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Bi-objective traffic count location model for mean and covariance of origin-destination estimation

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Abstract: This paper describes a bi-objective optimization model for the traffic count location problem in stochastic origin-destination (OD) traffic demand estimation. Two measures are defined to capture the maximum possible absolute error of the mean and the covariance of the estimated OD demand. The bounds of these two measures are mathematically deduced, and then the bi-objective optimization model is formulated to minimize the two upper bounds simultaneously. A surrogate-assisted genetic algorithm is proposed to solve this model, and a series of numerical examples are presented to demonstrate the applicability of the proposed model and the efficiency of the proposed algorithm.

Keywords: traffic count location; origin–destination estimation; covariance matrix; bi-objective optimization; surrogate-assisted genetic algorithm

1 Introduction

Due to the development of communication technology, collecting traffic data through traffic sensor systems becomes more and more popular in the field of transportation. Different types of traffic sensors (e.g., counting sensors, path-ID sensors, image sensors, and vehicle sensors) are installed for various purposes, such as OD estimation (Castillo et al., 2013), travel time estimation (Tang et al., 2018), link flow inference (Xu et al., 2016), path flow reconstruction (Castillo et al., 2012), and so on. On the one side, the traffic data collected by sensor systems are valuable and helpful for improving the service of transportation. On the other side, the sensor systems are usually impossible to cover the whole network due to physical limitations and budget constraints. Thus, the optimization of sensor locations has attracted increasing attention in the field of transportation.

Compared with other traffic information, the origin-destination (OD) matrix describes the traffic flow between a set of origins and a set of destinations, providing an important input parameter for urban traffic planning (González et al., 2019). In many applications, traffic counting sensor with the cheaper cost is widely used for OD estimation. Thus, the problem of optimizing the traffic count location has attracted increasing attention. However, most existing traffic count location models ignore the following two important features required to determine OD demand.

- The stochastic characteristics of OD demand, particularly the spatial correlations among the traffic demands of different OD pairs, have not been fully considered in traffic count location problems.
- The estimation errors for both the mean and covariance of stochastic OD demands have not been explicitly investigated in traffic count location problems.

In view of these limitations, this paper describes a new bi-objective model for optimizing the traffic count location to estimate the mean and covariance of OD demand.

It should be noted that most existing studies assumed that OD demands are independent. However, the correlation of OD demand should not be ignored. Generally, there are two categories of OD demand correlations, i.e., the spatial and temporal correlation (Fu and Lam, 2018). Firstly, spatial OD demand correlation refers to the correlations (or dependency to some extent) of the OD demands during the same hourly periods between different OD pairs in a spatial manner. Secondly, temporal OD demand correlation represents the correlations of OD demands for the same OD pair between different periods (e.g., 8:00 am–9:00 am, and 9:00 am–10:00 am). This paper aims to optimize the traffic count location for the estimation of the mean and spatial covariance of the OD demand between different OD pairs.

1.1 Literature review

A large number of OD demand studies have shown that the estimation accuracy is related to both the model being used and the number and location of sensors (Lu et al., 2013). Therefore, determining the number and location of sensors to cover the entire road network better is a core problem in transportation networks. In general, the number of counting sensors determines the scale of the survey, and their location determines the amount of information that can be collected. Therefore, the problem of traffic count location based on OD estimation is mainly concerned with the following two aspects: 1) Minimizing the cost of data collection, such as by minimizing the number of sensors or minimizing the total cost of their locations (Zanguia et al., 2015; Chen et al., 2016); 2) Maximizing the amount of information that can be obtained under the condition of limiting the number of sensors (Park and Haghani, 2015; Geetla et al., 2014). In short, an optimal traffic count location scheme must consider the limited resources available for data collection and collect as much traffic information as possible to improve the estimation accuracy of the OD matrix.

In the literature, the problem of traffic count location for OD estimation has been extensively studied. Existing traffic count location models concentrate on various modeling purposes. These models are summarized in Table 1 in terms of their objective function, consideration of network uncertainty, and modeling approach.

- **Objective function.** The traffic sensor location model for OD estimation is divided into the following two categories with respect to the types of objective functions: single-objective function and multiple-objective function. In the single-objective function model, the sensor location scheme is determined by solving an optimization problem with one objective function, such as minimizing the cost of sensor location or maximizing the amount of information gains. For example, Vieira et al. (2020) proposed a single-objective optimization model to tackle the traffic counting location problem (TCLP) with the purpose of the minimum number of sensors. Then, a progressive hybrid algorithm based on set covering was developed to solve the TCLP. Hadavi and Shafahi (2016) established a single-objective location model of vehicle identification sensor that aimed for the maximum number of uniquely identified OD pairs under budget constraints, and used GAMS software to solve the model. Regarding multi-objective function model, researchers considered different objectives to determine the sensor location schemes. For example, Fei and Mahmassani (2011) studied a bi-objective optimal sensor location model by integrating road information with coverage data for OD pairs. A greedy heuristic algorithm was then proposed to find the Pareto optimal solution. Owais et al. (2019) presented a multi-objective sensor location model that addressed a trade-off between the accuracy of OD estimation and the cost of sensor location. Then a multi-criteria meta-heuristics algorithm was designed to solve the proposed model with a polynomial time complexity.

- **Consideration of network uncertainty.** In the real world, the network uncertainty refers to the disturbances that affect transportation networks, such as adverse weather conditions, peak hour traffic flow fluctuations, signal failures and so on (Chen et al., 2010). Some Researchers have taken account of different uncertain circumstance to provide different types of sensor location models for OD estimation, including sensor failure model, human behavior stochasticity, and error propagation model. For example, considering the sensor failure probability, An et al. (2018) formulated a reliable sensor location model as a mixed-integer linear program that aimed at maximizing the accuracy of object surveillance under the risk of possible sensor disruptions. Danczyk et al. (2016) proposed a sensor configuration model to minimize overall freeway performance monitoring errors while considering probabilistic sensor failures. Concerning human behavior stochasticity, Moreira-Matias et al. (2016) studied the location of portable digital devices equipped with GPS for estimating the time-evolving OD demand, which follows the stochastic dynamics on the human behavior. In relation to the error propagation issue, Zhou and List (2010) proposed a scenario-based stochastic sensor location model that can recognize different uncertainty sources, such as the uncertainty in historical data, sensor measurement errors, and approximation errors.
- **Modeling approach.** As well as the mathematical programming approach used in the studies above, algebraic methods have been employed to solve the problem of traffic count location for OD estimation. For example, Rodriguez-Vega et al. (2019) solved the location problem of flow and turning ratio sensors by using the method of the rank of matrix, which can reduce the total number of sensors. Fu et al. (2016) developed a two-stage heterogeneous sensor location model, in which the first stage uses path reconstruction to determine the optimal locations of active sensors in the network and the second stage applies the theory of maximum clique to replace some active sensors with passive sensors strategically.

Table 1 Literature summary on the problem of traffic count location for OD estimation

Literature	Objective function	Consideration of network uncertainty	Modeling approach	Solution methods
Vieira et al. (2020)	Single-objective	No	Mathematical programming	Hybrid algorithm based on set covering
Hadavi and Shafahi (2016)	Single-objective	No	Mathematical programming	GAMS software
Fei and Mahmassani (2011)	Multi-objective	No	Mathematical programming	Greedy algorithm
Owais et al. (2019)	Multi-objective	No	Mathematical programming	Meta-heuristics algorithm
An et al. (2018)	Multi-objective	Sensor failure	Mathematical programming	Customized Lagrangian relaxation algorithm
Danczyk et al. (2016)	Single-objective	Sensor failure	Mathematical programming	Heuristic algorithm using *k*-shortest path search
Moreira-Matias et al. (2016)	Single-objective	Human behavior stochasticity	Mathematical programming	Incremental discretization framework
Zhou and List (2010)	Single-objective	Error propagation	Mathematical programming	Beam search algorithm
Rodriguez-Vega et al. (2019)	Single-objective	No	Algebraic method	Heuristic algorithm using the rank of matrix
Fu et al. (2016)	Single-objective	No	Graph theory	Heuristic algorithm using the maximum clique
This paper	**Multi-objective**	**Spatial covariance of OD demand**	**Mathematical programming**	**Surrogate-assisted genetic algorithm**

Although many studies on traffic count locations have been reported, few have considered the stochastic features of OD demand, instead of focusing on the mean estimation of OD demand. In fact, the OD traffic flow fluctuates randomly over time and should not be regarded as a deterministic variable. Thus, the OD demand has been treated as a random variable in network modeling (Chen and Xu, 2012). Some researchers have identified the importance of stochastic OD demand in traffic planning. For example, Waller et al. (2001) found that the relevant degree of OD demand plays an important role in estimating errors in the expected travel time. According to Zhao and Kockelman (2002), ignoring the correlation of OD demand reduces the reliability of traffic forecasts and affects infrastructure layout. To describe the stochasticity of OD demand, traditional OD estimation models need to be extended to estimate the mean and covariance of OD demand (Shao et al., 2015; Ji et al., 2011). Shao et al. (2014) proposed a bi-level programming model in which the weighted least-squares problem is adopted to estimate the OD mean and covariance in the upper level. The lower level is a reliability-based traffic assignment problem that is used to represent the risk-taking behavior of travelers under stochastic OD demand. Xing et al. (2013) proposed an information-theoretic sensor location model that aims to minimize total travel time uncertainties, including the uncertainty associated with prior travel time estimates, sensor measurement errors, and sampling errors. However, few studies have focused on the traffic count location for stochastic OD demand estimation. Specifically, current traffic count location models ignore the estimation of OD demand covariance when determining the optimal traffic count location scheme. To this end, this paper proposes a new traffic count location model that accounts for both the mean and covariance of OD demand under network uncertainty.

1.2 Contribution statement

This paper studies the traffic count location problem to obtain the mean and covariance of OD demand using traffic count data. The proposed model extends the existing works with the following new features.

- The proposed model considers the stochasticity of OD demand. Indeed, the traffic count location model is developed to provide the optimal traffic count location scheme that minimizes the estimation error of both the mean and covariance matrix of stochastic OD demand.
- To measure the accuracy of the estimated stochastic OD demand, the concept of maximum possible relative error (MPRE, Yang et al., 1991) is extended to a new concept named maximum possible absolute error (MPAE). MPAE is applicable to cases where the estimated values (e.g., mean or covariance of OD demand) are zero, whereas MPRE cannot be used in such cases.

The traffic count location model is formulated as a bi-objective optimization model with nonlinear constraints based on the concept of MPAE. Moreover, a surrogate-assisted genetic algorithm is adopted to solve the proposed model. Numerical examples are presented to demonstrate the applicability of the proposed model and solution algorithm, leading to some insightful discussions.

The remainder of this paper is organized as follows. In section 2, the MPAE is derived for the mean and covariance of OD demand estimation. Moreover, a rigorous mathematical proof of their bounds is also given. In section 3, a bi-objective traffic count location model is established. Section 4 introduces a surrogate-assisted genetic algorithm for solving this model by transforming the bi-objective optimization problem to a single-objective one. In section 5, two transportation networks are used to illustrate the applicability of the proposed model and the efficiency of the proposed algorithm. Finally, our conclusions and ideas for future studies are presented in section 6.

2 MPAE for estimating the mean and covariance of OD demand

2.1 Maximum Possible Relative Error

It is difficult to obtain the true OD matrix. Thus, accurately calculating the error in the OD estimation deserves further investigation. Yang et al. (1991) proposed the MPRE concept to represent the maximum possible deviation between the estimated and true mean OD demand. In the case of traffic constraints and the prior OD matrix, MPRE is unique and can evaluate the reliability of the resulting OD matrix. MPRE can usually be described as a special quadratic optimization problem. A brief explanation of MPRE is given below.

Consider a traffic network $\mathbf{G(N,A)}$, where $\mathbf{N}$ represents the node set and $\mathbf{A}$ represents the link set. $\tilde{\mathbf{A}}$ is the set of observed links, $\tilde{\mathbf{A}} \subset \mathbf{A}$, and $\mathbf{W}$ is the set of all OD pairs. q_w is the estimated mean traffic demand of OD pair $w \in \mathbf{W}$, and q_w^* is the true mean traffic demand of OD pair w. p_{aw} is the choice proportion of OD pair w through link $a \in \tilde{\mathbf{A}}$. v_a is the observed traffic flow on link a. It is assumed that the choice proportion p_{aw} is given and fixed. Thus, for all $a \in \tilde{\mathbf{A}}$, the estimated and true mean OD demand should satisfy the following equations.

$$\sum_{w\in W} p_{aw}q_w = v_a \qquad \forall\ a\in\tilde{\mathbf{A}} \tag{1}$$

$$\sum_{w\in W} p_{aw}q_w^* = v_a \qquad \forall\ a\in\tilde{\mathbf{A}} \tag{2}$$

It follows from Eqs. (1) and (2) that

$$\sum_{w\in W} p_{aw}(q_w^* - q_w) = 0 \quad \forall\ a\in\tilde{\mathbf{A}} \tag{3}$$

Define $\lambda_w = \left(q_w^* - q_w\right)\big/ q_w$ as the relative error between q_w^* and q_w. Substituting λ_w into Eq. (3), it follows that

$$\sum_{w\in W} p_{aw}q_w\lambda_w = 0 \quad \forall\ a\in\tilde{\mathbf{A}} \tag{4}$$

Let $\boldsymbol{\lambda} = \left(\lambda_w\right)_{w\in\mathbf{W}}$. Then, $\varphi(\boldsymbol{\lambda}) = \sum_w \left(\lambda_w\right)^2$ represents the relative deviation between the estimated and true mean OD demand. The smaller the value of $\varphi\left(\boldsymbol{\lambda}\right)$, the more accurate the estimated mean OD demand. Thus, MPRE is defined as the maximum value of $\varphi\left(\boldsymbol{\lambda}\right)$:

$$\text{MPRE} = \max_{\boldsymbol{\lambda}} \varphi\left(\boldsymbol{\lambda}\right) \tag{5}$$

Note that MPRE represents the maximum possible deviation between the estimated and true mean OD demand. Thus, smaller values of MPRE indicate a smaller estimation error for the mean OD demand. In view of this, MPRE can be treated as the objective of the traffic count location problem for estimating the mean OD demand.

2.2 Advantages of the Maximum Possible Absolute Error

MPRE was developed based on the popular statistical concept of relative error. Similarly, the absolute error can characterize the degree of similarity between an estimate and the true value. The relative error is independent of the scale of the estimate, but inapplicable if the estimate is zero. This is mainly because the concept of relative error $\lambda_w = \left(q_w^* - q_w\right)\big/ q_w$ requires the denominator in the above fraction is non-zero, i.e. $q_w \neq 0$. If the estimated OD demand is zero (i.e. $q_w = 0$), the definition of λ_w is not well defined mathematically. Thus, MPRE cannot be applied in such cases where the estimated mean and covariance of OD demand is zero. Actually, the estimated OD demand could be zero, as the true value is unknown. To overcome this difficulty, the concept of absolute error is used to derive the maximum possible absolute error (MPAE), which can handle cases in which the estimate is zero. Moreover, the absolute error and relative error are not significantly different when the precision of the OD matrix estimation is well controlled (Gan et al., 2005). Thus, the proposed MPAE is applicable to cases in which the estimated parameters are zero. In this paper, the estimated parameters are the mean and covariance of the OD demand. These two parameters could be zero. For example, if the traffic demand of two OD pairs is independent, the corresponding covariance between the two OD demands will be zero.

Different from the concept of relative error $\lambda_w = \left(q_w^* - q_w\right)\big/ q_w$, let $\eta_w^{mean} = q_w^* - q_w$ represent the absolute error between the true and estimated mean demand of OD pair w. It is noted that the absolute error η_w^{mean} does not include the modulus operator and may be positive or negative. Then, Eq. (3) can be rewritten as

$$\sum_{w\in W} p_{aw}\eta_w^{mean} = 0 \qquad \forall\ a\in\tilde{\mathbf{A}} \tag{6}$$

Define

$$G(\eta^{mean}) = \sqrt{\frac{\sum_w \left(\eta_w^{mean}\right)^2}{n}} \tag{7}$$

where $\eta^{mean} = \left(\text{L}\ , \eta_w^{mean}, \text{L}\ \right)^T$ and n is the total number of OD pairs.

As a measure of the mean estimation error of OD demand, small values of $G(\eta^{mean})$ represent highly accurate estimation of the mean OD demand. Inspired by the concept of MPRE, the MPAE of mean OD demand (MPAEM) is defined as the following optimization problem (Yuan, 2009):

$$\text{MPAEM}(\eta^{mean}) \ = \ \max \ \ G(\eta^{mean}) \tag{8}$$

$$\text{s.t.} \quad \sum_{w \in W} p_{aw} \eta_w^{mean} = 0 \quad \forall \ a \in \tilde{\mathbf{A}}$$

The true and estimated covariance of OD demand should satisfy the following conditions:

$$\sum_{w \in \mathbf{W}} \sum_{w' \in \mathbf{W}} p_{aw} p_{bw'} \sigma_{ww'}^{q} = \sigma_{ab}^{v} \qquad \forall \ a,b \in \tilde{\mathbf{A}} \tag{9}$$

$$\sum_{w \in \mathbf{W}} \sum_{w' \in \mathbf{W}} p_{aw} p_{bw'} \sigma_{ww'}^{q^*} = \sigma_{ab}^{v} \qquad \forall \ a,b \in \tilde{\mathbf{A}} \tag{10}$$

where $\sigma_{ww'}^{q}$ and $\sigma_{ww'}^{q^*}$ are the estimated and true covariance of traffic demand between OD pairs w and w', respectively; σ_{ab}^{v} is the covariance of observed traffic flow on links a and b.

It follows from Eqs. (9) and (10) that

$$\sum_{w \in \mathbf{W}} \sum_{w' \in \mathbf{W}} p_{aw} p_{bw'} (\sigma_{ww'}^{q^*} - \sigma_{ww'}^{q}) = 0 \quad \forall \ a,b \in \tilde{\mathbf{A}} \tag{11}$$

Similar to the definition of η_w^{mean}, let $\eta_{ww'}^{\text{cov}} = \sigma_{ww'}^{q*} - \sigma_{ww'}^{q}$ represent the absolute error between the true and estimated covariance of traffic demand between OD pairs w and w'. The absolute error of covariance terms $\eta_{ww'}^{\text{cov}}$ can also be positive or negative. Then, Eq. (11) can be rewritten as

$$\sum_{w \in \mathbf{W}} \sum_{w' \in \mathbf{W}} p_{aw} p_{bw'} \eta_{ww'}^{\text{cov}} = 0 \quad \forall \ a,b \in \tilde{\mathbf{A}} \tag{12}$$

Define

$$G(\eta^{\text{cov}}) = \sqrt{\frac{\sum_{w,w'} \left(\eta_{ww'}^{\text{cov}}\right)^2}{n^2}} \tag{13}$$

Thus, the MPAE of the OD covariance matrix (MPAEC) is defined as the following optimization problem:

$$\text{MPAEC}(\eta^{\text{cov}}) \ = \ \max \ \ G(\eta^{\text{cov}}) \tag{14}$$

$$\text{s.t.} \quad \sum_{w \in \mathbf{W}} \sum_{w' \in \mathbf{W}} p_{aw} p_{bw'} \eta_{ww'}^{\text{cov}} = 0 \quad \forall \ a,b \in \tilde{\mathbf{A}}$$

2.3 Properties of MPAEM and MPAEC

The OD covering rule is widely used in dealing with traffic count location problems. Yang et al. (1991) proved that the OD covering rule was a necessary condition for ensuring the boundedness of MPRE. Thus, the OD covering rule is usually regarded as the basic principle of the traffic count location problem.

(OD Covering Rule): The traffic count locations on a road network should be located on links so that a certain portion of trips between any OD pair will be observed.

Proposition 1: If the traffic count location scheme satisfies the OD covering rule, then the values of both MPAEM and MPAEC are bounded.

Proof: If the OD covering rule holds, the traffic demand for any OD pair can be observed. In particular, for any two OD pairs with non-zero demands $w, w' \in \mathbf{W}$, there are two links $a, b \in \tilde{\mathbf{A}}$ such that $p_{aw} \neq 0$ and $p_{bw'} \neq 0$.

In Eqs. (1) and (2), because $p_{aw} > 0$ and $q_w, q_w^* \geq 0$, we have

$$0 \leq q_w \leq \frac{v_a}{p_{aw}}, \quad 0 \leq q_w^* \leq \frac{v_a}{p_{aw}} \tag{15}$$

Then,

$$\left|\eta_w^{mean}\right| = \left|q_w^* - q_w\right| \leq \frac{v_a}{p_{aw}} \tag{16}$$

It follows from Eq. (16) that η_w^{mean} is bounded.

Assume that the OD demand covariance is greater than or equal to zero. According to Eqs. (9) and (10), it follows from $p_{aw} \neq 0$ and $p_{bw'} \neq 0$ that

$$0 \leq \sigma_{ww'}^{q} \leq \frac{\sigma_{ab}^{v}}{p_{aw} p_{bw'}}, \qquad 0 \leq \sigma_{ww'}^{q*} \leq \frac{\sigma_{ab}^{v}}{p_{aw} p_{bw'}} \tag{17}$$

Then,

$$\left|\eta_{ww'}^{\mathrm{cov}}\right| = \left|\sigma_{ww'}^{q*} - \sigma_{ww'}^{q}\right| \leq \frac{\sigma_{ab}^{v}}{p_{aw} p_{bw'}} \tag{18}$$

Thus, $\eta_{ww'}^{\mathrm{cov}}$ is also bounded. According to Eqs. (16) and (18), the values of both MPAEM and MPAEC are bounded. This completes the proof.

Remark: According to Eq. (16), for a specific OD pair w, the upper bound of the absolute error for the mean OD demand is $\frac{v_a}{p_{aw}}$. That is, the upper bound of η_w^{mean} takes a different value $\frac{v_a}{p_{aw}}$ in each link a. Intuitively, the smaller the value of η_w^{mean}, the more accurate the mean OD demand estimation. Thus, the minimal upper bound of η_w^{mean} is needed. This can be expressed as

$$\overline{\eta_w^{mean}} = \min_{a \in A, p_{aw} > 0} \left\{ z_a \left(\frac{v_a}{p_{aw}} \right) \right\}, \quad \forall w \in W \tag{19}$$

where $\overline{\eta_w^{mean}}$ represents the minimal upper bound of the mean absolute error of OD pair w; z_a is a 0-1 decision variable. $z_a = 1$ indicates that link a has a couting sensor, and otherwise zero. It should be pointed out that z_a represents the sensor location scheme. The objective function in Eq. (19) should be minimized over only links $a \in \tilde{\mathbf{A}}$.

Similarly, the minimal upper bound of $\eta_{ww'}^{\mathrm{cov}}$ can be expressed as follows:

$$\overline{\eta_{ww'}^{\mathrm{cov}}} = \min_{\substack{a, b \in A \\ p_{aw} > 0, p_{bw'} > 0}} \left\{ z_a z_b \left(\frac{\sigma_{ab}^{v}}{p_{aw} p_{bw'}} \right) \right\}, \quad \forall w, w' \in W \tag{20}$$

where $\overline{\eta_{ww'}^{\mathrm{cov}}}$ represents the minimal upper bound of the absolute error in the covariance between OD pairs w and w'.

3 Bi-objective optimization for traffic count location model

Traffic count location models attempt to estimate the stochastic OD demand as accurately as possible. In view of OD demand variations, the traffic count location scheme should account for the estimation error in both the mean and the covariance of the estimated stochastic OD demand. According to the definition of MPAEM and MPAEC, the lower the upper bound of MPAEM and MPAEC, the more accurate the estimates for the mean and covariance of stochastic OD demand. Moreover, the dimensions of MPAEM and MPAEC are also different. It is hard to combine them into one objective function using a weighted summation method. Therefore, a bi-objective optimization for traffic count location model is proposed to minimize the upper bounds of MPAEM and MPAEC simultaneously as follows:

$$\min \quad f_1(z) = \sum_{w \in W} \lambda_w \overline{\eta_w^{mean}} \tag{21}$$

$$\min \quad f_2(z) = \sum_{w,w' \in W} \lambda_{ww'} \overline{\eta_{ww'}^{\mathrm{cov}}} \tag{22}$$

$$s.t. \quad \overline{\eta_w^{mean}} = \min_{a \in A,\, p_{aw} > 0} \left\{ z_a \left(\frac{v_a}{p_{aw}} \right) \right\}, \quad \forall w \in W \tag{23}$$

$$\overline{\eta_{ww'}^{\mathrm{cov}}} = \min_{\substack{a,b \in A \\ p_{aw} > 0,\, p_{bw'} > 0}} \left\{ z_a z_b \left(\frac{\sigma_{ab}^{v}}{p_{aw} p_{bw'}} \right) \right\}, \quad \forall w, w' \in W \tag{24}$$

$$\sum_{a \in A} z_a = \tilde{l} \tag{25}$$

$$\sum_{a \in A} \delta_{aw} z_a \ge 1, \quad \forall w \in W \tag{26}$$

$$z_a = 0,1 \qquad \forall a \in A \tag{27}$$

where λ_w and $\lambda_{ww'}$ are positive weighting parameters indicating the importance of $\overline{\eta_w^{mean}}$ and $\overline{\eta_{ww'}^{\mathrm{cov}}}$, respectively. δ_{aw} is a 0-1 parameter. $\delta_{aw} = 1$ indicates that some trips of OD pair w pass over link a, and otherwise zero. Clearly, if $p_{aw} > 0$, then $\delta_{aw} = 1$; else, $\delta_{aw} = 0$.

Equation (26) ensures compliance with the OD covering rule, and Eq. (25) indicates that the total number of traffic counting sensors should be equal to a given fixed parameter $\tilde{l}$. Note that the parameter $\tilde{l}$ must be greater than or equal to the minimum number of counting sensors l_0 required to satisfy the OD covering rule. If $\tilde{l} < l_0$, there is no feasible solution for the proposed model. l_0 can be determined by the following model (Yang and Zhou, 1998):

$$\min \quad \sum_{a \in A} z_a \tag{28}$$

$$s.t. \quad (26) \sim (27)$$

For the implementation of this model, the weighting parameter λ_w and $\lambda_{ww'}$ can be set as follows.

$$\lambda_w = \frac{\tilde{q}_w}{\sum_{w \in W} \tilde{q}_w} \tag{29}$$

$$\lambda_{ww'} = \frac{\tilde{\sigma}_{ww'}^{q}}{\sum_{w \in W} \sum_{w' \in W} \tilde{\sigma}_{ww'}^{q}} \tag{30}$$

where $\tilde{q}_w$ is the prior mean traffic demand of OD pair w. $\tilde{\sigma}_{ww'}^{q}$ is the prior covariance of traffic demand between OD pairs w and w'.

According to Khoo et al. (2017), OD pairs with higher traffic demand should be assigned a higher ratio in the corresponding estimation. In line with this idea, Eq. (29) means that the more prior OD demand information is available, the higher the value of the weighting parameter. That is, the estimation error of OD pairs with higher traffic demand is more likely to be minimized. The following example illustrates the performance of Eq. (29). Suppose that there are two OD pairs in a transportation network, with prior mean OD demands of 990 passenger car units/hour (pcu/h) and 10 pcu/h. This condition means that the first OD pair has much more traffic flow information than the second OD pair. Therefore, it is unreasonable for the weighting parameters of these two OD pairs to be the same. According to Eq. (29), the weighting parameters for the two OD pairs are

0.99 and 0.01, respectively. Having different weighting parameters is more reasonable than having identical weighting parameters, as it addresses the different traffic flow information of the two OD pairs.

4 Surrogate-assisted Genetic Algorithm for bi-objective traffic count location model

In the bi-objective optimization model, there may be conflicts between the two objectives. In general, there would then be no optimal solution for the two objective functions. In view of this, we attempt to find the Pareto solutions (Grasso et al., 2018). One common approach to finding the Pareto solutions is to transform the bi-objective optimization problem into a single-objective optimization problem using the weighted summation method (Kolak et al., 2018). Note that the dimensions of the two objective functions in the proposed model are inconsistent. The dimension of $\overline{\eta_w^{mean}}$ is pcu/h, while the dimension of $\overline{\eta_{ww'}^{\text{cov}}}$ is (pcu/h)^2. To address this inconsistency, the average values of $\sum_{w\in W}\lambda_w\overline{\eta_w^{mean}}$ and $\sum_{w,w'\in W}\lambda_{ww'}\overline{\eta_{ww'}^{\text{cov}}}$ are adopted following the work of Shao et al. (2014). Above all, the following single-objective optimization model with the objective function of weighted maximum possible absolute error (WMPAE) is used to obtain the Pareto solutions of the bi-objective optimization model.

$$\begin{aligned} \min \quad & y=\alpha\sum_{w\in W}\frac{\lambda_w\overline{\eta_w^{mean}}}{n}+(1-\alpha)\sum_{w,w'\in W}\frac{\lambda_{ww'}\overline{\eta_{ww'}^{\text{cov}}}}{n^2} \\ s.t. \quad & (23)\sim(27),(29)\sim(30) \end{aligned} \tag{31}$$

where the weighting parameter $0\le\alpha\le1$ is used to measure the relative importance of MPAEM and MPAEC in the model.

The above optimization model is a 0-1 programming problem. The decision variable z_a is the sensor location scheme. For a transportation network with n links and m sensors, the total number of possible sensor location schemes for z_a is C_n^m. Under this circumstance, the time complexity of this problem is $O(C_n^m)$. That is to say all the possible sensor location schemes need to be enumerated to verify the optimality of the solution. Therefore, the above optimization model is a NP-hard problem. It is known in the literature that the computational time of enumeration algorithm exponentially increases with respect to the scale of decision variable. As such, the enumeration algorithm is very time consuming for large scale network.

Various modified enumeration methods have been developed to find the optimal or sub-optimal solution of such problems by enumerating some or all feasible solutions. For example, implicit enumeration methods such as the branch and bound method (Laporte and Louveaux, 1993; Karimi and Davoudpour, 2015) and genetic algorithm (Holland, 1975; Fogue et al., 2013) are extensively used. The genetic algorithm (GA) is a widely used robust evolutionary technique that can find the global optimum of nonlinear multi-objective optimization. Genetic algorithms represent the process of solving optimization problems via the evolutionary theory of “survival of the fittest”. In other words, through the selection, crossover, and mutation of chromosomes in a population, relatively good chromosomes are preserved and combined to evolve better populations and determine the optimal or near-optimal solution (Chiappone, et al., 2006).

Note that the calculation of the objective function (Eq. (31)) involves two sub-optimization problems (Eqs. (23) and (24)), which are two nonlinear optimization models and may require considerable computation time. Such property may lead to slow convergence and "premature" phenomenon in the traditional genetic algorithm. In view of this, the surrogate model (Datta and Regis, 2016) is incorporated into traditional genetic algorithm. Then, this paper proposes a surrogate-assisted genetic algorithm (SGA) to solve the proposed model.

SGA can be regarded as a modified GA. Specifically, compared with the original GA method, SGA adds the operations of approximating the objective function and correcting the individual, as shown in Table 2. The surrogate model approximates the objective function without directly solving the optimization problem (Eq. (31)). Such an approximation could considerably reduce the computational load of genetic algorithm. However, if the objective function of each individual in the population is calculated by the surrogate model, the accuracy of the solution may be influenced by the approximation errors. In view of this, for some individuals, the original model (Eq. (31)) still needs to be solved in order to correct the value of objective functions by removing the inaccurate information due to the approximation of surrogate model. In a word, the approximation and correction operations are aimed at reducing the amount of calculation and trying to alleviate the negative effects of approximation errors caused by surrogate model.

Table 2 Comparison of SGA and GA

Algorithm	Process
GA	Code → Selection → Crossover → Mutation
SGA	Code → [Approximation→ Correction]→Selection → Crossover → Mutation

The steps of SGA can be explained as follows:

(1) Coding for the representation of candidate solutions

According to the characteristics of the model proposed in this paper, a simple and intuitive binary code is chosen to represent the solution of the problem. In a chromosome, the index of a link in the road network is sorted to indicate whether the link contains a sensor. If so, the link code is set to 1; otherwise, it is set to 0. For example, x = 1 1 0 1 0 1 means that there are sensors on links 1, 2, 4, and 6, while there are no sensors on links 3 and 5.

(2) Initial population generation

The N initial feasible solutions are randomly selected from the feasible fields that satisfy the constraint conditions, and an initial population is constituted. The length of each chromosome is the number of links. We set the maximum evolution time T and error accuracy ε.

(3) Surrogate model for approximating the objective function

Equation (31) is used to randomly generate M initial sample points $\{\mathbf{z}^{(i)}, y^{(i)}\}$, $i=1,2,\ldots,M$, where $\mathbf{z}^{(i)}=[z_1^{(i)},\ldots,z_a^{(i)},\ldots,z_{|A|}^{(i)}]^T$, and $|A|$ is the number of links.

The Kriging surrogate model (Krige, 1951; Chen et al., 2016) assumes that the response value is related to the input variable as follows:

$$y = F(\mathbf{z}) + \alpha(\mathbf{z}) \tag{32}$$

where $F(\mathbf{z})$ is a regression model; $\alpha(\mathbf{z})$ is a random process with mean 0, variance σ^2, and covariance

$$\mathrm{cov}(\alpha(\mathbf{z}^{(i)}), \alpha(\mathbf{z}^{(j)})) = \sigma^2 R(\mathbf{z}^{(i)}, \mathbf{z}^{(j)}) \tag{33}$$

where $R(\mathbf{z}^{(i)}, \mathbf{z}^{(j)})$ is the correlation function of two sample points $\mathbf{z}^{(i)}$ and $\mathbf{z}^{(j)}$:

$$R(\mathbf{z}^{(i)}, \mathbf{z}^{(j)}) = \exp\left[-\sum_{a\in A}\theta_a \left|z_a^{(i)} - z_a^{(j)}\right|^2\right] \tag{34}$$

The estimated value $\hat{y}(\mathbf{z_0})$ of the response $y(\mathbf{z_0})$ at the unknown point $\mathbf{z_0}$ is given by

$$\hat{y}(\mathbf{z_0}) = \hat{\beta} + r^T(\mathbf{z_0})R^{-1}(Y - I\hat{\beta}) \tag{35}$$

where $\hat{\beta}=\frac{I^T R^{-1} Y}{I^T R^{-1} I}$, $I=[1,1,\ldots,1]^T{}_{1\times M}$, $R=[R(\mathbf{z}^{(i)},\mathbf{z}^{(j)})]_{M\times M}$, $Y=[y^{(1)},y^{(2)},\ldots,y^{(M)}]^T$; $r(\mathbf{z_0})$ is an M-dimensional vector whose elements reflect the correlation function between point $\mathbf{z_0}$ and the sample point, and the estimated variance is $\hat{\sigma}^2=\frac{(Y-I\hat{\beta})^T R^{-1}(Y-I\hat{\beta})}{M}$.

(4) Individual correction

First, the objective function of each individual in the population is calculated by the surrogate model (Eq. (35)). Then some individuals are selected to use the original model (Eq. (31)) to correct their objective function values. This paper adopts Expected Improvement (EI) add point correction method (Donald et al., 1998). EI value of each individual is calculated as follows.

$$E[I(\mathbf{z})]=(F_{\min}-\hat{y})\Phi\left(\frac{F_{\min}-\hat{y}}{\hat{\sigma}}\right)+\hat{\sigma}\phi\left(\frac{F_{\min}-\hat{y}}{\hat{\sigma}}\right) \tag{36}$$

where $F_{\min}$ is the minimum value of the objective function for all sample points; $\hat{y}$ is the predicted value of the Kriging model at point $\mathbf{z}$; $\hat{\sigma}$ is the predicted standard deviation of the Kriging model; and $\phi(\cdot)$, $\Phi(\cdot)$ are the standard normal density and distribution functions, respectively. Select the individual with the maximum EI from the population and determine whether this individual is in the sample set. If so, no correction is made and the subsequent genetic algorithm steps are performed. Otherwise, the original model is used for correction. The corrected individual is added to the sample set, and the surrogate model is updated.

(5) Fitness function

The fitness function reflects the relative superiority of each individual in the population. Higher values of the fitness function indicate that the chromosome is closer to the optimal solution. Moreover, the proposed traffic count location model aims to minimize the estimation error. Thus, the fitness function of individual i is defined as

$$f_i=\frac{1}{\hat{y}^{(i)}} \tag{37}$$

(6) Selection

The selection mechanism compares the fitness function values. The higher the degree of individual fitness, the greater the likelihood of selection. This paper adopts the roulette selection method. To ensure the optimal solution of each generation is not destroyed by the crossover and mutation operations during the search evolution, the best individual of the current generation is also preserved.

(7) Crossover

The crossover operation exchanges several genes between two selected chromosomes to form two new chromosomes with high crossover probability p_c. In this paper, we use the single-point crossover, commonly used in binary coding, which selects a cross-point at random and then switches the genes of two parent chromosomes at that point.

(8) Mutation

The mutation operation changes some sub-strings value in an individual coding with low mutation probability p_m to form new individuals. For the binary encoding, the random selection of a point changes the value of that point from 1 to 0 or from 0 to 1.

5 Numerical experiments

This section presents the results of two numerical experiments. A small transportation network is used to demonstrate the following properties of the proposed model: (a) the effects of OD demand covariance on the traffic count location problem; (b) the effects of prior OD demand information on the traffic count location problem; (c) modeling behavior in the case where the stochastic OD demand estimation is zero; and (d) the tradeoff between the two objective functions by providing all Pareto solutions. The second example utilizes a real transportation network to demonstrate the convergence of the proposed surrogate-assisted genetic algorithm.

5.1 Small transportation network

Figure 1 depicts a small transportation network with 14 links and 6 OD pairs, i.e., (1–6), (1–8), (1–9), (2–6), (2–8), (2–9). The true and prior mean OD demand are listed in Table 3, and the link-OD choice proportion is presented in Table 4. The prior OD demand covariance matrix and observed link flow covariance matrix are presented in Tables 5 and 6, respectively.

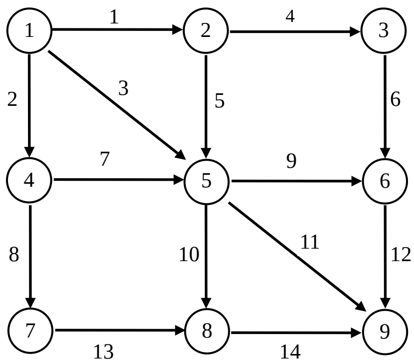

Fig. 1 Small network

Table 3 Mean OD demand (pcu/hour)

OD pair	1–6	1–8	1–9	2–6	2–8	2–9
Prior demand	100	130	120	120	170 (10)	140
True demand	120	150	100	130	200 (0)	90

Note: The numbers in parentheses represent the special case discussed in sub-section 5.1.2, where the demand of OD pair 2–8 is zero.

Table 4 Link-OD choice proportion

Link \ OD pair	1–6	1–8	1–9	2–6	2–8	2–9	Observed mean link flow
1	0.5						60
2		1	0.4				190
3	0.5		0.6				120
4				0.6		0.6	132
5	0.5			0.4	1 (0)	0.4	348 (148)
6				0.6		0.6	132
7			0.4				40
8		1					150
9	1			0.4			172
10			0.2		1 (0)		220 (20)
11			0.8			0.4	116
12						0.6	54
13		1					150
14			0.2				20
Mean OD demand	120	150	100	130	200 (0)	90	

Note: The numbers in parentheses represent the special case discussed in sub-section 5.1.2, where the demand of OD pair 2–8 is zero.

Table 5 Prior OD demand covariance matrix (pcu/hour)

OD pair	1–6	1–8	1–9	2–6	2–8	2–9
1-6	1296					
1-8	56.7	2025				
1-9	27	29.7	900			
2-6	36.5	33.3	23.4	1521		
2-8	54 (0)	56.7 (0)	30.6 (0)	35.1 (0)	3600 (0)	
2-9	44.7	45	23.5	34.7	47 (0)	729

Note: The numbers in parentheses represent the special case discussed in sub-section 5.1.2, where the demand of OD pair 2–8 is zero.

Table 6 Observed link flow covariance matrix (pcu/hour^2)

Link	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1296													
2	83.7	2984.4												
3	1323	1013.4	2250											
4	81.2	125.2	128.1	2319.4										
5	1431.2	296.2	1535.7	2482.7	7650									
6	81.2	125.2	128.1	2319.4	2482.7	2319.4								
7	27	929.7	927	46.9	1045	46.9	900							
8	56.7	2054.7	86.4	78.3	191.7	78.3	29.7	2025						
9	1332.5	140.4	1382.9	1636.9	3058.5	1636.9	50.4	90	2890					
10	81	1017	1011.6	129	3840.6	129	930.6	86.4	139.5	4561.2				
11	71.7	998.2	995.2	810.6	959.9	810.6	923.5	74.7	129.8	1001.1	1676			
12	44.7	68.5	68.2	763.7	855.4	763.7	23.5	45	79.4	70.5	752.5	729		
13	56.7	2054.7	86.4	78.3	191.7	78.3	29.7	2025	90	86.4	74.7	45	2025	
14	27	929.7	927	46.9	104.5	46.9	900	29.7	50.4	930.6	923.5	23.5	29.7	900

5.1.1 Effects of different objective functions and the number of sensors

In this example, the performance of the newly proposed measures is examined in terms of determining the optimal traffic count locations. To this end, four scenarios with different weighting parameters are used, and the scenarios are summarized in Table 7. Each scenario has a different objective function to illustrate the difference between the proposed measures and the existing MPAEM. Scenario An only considers the mean of OD demand (MPAEM), Scenario B only considers the covariance of OD demand (MPAEC), Scenario C takes both the mean and covariance of OD demand into account (MPAE), and Scenario D is a comprehensive case in which the estimation error of both the mean and covariance of OD demand as well as the prior traffic demand information are simultaneously considered in the objective function (WMPAE).

Table 7 Four scenarios with different weighting parameters

Scenario	Consideration of mean OD demand	Consideration of OD demand covariance	Consideration of traffic demand information for different OD pairs	Parameter values
A. MPAEM	√	×	×	$\alpha = 1$ $\lambda_w = \lambda_{ww'} = 1$
B. MPAEC	×	√	×	$\alpha = 0$ $\lambda_w = \lambda_{ww'} = 1$
C. MPAE	√	√	×	$\alpha = 0.5$ $\lambda_w = \lambda_{ww'} = 1$
D. WMPAE	√	√	√	$\alpha = 0.5$ $\lambda_w = \frac{\tilde{q}_w}{\sum_w \tilde{q}_w}$, $\lambda_{ww'} = \frac{\tilde{\sigma}_{ww'}}{\sum_{w\in W}\sum_{w'\in W}\tilde{\sigma}_{ww'}}$

Even when using the same objective function, the resulting location will depend on the number of sensors. As discussed in section 3, the limited number of counting sensors $\tilde{l}$ must be greater than the minimum counting sensor number l_0 that satisfies the OD covering rule. Solving the optimization model (Eqs. (26)~(28)) yields $l_0 = 2$. Therefore, it is assumed that l% ≥ 3 in this example. As the scale of this network is small, the enumeration method can be used to obtain the global optimum of the traffic count location scheme. The corresponding results are shown in Tables 8–11.

Table 8 Results of Scenario A

Number of sensors	**MPAEM**	MPAEC	MPAE	WMPAE	Optimal sensor location scheme
3	***352.64***	***762.20***	352.64	304.44	2, 3, 5
4	204.03	497.03	204.03	190.70	2, 9, 10, 11
5	214.67	321.10	214.67	145.92	***2, 3, 5, 9, 11***
6	0	0	0	0	All feasible solutions

Table 9 Results of Scenario B

Number of sensors	MPAEM	**MPAEC**	MPAE	WMPAE	Optimal sensor location scheme
3	***568.18***	***684.41***	684.41	678.84	2, 5, 7
4	358.05	404.67	404.67	352.06	2, 5, 7, 9
5	310.91	310.42	310.42	279.35	***2, 3, 4, 9, 10***
6	0	0	0	0	All feasible solutions

Table 10 Results of Scenario C

Number of sensors	MPAEM	MPAEC	**MPAE**	WMPAE	Optimal sensor location scheme	Total covering flow by optimal sensor location
3	***384.73***	735.52	560.13	553.49	2, 5, 14	558
4	236.33	495.49	365.91	294.86	2, 3, 5, 7	**698**
5	239.45	311.91	275.68	252.47	***5, 7, 8, 12, 14***	612
6	0	0	0	0	All feasible solutions	

Table 11 Results of Scenario D

Number of sensors	MPAEM	MPAEC	MPAE	**WMPAE**	Optimal sensor location scheme	Total covering flow by optimal sensor location
3	***358.17***	778.41	568.29	464.77	2, 5, 9	710
4	297.98	486.08	392.03	291.47	2, 3, 5, 9	**830**
5	262.74	322.22	292.48	216.55	***3, 5, 7, 9, 13***	830
6	0	0	0	0	All feasible solutions	

As can be seen from Tables 8–11, the traffic count location schemes of the four scenarios are different even when the number of sensors is the same. For example, with five counting sensors, the traffic count location schemes are (2, 3, 5, 9, 11), (2, 3, 4, 9, 10), (5, 7, 8, 12, 14), and (3, 5, 7, 9, 13) for scenarios A, B, C, and D, respectively. That is, the result given by the proposed traffic count location model is different from that of the existing model using MPAEM.

As the number of sensors increases, the magnitude of all four measures decreases. This phenomenon is in line with conventional traffic count location models. More counting sensors used for OD estimation means that more information is acquired, which leads to a decrease in the estimation error. When there are six counting sensors, all four measures return a value of zero because the number of OD pairs is equal to the number of counting sensors. In this case, the observed traffic flow information can uniquely identify the OD demand without estimation errors. Similar conclusions have been found in many OD demand estimation studies (Gentili and Mirchandani, 2018).

An interesting finding is that the four measures cannot simultaneously reach their own minimum using any one traffic count location scheme. For example, with three counting sensors, the minimum MPAEM value of 352.64 is achieved in scenario A. However, the corresponding measure of MPAEC in scenario A is 762.20, which is greater than that in scenario B (684.41). That is, the absolute optimum concerning the MPAEM and MPAEC measures cannot be obtained. For the case of stochastic OD demand estimation, the proposed traffic count location model is needed as it accounts for estimation errors in both the mean and covariance of OD demand.

To demonstrate the effects of weighting parameters λ_w and $\lambda_{ww'}$, the total traffic flow covered by the sensor location scheme is given in Tables 10–11. The results indicate that the use of weighting parameters helps cover more of the traffic flow. For example, for the case of four counting sensors, there is not much difference between the MPAE and WMPAE values in scenarios C and D. However, the total traffic flow covered by the sensor location scheme in scenario D is 830 pcu/h, which is somewhat higher than that in scenario C (698 pcu/h). Therefore, scenario D (WMPAE) is a better optimal model for the counting sensor location problem.

5.1.2 Handling the case of zero estimation of stochastic OD demand

This example is used to demonstrate that the proposed model is capable of determining the traffic count location scheme when the estimated mean and covariance of OD demand are zero. In real life, the traffic demand of some OD pairs may be zero because of changes in traffic control policies or land use development. The concepts of MPAEM and MPAEC can address cases where the estimated mean and covariance of OD demand are zero, whereas the widely used MPRE cannot deal with this case. It is assumed that the true mean demand of OD pair 2–8 is zero ($q_{28}^* = 0$), as shown in parentheses in Table 3. Under this assumption, the corresponding link-OD choice proportion and prior OD demand covariance of OD pair 2–8 is also assumed to be 0, as indicated by parentheses in Tables 4 and 5, respectively. The optimal location scheme of the counting sensors is presented in Table 12.

Table 12 Optimal location of counting sensors with zero OD demand

Number of sensors	Scenario A		Scenario B		Scenario C		Scenario D	
	MPAEM	Location scheme	MPAEC	Location scheme	MPAE	Location scheme	WMPAE	Location scheme
3	286.53	2, 5, 7	609.76	2, 5, 10	447.27	3, 5, 8	371.82	5, 10, 13
4	189.06	2, 5, 10, 11	323.74	2, 3, 5, 9	256.40	2, 5, 9, 10	232.93	2, 3, 5, 10
5	171.73	3, 4, 5, 10, 13	256.88	2, 3, 9, 10, 11	240.44	2, 4, 7, 9, 14	205.24	3, 7, 9, 12, 13

In Table 12, the value of the objective function in all four scenarios decreases with the increase in the number of sensors. For example, the value of MPAEM in Scenario A decreased from 286.53 to 171.73 as the number of sensors increases from 3 to 5. This phenomenon is similar to that of non-zero OD demand (see Tables 8–11). Note that the observed mean and covariance matrix of link flow is different from that of non-zero OD demand. As a result, the optimal counting sensor location scheme under zero OD demand is different from that of the non-zero OD demand case. This can be inferred from the results in Tables 10 and 12. For Scenario C, when there are five sensors, the optimal location scheme with zero OD demand is (2, 4, 7, 9, 14), whereas that for the non-zero OD demand case is (5, 7, 8, 12, 14). Moreover, an interesting finding is that the estimation error of the four scenarios with zero OD demand is smaller than in the non-zero OD demand case. The reason is that the total observed traffic flow in the zero OD demand case is smaller than that in the non-zero OD demand case.

The proposed modeling approach can also be used to identify the critical roads (or links) in terms of OD demand estimation. Following the work of Bagloee et al. (2017), a link is defined as a critical link if it is determined as a traffic count location in more than or equal to 6 out of 12 times of the traffic location schemes in Table 12. A link is defined as a noncritical link if it is determined as a traffic count location in fewer than two times. The first row of Table 13 shows the times that each link is equipped with a sensor for the case $q_{28}^{*}=0$. And then the critical and noncritical links are marked in Fig. 2(a). We find that most of the critical links are located in the center of the network, whereas noncritical links are located at the edges of the network. This is an interesting test, but such a conclusion may highly depend on the parameters set in Table 3-6. In view of this, we additionally set two other cases: (1) the true mean demand of OD pair 1–9 is zero ($q_{19}^{*}=0$); (2) set another non-zero case of OD demand randomly in Table 3, for example $q^{*}=[130,200,90,160,220,100]$. We also present the times each link counted in different location schemes for the two cases as shown in the last two rows of Table 13. And the corresponding layout of the critical and noncritical links is shown in Figs. 2(b) and 2(c). Overall, the three different cases show a similar pattern that critical links are located at the center and noncritical links are located at the edge. The underlying reasons for such this interesting finding deserve further investigation.

Table 13 Times of each link in different location schemes

Case \ Link	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A. zero demand of OD pair 2-8 $q_{28}^{*}=0$	0	8	6	2	9	0	3	1	5	7	2	1	3	1
B. zero demand of OD pair 1-9 $q_{19}^{*}=0$	0	10	6	1	10	0	5	1	7	2	2	1	1	2
C. non-zero OD demand $q^{*}=[130,200,90,160,220,100]$	2	5	7	1	11	1	7	5	2	2	1	0	3	1

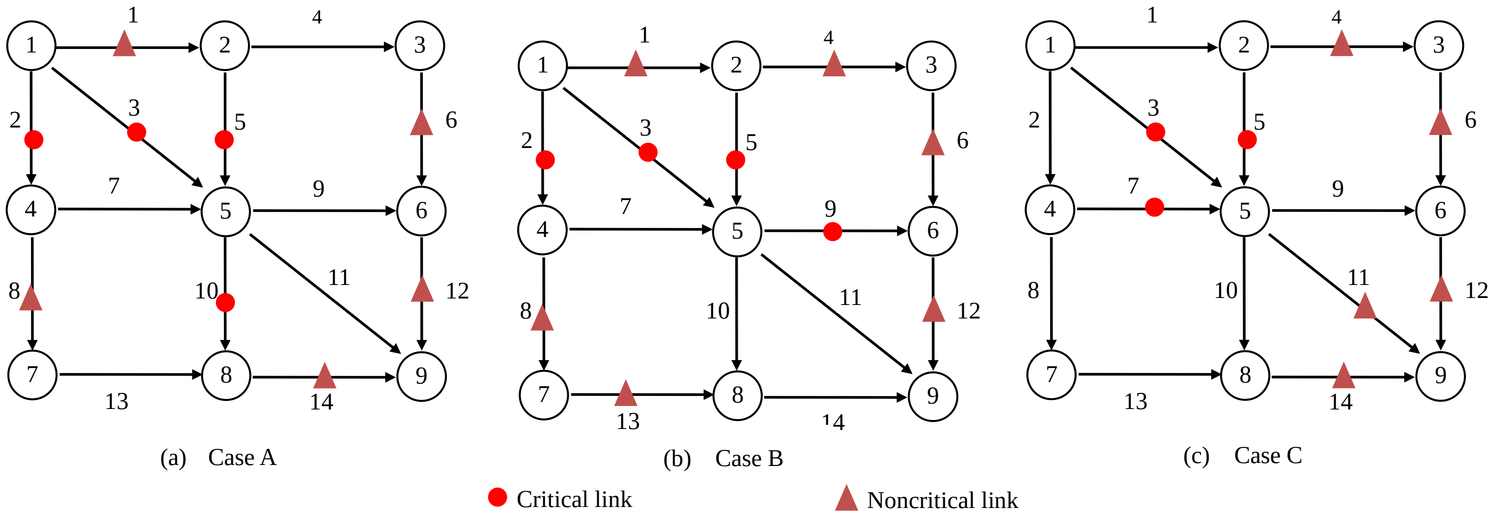

Fig. 2 Critical and noncritical links

5.1.3 Pareto solutions of the proposed model

In this paper, the proposed optimal model for the traffic count sensor locations is formulated as a bi-objective optimization problem. The two objective functions are generally in conflict with one another. The Pareto solutions (non-dominated solutions) shown in Fig. 3 correspond to the case of five counting sensors. As there are a total of 14 links in this network, the total number of possible sensor location schemes is $C_{14}^{4} = 1001$. The constraint in Eq. (26) reduces the number of feasible solutions to 49. Finally, there are 11 Pareto optimal solutions for this example. The Pareto front is moving toward the ideal front.

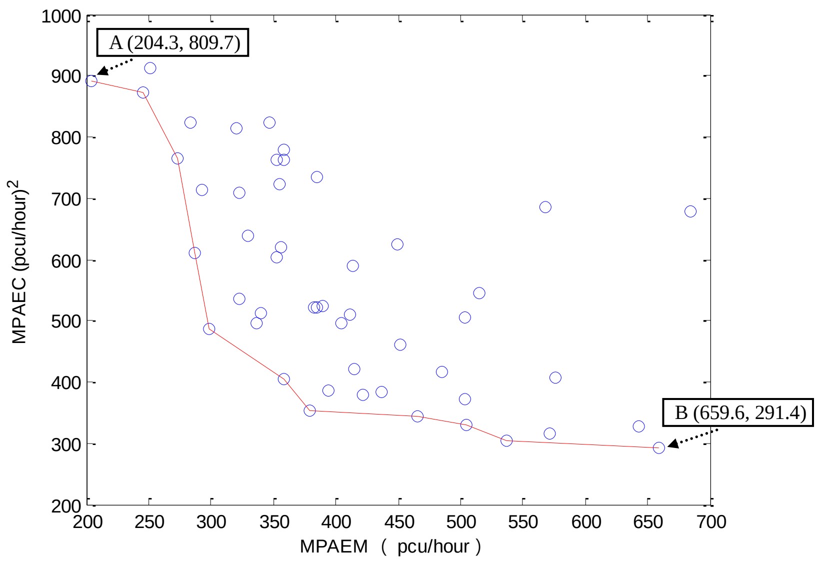

Fig. 3 Pareto optimal solutions of small network

In addition, comparing the two special Pareto solutions A and B in Fig. 3, it is apparent that solution A performs best in terms of minimizing the error in the mean OD estimation, whereas solution B performs best in terms of minimizing the error in the OD covariance estimation. It is hard to determine which of these is the best solution. For implementation purposes, determining the most suitable counting sensor location scheme for different purposes is worthy of further investigation.

5.2 Real transportation network

To demonstrate the efficiency of the proposed surrogate-assisted genetic algorithm for solving the optimization problem of traffic count location, we consider a real network in the City of Irvine, Orange County, California. The Irvine network, as shown in Fig. 4, consists of 39 zones, 162 nodes, 496 links, and 108 OD pairs (Xu et al., 2016). The OD demand is set to follow a multivariate normal distribution. The traffic flow covariance ranges from 21.7 (pcu/hour)^2 to 4086.3 (pcu/hour)^2. The link choice proportion matrix is given and fixed.

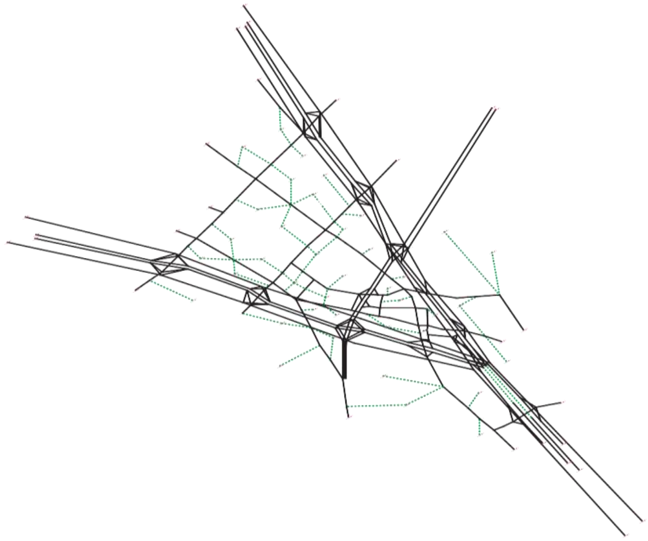

Fig. 4 Irvine network

5.2.1 Convergence of the surrogate-assisted genetic algorithm

In order to present the merit of the proposed surrogate-assisted genetic algorithm (SGA) compared with traditional genetic algorithm (GA), we solve the WMPAE model when $\alpha = 0.5$ with SGA and GA, respectively. The experiment was implemented in Matlab on the Windows 10 platform and executed on a personal computer with an Intel Core^{TM} i7-6500U 2.5 GHz CPU and 4 GB memory. The limited sensor number was set to 38, which is the minimum number satisfying the OD covering rule. The surrogate-assisted genetic algorithm used the number of initial samples $M = 70$, population size $N = 50$, the maximum number of iterations $T = 1000$, error accuracy $\varepsilon = 10^{-3}$, crossover probability $p_c = 0.7$, and mutation probability $p_m = 0.05$.

Figure 5 illustrates the convergence of two algorithms SGA and GA. It is clear that the fitness function of SGA increases over the first 148 generations and becomes stable from the 149th generation. Nevertheless, GA takes 262 generations to converge, nearly twice the number of iterations for SGA. Meanwhile, the fitness growth of GA is very slow in the first 250 generations and may be caught in the local optimum. Besides, the fitness value of SGA is 0.0486, slightly higher than the 0.0449 of GA. The higher the fitness value, the smaller the corresponding objective function value. Therefore, the OD estimation error obtained by SGA is smaller than GA. On the other hand, the computation time of SGA is 1983.472 s, which is also less than that of GA (2705.926 s). In summary, this experiment shows that the proposed surrogate-assisted genetic algorithm converges faster than genetic algorithm.

Although SGA is more efficient than GA, it is worth noting that the fitness values of SGA still has small fluctuations when it tends to be stable. As discussed in section 4, SGA can be regarded as a modified GA. In the selection operation, SGA preserves the best individual of the current generation and sends it to next generation. In the next generation, new individuals are generated in the population through "crossover and mutation" operation. These new individuals may be better than “the best” of last generation. That is the main reason for the fluctuations of fitness value in Figure 5.

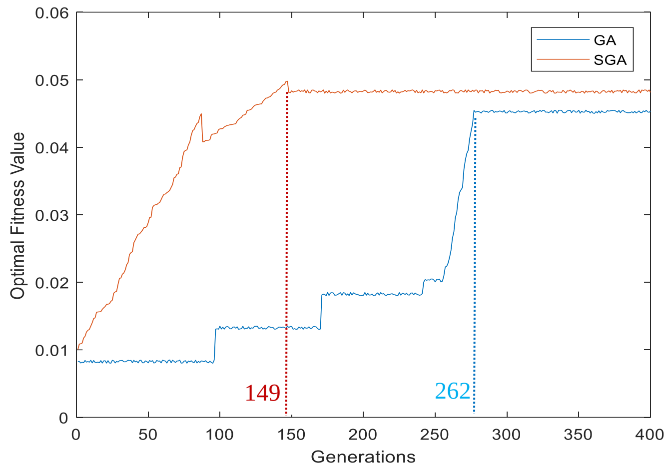

Fig. 5 Convergence of SGA and GA

5.2.2 Sensitivity analysis of surrogate-assisted genetic algorithm

In this section, we study the effect of different parameters on the proposed surrogate-assisted genetic algorithm. First of all, the number of sensors is set to range from 40 to 80 with the step size of 10, as shown in Figure 6. Figure 6 also shows that when the number of sensors increases, the estimated error of WMPAE sharply decreases from 685.78 to 203.41. However, the computation time has not increased much from 2298.09 s to 2965.20 s, about 10 minutes. Therefore, if the cost of sensor location allows, the number of sensors should be as large as possible to obtain more accurate OD estimation, which is not at the expense of computation time.

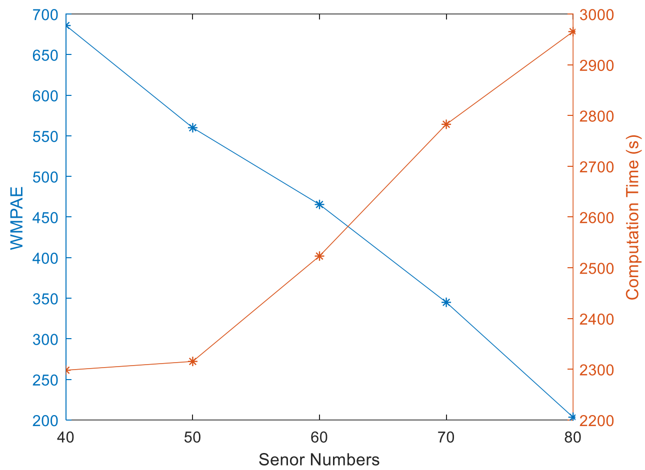

Fig. 6 Effect of different sensor numbers on WMPAE and computational time

In addition, we consider the effect of other cases on the algorithm: (a) Randomly select 40, 60, and 90 OD pairs from the Irvine network; (b) The weight α of the objective function of

WMPAE are set to 0.1, 0.3, 0.5 and 0.8, respectively; (c) Consider the traffic congestion, namely increase the prior OD demand by 20 percent. The value of WMPAE associated with these different cases is shown in Table 14. When the weight α increases from 0.1 to 0.3, WMPAE remains almost unchanged. However, as the weight α increases further, WMPAE decreases significantly. On the other hand, it can be seen that the two cases of 40 and 60 OD pairs have similar results on WMPAE. While the number of OD pairs increases to 90, WMPAE is twice of that in cases of 40 or 60 OD pairs. For example, when the weight α is 0.3 for 40 OD pairs, the WMPAE is 303.66 for non-congestion condition, while it increases to 597.53 when the weight α is 0.3 for 90 OD pairs. This shows that the scale of the network has a significant impact on the estimation error. The larger the scale of the network, the larger the estimation error. At the same time, we note that under the same weight α and number of OD pairs, the estimation error is not sensitive to traffic congestion. For example, when the weight α is 0.8 for 90 OD pairs, the WMPAE is 318.83 for non-congestion condition, while it slightly increases to 325.34 for congestion condition.

Table 14 WMPAE with different weights and OD pairs

Weight α	40 OD pairs		60 OD pairs		90 OD pairs	
	Non-congestion	Congestion	Non-congestion	Congestion	Non-congestion	Congestion
0.1	306.52	320.56	315.48	318.67	595.19	609.73
0.3	**303.66**	318.59	310.79	323.74	**597.53**	607.34
0.5	235.53	255.59	243.54	251.47	431.52	441.77
0.8	235.49	237.62	234.22	239.85	***318.83***	***325.34***

6 Conclusions and further studies

In this paper, we addressed an issue on the optimal location of traffic counting sensor on the links to estimate the stochastic OD demand. To this end, we proposed a new measure to evaluate the estimation error of the stochastic OD demand. The maximum possible absolute error of OD demand covariance (MPAEC) was defined to quantify the estimation error of traffic count location schemes for OD demand covariance estimation. This measure can be regarded as an extension of the conventional maximum possible absolute error of mean OD demand (MPAEM). After deriving the bounds of MPAEM and MPAEC, a bi-objective optimization model was proposed for the traffic count location problem. The two objectives were designed to minimize the upper bounds of MPAEM and MPAEC simultaneously. The proposed model is capable of handling zero estimates, unlike conventional models. A surrogate-assisted genetic algorithm was adopted to solve the proposed bi-objective optimization model. Two numerical examples have been presented to illustrate the applicability of the proposed model and the efficiency of the proposed algorithm. The results show that (a) the proposed MPAEC measure can be used to capture the estimation error of OD demand covariance in traffic count location schemes; (b) the proposed model is capable of solving cases in which the estimated mean and covariance of OD demand are zero; and (c) the proposed algorithm can solve the traffic count location model for a real network with 162 nodes, 496 links, and 108 OD pairs.

Several areas of research are worthy of further investigation.

(1) This paper discusses the optimal sensor location model for estimating the OD matrix. Extending the proposed model for purposes such as travel time estimation and path reconstruction requires further study.

(2) It is assumed that the covariance between the OD demand and link flow is non-negative. Relaxing this non-negative assumption deserves further investigation.

(3) The proposed model assumes that the traffic count sensors do not return any measurement errors. In reality, there are likely to be measurement errors or sensor failures. From this point of view, it is important to account for these issues effectively in the modeling approach to achieve accurate estimations.

(4) Current research on traffic count location optimization mainly focuses on static OD demand estimation for strategic planning purposes. Further studies could be carried out to extend the proposed model to the dynamic case for traffic control and management.

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