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A household optimum utility approach for modeling joint activity-travel choices in congested road networks

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Abstract

This study proposes a new household optimum (HO) utility approach to model the intra-household interactions between household members by heterogeneous household type with different size in deciding their daily joint/solo activities and travel in congested road networks. In contrast to the conventional approach based on selfish choices of individuals to maximize their own utility, the proposed approach considers the activity-travel choices of all household members to maximize their household utility. Based on the HO utility approach, a new household activity-based network equilibrium model is proposed to simultaneously take into account the time-dependent household daily activity-travel scheduling and traffic assignment problems within a unified modeling framework. Two new household-oriented network equilibrium principles, namely, HO and household-based system optimum (HSO), are introduced together with the formulations of their equivalent mathematical programming problems. The analytical relationships between HO, HSO, conventional user equilibrium and individual-based system optimum, and their properties are then investigated. The proposed model is formulated as an equivalent variational inequality problem and solved by a diagonalization method in a supernetwork platform. Numerical examples are provided to illustrate the merits of the proposed model, together with the key insights of the results that highlight the importance of considering joint activities and travel in travel demand forecasting and network design.

Keywords: joint activity-travel choices, intra-household interactions, household optimum, supernetwork

1. Introduction

1.1. Motivation

It has been widely acknowledged in the transportation literature that individuals often jointly conduct their daily activities and travel with other household members (Gliebe and Koppelman (2005); Bradley and Vovsha (2005); Bhat et al. (2013); Lin and Wang (2014); Lai et al. (2019)). Underlying the joint activities and travel of household members is the coordination and synchronization in time and space of the activity-travel schedules of the individuals involved (Zhang et al. (2005); Habib et al. (2008)). For example, cancellation of the personal activity of an individual after work may shift his/her preference for transit modes; he/she also might spend more time with other household members. Therefore, activity-travel scheduling should be considered from the household decision-making perspective rather than at the individual level because of possible intra-household interactions (Gupta and Vovsha (2013)).

Recognition of intra-household interactions has motivated a growing number of studies as found in two well-known special issues edited by Bhat and Pendyala (2005) and Timmermans and Zhang (2009), and two recent reviews by de Palma et al. (2014); Ho and Mulley (2015). Intra-household interactions have been also integrated into many operational activity-based travel demand systems, such as ALBATROSS (Arentze and Timmermans (2004a)), TASHA (Roorda et al. (2009)), ADAPTS (Auld and Mohammadian (2012)), CT-RAMP (Vovsha et al. (2011)), and MATSim (Dubernet and Axhausen (2015)). However, the activity-travel choices of individuals and their interactions in these systems are typically predicted by different separate sub-modes. Besides, to consider network congestion effects, these systems require external trip-based traffic assignment or simulation models in which feedback between activity-travel scheduling of individuals and traffic assignment is iteratively updated in a separate and ad hoc manner. This could lead to an inconsistent equilibrium solution.

Many researchers have investigated the integration of activity-based modeling and dynamic traffic assignment, referred to as the activity-based network equilibrium models, such as the models developed by Lam and Yin (2001); Lam and Huang (2002), the extensions of Recker (1995)'s model (Chow and Djavadian (2015); Liu et al. (2017)), or the models using the supernetwork approach (Ramadurai and Ukkusuri (2010); Liu et al. (2015); Li et al. (2018); Ouyang et al. (2011); Fu and Lam (2014); Fu et al. (2014); Fu and Lam (2018)). Nevertheless, in most of these existing studies, the

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activity-travel choices are determined independently for each individual. Not much effort has been paid to understanding how intra-household interactions, especially joint activity and travel participation, could affect the activity-travel scheduling of household members.

In this study, we propose a new household optimum (HO) utility approach to model the interactions between household members from heterogeneous household types with different sizes for making choices on daily joint/solo activities and travel in congested road networks. The proposed approach considers the activity-travel choices of all household members to maximize the household utility. As some empirical studies (see e.g., Bradley and Vovsha (2005); Lin and Wang (2014); Lai et al. (2019)) have shown the evidence of extra benefits for making intra-household interactions on joint activities and travel, the household utility includes the marginal utility of performing activities and cost of travel by mode of individual household members, and extra utility for joint activity participation and travel. Based on the HO utility approach, we propose a new household activity-based network equilibrium model that simultaneously takes into account the time-dependent household daily activity-travel scheduling and traffic assignment problems within a unified modeling framework.

1.2. Literature review

Given the broad scope of activity-based modeling literature, the main focus of this literature review is limited to activity-based models with intra-household interactions and activity-based network equilibrium models.

1.2.1. Activity-based models with intra-household interactions

Within activity-based models explicitly considering intra-household interactions, there is commonly an underlying assumption regarding how to deal with the possibly conflicting objectives of different household members. One of the most widely adopted approaches is the utility-maximizing approach in which household members make activity-travel choices to maximize their household utility, which commonly includes the individual-specific utility and the utility of intra-household interactions. This approach has been adopted in many time allocation/activity generation models (Srinivasan and Bhat (2005); Kato and Matsumoto (2009); Bhat et al. (2013); Bernardo et al. (2015); Lai et al. (2019)), or discrete choice models to explore various aspects of intra-household interactions, such as daily activity pattern types (Bradley and Vovsha (2005)), travel arrangement (mode choice and/or choice of ridesharing to joint activities or chauffeuring) (Gliebe and Koppelman (2005); Gupta et al. (2014); Weiss and Habib (2018)), heterogeneous group decision-making mechanisms (Zhang et al. (2009)), or synchronization of work tour departure and arrival times (Gupta and Vovsha (2013)). Nevertheless, none of these models account for full-day household activity-travel schedules as they are commonly embedded in an activity-based microsimulation framework (e.g., MATSim (Balmer et al. (2006))) or serve as hierarchical sub-models in an existing activity-based system (e.g., CT-RAMP (Vovsha et al. (2011))) to generate full-day schedules.

Another class of activity-based models using the utility-maximizing approach is based on optimization algorithms to generate full-day household activity-travel schedules. These include Recker (1995)'s model and its extensions (Gan and Recker (2008, 2013); Kang and Recker (2013); Chow and Nurumbetova (2015)), which formulate the household activity-travel scheduling problem as a pickup and delivery problem with time windows. Other types of algorithms can also be adopted. For instance, Meister et al. (2005) propose a genetic algorithm to generate household daily schedules. Liao et al. (2013b); Liao (2019) use a multistate supernetwork approach, which converts the activity-travel scheduling problem into a shortest path-finding problem, to model joint travel decisions in a multi-modal system. Their proposed supernetwork for modeling joint travel is lied on the earlier comprehensive work of the same authors on scheduling individual activity-travel choices (see e.g., Liao et al. (2013a); Liao (2016)). However, these aforementioned studies assume that the trip chain and/or activity duration are given and fixed. Besides, joint activity participation choices of household members are not explicitly modeled.

There are also several other approaches beyond the utility-maximizing approach to model intra-household interactions through an agent-based micro-simulation framework, such as ALBATROSS (Arentze and Timmermans (2004a)), TASHA (Roorda et al. (2009)), ADAPTS (Auld and Mohammadian (2012)), and MATSim (Dubernet and Axhausen (2015)). In these models, activity-travel choices of individuals are generated using various approaches, including computational process, production rules, and/or utility maximization to handle complexity of temporal-spatial constraints in intra-household interactions. The activity-travel scheduling processes are typically based on the concept of skeleton schedules with given and fixed attributes, such as activity starting time, duration, destination and location (Habib et al. (2008)).

1.2.2. Activity-based network equilibrium models

Many activity-based traffic assignment models simultaneously consider activity-travel scheduling and trip assignment, such as those developed by Lam and Yin (2001); Lam and Huang (2002) or the extensions of Recker (1995)'s model (see e.g., Chow and Djavadian (2015); Liu et al. (2017)). Although these studies provide a better understanding of the interactions between the activity-travel scheduling behavior of travelers and network congestion effects, they do not fully examine the interdependency of activity and travel choices as well as intra-household interactions. In these models, the trip chain and/or activity duration are, to some extent, fixed and given. Some researchers (e.g., Li et al. (2014, 2017); Cantelmo and Viti (2018)) revisit the bottleneck model from an activity-based approach perspective with the focus on activity duration and scheduling utility. These activity-based bottleneck models can provide significant new insights into the activity scheduling utilities of travelers with a closed-form solution; they are, however, restricted to a specific network with one origin-destination (OD) pair and activity scheduling at the individual level.

To simultaneously model multi-dimensional choice facets of activity-travel scheduling with trip assignment, various activity-based network equilibrium models have adopted the supernetwork approach, such as activity-travel networks (Ramadurai and Ukkusuri (2010)), multistate supernetworks (Liu et al. (2015); Li et al. (2018)), and activity-time-space (ATS) supernetworks (Ouyang et al. (2011); Fu and Lam (2014); Fu et al. (2014)). In these models, the feasible activity-travel choices of an individual are consistently represented by a path in the supernetwork. The column generation method is commonly adopted to generate the path set. These models, however, only focus on activity-travel choices at the individual level and ignore the possible intra-household interactions.

Fu and Lam (2018) recently develop an activity-based network equilibrium model in a joint-activity-time-space (JATS) supernetwork platform for modeling the joint activity-travel path (JATP) choices of two household members. Nevertheless, their model only considers the joint activity and travel choices of full-time worker-couple households using public transit. The household activity-travel schedule is restricted to at most one joint travel episode before and one after work, and only one joint activity (i.e., shopping) episode after work. These limitations could hinder the applicability of their model in practice.

1.3. Contributions

The main contributions of this paper from the perspectives of methodological and theoretical development are described below.

For the methodological development, we propose a new household activity-based network equilibrium model for a time-dependent household activity-travel scheduling problem for heterogeneous household types with different sizes in congested road networks. The choice facets of a household daily activity-travel schedule includes the choices of all members in the household for activity participation (activity type, location, starting time and duration choices), the choices for travel between activity locations (transportation mode, i.e., either private car or public transit, departure time and path choices), and the choices for intra-household interactions (joint activity participation, car allocation and ridesharing choices, i.e., either ridesharing to a joint activity or chauffeuring). To the best of our knowledge, we are the first in the literature to model such choice facets simultaneously with traffic assignment. The activity-travel choices of the household daily schedule are then represented by a unified joint-activity-travel path (JATP) choice in a proposed joint-activity-time-space (JATS) supernetwork platform. The proposed model is formulated as an equivalent variational inequality (VI) problem and solved by a diagonalization method that converts the time-dependent household activity-travel scheduling problem into an equivalent static traffic assignment problem on the supernetwork platform. Based on the proposed supernetwork platform, any of the conventional path-finding algorithms, e.g., Dijkstra's algorithm, can be adopted to generate the JATP choice set with use of the column generation method.

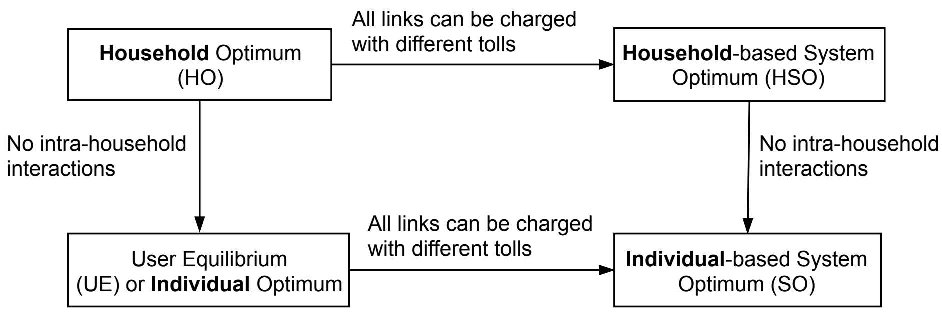


Figure 1: The relationships between the HO, HSO, UE and SO principles

For theoretical development, we introduce two new household-oriented network equilibrium principles based on HO and household-based system optimum (HSO) together with the formulations of their equivalent mathematical programming (MP) problems. We then investigate the analytical relationships between HO, HSO, conventional user equilibrium (UE), and individual-based system optimum (SO), together with their properties. Fig. 1 summarizes the relationships between the HO, HSO, UE, and SO principles, in which UE (or individual optimum) refers to the selfish choices of travelers to maximize their own utility, HSO refers to the full cooperation (in terms of departure-time choices and intra-household interactions) of all travelers to maximize the system social benefit, and HO is intermediate between UE and HSO in that there is a certain coordination (i.e., only intra-household interactions) among travelers from the same household, while SO is a special case of HSO without intra-household interactions. These two coherent household-oriented network equilibrium principles (HO and HSO) can provide new insights on the Braess paradox for evaluation of new transport policies and alternative road toll schemes. The conventional UE and SO solutions may lead to biased results when there is strong interaction between household members with respect to their daily activity scheduling and travel choices.

Compared to Fu and Lam (2018)'s model, we provide a comprehensive modeling framework to investigate the household interactions with the generalized model formulation and the associated properties. First, our proposed HO and HSO models can explicitly take into account multiple joint activity and travel episodes in daily activity-travel schedules of heterogeneous household types with more than two persons, together with additional car allocation and ridesharing choices of private car users. This facilities the applicability of our model for more generalized intra-household interactions on joint activity and travel choices of different household types using different transport modes (public transit or private car). Second, the model relationships and properties between HO, HSO, UE and SO are overlooked by Fu and Lam (2018) while the equivalent MP problems derived in our study allow us to solve the problem more efficiently. Third, we also provide a benchmark comparison of HO, HSO, UE and SO results with key findings that highlight the importance of considering joint activities and travel in travel demand forecasting and network design problems.

2. Basic considerations

2.1. Model assumptions

To present the main ideas, we make the following assumptions:

A1 The proposed model falls within the category of models for *long-term planning* at a strategic level. Travelers have perfect knowledge of traffic conditions throughout the whole road network (Ouyang et al. (2011); Fu and Lam (2014, 2018)).

A2 The daily time period $[0, T]$ is discretized into a finite set of constant time intervals $k = 1 \dots K$ with duration σ such that $\sigma K = T$ (Lam and Yin (2001); Lam and Huang (2002); Ouyang et al. (2011); Fu and Lam (2014, 2018)).

A3 A traveler conducts his/her trip between each origin-destination (OD) pair using one of two modes, i.e., either private car or public transit, and under one of four roles, i.e., solo driver (SD), ridesharing driver (RD), or ridesharing passenger (RP) when using a private car, or transit passenger (TP) when using public transit. On different trip legs of the trip chain, a driver can interchange between the SD and RD roles while a passenger chooses between the RP and TP roles. Fig. 2 illustrates the feasible mode choice and travel role on each trip of the trip chains of a 3-person household with a worker couple and a school child. The artwork is borrowed from Gliebe and Koppelman (2005). In this example, the child (with trip chain H-S-H) is escorted to school by the husband (with trip chain H-W2-H) by a private car while he/she returns home with the wife (with trip chain H-W1-H) by public transit. Note that a joint trip by private car users can only happen with the participation of one RD and at least one RP. These feasible mode choices and travel roles on separate trips of the trip chains are not given but generated endogenously in the proposed model thanks to the supernetwork platform.

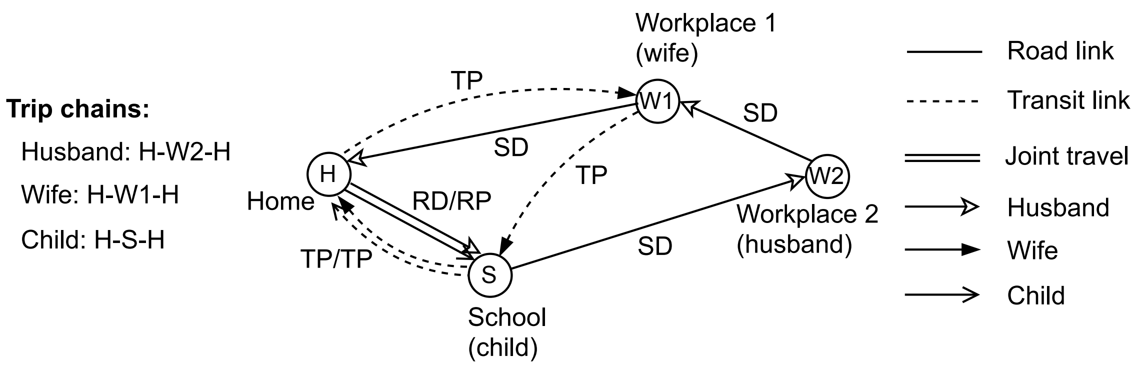


Figure 2: Feasible mode choice and travel role on each trip of the trip chains of a 3-person household

A4 An RD may pick up or drop off RPs of the same household at pre-specified locations, and car capacity is always sufficient for all household members. In addition, parking costs and restrictions on parking and activity capacity are excluded.

A5 Without loss of generality of the proposed model's capability, it is assumed that each OD pair is connected by public transit via a single *dummy link* whose travel cost reflects the transit level of service between the OD pair. The transit cost can be interpreted as the excess-demand function in elastic-demand traffic assignment problems (Florian (1977); Gartner (1980); Sheffi (1985); Ryu et al. (2017)). The transit demand then reflects the shift of private car users to public transit in response to congestion and operational changes in the road network. To present the main ideas of the household ridesharing problem, the transit cost (a function of the transit waiting, walking and in-vehicle times) is assumed to be *constant* for a given OD pair but to vary among OD pairs. This assumption is valid in situations where transit vehicles move on dedicated lanes (e.g., for metro rail or bus rapid transit) and where the transit capacity is large enough that congestion effects on transit lines are negligible (Florian (1977); Sheffi (1985);

Ryu et al. (2017)). Such a constant excess-demand function has also been adopted widely in the literature for other transportation problems, particularly for automobile-oriented cities in North America, e.g., Zhou et al. (2006); Sohn (2011); Xie and Duthie (2015).

A6 Road congestion is taken into account, and the travel time by private car for each link is separable and strictly increasing with the vehicle flow for that link (Lam and Yin (2001); Lam and Huang (2002); Liu et al. (2015)). The first-in-first-out condition, which prohibits private cars from arriving at a destination earlier by leaving later, is not considered (Janson (1991); Chen and Hsueh (1998); Boyce et al. (2001)).

The rest of the paper is organized as follows. The model assumptions and some useful concepts, such as JATP and its utility, are discussed in Sec. 2. The proposed household activity-based network equilibrium model and its formulation are presented in Sec. 3. This is followed by a solution method in Sec. 4. Numerical results are used to highlight the merits of the proposed model together with the key insights in Sec. 5. Finally, in Sec. 6, conclusions are given together with recommendations for further study.

2.2. *Daily joint activity-travel path*

We extend the concept of JATP, introduced by Fu and Lam (2018), to simultaneously represent the multi-dimensional choice facets of the daily joint/solo activity-travel choices of all members in a same household and their interactions. Fig. 3 illustrates the choices included in the JATP choice of a 3-person household with a worker couple and a schoolchild whose mode choices and travel roles on separate trips of the trip chains are shown in Fig. 2. Note that for illustration purposes, there is only one path for each OD pair between two activity locations; and the period from 9:00 to 15:00 during which household members stay at work or school is discarded. Obviously, we can trace all of the daily joint/solo activity-travel choices of household members and their interactions via a unified JATP choice.

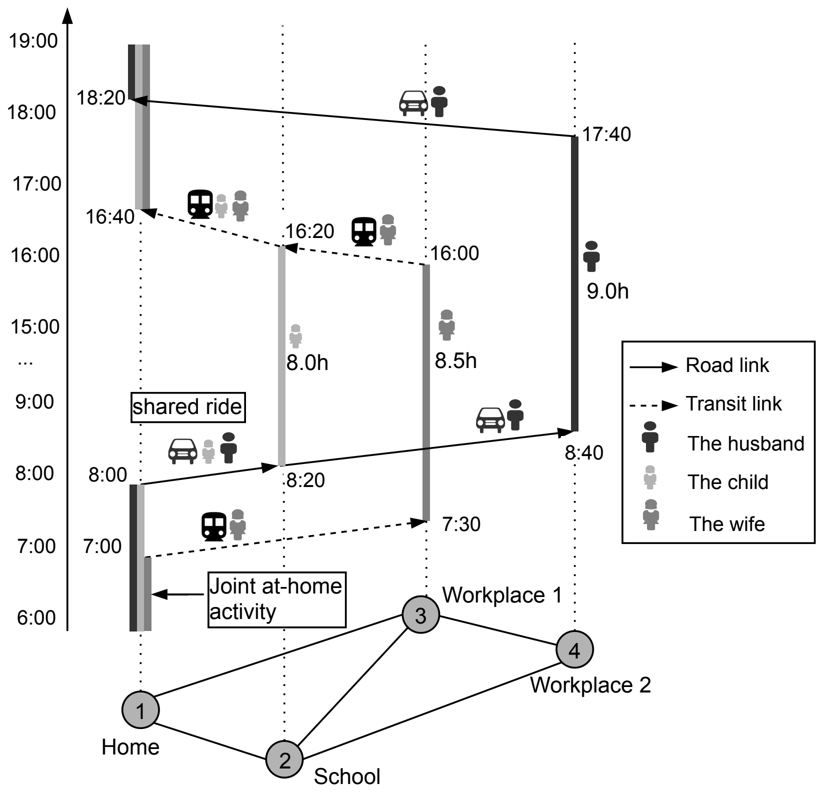


Figure 3: A JATP choice of a 3-person household

2.3. *Road network with heterogeneous household types*

Consider a road network (base network) $B(S, A)$ where S and A are the sets of nodes and links, respectively. A node $s \in S$ could be a road intersection or a location for activities or drop-off/pickup. A link $a \in A = A_1 \cup A_2$ represents either a general-purpose or a high-occupancy toll (HOT) lane (i.e., tolled lanes with no charge for high-occupancy vehicles) on a directed road link. Let I denote the set of activities, W a set of OD pairs connecting two locations for activities or drop-off/pickup, and P^w the set of feasible paths for private cars between OD pair w.

Consider multiple heterogeneous household types $h = 1 \dots H$, where each household type h is associated with multiple persons $m = 1 \dots M^h$, a household travel demand $D^h > 0$ (the number of households with type h), and a set of JATP choices

Q^h. Household type h represents D^h specific households with the same household demographic characteristic, such as household size, home location, workplace, and car ownership. Person m of household type h can take part in activities and travel in multiple groups $g = 1 \ldots G^h$ in which each group g is a set of persons from the same household with group size $|g|$ (the number of persons in the group).

2.4. JATP utility

Let u_q^h denote the daily net utility for a household of type h choosing JATP q, calculated as the difference between the total utility of activities and the travel cost (by private car and public transit) for all members in the household during a day. It is given by

$$u_q^h = \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} \sum_{k=1}^{K} \left(\underbrace{\sum_{i\in I} \sum_{s\in S} u_{is}^{hmg}(k)\delta_{qis}^{hmg}(k)}_{\text{activity utility}} - \underbrace{\sum_{w\in W} \sum_{p\in P^w} c_{pw}^{hmg}(k)\xi_{qpw}^{hmg}(k)}_{\text{travel cost by private car}} - \underbrace{\sum_{w\in W} \hat{c}_w(k)\hat{\xi}_{qw}^{hmg}(k)}_{\text{travel cost by public transit}} \right) \quad \forall q, h, \tag{1}$$

where $u_{is}^{hmg}(k)$ is the utility for person m in group g of household type h conducting activity i at location s during interval k, $c_{pw}^{hmg}(k)$ the travel cost for person m in group g of household type h entering path p between OD pair w by private car (taking either the SD, RD, or RP role) during interval k, and $\hat{c}_w(k)$ the transit travel cost for departing from the origin of OD pair w during interval k.

In Eq. (1), $\delta_{qis}^{hmg}(k)$ equals 1 if person m in group g of household type h choosing JATP q performs activity i at location s during interval k and 0 otherwise, $\xi_{qpw}^{hmg}(k)$ equals 1 if person m in group g of household type h choosing JATP q enters path p between OD pair w by private car during interval k and 0 otherwise, and $\hat{\xi}_{qw}^{hmg}(k)$ equals 1 if person m in group g of household type h choosing JATP q uses public transit for departing from the origin of OD pair w during interval k and 0 otherwise.

2.4.1. Activity utility

Empirical studies show that the utility of an activity depends on its location and time of day (Ettema and Timmermans (2003); Ettema et al. (2007)), and varies depending on the group of persons taking part in the activity (Zhang et al. (2005, 2009)). Studies also show that household members can gain extra utility for conducting activities together (Bradley and Vovsha (2005); Lai et al. (2019)). The utility for person m in group g of household type h performing activity i at location s during interval k is formulated as

$$u_{is}^{hmg}(k) = \left(1 + \alpha_{is}^{hg}(k)\right) u_{is}^{hm}(k) \quad \forall i, s, g, h, k, \tag{2}$$

where $u_{is}^{hm}(k)$ is the marginal utility for person m of household type h conducting activity i alone at location s during interval k, and $\alpha_{is}^{hg}(k)$ the household preference parameter for joint activity participation by group g of household type h in activity i at location s during interval k:

$$\begin{cases} \alpha_{is}^{hg}(k) = 0 & \text{if } |g| = 1 \\ \alpha_{is}^{hg}(k) \geq 0 & \text{if } |g| > 1 \end{cases} \quad \forall i, s, g, h, k. \tag{3}$$

The positive sign of $\alpha_{is}^{hg}(k)$ implies the preference of household members for joint activity participation, and the extra utility is a factor of $\alpha_{is}^{hg}(k)$.

The marginal utility in Eq. (4) can be formulated as the following bell-shaped function (Ettema and Timmermans (2003); Ashiru et al. (2004)):

$$u_{is}^{hm}(k) = \bar{u}_{is}^{hm} + \int_{\sigma k}^{\sigma(k+1)} \left(\frac{\rho_i^{hm} \upsilon_i^{hm} \tilde{u}_{is}^{hm}}{\exp\left[\rho_i^{hm}\left(t - \tilde{t}_i^{hm}\right)\right] \left\{1 + \exp\left[-\rho_i^{hm}\left(t - \tilde{t}_i^{hm}\right)\right]\right\}^{\upsilon_i^{hm}+1}} \right) \mathrm{d}t \quad \forall h, m, i, s, k, \tag{4}$$

where $\bar{u}_{is}^{hm}$ and $\tilde{u}_{is}^{hm}$ are respectively the baseline utility and the total utility of person m of household type h performing activity i at location s, $\tilde{t}_i^{hm}$ determines the time at which the marginal utility reaches its maximum value, ρ_i^{hm} and υ_i^{hm} are calibrated parameters. Fig. 4 illustrates the marginal utility functions for the typical home and working activities by time of day of a worker couple (1.0 USD = 7.8 HKD). The gray area below the curve shows the maximum utility for the husband, who starts the work at 9:00 a.m. and works for 8 hours.

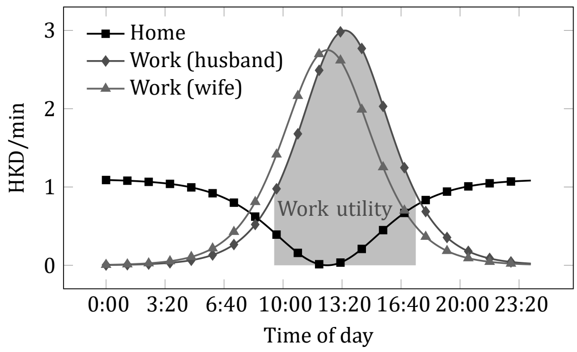


Figure 4: The marginal utility functions for home and working activities of a worker couple

2.4.2. Travel cost

The travel cost for person m in group g of household type h entering path p between OD pair w by private car during interval k is given by

$$c_{pw}^{hmg}(k) = \sum_{a \in A} \sum_{l=k}^{K} c_a^{hmg}(l) \zeta_{ap}^{wl}(k) \quad \forall p, w, g, m, h, k, \tag{5}$$

where $c_a^{hmg}(l)$ is the travel cost for person m in group g of household type h traversing link a by private car during interval l, and $\zeta_{ap}^{wl}(k)$ equals 1 if the private car entering path p between OD pair w during interval k arrives at link a during interval l and 0 otherwise. Eq. (5) is the sum of link costs taking into account the interval during which the private car enters each link along the path (Janson (1991); Chen and Hsueh (1998); Boyce et al. (2001)).

Unlike solo trips, private car users can divide the operating cost (e.g., toll and fuel cost) among themselves for taking part in a joint trip, but they cannot do the same with the travel time as all of them experience the same travel time for the trip (Daganzo (1982)). Thus, the link cost in Eq. (5) includes two components: the travel time cost, which cannot be divided, and the operating cost, which can be divided among individuals. In particular,

$$c_a^{hmg}(k) = \underbrace{\gamma \left(1 - \beta_a^{hg}(k)\right) t_a(k)}_{\text{travel time cost}} + \underbrace{\frac{1}{|g|} \phi_a^{hg}(k)}_{\text{shared operating cost}} \qquad \forall a, g, m, h, k, \tag{6}$$

where γ is the value of travel time for each private car user, $\beta_a^{hg}(k)$ and $\phi_a^{hg}(k)$ respectively the household preference parameter for joint travel and the operating cost for group g of household type h traversing link a by private car during interval k, $t_a(k)$ the travel time for private cars on link a during interval k. For the sake of simplicity, we assume that the operating cost is constant. Note that it is straightforward to relax this assumption to the flow-dependent case.

The household preference parameter for joint travel in Eq. (6) is given by

$$\begin{cases} \beta_a^{hg}(k) = 0 & \text{if } |g| = 1 \\ 0 \le \beta_a^{hg}(k) < 1 & \text{if } |g| > 1 \end{cases} \quad \forall a, g, m, h, k. \tag{7}$$

The positive sign of $\beta_a^{hg}(k)$ implies the preference of household members for joint travel, and the extra utility (or the reduced travel time cost) for household members spending travel time together is a factor of $\beta_a^{hg}(k)$. Such a household preference for joint travel could result from the psychological and demographic characteristics of the household members (Srinivasan and Bhat (2008); Lin and Wang (2014)). For example, some household members may value companionship more than others, or one spouse may depend on the other for rides (Gliebe and Koppelman (2005)). We can also interpret $\gamma(1 - \beta_a^{hg}(k))$ in Eq. (6) as the reduced value of travel time for each person in the case of joint travel. Note that $\beta_a^{hg}(k) < 1$ ensures a nonnegative travel time cost.

2.5. Household optimum

In this study, individuals conduct activity-travel choices to maximize their household daily net utility, which leads to the following Household Optimum (HO) condition:

Definition 1. *At equilibrium, no household can improve daily net utility by changing its JATP choice to any other feasible JATP choice for that household type. Under this condition,*

$$\begin{cases} u_q^h - \mu^h = 0 & \text{if } f_q^h > 0 \\ u_q^h - \mu^h \le 0 & \text{if } f_q^h = 0 \end{cases} \quad \forall q, h, \tag{8}$$

where μ^h is the maximum daily net utility at equilibrium for household type h, and f_q^h is the flow (number of households) for household type h choosing JATP q.

3. Model formulation

3.1. Variational inequality problem

The HO problem (8) can be written as the following VI formulation problem (see e.g., Smith (1979)): finding a JATP flow vector $\mathbf{f}^* \in \Omega$ such that

$$\sum_{h=1}^{H} \sum_{q \in Q^h} u_q^{h*} \left(f_q^{h*} - f_q^h\right) \geq 0 \quad \forall \mathbf{f} \in \Omega, \tag{9}$$

where $*$ denotes equilibrated values, $\mathbf{f} = (\dots, f_q^h, \dots)^T$, and $\Omega = \{\mathbf{f} : (10){-}(11)\}$ is the feasible region for JATP flows, defined by the following constraints:

First, to conserve the number of households in the network, the following flow conservation constraints apply:

$$\sum_{q \in Q^h} f_q^h = D^h \quad \forall h. \tag{10}$$

where the demand for each household type, i.e., D^h, is given and fixed.

Second, the following nonnegative constraints prevent negative JATP flows:

$$f_q^h \geq 0 \quad \forall q, h. \tag{11}$$

Third, the following flow propagation (or network loading) constraints are applied to ensure the consistent movements of travelers forward in space and time through activity locations, road links, and transit dummy links during a day:

$$v_{is}^{hmg}(k) = \sum_{q \in Q^h} f_q^h \delta_{qis}^{hmg}(k) \quad \forall i, s, g, m, h, k, \tag{12}$$

$$v_a^{hmg}(k) = \sum_{q \in Q^h} \sum_{w \in W} \sum_{p \in P^w} \sum_{l=1}^{k} f_q^h \xi_{qpw}^{hmg}(l) \zeta_{ap}^{wk}(l) \quad \forall\, a, g, m, h, k, \tag{13}$$

$$e_w(k) = \sum_{h=1}^{H} \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} \sum_{q \in Q^h} f_q^h \hat{\xi}_{qw}^{hmg}(k) \quad \forall\, w, k, \tag{14}$$

where $v_{is}^{hmg}(k)$ is the flow for person m in group g of household type h performing activity i at location s during interval k, $v_a^{hmg}(k)$ the flow for person m in group g of household type h traversing link a by private car during interval k, and $e_w(k)$ the transit passenger flow departing from the origin of OD pair w during interval k. The above constraints ensure the feasibility of time–space trajectories of travelers through JATPs via the use of 0–1 integer variables $\delta_{qis}^{hmg}(k)$, $\xi_{qpw}^{hmg}(k)$, $\hat{\xi}_{qw}^{hmg}(k)$, and $\zeta_{ap}^{wk}(l)$. These constraints are not stationary but depend on the (continuous) path and link travel times during these intervals. For the relationship of the 0–1 integer variables to the path and link travel times, readers can refer to Appendix A.

Finally, the definitional constraints for vehicle and person flows of private car users on road links are as follows:

$$\tilde{v}_a(k) = \sum_{h=1}^{H} \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} \frac{1}{|g|} v_a^{hmg}(k) \quad \forall a, k, \tag{15}$$

$$v_a(k) = \sum_{h=1}^{H} \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} v_a^{hmg}(k) \quad \forall a, k. \tag{16}$$

3.2. Mathematical programming problem

In this section, we express the HO problem (8) as an equivalent MP problem. The use of an equivalent MP problem is also effective for investigating the analytical relationships between HO, HSO, UE, and individual-based SO.

First, we examine the partial derivative, or Jacobian matrix, of the vector of link costs (6) for private car users on link a during interval k, $\mathbf{c}_a(k) = (\dots, c_a^{hmg}(k), \dots, c_a^{jnz}(k), \dots)^T$, and obtain that

$$\nabla \mathbf{c}_a(k) = \begin{pmatrix} \ddots & \vdots & \ddots & \vdots & \ddots \\ \cdots & \frac{\partial c_a^{hmg}(k)}{\partial v_a^{hmg}(k)} & \cdots & \frac{\partial c_a^{hmg}(k)}{\partial v_a^{jnz}(k)} & \cdots \\ \ddots & \vdots & \ddots & \vdots & \ddots \\ \cdots & \frac{\partial c_a^{jnz}(k)}{\partial v_a^{hmg}(k)} & \cdots & \frac{\partial c_a^{jnz}(k)}{\partial v_a^{jnz}(k)} & \cdots \\ \ddots & \vdots & \ddots & \vdots & \ddots \end{pmatrix}. \tag{17}$$

We can verify that the Jacobian matrix $\nabla \mathbf{c}_a(k)$ is not symmetric by

$$\frac{\partial c_a^{hmg}(k)}{\partial v_a^{jnz}(k)} \neq \frac{\partial c_a^{jnz}(k)}{\partial v_a^{hmg}(k)} \quad \forall h, m, g, j, n, z, a, k \quad \text{for} \quad |g| \neq |z| \tag{18}$$

due to $\frac{\partial c_a^{hmg}(k)}{\partial v_a^{jnz}(k)} = \frac{\gamma}{|z|}\left(1 - \beta_a^{hg}(k)\right)\frac{\partial t_a(k)}{\partial \tilde{v}_a(k)}$. Because of the asymmetric interactions among private car users, we can regard the HO problem (8) as a multiclass-users network equilibrium problem with asymmetric travel costs. If the Jacobian matrix is not symmetric, there does not exist an equivalent convex minimization problem (Dafermos (1971, 1972)). Using a normalization procedure, Lam and Huang (1992) converted the original asymmetric link cost functions to symmetric forms, and hence formulated an equivalent convex MP model. We adopt a similar procedure in this study.

Consider the modified travel cost function for person m in group g of household type h traversing link a by private car during interval k:

$$\bar{c}_a^{hmg}(k) = \frac{\gamma}{|g|} t_a(k) + \gamma\left(1 - \beta_a^{hg}(k) - \frac{1}{|g|}\right)\bar{t}_a(k) + \frac{1}{|g|}\phi_a^{hg}(k) \quad \forall a, g, m, h, k, \tag{19}$$

where $\bar{t}_a(k)$ is the *estimated* travel time by private car at equilibrium on link a during interval k, i.e., $\bar{t}_a(k) = t_a(k)^*$. We can verify that (i) the link cost function (19) ensures the symmetry of the Jacobian matrix (17), and (ii) it will equals the function (6) at equilibrium. Thus, the link cost function (19) can replace the function (6) without changing the equilibrium results. Using the link cost function (19), we can now derive the equivalent MP problem for HO.

Consider the MP problem for HO: finding a flow vector $\mathbf{v}^* = (\dots, v_{is}^{hmg}(k)^*, v_a^{hmg}(k)^*, e_w(k)^*, \dots)^T$ such that

$$\begin{aligned} \max Z(\mathbf{v}) = &\sum_{h=1}^{H}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{k=1}^{K}\sum_{i\in I}\sum_{s\in S}\left(1 + \alpha_{is}^{hg}(k)\right)u_{is}^{hm}(k)v_{is}^{hmg}(k) - \sum_{k=1}^{K}\sum_{w\in W}\hat{c}_w(\omega)e_w(k) \\ &- \sum_{k=1}^{K}\sum_{a\in A}\int_0^{\tilde{v}_a(k)}\gamma t_a(\omega)\mathrm{d}\omega - \sum_{h=1}^{H}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{k=1}^{K}\sum_{a\in A}\left(\gamma\left(1 - \beta_a^{hg}(k) - \frac{1}{|g|}\right)\bar{t}_a(k) + \frac{1}{|g|}\phi_a^{hg}(k)\right)v_a^{hmg}(k) \end{aligned} \tag{20}$$

subject to (10)–(16).

Proposition 1. *The solution to the MP problem* (20) *corresponds to the HO condition* (8)*.*

In the objective function (20), the first term is the total utility of activity participation in the system. The second term is the total travel cost of transit passengers. The integral term (with respect to the vehicle flow of private car users) is to equilibrate the monetary value of the total travel time of private cars (or private car drivers). This is because only private car drivers affect road link travel times while private car passengers do not. The last term includes the monetary value of the total travel time of private car passengers and the total operating cost of private cars (e.g., fuel cost and toll shared among private car drivers and passengers).

Note that $\delta_{qis}^{hmg}(k)$, $\xi_{qpw}^{hmg}(k)$, $\hat{\xi}_{qw}^{hmg}(k)$, and $\zeta_{ap}^{wl}(k)$ in constraints (10)–(14) are not just indexes but are dependent on the path and link travel times (see constraints (42a)–(42e)). We can only derive the HO condition (8) from the MP problem (20) under a fixed flow propagation relationship given constraints (10)–(14) (Janson (1991); Chen and Hsueh (1998); Boyce et al. (2001)). When the fixed flow propagation relationship is also the relationship realized at equilibrium, the sub-problem leads to the HO condition (8).

3.3. Network equilibrium relationships

Based on the MP problem (20), we now derive the analytical relationships of HO with HSO, UE, and SO, together with their properties. First, we investigate the relationship between HO and UE. Proposition 2 shows that HO includes UE as a special case where there are no intra-household interactions.

Proposition 2. *The solution to the HO problem* (20) *corresponds to the UE problem where there are no intra-household interactions.*

Let $\tilde{Z}(\mathbf{v})$ be the total net utility of all households in the system under flow vector $\mathbf{v}$:

$$\tilde{Z}(\mathbf{v}) = \sum_{h=1}^{H}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{k=1}^{K}\sum_{i\in I}\sum_{s\in S}\left(1+\alpha_{is}^{hg}(k)\right)u_{is}^{hm}(k)v_{is}^{hmg}(k) - \sum_{k=1}^{K}\sum_{w\in W}\hat{c}_w(\omega)e_w(k)$$
$$- \sum_{h=1}^{H}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{k=1}^{K}\sum_{a\in A}\left(\gamma\left(1-\beta_a^{hg}(k)\right)t_a(k) + \frac{1}{|g|}\phi_a^{hg}(k)\right)v_a^{hmg}(k). \tag{21}$$

We can rewrite the total net utility at equilibrium with respect to the maximum net utilities of the household types:

$$\tilde{Z}(\mathbf{v}) = \sum_{h=1}^{H} D^h\mu^h. \tag{22}$$

The HO condition (8) is only applied within each household type. In a system with only one homogeneous household type, the HO condition always ensures that at equilibrium the net utility of each chosen JATP is maximized or its corresponding flow is zero. Because the UE solution is one of special cases of the HO solution in which each individual has no interactions with other household members, the maximum net utility for each household under HO is always greater than or equal to that under UE. Following Eq. (22), we always have $\tilde{Z}(\mathbf{v}^{\text{ho}}) \geq \tilde{Z}(\mathbf{v}^{\text{ue}})$ where $\mathbf{v}^{\text{ho}}$ and $\mathbf{v}^{\text{ue}}$ are the flow vectors under HO and UE, respectively.

In a system with many heterogeneous household types, the HO condition requires only that the net utilities within each household type are *equilibrated*, but the maximum net utility at equilibrium for a given household type can be influenced (improved or worsened) by the choices of other household types. Thus, we cannot compare the maximum net utilities under HO and UE for each household type because of the asymmetric interactions of different household types through their travel costs (see Eq. (18)). Toint and Wynter (1996) reported such a phenomenon in multiclass-users traffic assignment problems. Following Eq. (22), $\tilde{Z}(\mathbf{v}^{\text{ho}})$ and $\tilde{Z}(\mathbf{v}^{\text{ue}})$ are non-comparable.

Consider the following condition:

$$\sum_{n=1}^{M^h}\sum_{z=1}^{G^h}\frac{\partial c_a^{hmg}(k)}{\partial v_a^{hnz}(k)} \gg \sum_{j=1,j\neq h}^{H}\sum_{n=1}^{M^j}\sum_{z=1}^{G^j}\frac{\partial c_a^{hmg}(k)}{\partial v_a^{jnz}(k)} \quad \forall a,g,m,h,k. \tag{23}$$

Generally speaking, the condition (23) implies that the activity-travel choices of a household type have marginal effects on the choices of other household types, so that the maximum net utility at equilibrium for each household type cannot be worsened by the choices of other household types. Under this condition, we can have $\tilde{Z}(\mathbf{v}^{\text{ho}}) \geq \tilde{Z}(\mathbf{v}^{\text{ue}})$. The condition (23) always holds in a system with only one homogeneous household type. This condition is strong and rarely holds in practice. Further studies should be carried out to investigate the impact of the complex interactions among households on the total net utility in the system.

Next, we investigate the relationships between HO, HSO and SO. Let $\tau_a(k)^{\text{hso}}$ be the toll for each private car on link a during interval k as follows:

$$\tau_a(k)^{\text{hso}} = \gamma\frac{\partial t_a(k)}{\partial\tilde{v}_a(k)}\sum_{h=1}^{H}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\left(1-\beta_a^{hg}(k)\right)v_a^{hmg}(k) \quad \forall a,k. \tag{24}$$

Proposition 3. *The solution to the HO problem* (20) *with the link cost functions for private cars given by*

$$c_a^{hmg}(k) = \gamma\left(1-\beta_a^{hg}(k)\right)t_a(k) + \frac{1}{|g|}\left(\phi_a^{hg}(k) + \tau_a(k)^{\text{hso}}\right) \quad \forall a,h,m,g,k, \tag{25}$$

results in the solution to the HSO problem: finding a flow vector $\mathbf{v}^* = (\ldots, v_{is}^{hmg}(k)^*, v_a^{hmg}(k)^*, e_w(k)^*, \ldots)^T$ *such that the maximum* $\tilde{Z}(\mathbf{v})$ *in* (21) *is subject to* (10)–(16).

Proposition 3 shows the relationship between HO and HSO under marginal cost pricing when a charge is imposed for all road links. Based on Eqs. (15) and (16), we can rewrite the link toll (24) as the following form:

$$\tau_a(k)^{\text{hso}} = \bar{\gamma}_a(k)\frac{\partial t_a(k)}{\partial \tilde{v}_a(k)}\varphi_a(k)\tilde{v}_a(k) \quad \forall a, k. \tag{26}$$

where $\varphi_a(k)$ and $\bar{\gamma}_a(k)$ are the average car occupancy level and the average value of travel time for all private car users on link a during interval k, given by

$$\varphi_a(k) = \frac{v_a(k)}{\tilde{v}_a(k)} \quad \forall a, k, \tag{27}$$

$$\bar{\gamma}_a(k) = \gamma \sum_{h=1}^{H}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\left(1-\beta_a^{hg}(k)\right)\frac{v_a^{hmg}(k)}{v_a(k)} \quad \forall a, k. \tag{28}$$

It is well-known that each additional traveler on a congested road imposes a congestion cost (known as an externality) on others (Yang and Meng (1998); Huang et al. (2000)). In the link toll (26), $\bar{\gamma}_a(k)\partial t_a(k)/\partial \tilde{v}_a(k)$ is the externality of each private car user and $\varphi_a(k)\tilde{v}_a(k)$ is the total number of private car users on link a during interval k. Thus the link toll to drive the HO solution to an HSO equals the externality of all private car users on the link (Yang and Meng (1998)). In addition, the link toll under HSO is uniform, i.e., all travel groups on a link are charged the same toll. The link toll (26) includes the link tolls in Yang and Huang (1999); Huang et al. (2000) for carpooling problems as special cases in which the car occupancy level on the link is at most two.

Note that the HSO principle defined in this study differs from the conventional individual-based SO principle. By ignoring intra-household interactions, we can reduce the HSO problem to a SO problem. Thus, we always have $\tilde{Z}(\mathbf{v}^{\text{hso}}) \geq \tilde{Z}(\mathbf{v}^{\text{so}})$ where $\mathbf{v}^{\text{hso}}$ and $\mathbf{v}^{\text{so}}$ are the flow vectors under HSO and SO, respectively. Simplifying the link toll (26) by setting $\varphi_a(k) = 1$ and $\bar{\gamma}_a(k) = \gamma$, the link toll under SO is given by the following well-known marginal cost function:

$$\tau_a(k)^{\text{so}} = \gamma\frac{\partial t_a(k)}{\partial \tilde{v}_a(k)}\tilde{v}_a(k) \quad \forall a, k. \tag{29}$$

Similar to Proposition 3, we can easily verify that under a uniform link toll (29), the solution to the HO problem ignoring intra-household interactions corresponds to the SO solution. Thus, the toll pattern under SO is a special case of that under HSO in which joint activities and travel are ignored. The link toll pattern under SO is biased, in that it assigns the same congestion cost and the same value of travel time γ to all private cars, regardless of occupancy level, and fails to take into account the dependence of the congestion cost on the ridesharing passengers in each car.

3.4. Existence and uniqueness conditions

In this section, we discuss the existence and uniqueness of the HO problem based on the VI problem (8). According to Theorem 1.4 in Nagurney (1993), the continuity of the JATP utility function (1) with JATP flows implies that the solution to the VI problem (9) exists. Based on **A6**, the link travel time function for private cars is strictly increasing with the vehicle flow of that link, and thus it is also continuous. As a result, the JATP utility function (1) is continuous, and a solution to the VI problem (9) exists.

According to Theorem 1.6 in Nagurney (1993), the solution to the VI problem (9) is unique if the JATP utility function (1) is strictly monotonic with the JATP flows. As mentioned, the 0–1 integer variables in the JATP utility function are not just indexes but are dependent on the link vehicle flows of private car users. Thus, the JATP utility function is non-concave with the JATP flows. This means that the solution to the VI problem is not unique. However, for any fixed flow propagation relationship, the JATP utility function strictly increases with the JATP flows (see Proposition 4), and the VI sub-problem has a unique solution. This property is of use for developing a solution method for the VI problem.

Proposition 4. *Under the fixed flow propagation relationship in constraints* (12)–(14)*, the JATP utility function* (1) *is strictly decreasing with the JATP flows.*

4. Solution method

The VI problem (9) requires the enumeration of feasible JATPs in advance. Based on a proposed supernetwork platform, we adopt the column generation method to generate feasible JATPs when needed. We then develop a modified diagonalization method to solve the VI problem (9).

4.1. Joint-activity-time-space supernetwork platform

With use of the column generation method, finding the maximum utility JATP satisfying constraints (42a)–(42m) is an integer problem and not straightforward. In this section, we propose a new JATS supernetwork platform to overcome this issue.

The supernetwork approach has been extensively adopted in the literature to model activity-travel choices at the individual level (see e.g., Ouyang et al. (2011); Fu and Lam (2014); Fu et al. (2014)). Fu and Lam (2018) recently proposed an supernetwork platform for modeling joint daily activity-travel choices of household members. However, the joint activity-travel choices in their model is very limited. In particular, their supernetwork platform only allows for the schedules of full-time worker-couple households using public transit with at most one joint travel episode before and one after work, and one joint activity episode after work. A dedicated path-finding algorithm is then developed to generate the JATP choice set with use of the column generation method. Their proposed algorithm is restricted to a specific household schedule (the schedule of a full-time worker couple with joint shopping after work).

In contrast, our proposed JATS supernetwork allows for multiple joint activities and travel for heterogeneous household types with more than two persons using different transport modes. Any conventional path-finding algorithm, e.g., Dijkstra's algorithm, can be used during column generation. In what follows, we present the properties of our proposed JATS supernetwork platform. Details of the supernetwork construction are given in Appendix C.

Let $\mathcal{G}^m(\mathcal{N}^m, \mathcal{L}^m)$ be an ATS supernetwork of person m where $\mathcal{N}^m$ and $\mathcal{L}^m$ are the sets of ATS nodes and links, respectively. We decompose the sets of ATS nodes and links as

$$\mathcal{N}^m = \bigcup_{k=1}^{K+1} \mathcal{N}^m(k) \quad \text{and} \quad \mathcal{L}^m = \bigcup_{k=1}^{K} \mathcal{L}^m(k) \quad \forall m, \tag{30}$$

where $\mathcal{N}^m(k)$ is the set of ATS nodes at interval k, and $\mathcal{L}^m(k) = \mathcal{N}^m(k) \times \mathcal{N}^m(k+1)$ the set of ATS links during interval k.

Property 1. An ATS link $l \in \mathcal{L}^m(k)$ represents an activity *or* travel choice of person m during interval k.

Let $\mathbf{G}^h(\mathbf{N}^h, \mathbf{L}^h)$ be the JATS supernetwork of household type h, where $\mathbf{N}^h$ and $\mathbf{L}^h$ are the sets of JATS nodes and links, respectively. Similar to the sets of ATS nodes and links, we decompose the sets of JATS nodes and links as

$$\mathbf{N}^h = \bigcup_{k=1}^{K+1} \mathbf{N}^h(k) \quad \text{and} \quad \mathbf{L}^h = \bigcup_{k=1}^{K} \mathbf{L}^h(k) \quad \forall h, \tag{31}$$

where $\mathbf{N}^h(k)$ is the set of JATS nodes at interval k, and $\mathbf{L}^h(k) = \mathbf{N}^h(k) \times \mathbf{N}^h(k+1)$ the set of JATS links during interval k. We can express the sets of JATS nodes and links with respect to the sets of ATS nodes and links as

$$\mathbf{N}^h(k) = \left\{ \mathbf{n} \in \bigtimes_{m=1}^{M^h} \mathcal{N}^m(k) \right\} \quad \text{and} \quad \mathbf{L}^h(k) = \left\{ \mathbf{l} \in \bigtimes_{m=1}^{M^h} \mathcal{L}^m(k), \mathbf{l} \text{ is feasible} \right\} \quad \forall h, k, \tag{32}$$

where "×" is the Cartesian product. The feasibility of a JATS link in Eq. (32) can be determined by the *pre-defined* rules based on the household context, such as the number of cars and/or car licenses owned by the household. Further details on the feasibility of JATS links are given in Appendix C.

Property 2. A JATS link $\mathbf{l} \in \mathbf{L}^h(k)$ represents the activity *and* travel choices of *all* persons of household type h and their interactions during interval k.

Let $\mathbf{X}^h$ be the set of feasible paths in JATS supernetwork $\mathbf{G}^h$ for household type h.

Property 3. A JATS path $\mathbf{x} \in \mathbf{X}^h$ represents a JATP choice $q \in Q^h$.

The utility of JATS path $\mathbf{x} \in \mathbf{X}^h$ is calculated by

$$\psi_{\mathbf{x}}^h = \sum_{m=1}^{M^h} \sum_{\mathbf{l} \in \mathbf{L}^h} \psi_{\mathbf{l}}^{hm} \xi_{\mathbf{l}}^{\mathbf{x}} \quad \forall \mathbf{x}, h, \tag{33}$$

in which $\xi_{\mathbf{l}}^{\mathbf{x}}$ equals 1 if JATS link $\mathbf{l}$ is on path $\mathbf{x}$ and 0 otherwise, and $\psi_{\mathbf{l}}^{hm}$ is the utility of JATS link $\mathbf{l}$ for person m of household type h using JATS path $\mathbf{x}$.

Property 4. Given that a JATS path $\mathbf{x} \in \mathbf{X}^h$ represents a JATP choice $q \in Q^h$, JATS path utility $\psi^h_{\mathbf{x}}$ equals JATP utility u^h_q.

Next, we present the mapping of JATS link utility $\psi^{hm}_{\mathbf{l}}$ to the activity utility and travel cost in the JATP utility function (1).

Case 1. If JATS link $\mathbf{l} \in \mathbf{L}^h(k)$ includes the participation of person m in group g of household type h during interval k in activity i at location s, we have

$$\psi^{hm}_{\mathbf{l}} = u^{hmg}_{is}(k) \quad \forall \mathbf{l}, m, h, \tag{34}$$

where $u^{hmg}_{is}(k)$ is the activity utility given by Eq. (2).

Case 2. If JATS link $\mathbf{l} \in \mathbf{L}^h(k)$ includes the travel of person m in group g of household type h during interval k for entering path p between OD pair w by private car during interval l, then

$$\psi^{hm}_{\mathbf{l}} = -\frac{c^{hmg}_{pw}(l)}{\lceil \bar{t}^w_p(l) \rceil} \quad \forall \mathbf{l}, m, h, \tag{35}$$

where $c^{hmg}_{pw}(l)$ is the path travel cost given by Eq. (5), $\lceil . \rceil$ a function used to convert travel times to an integer time interval in which $\lceil t \rceil = k$ if $l \le t/\sigma < l+1$, and $\bar{t}^w_p(l)$ the estimated travel time at equilibrium for the private car entering path p between OD pair w during interval k, given by

$$\bar{t}^w_p(l) = \sum_{a \in A} \sum_{e=l}^{K} \bar{t}_a(e) \bar{\zeta}^{we}_{ap}(l) \quad \forall p, w, l, \tag{36}$$

where $\bar{\zeta}^{we}_{ap}(l)$ equals 1 if the private car entering path p between OD pair w during interval l arrives at link a during interval e under estimated link travel times and 0 otherwise. Note that because $\lceil \bar{t}^w_p(l) \rceil$ ATS links are used to model the travel of group g during interval k for entering path p between OD pair w during interval l, Eq. (35) includes the term $1/\lceil \bar{t}^w_p(l) \rceil$.

Case 3. If JATS link $\mathbf{l} \in \mathbf{L}^h(k)$ includes the travel of person m in group g of household type h during interval k for departing from the origin of OD pair w by public transit during interval l, we have

$$\psi^{hm}_{\mathbf{l}} = -\frac{\hat{c}_w(l)}{\lceil \hat{t}_w(l) \rceil} \quad \forall \mathbf{l}, m, h, \tag{37}$$

where $\hat{t}_w(l)$ is the constant and given transit travel time for departing from the origin of OD pair w during interval l.

Note that the JATS supernetwork construction requires a pre-defined path set for each OD pair. We can obtain such a path set using a choice set generation method, such as those of Bekhor et al. (2008); Bovy (2009). The advantage of using a behaviorally generated path set is that they have a better chance of actually being used in practice as these paths can be generated according to a calibrated behavioral path choice model.

4.2. Diagonalization method

In this section, we develop a modified diagonalization method to solve the VI problem (9). The diagonalization method was first proposed by Dafermos (1982) to solve the static traffic assignment with asymmetric travel cost functions, and later adapted to solve various problems, such as the static traffic assignment with asymmetric interactions between cars and trucks (Mahmassani and Mouskos (1988)), dynamic traffic assignment (Chen and Hsueh (1998)) and dynamic activity-based traffic assignment (Lam and Yin (2001)) with temporal link flow interactions. The diagonalization method relaxes some link flow interactions in the original problem to yield sub-problems that can be solved more effectively.

The link flow interactions in the VI problem (9) result from constraints (42a)–(42e) in which the vehicle flow of private car users entering a link along a path during a certain interval cannot get onto the next link on the path until the *actual* link travel time elapses. The modified diagonalization method developed in this paper relaxes these link interactions by assuming that the conditions in Proposition 4 hold. This treatment yields diagonalized VI sub-problems with (i) a fixed and flow-independent flow propagation relationship, and (ii) a strictly increasing (and continuous) JATP utility function. The VI problem (9) is then solved by a series of VI sub-problems, each of which is a static traffic assignment problem with strictly monotonic cost functions.

The diagonalization method developed in this paper is outlined as follows.

Step 0: Initialization. Let $z = 0$.

Step 0.1: Estimate link travel times $\bar{t}_a(k)^0 = t^0_a, \forall a, k$, where t^0_a is the link free-flow travel time.

Step 0.2: Estimate path travel times $\bar{t}_p^w(k)^0, \forall w, p, k$, based on estimated link travel times $\bar{t}_a(k)^0, \forall a, k$.
Step 0.3: Construct a JATS supernetwork $(\mathbf{G}^h)^0, \forall h$, based on estimated path travel times $\bar{t}_p^w(k)^0, \forall p, w, k$.
Step 0.4: Solve the VI problem (9) using $(\mathbf{G}^h)^0$ to find an initial feasible flow pattern $\mathbf{f}^0$, and reproduce actual link travel times $t_a(k)^0, \forall a, k$.

Step 1: Update estimated path travel times. Let $z = z + 1$.

Step 1.1: Update estimated link travel times using method of successive averages (MSA):

$$\bar{t}_a(k)^z = (1 - \frac{1}{z})\bar{t}_a(k)^{z-1} + \frac{1}{z}t_a(k)^z \quad \forall a, k.$$

Step 1.2: Update estimated path travel times $\bar{t}_p^w(k)^z, \forall p, w, k$, based on estimated link travel times $\bar{t}_a(k)^z$, $\forall a, k$.

Step 2: The VI problem (9) with estimated travel times.

Step 2.1: Construct a new JATS supernetwork $(\mathbf{G}^h)^z, \forall h$, based on estimated path travel times $\bar{t}_p^w(k)^0, \forall p, w, k$.
Step 2.2: Solve the VI problem (9) using $(\mathbf{G}^h)^z$ to find a feasible flow pattern $\mathbf{f}^z$, and reproduce actual link travel times $t_a(k)^z, \forall a, k$.

Step 3: Convergence check. If $\bar{t}_a(k)^z \approx \bar{t}_a(k)^{z-1}$, $\forall a, k$, and $\mathbf{f}^z \approx \mathbf{f}^{z-1}$, stop. The current solution is optimal. Otherwise, go to **Step 1**.

In Step 0, an initial feasible flow pattern is generated by solving the VI sub-problem with the given free-flow link travel times for private cars. In Step 1, the link travel times for private cars are estimated by the MSA to derive the estimated path travel times. Given the estimated path travel times, Step 2 solves the VI sub-problem, based on the newly constructed JATS supernetworks, and reproduces the new actual link travel times. Step 3 checks at equilibrium whether the resultant estimated link travel times for private cars and the flow patterns of two consecutive iterations are equal or not.

At each iteration of the diagonalization method, the VI sub-problem is solved by a path-swapping algorithm adapted from the path-swapping algorithm by Huang and Lam (2002). A column generation technique is integrated to solve the VI sub-problem without the need to enumerate the JATPs in advance. Under the assumption that the path cost function, equivalent to the JATP utility function in this paper, is strictly monotonic, Huang and Lam (2002) proved that the path-swapping algorithm produces a converged and stable solution. Indeed, the JATP utility function (1) is strictly increasing in the VI sub-problem (Proposition 4). The path-swapping algorithm is presented below.

Step 0: Initialization.

Step 0.1: Let $n = 0$. Set move size parameters $\varpi > 0$ and $\eta > 0$; and tolerance $\varepsilon > 0$.
Step 0.2: Under *free-flow* condition, find the maximum utility JATP q using JATS supernetwork $\mathbf{G}^h, \forall h$.
Step 0.3: Let $Q^h = \{q\}, \forall h$. Assign the flow $(f_q^h)^0 = D^h, \forall h$.

Step 1: Column generation.

Step 1.1: Find the maximum utility JATP q using JATS supernetwork $\mathbf{G}^h, \forall h$.
Step 1.2: Remove unused JATPs in Q^h and update $Q^h = Q^h \cup \{q\}, \forall h$.

Step 2: Convergence test. Stop if the relative gap (RGAP) satisfies

$$RGAP = \left(\sum_{h=1}^{H} \sum_{q \in Q^h} (f_q^h)^n \left[(\mu^h)^n - (u_q^h)^n \right] \right) \Big/ \left(\sum_{h=1}^{H} \sum_{q \in Q^h} (f_q^h)^n (\mu^h)^n \right) < \varepsilon.$$

Step 3: JATP flow swapping.

$$(f_q^h)^{n+1} = \begin{cases} \max\left\{0, (f_q^h)^n - \varpi_n (f_q^h)^n \left[(\mu^h)^n - (u_q^h)^n \right] \right\} & \forall h, q \notin E_n^h, \\ (f_q^h)^n + \Psi_n^h / \left| E_n^h \right| & \forall h, q \in E_n^h, \end{cases}$$

$$\text{where } \Psi_n^h = \sum_{q \notin E_n^h} \left[(f_q^h)^n - (f_q^h)^{n+1} \right], \text{ and } E_n^h = \left\{ q \in Q^h : (u_q^h)^n = (\mu^h)^n \right\} \quad \forall h,$$

Let $n = n + 1$. Go to **Step 1**.

Dafermos (1982) gives formal sufficient conditions for convergence of the diagonalization algorithm. Specifically, in addition to the continuity of the link cost function, the effects of link flow interactions on a link cost are *"weak"* compared with the effects of the flow on that link. When the diagonalization method is adapted to solve different problems in practice, these conditions are stringent and unnecessary. In particular, Friesz et al. (1984); Mahmassani and Mouskos (1988) observed that the diagonalization algorithm converges despite those conditions being violated. Hence, we can expect the method to converge to a good-quality solution for achieving network equilibrium. For the convergence of the path-swapping algorithm for the VI sub-problems, Huang and Lam (2002) provides conditions for ϖ_n under which the algorithm converges to an equilibrium as $\lim_{n\to+\infty} \varpi_n (f_q^h)^n = 0$ and $\sum_n \varpi_n (f_q^h)^n = +\infty$. In this study, $\varpi_n = \frac{0.0005}{1+n/10}$.

4.3. Time resolution choice

According to **A2**, flow propagation (or network loading) in this study is exerted with respect to the discrete-time intervals with duration σ, which influences the estimated path travel times for private cars when constructing the JATS supernetwork at each iteration of the proposed diagonalization method. Higher time resolution yields a more accurate numerical solution, but at the price of longer computational time. In practice, we choose the time resolution based on the purposes of the model. For example, the time resolution could be a few minutes for short-term traffic operation management and control (e.g., MATSim (Dubernet and Axhausen (2013))), but up to an hour or four or five broad departure time periods of the day for long-term transportation planning (e.g., a half hour in SACSIM (Bradley and Vovsha (2005)); an hour in TASHA (Roorda et al. (2008)) and Lam and Yin (2001); Gupta and Vovsha (2013); and two or three hours per time interval in ALBATROSS (Arentze and Timmermans (2004a)). In addition, the time resolution must also be based on empirical results so that it accurately reflects the activity durations (see Srinivasan and Bhat (2008); Lai et al. (2019)). We also acknowledge recent activity-based travel demand models with continuous time specifications for activity starting time and duration, and tour-timing choices (see e.g., CUSTOM (Habib (2018); Hasnine and Habib (2019))). However, these models focus on activity-travel choices at the individual level. Modeling intra-household interactions under the continuous-time setting could be challenging for further study in future.

5. Numerical examples

In this section, numerical examples are presented to illustrate the followings: (1) the merits of the proposed model based on two new household-oriented network equilibrium principles, namely HO and HSO, together with new insights on the Braess paradox; (2) the applicability of the proposed model for evaluating ridesharing policies; and (3) the performance and convergence of the proposed solution method.

5.1. Example 1: small network

5.1.1. Settings

To highlight the merits and applicability of the proposed model, we tested a small network with a homogeneous two-person household type. Fig. 5 shows the small test network which comprises four nodes, twelve links, three activity types, i.e., home (H), work (W1 or W2)) and shopping (S), and four activity locations. We assumed that 20,000 worker couples started and ended their daily schedules at home, and worked at workplaces 1 and 2. Table 1 shows the input parameters for the activity utility functions. The household preference parameters for joint activities and travel were $\alpha_{is}^{hg}(k) = \beta_a^{hg}(k) = 0.4$. We also assumed that households had no preference for joint activities at home, and each time interval was 15 min. All paths for private cars connecting activity and drop-off/pickup locations were enumerated.

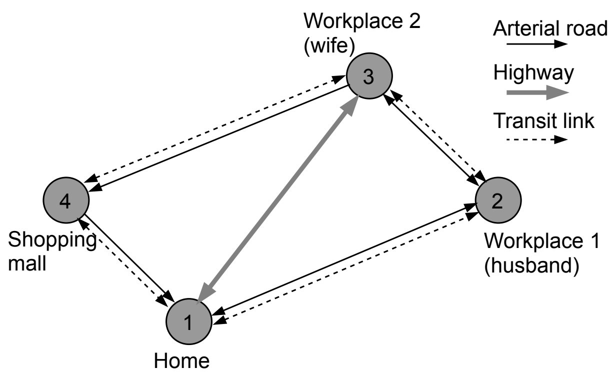


Figure 5: A small test network for Example 1

Table 1: Input parameters for the utility functions of worker couples for Example 1

Activity	$\bar{u}_{is}^{hm}$ (HKD/min)	$\tilde{u}_{is}^{hm}$ (HKD)	ρ_i^{hm}	υ_i^{hm}	$\tilde{t}_i^{hm}$ (min)
Home	0.425	300	−0.0055	1	780
Work (husband)	0.000	375	0.0100	1	810
Work (wife)	0.000	375	0.0100	1	780
Shopping (husband)	0.000	100	0.0250	1	1120
Shopping (wife)	0.000	150	0.0250	1	1120

The link travel time by private car was based on a Bureau of Public Roads (BPR) function as follows:

$$t_a(k) = t_a^0\left[1 + 1.5\left(\frac{4 \times \tilde{v}_a(k)}{\tilde{C}_a}\right)^4\right] \quad \forall a, k, \tag{40}$$

where t_a^0 is the link free-flow travel time, and $\tilde{C}_a$ (veh/hr) is the remaining link capacity for private cars after excluding fixed transit vehicle flow on the link. Note that the link flow during each interval in Eq. (40) was multiplied by four because each hour includes four intervals. We assumed that a transit vehicle was equivalent to four private cars. Table 2 shows the input parameters for the link travel times for private cars. The value of travel time for private car users was $\gamma = 60$ HKD/hr. The operating cost $\phi_a^{hg}(k)$ includes only the fuel cost, 1.4 HKD/km.

Table 2: Input parameters for link travel times for private cars for Example 1

Road link*	Free-flow travel time (hr)	Free-flow speed (km/hr)	Link length (km)	Link capacity (veh/hr)	Fixed transit vehicle flow (veh/hr)
1-2	0.25	50	12.50	1800	100
1-3	0.17	70	11.67	3600	0
2-3	0.17	50	8.33	1800	100
1-4	0.17	50	8.33	1800	100
3-4	0.25	50	12.50	1800	100

*: only one direction of the link is shown.

Table 3: Input parameters for transit travel times for Example 1

OD pair*	Sequence of road links	Travel distance (km)	Walking and waiting time (hr)	In-vehicle time (hr)
1-2	1-2	12.50	0.25	0.58
1-3	1-2, 2-3	20.83	0.42	0.97
1-4	1-4	8.33	0.17	0.39
2-3	2-3	8.33	0.17	0.39
2-4	2-3, 3-4	20.83	0.42	0.97
3-4	3-4	12.50	0.25	0.58

*: only one direction of the OD pair is shown.

The transit travel cost was

$$\hat{c}_w(k) = \gamma_1[\text{walking and waiting time}] + \gamma_2[\text{in-vehicle time}] + [\text{fare}] \quad \forall w, k, \tag{41}$$

where $\gamma_1 = 120$ HKD/hr is the value of walking and waiting time, and $\gamma_2 = 60$ HKD/hr the value of in-vehicle time. Table 3 shows the input parameters for the transit travel times in which transit travel time is travel distance (based on road length) divided by transit speed. We assumed that the transit travel time comprised 30% of walking and waiting time and 70% of in-vehicle time. The transit speed was 15 km/hr and the transit fare was 10 HKD for each OD pair.

5.1.2. The merits of the proposed model

To highlight the merits of the proposed model, we provide a benchmark comparison of the household optimum utility approach based on HO and HSO with the conventional approach based on UE and SO, which ignore intra-household interactions on joint activities and travel.

First, we compare the average daily time allocation (24 hr) of activities per person under the four above-stated network equilibrium principles. It can be seen from Table 4 that HO and HSO can explicitly capture multiple joint/solo activity and travel episodes. UE and SO disregard both joint out-of-home (i.e., shopping) and at-home activities. Such at-home activity participation is commonly ignored or not properly modeled by existing household-level activity-based travel demand models. UE and SO could also underestimate the total (both solo and joint) travel time duration as they ignore additional drop-off/pickup trips for joint travel.

Table 4: Average daily time allocation (24 hr) of activities per person under four network equilibrium principles

Average duration (hr/person)	HO	HSO	UE	SO
Solo travel time	0.9	0.6	1.6	1.4
Joint travel time	**1.1**	**0.9**	**0.0**	**0.0**
Solo at-home	0.5	0.3	1.7	1.4
Joint at-home	**11.9**	**12.5**	**11.5**	**12.0**
Work	8.8	8.9	8.6	8.8
Solo shopping	0.0	0.0	0.5	0.4
Joint shopping	**0.8**	**0.8**	**0.0**	**0.0**

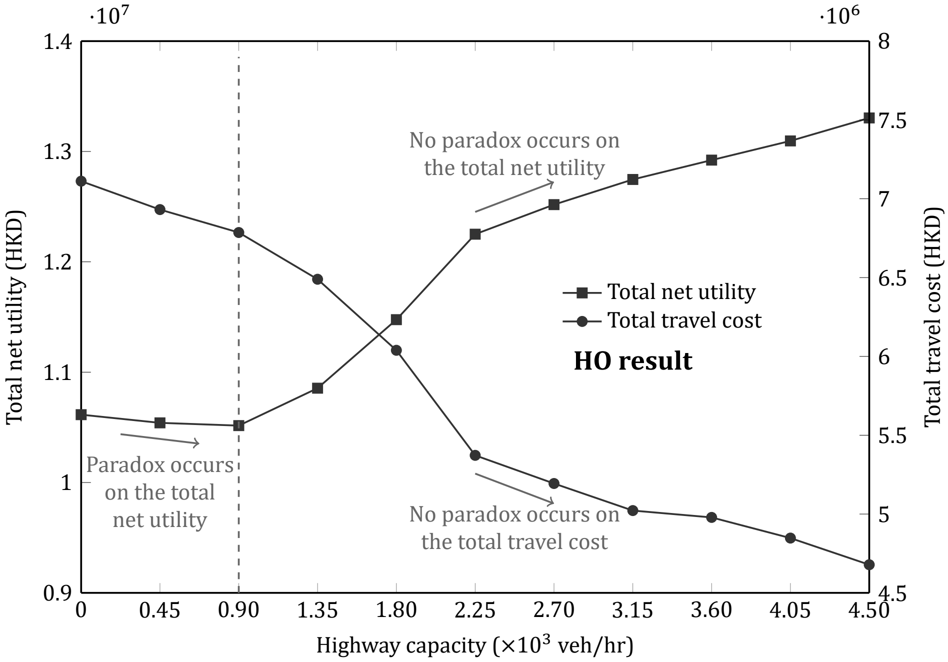
By investigating the extra benefits of joint activities and travel, the proposed approach based on HO and HSO can be used to coherently investigate the correlation among solo/joint activity-travel choices (see e.g., Lai et al. (2019)). Such correlation among joint/solo activity-travel choices could be difficult to simultaneously examine by existing activity-based travel demand models in a consistent manner. For instance, the results in Table 4 show that, under HO and HSO, when household members conduct joint travel and joint shopping, they tend to spend more time together at home. The longer joint shopping, at-home, and travel duration also leads to the shorter solo at-home duration.

Table 5: Total number of trips by different modes of the system under four network equilibrium principles

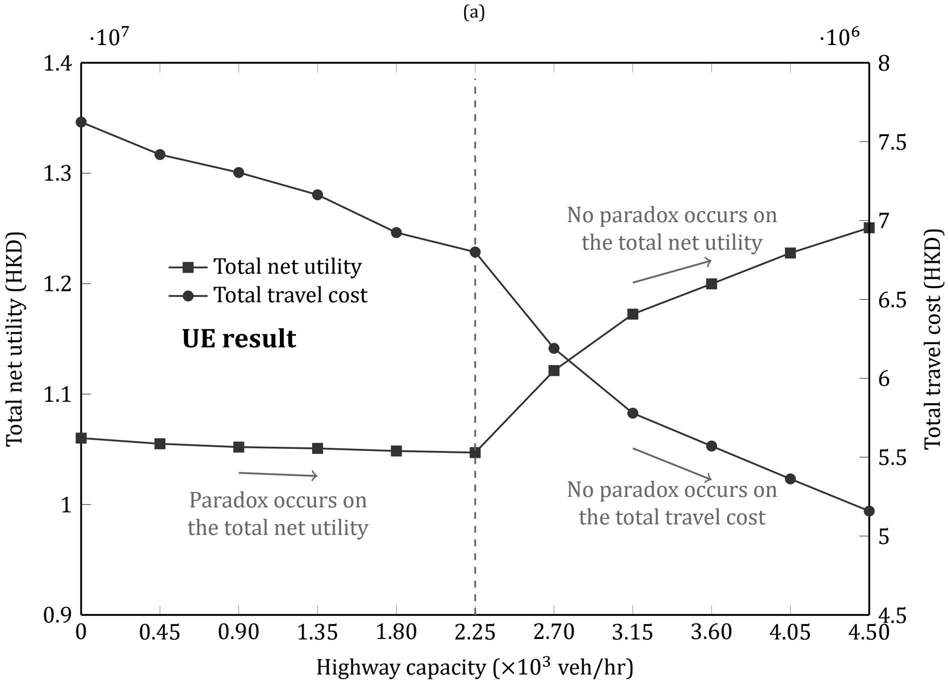
	HO	HSO	UE	SO
Total number of joint private car trips (I)	32 691	38 205	0	0
Total number of solo private car trips (II)	43 504	29 240	69 958	56 305
Total number of private car trips (I)+(II)	**76 195**	**67 445**	**69 958**	**56 305**
Total number of transit public trips	10 347	18 233	17 853	29 152

We further elaborate the total number of trips by different modes generated under four network equilibrium principles. The results are shown in Table 5. Note that the total number of trips under different principles is different because the travel demands between OD pairs are generated endogenously in the activity-based approach rather than given and fixed as assumed in the trip-based approach. The results indicate that even though a large number of joint trips shared among private car users under HO and HSO, the proposed approach tends to lead to a larger total number of private car trips than that of the conventional approach based on UE and SO. This is reasonable as joint trips by private car, in most cases, requires additional solo drop-off/pickup trips. In addition, the use of public transit under HO and HSO decreases because household members have more flexible travel choices with ridesharing. The results imply that ignoring joint activities and travel could result in a biased estimation of the numbers of trips between OD pairs and the use of different transport modes.

The selfish behaviors of travelers under UE in trip-based models could lead to a Braess paradox in which adding a new road link may increase the total travel cost of the system (Sheffi (1985)). We next show the occurrence of a Braess-like paradox in activity-based models in which the total net utility of the system under HO and UE may decrease when the road network is expanded. In this example, we adjusted the highway capacity between home and workplace 2 from 0 veh/hr to 4,500 veh/hr. The zero capacity then indicates that the highway is not added to the network. The paradox is illustrated in Fig. 6. When the highway is added and its capacity is less than 900 veh/hr, the paradox occurs under both HO and UE. However, when the highway capacity is greater than 900 veh/hr, the paradox disappears under HO; but it still exists under UE until the highway capacity reaches 2,250 veh/hr (see Fig. 6b). It seems that the cooperation of household members in taking part in joint activities and travel under HO, to some extent, reduces the paradox region. Unlike the Braess paradox in trip-based models, the paradox in activity-based models occurs even though the total travel cost of the system is reduced, i.e., no Braess paradox from the trip-based approach perspective. This is because, in the activity-based approach, the activity utility of individuals could decrease when the traffic is improved. The results suggest that network design should be investigated from both household-level and activity-based approach perspectives. This calls for future studies.



(a)



(b)

Figure 6: The Braess-like paradox under (a) HO and (b) UE

Next, we investigate the properties concerning the total net utility of the system and road toll under different network equilibrium principles, as discussed in Sec. 3.3. Given there is only one homogeneous household type, Table 6 shows that HO and HSO always lead to higher total net utilities of the system than those under UE and SO ($\tilde{Z}(\mathbf{v}^{\text{ho}}) \geq \tilde{Z}(\mathbf{v}^{\text{ue}})$ and $\tilde{Z}(\mathbf{v}^{\text{hso}}) \geq \tilde{Z}(\mathbf{v}^{\text{so}})$). The increasing total net utility of the system arises from the higher total utility of activities and the lower total travel cost. This is because of the extra utilities of both joint activities and travel, as well as shared operating cost of private car users. The results show that ignoring intra-household interactions could greatly underestimate the total net utility of the system, or the impacts of a new transport policy.

Fig. 7a shows the relationship between the number of private car trips, car occupancy, and road toll by time of the day under HSO and SO on the highway (links 1-3 in the morning peak and 3-1 in the evening peak). Note that the toll patterns under HSO and SO, as discussed in Sec. 3.3, are uniform. The results indicate that the number of private car trips under SO is relatively higher than that under HSO because a large amount of traffic is reduced by ridesharing (a high occupancy level under HSO shown in Fig. 7b). Consequently, the road toll under SO shown in Fig. 7c is significantly overestimated

Table 6: Total net utility of the system under four network equilibrium principles

Network performance ($\times 10^6$ HKD)	HO	HSO	UE	SO
Total utility of activities (I)	17.90	18.08	17.57	17.70
Total travel cost (II)	4.98	3.68	5.57	4.52
Total net utility (I)-(II)	**12.92**	**14.39**	**12.00**	**13.18**

compared with that under HSO. Note that the reduced road toll under HSO could be also because of the extra utility for joint travel (see Eq. (24)). These results suggest that the introduction of road pricing schemes should consider interactions of household members as road tolls could be biased if joint activities and travel are ignored.

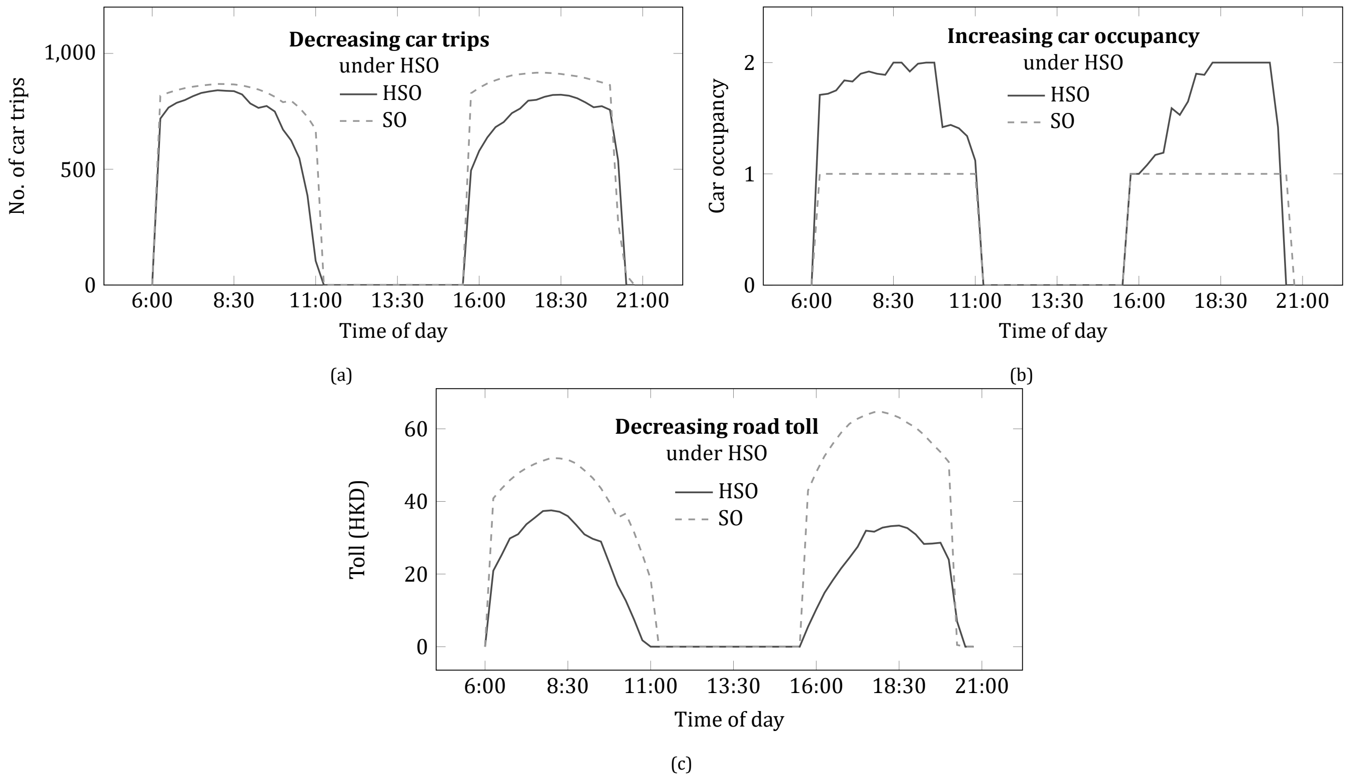


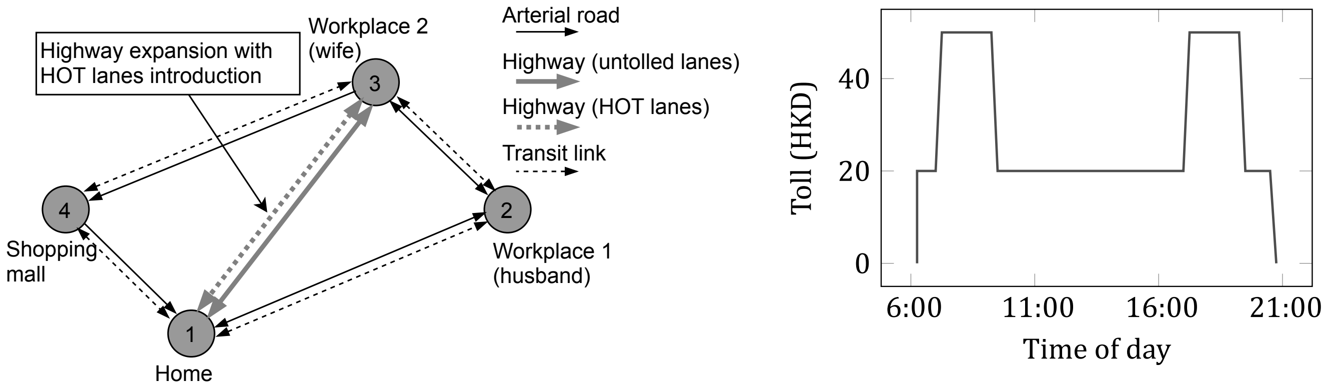
Figure 7: The relationship between the (a) number of private car trips, (b) car occupancy, and (c) road toll by time of the day under HSO and SO on the highway

5.1.3. The applicability of the proposed model for evaluating ridesharing policies

The purpose of the proposed model is to assess coherently the effects of alternative transport policies on long-term strategic planning, particularly in road networks with HOT lanes. In this section, transport policies are evaluated by the proposed model using the HO principle, under the same base network and settings as those in Sec. 5.1.1.

Suppose there is a project to expand the highway between home and workplace 2. The road capacity of the highway will be expanded by three times, from 3,600 veh/hr to 10,800 veh/hr. To encourage joint travel, half of the highway capacity is converted into HOT lanes, and the remainder is used for untolled lanes. The expanded network is shown in Fig. 8a. A time-varying toll scheme is introduced on the HOT lanes in which the tolls in the morning peak period [7:00, 9:00] and the evening peak period [17:00, 19:00] are 50 HKD, and those of the other periods are 20 HKD. The time-varying toll scheme is illustrated in Fig. 8b.

Lam and Yin (2001) suggested a method to evaluate such network expansion using the activity-based approach by examining how much travel time is saved and how the saved travel time can be productively used. Given the existence for the extra benefits of joint activities and travel (Bradley and Vovsha (2005); Lin and Wang (2014); Lai et al. (2019))), joint and solo activity/travel participation has different utility/travel cost. Consequently, conventional activity-based models, which disregard joint activity and travel, could lead to a biased estimation of the benefit induced by travel time saving. The results in Table 7 show that the total travel time saving is 0.2 hr/person for solo travel and 0.5 hr/person for joint travel. The saved travel time leads to a decrease of 0.1 hr/person for solo at-home activities, and the increases of 0.4 hr/person for joint at-home activities, 0.3 hr/person at work, and 0.1 hr/person for joint shopping. Note that the decrease in solo



(a) (b)

Figure 8: The (a) expanded test network and (b) toll by time of day on HOT lanes

at-home duration is reasonable as joint travel negatively correlates with solo at-home activities (see Table 4). Based on this time allocation with specific joint/solo activity/travel participation, the estimation of the benefit of travel time saving can be derived by the proposed model.

Table 7: Effects of the network expansion on the average daily time allocation (hr/person) of activities

Average duration (hr/person)	Base network (I)	Expanded network (II)	Difference (II)-(I)
Solo travel time	0.9	0.7	**−0.2**
Joint travel time	1.1	0.6	**−0.5**
Solo at-home	0.5	0.4	**−0.1**
Joint at-home	11.9	12.3	**0.4**
Work	8.8	9.1	**0.3**
Solo shopping	0.0	0.0	0.0
Joint shopping	0.8	0.9	**0.1**

5.1.4. Effects of the time interval choice

As mentioned, the shorter time interval choice yields a more accurate numerical solution, but at the price of longer computational time. In this section, we investigate the effects of the time interval choice on the average daily time allocation of activities. The settings are the same as those used in Sec. 5.1.2. The results in Table 8 show that the longer time interval choice (i.e., 60-min interval) leads to the more biased time allocation. Joint activities seem to be the most affected as the longer time interval choice could reduce the flexibility in synchronizing joint activities. The results suggest that the time interval choice should be chosen carefully, based on the purpose of the model as well as empirical results regarding the duration of activity and travel episodes.

Table 8: Effects of the time resolution choice on the average daily time allocation (24 hr) of activities per person

Average duration (hr/person)	15-min interval	30-min interval	60-min interval
Solo travel time	0.9	1.0	1.5
Joint travel time	**1.1**	**1.2**	**1.5**
Solo at-home	0.5	0.6	0.7
Joint at-home	**11.9**	**11.3**	**10.2**
Work	8.8	8.6	8.2
Solo shopping	0.0	0.0	0.0
Joint shopping	**0.8**	**1.2**	**1.9**

5.2. Example 2: Sioux-Falls network

5.2.1. Settings

The second example is presented to illustrate the performance and convergence of the proposed solution method. The example network is the medium-size Sioux Falls network (Leblanc (1973)) with 24 nodes, 76 links, 3 activity types (home, out-of-home mandatory (work or school), and out-of-home non-mandatory (shopping)), and 6 activity locations as shown in Fig. 9. The system comprised two household types: $h1$, i.e., worker couple with a schoolchild, and $h2$, i.e., worker couple. We assumed that 15,000 households of each type started and ended their schedules at home. For each worker couple with a schoolchild, the husband and wife worked at workplaces 1 and 2, respectively; meanwhile, for the worker couple, the husband and wife worked at workplaces 1 and 3. We also assumed that each household owned two cars. The input parameters for the activity utility functions of the wives and husbands were the same as those used in Example 1, and the input parameters for the activity utility functions of the child were analogous to those of the wife, in that school utility was equivalent to work utility.

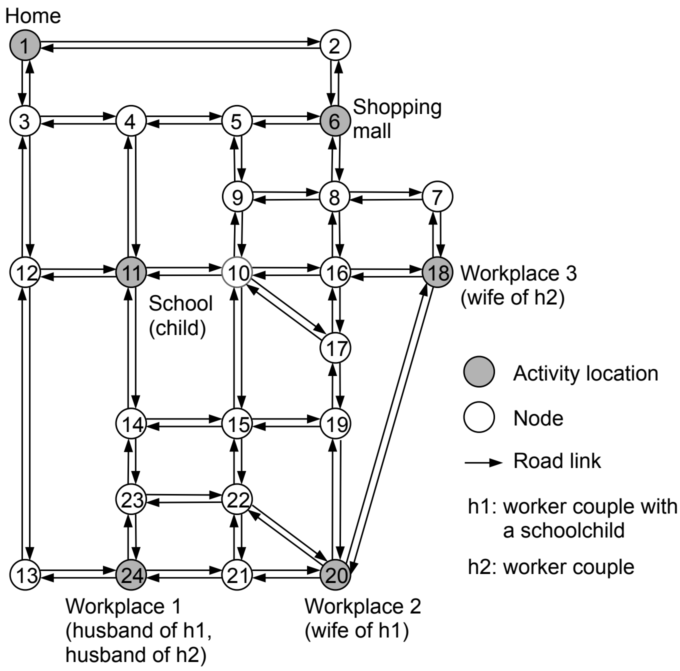


Figure 9: Sioux Falls network for Example 2

The working path set for private cars between OD pairs used in this example was from Bekhor et al. (2008). This path set was behaviorally generated by using a combination of the link elimination method and the penalty method. Inspection of the paths for different OD pairs revealed that this path set included both completely disjointed paths and very similar paths. Given that this example only considered six activity locations, there were 30 OD pairs and 217 paths with an average of 7.2 paths for each OD pair, and the maximum number of paths generated for any OD pair was 12. Note that this path set has been adopted extensively in the literature, for example, to examine the cross-nested logit stochastic user equilibrium (SUE) model in Bekhor et al. (2008), the α-reliable mean-excess traffic equilibrium model in Chen and Zhou (2010), the length-based and congestion-based C-logit SUE models in Zhou et al. (2012), the C-logit SUE model with elastic demand in Xu and Chen (2013), and the multiclass mean-excess traffic equilibrium model with elastic demand in Xu et al. (2014). The transit path for each OD pair is the longest-distance path for private cars of the OD pair. The link capacity for each link was 1,800 veh/hr and the fixed transit vehicle flow on each link was 100 veh/hr. All other parameters were the same as those used in Example 1.

5.2.2. Results

Fig. 10 shows the convergence of the developed diagonalization method under the HO principle for the Sioux Falls network. The gap is the relative difference in minutes between the estimated link travel times (for constructing the supernetwork platform) of two consecutive iterations. The method required around 60 iterations and an average of 300 seconds per iteration to get the converged solution. The computer program was coded with Java SE 8 on Window 10, Intel Core i5 3.50 GHz, 8 GB RAM. The threshold for the gap in the VI sub-problem was 0.001.

5.3. Remarks on the applicability of the proposed model

The above numerical examples were solved to investigate small- and medium-sized road networks under simplified setups to facilitate the representation of the essential ideas. The application in practice of the proposed model would require information on the daily activity-travel programs of households, e.g., daily activities, activity locations, path choices, network data, and household types. For instance, we can derive the daily activity-travel programs of households in Hong Kong from the Hong Kong Travel Characteristics Survey (2011) (TCS). The Hong Kong TCS includes four typical activity types, namely, work, school, home, and non-home, and further activities for car-owning and non-car owning households

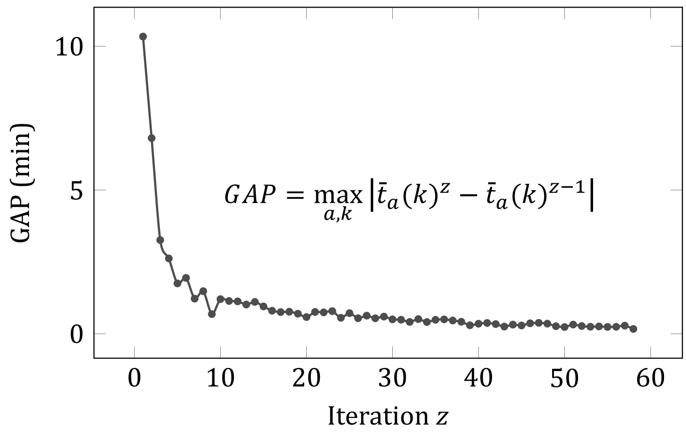


Figure 10: Convergence gap of the developed diagonalization method

of different income levels. For strategic transport planning purposes in Hong Kong, the Third Comprehensive Transport Study (CTS-3) road network comprises 338 zones and about 3,700 road links.

As real road networks are generally larger than the Sioux Falls network, the computational feasibility of the proposed model is of concern for practical applications. According to Eq. (47b), the size of the supernetwork depends on the household size, the number of daily activities, activity locations, and path choices. In practice, however, the daily activity-travel program of an individual consists only of a relatively small number of daily activities and is relevant to only a small part of the real transport network (Arentze and Timmermans (2004b); Liao et al. (2011)). Besides, empirical studies have shown that the daily activity-travel program of a household can be simplified to just a few activities. For example, Gliebe and Koppelman (2002) classified household activities into four types: subsistence, e.g., out-of-home work, school, or college; maintenance, e.g., out-of-home shopping, personal, and appointments; leisure, e.g., out-of-home free-time and visiting; and home, e.g., unspecified home activities. Similarly, Bradley and Vovsha (2005) divided daily activities into three types: mandatory, non-mandatory, and home.

For the above reasons, the JATS supernetwork for each household type is related to only small numbers of activities, locations, and path choices. Besides, most households in practice have less than four persons. For example, according to the Hong Kong Census and Statistics Department (2017), around 69.8% of households in Hong Kong have less than four members. The figure for the US is 77.4% (US National Household Travel Survey (2017)). Thus, for strategic planning, the settings in the Sioux Falls network example (i.e., 24 nodes, 76 links, 3 activity types, 6 activity locations, 30 OD pairs, 217 paths, 30-min interval, and 2 typical household types with 2 and 3 persons) may indeed be sufficient for demonstrating the computational feasibility of the proposed model.

Because a JATS supernetwork is constructed for each household type, the computational time of the proposed model increases linearly with the number of household types. For realistic networks with numerous household types, the computational time required for solving the proposed problem could become impracticably long. However, the increase in computational time is linear, and would not necessarily become unacceptable for long-term planning purposes. Further studies on efficient solution algorithms and data structures for large JATS supernetworks are necessary.

6. Conclusions and further studies

This study proposed a new HO utility approach, which simultaneously considers the choices of all household members to maximize the household utility, to model the intra-household interactions on joint activity-travel choices for heterogeneous household types with different sizes in congested road networks. The proposed household activity-based network equilibrium model simultaneously takes into account time-dependent household daily activity-travel scheduling and traffic assignment problems within a unified modeling framework. The proposed model was then formulated as an equivalent VI problem and solved by a diagonalization method.

Two new household-oriented network equilibrium principles based on HO and HSO were introduced together with the formulations of their equivalent MP problems. The analytical relationships between HO, HSO, the conventional UE and individual-based SO, and their properties were then investigated. It was proved that

- The HO solution is equivalent to that of UE when there are no intra-household interactions.
- In a system with one homogeneous household type, the total net utility of the system under HO is always greater than or equal to that under UE.
- In a system with many heterogeneous household types, the total net utilities of the system under HO and UE are non-comparable.
- There exists a uniform toll pattern to drive the HO solution to an HSO.

- There exists a uniform toll pattern to drive the HO solution with no intra-household interactions to an SO.
- The link toll pattern under HSO is equivalent to that under SO when there are no intra-household interactions.
- The total net utility of the system under HSO is always greater than or equal to that under SO.

These two coherent household-oriented network equilibrium principles (HO and HSO) can provide new insights on the Braess paradox for evaluation of new transport policies and alternative road toll schemes. The conventional UE and SO solutions may lead to biased results when there is strong interaction between household members with respect to their daily activity scheduling and travel choices.

The numerical results showed that intra-household interactions regarding joint activity-travel choices have significant impacts on time allocation, correlation between joint/solo activities and travel, trip generation, modal share, occurrence of Braess-like paradox, total net utility of the system, and road toll pattern. The results suggest that travel demand forecasting and network design should be investigated from both household-level and activity-based approach perspectives. In addition, the results also showed the applicability of the proposed model for coherently assessing the effects of various transport policies on long-term strategic planning, especially in road networks with HOT lanes.

It could be argued that the combination exploitation of joint activity and travel participation in a general case could hinder the applicability of the proposed model in reality. In practice, we can, however, adopt the concept of skeleton activity-travel schedules for full-time workers (Habib and Miller (2006)) to reduce the complexity of the supernetwork. Such skeleton concept has been widely used in most operational disaggregate activity-based travel demand models in which a skeleton schedule refer to the schedule with fixed attributes, such as activity starting time, duration and location (Habib (2018)). Joint activity-travel participation is then only scheduled within the time gaps available in the skeleton schedule. Besides, the proposed model can be used as a platform for integration of the activity-based approach and traffic assignment model in which some insightful results from the existing time allocation/activity generation models in literature, such as Srinivasan and Bhat (2005); Kato and Matsumoto (2009); Bhat et al. (2013); Bernardo et al. (2015); Lai et al. (2019), can also be embedded into to construct the proposed supernetwork for practical applications. The combination exploitation of joint activity and travel participation could be significantly reduced in practice so as to enable the potential applications of the proposed model in reality.

In this study, the proposed model has omitted some realistic aspects which could be addressed in further studies, such as including multimodal transport modes with interactions between modes, physical capacity constraints on roads and activity locations, and ridesharing between persons in different households into the proposed model; calibrating the parameters of the activity utility functions with empirical data; or developing efficient solution algorithms and data structures for solving the JATP problems in supernetworks. The proposed model can also lead to a new avenue of research based on the two proposed household-oriented network equilibrium principles (HO and HSO), such as network design (Kang et al. (2013)), HOT lane design (Di et al. (2018)), and integrated land-use and transport optimization problems (Yim et al. (2011)). It can also be used to investigate various activity-travel behaviors of household members from the group decision perspective, such as altruism, and the correlation between joint/solo activities and travel (Lai et al. (2019)).

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Commonly-used notation

Sets	
A	set of links for private car users
S	set of nodes (including intermediate nodes and activity and/or drop-off/pickup locations)
I	set of activities
P^w	set of feasible paths between OD pair w for private car users
Q^h	set of feasible JATPs for household type h
Indices	
k	a time interval
m	a person

h a household type

g a group of persons

i an activity

s a location

p a path for private car users

a a link for private car users

w an OD pair

q a JATP

K number of time intervals in the study period

H number of household types

G^h number of groups in household type h

M^h number of persons in household type h

Parameters

σ time interval duration

$\alpha_{is}^{hg}(k)$ household preference parameter for joint activity participation by group g of household type h in activity i at location s during interval k

$\beta_a^{hg}(k)$ household preference parameter for joint travel by group g of household type h on link a during interval k

$\hat{c}_w(k)$ constant transit travel cost for departing from the origin of OD pair w during interval k

$\hat{t}_w(k)$ constant transit travel time for departing from the origin of OD pair w during interval k

$\phi_a^{hg}(k)$ operating travel cost by private car on link a during interval k

γ value of travel time for private car users

D^h travel demand for household type h

Variables

f_q^h flow for household type h choosing JATP q

$\tilde{v}_a(k)$ vehicle flow of private car users on link a during interval k

$v_a(k)$ person flow of private car users on link a during interval k

$v_{is}^{hmg}(k)$ flow of person m in group g of household type h performing activity i at location s during interval k

$v_a^{hmg}(k)$ flow of person m in group g of household type h traversing link a by private car during interval k

$e_w(k)$ transit passenger flow between OD pair w during interval k

$\delta_{qis}^{hmg}(k)$ equals 1 if person m in group g of household type h choosing JATP q performs activity i at location s during interval k and 0 otherwise

$\xi_{qpw}^{hmg}(k)$ equals 1 if person m in group g of household type h choosing JATP q enters path p between OD pair w by private car during interval k and 0 otherwise

$\hat{\xi}_{qw}^{hmg}(k)$ equals 1 if person m in group g of household type h choosing JATP q uses public transit between OD pair w during interval k and 0 otherwise

$\zeta_{ap}^{wl}(k)$ equals 1 if private cars entering path p between OD pair w during interval k arrive at link a during interval l and 0 otherwise

Functions

u_q^h daily net utility for household type h choosing JATP q

$u_{is}^{hmg}(k)$ utility for person m in group g of household type h performing activity i at location s during interval k

$u_{is}^{hm}(k)$ marginal utility for person m of household type h performing activity i alone at location s during interval k

$c_a^{hmg}(k)$ travel cost for person m in group g of household type h traversing link a by private car during interval k

$c_{pw}^{hmg}(k)$ travel cost for person m in group g of household type h entering path p between OD pair w by private car during interval k

$t_p^w(k)$ travel time for private cars entering path p between OD pair w during interval k

$\bar{t}_p^w(k)$ estimated travel time at equilibrium for private cars entering path p between OD pair w during interval k

$t_a(k)$ travel time by private car on link a during interval k

$\bar{t}_a(k)$ estimated travel time at equilibrium by private car on link a during interval k

$\lceil . \rceil$ function to converse the travel time to the integer time interval unit

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Appendix A. Feasibility of JATPs

In this appendix, we formulate the flow propagation relationship between JATPs, paths, and links. The 0–1 integer variables in constraints (10)–(14) satisfy the following constraints:

$$\underbrace{\sum_{g=1}^{G^h}\sum_{i\in I}\delta_{qis}^{hmg}(k)}_{\text{person } m \text{ at } s \text{ during } k} = \underbrace{\sum_{g=1}^{G^h}\sum_{i\in I}\delta_{qis}^{hmg}(k-1)}_{\text{person } m \text{ at } s \text{ during } k-1} + \underbrace{\sum_{g=1}^{G^h}\sum_{w\in W^+(s)}\xi_{qw}^{hmg}(k-\lceil\hat{t}_w(k)\rceil-1) - \sum_{g=1}^{G^h}\sum_{w\in W^-(s)}\xi_{qw}^{hmg}(k-1)}_{\text{person } m \text{ using public transit arrives at and departs from } s \text{ during } k-1}$$

$$+\underbrace{\sum_{g=1}^{G^h}\sum_{w\in W^+(s)}\sum_{p\in P^w}\xi_{qpw}^{hmg}(k-\lceil t_p^w(k)\rceil-1) - \sum_{g=1}^{G^h}\sum_{w\in W^-(s)}\sum_{p\in P^w}\xi_{qpw}^{hmg}(k-1)}_{\text{person } m \text{ using private car arrives at and departs from } s \text{ during } k-1} \quad \forall s,q,m,h,k, \tag{42a}$$

$$\underbrace{\sum_{g=1}^{G^h}\sum_{i\in I}\frac{1}{|g|}\delta_{qis}^{hmg}(k)}_{\text{the private car carrying person } m \text{ at } s \text{ during } k} = \underbrace{\sum_{g=1}^{G^h}\sum_{i\in I}\frac{1}{|g|}\delta_{qis}^{hmg}(k-1)}_{\text{the private car carrying person } m \text{ at } s \text{ during } k-1}$$

$$+\underbrace{\sum_{g=1}^{G^h}\sum_{w\in W^+(s)}\sum_{p\in P^w}\frac{1}{|g|}\xi_{qpw}^{hmg}(k-\lceil t_p^w(k)\rceil-1) - \sum_{g=1}^{G^h}\sum_{w\in W^-(s)}\sum_{p\in P^w}\frac{1}{|g|}\xi_{qpw}^{hmg}(k-1)}_{\text{the private car carrying person } m \text{ arrives at and departs from } s \text{ during } k-1} \quad \forall s,q,m,h,k. \tag{42b}$$

Constraints (42a) and (42b) ensure the consistent movement of the flows for person m of household type h and his/her private car on JATP q forward in space and time through location s during interval k where $W^-(s)$ and $W^+(s)$ are respectively the sets of OD pairs with s being destinations and origins. Recall that $\hat{t}_w(k)$ is the constant transit travel time for departing from the origin of OD pair w during interval k, $t_p^w(k)$ the travel time for private cars entering path p between OD pair w during interval k, $\lceil . \rceil$ a function used to convert travel time to an integer time interval in which $\lceil t \rceil = k$ if $k \le t/\sigma < k+1$.

The path travel time in constraints (42a) and (42b) is calculated by

$$t_p^w(k) = \sum_{a\in A}\sum_{l=k}^{K} t_a(l)\zeta_{ap}^{wl}(k) \quad \forall p,w,k, \tag{42c}$$

in which $\zeta_{ap}^{wl}(k)$ satisfies

$$(k+\lceil t_{pa}^w(k)\rceil - l)\zeta_{ap}^{wl}(k) \le 0 \quad \forall a,p,w,k,l, \tag{42d}$$

$$(k+\lceil t_{pa}^w(k)\rceil - l + 1)\zeta_{ap}^{wl}(k) \ge 0 \quad \forall a,p,w,k,l. \tag{42e}$$

Constraints (42d) and (42e) force each path to use its links during the time intervals compatible with the link travel times, $t_{pa}^w(k)$ is the travel time for private cars entering path p between OD pair w during interval k to the tail node of link a (Janson (1991))

$$t_{pa}^w(k) = t_a(k+\lceil t_{pb}^w(k)\rceil) + t_{pb}^w(k) \quad \forall a,b,p,w,k, \tag{42f}$$

where b is the preceding link of link a on path p. When a is the last link on path p, $t_p^w(k) = t_a(k+\lceil t_{pa}^w(k)\rceil) + t_{pa}^w(k)$.

In addition, the following constraints must hold.

$$\sum_{n\in g}\xi_{qpw}^{hng}(k)\varsigma_q^{hn} = \xi_{qpw}^{hmg}(k) \quad \forall p,w,q,g,m,h,k, \tag{42g}$$

$$\sum_{i\in I}\delta_{qis}^{hmg}(k) \le 1 \quad \forall s,q,g,m,h,k, \tag{42h}$$

$$\sum_{k=1}^{K} \xi_{qpw}^{hmg}(k) \leq 1 \quad \forall p, w, q, g, m, h, k, \tag{42i}$$

$$\sum_{k=1}^{K} \hat{\xi}_{qw}^{hmg}(k) \leq 1 \quad \forall w, q, g, m, h, k, \tag{42j}$$

$$\sum_{l=k}^{K} \zeta_{ap}^{wl}(k) \leq 1 \quad \forall a, w, p, k, l, \tag{42k}$$

$$\sum_{m=1}^{M^h} \varsigma_{q}^{hm} \leq V^h \quad \forall q, h. \tag{42l}$$

Constraint (42g) is the ridesharing constraint, which forces each travel group of private car users to have at least one driver from the household. Constraint (42h) allows each JATP to have only one activity during one interval. Constraints (42i) and (42j) allow each JATP to use each of its private car paths and transit dummy links during only one departure interval. Constraint (42k) allows each private car path to use each of its links during only one interval for each departure time. Constraint (42l) is the car allocation constraint where ς_q^{hm} equals 1 if household type h assigns a private car to person m choosing JATP q during the day and 0 otherwise.

Finally, the boundary constraints are

$$\delta_{qi_0s_0}^{hmg}(1) = \delta_{qi_0s_0}^{hmg}(K) = 1 \quad \forall q, g, m, h. \tag{42m}$$

Constraint (42m) shows that each person m of household type h choosing JATP q take part in activity i_0 at location s_0 during the initial and final intervals.

Appendix B. Proofs

Proof of Proposition 1. For a fixed flow propagation relationship realized under HO in constraints (10)–(14), i.e., $\delta_{qis}^{hmg}(k) = \delta_{qis}^{hmg}(k)^*$, $\xi_{qpw}^{hmg}(k) = \xi_{qpw}^{hmg}(k)^*$, $\hat{\xi}_{qw}^{hmg}(k) = \hat{\xi}_{qw}^{hg}(k)^*$, and $\zeta_{ap}^{wl}(k) = \zeta_{ap}^{wl}(k)^*$, the Lagrangian of Eqs. (9)–(11) is expressed as

$$L(\mathbf{v}, \mu, \lambda) = Z(\mathbf{v}) - \sum_{h=1}^{H} \mu^h \left(\sum_{q \in Q^h} f_q^h - D^h \right) - \sum_{h=1}^{H} \sum_{q \in Q^h} \lambda_q^h (-f_q^h), \tag{43a}$$

where $\mu^h \geq 0$ and $\lambda_q^h \geq 0$ are the Lagrangian multipliers of Eqs. (10)and (11), respectively.

Based on Eqs. (12), (13) and $\bar{t}_a(k) = t_a(k)^*, \forall a, k$, at equilibrium, one can derive

$$\frac{\partial Z(\mathbf{v})}{\partial v_{is}^{hmg}(k)} = \left(1 + \alpha_{is}^{hg}(k)\right) u_{is}^{hm}(k) \quad \forall i, s, g, m, h, k, \tag{43b}$$

$$\frac{\partial v_{is}^{hmg}(k)}{\partial f_q^h} = \delta_{qis}^{hmg}(k)^* \quad \forall i, s, g, m, h, k, \tag{43c}$$

$$\frac{\partial Z(\mathbf{v})}{\partial v_a^{hmg}(k)} = \gamma \left(1 - \beta_a^{hg}(k)\right) \bar{t}_a(k) + \frac{1}{|g|} \phi_a^{hg}(k) \quad \forall a, g, m, h, k, \tag{43d}$$

$$\frac{\partial v_a^{hmg}(k)}{\partial f_q^h} = \sum_{w \in W} \sum_{p \in P^w} \sum_{l=k}^{K} \xi_{qpw}^{hmg}(k)^* \zeta_{ap}^{wl}(k)^* \quad \forall a, g, m, h, k, \tag{43e}$$

$$\frac{\partial Z(\mathbf{v})}{\partial e_w(k)} = \hat{c}_w(k) \quad \forall w, k, \tag{43f}$$

$$\frac{\partial e_w(k)}{\partial f_q^h} = \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} \hat{\xi}_{qw}^{hmg}(k)^* \quad \forall w, q, g, m, h, k. \tag{43g}$$

The above results yield

$$\frac{\partial Z(\mathbf{v})}{\partial f_q^h} = u_q^h \quad \forall q, h, \tag{43h}$$

$$\frac{\partial L(\mathbf{v},\mu,\lambda)}{\partial f_q^h} = u_q^h - \mu^h + \lambda_q^h \quad \forall q,h. \tag{43i}$$

The Kuhn-Tucker conditions are then given by

$$u_q^h - \mu^h + \lambda_q^h = 0 \quad \forall q,h, \tag{43j}$$

$$-f_q^h \lambda_q^h = 0 \quad \forall q,h. \tag{43k}$$

Following Eq. (43k), if $f_q^h > 0$ then $\lambda_q^h = 0$, and $f_q^h = 0$ then $\lambda_q^h \geq 0$. Hence, Eqs. (43j) and (43k) lead to the following condition:

$$\begin{cases} u_q^h - \mu^h = 0 & \text{if } f_q^h > 0 \\ u_q^h - \mu^h \leq 0 & \text{if } f_q^h = 0 \end{cases} \quad \forall q,h. \tag{43l}$$

The condition (43l) is indeed the HO condition (8). The proof is completed. $\square$

Proof of Proposition 2. Because of no intra-household interactions among individuals, we can treat each individual in the system as a separate household. Then, the sizes of all households and all travel groups equal to one, i.e., $M^h = 1, G^h = 1, |g| = 1, \forall h$. According to Eqs. (3) and (7), $\alpha_{is}^{hg}(k) = 0$ and $\beta_a^{hg}(k) = 0$ when $|g| = 1$, and $\bar{t}_a(k) = t_a(k)^*, \forall a,k$, at equilibrium. The HO objective function (20) is simplified as

$$\begin{aligned} \max Z(\mathbf{v}) = &\sum_{h=1}^{H} \sum_{m=1}^{1} \sum_{g=1,|g|=1}^{1} \sum_{k=1}^{K} \sum_{i \in I} \sum_{s \in S} u_{is}(k) v_{is}^{hmg}(k) - \sum_{k=1}^{K} \sum_{w \in W} \hat{c}_w(k) e_w(k) \\ &- \sum_{k=1}^{K} \sum_{a \in A} \int_0^{\tilde{v}_a(k)} \gamma t_a(\omega) \mathrm{d}\omega - \sum_{h=1}^{H} \sum_{m=1}^{1} \sum_{g=1,|g|=1}^{1} \sum_{k=1}^{K} \sum_{a \in A} \phi_a^{hg}(k) v_a^{hmg}(k). \end{aligned} \tag{44a}$$

Similar to the equivalence analysis of Proposition (1), we can derive

$$\begin{aligned} \frac{\partial Z(\mathbf{v})}{\partial f_q^h} = &\sum_{m=1}^{1} \sum_{g=1,|g|=1}^{1} \sum_{k=1}^{K} \sum_{i \in I} \sum_{s \in S} u_{is}^{hm}(k) \delta_{qis}^{hmg}(k)^* - \sum_{m=1}^{1} \sum_{g=1,|g|=1}^{1} \sum_{k=1}^{K} \sum_{w \in W} \hat{c}_w(k) \hat{\xi}_{qw}^{hmg}(k)^* \\ &- \sum_{m=1}^{1} \sum_{g=1,|g|=1}^{1} \sum_{k=1}^{K} \sum_{w \in W} \sum_{p \in P^w} \sum_{a \in A} \sum_{l=k}^{K} \left(\gamma t_a(l) + \phi_a^{hg}(l)\right) \xi_{qpw}^{hmg}(k)^* \zeta_{ap}^{wl}(k)^* = u_q^h \quad \forall q,h, \end{aligned} \tag{44b}$$

to yield the following HO condition

$$\begin{cases} u_q^h - \mu^h = 0 & \text{if } f_q^h > 0 \\ u_q^h - \mu^h \leq 0 & \text{if } f_q^h = 0 \end{cases} \quad \forall q,h. \tag{44c}$$

According to Eq. (44b), the net utility u_q^h for household type h choosing JATP q only corresponds to the only person of the household. Thus the HO condition (44c) is equivalent to the UE condition. The proof is completed. $\square$

Proof of Proposition 3. Similar to the equivalence analysis of Proposition (1), taking the partial derivative of $\tilde{Z}(\mathbf{v})$ in (21), one can obtain

$$\frac{\partial \tilde{Z}(\mathbf{v})}{\partial v_a^{hmg}(k)} = \gamma \left(1 - \beta_a^{hg}(k)\right) t_a(k) + \frac{1}{|g|} \left(\phi_a^{hg}(k) + \gamma \frac{\partial t_a(k)}{\partial \tilde{v}_a(k)} \sum_{j=1}^{H} \sum_{n=1}^{M^j} \sum_{z=1}^{G^j} \left(1 - \beta_a^{jz}(k)\right) v_a^{jnz}(k) \right) \quad \forall a,g,m,h,k. \tag{45a}$$

Denote

$$\tau_a(k) = \gamma \frac{\partial t_a(k)}{\partial \tilde{v}_a(k)} \sum_{j=1}^{H} \sum_{n=1}^{M^j} \sum_{z=1}^{G^j} \left(1 - \beta_a^{jz}(k)\right) v_a^{jnz}(k) \quad \forall a,k. \tag{45b}$$

Substituting $\frac{\partial \tilde{Z}(\mathbf{v})}{\partial v_a^{hmg}(k)}$ into $\frac{\partial \tilde{Z}(\mathbf{v})}{\partial f_q^h}$, we have

$$\frac{\partial \tilde{Z}(\mathbf{v})}{\partial f_q^h} = \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} \sum_{k=1}^{K} \sum_{i \in I} \sum_{s \in S} \left(1 + \alpha_{is}^{hg}(k)\right) u_{is}^{hm}(k) \delta_{qis}^{hmg}(k)^* - \sum_{m=1}^{M^h} \sum_{g=1}^{G^h} \sum_{k=1}^{K} \sum_{w \in W} \hat{c}_w(k) \hat{\xi}_{qw}^{hmg}(k)^* \tag{45c}$$

$$-\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{k=1}^{K}\sum_{w\in W}\sum_{p\in P^w}\sum_{a\in A}\sum_{l=k}^{K}\left(\gamma\left(1-\beta_a^{hg}(l)\right)t_a(l)+\frac{1}{|g|}\left(\phi_a^{hg}(l)+\tau_a(l)\right)\right)\xi_{qpw}^{hmg}(k)^*\zeta_{ap}^{wl}(k)^* \quad \forall q,h.$$

From Eq. (45c), by letting

$$c_a^{hmg}(k)=\gamma\left(1-\beta_a^{hg}(k)\right)t_a(k)+\frac{1}{|g|}\left(\phi_a^{hg}(k)+\tau_a(k)\right) \quad \forall a,h,m,g,k, \tag{45d}$$

the HO problem (20) is reduced to the HSO problem. The proof is completed. □

Proof of Proposition 4. To prove that u_q^h is strictly decreasing with JATP flows, we have to prove that $\frac{\partial u_q^h}{\partial f_q^h}<0, \forall q,h$. Due to the fixed flow propagation relationship, we can derive

$$\frac{\partial u_q^h}{\partial f_q^h}=-\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{k=1}^{K}\sum_{w\in W}\sum_{p\in P^w}\sum_{a\in A}\sum_{l=k}^{K}\left(\gamma\left(1-\beta_a^{hg}(l)\right)\frac{\partial t_a(l)}{\partial f_q^h}\right)\xi_{qpw}^{hmg}(k)^*\zeta_{ap}^{wl}(k)^* \quad \forall q,h, \tag{46a}$$

where

$$\frac{\partial t_a(l)}{\partial f_q^h}=\frac{\partial t_a(l)}{\partial \tilde{v}_a(l)}\frac{\partial \tilde{v}_a(l)}{\partial f_q^h}=\frac{\partial t_a(l)}{\partial \tilde{v}_a(l)}\sum_{m=1}^{M^h}\sum_{g=1}^{G^h}\sum_{w\in W}\sum_{p\in P^w}\sum_{k=1}^{l}\frac{1}{|g|}\xi_{qpw}^{hmg}(k)^*\zeta_{ap}^{wl}(k)^* \quad \forall a,q,h,l. \tag{46b}$$

According to **A6**, $\frac{\partial t_a(l)}{\partial \tilde{v}_a(l)}>0$. We also have $\frac{\partial \tilde{v}_a(l)}{\partial f_q^h}\geq 0$ in Eq. (46b). Finally, due to $\gamma>0$ and $0\leq\beta_a^{hg}(l)<1$, $\frac{\partial u_q^h}{\partial f_q^h}<0$ in Eq. (46a). The proof is completed. □

Appendix C. The JATS supernetwork construction

In this appendix, details of the JATS supernetwork construction are presented. First, we show the procedure of the ATS supernetwork expansion. Given an ATS supernetwork for person m, the sets of ATS nodes and links are partitioned as follows.

$$\mathcal{N}^m=\mathcal{N}_1^m\bigcup\mathcal{N}_2^m, \quad \forall m,$$
$$\mathcal{L}^m=\mathcal{L}_1^m\bigcup\mathcal{L}_2^m\bigcup\mathcal{L}_3^m\bigcup\mathcal{L}_4^m\bigcup\mathcal{L}_5^m\bigcup\mathcal{L}_6^m, \quad \forall m,$$

where $\mathcal{N}_1^m$ and $\mathcal{N}_2^m$ are the sets of ATS nodes for person m under the driver (SD and RD) and passenger (RP and TP) roles, respectively; $\mathcal{L}_1^m$ and $\mathcal{L}_2^m$ are the sets of *activity* links for person m under the driver and passenger roles, respectively; $\mathcal{L}_3^m$, $\mathcal{L}_4^m$, $\mathcal{L}_5^m$ and $\mathcal{L}_6^m$ are the sets of *travel links* for person m under the SD, RD, RP and TP roles, respectively. The steps in the ATS supernetwork construction are given as follows.

ATS nodes.

- For each activity location $s\in S$ and interval $k=1\ldots K+1$, construct
  - one ATS node $D(m,s,k)\in\mathcal{N}_1^m$ for person m under the driver role,
  - one ATS node $P(m,s,k)\in\mathcal{N}_2^m$ for person m under the passenger role.
- For each path $p\in P^w, w\in W$, let $\kappa=\lceil\bar{t}_p^w(k)\rceil$. If $\kappa>1$, construct
  - $(\kappa-1)$ ATS nodes $SD(m,p,k,k+1),\ldots,SD(m,p,k,k+\kappa-1)\in\mathcal{N}_1^m$ representing the travel of person m under the SD role,
  - $(\kappa-1)$ ATS nodes $RD(m,p,k,k+1),\ldots,RD(m,p,k,k+\kappa-1)\in\mathcal{N}_1^m$ representing the travel of person m under the RD role,
  - $(\kappa-1)$ ATS nodes $RP(m,p,k,k+1),\ldots,RP(m,p,k,k+\kappa-1)\in\mathcal{N}_2^m$ representing the travel of person m under the RP role.
- For each OD pair $w\in W$, let $\kappa=\lceil\hat{t}_w(k)\rceil$. If $\kappa>1$, construct $(\kappa-1)$ ATS nodes $TP(m,w,k,k+1),\ldots,TP(m,w,k,k+\kappa-1)\in\mathcal{N}_2^m$ representing the travel of person m under the TP role.

ATS activity links.

- Construct two activity links $D(m,s,k)\to D(m,s,k+1)\in\mathcal{L}_1^m$ and $P(m,s,k)\to P(m,s,k+1)\in\mathcal{L}_2^m$ that represent the activity participation of person m under the driver and passengers roles at location s during interval k, respectively.

ATS travel links.

- For the SD role: let $\kappa = \lceil \bar{t}_p^w(k) \rceil$, if $\kappa > 1$, construct κ travel links $D(m,r,k) \to SD(m,p,k,k+1)$, $SD(m,p,k,k+1) \to SD(m,p,k,k+2), \ldots, SD(m,p,k,k+\kappa-1) \to D(m,s,k+\kappa) \in \mathcal{L}_3^m$ that represent the travel of person m during intervals $k, k+1, \ldots, k+\kappa$ for entering path p between OD pair w during interval k. Otherwise, construct a travel link $D(m,r,k) \to D(m,s,k+\kappa) \in \mathcal{L}_3^m$.
- For the RD role: let $\kappa = \lceil \bar{t}_p^w(k) \rceil$, if $\kappa > 1$, construct κ travel links $D(m,r,k) \to RD(m,p,k,k+1)$, $RD(m,p,k,k+1) \to RD(m,p,k,k+2), \ldots, RD(m,p,k,k+\kappa-1) \to D(m,s,k+\kappa) \in \mathcal{L}_4^m$ that represent the travel of person m during intervals $k, k+1, \ldots, k+\kappa$ for entering path p between OD pair w during interval k. Otherwise, construct a travel link $D(m,r,k) \to D(m,s,k+\kappa) \in \mathcal{L}_4^m$.
- For the RP role: let $\kappa = \lceil \bar{t}_p^w(k) \rceil$, if $\kappa > 1$, construct κ travel links $P(m,r,k) \to RP(m,p,k,k+1)$, $RP(m,p,k,k+1) \to RP(m,p,k,k+2), \ldots, RP(m,p,k,k+\kappa-1) \to P(m,s,k+\kappa) \in \mathcal{L}_5^m$ that represent the travel of person m during intervals $k, k+1, \ldots, k+\kappa$ for entering path p between OD pair w during interval k. Otherwise, construct a travel link $P(m,r,k) \to P(m,s,k+\kappa) \in \mathcal{L}_5^m$.
- For the TP role: let $\kappa = \lceil \hat{t}_w(k) \rceil$, if $\kappa > 1$, construct κ travel links $P(m,r,k) \to TP(m,w,k,k+1)$, $TP(m,w,k,k+1) \to TP(m,w,k,k+2), \ldots, TP(m,w,k,k+\kappa-1) \to P(m,s,k+\kappa) \in \mathcal{L}_6^m$ that represent the travel of person m during intervals $k, k+1, \ldots, k+\kappa$ between OD pair w during interval k. Otherwise, construct a travel link $P(m,r,k) \to P(m,s,k+\kappa) \in \mathcal{L}_6^m$.

ATS start/end links.

- For a given start location $s \in S$ of person m, construct two start links Start $\to D(m,s,1)$ under the driver role and Start $\to P(m,s,1)$ under the passenger role.
- For a given end location $s \in S$ of person m, construct two end links $D(m,s,K+1) \to$ End under the driver role and $P(m,s,K+1) \to$ End under the passenger role.

Recall that in the above ATS supernetwork construction, $\bar{t}_p^w(k)$ is the estimated path travel time for private cars at equilibrium and $\hat{t}_w(k)$ the constant and given transit travel time.

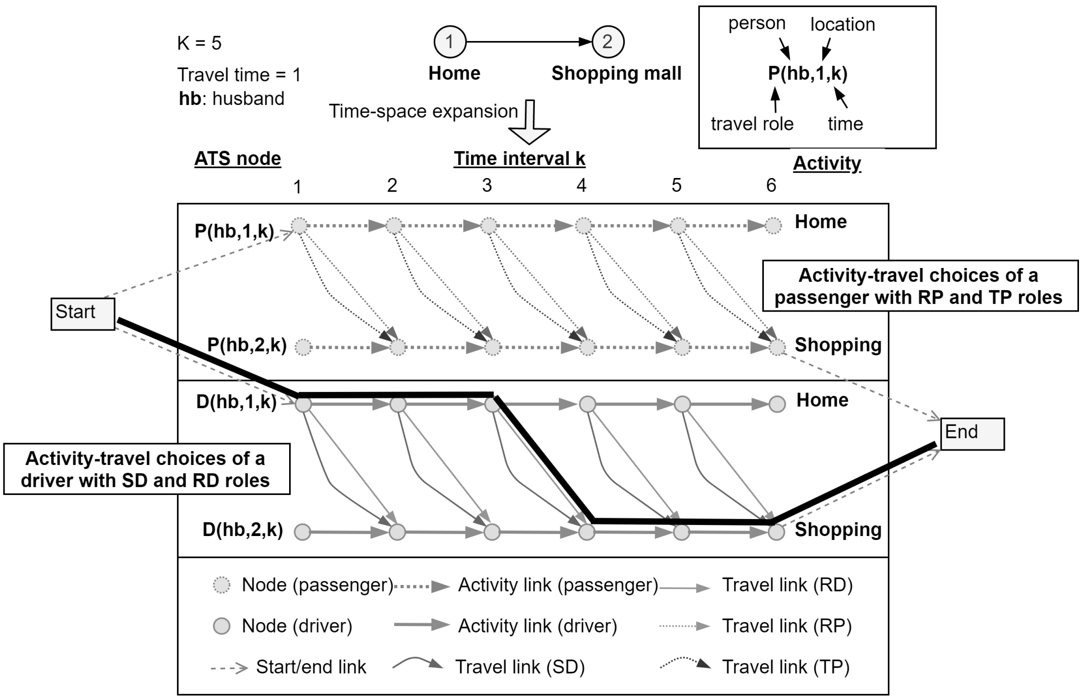


Figure 11: A simple base network and the constructed ATS supernetwork of the husband of a worker couple

Consider an example road network, which comprises two locations with two activities, i.e., home and shopping. We assume that a worker couple starts their schedules at home and ends at the shopping mall, and the household has two cars. There are five time intervals. Travel times by private car or public transit are one interval. Fig. 11 shows the base network and the ATS supernetwork of the husband. Note that because of the symmetry of the two household members, only the ATS supernetwork of the husband is presented. The ATS supernetwork for the wife is the same. A path from Start to End in the ATS supernetwork represents the daily activity-travel choices of the husband under either the driver or passenger role. For example, the highlighted path in Fig. 11 represents the following choices of the husband: (i) stay at home during [1, 3], (ii) travel to the shopping mall under the RD role during [3, 4], and (iii) conduct shopping during [4, 6]. Note that to simplify the illustration, the travel time in this example is only one interval. For longer travel times, we would have to decompose the travel time into one-interval travel links in the supernetwork (see the procedure of the ATS supernetwork expansion presented above).

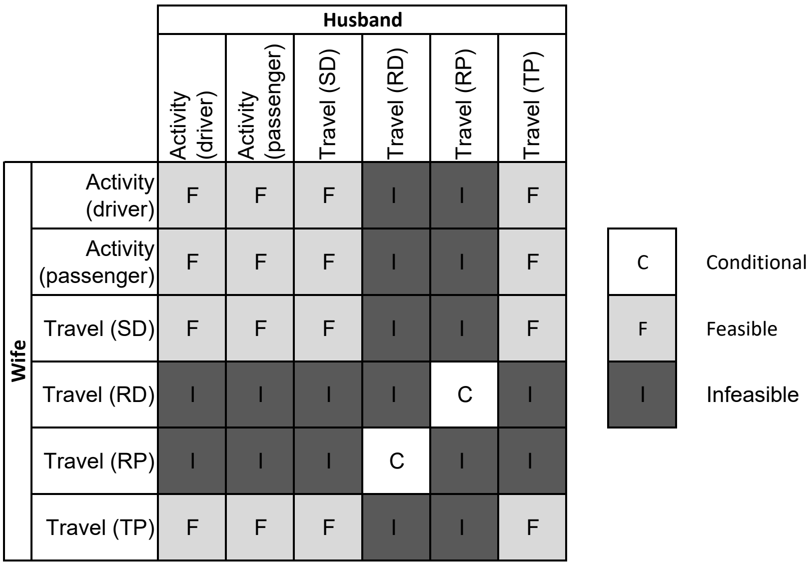


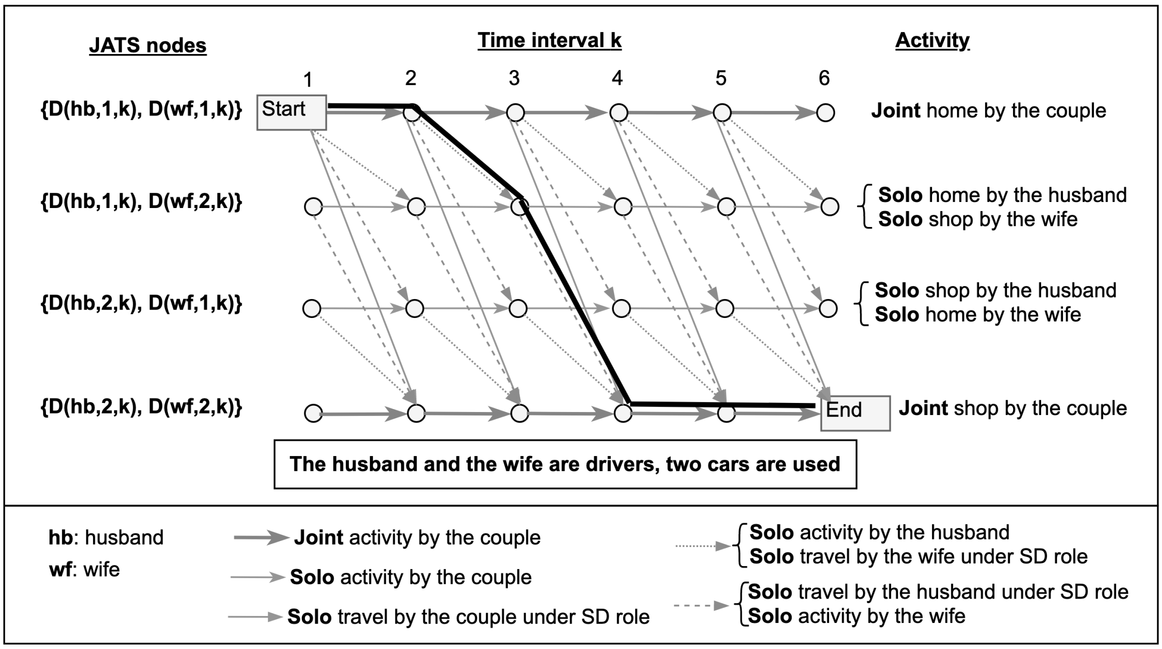
Figure 12: Feasibility of a JATS link for a worker couple

Following Eq. (32), a JATS link during an interval comprises a vector of ATS links during that interval, and not all JATS links are feasible. Fig. (12) illustrates the feasibility of a JATS link, which is a combination of two ATS links, for the worker couple. In this figure, dark gray cells show infeasible combinations (I), light gray cells show feasible combinations (F), and white cells indicate *conditional* feasible combinations (C), which are only feasible under a certain condition. For instance, if the ATS link of the husband is an activity link (either under the driver or passenger role) or a travel link under the TP role, we can combine the link with any ATS link of the wife, except for travel links under the roles RD and RP. This is because travel links under the RD and RP roles are governed by the ridesharing constraint, in which a shared ride can only happen if there is one RD and one RP. If the ATS link of the husband is a travel link under the role RD, we can only combine the link with a travel link under the RP role of the wife, provided that the two ATS links use the same path and same departure time.

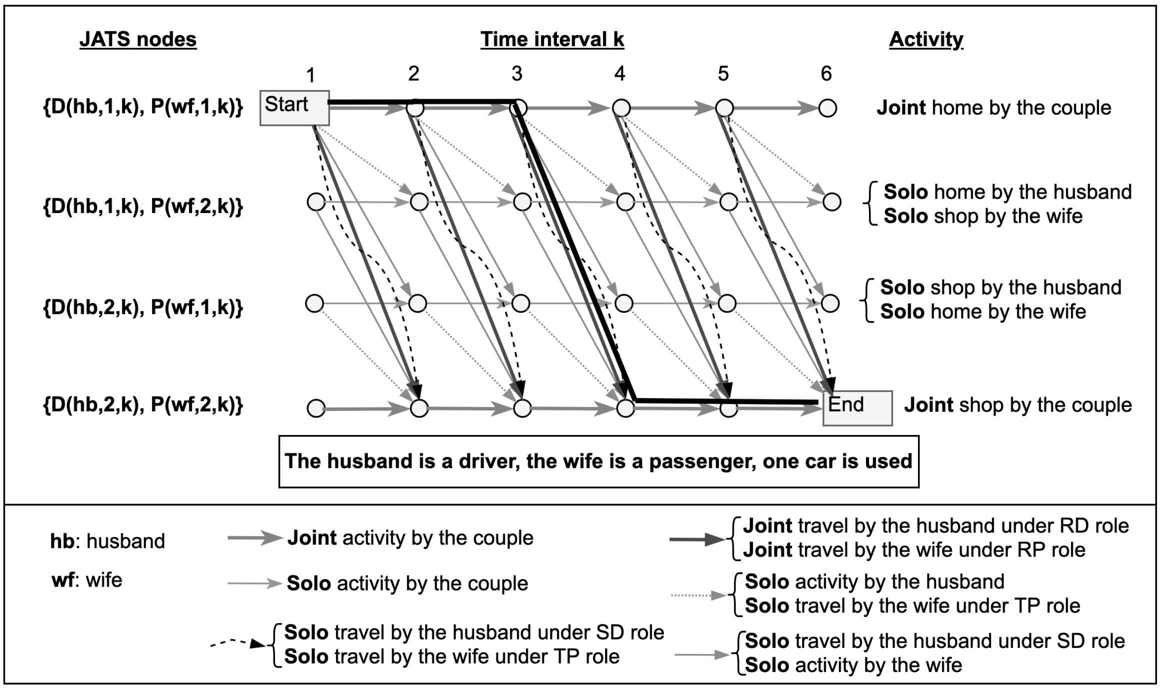
Fig. 13 illustrates the JATS supernetwork of the worker couple made by joining the two ATS supernetworks of the husband (shown in Fig. 11) and the wife. The figure presents two cases: in Case 1 the husband and the wife are drivers and in Case 2 the husband is a driver and the wife is a passenger. Note that two other cases are also possible: Case 3, in which the husband and the wife are passengers, and Case 4, in which the husband is a passenger and the wife is a driver. We excluded the presentation of Cases 3 and 4 because they are similar to Cases 1 and 2, respectively. A JATS path from Start to End in each case represents the daily activity-travel choices of the couple under different travel roles and their interactions.

For example, the highlighted path in Fig. 13a represents the following choices: (i) the couple stays at home together during [1, 2], (ii) the husband stays home alone and the wife travels solo to the shopping mall under the SD role during [2, 3], (iii) the husband travels solo to the shopping mall under the SD role and the wife conducts solo shopping during [3, 4], and (iv) the couple conducts joint shopping during [4, 6]. Similarly, the highlighted path in Fig. 13b represents the following: (i) the couple jointly stays at home during [1, 3], (ii) the couple jointly travels to the shopping mall, in which the husband is RD and the wife is RP, during [3, 4], (iii) the couple performs joint shopping during [4,6]. Depending on which case is used, we can derive the household car use and car allocation. For instance, two cars are used and each person is allocated a car in Case 1, while one car is used and only the husband is assigned a car in Case 2. Additionally, a household context, such as the number of cars or car licenses owned by the household, can be imposed. For example, if the household has only one car, Case 1 is infeasible; if only the husband owns a car license, Cases 1 and 4 are infeasible.

Given the JATS supernetwork construction above, we can analyze the size of the supernetwork as follows. Let $S^m \subseteq S$ be the set of activity and drop-off/pickup locations, and $W^m \subseteq W$ the set of OD pairs of person m. For each interval k, there are $|S^m|$ activity links for person m under SD and RD roles, $|S^m|$ activity links for person m under the RP and TP roles, $|P^w|$ travel links for person m under the SD role, $|P^w|$ travel links for person m under the RD role, $|P^w|$ travel links for person m under the RP role, and one travel link for person m under the TP role for each OD pair $w \in W^m$. Each private car trip using path p during interval k is then decomposed into $\lceil \bar{t}_p^w(k) \rceil$ (or $\lceil \hat{t}_w(k) \rceil$ for each transit trip) travel links. The



(a) Case 1: the husband, and the wife are drivers



(b) Case 2: the husband is a driver, and the wife is a passenger

Figure 13: The JATS supernetwork of a worker couple

number of links in ATS supernetwork $\mathcal{G}^m$ for person m is given by

$$|\mathcal{L}^m| = \sum_{k=1}^{K}\sum_{j=1}^{6}|\mathcal{L}_j^m(k)| = \sum_{k=1}^{K}\left(2|S^m| + 3\sum_{w\in W^m}\sum_{p\in P^w}\lceil \bar{t}_p^w(k)\rceil + \sum_{w\in W^m}\lceil \hat{t}_w(k)\rceil\right) \quad \forall m. \tag{47a}$$

Based on Eq. (32), the number of links in JATS supernetwork $\mathbf{G}^h$ for household type h is calculated as

$$|\mathbf{L}^h(k)| = \frac{1}{\kappa}\sum_{k=1}^{K}\prod_{m=1}^{M^h}|\mathcal{L}^m(k)| \quad \forall h, \tag{47b}$$

where $\kappa > 1$ in practice because JATS links only represent *feasible* combinations of ATS links rather than all possible combinations. As shown in Fig. 11, the average number of ATS links during each interval is 8 for one person. Fig. 13 shows that the average numbers of JATS links during each interval are 9 in Case 1 and 10 in Case 2. There are also two other similar cases. The average number of feasible JATS links for the couple during each interval is $2(9 + 10) = 38$, while the average number of all possible JATS links can be approximated as $8 \times 8 = 64$. Hence, the value of κ in Eq. (47b) is $\kappa \approx 64/38 \approx 1.68$.

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