A New Simplified Hypothesis B Method for Calculating Consolidation Settlements of Double Soil Layers Exhibiting Creep

by

Wei-Qiang FENG[1,2]
[1] PolyU Shenzhen Research Institute, Shenzhen, China.
[2] Department of Civil and Environmental Engineering,
The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong.
Email: 11901182r@connect.polyu.hk

Jian-Hua YIN[1,2]
[1] PolyU Shenzhen Research Institute, Shenzhen, China.
[2] Department of Civil and Environmental Engineering
The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China
Tel: (852) 2766-6065, Fax: (852) 2334-6389, Email: cejhyin@polyu.edu.hk

Manuscript submitted to International Journal for Numerical and Analytical Method in Geomethanics for possible publication as an Article

February 2016

Abstract:

This paper presents a new simplified method, based on Hypothesis B, for calculating the consolidation settlements of double soil layers exhibiting creep. In the new simplified Hypothesis B method, different stress-strain states including over-consolidation and normal consolidation states can be considered with the help of the "equivalent time" concept. Zhu and Yin method and US Navy method are adopted to calculate the average degree of consolidation for a double soil layer profile. This new simplified Hypothesis B method is then used to calculate the consolidation settlements of double soil layers, which have two different total thicknesses of soil layer (4m and 8m) and three different OCR values (Over-Consolidation Ratio, $OCR = 1$, 1.5 and 2). The accuracy and verification of this new simplified method are examined by comparing the calculated results with simulation results from a fully coupled finite element (FE) program using a soft soil creep model. Four cases of double layer soil profiles are analyzed. Hypothesis A method with US Navy method for the average degree of consolidation has also been used to for calculating consolidation settlements of the same cases. For $Case\, I(4m)$ and $Case\, III(8m)$, it is found that curves of the new simplified Hypothesis B method using both Zhu and Yin method and US Navy method are very close to the results from FE simulations with the *relative errors* within 8.5%. For $Case\, II(4m)$ and $Case\, IV(8m)$, it is found that curves of the new simplified Hypothesis B method using Zhu and Yin method agrees better with results from FE simulations with the *relative errors* within 11.7% than curves of the new simplified Hypothesis B method adopting US Navy method with the *relative error* up to 36.1%. Curves of Hypothesis A method adopting US Navy method have the *relative error* up to 55.0% among all four cases. In overall, the new simplified Hypothesis B method is suitable for calculation of consolidation settlements of double soil layers exhibiting creep, in which, Zhu and Yin method is recommended to obtain the average degree of consolidation.

Keywords: double soil layers, consolidation settlement, creep, visco-plastic

1. Introduction

The time-dependent phenomenon of soils can be attributed to hydrodynamic lags (consolidation) and viscous deformation of the soil skeleton [1]. The consolidation of a clayey soil is caused by the dissipation of the excess pore water pressure while viscous deformation is due to the viscosity of the soil skeleton, including the creep, stress relaxation, and strain rate dependency. The design of geotechnical projects, such as the reclamation, needs to consider the consolidation settlement of the soil exhibiting creep [2, 3].

Ladd *et al.* [4] questioned whether the creep occurs during "primary" consolidation, which led to two extreme methods in terms of Hypotheses A and B: Hypothesis A assumes that creep contribution can be included independently after "primary" consolidation stage, whereas Hypothesis B assumes that creep contribution should be included throughout the consolidation and compression process. This question remains controversial among researchers. Mesri and Godlewski [5], Choi [6], Feng [7], Mesri and Vardhanabhuti [8], and Mesri [9], supporting Hypothesis A, believed that soil is compressed for two interrelated reasons: (*i*) the change of effective stress, (*ii*) the change of time. Meanwhile, Bjerrum [10], Stolle *et al.* [11], Vermeer and Neher [12], Nash and Ryde [13], Yin *et al.* [14], Leroueil [15], Leoni *et al.* [16], Karim *et al.* [17], Nash and Brown [18] used Hypothesis B to consider that the creep occurs during the consolidation stage. According to the definition, creep is a continuous deformation of soil under a constant load (or an incremental creep under an incremental constant load) [1][14]. It is reasonable to say that creep always exists under the action of varying effective stress, which means that Hypothesis B is logically correct.

Based on Hypothesis B and the "equivalent time" concept [10, 19, 20], Yin [2], Yin and Feng [3] presented a new simplified Hypothesis B method for the handy calculation of the consolidation settlement of a single soil layer exhibiting, considering different stress-strain states. Yin and Feng [3] verified the accuracy of this new simplified method by comparing calculated values with results from fully coupled finite element (FE) simulations. In reality, due to the geological history, a soil profile has layers more than one layer [21, 22]. The consolidation problem of multiple soil layers was extensively studied before. Schiffman and Stein [23] obtained a mathematical solution for a layered consolidation problem. US Department of the Navy [24] proposed a simplified procedure to convert multiple soil layers into one single soil layer. Details of this procedure will be presented later. Zhu and Yin [22, 25] presented an analytical solution and solution charts for double soil layers under the ramp loading with different depths, and demonstrated the different consolidation behaviors between the double soil layers and a simplified one single soil layer [24]. Meanwhile, Xie *et al.* [26] introduced an analytical solution for the two-layered soil with partially drained boundaries. Xie *et al.* [27] considered the nonlinear properties of double layered soils. Related problems such as double layered soils with vertical drains [28, 29], soft clayey soils reinforced by floating stone columns [30, 31] and double layered system for unsaturated soil [32] have been widely studied without considering creep.

This paper aims to generalize a new simplified Hypothesis B method (Yin anf Feng 2016) for a single soil layer to double soil layers for calculating consolidation settlement of soils with creep for different stress-strain states under instant loading. Examples with two different total thickness values (4m and 8m) and three different stress-strain states (Over-Consolidation Ratio, $OCR = 1$, 1.5 and 2) are presented to illustrate the accuracy of this simplified Hypothesis B method when using Zhu and Yin method [22, 25] and US Navy method [24] for determination of the average degree of consolidation. The accuracy (or relative errors) of this simplified Hypothesis B method is examined by comparing calculated results with simulation results from a fully coupled finite element software with an elastic visco-plastic constitutive model for the clayey soil used in the examples. As a benchmark comparison, the conventional Hypothesis A method with US Navy method for the average degree of consolidation has also been used to calculate consolidation settlements of the same cases.

2. Brief Review of the New Simplified Method Based on Hypothesis B for a Single Soil Layer

Based on Hypothesis B and “equivalent time” concept [19, 20, 33], Yin [2], Yin and Feng [3] proposed a new simplified Hypothesis B method for 1-D consolidation settlement prediction for one single layer of a clayey soil as follows:

$$\begin{aligned} S_{totalB} &= S_{"primary"} + S_{creep} \\ &= U_v S_f + [\alpha S_{creep,f} + (1-\alpha) S_{"secondary"}] \quad for \quad t \geq 1\,day \quad (t \geq t_{EOP,field} \; for \; S_{"secondary"}) \end{aligned} \tag{1}$$

where $S_{"primary"} = U_v S_f$ denotes the settlement of “primary” consolidation at any time *t*,. U_v is the average degree of consolidation for the soil layer, S_f represents the final settlement at the end of “primary” consolidation. It is noted that $S_f = \varepsilon_f H$ where ε_f is the considering vertical strain and H is the thickness of a single soil layer. In Eq.(1), S_{creep} is the creep settlement during and after “primary” consolidation, The subscript “*creep*” indicates that the settlement is related to creep. In Eq.(1) α is a constant parameter to reasonably consider the creep settlement during and after the consolidation, and its value should be in the range from 0 to 1. $S_{creep,f}$ in Eq.(1) is the creep settlement calculated at the final effective stress ignoring the coupling of the excess pore water pressure and $S_{creep,f} = \varepsilon_{creep,f} H$ where $\varepsilon_{creep,f}$ is the corresponding final creep strain. In Eqe.(1) $S_{"secondary"}$ is the “secondary” consolidation settlement based on Hypothesis A, $S_{"secondary"} = \varepsilon_{"secondary"} H$ $= \frac{C_{\alpha e}}{1+e_0} \log \frac{t}{t_{EOP,field}} H$ where $\varepsilon_{"secondary"}$ the corresponding “secondary” strain, $C_{\alpha e}$ is the “secondary” consolidation coefficient, e_0 is the initial void ratio, and $t_{EOP,field}$ is the time at the End-Of-Primary (EOP) consolidation in the field and can be calculated using the time at $U_v = 98\%$. It is noted that when $\alpha = 0$, Eq.(1) is reduced to the equation of Hypothesis A method: $S_{totalA} = U_v S_f + S_{"secondary"} \quad for \quad t \geq t_{EOP,field} \; for \; S_{"secondary"}$. We use S_{totalA} to denote the settlement calculated using Hypothesis A method here, rather than still using S_{totalB}.

The key issue of Eq.(1) is how to accurately determine the creep strain under different stress-strain states including the normal consolidation and over-consolidation states in the new simplified Hypothesis B method. Figure 1 shows the relationship of vertical strain versus log(vertical effective stress) with different stress-strain states. The initial stress-strain state, $Point\ 1\,(\sigma'_{z1}, \varepsilon_{z1})$, and pre-consolidation stress-strain state, $Point\ 3\,(\sigma'_{zp}, \varepsilon_{zp})$, are already known. The slope of unloading-reloading line is $C_e/(1+e_0)$, and the slope of normal consolidation line is $C_c/(1+e_0)$, which are obtained from oedometer tests with duration of 24 hours (1 day) as a common approach. When the initial point is at $Point$ 1 and the final effective stress state is at $Point$ 2 on the over-consolidation line, the final "primary" consolidation and creep strains are calculated with the following equations (Yin and Feng 2016):

$$\begin{aligned}
&\varepsilon_f = \varepsilon_{z2} = \frac{C_e}{1+e_o}\log\left(\frac{\sigma'_{z2}}{\sigma'_{z1}}\right) + \varepsilon_{z1} \\
&\varepsilon_{creep,f} = \frac{C_{\alpha e}}{1+e_0}\log\left(\frac{t+t_{e2}}{t_0+t_{e2}}\right) \\
&t_{e2} = t_0 \times 10^{(\varepsilon_{z2}-\varepsilon_{zp})\frac{(1+e_0)}{C_{\alpha e}}}\left(\frac{\sigma'_{z2}}{\sigma'_{zp}}\right)^{-\frac{C_c}{C_{\alpha e}}} - t_0 \quad for\ t \geq 1\ day
\end{aligned} \tag{2}$$

The t_0 is a material parameter and shall be taken as 1 (day) since $C_e/(1+e_0)$ and $C_c/(1+e_0)$ are obtained from oedometer tests with duration of 24 hours (1 day). Eq.(2) is valid for time *t* equal to or larger than 1 day since the data points in Figure 1 all have 1 day duration already.

When the initial point is at $Point\ 1$ and the final effective stress state is at $Point\ 4$ on the normal consolidation line, the final "primary" consolidation and creep strains are expressed as:

$$
\begin{aligned}
\varepsilon_f &= \varepsilon_{z4} = \left(\frac{C_e}{1+e_o} \log\left(\frac{\sigma'_{zp}}{\sigma'_{z1}} \right) + \varepsilon_{z1} \right) + \frac{C_c}{1+e_o} \log\left(\frac{\sigma'_{z4}}{\sigma'_{zp}} \right) \\
\varepsilon_{creep,f} &= \frac{C_{\alpha e}}{1+e_0} \log\left(\frac{t}{t_0} \right) \quad for\ t \geq 1\, day
\end{aligned}
\tag{3}
$$

We prefer to call $C_{\alpha e}$ a creep coefficient, rather than the "secondary" consolidation coefficient since Eq.(2) and Eq.(3) consider creep occurs during and after "primary" consolidation. Eq.(2) and Eq.(3) are derived by using the equivalent time t_e proposed by Yin and Graham [19, 20], Yin *et al.* [14].

3. A New Simplified Hypothesis B Method for Calculating Consolidation Settlement of Multiple Layers of Soils Exhibiting Creep

In many cases, there are more than one layer of soils in the field and each stratum is influenced by the other layer[25]. To consider double soil layers condition, a new simplified Hypothesis B method is proposed:

$$
\begin{aligned}
S_{totalB} &= \sum_{i=1}^{n} S_{"primary"i} + \sum_{i=1}^{n} S_{creepi} = U_a \sum_{i=1}^{n} S_{fi} + \sum_{i=1}^{n} [\alpha S_{creep,fi} + (1-\alpha) S_{"secondary"i}] \\
&= U_a \sum_{i=1}^{n} \varepsilon_{fi} H_i + \sum_{i}^{n} \{[\alpha \varepsilon_{creep,fi} + (1-\alpha)\varepsilon_{"secondary"i}] H_i\} \quad for\ t \geq 1 day (t \geq t_{EOP,field}\ \ for\ S_{"secondary"i})
\end{aligned}
\tag{4}
$$

where $\sum_{i=1}^{n} S_{"primary"i}$ is the "primary" consolidation settlement of *n* soil layers, U_a and $\sum_{i=1}^{n} S_{fi}$ are the average degree of consolidation and the total "primary" consolidation settlement of *n* soil layers, $\sum_{i=1}^{n} S_{creepi}$ is the total creep settlement of *n* soil layers, $\sum_{i=1}^{n} S_{creep,fi}$

$\sum_{i=1}^{n} S_{\text{"secondary"}i}$ are the total final creep settlement and the total “secondary” consolidation settlement of *n* soil layers. Eq.(4) is an extension of Eq.(1) to consider multiple soil layers. Eq.(4) can be reduced to Eq.(1) when $n = 1$. In previous study, Yin and Feng [3] suggested α 0.8 for a single soil layer. In the study of this paper, the authors have found that α is related to Over-Consolidation Ratio (OCR) and can be taken as $\alpha = 0.4 + 0.2\text{OCR}$. For OCR=1, 1.5, and 2, we have $\alpha = 0.6, 0.7, 0.8$. The verification of these α values can be seen in Figures 3 to 8 later with a comparison with FE simulation results. Eqs. (2) and (3) are also used to determine the final creep compression $\varepsilon_{creep,f}$ and then creep settlement $S_{creep,f}$ of a soil in each layer under different stress-strain states. Another important issue is how to correctly determine the average degree of consolidation, U_a for multiple soil layers.

In this paper, we use Eq.(4) to analyze a double soil layer system which was studied before by Zhu and Yin [22, 25], Xie *et al.* [21, 26] without considering creep. In this analysis, the solution derived by Zhu and Yin [22, 25] for double soil layer consolidation analysis is adopted (denoted Zhu and Yin method) for calculating the average degree of consolidation U_a. Zhu and Yin [22, 25] provided charts for calculating the average degree of consolidation U_a. In the solution and charts, Zhu and Yin [22, 25] introduced two independent parameters (p, q), construction time factor (T_c) and time factor (T) for the consolidation settlement calculation. Key equations are summarized as follows:

$$
\begin{aligned}
p &= \frac{\sqrt{k_2 m_{v2}} - \sqrt{k_1 m_{v1}}}{\sqrt{k_2 m_{v2}} + \sqrt{k_1 m_{v1}}} \\
q &= \frac{H_1\sqrt{c_{v2}} - H_2\sqrt{c_{v1}}}{H_1\sqrt{c_{v2}} + H_2\sqrt{c_{v1}}} \\
\omega &= \frac{(1+q)}{2} \\
\xi &= \frac{(1-q)}{2} \\
T_c &= \frac{c_{v1}c_{v2}t_c}{\left(H_1\sqrt{c_{v2}} + H_2\sqrt{c_{v1}}\right)^2} \\
T &= \frac{c_{v1}c_{v2}t}{\left(H_1\sqrt{c_{v2}} + H_2\sqrt{c_{v1}}\right)^2}
\end{aligned} \tag{5}
$$

$$
U_a(T,T_c) = \begin{cases} \dfrac{T_c}{T} - \displaystyle\sum_{n=1}^{\infty} \frac{c_n}{\lambda_n^4 T_c}[1-\exp(-\lambda_n^2 T)] & T \le T_c \\ 1 - \displaystyle\sum_{n=1}^{\infty} \frac{c_n}{\lambda_n^4 T_c}[1-\exp(-\lambda_n^2 T_c)] \times \exp[-\lambda_n^2 (T-T_c)] & T \ge T_c \end{cases} \tag{6}
$$

where λ_n is the root of the equation $sin\theta + psin(q\theta) = 0$ for both top and bottom drained condition (*condition1*) and the equation $cos\theta - pcos(q\theta) = 0$ for one side drained condition (*condition2*). Values of c_n are determined by the following equation:

$$
c_n = \begin{cases} \dfrac{2[m_{v1}H_1\xi\sin(\lambda_n\xi) + m_{v2}H_2\omega\sin(\lambda_n\omega)]^2}{\omega^2\xi^2(m_{v1}H_1 + m_{v2}H_2)[m_{v1}H_1\xi\sin^2(\lambda_n\xi) + m_{v2}H_2\omega\sin^2(\lambda_n\omega)]} & for\,condition1 \\ \dfrac{2[m_{v1}H_1\xi\cos(\lambda_n\xi)]^2}{\omega^2(m_{v1}H_1 + m_{v2}H_2)[m_{v1}H_1\xi\cos^2(\lambda_n\xi) + m_{v2}H_2\omega\sin^2(\lambda_n\omega)]} & for\,condition2 \end{cases} \tag{7}
$$

Details of the derivation could be found in Zhu and Yin [22, 25] and the solution is valid for the uniform vertical stress under the ramp loading on double soil layers. The procedures of a step-by-step calculation are provided later.

US department of the Navy [24] proposed a simplified procedure for consolidation analysis of multiple soil layers. For double soil layers, we can convert soil layer 2 to an equivalent thickness of soil layer 1, using:

$$H_2' = H_2\left(c_{v1}/c_{v2}\right)^{1/2} \qquad T = \frac{c_{v1}t}{(H_1 + H_2')^2} \tag{8}$$

where H_2 is the height of the soil layer 2, H_2' is the equivalent thickness of soil layer 2 as if it is made up of soil layer 1, c_{v1} and c_{v2} are the coefficients of consolidation for layers 1 and 2, respectively. T is the overall time factor of the whole deposit. After the conversion, the average degree of consolidation, U_a, can be determined as one single soil layer. This method is named as US Navy method in this paper.

4. Four Cases of Double Soil Layers and Finite Element Modelling Approach

In this section, we have selected the geologic profile of the Hong Kong International Airport (HKIA) in Lantau Island, Hong Kong as an example to apply the new simplified Hypothesis B method for consolidation analysis of double soil layers. The representative values of soil parameters are adopted for using this new simplified method to calculate the consolidation settlement of soils with creep. Plaxis (2D 2015 version) is also used to analyze the consolidation settlement of the same soil layers. The corresponding results will be presented and compared in the next section to verify the applicability and accuracy of the new simplified Hypothesis B method.

4.1 Description of the double soil layers

There is more than one soil layer at the site of HKIA [34, 35]. "Upper Marine Clay" is at the top of the soil layer with 2m~8m in thickness and "Upper Alluvium" layer underlays the "Upper Marine Clay". The base of "Upper Alluvium" is regarded to be impermeable, and the top of "Upper Marine Clay" is seabed and normally filled by sand so that the top is considered free drained [36]. Detailed description of "Upper Marine Clay" can be found in [3], [37], [38]. The void ratio of "Upper Alluvium" is 1. Both "Upper Marine Clay" and "Upper Alluvium" are considered to have three different Over-Consolidation Ratio (OCR) values ($OCR = 1,\ 1.5\ \text{and}\ 2$) as a parametric study.

Figure 2 shows the profile of four cases of double soil layers. Table 1 presents values of all parameters of four cases of double soil layers used for consolidation analysis using the new simplified Hypothesis B method and Finite Element Modelling (FEM) using Plaxis (2D version 2015). The total thickness of $Case\,I\ (4m)$ and $Case\,II\ (4m)$ is 4m with 2 m "Upper Marine Clay" in the top followed by 2 m "Upper Alluvium", the bottom of which is impermeable. Comparing to $Case\,I\ (4m)$, the difference of $Case\,II\ (4m)$ is that the permeability value of "Upper Marine Clay" is increased by one order and the permeability value of "Upper Alluvium" is decreased by one order. The total thickness of and $Case\,IV\ (8m)$ is 8 m with 4 m "Upper Marine Clay" in the top followed by 4 m "Upper Alluvium", the bottom of which is impermeable. Comparing to $Case\,III\ (8m)$, the difference of $Case\,IV\ (8m)$ is that the permeability value of "Upper Marine Clay" is increased by one order and the permeability value of "Upper Alluvium" is decreased by one order. A vertical stress of $20kPa$ is assumed suddenly applied on the two layers in Figure 2 [3].

4.2 Description of a Finite Element Modelling Approach

In order to verify the accuracy of the new simplified Hypothesis B method for double soil layers, the Finite Element (FE) software Plaxis (2D version 2015) is used for the numerical simulation adopting the soft soil creep (SSC) model [12], [39], which is, in fact, an non-linear Elastic Visco-Plastic constitutive model [40][41]. A two-dimensional plane strain finite element mesh with 15-node triangular elements is used in Plaxis simulation.

As illustrated in Figure 2, the top elements of the soil have free drainage and the bottom elements are impermeable when conducting the consolidation analysis. The left and right vertical boundaries in Figure 2 are impermeable and are confined to have vertical movements only. A vertical stress of $20kPa$ is instantly applied on the top of all FE simulation models and the loading period is up to 100000 days to make sure that consolidation is totally completed in all simulation cases. The monitoring point for the settlement is at the top surface of the FE model, as illustrated in Figure 2. The definition of the SSC parameters can be found in the Plaxis manual (2D version 2015), and values of parameters used in Plaxis are listed in Table 1. The initial pre-consolidation stress plays an important role in the ground settlement prediction when adopting the SSC model [39]. When considering OCR value effects, OCR values of "Upper Marine Clay" and "Upper Alluvium" are set to be 1, 1.5 and 2. Initial stress-strain condition before adding the vertical loading and consolidation is generated with the in-situ K_0 condition.

5. Application and Verification of the New Simplified Hypothesis B Method for Consolidation Analysis of Double Soil Layers with Creep

This section presents the detailed procedures of applying the new simplified Hypothesis B method for consolidation analysis of four cases of double layers of soils exhibiting creep and calculated results.

5.1 Procedures of applying the new simplified Hypothesis B method for consolidation settlement calculations

In order to calculate the consolidation settlement of soils with creep, the initial and final effective stress states should be firstly determined. It is suggested that the total thickness of 4m or 8m shall be divided into a number of sub-layers with 0.5m thickness in order to calculate the final primary settlement S_{fi} more accurately for each soil type layer.

Secondly, values of the initial effective stress ($\sigma'_{z1,j}$), that is, $Point\ 1\,(\sigma'_{z1}, \varepsilon_{z1})$ in Figure 1, pre-consolidation stress state ($\sigma'_{zp,j}$) and final effective stress ($\sigma'_{zf,j}$) for each sub-layer *j* after loading are calculated below:

$$\begin{aligned} \sigma'_{z1,j} &= (\gamma_{soil,j} - \gamma_w) z_j \\ \sigma'_{zp,j} &= OCR \times \sigma'_{z1,j} \\ \sigma'_{zf,j} &= \sigma'_{z1,j} + \Delta\sigma'_z \end{aligned} \tag{9}$$

where z_j is the sub-layer middle location, $\gamma_{soil,j}$ is the saturated weight of the soil in the sub-layer, as listed in Table 1, γ_w is water unit weight, taken as *9.81 kN/m*^{3}, $\Delta\sigma'_z$ is the vertical loading, taken as *20 kPa* in the calculation. In Eq.(9), we introduce a new index "*j*" for sub-layers (up to a total of *m* sub-layers) of 0.5m thick only for each soil type. This index "*j*" is different from the index "*i*" in Eq.(4) which is for layers of different soils.

It should be noted that the unit weight of "Upper Alluvium" is different from that of "Upper Marine Clay" soil and the initial effective stress should be determined carefully for each layer in Figure 2. As shown in Figure 1, the initial effective stress state is at $Point\ 1\,(\sigma'_{z1}, \varepsilon_{z1})$ for $OCR = 1.5\ or\ 2$, and at $Point\ 3\,(\sigma'_{zp}, \varepsilon_{zp})$ for $OCR = 1$. Assuming the initial strain is zero for all four cases. Final effective stress state is at $Point\ 4\,(\sigma'_{z4}, \varepsilon_{z4})$ after the loading of *20kPa* for all the sub-layers of "Upper Marine Clay" with two different thicknesses 2m or 4m and *OCR* values. After the loading of *20kPa,* the final effective stress state after the stress increment is at $Point\ 4\,(\sigma'_{z4}, \varepsilon_{z4})$ for all sub-layers of "Upper Alluvium" with $OCR = 1$, but at $Point\ 2\,(\sigma'_{z2}, \varepsilon_{z2})$ for some sub-layers of "Upper Alluvium" when 8m layer when $OCR = 1.5\ or\ 2$ and 4m soil layer with $OCR = 2$. All these final effective stresses can be calculated using the parameter values in Table 1.

Thirdly, Eq. (3) is used after the stress increment of *20kPa* to determine "primary" consolidation final settlement $s_{fi,j}$, for each sub-layer *j* of 0.5m thick with $OCR=1$ of a soil type layer *i*. Since all the sub-layers of "Upper Marine Clay" and "Upper Alluvium" are at $Point\ 4\,(\sigma_{z4}^{'},\varepsilon_{z4})$ for final effective stress state, Eq. (3) is used for $s_{fi,j}$. When the final effective stress state is at $Point\,2\,(\sigma_{z2}^{'},\varepsilon_{z2})$, Eq. (2) is adopted for some sub-layers of "Upper Alluvium". The total final "primary" consolidation settlements of "Upper Marine Clay" and "Upper Alluvium" can be obtained by summing those of all sub-layers. Afterwards, the total final "primary" consolidation settlement S_{fi}, the *coefficient of volume compressibility*, m_{vi}, and the *coefficient of consolidation,* c_{vi}, for the whole "Upper Marine Clay" or the whole "Upper Alluvium" can be obtained as follows:

$$\begin{aligned} S_{fi} &= \sum_{j=1}^{m} s_{fi,j} \\ m_{vi} &= \frac{1}{H_i}\frac{S_{fi}}{\Delta\sigma_z^{'}} \\ c_{vi} &= \frac{k_{vi}}{m_{vi}\gamma_w} \end{aligned} \qquad (10)$$

where H_i is the total thickness of "Upper Marine Clay" or "Upper Alluvium". Calculated values of S_{fi}, m_{vi}, and c_{vi} for "Upper Marine Clay" and "Upper Alluvium" are listed in Table 2.

Fourthly, the factors for double soil layers, p and q, can be calculated with Eq. (5) by substituting the values of m_v and c_v, and the corresponding values are also listed in Table 2. Take $Case\,I(4m)$ with $OCR=1$ as an example: $H_1 = H_2 = 2m$, c_{v1}= 0.00122 m^2/day and c_{v2}= 0.02208 m^2/day in Table 2, the time factor, T, after a loading time of 100 days with one-way drainage condition could be determined as follows:

$$T = \frac{c_{v1}c_{v2}t}{\left(H_1\sqrt{c_{v2}} + H_2\sqrt{c_{v1}}\right)^2} = \frac{0.0012 \times 0.02208 \times 100}{4(\sqrt{0.02208} + \sqrt{0.0012})^2} = 0.02 \qquad (11)$$

From the solution charts for one-way drainage condition [25], the average degree of consolidation U_a is 18% for $p = -0.3$ and $q = 0.62$ (from solution charts in Zhu and Yin [22, 25]) and is 13% for $p = 0.3$ and $q = 0.62$. It is noted that $T_c = 0$ since the loading is suddenly applied. With the help of the interpolation method for $p = -0.22$ in Table 2, the average degree of consolidation U_a at time of 100 days and for $p = -0.22$ and $q = 0.62$ could be obtained:

$$U_a = \frac{[0.3-(-0.22)]\times 18\% + [(-0.22)-(-0.3)]\times 13\%}{[0.3-(-0.3)]} = 17.3\% \tag{12}$$

Similarly, the average degree of consolidation, U_a, for double soil layers in other different times or other conditions can also be determined.

In order to compare with the US Navy method [24], the average degree of consolidation, U_a, is also calculated by transferring the “Upper Alluvium” into “Upper Marine Clay” soil considering the difference of coefficient of consolidation, c_v, with Eq.(8). Then, the average degree of consolidation, U_a, could be easily determined as one equivalent single layer.

Lastly, for “Upper Marine Clay”, the creep compression $\varepsilon_{creep,f}$ is calculated by adopting Eq.(3) when the final effective stress state is in a normal consolidation state. Eq.(2) shall be used for calculating the creep compression $\varepsilon_{creep,f}$ when the final effective stress state of some sub-layers is in over-consolidation state. The equivalent times, t_{e2}, in Eq.(2) shall also be calculated first using the third row equation in Eeq.(2) for a few sub-layers of “Upper Alluvium” with 8m and listed in Table 3 for $OCR = 2$. Values of $t_{EOP,field}$ are determined to be the time when the average degree of consolidation is 98% for double soil layers.

5.2 Comparison of results from the new simplified Hypothesis B method, FE simulations, and Hypothesis A method

The finite element software Plaxis (2015 version) are used to simulate the same four cases of double soil layers and results are used to verify the accuracy of calculated results using the new simplified Hypothesis B method. Since the conventional Hypothesis A method is still used by some people. The limitations of this method are not well understood. It is also good to know the difference between the new simplified Hypothesis B method and conventional Hypothesis A method. Therefore, the conventional Hypothesis A method with US Navy method for the average degree of consolidation has also been used to calculate consolidation settlements of the same cases.

Curves of the new simplified Hypothesis B method are compared with curves from FE simulations with a SSC model and from Hypothesis A method in Figures 3~8 for different layer thickness and OCR values. Dot symbols are results from the Plaxis FE simulations. Solid lines represent calculation results of the new simplified Hypothesis B method with Zhu and Yin method for U_a. Dashed lines are the calculation results of the new simplified Hypothesis B method with US Navy method for U_a., Doted lines are the calculation results of the Hypothesis A method with US Navy method for U_a.

(a) $Case\,I(4m)$ and $Case\,II(4m)$

A comparison of FE simulation results with SSC model and the new simplified Hypothesis B method using Zhu and Yin method and US Navy method is shown in Figure 3 for 4m thick double soil layers with $OCR=1$ ($\alpha=0.6$). For $Case\,I(4m)$, it can be observed that calculated results of the new simplified Hypothesis B method using US Navy method and Zhu and Yin method for U_a are almost the same as illustrated in Figure 3(a) and are all very close to FE simulation results. The calculated curves with Zhu and Yin method for U_a are overlapped by those with US Navy method for U_a when the primary consolidation is completed. For $Case\,II(4m)$, there is an obvious gap between the calculated results using Zhu and Yin method for U_a and those adopting US Navy method for U_a. It is seen clearly from Figurer 3(b) that calculated curves of the new simplified Hypothesis B method using Zhu and Yin method for U_a are in a good agreement with FE simulation results. However, the obvious difference between FE simulation results and calculated results using US Navy method is observed during the consolidation stage. After the consolidation stage, results of the new simplified Hypothesis B method using both Zhu and Yin method and US Navy method are very close to FE simulation results. By comparing the results between the new simplified Hypothesis B method using Zhu and Yin method, US Navy method and FE simulation results, it can be deduced that US Navy method predicts the wrong average degree of consolidation, U_a, for double soil layers in $Case\,II(4m)$. It is seen from Figure 3 that Hypothesis A method gives much less settlement compared to results from the FE simulation and the new simplified Hypothesis B method.

Yin and Feng [3] defined the parameter, *relative error*, to evaluate the accuracy of the new simplified Hypothesis B method at a certain time *t*. The *relative error* is defined as:

$$relative\ error = \left|(S_{totalB} - S_{Plaxis})/S_{Plaxis}\right| \times 100\% \tag{13}$$

where S_{Plaxis} is the predicted settlement from Plaxis at time *t*. In this paper, we take two times at $U_a = 50\%$, $U_a = 80\%$ (from Hypothesis A method using US Navy method for U_a) and time of 100000 days. S_{totalB} is the total settlement calculated from the new simplified Hypothesis B method. Eq.(3) can also be used to calculate *relative error* for Hypothesis A method, in which S_{totalB} is replaced by S_{totalA}. As a result, values of *relative error* for all double soil layer conditions are listed in Table 4(a).

Figure 4 shows the comparison of settlement-log(time) curves from FE simulation, the new simplified Hypothesis B method, and Hypothesis A method for 4m double layers of soil profile with $OCR = 1.5$ ($\alpha = 0.7$). Figure 5 shows the comparison of settlement-log(time) curves from FE simulation, the new simplified Hypothesis B method, and Hypothesis A method for 4m double layers of soil profile with $OCR=2$ ($\alpha = 0.8$). Similar characteristics are observed for the new simplified Hypothesis B method with Zhu and Yin method and US Navy method for U_a in the two cases in Figures 4 and 5.

For 4m thick double soil layer with $OCR=1$, 1.5, and 2 it can be observed in Table 4(a) that the values of *relative error* are from 0.9% to 10.3% for the new simplified Hypothesis B method using Zhu and Yin method for U_a and from 0.7% to 35.5% for the new simplified Hypothesis B method using US Navy method for U_a. However, values of *relative error* are from 10.1% to 50.4% for the Hypothesis A method with US Navy method for U_a. The Hypothesis A method underestimates the consolidation settlement a lot.

(b) $Case\,III(8m)$ and $Case\,IV(8m)$

For 8m thick double layers of soil profile, Figures 6, 7, 8 show comparisons of curves from the FE simulation, the new simplified Hypothesis B method with Zhu and Yin method and US Navy method for U_a, and from Hypothesis A method with US Navy method for U_a for $OCR = 1, 1.5$ and 2, respectively. Characteristics of these curves are similar to those in Figures 3, 4, and 5. In overall, the curves from the new simplified Hypothesis B method with Zhu and Yin method for U_a are closer to the dot lines from the Plaxis FE simulations than those from other two simple methods. Again, Hypothesis A method underestimates the settlement a lot.

All values of the *relative error* are listed in Table 4(b). In both cases of $Case\,III(8m)$ and $Case\,IV(8m)$ with $OCR=1$, 1.5, and 2, it can be observed in Table 4(b) that the values of *relative error* are from 0.7% to 11.7% for the new simplified Hypothesis B method using Zhu and Yin method for U_a and 0.1% to 36.1% for the new simplified Hypothesis B method using US Navy method for U_a. The values of *relative error* are from

21.1% to 55.0% for the Hypothesis A method with US Navy method for U_a. Again, the Hypothesis A method underestimates the consolidation settlement a lot.

Some errors are caused by the approximation of US Navy method for estimating U_a. Zhu and Yin [25] found that errors of US Navy method are very significant in some simplification cases of double soil layers which are converted into one single soil layer. In order to predict the long term consolidation settlement as accurately as possible, Zhu and Yin method is recommended for calculating U_a of a double soil layer profile.

6. Conclusions

Based on Hypothesis B and the "equivalent time" concept [19, 20], a new simplified method is presented to calculate the consolidation settlement of double layers of soils with creep for different stress-strain states. Two idealized soil layers with different total thickness values (4m and 8m) and three different OCR values are considered for consolidation analysis to illustrate the applicability of this new simplified Hypothesis B method. Zhu and Yin method and US Navy method are adopted to obtain the average degree of consolidation for double soil layers. Four cases of the consolidation of the double soil profile have been analyzed using a Finite Element (FE) method with an elastic visco-plastic constitutive model, the new simplified Hypothesis B method, and Hypothesis A method. Results are presented and discussed. Main conclusions are drawn as follows:

(a) It is found that the curves from the new simplified Hypothesis B method adopting Zhu and Yin method for U_a are generally in good agreement with results from FE simulation. The *relative error* of this new method with Zhu and Yin method for U_a is from 0.9% to 8.5% for $Case\,I(4m)$ and $Case\,III(8m)$, and from 1.0% to 11.7% for $Case\,II(4m)$ and $Case\,III(8m)$.

(b) Curves from the new simplified Hypothesis B method adopting US Navy method for U_a are close to those from the new method with Zhu and Yin method U_a in $Case\,I(4m)$ and $Case\,III(8m)$ and those from the FE simulations. The differences and *relative errors* are big in $Case\,II(4m)$ and $Case\,IV(8m)$. The *relative error* of this new method adopting US Navy method for U_a is from 0.1% to 7.3% for $Case\,I(4m)$ and $Case\,III(8m)$, and from 0.5% to 36.1% for $Case\,II(4m)$ and $Case\,III(8m)$.

(c) The consolidation settlements are all underestimated by using Hypothesis A method adopting US Navy method for U_a for all cases. The *relative error* of the Hypothesis A method adopting US Navy method for U_a is from 10.1% to 55.0% for all four cases.

(d) According to the study in this paper, this new simplified Hypothesis B method adopting Zhu and Yin method for calculating the average degree of consolidation is the most accurate method for calculating the consolidation settlements of double layers of soils exhibiting creep.

Acknowledgement

The work in this paper is supported by a research grant (project No. 51278442) from National Natural Science Foundation of China (NSFC), a general research fund (GRF) (project No. PolyU 152196/14E) from Research Grants Council (RGC) of Hong Kong Special Administrative Region Government of China, grants (4-BCAW, G-YM28, G-YN97, B-Q43L) from The Hong Kong Polytechnic University, China.

References

1 Mitchell, K J, Soga. K. Fundamentals of soil behavior. 3th ed. *John Wiley & Sons, Inc., New York*. 2005.

2 Yin, J-H. From constitutive modeling to development of laboratory testing and optical fiber sensor monitoring technologies. *Chinese Journal of Geotechnical Engineering* 2011; **33**(1): 1-15.

3 Yin, J-H, Feng, W-Q. A new simplified method and its verification for calculation of consolidation settlement of a clayey soil with creep. *Canadian Geotechnical Journal*, 2016; in press.

4 Ladd, C. C. Stress-deformation and strength characteristics, state of the art report. *Proc. of 9th ISFMFE*, 1977; **4**: 421-494.

5 Mesri, G, Godlewski, P M. Closure of "Time and Stress-Compressibility Interrelationship". *Journal of the Geotechnical Engineering Division* 1979; 105(1).

6 Choi, Y-K. Consolidation behavior of natural clays. *University of Illinois at Urbana-Champaign*, 1982.

7 Feng, T-W. Compressibility and permeability of natural soft clays and surcharging to reduce settlements. *Diss. University of Illinois at Urbana-Champaign*, 1991.

8 Mesri G. and Vardhanabhuti, B. Closure of 'Secondary compression' by Mesri and Vardhanabhuti (2005). *J.Geotech. Geoenviron. Engng* 2006; **132**: 817-818.

9 Mesri, G. Effects of friction and thickness on long-term consolidation behavior of Osaka Bay clays. *Soils and Foundations* 2009; **49**(5): 823-824.

10 Bjerrum, L. Engineering geology of Norwegian normally-consolidated marine clays as related to settlements of buildings. *Géotechnique* 1967; **17**(2): 83-118.

11 Stolle, D F E, Vermeer P. A. and Bonnier P G. A consolidation model for a creeping clay. *Canadian geotechnical journal* 1999; **36**(4): 754-759.

12 Vermeer, P A, and Neher H P. A soft soil model that accounts for creep. *Proceedings of the International Symposium "Beyond 2000 in Computational Geotechnics10 Years of PLAXIS International*, Balkema. 1999: 249-261.

13 Nash, D F T, and Ryde S J. Modelling consolidation accelerated by vertical drains in soils subject to creep. *Géotechnique* 2001; **51**(3): 257-273.

14 Yin, J-H, Zhu, J-G, and Graham J. A new elastic visco-plastic model for time-dependent behaviour of normally and over-consolidated clays: theory and verification. *Canadian Geotechnical Journal* 2002; **39**(1): 157-173.

15 Leroueil, S. The Isotache approach. Where are we 50 years after its development by Professor Šuklje? 2006 Prof. Šuklje's Memorial Lecture. *Proceedings of the XIII Danube-European Conference on Geotechnical Engineering*, Ljubljana, Slovenia. 2006(2): 55-88.

16 Leoni, M, Karstunen, M, and Vermeer, P. A.. Anisotropic creep model for soft soils. *Géotechnique* 2008; **58**(3): 215-226.

17 Karim, M. R., et al. Predicting the long-term performance of a wide embankment on soft soil using an elastic-viscoplastic model. *Canadian Geotechnical Journal* 2010; **47**(2): 244-257.

18 Nash, D, and Brown M. Influence of Destructuration of Soft Clay on Time-Dependent Settlements: Comparison of Some Elastic Viscoplastic Models. *International Journal of Geomechanics* (2013): A4014004.

19 Yin, J-H, and Graham J. Viscous-elastic-plastic modelling of one-dimensional time-dependent behaviour of clays. *Canadian Geotechnical Journal* 1989; **26**(2): 199-209.

20 Yin, J-H, and Graham J. Equivalent times and one-dimensional elastic viscoplastic modelling of time-dependent stress-strain behaviour of clays. *Canadian Geotechnical Journal* 1994; **31**(1): 42-52.

21 Xie, K-H. Theory of One Dimensional Consolidation of Double-Layered Ground and its Applications [J]. *Chinese Journal of Geotechnical Engineering* 1994; **5**: 24-35.

22 Zhu, G-F, Yin, J-H. Consolidation of double soil layers under depth-dependent ramp load. *Géotechnique* 1999; **49**(3): 415-421.

23 Schiffmann, R. L., and Stein J R.. One-dimensional consolidation of layered systems. *Journal of Soil Mechanics & Foundations Div*. 1970: 1499-1504

24 US Department of the Navy. Soil mechanics design manual 7.1. NAVFAC DM-7.1. *US Department of the Navy*, Arlington, Va. 1982.

25 Zhu, G-F, and Yin, J-H. Solution charts for the consolidation of double soil layers. *Canadian geotechnical journal* 2005; **42**(3): 949-956.

26 Xie, K-H, Xie, X-Y and Gao X. Theory of one dimensional consolidation of two-layered soil with partially drained boundaries. *Computers and Geotechnics* 1999; **24**(4): 265-278.

27 Xie, K-H, Xie, X-Y, and Jiang, W. A study on one-dimensional nonlinear consolidation of double-layered soil. *Computers and Geotechnics* 2002; **29**(2): 151-168.

28 Tang, X-W and Onitsuka K. Consolidation of double-layered ground with vertical drains. *International Journal for Numerical and Analytical Methods in Geomechanics* 2001; **25**(14): 1449-1465.

29 Wang, X-S, and Jiao J J. Analysis of soil consolidation by vertical drains with double porosity model. *International Journal for Numerical and Analytical Methods in Geomechanics* 2004; **28**(14): 1385-1400.

30 Han, J, and Ye, S-L. A theoretical solution for consolidation rates of stone column-reinforced foundations accounting for smear and well resistance effects. *The International Journal Geomechanics* 2002; **2**(2): 135-151.

31 Wang, G-C. Consolidation of soft clay foundations reinforced by stone columns under time-dependent loadings. *Journal of geotechnical and geoenvironmental engineering* 2009; **135**(12): 1922-1931.

32 Zhou, W-H, Zhao, L-S and Li, X-B. A simple analytical solution to one-dimensional consolidation for unsaturated soils. *International Journal for Numerical and Analytical Methods in Geomechanics* 2014; **38**(8): 794-810.

33 Yin, J-H and Graham J. Elastic visco-plastic modelling of one-dimensional consolidation. *Géotechnique* 1996; **46**(3): 515-527.

34 Koutsoftas, Demetrious C., Roger Foott, and Handfelt Leo D.. Geotechnical investigations offshore Hong Kong. *Journal of Geotechnical Engineering* 1987; **113**(2): 87-105.

35 Handfelt, Leo D., Demetrious C. Koutsoftas, and Roger Foott. Instrumentation for test fill in Hong Kong. *Journal of geotechnical engineering* 1987; **113**(2): 127-146.

36 Degago, S. A., *et al.* Use and misuse of the isotache concept with respect to creep hypotheses A and B. *Géotechnique* 2011; **61**(10): 897-908.

37 Zhu, G-F, Yin, J-H and Graham J. Consolidation modelling of soils under the test embankment at

Chek Lap Kok International Airport in Hong Kong using a simplified finite element method. *Canadian Geotechnical Journal* 2001; **38**(2): 349-363.

38 Pyrah, I. C. One-dimensional consolidation of layered soils. *Géotechnique* 1996; **46**(3): 555-560.

39 Waterman, D, and Broere W. Practical application of the Soft Soil Creep model. Plaxis Bulletin 2011; **15**: 1-15.

40 Yin, J-H and Graham, J. (1999). Elastic visco-plastic modelling of the time-dependent stress-strain behavior of soils. Canadian Geotechnical Journal, Vol.36, No.4, pp.736-745.

41 Yin, J-H (2015), Fundamental Issues on Constitutive Modelling of the Time-dependent Stress-Strain Behaviour of Geomaterials. International Journal of Geomechanics, 15(5), A4015002, P1 to P9.