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How (Grandfathered) Slots Can Be a First-best Policy for a Congested Airport Whereas Prices Cannot

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Abstract

This study considers a stylised airport network, designed to identify clearly the role of local and non-local passengers for the assessment of local welfare-maximising airport congestion policies. The analysis shows that, in our framework, the local welfare-maximising slot quantity can coincide with the first-best outcome, whereas this is impossible in the case of pricing policy. Whether the outcomes coincide in the case of slot policy depends on the shares of locals and non-locals in terms of inframarginal and marginal passengers.

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1.0 Introduction

The aviation passenger number is growing and will probably outpace growth in the physical airport capacity in the long term, which will increase future airport congestion and airline delays via an increasing ratio of air traffic over the physical airport capacity. Therefore, airport congestion is a relevant policy problem in practice. This is true despite the devastating effects of the Coronavirus outbreak and the corresponding international lockdowns in 2020, which will slow down the growth of the aviation industry for years (for example, Czerny *et al.*, 2020; Pearce, 2020). Cirium (2020)'s record of global departure performance in 2019 indicates that the average delay time for the top 20 global airports was 50.7 minutes for around 364,000 delayed flights. The Federal Aviation Administration (2020a) estimated that the airline cost for an hour of delay ranges from about $1,400 to $4,500, and that passenger time valuations range from $35 to $63 per hour. These numbers indicate that air transport delays are costly, which highlights the importance of airport congestion policies.

Many airports are utilising 'slots' to mitigate delays by imposing an upper limit on the number of flight movements per period of time (for example, 30 minutes). An airport slot (or simply 'slot') is 'a permission given by a coordinator for a planned operation to use the full range of airport infrastructure necessary to arrive or depart at a Level 3 airport on a specific date and time' (International Air Transport Association, 2019b). Airports can control delays and congestion effects by ensuring that the scheduled air traffic does not exceed the 'declared airport capacity'.[1] There are around 204 airports using slots worldwide, serving around 43 per cent of the passengers (International Air Transport Association, 2019a). Interestingly, only three airports in the USA — John F. Kennedy International Airport (JFK), LaGuardia Airport (LGA), and Ronald Reagan Washington National Airport (DCA) — currently make use of slots (Federal Aviation Administration, 2020b). Most airports in the USA allocate their capacity on a first-come-first-serve basis. This includes Hartsfield-Jackson Atlanta International Airport (ATL), which was the biggest airport in the world in terms of passenger numbers prior to the Coronavirus outbreak.

Alternatively, airports may increase airport charges to suppress demand and reach desirable levels of delay (Daniel, 1995; Brueckner, 2002; Pels and Verhoef, 2004; Zhang and Zhang, 2006; Basso, 2008; Czerny and Zhang, 2011, 2014a and 2014b). In contrast to quantity-based policies such as slots, the proposed pricing policies can hardly be found in practice (Zhang and Czerny, 2012). This could be a consequence of the distributional effects of airport congestion policies. Whereas pricing policies will lead to higher payments from the airlines to airports, slot policies can avoid such distributional effects by using grandfather rights (for example, Czerny and Lang, 2019).

Another defining feature of the air transport industry, besides the importance of airport slots and grandfathering, is that airports are typically owned and controlled by local governments.[2] This suggests that airports presumably distinguish between local and

[1] A few factors, including the number and layout of runways, meteorological conditions, traffic mix characteristics, and the regulatory environment (Gillen *et al.*, 2016), determine the declared airport capacity per time unit.

[2] In North America, only 1 per cent of airports involve private sector participation, whereas this share reaches between 11 and 75 per cent outside North America (Airport Council International, 2017).

non-local passengers, leading to the maximisation of the local airport's welfare — as opposed to the first-best outcome that maximises the total welfare across all airport regions.

This study develops a stylised but rich enough model to analyse the role of the shares of local and non-local passengers (simply called locals and non-locals) for the assessment of local welfare-maximising airport congestion policies by comparing the local welfare-maximising solutions with the first-best outcome. To concentrate on the role of locals and non-locals and assess the congestion policies, the model is stripped down to considering a simple airport network with one congested airport and one uncongested airport (it will be shown that the results derived in this reduced framework carry over to the more complicated case with multiple congested airports). Airline markets are assumed to be perfectly competitive in the model and airlines are assumed to serve their locals only, which we call exclusive airline services, in the basic model version. Some model results are sensitive with respect to the consideration of exclusive or inclusive airline services. The latter, inclusive airline services, involves a more realistic market scenario in which local airlines serve locals and non-locals — as will be shown in Section 3.4. The model abstracts away from all airline costs except airport charges.

The congested airport can use slot or pricing policies to mitigate the congestion problem for locals by choosing the slot quantity or the airport charge, respectively. In the case of slot policy, the airport does not earn from selling slots. This captures the notion of grandfather rights established by the Worldwide Scheduling Guidelines of the International Air Transport Association (IATA) and that, because of the grandfathering slots, contribute to airline profits. Efficient rationing is assumed to hold in this study in the sense that slots are allocated to passengers with the highest willingness to pay. This implies a conservative assessment of pricing policies because the efficient allocation of slots among airlines cannot be guaranteed in reality (Czerny and Lang, 2019).[3] In the case of pricing policy, the airport generates a positive profit from locals and non-locals. However, whereas the positive profit derived from non-locals matters to the local welfare-maximising airport, the consumer surplus from non-locals is ignored by the local welfare-maximising airport. Altogether, the present study analyses whether, from a social welfare perspective and when the airport is congested *and* serves locals and non-locals, society is better off when airlines make money (slot policy) or when the airport makes money (pricing policy).

The main part of the analysis is based on the consideration of general functional forms. The analysis shows that, in our framework, the local welfare-maximising slot quantity can coincide with the first-best outcome, whereas this is impossible in the case of pricing policy. The main result is to show that whether the outcomes coincide in the case of slot policy depends on the relationship between two types of shares of locals. The first type represents the share of locals relative to the total number of passengers, which we call the share of inframarginal locals. The second type is related to the effect of a marginal increase in the slot quantity on the quantities of locals, which we call the share of marginal locals. More specifically, the second type of share is equal to the increase in locals relative to

[3] Efficient rationing of slots can be achieved, for example, by slot trading (Brueckner, 2009). Alternatively, slot auctions may be used for slot allocation (for example, Rassenti *et al.*, 1982). To our knowledge, the first example for real-world slot auctions is Guangzhou Baiyun airport, where nine airport slot pairs were sold to airlines via an auction mechanism in 2015. A US Department of Transportation (DOT)'s proposal to auction slots at three New York airports in 2008 was unsuccessful and postponed indefinitely.

the increase in the total number of passengers or, equivalently, the increase in the slot quantity.

Using these concepts, the analysis shows that the first-best outcome coincides with the local welfare-maximising slot policy if the implied share of inframarginal locals is equal to the implied share of marginal locals. The intuition is developed with the help of cost-benefit ratios associated with a marginal increase in the slot quantity. The cost-benefit ratios are measured by the marginal external congestion cost divided by the ticket price. If the shares of inframarginal and marginal locals implied by the local welfare-maximum are equal, then the cost-benefit ratios associated with a marginal increase in the slot quantity are equal from the local and the first-best viewpoints. From a managerial point of view, the application of the shares of inframarginal and marginal locals in practice is discussed, showing that they can be used to assess the incentives of a local welfare-maximising airport from the first-best viewpoint. It is also shown that the intuition based on cost-benefit ratios carries over to the more complicated case with multiple congested airports. The basic model considers exclusive airline services, whereas inclusive airline services where airlines serve locals and non-locals are discussed as a robustness check. Linear functional forms are used to further illustrate the role of locals and non-locals for the policy comparison, and derive analytical solutions.

The present study contributes to various strands of the literature. It contributes to the literature on slots versus pricing policies (Brenck and Czerny, 2002; Czerny, 2008, 2010; Brueckner, 2009; Basso and Zhang, 2010; Daniel, 2014; Basso *et al.*, 2017; Czerny and Lang, 2019; De Palma and Lindsey, 2020) by considering decentralised slot and pricing policies in an airport network. It contributes to the literature on localised decision makers and the tolling of non-locals (De Borger *et al.*, 2005; De Borger and Van Dender, 2006; De Borger *et al.*, 2007; Mantin, 2012; Wan and Zhang, 2013; Czerny *et al.*, 2014; Wan *et al.*, 2016; Fung and Proost, 2017; Wan *et al.*, 2018). It also contributes to the growing literature on congestion management in airport networks (Pels and Verhoef, 2004; Button, 2008; Berardino, 2009; Benoot *et al.*, 2012; Silva *et al.*, 2014; Gillen and Starkie, 2016; Lin and Zhang, 2016, 2017; Arcelus *et al.*, 2019).[4] It finally contributes to the literature on the role of locals and non-locals (Mun, 2019; Lin, 2020).

The studies most closely related to the present study are the ones by Czerny and Lang (2019), and Lang and Czerny (2021). They considered congestion policy games in airport networks with multiple congested airports in the absence or presence of substitute destination choices for origin-destination passengers, respectively. This study's main contribution is to analyse, in a clear and transparent way, the role of locals and non-locals for the assessment of local welfare-maximising airport congestion policies and airport versus airline profits, by additionally distinguishing between inframarginal and marginal passengers.

The paper is organised as follows. The model will be presented in Section 2. Section 3 analyses airport policies based on general functional forms. More specifically, the role of locals and non-locals, and the concepts of the shares of inframarginal and marginal locals, will be discussed in detail in this section. At the end of Section 3, Czerny and Lang (2019)'s study is discussed to show that the results derived in this study are fundamental in the sense that they carry over to a congestion-policy game in a network with

[4]See Czerny and Lang (2019)'s detailed discussion on the various strands of mentioned literature.

multiple congested airports. The difference between the exclusive and inclusive airline services is discussed to shed light on how the share of passengers served by the local airlines affects the assessment of local welfare-maximising policies. In Section 4, linear functional forms are used to illustrate the relationship between the cost-benefit ratios of a marginal increase in the slot quantity from the local and the first-best viewpoints. Section 5 concludes the paper and discusses avenues for future research.

2.0 The Model

There are two airports. One airport is congested in the sense that the physical capacity is low relative to the passenger or flight volume so that airline delays occur. The other airport is uncongested in the sense that the physical airport capacity is large enough to serve flights and passengers without any delay. This uncongested airport could represent an arbitrary number of uncongested airports (Brueckner, 2002). Locals from both airports fly between the two airports and take return flights. The following concentrates on policy decisions of the congested airport by referring to this airport as 'the airport'. The uncongested airport is a passive airport in the sense that it provides infrastructure services and does not implement any policies. The operating costs and user costs of this uncongested airport are normalised to zero. This uncongested airport will be referred to as 'the other airport'.

The number of locals is denoted as q_l and the number of non-locals is denoted as q_{nl}. The numbers of passengers are non-negative; that is, $q_i \geqslant 0$ for $i = l, nl$. The strictly concave travelling benefits of airport i's passengers are denoted as $B_i(q_i)$ with $B_i''(q_i) < 0$. The total number of passengers at the airport is denoted as Q, with $Q = q_l + q_{nl}$.

This study assumes that local and non-local airlines exist, that airline markets are perfectly competitive, and that besides airport charges, all other airline costs are normalised to zero. This basic model version further assumes that airlines offer exclusive services in the sense that locals only fly with their local airlines (a relaxed model with inclusive airline services will be considered in Section 3.4). Airline load factors are assumed to be given by 100 per cent, and aircraft sizes are fixed and normalised to one unit of passenger.[5] The latter implies that the number of flights is equal to the number of passengers. Let the total number of passengers Q determine the convex average passenger delay, denoted as $T(Q)$ with $T'(Q) > 0$ and $T''(Q) \geqslant 0$. Because flight and passengers are equal in number, the average delay function can represent delays caused by flights due to limited runway capacity and delays caused by passengers due to limited terminal capacity. The passengers' time valuations are denoted as v.

The airport can choose between two policy measures, which are slot policy, denoted as S, and pricing policy, denoted as P. Let ϕ with $\phi = S, P$ denote the policy variable. In the case of slot policy $\phi = S$, the airport sets an upper limit on the number of flights, which is equal to the number of passengers, at the airport. For convenience, this study considers the upper limit on the number of passengers, denoted as $\bar{Q}$ and called the slot quantity. In the case of pricing policy $\phi = P$, the airport charges the local and non-local airlines a

[5]Endogenous load factors and aircraft sizes have recently been considered by Czerny *et al.* (2016).

non-discriminatory per-passenger airport charge, denoted as R with $R = R(\phi) \geqslant 0$.[6] In the case of slot policy and to capture the notion of grandfather rights, the airport charge is assumed to be zero; that is, $R(S) = 0$. This means the airport cannot earn from selling slots, leading to zero airport revenue in the case of slot policy. This is an important feature of the present model because it captures the IATA's Worldwide Scheduling Guidelines requiring that airport slot allocation is based on historic precedence. The airports' costs are all normalised to zero. Together with the assumption of a non-negative airport charge, this means that airport cost recovery is always ensured.

The ticket price is denoted as r with $r = r(\phi) \geqslant 0$. In the case of slot policy, airlines can generate positive profits because slots are provided for free, whereas the ticket price is positive; that is, $r(S) \geqslant 0 = R(S)$. In the case of pricing policy, airlines generate zero profits because they have to pay the airport charge, which is exactly equal to the ticket price because of perfect competition; that is, $r(P) = R(P) \geqslant 0$. In this scenario, all the producer surplus is internalised by the airport.

Efficient rationing ensures that passengers with the highest willingness to pay are served first. This can be guaranteed under pricing policy and is also assumed to be guaranteed in this study in the case of slot policy. However, the current slot allocation practice based on grandfather rights cannot guarantee efficient rationing under slots. Therefore, this study provides a conservative assessment of pricing policy relative to slot policy.

The model compares the local welfare-maximising outcomes where airports are assumed to attach a unit weight to both local consumer surplus, and the profits of their local airlines under slot and pricing policies with the first-best outcome. Czerny and Lang (2019) demonstrated that the consideration of slot and pricing policies can be justified in the sense that slot policy is relevant when airport profits do not matter for local governments, whereas pricing policy is relevant when airport profits matter.

3.0 Policy Assessment Based on General Functional Forms

This section starts with a discussion of passenger demands, and how they relate to the slot quantity and the ticket price. This step provides crucial insights that will be used for the assessment of slot and pricing policies. The section continues with a discussion of the optimal slot and pricing policies from the local airport's viewpoint, and compares them with the first-best outcome. Special attention will be given to the role of locals and non-locals for the results.

3.1 Demand relationships

The generalised price of travelling, denoted as η, is given by:

$$\eta = r + vT(Q). \tag{1}$$

The right-hand is the sum of the ticket price r and the average congestion costs $vT(Q)$. Passengers consider the generalised price as given. Demands for locals and non-locals depending on r are denoted as $D_l(r)$ and $D_{nl}(r)$, respectively. They are determined by the

[6]Price discrimination violates the rules of the World Trade Organization (WTO) for free transit. Detailed regulations can be found in Article 5 of GATT 1994.

conditions:

$$B_l'(q_l) = B_{nl}'(q_{nl}) = \eta. \tag{2}$$

Passengers will travel if their marginal benefit from travelling is at least as high as the generalised price. Applying Cramer's rule to the system of equations in equation (2) yields the following (all the proofs are relegated to the Appendix):

Lemma 1. *The effect of a marginal increase in r on demands can be characterised as:*

$$D_l'(r), D_{nl}'(r) < 0. \tag{3}$$

This lemma shows that both locals and non-locals' demands are decreasing in the ticket price. This implies that the total demand is also decreasing in the ticket price.

The welfare assessment of slot policy requires an understanding of the relationship between the slot quantity $\bar{Q}$ and the demands D_l and D_{nl}. Here and hereafter, it is assumed that the slot constraint is always binding.[7] Let $D(r)$ denote the sum of the locals' and non-locals' demands depending on r with $D(r) = D_l(r) + D_{nl}(r)$. By Lemma 1, $D'(r) < 0$. The ticket price $r(S)$ in the case of slot policy is implicitly determined by:

$$\bar{Q} - D(r) = 0. \tag{4}$$

Let $r(\bar{Q})$ denote the ticket price depending on the slot quantity. Totally differentiating equation (4) yields the following:

Lemma 2. *The ticket price is decreasing in the slot quantity; that is:*

$$r'(\bar{Q}) < 0. \tag{5}$$

By Lemma 1, an increase in the slot quantity is associated with a reduction in the ticket price to ensure that the passenger demand is equal to the slot quantity. Substituting $r(\bar{Q})$ for r in demands $D_l(r)$ and $D_{nl}(r)$ yields the demands depending on the slot quantity — that is, $D_l(r(\bar{Q})) = D_l(\bar{Q})$ and $D_{nl}(r(\bar{Q})) = D_{nl}(\bar{Q})$, leading to $D(r(\bar{Q})) = D(\bar{Q}) = \bar{Q}$. Using Lemmas 1 and 2, the relationships between the slot quantity and demands can be described in the following way:

Lemma 3. *The effect of a marginal increase in slot quantity $\bar{Q}$ on demands can be characterised as:*

$$0 < D_l'(\bar{Q}), D_{nl}'(\bar{Q}) < D'(\bar{Q}) = 1. \tag{6}$$

This lemma shows that the demands of locals and non-locals, and thus the total demand, are increasing in the slot quantity. It further shows that an increase in the slot quantity increases the total demand by an equal amount (a natural result).

For the comparison of the local welfare-maximising ticket price with the first-best price, it is useful to characterise the relationship between the ticket price r and the generalised price η. Using equation (1) and substituting Q by $D(r)$, the generalised price depending on r can be written as $\eta(r) = r + vT(D(r))$. Taking the derivative of the right-hand side

[7] Airports may not always operate at full capacity. Gillen *et al.* (2016) pointed out that meteorological and other stochastic factors may contradict the current assumption.

with respect to r and rearranging yields the following:

Lemma 4. *The effect of a marginal increase in the ticket price r on the generalised price η can be characterised as:*

$$0 < \eta'(r) < 1. \tag{7}$$

This lemma shows that an increase in r leads to an increase in the generalised price that is smaller than the increase in r. This is related to the structure of the generalised price. Equation (1) shows that the generalised price is the sum of the ticket price r and the congestion cost $vT(Q)$. An increase in the price directly increases the generalised price because it is part of the generalised price. However, by Lemma 1, it also leads to a reduction in the total number of passengers, thus reducing the generalised price by reducing congestion and average congestion cost. This explains the inequality $\eta'(r) < 1$.

3.2 Slots versus pricing policy

The consumer surpluses of locals and non-locals, denoted as CS_i for $i = l, nl$, are equal to the differences between the benefits and the sum of ticket price payments and delays costs, which can be written as:

$$CS_i(q_i) = B_i(q_i) - q_i \cdot \eta. \tag{8}$$

The first term on the right-hand side is the benefit of passengers. The second term is the passengers' total costs for travelling, including the total payment to their local airlines and the total delay costs.

The welfares of the airport and the other airport are denoted as W_i for $i = l, nl$. The airport's welfare W_l is equal to the sum of the locals' consumer surplus, and the joint profit of the local airlines and the airport (that is, $q_l \cdot r(\phi) + q_{nl} \cdot R(\phi)$), which can be simplified and written as:

$$W_l(q_l, q_{nl}) = B_l(q_l) + q_{nl} \cdot R(\phi) - q_l \cdot vT(Q). \tag{9}$$

The first term on the right-hand side is the benefit of locals. The second term is the airport profit derived from non-locals. The third term represents the total delay costs of locals. Comparing the airport's welfare in equation (9) with the consumer surplus of locals in equation (8), the difference is the second term. It captures that in the case of pricing policy $\phi = P$, there will be extra profit from charging non-local airlines. The extra profit, however, would be absent in the case of slot policy $\phi = S$ because $R(S) = 0$ by assumption.

The other airport's welfare can be written as:

$$W_{nl}(q_l, q_{nl}) = B_{nl}(q_{nl}) - q_{nl} \cdot R(\phi) - q_{nl} \cdot vT(Q). \tag{10}$$

The first term on the right-hand side is the benefit of non-locals. The second term is the total payment to the airport, which is zero in the case of slot policy because $R(S) = 0$ by assumption. The third term represents the total delay costs of non-locals.

3.2.1 First-best outcome as a benchmark

The assessment of the congestion policies is based on a comparison of the local welfare-maximising solutions under slot and pricing policies with the first-best outcome. The total welfare generated by the two airports, denoted as W with $W = W_l + W_{nl}$, is given by the difference between the sum of benefits and the sum of delay costs, which can be written as:

$$W(q_l, q_{nl}) = B_l(q_l) + B_{nl}(q_{nl}) - QvT(Q). \tag{11}$$

Let the first-best solution be indicated by a double asterisk '**'. The first-best number of passengers are determined by the first-order condition $\partial W(q_i^{**})/\partial q_i = 0$, which can be written as:

$$B_i'(q_i^{**}) - (vT(Q^{**}) + Q^{**} \cdot vT'(Q^{**})) = 0.\text{[8]} \tag{12}$$

The first term on the left-hand side is the marginal benefit of either locals or non-locals. The sum of the two terms inside the parentheses is equal to the marginal congestion cost. The first-best outcome is achieved when the marginal benefits are equal to the marginal congestion costs.

Consider a laissez faire situation in which there are no slot controls and the airport charge is equal to zero. In this scenario, the generalised price is equal to the average congestion cost $vT(Q^{**})$, which is equal to the first part of the marginal congestion costs in the parentheses. The second term, $Q^{**} \cdot vT'(Q^{**})$, describes the congestion cost that passengers impose on others when they travel. This part is not included in the generalised price in this scenario and, therefore, is external. Passengers will make excessive use of the congested airport because of the existence of the marginal external congestion costs. Both slot and pricing policies can be used to implement the first-best outcome by adjusting the generalised price via the ticket price that is associated with the slot quantity or the airport charge.

In the case of slot policy, $\phi = S$, the first-best outcome can be achieved by setting the slot quantity $\bar{Q}$ equal to the first-best number of passengers $\bar{Q}^{**}$. This leads to a first-best ticket price, $r^{**}(S)$, that is equal to the marginal external congestion costs; that is:

$$r^{**}(S) = \bar{Q}^{**} \cdot vT'(\bar{Q}^{**}). \tag{13}$$

In the case of pricing policy, $\phi = P$, the first-best outcome can be achieved by directly setting a first-best airport charge, $r^{**}(P)$, that is equal to the marginal external congestion costs; that is:

$$r^{**}(P) = D(r^{**}) \cdot vT'(D(r^{**})). \tag{14}$$

Together with equations (1) and (2), the first-best price in equation (13) and the first-best airport charge in equation (14) imply that the first-best outcome is achieved by these policies because they ensure that the marginal benefits of locals and non-locals are equal to the marginal congestion costs in these situations. This shows that both policies will indeed achieve the first-best outcome. The following analyses the decentralised decision making of the congested airport and compares the local welfare-maximising solutions with the first-best outcome.

[8] The concavity of the benefit functions, together with the convexity of the delay function, imply that the Hessian of $W(q_l, q_{nl})$ in equation (11) is negative definite. Therefore, there exists a unique solution for the welfare-maximising quantities of locals and non-locals.

3.2.2 Local welfare-maximising slot policy

Consider $\phi = S$. Plugging the demands of locals and non-locals, depending on the slot quantity into the welfare function in equation (9), yields the local welfare depending on the slot quantity; that is, $W_l(\bar{Q}) = W_l(D_l(\bar{Q}), D_{nl}(\bar{Q}))$. Let the local welfare-maximising solution be indicated by a single asterisk '*'. Assume that the local welfare-maximising slot quantity is determined by the first-order condition, $W_l{}'(\bar{Q}^*) = 0$, which can be written as:

$$(B_l'(D_l(\bar{Q}^*)) - vT(\bar{Q}^*)) \cdot D_l'(\bar{Q}^*) - D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*) = 0. \tag{15}$$

The first term on the left-hand side is the product of $B_l'(D_l(\bar{Q}^*)) - vT(\bar{Q}^*)$, which is equal to the local welfare-maximising ticket price, $r^*(S)$, by equations (1) and (2), and the derivative, $D_l'(\bar{Q}^*)$. By Lemma 3, $D_l'(\bar{Q}^*)$ takes a value between 0 and 1 and describes a share of locals, in which the share captures the increase in the quantity of locals associated with a marginal increase in the slot quantity. The second term captures the marginal external congestion cost of locals. Altogether, the local welfare-maximising slot quantity ensures that the marginal increase in the benefit of locals, as measured by the weighted ticket price, $r^*(S) \cdot D_l'(\bar{Q}^*)$, is equal to the locals' marginal external congestion cost.

Using the conditions in equation (2) and $D_l'(\bar{Q}^*) + D_{nl}'(\bar{Q}^*) = D'(\bar{Q}^*) = 1$, the left-hand side of equation (15) can be rewritten as:

$$r^*(S) \cdot D_l'(\bar{Q}^*) - D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*) \cdot (D_l'(\bar{Q}^*) + D_{nl}'(\bar{Q}^*)) = 0. \tag{16}$$

Solving the first-order condition for the local welfare-maximising slot quantity, by dividing $D_l'(\bar{Q}^*)$ and rearranging equation (16), yields the local welfare-maximising ticket price in the case of slot policy, which can be written as:

$$r^*(S) = D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*) + \frac{D_{nl}'(\bar{Q}^*)}{D_l'(\bar{Q}^*)} \cdot D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*). \tag{17}$$

The first term on the right-hand side is the marginal external congestion cost of locals. The second term is a weighted marginal external congestion cost of locals. Lemma 3 mentions that $D_l'(\bar{Q}^*)$, $D_{nl}'(\bar{Q}^*) > 0$, which implies that the weight in the second term, $D_{nl}'(\bar{Q}^*)/D_l'(\bar{Q}^*)$, is positive. Therefore, the local welfare-maximising slot quantity leads to an over-internalisation of the locals' part of the marginal external congestion cost because $r^*(S) > D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*)$. The intuition can be described as follows.

In the presence of non-locals, they are taking up the airport's limited slot resources and, thus, benefit from the slot expansion; however, they are not contributing to the congested airport's welfare. This reduces the local welfare-maximising slot quantity relative to the case in which non-locals would be absent.

From the first-best viewpoint, both locals and non-locals make excessive use of the congested airport capacity in the case of laissez faire. The local airport's incentives to reduce the slot quantity in the presence of non-locals may, therefore, be desirable from the first-best viewpoint. The following proposition highlights the main result of the paper. It describes the condition under which the local welfare-maximising slot quantity implements the first-best outcome:

Proposition 1. *The consideration of general functional forms implies:*

(i) If in the local welfare-maximum the demand of locals and the total demand satisfy the equality:

$$\frac{D_l(\bar{Q}^*)}{\bar{Q}^*} = D_l'(\bar{Q}^*), \tag{18}$$

then the local welfare-maximising slot quantity is equal to the first-best slot quantity — that is, $\bar{Q}^* = \bar{Q}^{**}$;

(ii) if the left-hand side exceeds the right-hand side, then the local welfare-maximising slot quantity is too low relative to the first-best slot quantity — that is, $\bar{Q}^* < \bar{Q}^{**}$; and

(iii) if the left-hand side is smaller than the right-hand side, then the local welfare-maximising slot quantity is too high relative to the first-best slot quantity — that is, $\bar{Q}^* > \bar{Q}^{**}$.

The left-hand side of equation (18) shows the share of the locals' number of passengers relative to the total number of passengers, which we call the *share of inframarginal locals*. The right-hand side shows the increase in locals associated with a marginal increase in the slot quantity, which we call the *share of marginal locals*. If the local welfare-maximising slot quantity implies that these two shares are equal, then the local welfare-maximising slot quantity implies the first-best outcome.

For an intuition, expand equation (18) by multiplying both sides with $vT'(\bar{Q}^*)/r^*(S)$ and rearrange, which yields:

$$\frac{\bar{Q}^* \cdot vT'(\bar{Q}^*)}{r^*(S)} = \frac{D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*)}{D_l'(\bar{Q}^*) \cdot r^*(S)}. \tag{19}$$

Both the locals' and non-locals' marginal benefits and marginal external congestion costs are increasing in the slot quantity. The left-hand side shows the cost-benefit ratio of a marginal increase in the slot quantity in terms of the marginal external congestion cost and the ticket price from the first-best viewpoint. The right-hand side shows the cost-benefit ratio of a marginal increase in the slot quantity in terms of the marginal external congestion cost and the ticket price from the airport's viewpoint (which is equal to one in the local welfare-maximum). If these cost-benefit ratios are equal in the local welfare-maximum, which is true if the shares of inframarginal and marginal locals are equal in the local welfare-maximum as mentioned in equation (18), then the local welfare-maximising solution coincides with the first-best outcome.

If a marginal increase in the slot quantity leads to a higher cost-benefit ratio of the airport relative to the first-best viewpoint, then the airport is too reluctant to increase slot quantity. In local welfare-maximum, slot policy becomes too strict in the sense that slot quantity is too low relative to the first-best outcome. If a marginal increase in the slot quantity leads to a lower cost-benefit ratio of the airport relative to the first-best viewpoint, then the airport is too inclined to increase slot quantity. In local welfare-maximum, slot policy becomes too loose in the sense that slot quantity is too high relative to the first-best outcome.

The shares of inframarginal and marginal locals can be used to inform policy makers in real-world practice. The share of inframarginal locals can be identified by dividing, say, the yearly number of locals by the yearly total number of passengers given by the sum of locals and non-locals. The corresponding share of marginal locals can be identified by considering the changes in the number of passengers associated with an increase in the slot quantity. More specifically, this share can be estimated by dividing the increase in the yearly number of locals associated with an increase in the slot quantity by the increase in the yearly total number of passengers associated with an increase in the slot quantity. The relationship between the two estimates can then be used to assess the incentives of a local welfare-maximising airport from the first-best viewpoint. For instance, if the estimated share of inframarginal locals is higher than the share of marginal locals, then the airport's incentives to expand the slot quantity should be too high from the first-best viewpoint, whereas the airport's incentives to increase the slot quantity should be too low from the first-best viewpoint in the reverse case.

3.2.3 Local welfare-maximising pricing policy

Consider $\phi = P$. Substituting the demands of locals and non-locals, depending on the price into the welfare function in equation (9), yields the local welfare depending on the ticket price; that is, $W_l(r) = W_l(D_l(r), D_{nl}(r))$. Assume that the local welfare-maximising ticket price, $r^*(P)$, is determined by the first-order condition, $W_l'(r^*) = 0$, which can be written as:

$$(B_l'(D_l(r^*)) - vT(D(r^*))) \cdot D'(r^*) - D_l(r^*) \cdot vT'(D(r^*)) \cdot D'(r^*) + D_{nl}(r^*) = 0. \quad (20)$$

The first and second terms on the left-hand side capture the benefits and marginal external congestion cost of locals, respectively. The third term on the left-hand side captures the revenue gain from non-locals. Substituting $(B_l'(D_l(r^*)) - vT(D(r^*)))$ in the parentheses by the airport charge $r^*(P)$ in the local welfare-maximum and solving for the airport charge yields:

$$r^*(P) = D(r^*) \cdot vT'(D(r^*)) + \left|\frac{D(r^*)}{D'(r^*)}\right| \cdot \frac{D_{nl}(r^*)}{D(r^*)} \cdot \eta'(r^*). \quad (21)$$

The first term on the right-hand side is the marginal external congestion cost of all passengers, which is also the first-best price in equation (14). The second term is a positive markup, which is determined by the semi-price elasticity of demand, $D(r^*)$, with respect to the airport charge weighted by the share of non-locals and the marginal effect of a change in the airport charge on the generalised price. The semi-price elasticity, $|D(r^*)/D'(r^*)|$, represents the optimal charge in the case of profit maximisation. The notion of profit maximisation only applies to non-locals when the airport maximises local welfare, which is why the elasticity measure includes a weight that captures the share of (inframarginal) non-locals. However, the non-locals contribute not only to the airport's profit but also to congestion. Therefore, the marginal change in the generalised price is added as another weight. Altogether, this implies the following:

Proposition 2. *The local welfare-maximising airport charge never reaches the first-best outcome in the sense that* $r^*(P) > r^{**}(P)$.

In the absence of non-locals, the markup is zero in equation (21). Thus, the local welfare-maximising airport charge is equal to the first-best price. This implies that the airport's incentive to exploit the non-locals, by charging a positive markup on the marginal external congestion cost of locals, eliminates the possibility that pricing policy can achieve the first-best outcome.

3.3 A congestion game in an airport network

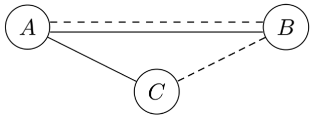
The present model considers the role of locals and non-locals, and evaluates the local policies in the context of one (active) congested airport and one (passive) uncongested airport. In practice, changes in the congestion policies of one airport are observed to propagate congestion to other airports (Federal Aviation Administration, 2010; Arykan *et al.*, 2013). This subsection will show that the insights derived from this simple two-airport network are robust in the sense that they carry over to a more realistic framework with two congested airports.

Consider Czerny and Lang (2019), who developed and studied a framework that can be considered as a variation of the present framework. They considered a three-airport network model. This network involved two symmetric airports, denoted as A and B, that were populated and congested. Furthermore, there was a third airport, denoted as C, that was unpopulated and uncongested. Passengers were origin-destination passengers and took return flights. They travelled to the other congested airport or the third airport. Figure 1 illustrates such a three-airport network in which flights for locals from airport A are depicted by solid lines and flights for locals from airport B are depicted by dashed lines. Airport i's demands of locals were denoted by D_{ij} and D_{iC} for those who travelled between airports A and B, and between i and C, respectively, with $i = A$, B and $i \neq j$. The total passenger demands at airports A and B were therefore given by $D_i = D_{ij} + D_{ji} + D_{iC}$. The local airports' slot quantities were denoted as $\bar{Q}_i$. Czerny and Lang (2019) found that equilibrium slot quantities were always too high relative to the first-best outcome. Equation (18) can be used to develop a better understanding of the fundamental drivers of this result.

Consider the left-hand side of equation (18), which shows the share of inframarginal locals. The value of the left-hand side that corresponds to this more complex framework can be written as $(D_{ij} + D_{iC})/D_i$. Consider the right-hand side of equation (18), which shows the share of marginal locals. In a first step, the value of the right-hand side that corresponds to the extended framework would be considered to be equal to the share of marginal locals $\partial D_{ij}/\partial \bar{Q}_i + \partial D_{iC}/\partial \bar{Q}_i$. However, remember that it is the cost-benefit ratios that ultimately matter.

A crucial difference between the present framework, and Czerny and Lang (2019)'s framework, is that locals who travelled between airports A and B had to pay ticket prices that reflect the use of two scarce airport infrastructures. Therefore, their marginal benefit of locals who travelled between the congested airports was twice as high as the

Figure 1
A Three-Airport Network. Solid Lines: Flights for Locals from Airport A. Dashed Lines: Flights for Locals from Airport B.



marginal benefit of locals who travelled between airports i and C, who utilised only one congested airport. The locals who travelled between the congested airports thus count twice to capture the difference in marginal benefits (and marginal congestion costs). The right-hand side in equation (16), therefore, correctly translates into $2\partial D_{ij}/\partial\bar{Q}_i + \partial D_{iC}/\partial\bar{Q}_i$ in their framework with two congested airports. Czerny and Lang (2019) showed that the effect of a marginal increase in the local slot quantity on passengers who travelled between the congested airports was independent of their origins in the sense that $\partial D_{ij}/\partial\bar{Q}_i = \partial D_{ji}/\partial\bar{Q}_i$. Using this equality, $2\partial D_{ij}/\partial\bar{Q}_i + \partial D_{iC}/\partial\bar{Q}_i$ can be simplified as $\partial D_i/\partial\bar{Q}_i = 1$; however, this implies that the equality in equation (18) can never be satisfied in their framework because $(D_{ij} + D_{iC})/D_i < 1$. Altogether, the cost-benefit ratio associated with a marginal increase in the local slot quantity is always lower from the local viewpoint relative to the first-best viewpoint. Therefore, the local welfare-maximising slot quantity will be too high relative to the first-best outcome, as was shown by Czerny and Lang (2019).

The local welfare-maximising slot quantity can be first-best in the framework with one congested airport, whereas this is not possible in the framework considered by Czerny and Lang (2019). In this sense, the assessment of slot policies depends on the airport networks under consideration. The result that pricing policies lead to excessive pricing from the first-best viewpoint is, however, more robust because it carries over to the framework with two congested airports considered by Czerny and Lang (2019). This is because local airports fully internalise congestion costs and have the incentives to exploit the non-locals independent of the airport networks under consideration.

3.4 Exclusive versus inclusive airline services

The model has been focusing on exclusive airline services where local airlines only serve their locals. This subsection discusses a relaxed environment with inclusive airline services, in the sense that airlines can serve both locals and non-locals. Consider the region of the congested airport. Let θ_l denote the share of locals served by local airlines and θ_{nl} the share of non-locals served by local airlines. Previously, it was assumed that $\theta_l = 1$ and $\theta_{nl} = 0$. In this section, this assumption is relaxed in the sense that the two shares can take any value between 0 and 1; that is, $0 \leqslant \theta_i \leqslant 1$ for $i = l, nl$. With this notation, the number of the local airlines' passengers is given by $\theta_l q_l + \theta_{nl} q_{nl}$, and the airport's welfare can be written as:

$$W_l = CS_l + R(\phi) \cdot Q + (r(\phi) - R(\phi)) \cdot (\theta_l q_l + \theta_{nl} q_{nl}). \tag{22}$$

The first term on the right-hand side is the locals' consumer surplus. The second term is the airport profit. The third term is the local airlines' profit from both locals and non-locals.

In the case of pricing policy $\phi = P$, $r(P) = R(P)$. This implies that the third term on the right-hand side of equation (22), which is the local airlines' profit, is zero. The airport's welfare is equal to the sum of the locals' consumer surplus and the airport profit, which is identical to the airport's welfare in equation (9). Therefore, we have the following:

Proposition 3. *The local welfare-maximising pricing policies are independent of whether airline services are exclusive or inclusive, in the sense that the local welfare-maximising airport charge is given by* $r^*(P)$ *in equation (21) for all* $\theta_l, \theta_{nl} \in [0, 1]$.

In the case of pricing policy, perfectly competitive airline markets imply zero airline profits. Therefore, airlines do not contribute to local welfares and, hence, it is of no importance for the local welfare-maximiser whether airlines serve locals or non-locals.

The picture changes in the case of slot policy in which $\phi = S$ and $R(S) = 0$. In this scenario, the second term on the right-hand side of equation (22) representing the airport's profit is equal to zero, whereas the third term representing the local airlines' profit is positive. It is useful to recognise that what matters from the airport's viewpoint is the total number of passengers served by local airlines, not the shares of locals and non-locals. This is because this allows substituting the total number of flights $\theta_l q_l + \theta_{nl} q_{nl}$ by θQ, where θ represents the market share of local airlines. In the case of exclusive airline services, the total number of passengers is equal to the total number of locals; that is, $\theta Q = q_l$. In the case of inclusive airline services, the total number of flights may or may not be equal to the total number of locals, which is dependent on θ.

Consider the market share of local airlines, θ, as exogenously given.[9] Substituting $\theta_l q_l + \theta_{nl} q_{nl}$ by θQ and Q by $\bar{Q}$ in (22) yields the airport's welfare depending on the market share of the local airlines θ and slot quantity $\bar{Q}$, which can be written as:

$$W_l = CS_l + r(S) \cdot \theta \bar{Q}. \tag{23}$$

Assume that the local welfare-maximising slot quantity is determined by the first-order condition $W_l'(\bar{Q}^*) = 0$, which can be written as:

$$-\eta'(\bar{Q}^*) \cdot D_l(\bar{Q}^*) + r(\bar{Q}^*) \cdot \theta + r'(\bar{Q}^*) \cdot \theta \bar{Q}^* = 0. \tag{24}$$

The first term on the left-hand side captures the consumer surplus gain of locals associated with an increase in the slot quantity. This term is (the absolute value of) the product of the marginal change in the generalised price associated with an increase in the slot quantity and the locals' demand. Lemma 2 shows that $r'(\bar{Q}) < 0$ and Lemma 4 shows that $0 < \eta'(r) < 1$. Together this implies that $\eta'(\bar{Q}^*) < 0$ because $\eta'(\bar{Q}^*) = \eta'(r) \cdot r'(\bar{Q}^*)$, which means that an increase in the slot quantity increases consumer surplus. The second term captures the local airlines' gain in revenue associated with the increase in the slot quantity and the number of passengers. Lemma 2 shows that $r'(\bar{Q}) < 0$. Therefore, the third term captures the local airlines' loss in revenue from the decrease in the ticket price. Altogether, the local welfare-maximising slot quantity optimally balances consumer surplus and airline profit effects for the local economy.

Totally differentiating the first-order condition in equation (24) with respect to $\bar{Q}$ and θ yields the following:

Lemma 5. *In the case of inclusive airline services, for a given market share of the local airlines, θ, and given that the local welfare-maximising slot quantity is determined by the first-order condition in equation (24), the local welfare-maximising slot quantity is decreasing in θ; that is, $\partial \bar{Q}^* / \partial \theta < 0$.*

If θ decreases, then local airlines generate less profit and, therefore, the local airlines' profit becomes less significant relative to the locals' consumer surplus to the airport. The

[9] In the case of exclusive airline services, the airline market share can be considered as given only if the shares of inframarginal and marginal locals are equal.

lemma shows that in this scenario, the airport has the incentive to increase the slot quantity to allow more locals to travel because the locals' consumer surplus is increasing in the slot quantity, as implied by Lemma 3. This relationship exists if the local welfare-maximising slot quantity is determined by the first-order condition in equation (24), which does not always need to be true.

If θ is small enough, the local welfare-maximising slot quantity is so high that the non-negativity constraint associated with the ticket price becomes binding. Consider the extreme case in which θ is equal to zero. In this case, the local welfare is equal to the local consumer surplus that is strictly increasing in the number of passengers, implying a binding non-negativity constraint for the ticket price. In the case of a binding non-negativity constraint, the local welfare-maximising slot quantity is not unique because any high enough slot quantity can imply a zero ticket price and, therefore, be optimal.

Consider the extreme case in which the local airlines serve all passengers; that is, $\theta = 1$. In this case, the airport's welfare is independent of the policy choice because the airport can earn from all passengers by either the airport charge under pricing policy or the local airlines' ticket price under slot policy. This implies that the local welfare-maximising slot quantity is given by a local welfare-maximising ticket price that is equal to the local welfare-maximising airport charge in the case of pricing policy; that is, $r^*(S) = r^*(P)$ when $\theta = 1$ where $r^*(P)$ is given by equation (21). Together with Lemmas 2 and 5, this implies the following:

Proposition 4. *In the case of inclusive airline services and slot policy, there exists an upper bound for the local welfare-maximising ticket price $r^*(S)$ that is equal to $r^*(P)$ given by equation (21).*

4.0 Slot Policy: The Case of Linear Functional Forms

This section considers linear functional forms, and concentrates on the ambiguous relationships between the local welfare-maximising slot quantity and the first-best outcome. The discussion of pricing policy is omitted because it involves complex mathematical expressions, but provides little insight to justify the mention given that the consideration of general functional forms demonstrated that it unambiguously fails to implement the first-best outcome.

The benefits of travelling are given by the quadratic function:

$$B_i(q_i) = \alpha_i q_i - \frac{1}{2}\beta_i q_i^2, \tag{25}$$

with parameters α_i, $\beta_i > 0$ and $i = l, nl$, where parameters α_i are called the maximum reservation prices. Average delays are given by $T(Q)$ with $T(Q) = Q$. Demands $D_l(r)$ and $D_{nl}(r)$ are determined by the demand equilibrium conditions $B_i'(q_i) = r + vT(Q)$. Simultaneously solving these conditions yields the demands of locals and non-locals depending on the price r, which can be written as:

$$D_i(r) = \frac{\alpha_i\beta_j + (\alpha_i - \alpha_j)v - \beta_j \cdot r}{\beta_l\beta_{nl} + (\beta_l + \beta_{nl})v}. \tag{26}$$

The right-hand side shows that demands are decreasing in price r.[10] Using the demands in equation (26), the total demand at the airport is given by:

$$D(r) = \frac{\alpha_l \beta_{nl} + \alpha_{nl}\beta_l - (\beta_l + \beta_{nl}) \cdot r}{\beta_l \beta_{nl} + (\beta_l + \beta_{nl})v}. \tag{27}$$

Substituting $D(r)$ on the left-hand side of equation (27) with slot quantity $\bar{Q}$ and solving yields the ticket price in the case of slot policy, which can be written as:

$$r(\bar{Q}) = \frac{\alpha_l \beta_{nl} + \alpha_{nl}\beta_l}{\beta_l + \beta_{nl}} - \frac{\beta_l \beta_{nl} + (\beta_l + \beta_{nl})v}{\beta_l + \beta_{nl}} \cdot \bar{Q}. \tag{28}$$

The right-hand side shows that the ticket price $r(\bar{Q})$ is decreasing in the slot quantity $\bar{Q}$. Substituting price r on the right-hand side of equation (26) with the ticket price in equation (28) yields the demands depending on slot quantity, which can be written as:

$$D_i(\bar{Q}) = \frac{\alpha_i - \alpha_j + \beta_j \cdot \bar{Q}}{\beta_l + \beta_{nl}}. \tag{29}$$

The right-hand side shows that demands are increasing in slot quantity $\bar{Q}$.

Using $T'(Q) = 1$, the first-best price in equation (13) can be rewritten as $r^{**}(S) = v \cdot \bar{Q}^{**}$. Substituting the left-hand side in equation (28) by $r(\bar{Q})$, evaluated at the first-best slot quantity, yields the condition:

$$v \cdot \bar{Q}^{**} = \frac{\alpha_l \beta_{nl} + \alpha_{nl}\beta_l}{\beta_l + \beta_{nl}} - \frac{\beta_l \beta_{nl} + (\beta_l + \beta_{nl})v}{\beta_l + \beta_{nl}} \cdot \bar{Q}^{**}. \tag{30}$$

Solving for the first-best slot quantity $\bar{Q}^{**}$ yields:

$$\bar{Q}^{**} = \frac{\alpha_l \beta_{nl} + \alpha_{nl}\beta_l}{\beta_l \beta_{nl} + 2(\beta_l + \beta_{nl})v}.^{11} \tag{31}$$

The local welfare-maximising slot quantity in the case of slot policy is determined by the first-order condition $W_l'(\bar{Q}^{*}) = 0$ and can be written as:

$$\bar{Q}^{*} = \bar{Q}^{**} - (\alpha_l - \alpha_{nl}) \frac{(\beta_l + \beta_{nl})v}{\beta_{nl}(\beta_l \beta_{nl} + 2(\beta_l + \beta_{nl})v)}. \tag{32}$$

The first term on the right-hand side is given by the first-best slot quantity. The second term is the product of the difference in maximum reservation prices, $(\alpha_l - \alpha_{nl})$ and a positive term, which implies the following relationships between local welfare-maximising slot quantities and first-best number of passengers:

Proposition 5. *The consideration of linear functional forms implies:*

(i) If the maximum reservation prices of locals and non-locals are equal — that is, $\alpha_l - \alpha_{nl}$ *— then the local welfare-maximising slot quantity is equal to the first-best slot quantity; that is,* $\bar{Q}^{*} = \bar{Q}^{**}$*;*

[10] It is a necessary condition that $\alpha_i > 2v(\alpha_j - \alpha_i)/\beta_j$ to ensure that all demands are positive.

[11] Together with equation (14), the first-best airport charge is given by $r^{**}(P) = v \cdot \bar{Q}^{**}$.

(ii) if the maximum reservation price of locals is smaller than the maximum reservation price of non-locals — that is, $\alpha_l < \alpha_{nl}$ *— then the local welfare-maximising slot quantity exceeds the first-best slot quantity; that is,* $\bar{Q}^* > \bar{Q}^{**}$*; and*

(iii) if the maximum reservation price of locals exceeds the maximum reservation price of non-locals — that is, $\alpha_l > \alpha_{nl}$ *— then the local welfare-maximising slot quantity is smaller than the first-best slot quantity; that is,* $\bar{Q}^* < \bar{Q}^{**}$.

Proposition 5 highlights that the maximum reservation prices determine whether local welfare-maximising slot policy can reach first-best outcome. For an intuition, first consider the share of marginal locals, which can be written as:

$$D_l'(\bar{Q}) = \frac{\beta_{nl}}{\beta_l + \beta_{nl}}. \tag{33}$$

The right-hand side is positive and independent of slot quantity, which means that the share of marginal locals is independent of the slot quantity. Second, consider the share of inframarginal locals, which can be written as:

$$\frac{D_l(\bar{Q})}{\bar{Q}} = \frac{\alpha_l - \alpha_{nl}}{(\beta_l + \beta_{nl})\bar{Q}} + D_l'(\bar{Q}). \tag{34}$$

The first term on the right-hand side contains the difference in maximum reservation prices ($\alpha_l - \alpha_{nl}$). This difference determines the difference between the demand elasticities of locals and non-locals with respect to the generalised price because these demand elasticities are given by:

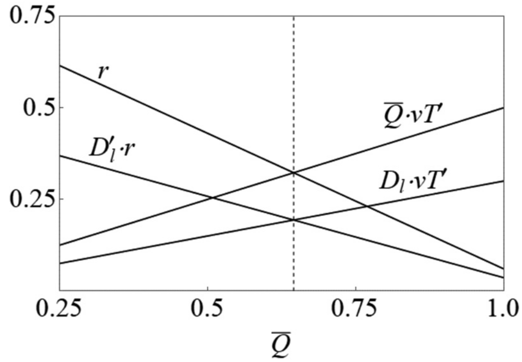
$$D_i'(\eta) \cdot \frac{\eta}{D_i(\eta)} = -\frac{\eta}{\alpha_i - \eta}. \tag{35}$$

If $\alpha_l = \alpha_{nl}$, then the first term on the right-hand side of equation (34) is zero and independent of slot quantity, implying that the share of inframarginal locals is always equal to the share of marginal locals. In this scenario, the local welfare-maximising slot policy reaches the first-best outcome. For an intuition, consider equation (35). The locals' and non-locals' demands are equally elastic in the generalised price. This means that in local welfare-maximum, a reduction in the generalised price that corresponds to an increase in slot quantity will lead to an equally proportional increase in the locals and non-locals' demands, which implies that the share of inframarginal and marginal locals are equal. These shares are not equal if the maximum reservation prices and the corresponding price elasticities of the passenger demands differ.

Figures 2 and 3 are used to illustrate why local welfare-maximising slot quantities can or cannot reach the first-best outcome depending on the difference between maximum reservation prices ($\alpha_l - \alpha_{nl}$). Parameters are given by $\alpha_l = 7/10$ (Figure 3 on the left), 4/5 (Figure 2), 9/10 (Figure 3 on the right), $\alpha_{nl} = 4/5$, $\beta_l = 2/5$, $\beta_{nl} = 3/5$, and $v = 1/2$. Slope parameters β_i are distinct in highlighting that only the differences in maximum reservation prices determine whether local welfare-maximisation can implement the first-best outcome.

In Figure 2, maximum reservation prices of locals and non-locals are equal and given by $\alpha_l = \alpha_{nl} = 4/5$. The upward sloping lines depict the marginal external congestion costs $\bar{Q} \cdot vT'$ and $D_l \cdot vT'$ from the first-best and the airport's viewpoints, respectively, depending on the slot quantity. The downward sloping lines depict the ticket price r and the weighted

Figure 2
Ticket Price r and the Weighted Ticket Price $D_l' \cdot r$, and Marginal External Congestion Costs $\bar{Q} \cdot vT'$ and $D_l \cdot vT'$, Depending on Slot Quantity when Maximum Reservation Prices are Equal; That is, $\alpha_l = \alpha_{nl}$.

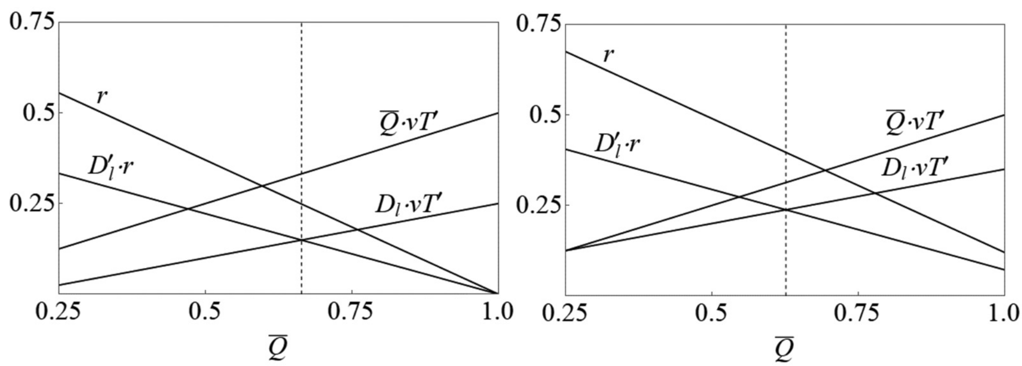


ticket price $D_l' \cdot r$, respectively, depending on the slot quantity. The minimum and maximum slot quantities are chosen at $\bar{Q} = 1/4$ and $\bar{Q} = 1$, respectively, to ensure that both locals' and non-locals' demands are non-negative across the figures.

The intersection point of the two lines on the top determines the first-best slot quantity. The intersection point of the two lines at the bottom determine the slot quantity that maximises the airport's welfare. The vertical dashed line depicts the local welfare-maximising slot quantity. The slot quantities determined by the intersection points of the two lines at the top and the two lines at the bottom are equal and given by $\bar{Q} = 20/31$. This shows that, in this scenario, local welfare-maximisation is consistent with the first-best outcome. The reason is that, evaluated at the optimal slot quantity, the cost-benefit ratios associated with an increase in the slot quantity are equal from the airport's and first-best viewpoints, as required by equation (19).

In Figure 3, maximum reservation prices of locals are given by $\alpha_l = 7/10$ ($<\alpha_{nl} = 4/5$) on the left and $\alpha_l = 9/10$ ($>\alpha_{nl} = 4/5$) on the right. The figure on the left shows that if

Figure 3
Ticket price r and the Weighted Ticket Price $D_l' \cdot r$, and Marginal External Congestion Costs $\bar{Q} \cdot vT'$ and $D_l \cdot vT'$, Depending on Slot Quantity with a Smaller ($\alpha_l < \alpha_{nl}$ on the Left) and a Bigger Locals' Maximum Reservation Price ($\alpha_l > \alpha_{nl}$ on the Right), Respectively.



$\alpha_l < \alpha_{nl}$, then the local welfare-maximising slot quantity, indicated by the dashed vertical line, implies that the marginal external congestion cost $\bar{Q} \cdot vT'$ exceeds the marginal benefit r from the first-best viewpoint. This leads to a higher cost-benefit ratio associated with an increase in the slot quantity from the first-best viewpoint relative to the airport's viewpoint. Therefore, the local welfare-maximising slot quantity exceeds the first-best slot quantity; that is, $\bar{Q}^* > \bar{Q}^{**}$. The figure on the right shows that if $\alpha_l > \alpha_{nl}$, then the local welfare-maximising slot quantity, indicated by the dashed vertical line, implies that the marginal external congestion cost $\bar{Q} \cdot vT'$ is smaller than the marginal benefit r from the first-best viewpoint. This leads to a lower cost-benefit ratio associated with an increase in the slot quantity from the first-best viewpoint relative to the airport's viewpoint. Therefore, the local welfare-maximising slot quantity is smaller than the first-best slot quantity; that is, $\bar{Q}^* < \bar{Q}^{**}$.

5.0 Conclusions

This study developed a stylised but rich enough model to analyse the role of locals and non-locals for the assessment of local welfare-maximising airport congestion policies by comparing the local welfare-maximising solutions with the first-best outcome. To identify the role of locals and non-locals for the assessment of congestion policies, the model was stripped down to the most essential parts required to make the main point, leading to the consideration of a simple airport network with one congested airport and one uncongested airport.

The congested airport could use slot or pricing policies to mitigate the congestion problem for locals by choosing the slot quantity or the airport charge, respectively. In the case of slot policy, the airport did not earn from selling slots. This captured the notion of grandfather rights established by the Worldwide Scheduling Guidelines of the IATA. In the case of pricing policy, the airport generated a positive profit from locals and non-locals. However, whereas the positive profit derived from non-locals mattered to the local welfare-maximising airport, the consumer surplus from non-locals was ignored by the local welfare-maximising airport.

The main part of the analysis was based on the consideration of general functional forms. The analysis showed that in our framework, the local welfare-maximising slot quantity could coincide with the first-best outcome, whereas, in our framework, this was impossible in the case of pricing policy. The main result was to show that whether the outcomes coincided in the case of slot policy depended on the relationship between two types of shares of locals. The first type represented the share of locals relative to the total number of passengers, which was called the share of inframarginal locals. The second type was related to the effect of a marginal increase in the slot quantity on the quantities of locals, which was called the share of marginal locals. More specifically, the second type of share was equal to the increase in locals relative to the increase in the total number of passengers or, equivalently, the increase in the slot quantity.

Using these concepts, the analysis showed that the first-best outcome coincided with the local welfare-maximising slot policy if the implied share of inframarginal locals was equal to the implied share of marginal locals. The intuition was developed with the help of cost-benefit ratios associated with a marginal increase in the slot quantity. The cost-benefit ratios were measured by the marginal external congestion cost divided by the ticket price. If the shares of inframarginal and marginal locals implied by the local welfare-maximum were equal, then the cost-benefit ratios associated with a marginal increase in the slot quantity were equal from the local and the first-best viewpoints. The application of the shares of inframarginal and marginal locals in practice was discussed to show that they could be estimated and used to assess the incentives of a local welfare-maximising airport from the first-best viewpoint. It was shown that the intuition based on cost-benefit ratios carried over to the more complicated case with multiple congested airports.

The difference between the exclusive and inclusive airline services were discussed to shed light on how the share of passengers served by the local airlines affects the assessment of local welfare-maximising policies. Linear functional forms were used to further illustrate the role of locals and non-locals for the policy comparison, and derive analytical solutions.

The model was stripped down to the most essential parts by employing a set of strong assumptions, which provides many opportunities to relax assumptions and test the results in settings that more closely resemble real-world market conditions. For instance, the current study considered the presence of atomistic airlines. Airline markets are typically classified as oligopolistic. Therefore, one avenue for future research could be looking into the role of airline market power for the welfare implications of slot and pricing policies. Another avenue for future research would be to investigate airport networks with (non-local) transfer passengers. Capturing transfer passengers requires a different set of assumptions because they use the congested airport twice as often as origin-destination passengers, and airports typically discriminate between origin-destination and transfer passengers in terms of airport charges (for example, Lin and Zhang, 2016). Finally, it would be useful to estimate empirically the shares of inframarginal and marginal locals of congested airports to derive a better understanding of their incentives to implement slot policies.

References

Airport Council International (2017): *Airport Ownership, Economic Regulation and Financial Performance*, Policy Brief 2017/01.

Arcelus, A., R. Booth, A. M. Fix, J. Y. K. Lehmann, F. G. Mantovanelli, and R. S. Pindyck (2019): 'Scarcity, market power, and prices at slot-constrained airports: evidence from Mexico City', *Journal of Transport Economics and Policy*, 53, 119–34.

Arykan, M., V. Deshpande, and M. Sohoni (2013): 'Building reliable air-travel infrastructure using empirical data and stochastic models of airline networks', *Operations Research*, 61, 45–64.

Basso, L. J. (2008): 'Airport deregulation: effects on pricing and capacity', *International Journal of Industrial Organization*, 26, 1015–31.

Basso, L. J. and A. Zhang (2010): 'Pricing vs. slot policies when airport profits matter', *Transportation Research Part B: Methodological*, 44, 381–91.

Basso, L. J., N. Figueroa, and J. Vásquez (2017): 'Monopoly regulation under asymmetric information: prices vs. quantities', *RAND Journal of Economics*, 48, 557–78.

Benoot, W., J. K. Brueckner, and S. Proost (2012): *Intercontinental Airport Competition*, Katholieke Universiteit Discussion Paper Series 12.03.

Berardino, F. (2009): 'New US airport slot policy in flux', *Journal of Transport Economics and Policy*, 43, 279–90.

Brenck, A. and A. I. Czerny (2002): *Allokation von Slots bei unvollständiger Information*, Wirtschaftswissenschaftliche Dokumentation Technische Universität Berlin.

Brueckner, J. K. (2002): 'Airport congestion when carriers have market power', *American Economic Review*, 92, 1357–75.
Brueckner, J. K. (2009): 'Price vs. quantity-based approaches to airport congestion management', *Journal of Public Economics*, 93, 681–90.
Button, K. (2008): 'Issues in airport runway capacity charging and allocation', *Journal of Transport Economics and Policy*, 42, 563–85.
Cirium (2020): *Cirium On-time Performance Review 2019*, available at https://cdn.pathfactory.com/assets/154/contents/108628/c3da9b59-4459-4b99-b47d-65bca23ce126.pdf?cmpid = EMC%7C%7CCIR%7CFGWWA-2020-0103-Monthly-Airport-OTP&sfid = 7014G0000019KUCQA2.
Czerny, A. I. (2008): 'Congestion management at airports under uncertainty', in A. I. Czerny, P. Forsyth, D. Gillen, and H.-M. Niemeier (ed.), *Airport Slots: International Experiences and Options for Reform*, Ashgate.
Czerny, A. I. (2010): 'Airport congestion management under uncertainty', *Transportation Research Part B: Methodological*, 44, 371–80.
Czerny, A. I. and A. Zhang (2011): 'Airport congestion pricing and passenger types', *Transportation Research Part B: Methodological*, 45, 595–604.
Czerny, A. I. and A. Zhang (2014a): 'Airport congestion pricing when airlines price discriminate', *Transportation Research Part B: Methodological*, 65, 77–89.
Czerny, A. I. and A. Zhang (2014b): 'Airport peak-load pricing revisited: the case of peak and uniform tolls', *Economics of Transportation*, 3, 90–101.
Czerny, A. I. and H. Lang (2019): 'A pricing versus slots game in airport networks', *Transportation Research Part B: Methodological*, 125, 151–74.
Czerny, A. I., F. Höffler, and S. Mun (2014): 'Hub port competition and welfare effects of strategic privatization', *Economics of Transportation*, 3, 211–20.
Czerny, A. I., V. A. van den Berg, and E. T. Verhoef (2016): 'Carrier collaboration with endogenous fleets and load factors when networks are complementary', *Transportation Research Part B: Methodological*, 94, 285–97.
Czerny, A. I., X. Fu, Z. Lei, and T. H. Oum (2020): 'Post pandemic aviation market recovery: experience and lessons from China', *Journal of Air Transport Management*, 90, 101971.
Daniel, J. I. (1995): 'Congestion pricing and capacity of large hub airports: a bottleneck model with stochastic queues', *Econometrica*, 63, 327–70.
Daniel, J. I. (2014): 'The untolled problems with airport slot restraints', *Economics of Transportation*, 3, 16–28.
De Borger, B. and K. Van Dender (2006): 'Prices, capacities and service levels in a congestible Bertrand duopoly', *Journal of Urban Economics*, 60, 264–83.
De Borger, B., S. Proost, and K. Van Dender (2005): 'Congestion and tax competition in a parallel network', *European Economic Review*, 49, 2013–40.
De Borger, B., F. Dunkerley, and S. Proost (2007): 'Strategic investment and pricing decisions in a congested transport corridor', *Journal of Urban Economics*, 62, 294–316.
De Palma, A. and R. Lindsey (2020): 'Tradable permit schemes for congestible facilities with uncertain supply and demand', *Economics of Transportation*, 21, 100149.
Federal Aviation Administration (2010): *New York Flight Delays Have Three Main Causes, but More Work Is Needed to Understand Their Nationwide Effect*, available at www.oig.dot.gov/sites/default/files/NY%20Delays%20Final.pdf.
Federal Aviation Administration (2020a): *Fact Sheet – Facts about the FAA and Air Traffic Control*, available at www.faa.gov/news/fact_sheets/news_story.cfm?newsId = 23315.
Federal Aviation Administration (2020b): *Slot Administration*, available at www.faa.gov/about/office_org/headquarters_offices/ato/service_units/systemops/perf_analysis/slot_administration.
Fung, C. M. and S. Proost (2017): 'Can we decentralize transport taxes and infrastructure supply?', *Economics of Transportation*, 9, 1–19.
Gillen, D., A. Jacquillat, and A. R. Odoni (2016): 'Airport demand management: the operations research and economics perspective', *Transportation Research Part A: Policy and Practice*, 94, 495–513.
Gillen, D. and D. Starkie (2016): 'EU slot policy at congested hubs, and incentives to add capacity', *Journal of Transport Economics and Policy*, 50, 151–63.
International Air Transport Association (2019a): *Progress Report on Airport Slot Allocation*, available at www.iata.org/contentassets/e45e5219cc8c4277a0e80562590793da/airport-slot-allocation.pdf.

International Air Transport Association (2019b): *Worldwide Slot Guidelines*, available at www.iata.org/contentassets/4ede2aabfcc14a55919e468054d714fe/wsg-edition-10-english-version.pdf.
Lang, H. and A. I. Czerny (2022): 'Airport pricing versus (grandfathered) slots: A generalization', *Economics of Transportation*, 29, 100249.
Lin, M. H. (2020): 'Congestion pricing and capacity for internationally interlinked airports', *Transportation Research Part B: Methodological*, 142, 126–42.
Lin, M. H. and A. Zhang (2016): 'Hub congestion pricing: discriminatory passenger charges', *Economics of Transportation*, 5, 37–48.
Lin, M. H. and Y. Zhang (2017): 'Hub-airport congestion pricing and capacity investment', *Transportation Research Part B: Methodological*, 101, 89–106.
Mantin, B. (2012): 'Airport complementarity: private vs. government ownership and welfare gravitation', *Transportation Research Part B: Methodological*, 46, 381–8.
Mun, S. I. (2019): 'Joint provision of transportation infrastructure', *Economics of Transportation*, 19, 100118.
Pearce, B. (2020): *COVID-19 Updated Assessment of Air Transport Markets and Airlines*, Presentation by Brian Pearce, Chief Economist of IATA, 8th GARS/EAC Online Panel Discussion 'Covid-19 — where is aviation now?', 15 September 2020.
Pels, E. and E. T. Verhoef (2004): 'The economics of airport congestion pricing', *Journal of Urban Economics*, 55, 257–77.
Rassenti, S. J., V. L. Smith, and R. L. Bulfin (1982): 'A combinatorial auction mechanism for airport time slot allocation', *Bell Journal of Economics*, 13, 402–17.
Silva, H. E., E. T. Verhoef, and V. A. C. van den Berg (2014): 'Airline route structure competition and network policy', *Transportation Research Part B: Methodological*, 67, 320–43.
Wan, Y. and A. Zhang (2013): 'Urban road congestion and seaport competition', *Journal of Transport Economics and Policy*, 47, 55–70.
Wan, Y., L. J. Basso, and A. Zhang (2016): 'Strategic investments in accessibility under port competition and inter-regional coordination', *Transportation Research Part B: Methodological*, 93, 102–25.
Wan, Y., A. Zhang, and K. X. Li (2018): 'Port competition with accessibility and congestion: a theoretical framework and literature review on empirical studies', *Maritime Policy & Management*, 45, 239–59.
Zhang, A. and Y. Zhang (2006): 'Airport capacity and congestion when carriers have market power', *Journal of Urban Economics*, 60, 229–47.
Zhang, A. and A. I. Czerny (2012): 'Airports and airlines economics and policy: an interpretive review of recent research', *Economics of Transportation*, 1, 15–34.

Appendix

A.1 Proof of Lemma 1

Let x_l and x_{nl} represent the conditions:

$$x_l = B_l'(q_l) - \eta = 0, \tag{A.1}$$

$$x_{nl} = B_{nl}'(q_{nl}) - \eta = 0. \tag{A.2}$$

Totally differentiating equations (a.1) and (a.2) leads to:

$$dx_l = \frac{\partial B_l'(q_l)}{\partial q_l} dq_l + \frac{\partial B_l'(q_l)}{\partial q_{nl}} dq_{nl} + \frac{\partial B_l'(q_l)}{\partial r} dr = 0, \tag{A.3}$$

$$dx_{nl} = \frac{\partial B_{nl}'(q_{nl})}{\partial q_l} dq_l + \frac{\partial B_{nl}'(q_{nl})}{\partial q_{nl}} dq_{nl} + \frac{\partial B_{nl}'(q_{nl})}{\partial r} dr = 0. \tag{A.4}$$

Using $\eta = r + vT(Q)$ and after rearranging, this system of equations can be written in matrix form as:

$$\begin{pmatrix} \frac{\partial x_l}{\partial q_l} & \frac{\partial x_l}{\partial q_{nl}} \\ \frac{\partial x_{nl}}{\partial q_l} & \frac{\partial x_{nl}}{\partial q_{nl}} \end{pmatrix} \begin{pmatrix} dq_l \\ dq_{nl} \end{pmatrix} = dr \begin{pmatrix} -\frac{\partial x_l}{\partial r} \\ -\frac{\partial x_{nl}}{\partial r} \end{pmatrix} = dr \begin{pmatrix} -\frac{\partial \eta}{\partial r} \\ -\frac{\partial \eta}{\partial r} \end{pmatrix} = dr \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{A.5}$$

Using symmetry and $\eta = r + vT(Q)$, the Jacobian on the left-hand side can be rewritten as:

$$\begin{pmatrix} \frac{\partial x_l}{\partial q_l} & \frac{\partial x_l}{\partial q_{nl}} \\ \frac{\partial x_{nl}}{\partial q_l} & \frac{\partial x_{nl}}{\partial q_{nl}} \end{pmatrix} = \begin{pmatrix} B_l''(q_l) - vT' & -vT' \\ -vT' & B_{nl}''(q_{nl}) - vT' \end{pmatrix}. \tag{A.6}$$

The determinant of the Jacobian, denoted as Ψ, can be written as:

$$\Psi = B_l''(q_l) \cdot B_{nl}''(q_{nl}) - vT'(B_l''(q_l) + B_{nl}''(q_{nl})), \tag{A.7}$$

in which the right-hand side is positive because $B_l''(q_l)$, $B_{nl}''(q_{nl})$ are negative, v is positive, and T' is positive by assumption. Applying Cramer's rule yields:

$$q_l'(r) = \frac{B_{nl}''(q_{nl})}{\Psi}, \tag{A.8}$$

$$q_{nl}'(r) = \frac{B_l''(q_l)}{\Psi}. \tag{A.9}$$

Both right-hand sides are negative. Substituting the number of passengers with demands completes the proof.

A.2 Proof of Lemma 2

Totally differentiating equation (4) yields:

$$d(\bar{Q} - D(r)) = \frac{\partial(\bar{Q} - D(r))}{\partial r} dr + \frac{\partial(\bar{Q} - D(r))}{\partial \bar{Q}} d\bar{Q} = -D'(r) \cdot dr + d\bar{Q} = 0. \tag{A.10}$$

An increase in slot quantity changes ticket price in the following way:

$$r'(\bar{Q}) = \frac{1}{D'(r)}. \tag{A.11}$$

Lemma 1 mentions that $D_l'(r)$, $D_{nl}'(r) < 0$, which implies that $D'(r) = D_l'(r) + D_{nl}'(r) < 0$. Therefore, the right-hand side of equation (a.11) is negative, which completes the proof.

A.3 Proof of Lemma 3

To show that the demands of both locals and non-locals are increasing in the slot quantity by less than 1, use Lemmas 1 and 2, which yields:

$$D_l'(\bar{Q}) = D_l'(r) \cdot r'(\bar{Q}) = \frac{B_{nl}''(q_{nl})}{B_l''(q_l) + B_{nl}''(q_{nl})}, \tag{A.12}$$

$$D_{nl}'(\bar{Q}) = D_{nl}'(r) \cdot r'(\bar{Q}) = \frac{B_l''(q_l)}{B_l''(q_l) + B_{nl}''(q_{nl})}. \tag{A.13}$$

Both right-hand sides are positive and less than 1.

To show that the total traffic is increasing in the slot quantity by 1, use equations (a.12) and (a.13), which yield:

$$D'(\bar{Q}) = D_l'(\bar{Q}) + D_{nl}'(\bar{Q}) = 1. \tag{A.14}$$

A.4 Proof of Lemma 4

To show that the generalised price is increasing in price r, substitute quantities with demands and use equations (a.8) and (a.9), which yield:

$$\eta'(r) = 1 + vT'(D(r)) \cdot D'(r) = \frac{B_l''(D_l(r)) \cdot B_{nl}''(D_{nl}(r))}{\Psi}, \tag{A.15}$$

where the right-hand side is positive.

To show that the generalised price is increasing in the price r by less than 1, consider $\eta'(r) = 1 + vT'(D(r)) \cdot D'(r)$. Lemma 1 mentions that $D'(r) = D_l'(r) + D_{nl}'(r) < 0$. Therefore, $vT'(D(r)) \cdot D'(r) < 0$ and $1 + vT'(D(r)) \cdot D'(r) < 1$ because $vT'(D(r)) > 0$ by assumption.

A.5 Proof of Proposition 1

Consider equation (17). Replace the local welfare-maximising ticket price $r^*(S)$ on the left-hand side by the marginal external congestion cost evaluated at the local welfare-maximising slot quantity, which yields:

$$\bar{Q}^* \cdot vT'(\bar{Q}^*) = D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*) + \frac{D_{nl}'(\bar{Q}^*)}{D_l'(\bar{Q}^*)} \cdot D_l(\bar{Q}^*) \cdot vT'(\bar{Q}^*). \tag{A.16}$$

Dividing $vT'(\bar{Q}^*)$ at both sides, replacing $\bar{Q}^*$ by $D_l(\bar{Q}^*) + D_{nl}(\bar{Q}^*)$, simplifying and rearranging yields:

$$\frac{D_l(\bar{Q}^*)}{D_{nl}(\bar{Q}^*)} = \frac{D_l'(\bar{Q}^*)}{D_{nl}'(\bar{Q}^*)}. \tag{A.17}$$

Adding one to both sides, substituting $D_l(\bar{Q}^*) + D_{nl}(\bar{Q}^*)$ with $\bar{Q}^*$ and rearranging yields equation (18). Too-high and too-low welfare-maximising slot quantity relative to the first-best slot quantity can be proved similarly by using greater-than and less-than signs in equation (a.16) respectively. This completes the proof.

A.6 Proof of Lemma 5

Totally differentiating the first-order condition in equation (24) yields:

$$d\frac{\partial W_l(\bar{Q}^*)}{\partial \bar{Q}} = \frac{\partial^2 W_l(\bar{Q}^*)}{\partial \bar{Q} \partial \theta} d\bar{Q}^* + \frac{\partial^2 W_l(\bar{Q}^*)}{\partial \bar{Q}^2} d\theta = 0. \tag{A.18}$$

Rearranging equation (a.18) and substituting $d\bar{Q}^*/d\theta$ by $\partial \bar{Q}^*/\partial \theta$ yields:

$$\frac{\partial \bar{Q}^*}{\partial \theta} = -\frac{\partial^2 W_l(\bar{Q}^*)/\partial \bar{Q} \partial \theta}{\partial^2 W_l(\bar{Q}^*)/\partial \bar{Q}^2}. \tag{A.19}$$

By assumption, the airport's welfare is concave in slot quantity, implying that $\partial^2 W_l(\bar{Q}^*)/\partial\bar{Q}^2 < 0$. To prove that the local welfare-maximising slot quantity is decreasing in θ, it is equivalent to proving that $\partial^2 W_l(\bar{Q}^*)/\partial\bar{Q}\partial\theta < 0$. Consider $\partial^2 W_l(\bar{Q}^*)/\partial\bar{Q}\partial\theta$, which can be written as:

$$\frac{\partial^2 W_l(\bar{Q}^*)}{\partial\bar{Q}\partial\theta} = r(\bar{Q}^*) + r'(\bar{Q}^*) \cdot \bar{Q}^*. \tag{A.20}$$

The right-hand side is negative. To see this, consider Lemma 2, which shows that $r'(\bar{Q}) < 0$, and Lemma 4, which shows that $0 < \eta'(r) < 1$. Together this implies that $\eta'(\bar{Q}) < 0$ because $\eta'(\bar{Q}^*) = \eta'(r(\bar{Q}^*)) \cdot r'(\bar{Q}^*)$. The first term on the left-hand side of equation (24) is positive, implying that the sum of the second and third terms, $\theta \cdot (r(\bar{Q}^*) + r'(\bar{Q}^*) \cdot \bar{Q}^*)$, is negative. This implies that $r(\bar{Q}^*) + r'(\bar{Q}^*) \cdot \bar{Q}^* < 0$ because $\theta > 0$. This completes the proof.