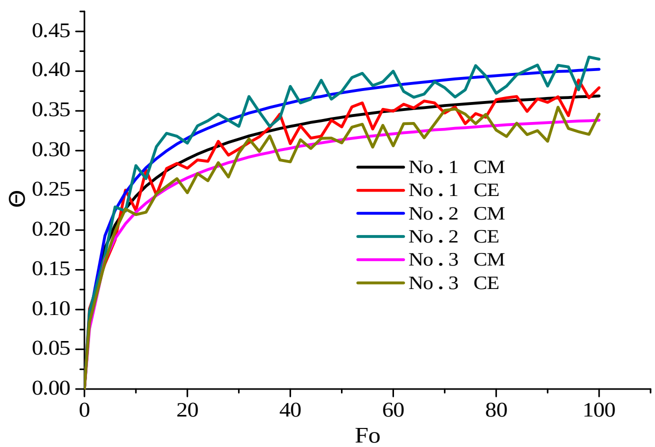

Fig.16 The temperature responses of both model calculation and experiments

5. Conclusions

With the development of GSHP technology, the research on groundwater seepage is becoming increasingly important, this is due to the fact that the advection of groundwater can improve the heat transfer ability of BGHE so that the performance of the whole system can be ameliorated. The paper analyzes the relevant characteristics involved in the heat exchanger process between BGHE and surrounding underground medium while groundwater flows though BGHE. The underground temperature field is unavoidably affected by seepage role. By means of comparison between pure conduction and combined heat transfer including conduction and groundwater advection, the significance of investigating the groundwater seepage can be proven. It goes without saying the most difficult task is to comprehend the groundwater velocity including value and orientation, therefore the back calculation method based on the finite heat source seepage model is proposed. Combined with the relevant knowledge of advanced mathematics, the goal function is established for acquiring the velocity. The derivative is utilized according to the principles of how to achieve

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the extreme value of multivariable function. Some characteristics derived from back calculation are investigated to explain the essence of method in detail. The back calculation method provide convenience for obtaining the groundwater velocity because only the temperature response is enough to achieve the groundwater velocity; this method is valuable for investigating the influence that groundwater seepage exerts on heat transfer of BGHE. The content of this paper mainly propose a theoretical method based on the reasonable principles, which can provide theoretical guidance for obtaining groundwater velocity while actual experiment is done.

Acknowledgement

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Figure Captions

Fig.1 The schematic diagram of groundwater which flows past BGHE

Fig.2 The comparison between pure conduction and combined heat transfer

Fig.3 The isothermals with the increase of U

Fig.4 The isothermals with the increase of Fo

Fig.5 The mean temperature response with the time when φ adopts different angles

Fig.6 The mean temperature response with the velocity value when φ adopts different angles

Fig.7 The schematic diagram of three even-distributed points around BGHE

Fig.8 Some distributing modes for three points around BGHE

Fig.9 The distributing diagram while there are different number points

Fig.10 The mean temperature response along Z-axis

Fig.11 The temperature responses of three points with U

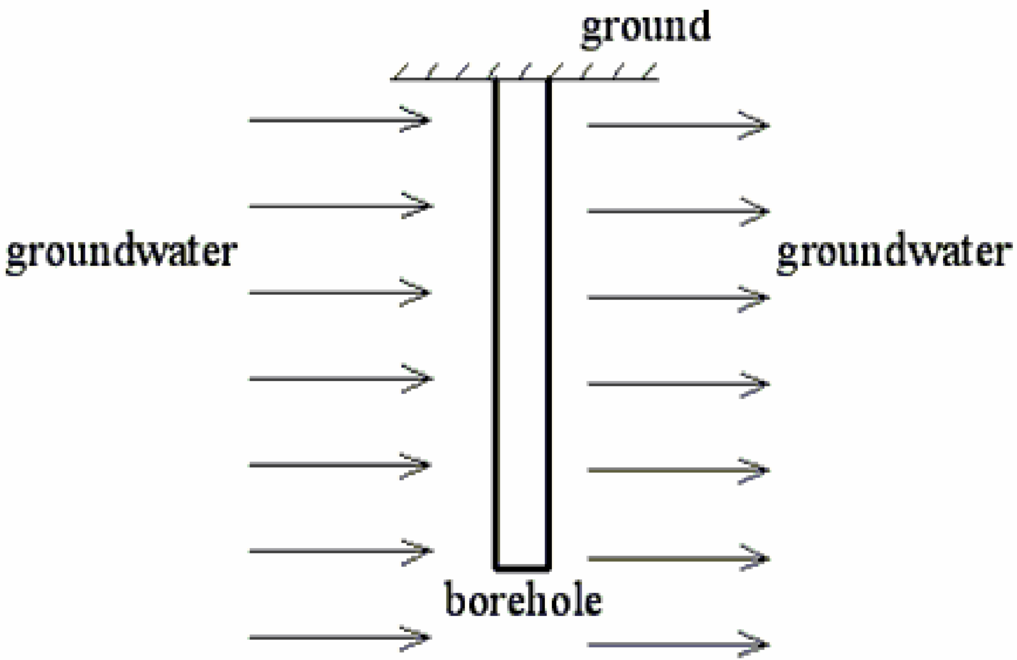
Fig.12 The temperature responses of three points with the time

Fig.13 The temperature responses of three points with the change of orientation

Fig.14 The variation trend of slopes of three pints while U and φ remain changeless

Fig.15 The variation trend of slop when orientation and velocity intensity varies respectively

Fig.16 The temperature responses of both model calculation and experiments



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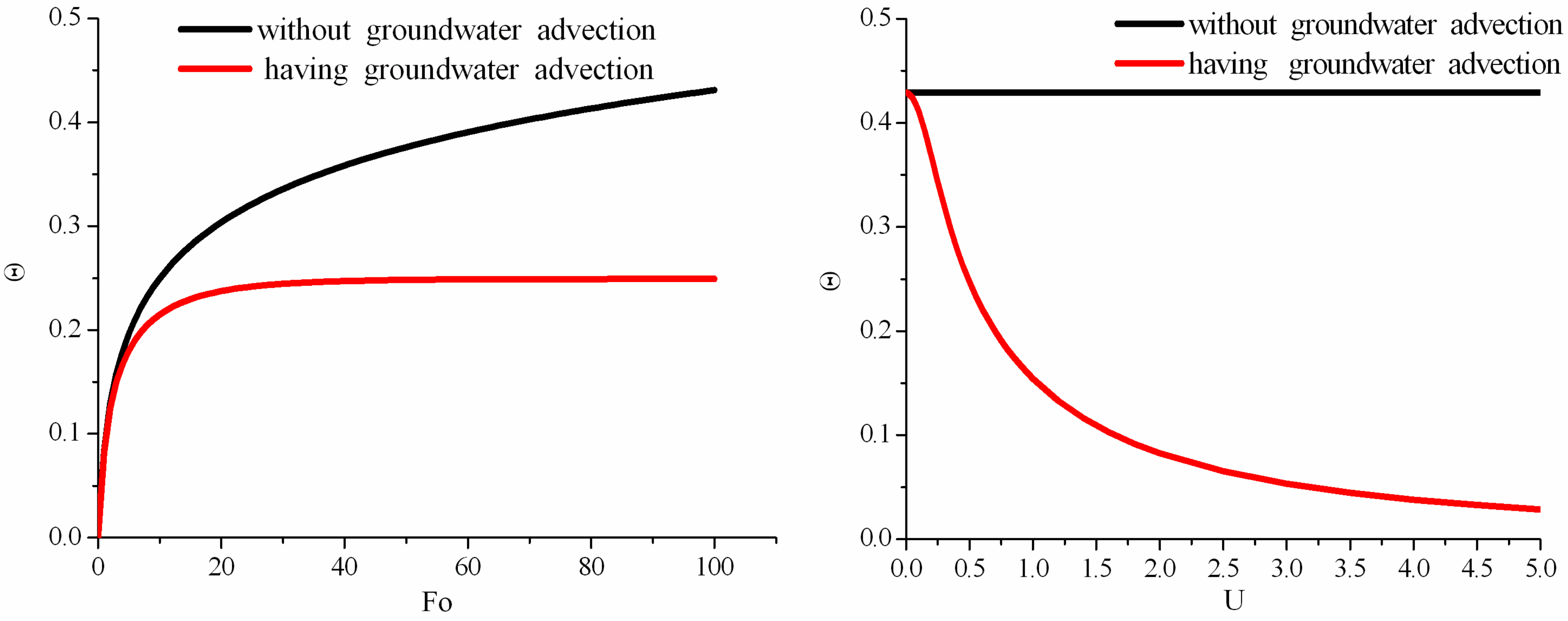

Fig.2 The comparison between pure conduction and combined heat transfer

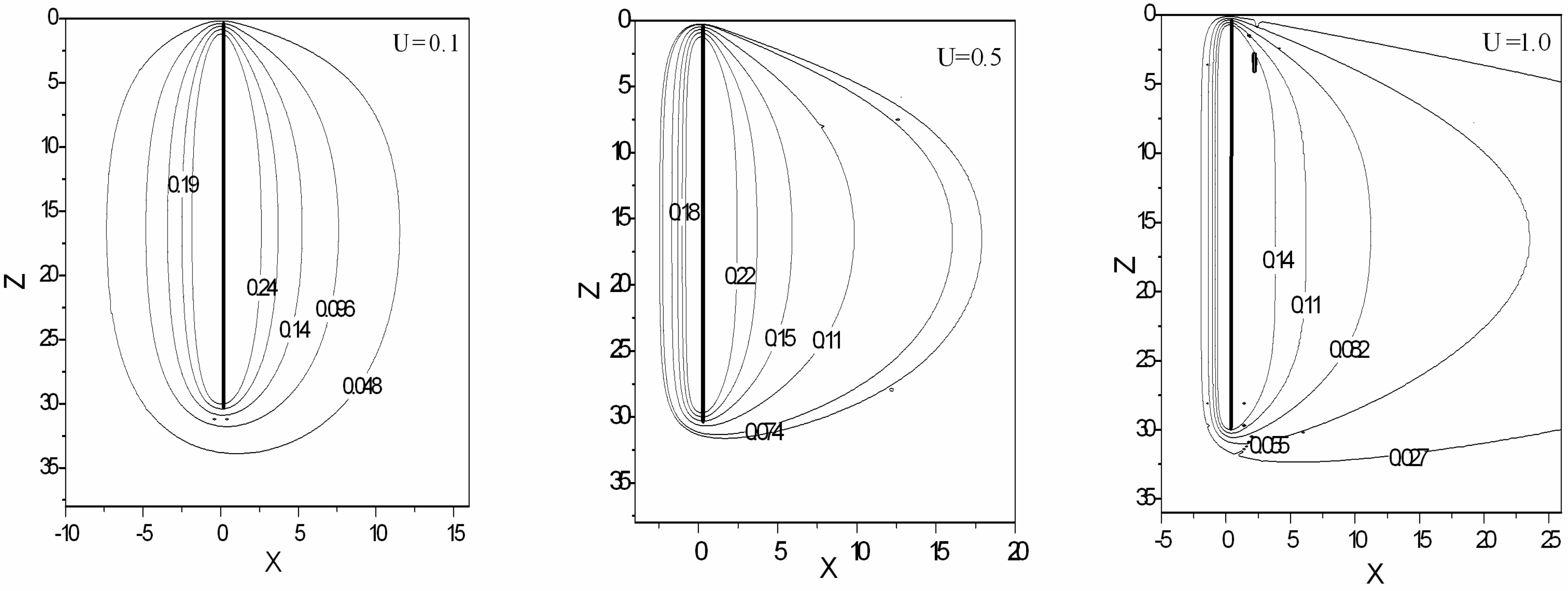

Fig.3 The isothermals with the increase of *U*

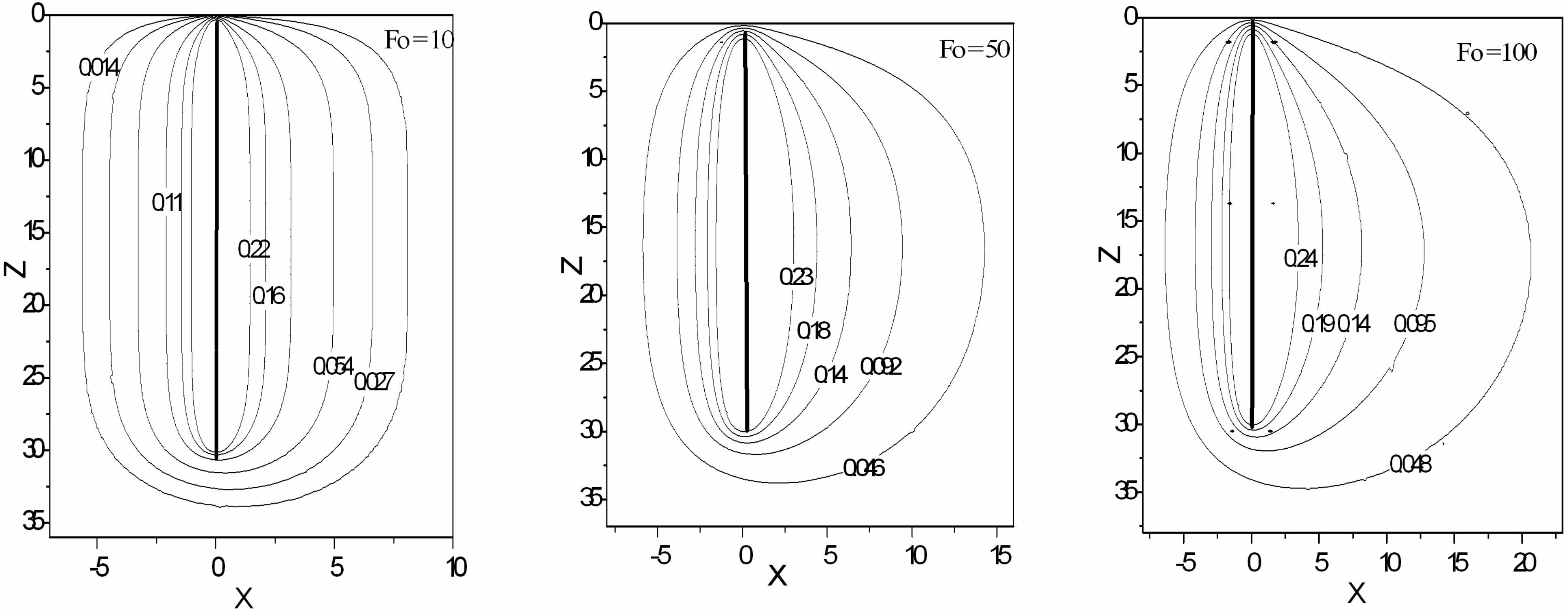

Fig.4 The isothermals with the increase of *Fo*

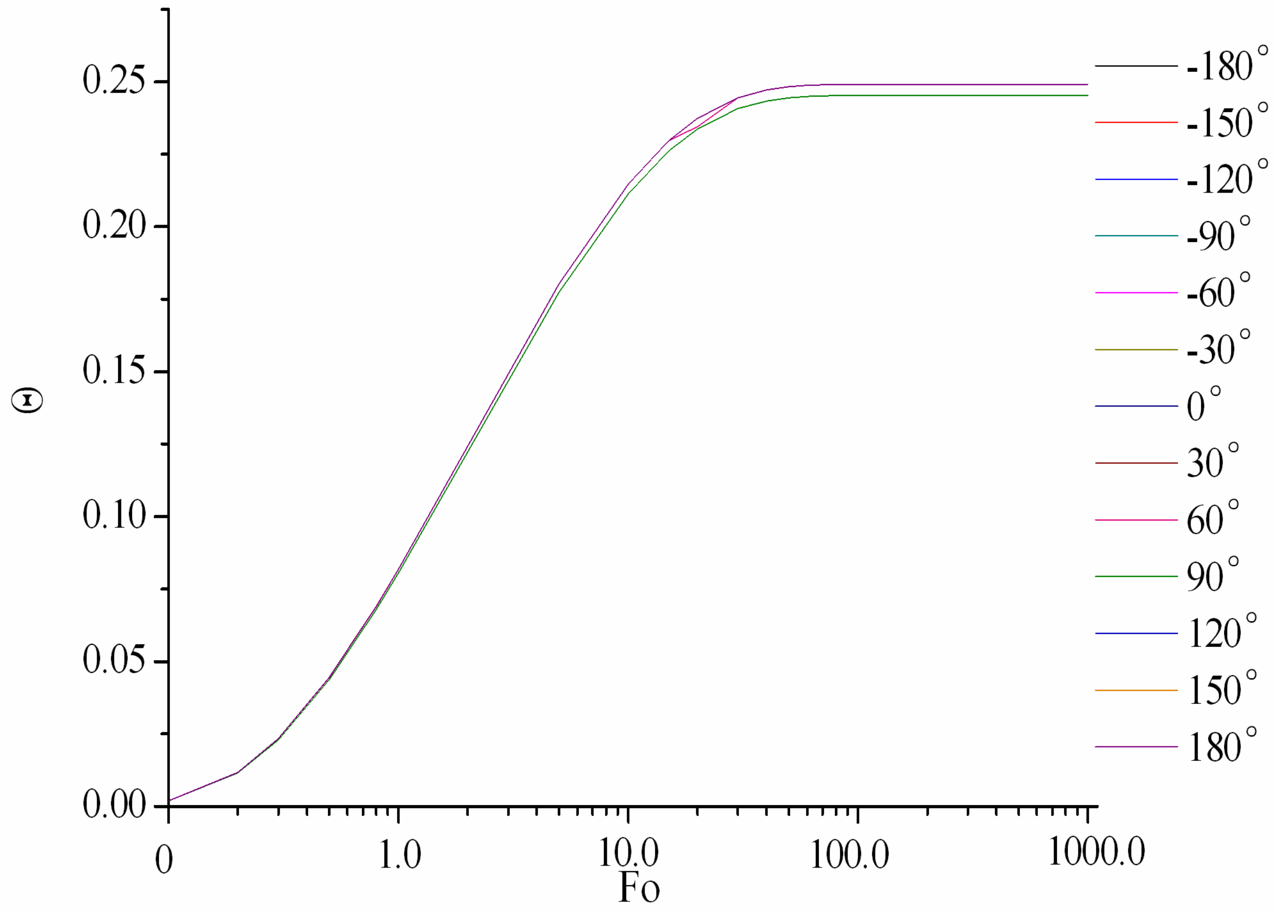

Fig.5 The mean temperature response with the time when φ adopts different angles

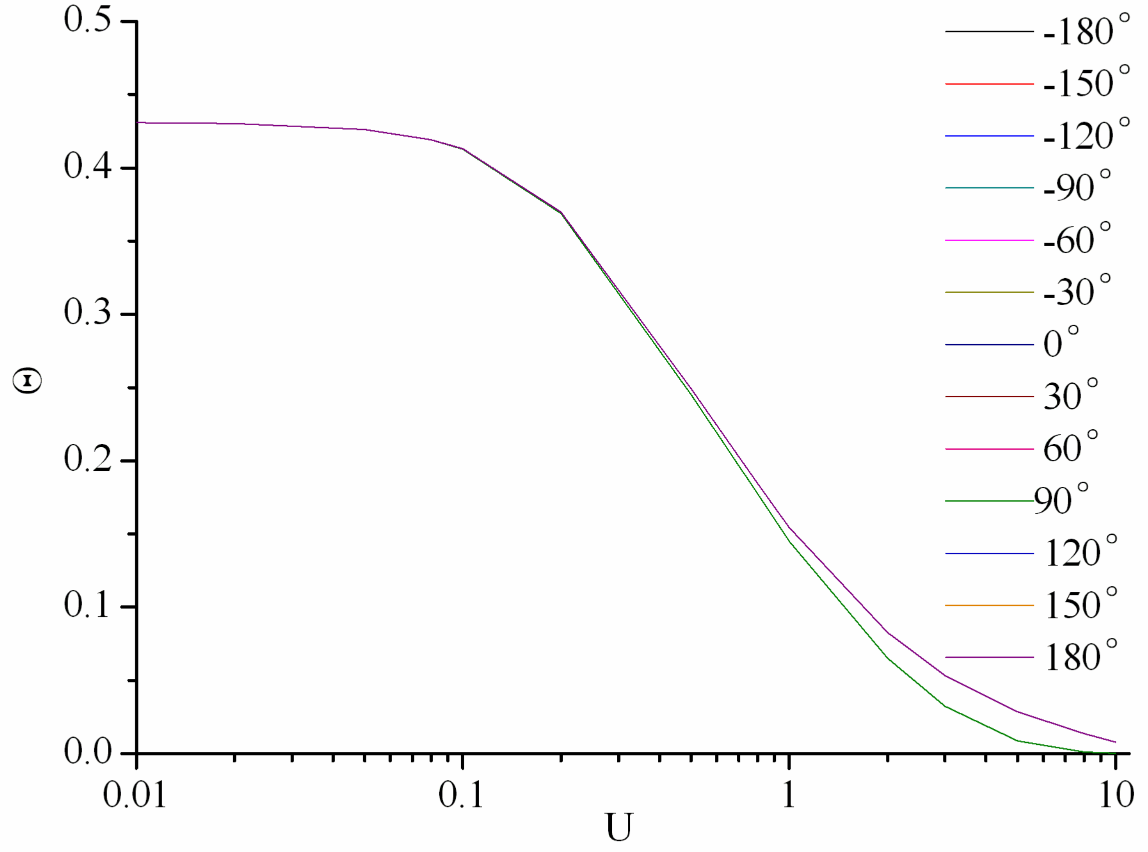

Fig.6 The mean temperature response with the velocity value when φ adopts different angles

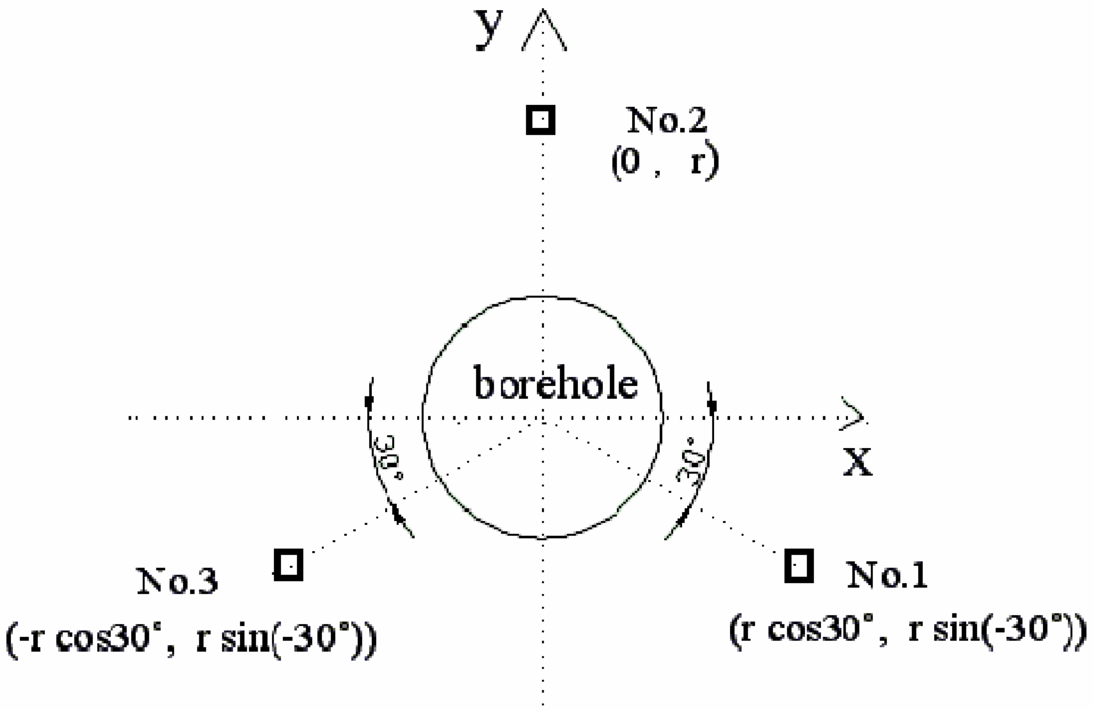

Fig.7 The schematic diagram of three even-distributed points around BGHE

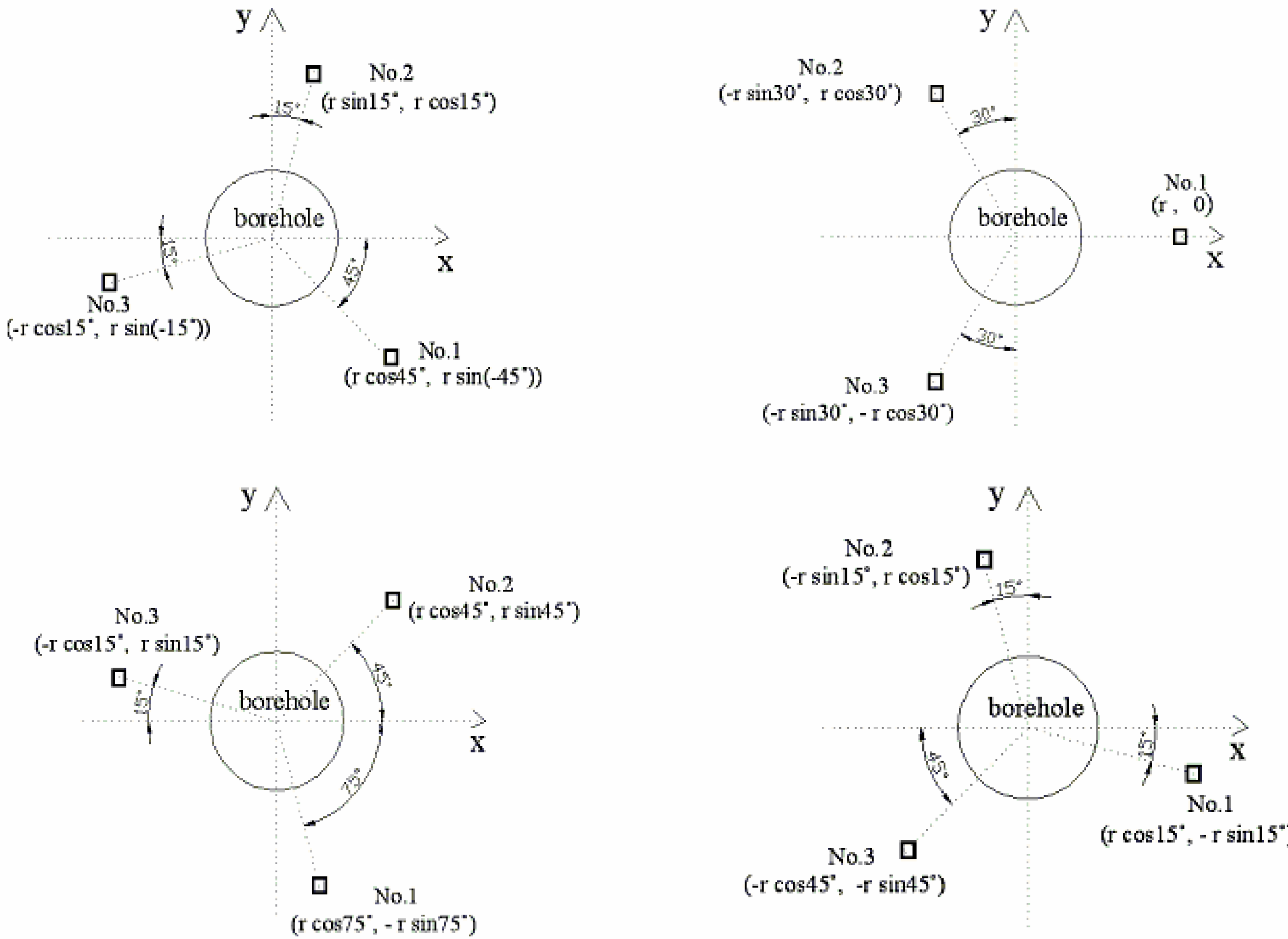

Fig.8 Some distributing modes for three points around BGHE

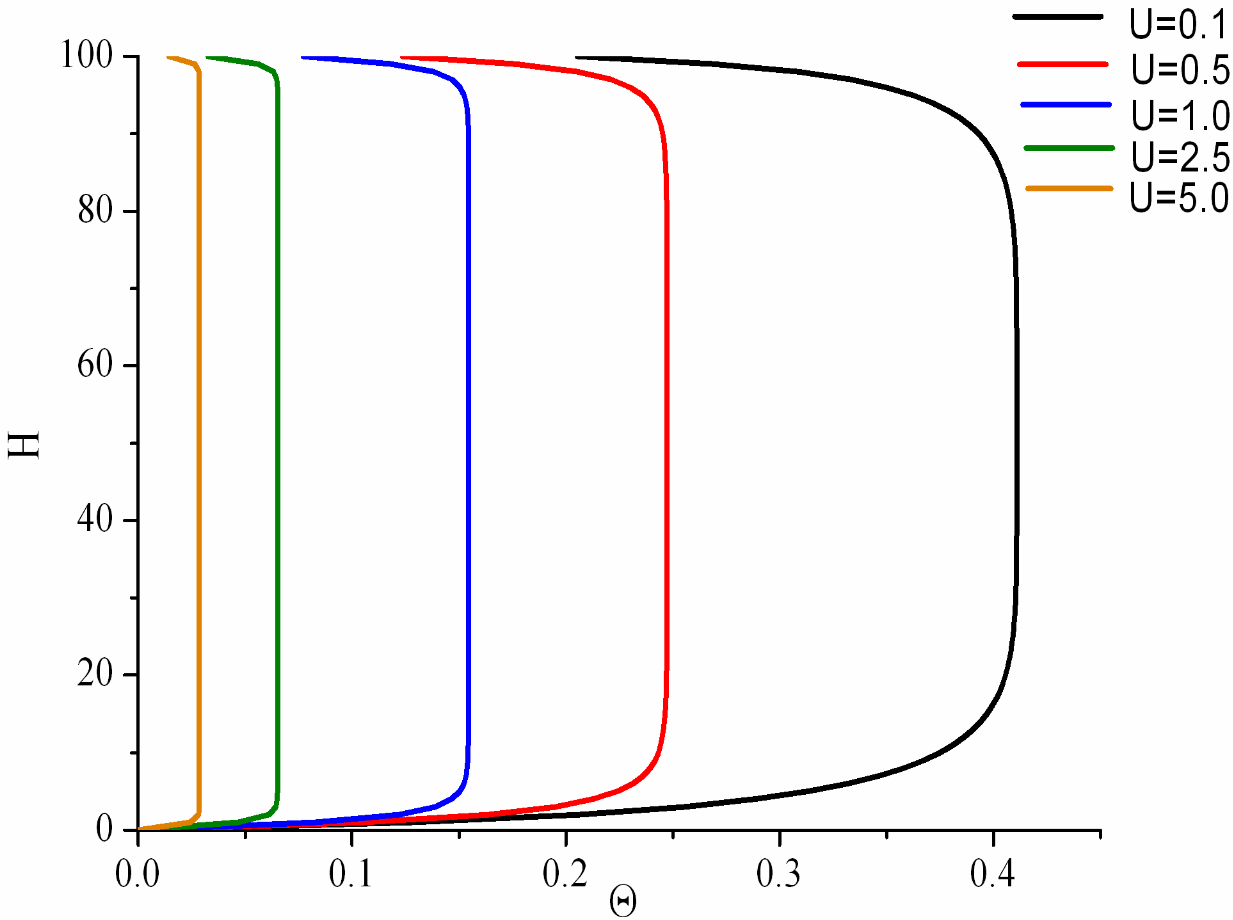

Fig.10 The mean temperature response along Z-axis

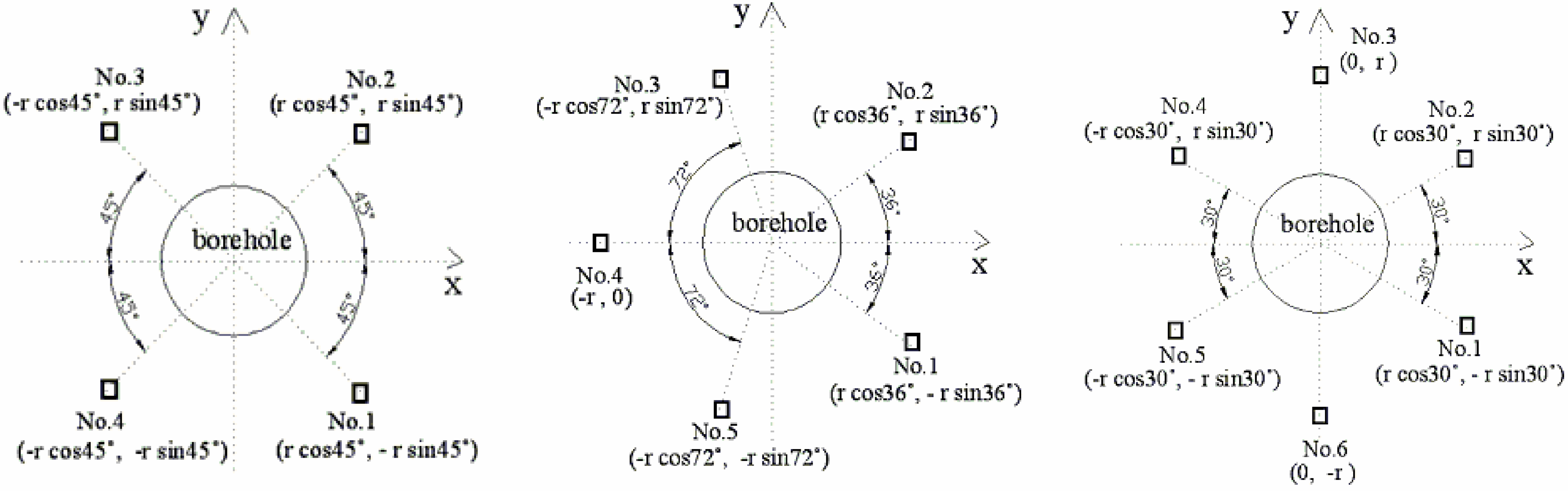

Fig.9 The distributing diagram while there are different number points

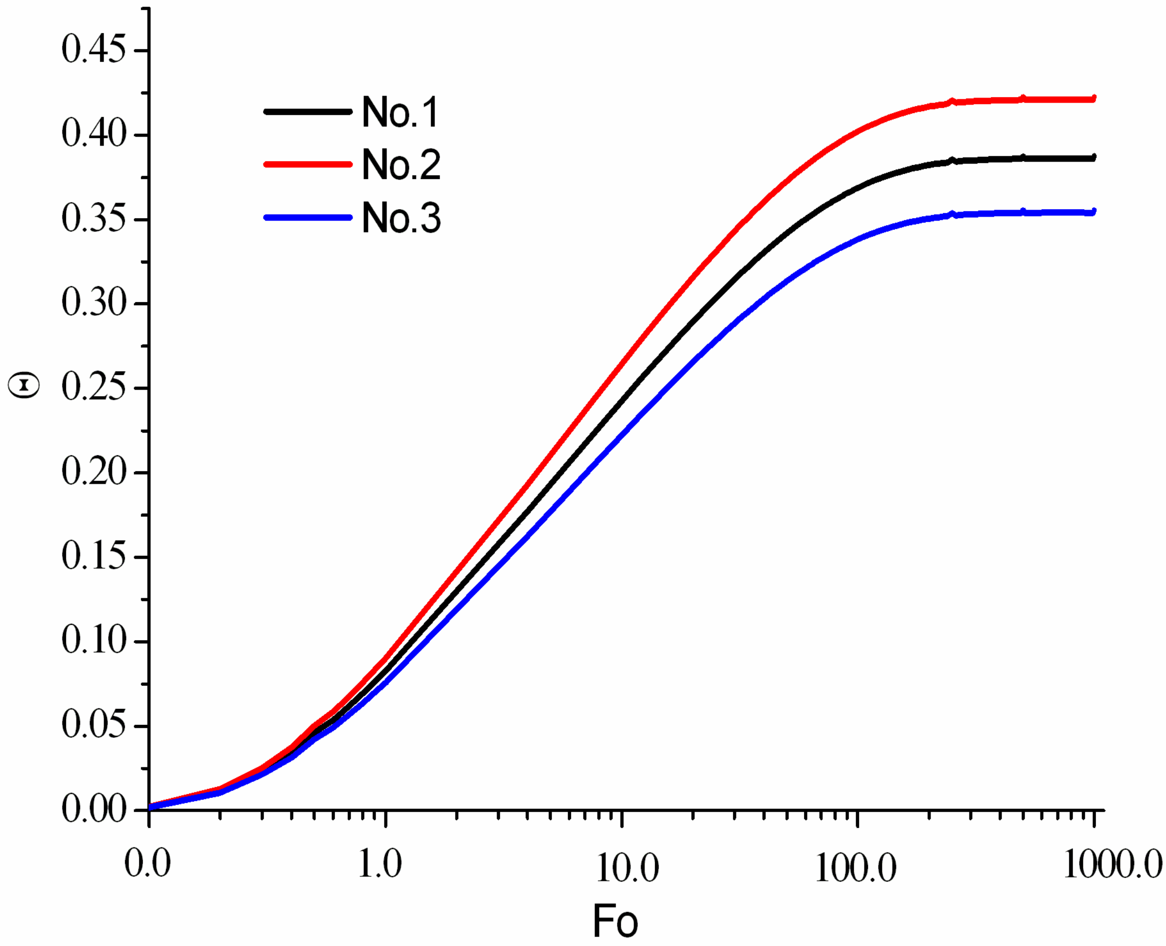

Fig.12 The temperature responses of three points with the time

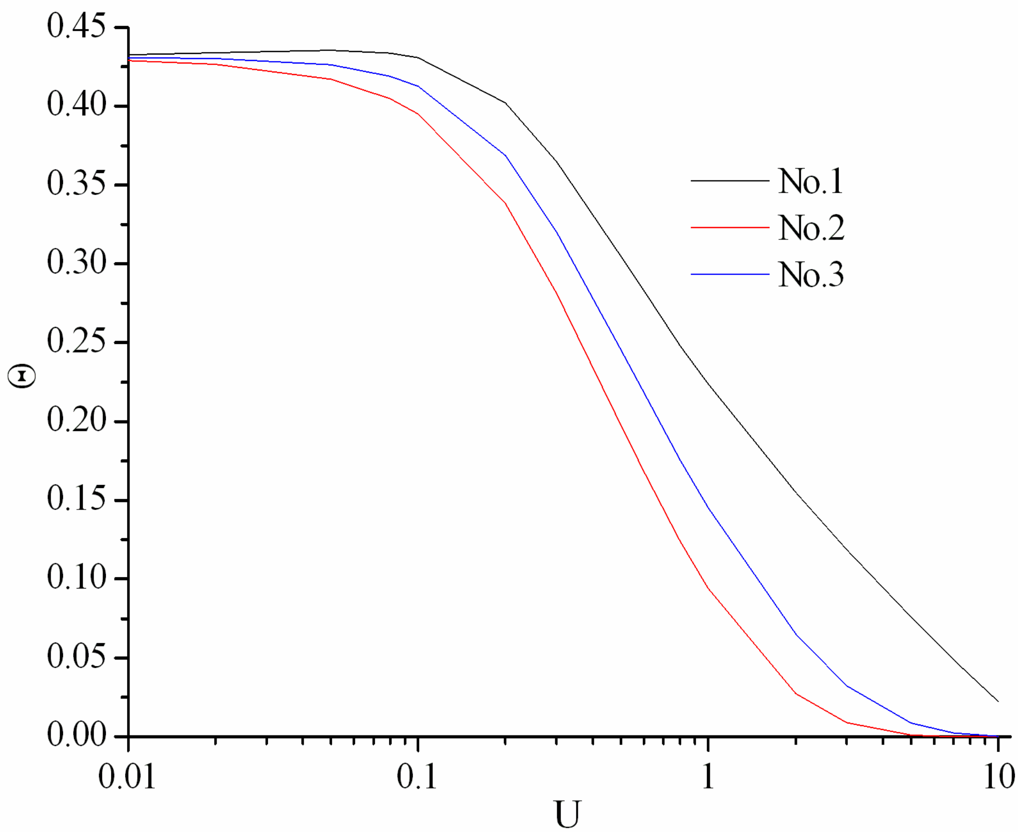

Fig.11 The temperature responses of three points with *U*

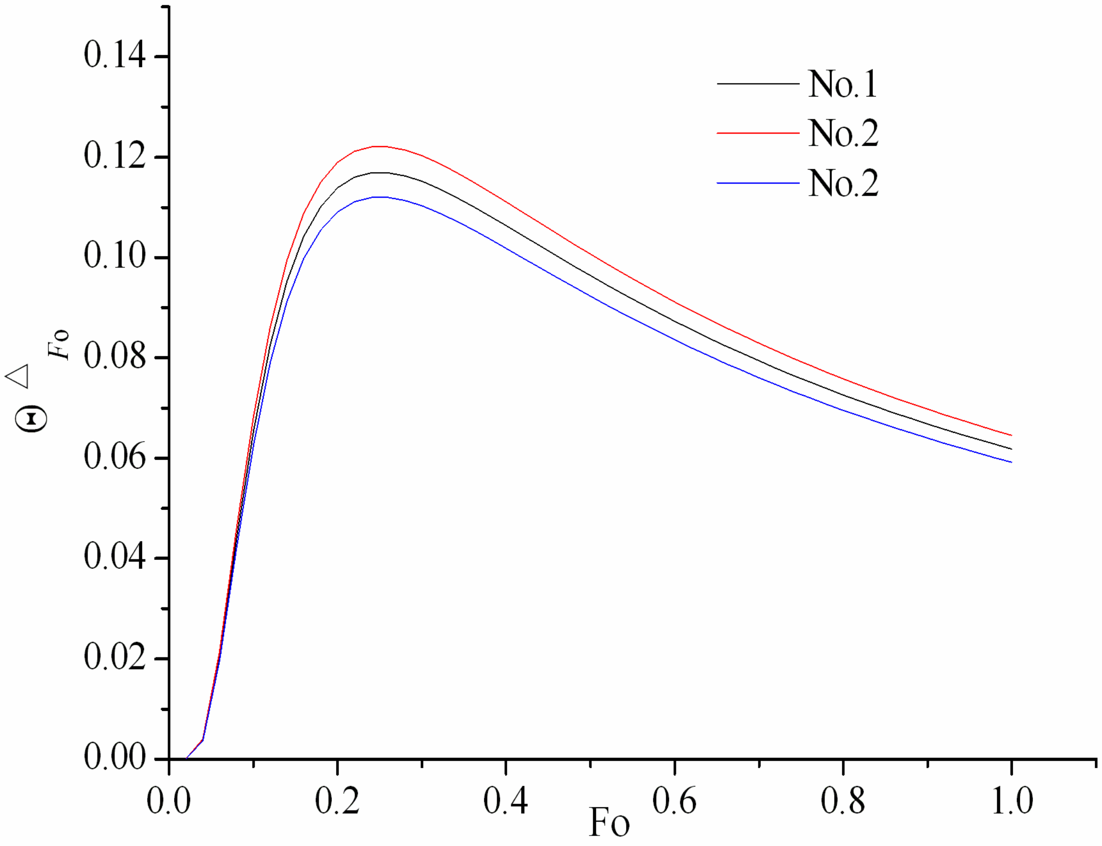

Fig.14 The variation trend of slopes of three pints while *U* and φ remain changeless

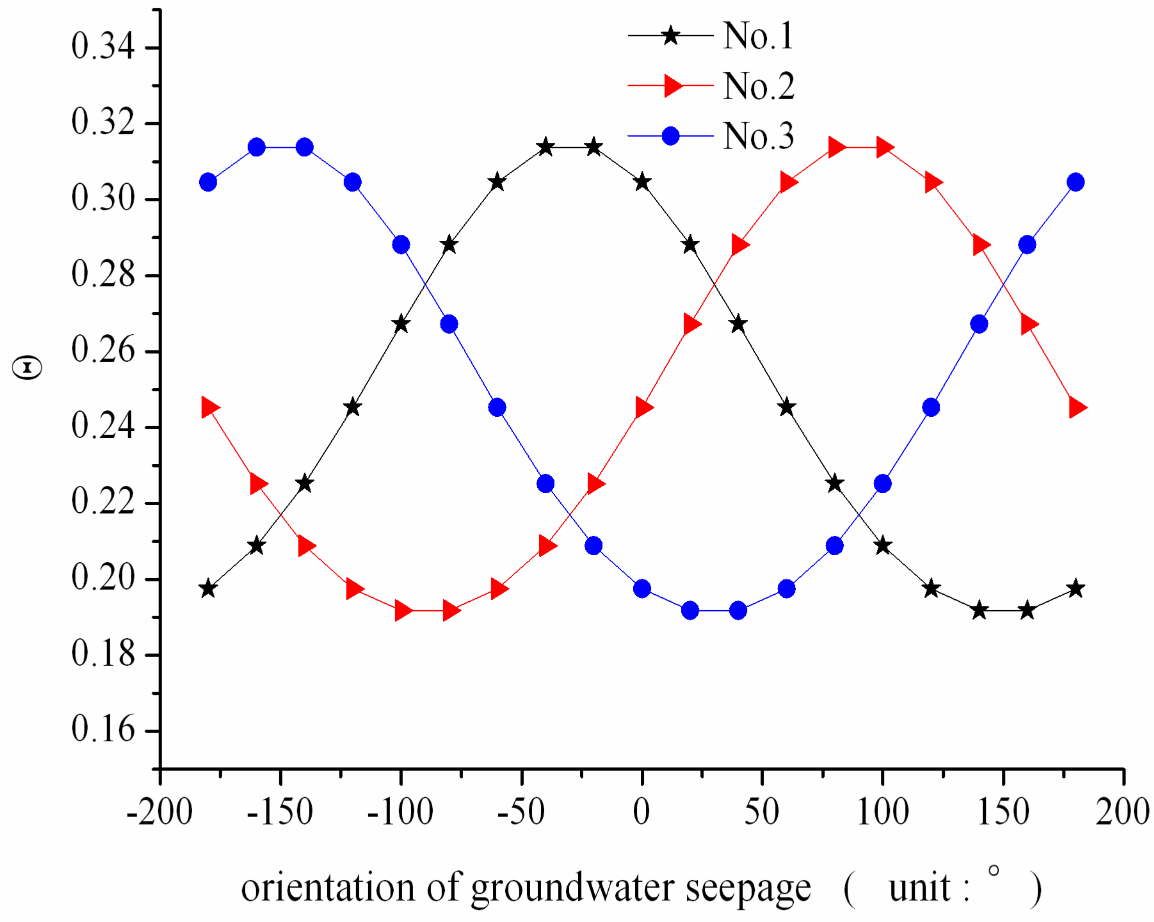

Fig.13 The temperature responses of three points with the change of orientation

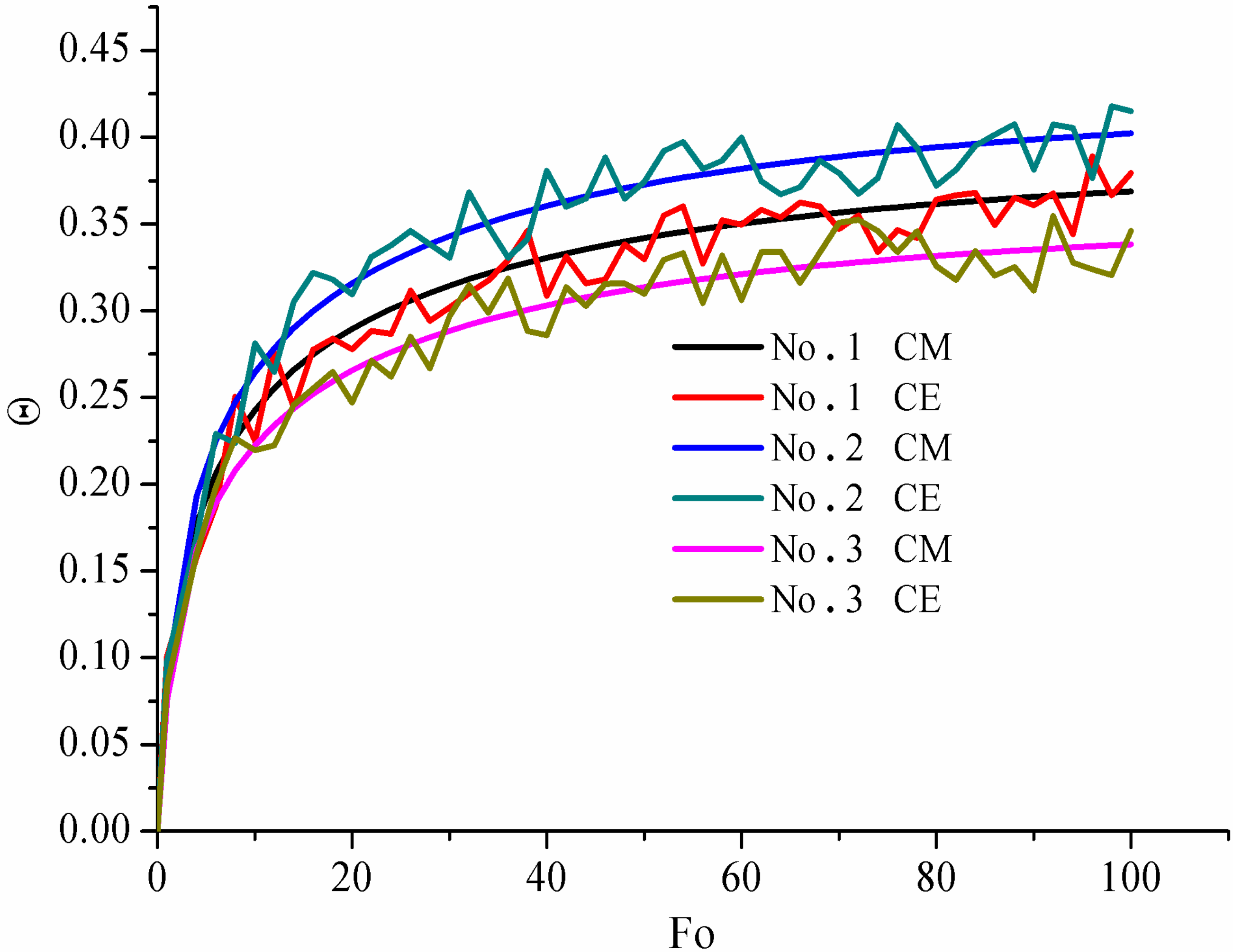

Fig.16 The temperature responses of both model calculation and experiments

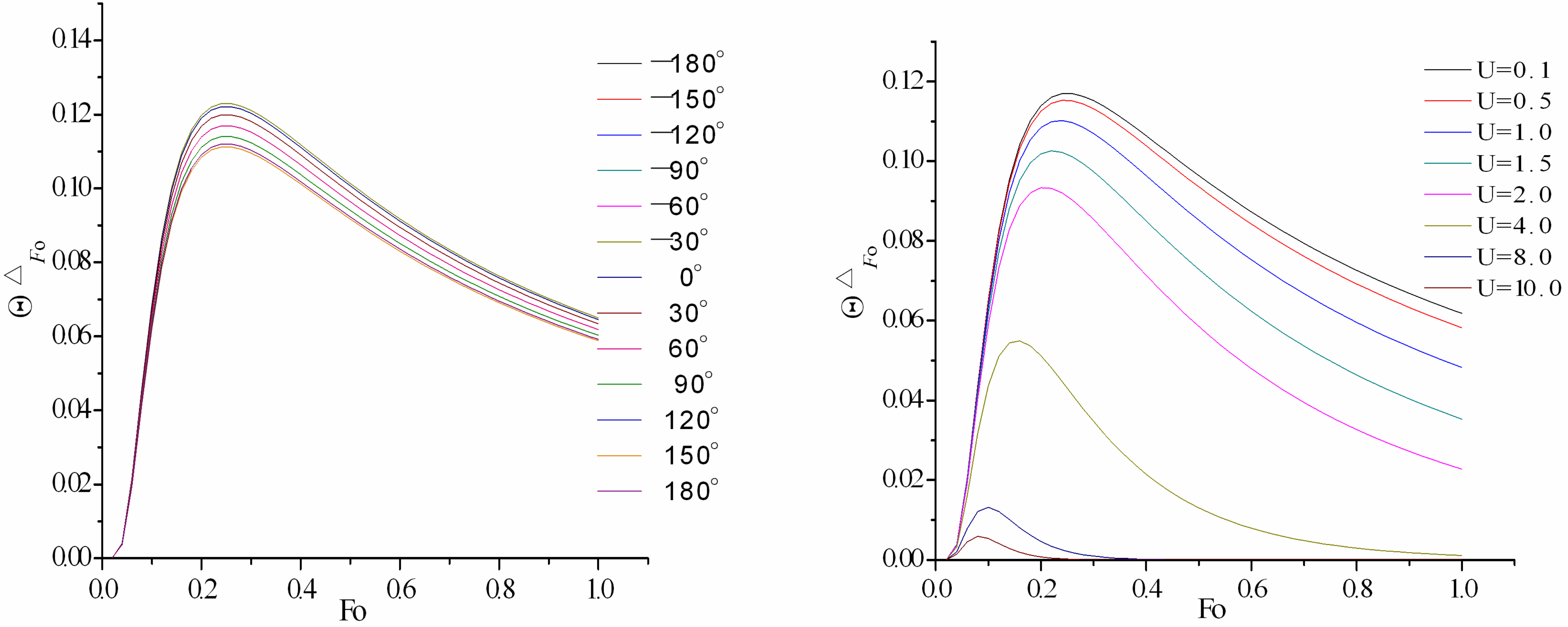

Fig.15 The variation trend of slope when orientation and velocity intensity varies respectively