

Phase-Field Modeling of Mechano–chemical-coupled Stress-Corrosion Cracking

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Abstract

A mechano–chemical coupling phase-field model is proposed to investigate stress-corrosion cracking (SCC). It is demonstrated that pit-to-crack transition occurs when the relative-rate parameter, $\kappa_v > 1$, which characterizes the critical scenario where stress-induced degradation occurs faster than electrochemical dissolution. Moreover, an exponential relationship between the stress intensity factor and cracking velocity is revealed, and it indicates an autocatalytic process resulting from the accelerations of stress and corrosion. We provide further details regarding the variation in the electrochemical environment, effect of mechanical loading, and significant role of the initial geometry in promoting SCC. The results obtained are useful for assessing critical structures in corrosive environments.

Keyword: Phase-Field Model; Stress-Corrosion Cracking; Mechano–chemical Coupling.

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1 Introduction

2 Repairs and replacements caused by corrosion have resulted in significant economic
3 burden in many countries. For example, in China, the cost incurred by corrosion amounted to
4 approximately 3.34% of its gross domestic product (GDP in 2018 [1], which is significantly
5 higher than the cost of all-natural disasters combined. Among various corrosion-induced
6 failures, stress-corrosion cracking (SCC), a progressive fracture caused by stress and the
7 electrochemical environment, has long been recognized as one of the most typical and
8 dangerous types of localized corrosion [2]. It significantly degrades the integrity and durability
9 of materials, thereby limiting the lifetime and reliability of key industrial equipment.

10 However, the mechanism of SCC, which results from the complicated conjoint actions of
11 stress and electrochemical fields, are difficult to understand. Among the various SCC
12 mechanisms proposed, three are referenced the most, *i.e.*, stress-sorption [3–4], film-rupture–
13 metal-dissolution [5–7], and hydrogen embrittlement [8–10]. The stress-sorption mechanism
14 ascribes SCC to adsorbed specific species that interact with strained chemical bonds at crack
15 tips, resulting in a decrease in the bond strength and surface energy, and consequently, a
16 reduction in the stress threshold for a brittle fracture. In the film-rupture–metal-dissolution
17 mechanism, as stress increases, a crack begins to develop at the tip of a corrosion pit, thereby
18 destroying the passive film and exposing the fresh material to the corrosive environment.
19 Consequently, the concentrated stresses promote the corrosion rate, and the metal dissolution
20 results in a further increase in the stresses at the crack tip. These factors result in an

1 autocatalytic process, which induces material failure. In the hydrogen embrittlement
2 mechanism, fracture occurs because of a brittle region at the crack tip caused by the entry of
3 hydrogen. Fontana and Greene [11], however, argued that because hydrogen embrittlement is
4 not a corrosion process, cracking occurring by this mechanism should not be considered as
5 SCC.

6 In-situ experiments using scanning electron microscopy (SEM) [12, 13], synchrotron X-
7 ray tomography [14], and the direct current potential drop (DCPD) method [15] have been
8 conducted to observe SCC. However, these methods have limitations. For example, the DCPD
9 method cannot directly characterize the morphology evolution of cracks, whereas SEM can
10 only offer a limited resolution; none of these methods can quantify the stress, electric, and
11 chemical fields inside a crack. SCC involves complex interactions among the stress state,
12 material microstructure, and electrochemical kinetics. The effect of each individual factor is
13 extremely difficult to experimentally differentiate and determine. Therefore, numerical
14 approaches have become essential for reliability analysis and anticorrosion design involving
15 SCC.

16 Theoretically, based on a predefined crack geometry, several analytical and numerical
17 models [16–19] have been proposed to predict the electrochemical evolution within a crack.
18 In these analyses, Nernst–Planck equations are employed to describe the diffusion and reaction
19 in an electrolyte, the Laplace or Poisson equation to describe the electric field, and the flux
20 boundary conditions imposed on the crack surfaces to account for metal dissolution. However,

1 when the corrosion rate is high, this type of model results in significant errors because
2 morphology evolution is not involved.

3 The migration of the metal–electrolyte interface (i.e., pitting and cracking) can be resolved
4 using the finite element method (FEM) [20–22], in which the position of the interface can be
5 determined based on the resolved cracking velocity supplemented with a remeshing or
6 moving-mesh technique (e.g., the arbitrary Lagrangian–Eulerian technique [23]) to
7 accommodate sharp interfaces. However, this method often results in many numerical errors,
8 a high computational cost, and mesh-dependent results; therefore, it is extremely difficult to
9 apply this method to complex problems. Additionally, the cracking problem can be managed
10 using the extended finite element method (XFEM) [24, 25]. However, the use of the XFEM
11 for SCC renders it difficult to solve the electrochemical governing equations on new surfaces
12 because the latter are still within the elements. Other solution approaches, such as the finite
13 volume method [26], cellular automata [27–29], and peridynamics [30–32], are also employed
14 in modeling corrosion; however, they are more complicated than the FEM and generally
15 require more computational resources.

16 Assuming diffusive interfaces, the phase field (PF) method avoids the difficulty in
17 accommodating moving interfaces; all governing equations become solvable when the FEM
18 is used (e.g., a commercial FEM package). Because of the convenience of incorporating the
19 effects of various physical, chemical, and mechanical fields, the PF method has been employed
20 to investigate various material processes. A few PF models have been proposed to model SCC.

1 In an early study involving a PF model, Stähle *et al.* [33] investigated the formation of
2 corrosion pits, the initiation and growth of cracks, and crack branching. However,
3 electrochemical processes, such as diffusion and reaction, were not incorporated into the
4 model. Based on the film-rupture–metal-dissolution mechanism, Mai *et al.* [34] proposed a PF
5 model to describe the SCC process, wherein a relationship between the stress field and
6 interface kinetics was assumed. Considering the contributions of chemical potential and elastic
7 energy to the free energy of an SCC system, Nguyen *et al.* [35–37] proposed another PF model,
8 in which the material dissolution rate was correlated with the fluxes of reactants and the release
9 of elastic energy after dissolution. In the models of Mai *et al.* and Nguyen *et al.*, corrosion was
10 regarded as a diffusion process; hence, the reaction kinetics were not considered to be rate-
11 limiting factors. Lin *et al.* [38, 39] formulated the reaction kinetics in the form of generalized
12 Butler–Volmer (BV) equations in their PF models. Hence, the overpotential can be correlated
13 with diffusivity, electric field, interfacial energy, and mechanical deformation, thereby
14 enabling stress-assisted local corrosion and the corresponding change in an aqueous
15 environment to be investigated.

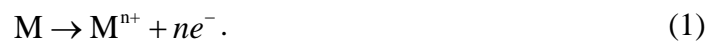
16 However, to the best of our knowledge, a numerical model describing all the complexities
17 of electrochemical and mechanical processes and their intrinsic interactions in SCC does not
18 exist. Therefore, we herein propose a new PF model that involve the formulations of chemical
19 potential, electrostatic potential, and mechanical and interfacial energies, whose summation
20 represents the Helmholtz free energy of the system; furthermore, the model can accommodate

1 the kinetics of diffusion, metallic dissolution, and cracking. In the PF framework, we describe
2 the cracking process using an Allen–Cahn-type equation to minimize the free energy of the
3 system, which is consistent with the theory of strain energy release rate in fracture mechanics.
4 The electrochemical kinetics of corrosion are described as a function of the electrochemical
5 potentials of the reactants and products, which involves the effects of stresses, electric fields,
6 and corrosive environments. Considering mass conservation and electroneutrality, a set of
7 Nernst–Planck–Poisson equations with reaction kinetics as the sink/source terms are
8 established to accommodate variations in the field variables. The proposed PF model is
9 solvable using a commercial FEM package, which is effective for investigating SCC and the
10 relationship between stress and corrosive species.

12 2. Methodology

13 2.1 SCC mechanism

14 The SCC process, as illustrated in Fig 1, begins with a local breakdown of the passive
15 film, which exposes a fresh material into a corrosive environment, e.g., saltwater. With the
16 applied anodic potential, denoted by φ , the metal (M) corrodes and releases metal cations (M^{n+})
17 into the electrolyte, whereas it releases electrons (e^-) into the metallic electrode, as follows:



19 When the metallic component is under mechanical loading, the stress concentration at the tip
20 of a corrosion pit accelerates local corrosion [38–42], resulting in the sharpening of the

1 corrosion pit [38, 39, 43, 44]. This yields a higher stress concentration, a higher anodic
 2 dissolution rate, and an autocatalytic process of crack propagation until catastrophic failure
 3 occurs.

4 Fig. 1

5 2.2. Thermodynamics

6 We begin by introducing a general expression of the Helmholtz free energy, denoted by
 7 Ψ , which is an integral of the density functional, ψ , over the domain, Ω , for a dissipative system,
 8 as follows:

$$9 \quad \Psi = \int_{\Omega} \psi dv = \int_{\Omega} (\psi^{\text{chem}} + \psi^{\text{elec}} + \psi^{\text{mech}} + \psi^{\text{int}}) dv. \quad (2)$$

10 As expressed in Eq. (2), the density functional ψ is expressed as four terms describing the
 11 chemical, electric, mechanical, and interfacial potentials (ψ^{chem} , ψ^{elec} , ψ^{mech} , and ψ^{int} ,
 12 respectively).

13 Using saltwater as an example electrolyte, the diffusible species are M^{n+} , Cl^- , and Na^+ ,
 14 whose concentrations are denoted by $c_{M^{n+}}$, c_{Cl^-} and c_{Na^+} , respectively. The chemical energy
 15 density ψ^{chem} is expressed as follows:

$$16 \quad \psi^{\text{chem}} = \sum_* \psi_*^{\text{chem}}(c_*) \quad (* = M^{n+}, Na^+, Cl^-), \quad (3)$$

17 where ψ_*^{chem} is the chemical energy density, where the subscript denotes the type of diffusible
 18 ion *. Because M^{n+} , Na^+ , and Cl^- only exist in the electrolyte, ψ_*^{chem} is defined based on a
 19 dilute solution, as follows:

$$20 \quad f_*^{\text{chem}} = c_* RT (\ln \bar{c}_* - 1) + c_* \mu_*^0 \quad (* = M^{n+}, Na^+, Cl^-), \quad (4)$$

1 where μ_*^0 , R , and T are the standard chemical potential, ideal gas constant, and
 2 thermodynamic temperature, respectively; $\bar{c}_* = c_*/c_*^{\text{ref}}$ is the dimensionless concentration
 3 with c_*^{ref} being the saturated concentration of ion *.

4 The electric potential energy density, ψ^{elec} , resulting from the charge density, is expressed
 5 as

$$\psi^{\text{elec}} = F \left(n\varphi_L c_{M^{n+}} + \varphi_L c_{Na^+} - \varphi_L c_{Cl^-} - \varphi_S c_{e^-} \right), \quad (5)$$

7 where F , φ_S , and φ_L are the Faraday constant and electrostatic potential in the metallic electrode
 8 and electrolyte, respectively. Because metal is a conductor, the distribution of φ_S is assumed
 9 to be uniform, with the magnitude being either zero (grounded) or an applied potential
 10 difference.

11 Considering elastoplastic deformation, the mechanical energy density ψ^{mech} can be written
 12 as the sum of the elastic and plastic contributions, as follows:

$$\psi^{\text{mech}} = \psi^{\text{elas}} + \psi^{\text{plas}}, \quad (6)$$

14 where the elastic term, employing the assumption of linear elasticity, is expressed as

$$\psi^{\text{elas}} = p(\phi) \frac{1}{2} \left((\boldsymbol{\varepsilon}^e)^T \cdot (\mathbf{D}^e \boldsymbol{\varepsilon}^e) \right). \quad (7)$$

16 Here, \mathbf{D}^e and $\boldsymbol{\varepsilon}^e$ are the stiffness matrix and elastic strain tensor of the metallic electrode,
 17 respectively. In Eq. (7), a function $p(\phi)$ is used to mollify the discontinuity from the solid to
 18 the liquid phase, as mechanical deformation only occurs in the solid. Using $p(\phi)$ is a
 19 fundamental technique in the PF model. In this study, $p(\phi)$ is a Hermitian interpolation function,
 20 i.e., $p(\phi) = \phi^3 (10 - 15\phi + 6\phi^2)$ (on the other forms of $p(\phi)$, cf. the discussion in [45]) with ϕ

$\phi \in [0, 1]$ being the order parameter of phase identification. In the present model, $\phi = 1$ indicates the M phase, which can signify $\phi = c^M / c_{\text{ref}}^M$, i.e., the normalized concentration of M in the metal anode is unity. Accordingly, $\phi = 0$ indicates the liquid electrolyte (no metallic atoms, only ions), and $0 < \phi < 1$ is in the interface, in which the reaction shown by Eq. (1) occurs.

The elastic strain is expressed as

$$\boldsymbol{\varepsilon}^e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p, \quad (8)$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}^p$ are the total and plastic strain tensors, respectively. We apply a linear isotropic hardening law with the von Mises yield criterion to describe the plastic deformation, which results in the following incremental expression of the plastic strain:

$$d\boldsymbol{\varepsilon}^p = d\varepsilon_{\text{eq}}^p \frac{\partial \sigma_{\text{eq}}}{\partial \boldsymbol{\sigma}}, \quad (9)$$

where $d\varepsilon_{\text{eq}}^p$ is the incremental equivalent plastic strain, $\boldsymbol{\sigma}$ is the stress tensor, and σ_{eq} is the equivalent (von Mises) stress, i.e., $\sigma_{\text{eq}} = \sqrt{3/2 \mathbf{s} : \mathbf{s}}$, with $\mathbf{s} = \boldsymbol{\sigma} - \text{tr}(\boldsymbol{\sigma})/3 \mathbf{I}$. Here, \mathbf{s} is the deviatoric stress tensor, $\text{tr}(\cdot)$ is used to obtain the trace of a tensor, and \mathbf{I} is the identity tensor.

The corresponding plastic energy density ψ^{plas} can be expressed as

$$\psi^{\text{plas}} = p(\phi) \left(\sigma_y^0 + \frac{1}{2} H \varepsilon_{\text{eq}}^p \right) \varepsilon_{\text{eq}}^p, \quad (10)$$

where σ_y^0 and H are the initial yield strength and hardening rate, respectively.

The last energetic term in Eq. (2) is the interface energy density ψ^{int} , which represents the additional energy due to the creation of new surfaces (e.g., cracking). It is expressed as

$$\psi^{\text{int}} = Wg(\phi) + \frac{\lambda}{2}|\nabla\phi|^2. \quad (11)$$

In Eq. (11), the first term on the right-hand side is a double-well function comprising $g(\phi) = (\phi)^2(1-\phi)^2$ and the energy barrier, W , to ensure that both the electrode ($\phi = 1$) and electrolyte phases ($\phi = 0$) are stable; meanwhile, the second term is the gradient energy density owing to the requirement of diffusive interfaces in a PF model [45], where λ is a constant to scale the magnitude of the gradient energy density. The two scaling factors W and λ can be correlated based on the variational principle proposed in classical fracture mechanics (i.e., the Griffith criterion is fulfilled) [46], as follows:

$$W = \frac{g_c}{2l}, \quad (12a)$$

$$\lambda = g_c l. \quad (12b)$$

In Eq. (12), g_c is the Griffith energy density, i.e., the energy dissipated upon the creation of a unit on the fracture surface; l denotes the thickness of the interface between the electrode and electrolyte, and it can be regarded as a pure numerical parameter or an actual material parameter, i.e., l can be determined from experiments (cf. [47]). However, although l should be minimal such that a fracture process can be approximated well, in a numerical implementation, l determines the mesh size, where a smaller l results in a higher computational cost.

In classical PF models [38, 39], W is correlated with λ [38], [39]. By introducing the interface energy density, s , and interface thickness, δ , W can be expressed as

$$W = 18\frac{s}{\delta}, \quad (13a)$$

$$\lambda = s\delta. \quad (13b)$$

In this case, the interface energy density, s , is the amount of energy released upon the creation of a new surface owing to an electrochemical process or phase transformation. Eqs. (12) and (13) are similar, although they imply different physical processes. Considering that the surface energy due to fracture is much larger than that due to corrosion, because the discontinuity caused by fracture is more abrupt, we used Eq. (12), which contains g_c and l , in the remainder of our study.

Mechano–electrochemical corrosion kinetics

For a generalized reaction, $\sum n_i R_i^{X_i} \rightarrow \sum m_j P_j^{Z_j}$, the reaction rate, denoted by r , can be defined as

$$r = -\frac{1}{n_i} \frac{\partial c_{R_i^{X_i}}}{\partial t} = \frac{1}{m_j} \frac{\partial c_{P_j^{Z_j}}}{\partial t}, \quad (14)$$

where R_i and P_j denote the reactants and products, respectively; n_i (or m_j) and X_i (or Z_j) are the stoichiometric number and charge number, respectively. We express r as follows (see our previous papers [38, 39] or Appendix A for derivation):

$$r = \frac{k^0}{a^{\text{TS}}} \left(a_{\text{R}} \exp\left(\frac{(1-\rho)(\mu_{\text{R}}^{\text{ex}} - \mu_{\text{P}}^{\text{ex}})}{RT}\right) - a_{\text{P}} \exp\left(-\frac{\rho(\mu_{\text{R}}^{\text{ex}} - \mu_{\text{P}}^{\text{ex}})}{RT}\right) \right). \quad (15)$$

In Eq. (15), k^0 is the rate coefficient; a^{TS} is the activity of the reaction in the transition state (a constant in the present study); $\rho \in (0, 1)$ is the asymmetry parameter [48]; a_{R} and a_{P} are the activities of the reactants and products, respectively; $\mu_{\text{R}}^{\text{ex}}$ and $\mu_{\text{P}}^{\text{ex}}$ are the excess chemical

1 potentials of the reactants and products, respectively.

2 For oxidation, $M \rightarrow M^{n+} + ne^-$ (Eq. (1)), the activities a_R and a_P , and the excess chemical
3 potential difference, $\mu_R^{\text{ex}} - \mu_P^{\text{ex}}$, can be expressed as

$$4 \quad a_R = \exp\left(\frac{1}{RTc_M^{\text{ref}}}\left(W\frac{\partial g}{\partial \phi} - \lambda\nabla\phi^2\right)\right), \quad (16a)$$

$$5 \quad a_P = \bar{c}_{M^{n+}}, \quad (16b)$$

$$6 \quad \mu_R^{\text{ex}} - \mu_P^{\text{ex}} = -\mu_{M^{n+}}^0 + nF(\varphi_S - \varphi_L) + \frac{\partial p(\phi)}{c_M^{\text{ref}}\partial\phi}\left(\frac{1}{2}(\boldsymbol{\varepsilon}^e)^T \cdot (\mathbf{D}^e \boldsymbol{\varepsilon}^e) + \left(\sigma_y + \frac{1}{2}H\varepsilon_{\text{eq}}^p\right)\varepsilon_{\text{eq}}^p\right). \quad (16c)$$

7 Considering an enhanced mechano-chemical coupling (MCC) in solid, Gutman [49]
8 suggested an additional term for the chemical potential, i.e., $\Delta\mu \approx V_M P$, with V_M and P the
9 molar volume and pressure, respectively. Because experiments show that tension and
10 compression both increase the chemical potential of solid materials [40, 42], we define

11 $P = |tr(\boldsymbol{\sigma})|/3$. Following Gutman [49], the effect of plastic deformation on corrosion due to
12 the multiplication of dislocation can be incorporated by adding another term in the form of
13 $RT \ln(\alpha v \varepsilon_{\text{eq}}^p / N_0 + 1)$, where α is a coefficient, v an orientation-dependent factor in the range
14 of 0.4–0.5 [50], and N_0 the initial dislocation density prior to plastic deformation. Therefore,

15 Eq. (16c) can be rewritten as

$$16 \quad \mu_1^{\text{ex}} - \mu_2^{\text{ex}} = -\mu_{M^{n+}}^0 + nF(\varphi_S - \varphi_L) + \frac{\partial p(\phi)}{c_M^{\text{ref}}\partial\phi}\left(\frac{1}{2}(\boldsymbol{\varepsilon}^e)^T \cdot (\mathbf{D}^e \boldsymbol{\varepsilon}^e) + \left(\sigma_y + \frac{1}{2}H\varepsilon_{\text{eq}}^p\right)\varepsilon_{\text{eq}}^p\right) + \frac{V_M}{3}|tr(\boldsymbol{\sigma})| + RT \ln\left(\frac{\alpha v \varepsilon_{\text{eq}}^p}{N_0} + 1\right). \quad (17)$$

17 In fact, the reaction rate equation (Eq. (15)) can be rewritten in the form of the BV equation,

1 in which the overpotential depends on the diffusivity, electric field, interfacial energy, and
 2 mechanical stress (see our previous papers [38, 39] or Appendix B).

3 4 2.4. Governing equations

5 During electrochemical corrosion, the mass conservation laws for the reactants and
 6 products result in the following expression:

$$7 \int_{\Omega} \frac{\partial c_*}{\partial t} d\omega = - \int_{\partial\Omega} \mathbf{j}_* \cdot \mathbf{n} da + \int_{\Omega} r_* d\omega, \quad (18)$$

8 where \mathbf{j}_* is the molar flux of species * per unit area, and \mathbf{n} is the outward unit vector normal
 9 to the boundary surface $\partial\Omega$. The terms on the right-hand side of Eq. (18) represent two
 10 approaches in which the concentration of * can be altered, i.e., diffusion across the boundary
 11 $\partial\Omega$ (the first term) and consumption (or production) resulting from the reaction (the second
 12 term). Using Gauss's divergence theorem, the mass-conservation equation (Eq. (18)) can be
 13 expressed in differential form as follows:

$$14 \frac{\partial c_*}{\partial t} = -\nabla \cdot \mathbf{j}_* + r_*. \quad (19)$$

15 Using the Onsager linear law [51], \mathbf{j}_* can be expressed as

$$16 \mathbf{J}_* = - \left(D_* \nabla c_* + \frac{D_*}{\delta^2 \Psi / \delta c_*^2} \frac{\delta^2 \Psi}{\delta c_* \delta \phi} \nabla \phi + \frac{D_*}{\delta^2 \Psi / \delta c_*^2} \frac{\delta^2 \Psi}{\delta c_* \delta \phi} \nabla \phi \right). \quad (20)$$

17 Using Eqs. (2) and (20), the governing equation of each concentration can be expressed as

$$18 \frac{\partial c_M}{\partial t} = \nabla \cdot \left(D_M \nabla c_M + \frac{D_M}{\partial^2 \psi^{\text{chem}} / c_M^2} \frac{\partial^2 \psi^{\text{chem}}}{\partial \phi \partial c_M} \nabla \phi \right) - r, \quad (21a)$$

$$19 \frac{\partial c_{M^{n+}}}{\partial t} = \nabla \cdot \left(D_{M^{n+}} \nabla c_{M^{n+}} + n \frac{D_{M^{n+}} c_{M^{n+}} F}{RT} \nabla \phi_L \right) + r, \quad (21b)$$

$$\frac{\partial c_{\text{Cl}^-}}{\partial t} = \nabla \left(D_{\text{Cl}^-} \nabla c_{\text{Cl}^-} - \frac{D_{\text{Cl}^-} c_{\text{Cl}^-} F}{RT} \nabla \phi_L \right), \text{ and} \quad (21c)$$

$$\frac{\partial c_{\text{Na}^+}}{\partial t} = \nabla \left(D_{\text{Na}^+} \nabla c_{\text{Na}^+} - \frac{D_{\text{Na}^+} c_{\text{Na}^+} F}{RT} \nabla \phi_L \right). \quad (21d)$$

As SCC generally involves a significant amount of time, electroneutrality in the electrolyte can be assumed, which results in zero net flow of charges and a zero net charge density in the electrolyte. The first condition is expressed as follows:

$$\sum_* \mathbf{i}_* = 0 \quad (* = \text{M}^{n+}, \text{Na}^+, \text{Cl}^-), \quad (22)$$

where \mathbf{i}_* is the flow of charges transported by the diffusion flux of ions $*$, expressed as

$$\mathbf{i}_* = Fn_* \left(D_* \nabla c_* + n_* \frac{D_* c_* F}{RT} \nabla \phi_L \right). \quad (23)$$

The second condition is expressed as

$$\sum_* n_* c_* = 0, \quad (* = \text{M}^{n+}, \text{Na}^+, \text{Cl}^-). \quad (24)$$

Eqs. (21–24), combined with the boundary conditions, yield the solution of the electric potential in the electrolyte ϕ_L . It is noteworthy that ϕ_L is generally non-uniform because of the difference in the mobilities of different types of ions; even in the absence of an externally applied electric field, the latter still results in the accumulation of an electric field.

Because both cracking and corrosion change the interface area, the migration rate of the electrode–electrolyte interface, delineated by the variation rate of the order parameter, $\partial\phi/\partial t$, can be segregated into two terms, as follows:

$$\frac{\partial\phi}{\partial t} = \left(\frac{\partial\phi}{\partial t} \right)_c + \left(\frac{\partial\phi}{\partial t} \right)_r. \quad (25)$$

1 The first term on the right-hand side of Eq. (25) describes cracking under mechanical loading,
 2 which can be expressed in the form of the classical Allen–Cahn equation, as follows:

$$\begin{aligned} \left(\frac{\partial\phi}{\partial t}\right)_c &= -M_\phi \frac{\delta\Psi}{\delta\phi} \\ &= M_\phi \left(\lambda \nabla^2 \phi - W \frac{\partial g(\phi)}{\partial \phi} \right) - M_\phi \frac{\partial p(\phi)}{\partial \phi} \left(\frac{1}{2} (\boldsymbol{\varepsilon}^e)^T \cdot (\mathbf{D}^e \boldsymbol{\varepsilon}^e) + \left(\sigma_y + \frac{1}{2} H \varepsilon_{\text{eq}}^p \right) \varepsilon_{\text{eq}}^p \right), \end{aligned} \quad (26)$$

4 where M_ϕ is the interfacial mobility. The second term on the right-hand side of Eq. (25)
 5 represents the electrode dissolution induced by corrosion. Based on our previous study [38],
 6 its evolution is associated with the corrosion reaction rate, which is expressed as follows:

$$\begin{aligned} \left(\frac{\partial\phi}{\partial t}\right)_r &= -\frac{\partial p(\phi)}{\partial \phi} \frac{r}{c_M^{\text{ref}}} \\ &= -\frac{\partial p(\phi)}{\partial \phi} L^{\text{bulk}} \left(a_R \exp\left(\frac{(1-\rho)(\mu_R^{\text{ex}} - \mu_P^{\text{ex}})}{RT}\right) - \bar{c}_{M^{n+}} \exp\left(-\frac{\rho(\mu_R^{\text{ex}} - \mu_P^{\text{ex}})}{RT}\right) \right), \end{aligned} \quad (27)$$

8 where the function $\partial p(\phi)/\partial \phi$ (nonzero in the interfaces) is used to ensure that the
 9 electrochemical process occurs only at the electrode–electrolyte interface, and
 10 $L^{\text{bulk}} = k_0 / (a_{\text{TS}} c_M^{\text{ref}})$ regulates the contribution of corrosion kinetics to interface migration.

11 Using Eqs. (26) and (27), Eq. (25) can be written as

$$\begin{aligned} \frac{\partial\phi}{\partial t} &= M_\phi \left(\lambda \nabla^2 \phi - W \frac{\partial g(\phi)}{\partial \phi} \right) - \frac{\partial p(\phi)}{\partial \phi} M_\phi \left(\frac{1}{2} (\boldsymbol{\varepsilon}^e)^T \cdot (\mathbf{D}^e \boldsymbol{\varepsilon}^e) + \left(\sigma_y + \frac{1}{2} H \varepsilon_{\text{eq}}^p \right) \varepsilon_{\text{eq}}^p \right) - \\ &\quad \frac{\partial p(\phi)}{\partial \phi} L^{\text{bulk}} \left(a_R \exp\left(\frac{(1-\rho)(\mu_R^{\text{ex}} - \mu_P^{\text{ex}})}{RT}\right) - \bar{c}_{M^{n+}} \exp\left(-\frac{\rho(\mu_R^{\text{ex}} - \mu_P^{\text{ex}})}{RT}\right) \right). \end{aligned} \quad (28)$$

13 Eq. (28) shows that mechanical stresses result in cracking (the second term on the right-hand
 14 side) and accelerated corrosion (the third term on the right-hand side). In addition, the local
 15 stress increases at an accelerated rate with the initiation and propagation of cracks. These two

1 effects are not the simple superposition shown in Eq. (28), as indicated by the numerical results
2 to be presented in the following section. It is noteworthy that the proposed model is different
3 from the existing PF models proposed by Mai *et al.* [34] and Nguyen *et al.* [35–37]. In their
4 models, the reaction kinetics are not formulated; therefore, a simulated corrosion process can
5 only be diffusion mediated. In addition, their models do not include the flow of electricity; as
6 such, the electrochemical process is difficult to describe.

8 3. Numerical results and discussion

9 To solve the governing equations above and present a detailed SCC process, we
10 considered a two-dimensional (2D) domain measuring $100/\mu\text{m} \times 1250/\mu\text{m}$, which comprises
11 a metallic electrode and an electrolyte measuring $100/\mu\text{m} \times 150/\mu\text{m}$ and $100/\mu\text{m} \times 1100/\mu\text{m}$,
12 respectively. Between the electrode–electrolyte binary system, a passive film with a thickness
13 of $0.5/\mu\text{m}$ was assumed; the film had a local breakdown with a triangular pit, representing a
14 surface notch (e.g., caused by scratch). Owing to symmetry, only half of the electrode–
15 electrolyte system was modeled, as shown in Fig. 2(a). It is noteworthy that if the passive film
16 is removed and the pit is flat, then the model reduces to a one-dimensional (1D) problem,
17 which can be employed to verify the governing equations and the code by comparing the
18 results with the experimental data of homogenous corrosion [52].

19 Fig. 2

20 Initially, the two phases were separated with $\phi = 1$ for the electrode and $\phi = 0$ for the

1 electrolyte. The zero-flux condition for the order parameter, $\nabla\phi=0$, was applied to all sides
2 of the domain. The initial concentrations in the electrolyte and the far field (i.e., the top
3 boundary) were set as $c_{Na^+} = 1/\text{mol L}^{-1}$ and $c_{Cl^-} = 1/\text{mol L}^{-1}$. For the other sides, zero-flux
4 conditions, $\nabla\mathbf{c}=0$, were applied. For the electrical potential, the Dirichlet boundary
5 conditions $\phi_L = 0$ and $\phi = \phi_M$ were set at the top of the electrolyte and the bottom of the
6 electrode, respectively; the zero-flux condition, $\nabla\phi=0$, was applied to the other sides. The
7 passive film was assumed to be a zero-flux boundary for the phase order parameter, ϕ ,
8 concentrations, c^* , and electrical field, ϕ . In terms of mechanical boundary conditions, the top
9 of the metal was unconstrained; the bottom and right sides were constrained along their normal
10 direction, and the left side was subjected to uniaxial tension under a constant traction, F_x , or a
11 displacement, u_x . The reference temperature was 15°C in the simulations.

12 The parameters used in the simulations are listed in Table 1. The COMSOL Multiphysics®
13 modeling software [53] was employed to solve the proposed PF model. To guarantee the
14 convergence of the solution and achieve a reasonable computational efficiency, the simulation
15 domain was segregated into two regions, as shown in Fig. 2(b). Region I contains both the
16 metallic region and the electrolyte, wherein a uniform square mesh with an element size of
17 $0.5/\mu\text{m}$ (i.e., five times smaller than the interface thickness) was adopted because the migration
18 of interfaces must be accurately described in this region. Region II is the far-field electrolyte,
19 and a uniform square mesh with a larger size of $2.5/\mu\text{m}$ was adopted to solve the ion diffusion.
20 Triangular elements with a maximum size of $2.5/\mu\text{m}$ were used to mesh the transition region

1 between Region I and Region II. The nonlinear governing equation were solved using the
2 Newton–Raphson method in COMSOL, in which the time-step was automatically refined to
3 ensure the convergence of the solution step. In our simulations, the initial and maximum time
4 steps were $t_{\text{ref}}/2000$ and $t_{\text{ref}}/100$ for temporal integration, respectively, and they were
5 sufficiently small to ensure a stable solution, where t_{ref} is the reference time. The evolution of
6 the reciprocal of the time step with the step number is plotted in Fig. C.1, which shows a rapid
7 decrease in the reciprocal of the time step from 5000 to $100/t_{\text{ref}}^{-1}$, demonstrating the stability
8 of the nonlinear solution.

9 Table 1

10 3.1. Numerical verification: homogenous corrosion simulation (without stress)

11 Fig. 3

12 By setting the mechanical loading as zero, a 1D numerical simulation of homogeneous
13 corrosion was first conducted to validate our model. Based on the experiments of
14 electrochemical corrosion of a type of stainless steel [52], the applied electrode potential, φ_M ,
15 was set from -0.4 to -0.25 V, which required the critical concentration, $\bar{c}_{M^{n+}}^{\text{cr}}$, to be in the range
16 of 7.6×10^6 – 1.1×10^{12} . Such a high concentration results in a high local (in the pit) M^{n+}
17 concentration. Therefore, the corrosion process is mediated by reaction kinetics, and the
18 corrosion rate (i.e., the velocity of interface migration), denoted by v_c , is constant for a
19 specified potential, φ_M . This relationship, as shown in Fig. 3, agrees well with the experimental
20 data [52]. In addition, this type of evolution is consistent with the 1D analytical solution

expressed in Eq. (C4). The numerical results show two different characteristic regimes. When the applied potential was low, the corrosion rate was linearly related to φ_M , which is the Tafel law; as φ_M increased, the v_c - φ_M relationship became exponential gradually.

Fig. 4

The 2D simulation of pitting can be (qualitatively) verified. Herein, we consider a small applied potential, $\varphi_M = -0.6/V$, which requires a low critical concentration $\bar{c}_{M^{n+}}^{cr} = 1$. To describe the relative importance between the reaction and diffusion, we use the dimensionless Damkohler number, D_a , which is expressed as [54]

$$D_a = \frac{(c_M^{ref}/c_{M^{n+}}^{ref})L^{bulk}l}{D^L/L}, \quad (32)$$

where $L = 1250/\mu m$ is the distance between the metallic surface and the far-field boundary of the electrolyte. The variation in D_a is selected from the range of 0.27–2.7 with the change in the kinetics coefficient, L^{bulk} , from 0.003 to 0.03/s⁻¹. Fig. 4(a) shows the variation in the pit depth, d_c , with time for $D_a = 0.27, 0.81, \text{ and } 2.7$. For a small D_a ($= 0.27$), d_c increased linearly with time, indicating that the corrosion process was mediated by the reaction kinetics. As D_a increased, the cation (M^{n+}) concentration at the corroded surface accumulated, thereby decelerating the reaction and resulting in a transition from kinetics- to diffusion-mediated corrosion. When $D_a = 2.7$, the increase in d_c became parabolic, which is consistent with the experimental observations [55] and analytical solutions [56]. The sensitivity of corrosion to D_a was further investigated based on the evolution of d_c at 100/min as a function of D_a , as plotted in Fig. 4(b). An approximate power-law relationship between d_c and D_a was revealed.

Fig. 4(c) shows further increases in the concentrations of cations M^{n+} and Cl^- anions at the tip of the pit. Because the diffusion pathway was constrained by the passive film, the locally released metallic cations could not fully diffuse into the environment, thereby resulting in a rapid accumulation near the tip of the pit. In addition, a rapid increase in c_{Cl^-} was observed, as shown in Fig. 4(c), because the transport of Cl^- anions from the outside neutralized the positive charges of M^{n+} . When the dimensionless D_a value was low ($= 0.27$), the increases in the cation and anion concentrations were more significant because of the longer diffusion time. The accumulation of M^{n+} near the tip of the pit resulted in an increase in the electrolyte potential ϕ_L . Correspondingly, the electric potential difference $\phi_S - \phi_L$ decreased, resulting in a gradual deceleration of pitting, as observed in experiments [57]. The increase in ϕ_L for different D_a values is shown in Fig. 4(d). When the D_a value was low ($= 0.27$), the kinetics-mediated corrosion process resulted in a gradual increase in ϕ_L . When the D_a value was high ($= 2.7$), the corrosion process was diffusion mediated and accompanied by a rapid surge in electric potential followed by a slow descent and then a plateau.

3.2. MCC in SCC

Fig. 5

Next, we consider a scenario where the metallic electrode is subject to uniaxial traction, $F_x = 140-160/MPa$, which induces SCC from the predefined triangular pit. The applied electrode potential, ϕ_M , was set as $-0.1/V$, and the kinetics coefficient, L^{bulk} , was set as $1.5 \times 10^{-7}/s^{-1}$ (as listed in table 1). Based on the analysis presented in the previous section, such a

1 low L^{bulk} resulted in a kinetics-mediated corrosion process in the absence of mechanical
2 loading and a semicircular pit (Fig. 4(b)). With mechanical loading, a crack developed, as
3 shown in Fig. 5(a), wherein the evolution of equivalent stress field is presented. The variations
4 in the crack depth, d_c , and the width of the pit (i.e., opening), w_c , as defined in Fig. 5(a), are
5 plotted in Fig. 5(b). It was observed that d_c increased at an accelerated rate because the stress
6 concentration at the tip of the pit not only induced damage (as expressed in Eq. (26)), but also
7 promoted corrosion by shifting the equilibrium electropotential (as expressed in Eq. B1),
8 resulting in the initiation of a crack. Owing to the crack, a higher stress concentration was
9 generated, and the cracking proceeded more rapidly upon the actions of stress and corrosion
10 (i.e., the autocatalytic effect). Meanwhile, the stress magnitude (i.e., the von Mises equivalent
11 stress) along the cracked surface was small, and the corrosion remained kinetics mediated;
12 therefore, the variation in w_c was linear, as indicated by the dashed lines in Fig. 5(b).

13 Because the present PF model can predict the entire process beginning from the growth
14 of the corrosion pit to the pit-to-crack transition and finally to the crack propagation (as shown
15 in Fig. 5(a)), some key questions pertaining to SCC may be answered. Next, we investigate (i)
16 the critical condition for the pit-to-crack transition, (ii) the relationship between stress and SSC
17 velocity, and (iii) the effect of stress on the electrochemical environment within the crack. It
18 is noteworthy that in previous studies involving PF models [34–37], a quantitative comparison
19 with experimental results (e.g., polarization curves) was difficult (if not impossible). However,
20 using our model, quantitative analysis becomes straightforward, and the critical condition for

1 the exponential development of SCC is relevant to that of an actual scenario.

2 Fig. 6

3 To identify quantitatively the critical condition for pit-to-crack transition, the rate
4 difference between SCC and mere corrosion can be defined as $\kappa_v = (v_{\text{tip}} - v_{\text{mouth}})/v_{\text{mouth}}$, where
5 $v_{\text{tip}} = \delta d_c / \delta t$ and $v_{\text{mouth}} = \delta w_c / \delta t$ are the speeds of crack deepening and opening, respectively.
6 These variables describe the additional contribution of stresses during cracking and
7 electrochemical corrosion, respectively. Fig 6 presents the evolutions of κ_v when $F_x = 140$ –
8 160/MPa; as shown, when κ_v exceeded 1 (or in the range of 1–2), a sharpened tip began to
9 develop at the corrosion pit. Hence, the critical condition for the pit-to-crack transition can be
10 identified as $\kappa_v > 1$. It is noteworthy that this condition is equivalent to the Tsujikawa–Kondo
11 criterion [58, 59], i.e., the stress-induced crack propagation is faster than pitting because of
12 corrosion. For the first time, we numerically confirmed this criterion and proposed a method
13 to predict the lifetime of a structure based on this criterion. As κ_v ($\kappa_v > 2$) increased, the crack
14 surface became wavy gradually (the blue and red lines at 40 and 60/min, respectively), which
15 was observed in the experiments, as exemplified by crack “A” in the inset [60]. When κ_v
16 increased further, e.g., when $\kappa_v > 15$ at the blue curve, the crack resembled a cylindrical (or
17 rectangular in 2D) hole, similar to crack “B” in the inset [60].

18 Fig. 7

19 Fig 7 (a) shows the evolutions of the von Mises stress at the crack tip, σ_{mises} , when $F_x =$
20 140–160/MPa, where an upward but non-smooth trend is exhibited. After the initial rapid

1 increase in stress owing to the elastic deformation and pitting, the subsequent elastoplastic
 2 deformation (e.g., tip blunting) reduced the rate of stress increase. After 60/min, the
 3 maximum equivalent stress was approximately 420/MPa for $F_x = 160$ /MPa (the solid blue line
 4 in Fig. 7(a)), which might be similar to the ultimate tensile strength of some steels. As the
 5 metal was subjected to uniaxial tension, Mode-I crack occurred. Hence, the stress-intensity
 6 factor for Mode-I crack, K_I , was employed to evaluate the failure induced by SCC. K_I can be
 7 calculated based on the strain energy release rate as follows:

$$K_I = \sqrt{-\frac{E}{t_s} \partial \left(\int_{\Omega} \psi^{\text{mech}} dv \right) / \partial d_c}, \quad (33)$$

8 where $t_s = l_{\text{ref}}$ is the thickness of the sample, and $-\partial \left(\int_{\Omega} \psi^{\text{mech}} dv \right) / (t_s \partial d_c)$ is the strain energy
 9 release rate with an increase in the crack depth, d_c . The dashed curves in Fig. 7(a) show that
 10 K_I increases at a gradually increasing rate. Fig. 7(b) shows the variations in the crack
 11 propagation speed, denoted by v_{tip} , against K_I . As the vertical axis is on a logarithmic scale,
 12 the straight lines in Fig. 7(b) indicate exponential relations, which suggest that the effect of K_I
 13 on the speed of SCC is significant after it exceeds a critical value. As K_I increased, the crack
 14 propagation speed increased exponentially until fracture, which is consistent with the
 15 experimental observations [57, 61].

17 Fig. 8

18 Based on an experiment [57], stress affects the electrochemical environment within the
 19 crack. To quantify this effect, the evolution of the difference in the concentration of M^{n+} at the
 20 tip and mouth of the crack, i.e., $C_{M^{n+}}^{\text{tip}} - C_{M^{n+}}^{\text{mouth}}$, is shown in Fig. 8(a), where $F_x = 140-160$ /MPa.

1 The gradual increase in the difference between $c_{M^{n+}}^{\text{tip}}$ and $c_{M^{n+}}^{\text{mouth}}$ was due to mechanical
2 loading because (i) the diffusion path to the tip of crack was longer than that to the mouth, and
3 (ii) the concentrated stress at the crack tip accelerated the metal dissolution. Furthermore, with
4 the increase in F_x , the distribution of metallic ions became more heterogeneous, indicating a
5 more localized corrosion induced by the local stress field. The local accumulation of positive
6 charges of M^{n+} resulted in a heterogeneous distribution of electropotential within the crack, as
7 shown in Fig. 8(b), wherein the electropotential difference between those at the tip and the
8 mouth of the crack is shown. These numerical results are consistent with the experimental
9 observation reported by MacDonald *et al.* [57].

10 Fig. 9

11 To separate the effects of stress and reaction kinetics and investigate their coupling effect
12 on SCC, the evolutions of the pit/crack depth, d_c , for five scenarios are presented in Fig. 9.
13 Scenario I involves all the couplings introduced above; in Scenario II, the enhanced stress
14 effect on the reaction kinetics (i.e., Eq. (20)) are disregarded; Scenarios IV and V represent the
15 cracks resulting from mere corrosion and stress only, respectively; Scenario III is a
16 superposition of curves IV and V. In Scenarios IV and V, a semi-circular pit (due to the uniform
17 reaction rate) and a wavy vertical crack (induced by stress) were observed, which resulted in
18 depths of 16 and $15/\mu\text{m}$ at 60/min (the end of simulation), respectively. These two depths were
19 much smaller than those due to coupling effects (I and II), indicating that neither stress nor
20 electrochemical process alone can be the primary cause of SCC. Moreover, their superposition,

1 represented by curve III, was much lower than curves I and II involving MCC, demonstrating
2 that the severe SCC was a result of the conjoint action of stress and the corrosive environment
3 rather than their superposition. Comparing curves I and II, it is clear that if the stress term
4 enhances corrosion kinetics by shifting the equilibrium potential to the left, as expressed in Eq.
5 (20), then SCC will be further improved.

6 Fig. 10

7 A metallic component is typically subjected to various types of mechanical loading,
8 resulting in SCC. A mechanical loading can be either a surface traction (F_x) or a surface
9 displacement (u_x). Using the predefined simulation model, we compared the cases of $F_x =$
10 $160/\text{MPa}$ and $u_x = 0.08/\mu\text{m}$, which initially resulted in similar far-field normal stresses. Fig.
11 10 shows the variations in crack depth with time for these two cases. Apparently, the surface
12 traction resulted in an accelerated cracking, whereas the displacement boundary condition
13 resulted in a stable crack growth, as clearly shown in the inset of Fig. 10, where the cracking
14 speed v_{tip} is plotted against time. Such a difference is characteristic of fracture mechanics: if
15 the sample is subjected to a displacement boundary condition, then the effect of crack
16 morphology on stress distribution is finite, and cracking is a stable and slow process. This
17 applies to SCC, as shown in Fig. 11(a), wherein the variations in the K_I can be compared. For
18 the displacement boundary condition, K_I increased much slower. In Fig. 11(b), the contour
19 plots of σ_{mises} are presented. As shown, the stress concentration area (e.g., the red and orange
20 areas) did not increase with time when the displacement boundary condition was applied;

1 however, it expanded rapidly when traction was applied.

2 Fig. 11

3 Fig. 12

4 Finally, we examined the effect of the initial geometry on the crack growth. It is
5 noteworthy that we set the initial pit to be a triangle with an initial depth of $b = 7.5/\mu\text{m}$. For
6 other depths, such as $b = 0, 2.5, \text{ and } 5/\mu\text{m}$, Fig. 12 (a) shows the evolutions of the pit/crack
7 when the metallic component was subjected to the traction boundary condition, $F_x = 160/\text{MPa}$.
8 As shown, for a smaller b , the early development of the pit was kinetically mediated, which
9 resulted in a semi-circular shape as the reaction kinetics was independent of orientation. With
10 the development of the pit, stresses became more concentrated at the tip, which ultimate
11 resulted in SCC. The variation in the rate difference between SCC and mere corrosion, κ_v , was
12 plotted, as shown in Fig 12(b). Similarly, it was indicated that the pit-to-crack transition
13 occurred when $\kappa_v > 1$. Next, we plotted the critical depth of the pit that enabled the occurrence
14 of the pit-to-crack transition, denoted by $d_c^{\text{pit-to-crack}}$, against b in the inset of Fig. 12(b). As
15 shown, $d_c^{\text{pit-to-crack}}$ decreased with b at a reducing rate, which implies the high sensitivity of
16 the SCC to the severity of the initial surface damage.

18 4. Conclusions

19 A new PF model was proposed to address the MCC in SCC. The interface migration
20 associated with pitting and cracking was described using the Allen–Cahn-type equation, which

1 incorporates the theory of strain energy release rate in fracture mechanics, and the generalized
2
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4 BV relation describing an electrochemical reaction. Furthermore, Gutman's expressions were
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6
7 used to address the enhanced MCC, where stress affected the reaction rate or the equilibrium
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10 potential (see Eq. B1) during oxidation. Finally, a set of governing equations, including the
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12 Nernst–Planck–Poisson model for diffusion and stress equilibrium equations for mechanical
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14 deformation, was solved using a commercial FEM package. The numerical model used can be
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16
17 corroborated based on the experimental results and analytical solutions of homogenous
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20 corrosion. Subsequently, we investigated the 2D SCC process. The main findings are as
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24 follows:

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27 (1) Based on the PF model, the autocatalytic effect resulting from the relationship
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30 between stress and corrosion was predicted. The effect resulted in the formation of a pit,
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33 initiation of a crack, and cracking at an accelerated rate. A parameter, κ_v , defined as $(v_{\text{tip}} -$
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35
36 $v_{\text{mouth}})/v_{\text{mouth}}$, was introduced to quantitatively analyze the additional contributions of stresses
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39 during cracking and electrochemical corrosion. Subsequently, the critical condition of pit-to-
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42 crack transition at $\kappa_v > 1$ was revealed, which, for the first time, confirmed the Tsujikawa–
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45 Kondo criterion [58, 59], i.e., when a pit-to-crack transition occurs, the stress-induced crack
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48 propagation should be faster than the pitting caused by corrosion. It was indicated that after
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51 the critical condition was fulfilled, an exponential relation between the cracking speed, v_{tip} ,
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54 and the stress-intensity-factor for Mode-I crack, K_I , was obtained, which agreed well with the
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57 experimental observation [57]; this similarly demonstrated the validity of the proposed PF
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1 model. Additionally, it was observed that an increase in the applied traction resulted in a more
2 heterogeneous distribution of cation concentration and electropotential and a more localized
3 corrosion, which were similarly observed in experiments [57].

4 (2) The comparison among different MCC scenarios indicated that the coupling effect
5 resulted in a more significant SCC than the linear combination of mechanical cracking and
6 corrosion. Nevertheless, the well-known difference between Dirichlet (displacement) and
7 Neumann (force) boundary conditions associated with the stability of crack propagation
8 remained valid, i.e., the SCC was more critical when a metallic component was subjected to
9 external forces. In this case, the effect of the initial pit depth or surface damage was
10 significant.

11 Acknowledgments

12 CL acknowledges the support from Guangdong Major Project of Basic and Applied Basic
13 Research (2019B030302011), International Sci & Tech Cooperation Program of Guangdong
14 Province (2019A050510022) and the support from Natural Science Basic Research Plan in
15 Shaanxi Province of China (No. 2019JQ-123). HHR acknowledges the support of the General
16 Research Fund of the Hong Kong Research Grants Council (Grant No.: 15213619, Account
17 code: Q73H).

1 **Author Contribution**

2 CL and HHR conceptualized the study. CL conducted PF modeling, simulation, and
3 drafted the manuscript; HHR edited and polished the manuscript.

4 **Additional Information**

5 **Competing Interest:** The authors declare no competing interests.

6 **Data Availability**

7 The data that support the findings of this study are available from the corresponding author
8 upon reasonable request.

9 **Appendix A**

10 Considering that the forward and backward reactions take place simultaneously, the net
11 reaction rate, expressed in the Arrhenius form, is:

$$12 \quad r = k^0 \left(\exp\left(-\frac{\mu_{\text{TS}} - \mu_{\text{R}}}{RT}\right) - \exp\left(-\frac{\mu_{\text{TS}} - \mu_{\text{P}}}{RT}\right) \right), \quad (\text{A.1})$$

13 where μ_{R} and μ_{P} are the chemical potentials of reactants and products, respectively; μ_{TS} is the
14 chemical potential at the transition state. For a reaction involving multiple reactants and
15 products, μ_{R} and μ_{P} can be expressed as:

$$16 \quad \mu_{\text{R}} = RT \ln \left(\prod_i (a_{\text{R}_i}^{x_i}) \right) + \sum_i n_i \mu_{\text{R}_i}^{\text{ex}}, \quad (\text{A.2a})$$

$$\text{and } \mu_p = RT \ln \left(\prod_j \left(a_{p_j^{z_j}} \right)^{n_j} \right) + \sum_j n_j \mu_{p_j^{z_j}}^{\text{ex}}, \quad (\text{A.2b})$$

where a_* and μ_*^{ex} are the activity and the excess chemical potential of $*$, respectively.

According to the definition of Bazant [62], the activity is only concentration dependent, defined as:

$$a_* = \exp \left(\frac{1}{RT} \left(\frac{\delta \left(\int_{\Omega} (\psi^{\text{chem}}) d\omega \right)}{\delta c_*} - \mu_*^0 \right) \right). \quad (\text{A.3a})$$

Correspondingly, the excess chemical potential, μ_*^{ex} , is:

$$\mu_*^{\text{ex}} = \frac{\delta \left(\int_{\Omega} \psi - \psi^{\text{chem}} d\omega \right)}{\delta c_*} + \mu_*^0, \quad (\text{A.3b})$$

which involves the contributions of mechanical energy, electric potential, and standard chemical potential. The chemical potential at the transition state, μ_{TS} , is defined as [62]:

$$\mu_{\text{TS}} = RT \ln a^{\text{TS}} + \rho \sum_i n_i \mu_{R_i^{x_i}}^{\text{ex}} + (1 - \rho) \sum_j n_j \mu_{p_j^{z_j}}^{\text{ex}}, \quad (\text{A.4})$$

Substituting Eqs. (A.2a), (A.2b) and (A.4) into Eq. (A.1), the reaction rate, r , can be finally derived in the form of Eq. (16).

Appendix B

If we set reaction rate $r = 0$, the electric potential difference at the electrode-electrolyte interface under an electrochemical equilibrium can be derived as:

$$\Delta \varphi_{\text{eq}} = (\varphi_{\text{S}} - \varphi_{\text{L}})_{\text{eq}} = \frac{1}{nF} \left(\begin{aligned} & RT \ln \left(\frac{a_{\text{P}}}{a_{\text{R}}} \right) + \mu_{\text{M}^{n+}}^0 - \frac{V_{\text{M}}}{3} |tr(\boldsymbol{\sigma})| - RT \ln \left(\frac{\nu \alpha \varepsilon_{\text{eq}}^{\text{P}}}{N_0} + 1 \right) \\ & - \frac{\partial p(\phi)}{c_{\text{M}}^{\text{ref}} \partial \phi} \left(\frac{1}{2} (\boldsymbol{\varepsilon}^{\text{e}})^T \cdot (\mathbf{D}^{\text{e}} \boldsymbol{\varepsilon}^{\text{e}}) + \left(\sigma_{\text{y}} + \frac{1}{2} H \varepsilon_{\text{eq}}^{\text{P}} \right) \varepsilon_{\text{eq}}^{\text{P}} \right) \end{aligned} \right). \quad (\text{B.1})$$

1 The reaction takes place under the overpotential, η , defined as $\eta = \Delta\phi - \Delta\phi_{\text{eq}}$. Substituting
 2 overpotential η into Eq. (16) leads to the generalized BV equation, expressed as:

$$r = \frac{k^0}{a_{\text{TS}}} (a_{\text{R}})^{\rho} (a_{\text{P}})^{(1-\rho)} \left(\exp\left(\frac{(1-\rho)nF\eta}{RT}\right) - \exp\left(-\frac{\rho nF\eta}{RT}\right) \right). \quad (\text{B.2})$$

5 Appendix C

6 Omitting the effect of concentration and stress, the reference chemical potential, $\mu_{\text{M}^{n+}}^0$,
 7 can be expressed as $\mu_{\text{M}^{n+}}^0 = nF\Delta\phi_{\text{eq}}$ (based on Eq. (B.1)), where n and F are listed in table 1
 8 and the equilibrium electric potential, $\Delta\phi_{\text{eq}}$, is about -0.6V [63]. Thus, the reference chemical
 9 potential, $\mu_{\text{M}^{n+}}^0$ can be determined to be about -100 kJ mol^{-1} .

10 Following Caginalp [64], 1D analytical solutions for the velocity of phase boundary
 11 migration can be obtained from Eq. (28) under the steady state conditions (i.e., the
 12 concentration, electric potential, and displacement field remaining unvaried) expressed as:

$$v = M_{\phi} l \left(\frac{1}{2} (\boldsymbol{\varepsilon}^e)^T \cdot (\mathbf{D}^e \boldsymbol{\varepsilon}^e) + \left(\sigma_y + \frac{1}{2} H \varepsilon_{\text{eq}}^p \right) \varepsilon_{\text{eq}}^p \right) + L^{\text{bulk}} l \left(a_{\text{R}} \exp\left(\frac{(1-\rho)(\mu_{\text{R}}^{\text{ex}} - \mu_{\text{P}}^{\text{ex}})}{RT}\right) - \bar{c}_{\text{M}^{n+}} \exp\left(-\frac{\rho(\mu_{\text{R}}^{\text{ex}} - \mu_{\text{P}}^{\text{ex}})}{RT}\right) \right), \quad (\text{C.1})$$

14 where v is the velocity of phase boundary migration.

15 If the mechanical loading is not applied, the effects of stress and the fracture energy are
 16 negligible, which leads to a 1D velocity of phase migration due to corrosion, given by:

$$v = L^{\text{bulk}} l \left[\begin{array}{c} \exp\left(\frac{(1-\rho)(nF(\varphi_s - \varphi_L) - \mu_{\text{M}^{n+}}^0)}{RT}\right) - \\ \bar{c}_{\text{M}^{n+}} \exp\left(-\frac{\rho nF(nF(\varphi_s - \varphi_L) - \mu_{\text{M}^{n+}}^0)}{RT}\right) \end{array} \right]. \quad (\text{C.2})$$

Letting $v = 0$, a critical concentration, $\bar{c}_{\text{M}^{n+}}^{\text{cr}}$, can be estimated from Eq. (C.2):

$$\bar{c}_{\text{M}^{n+}}^{\text{cr}} = \exp\left(\frac{nF(\varphi_s - \varphi_L) - \mu_{\text{M}^{n+}}^0}{RT}\right). \quad (\text{C.3})$$

It means that if the local concentration is larger than this critical value, the corrosion would be stopped, which results in a diffusion-controlled corrosion process. Hence, the increase in the applied potential leads to the more kinetics-mediated corrosion because the critical M^{n+} concentration is more difficult to achieve. Correspondingly, the higher diffusivity of M^{n+} in electrolyte reduces the accumulation of M^{n+} at the metal surface, resulting in the kinetics-mediated corrosion behavior.

Eq. (C.2) can be recast as:

$$v = L^{\text{bulk}} l \exp\left(\frac{(1-\rho)(nF(\varphi_s - \varphi_L) - \mu_{\text{M}^{n+}}^0)}{RT}\right). \quad (\text{C.4})$$

With Eq. (C.4), if the applied potentials, φ_s and φ_L , and the rate of homogenous corrosion, v , are known, the kinetic coefficient, L^{bulk} , can be estimated. Based on the experimental results provided in [52], wherein the applied potential in metallic electrode and electrolyte are 250/mV and 0/mV, respectively, the corrosion rate can be 11.6/mm year⁻¹. Thus, the kinetic coefficient, L^{bulk} , is determined to be $1.5 \times 10^{-7} / \text{s}^{-1}$ with $l = 2.5 / \mu\text{m}$, $\rho = 0.5$, $\varphi_s = -250 / \text{mV}$, $\varphi_L = 0 / \text{mV}$, $n = 2$ and $\mu_{\text{M}^{n+}}^0 = -100 / \text{kJ mol}^{-1}$.

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1 **Figure captions**

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4 Fig. 1 Schematics of SCC.

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6 Fig. 2 (a) The geometry and boundary conditions of the simulation domain and (b) FE mesh
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8
9 used in simulation.

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12 Fig. 3 The variation of corrosion rate v_c with the applied electrode potential ϕ_M resulting from
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15 a 1D simulation of homogeneous corrosion.

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18 Fig. 4 (a) The increase in the pit depth, d_c , with time; (b) the variation of d_c at 100/min against
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21 D_a ; the evolutions of (c) concentrations, $c_{M^{n+}}$ and c_{Cl^-} , and (d) potential, ϕ_L , in the
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24 electrolyte near the tip of pit, for different Damkohler number, D_a , in the range of 0.27
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27 – 2.7.

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30 Fig. 5 (a) Evolution of inhomogeneous distribution of stress as a sharp vertical crack
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33 development and (b) the evolutions of the crack depth normal to the tip of crack, d_c ,
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36 and the crack width normal to the mouth of crack, w_c .

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38 Fig. 6 Evolution of the variable, κ_v , with time for the metallic electrode subjected to different
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41 tractions ($F_x = 140/MPa - 160/MPa$). κ_v is defined as $\kappa_v = (v_{tip} - v_{mouth})/v_{mouth}$, which
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44 characterizes the rate difference between SCC and mere corrosion.

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47 Fig. 7 (a) evolutions of von Mises stress at the tip of the crack, σ_{mises} , and the stress-intensity-
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50 factor for Mode-I crack, K_I , and (b) variations of cracking velocity, denoted by v_{tip} , with
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53 K_I when $F_x = 140 - 160/MPa$.

1 Fig. 8 Evolutions of (a) the concentration (M^{n+}) and (b) electropotential differences at the tip
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4 and mouth of crack, denoted by $c_{M^{n+}}^{tip} - c_{M^{n+}}^{mouth}$ and $\varphi^{tip} - \varphi^{mouth}$, respectively, when $F_x =$
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7 140 – 160/MPa.

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9 Fig. 9 Evolutions of crack depth, d_c , with time for the sample subjected to the traction, F_x
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11 =160/MPa. Herein, curves I, IV and V represent respectively the scenarios that the
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14 crack results from the MCC, mere corrosion, and stress only. Curve III is a simple
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16
17 crack results from the MCC, mere corrosion, and stress only. Curve III is a simple
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20 superposition of lines IV and V, and curve II represents the scenario that the enhanced
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23 stress effect on reaction kinetics (i.e., Eq. (20)) are ignored.

24 Fig. 10 The difference between traction ($F_x = 160/MPa$) and displacement ($u_x = 0.08/\mu m$)
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26
27 boundary conditions in terms of the variation of crack depth, d_c , with time.

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29 Fig. 11 (a) Evolution of mises stress at the tip of crack, σ_{mises} , and the stress-intensity-factor
30
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32 for Mode-I crack, K_I , and (b) the corresponding contour plot of mises stress, when the
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35 sample is subjected to traction and displacement.

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38 Fig. 12 (a) The contour plot of crack and (b) the evolution of variable, κ_v , with time for the
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41 samples subjected to the traction, $F_x = 160/MPa$, having the different initial pit, $a =$
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44 7.5/ μm and $b = 0 - 7.5/\mu m$.

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47 Fig. C.1 Example of convergence curve for a simulation with the applied electrode potential,
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50 $\varphi_M = -0.1/V$, the kinetics coefficient, $L^{bulk} = 1.5 \times 10^{-7}/s^{-1}$, and uniaxial traction, $F_x =$
51
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53 140/MPa.

1 **Table**

	Parameter	Value
Reference length	l_{ref}	2.5/ μm
Reference time	t_{ref}	60/s
Interfacial mobility	M_{ϕ}	$1.5 \times 10^{-4} / \text{m}^3 \text{ J}^{-1} \text{ s}^{-1}$ [37]
Fracture energy density	g_c	120/ J m^{-2} [37]
Interface thickness	l	2.5/ μm [37]
Young's modulus of the metal	E	200/ GPa
Poisson's ratio of the metal	ν	0.33
Initial yield strength of metal	σ_y^0	320/ MPa
Linear strengthening modulus of metal	H	5/ GPa
Reference concentration of metal	C_M^{ref}	$1.4 \times 10^2 / \text{mol L}^{-1}$ [38]
Reference concentration of metal ion in electrolyte	$C_{M^{n+}}^{\text{ref}}$	5.436/ mol L^{-1} [38]
Reference chemical potential of M^{n+}	$\mu_{M^{n+}}^0$	-100/ kJ mol^{-1} (See Ref. [39] or Appendix D)
Coefficients to scale the contributions of corrosion kinetics to phase migration	L^{bulk}	$1.5 \times 10^{-7} / \text{s}^{-1}$ (See Appendix D)
Asymmetry factor	ρ	0.5 [38]

1	Diffusion coefficient in electrolyte	D^L	$1 \times 10^{-9} / \text{m}^2 \text{ s}^{-1}$ [38]
2			
3			
4	Electric conductivity of electrolyte	ε^L	$1 / \text{S m}^{-1}$ [38]
5			
6			
7	Electric conductivity of metal	ε^S	$10^7 / \text{S m}^{-1}$ [38]
8			
9			
10	Ideal gas constant	R	$8.314 / \text{J mol}^{-1} \text{ K}^{-1}$
11			
12	charge number of metallic ion M^{n+}	n	2
13			
14			
15	Faraday's constant	F	$96485 / \text{C mol}^{-1}$
16			
17			
18	Coefficient to scale the effect of plasticity	α	$1.67 \times 10^{11} / \text{cm}^2$ [50]
19			
20			
21	to electropotential		
22			
23			
24	Orientation-dependent factor	v	0.45 [50]
25			
26			
27	Initial dislocation density	N_0	$10^8 / \text{cm}^2$ [50]
28			

Table 1 Parameters used in simulation

Dear Editors:

We are grateful for the comments and have thoroughly revise and improve the manuscript according to your suggestion. The revised manuscript with modification marks (highlighted in yellow background) is now submitted for your further consideration. In the separate response letter, your comments have been responded item-by-item and the corresponding changes in the manuscript were indicated with the page and line numbers.

Thank you and best regards.

Yours sincerely,

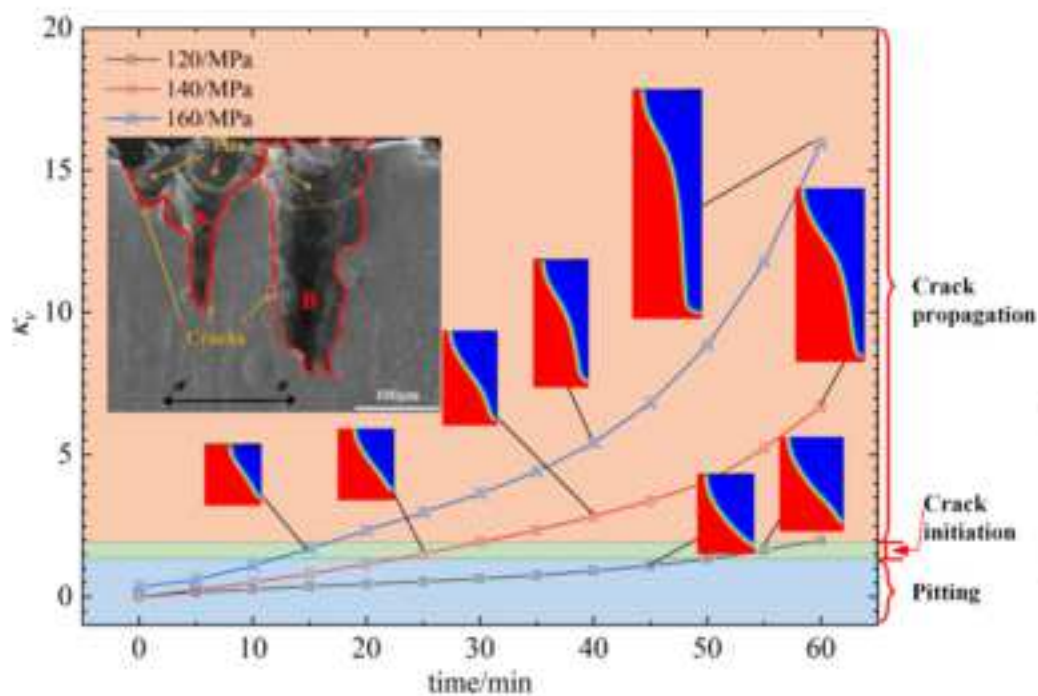
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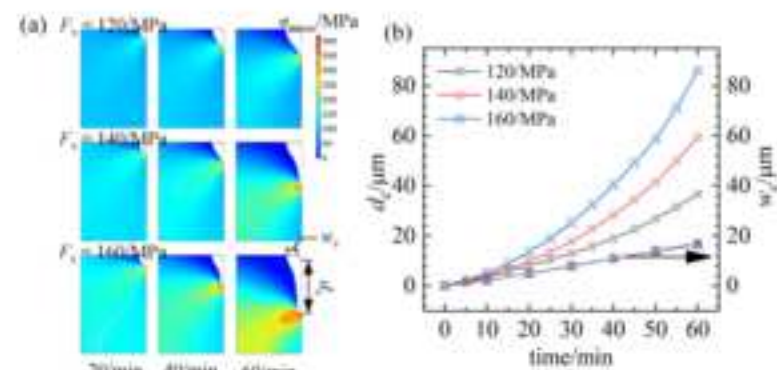
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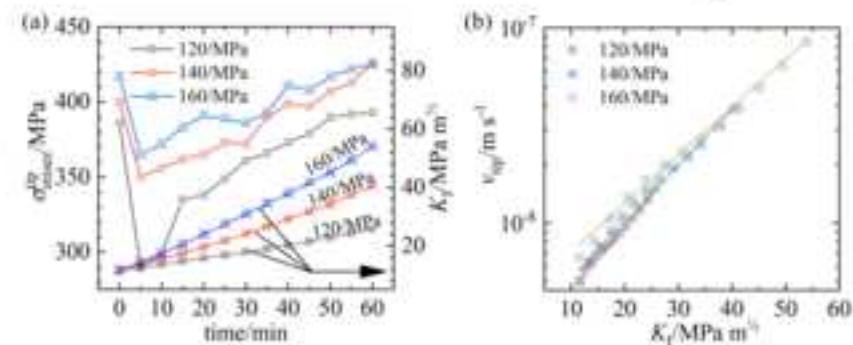
SSC process involving the growth of pit, pit-to-crack transition, and finally cracking

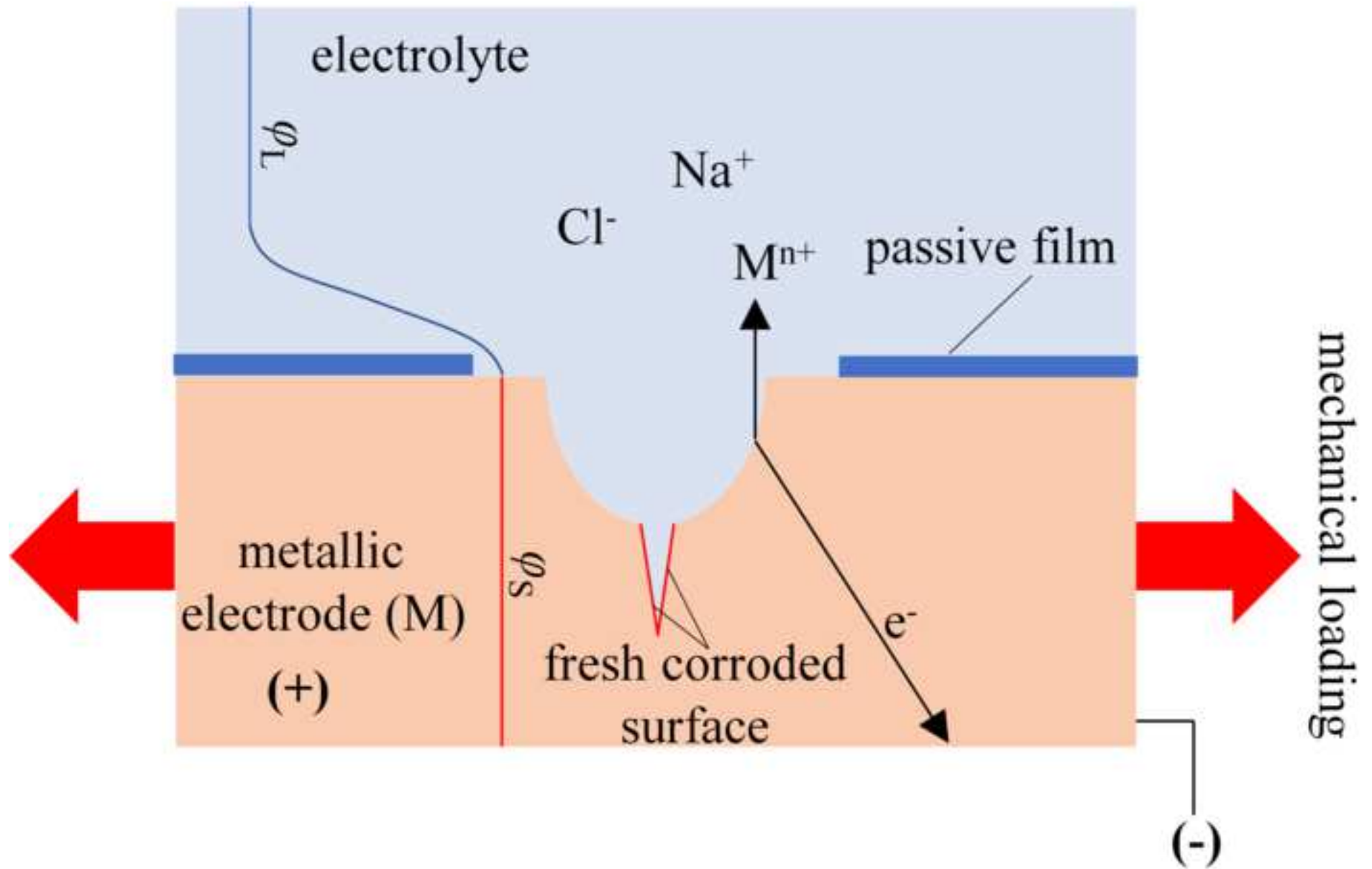


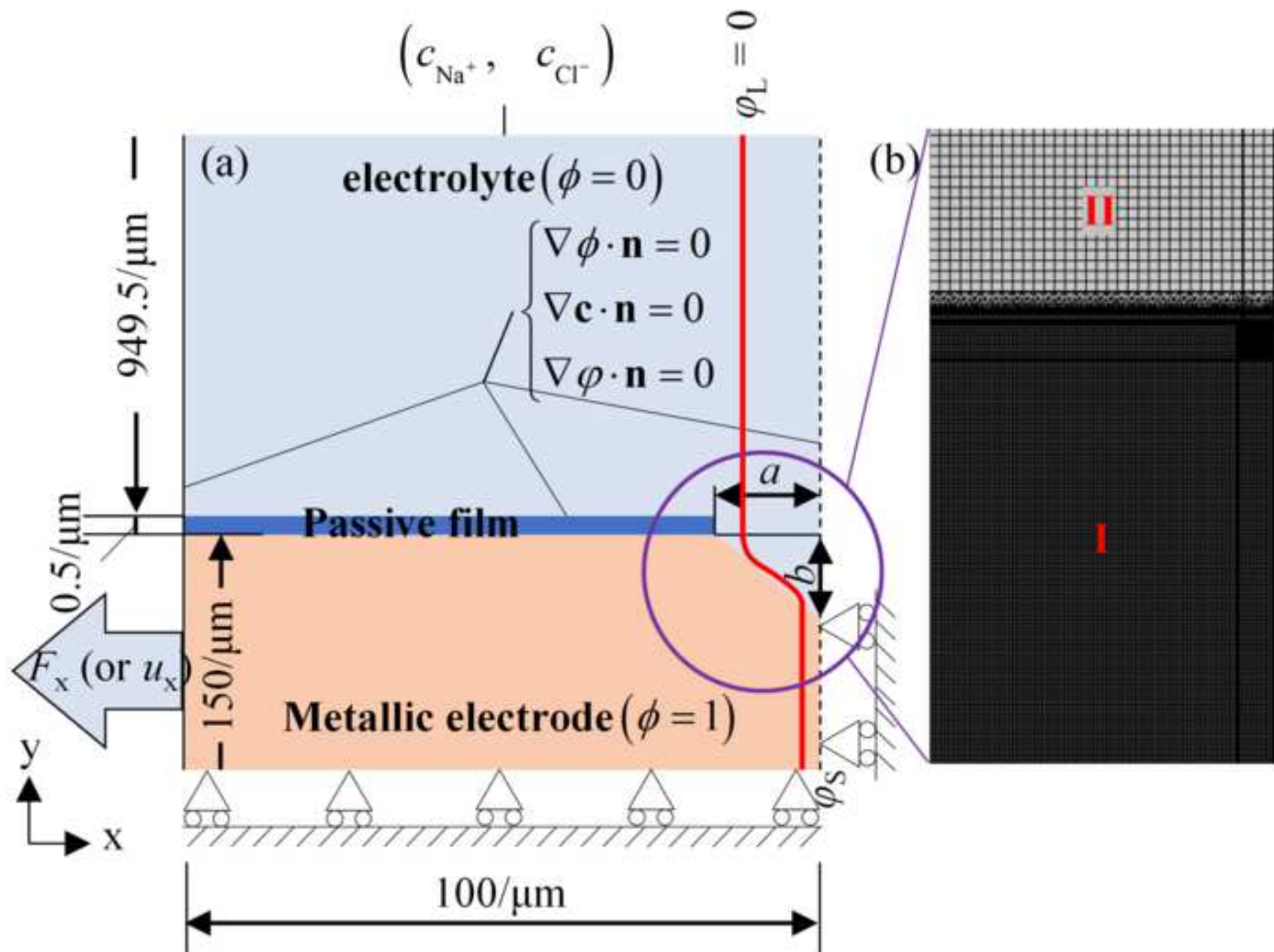
Evolution of depth of crack

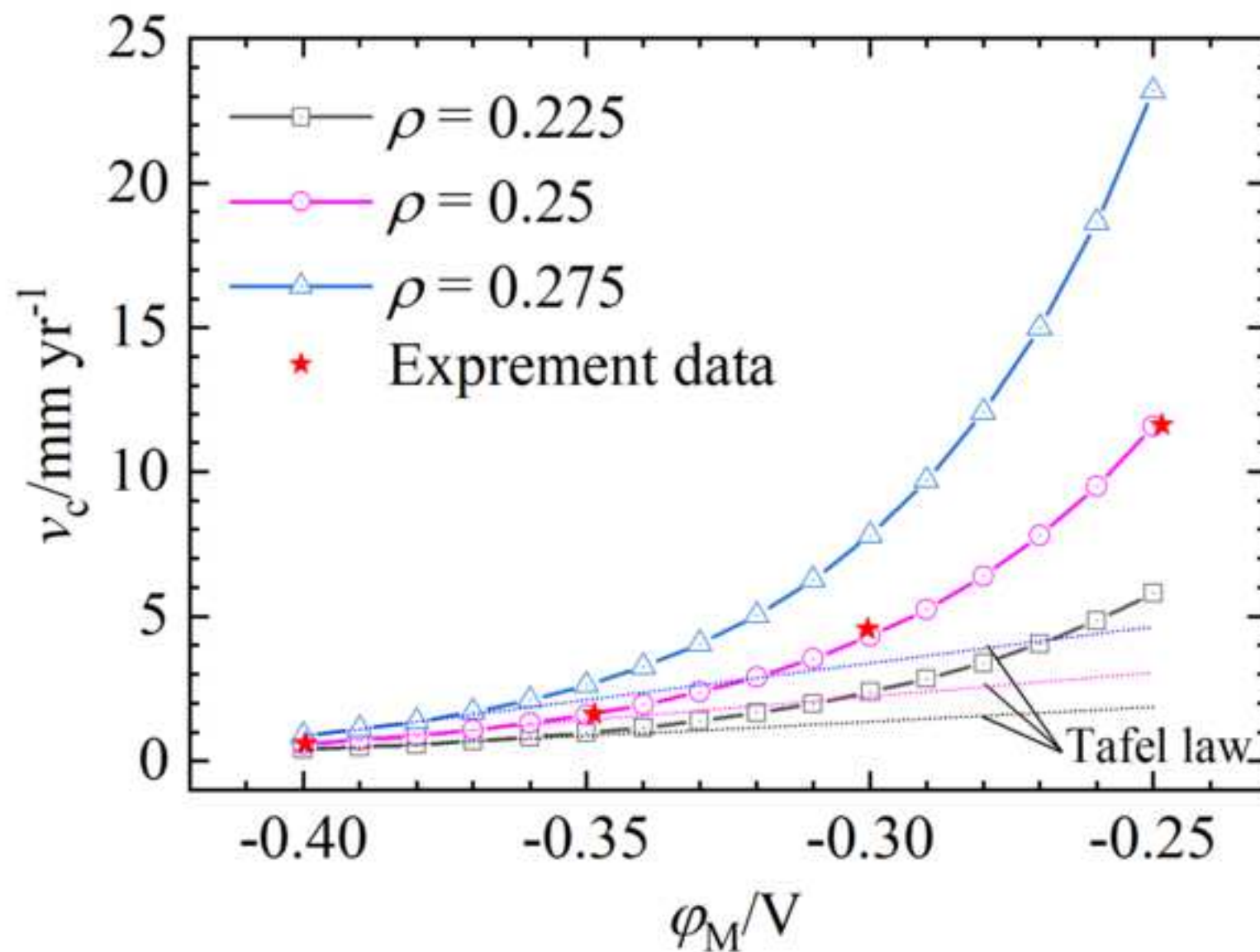


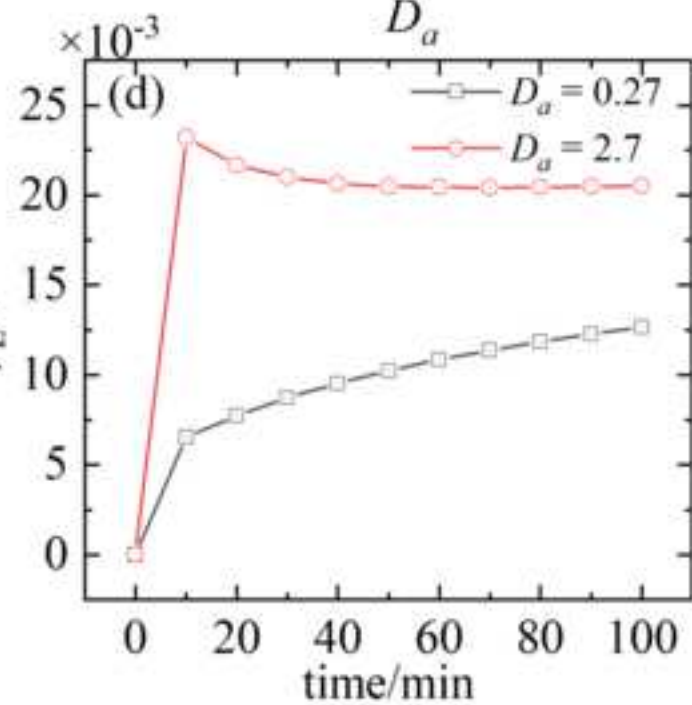
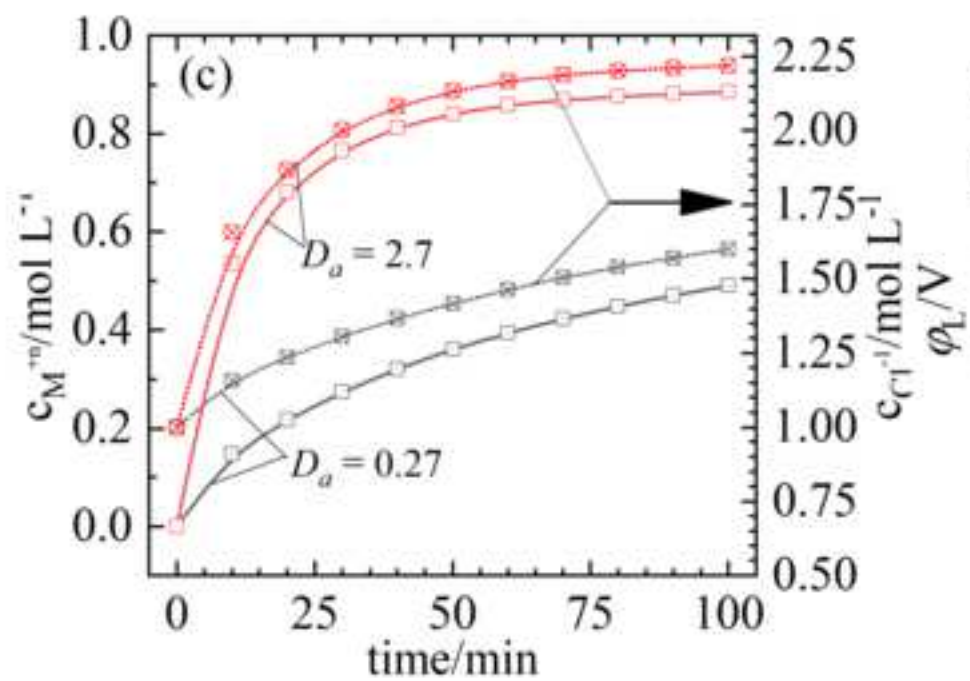
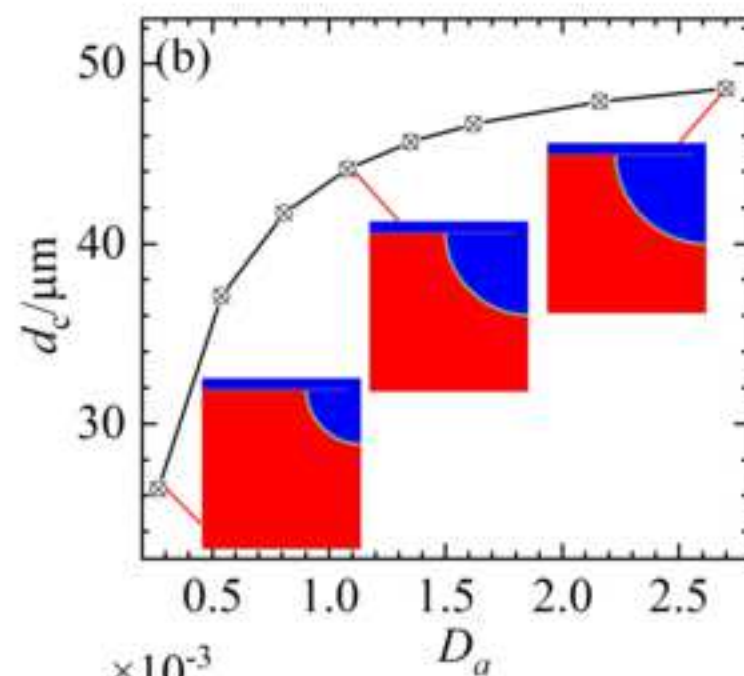
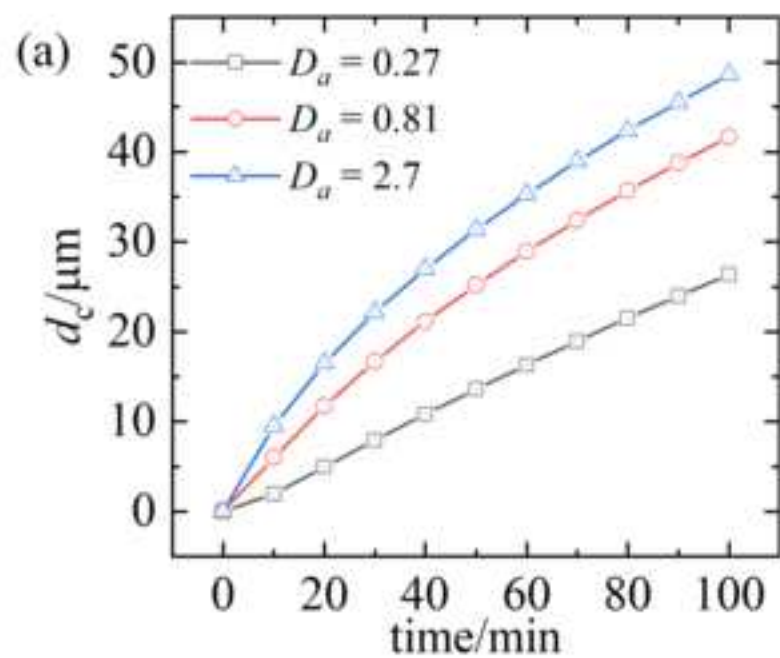
Evolution of stress and stress-intensity-factor

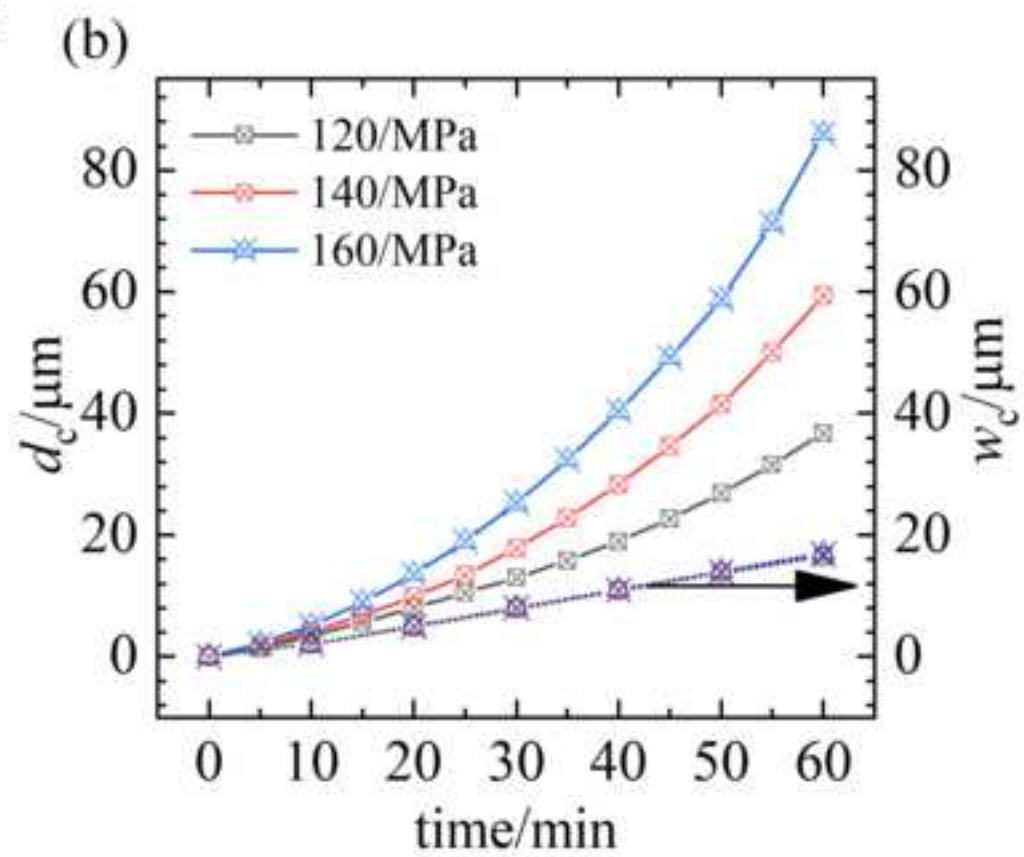
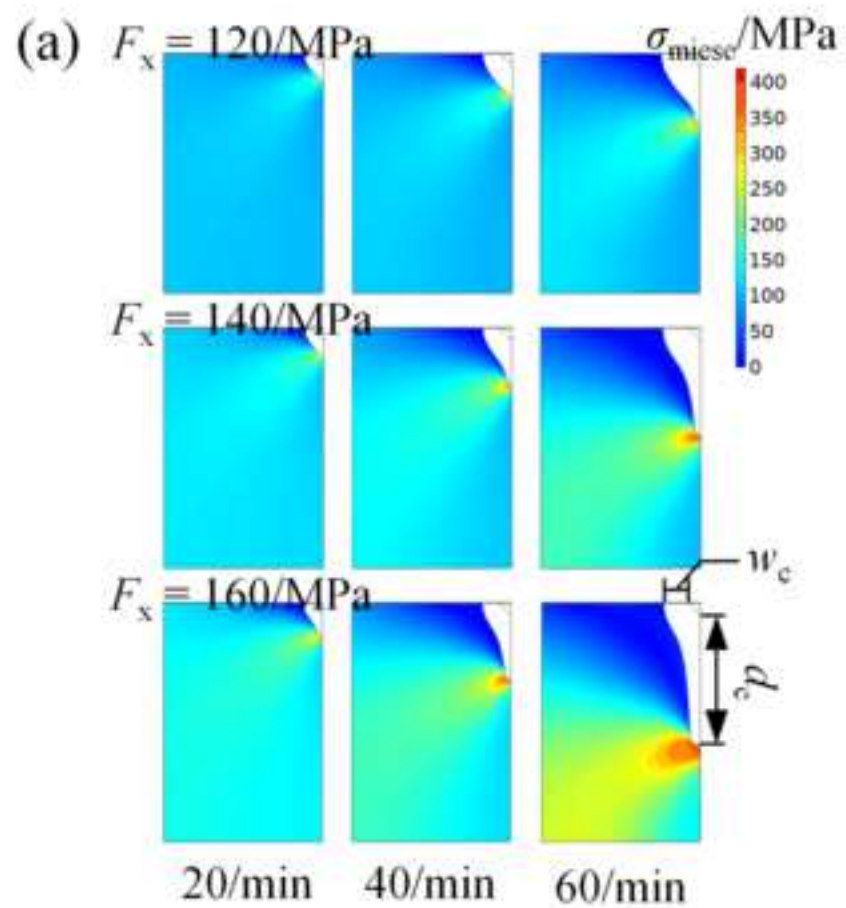


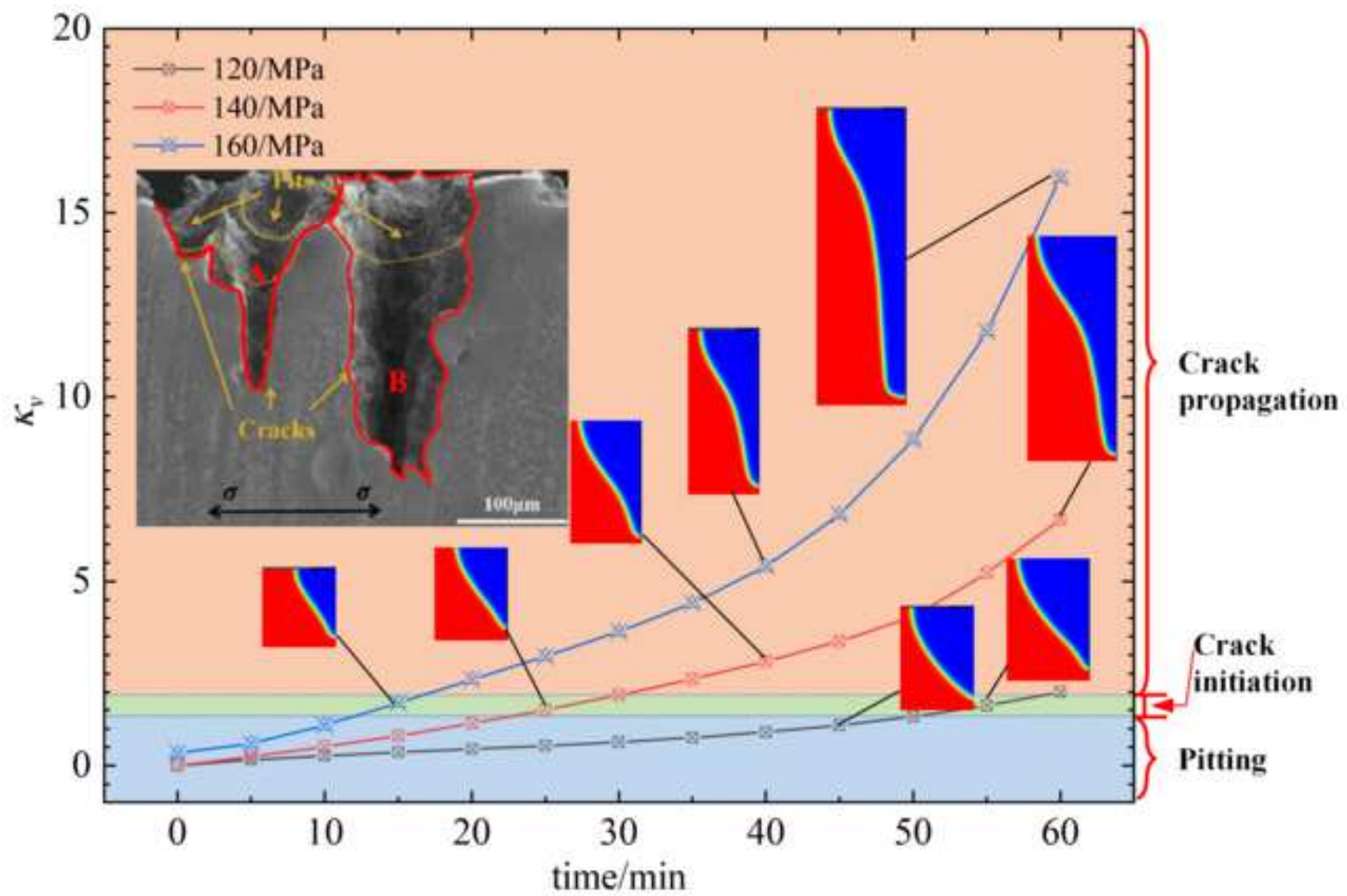


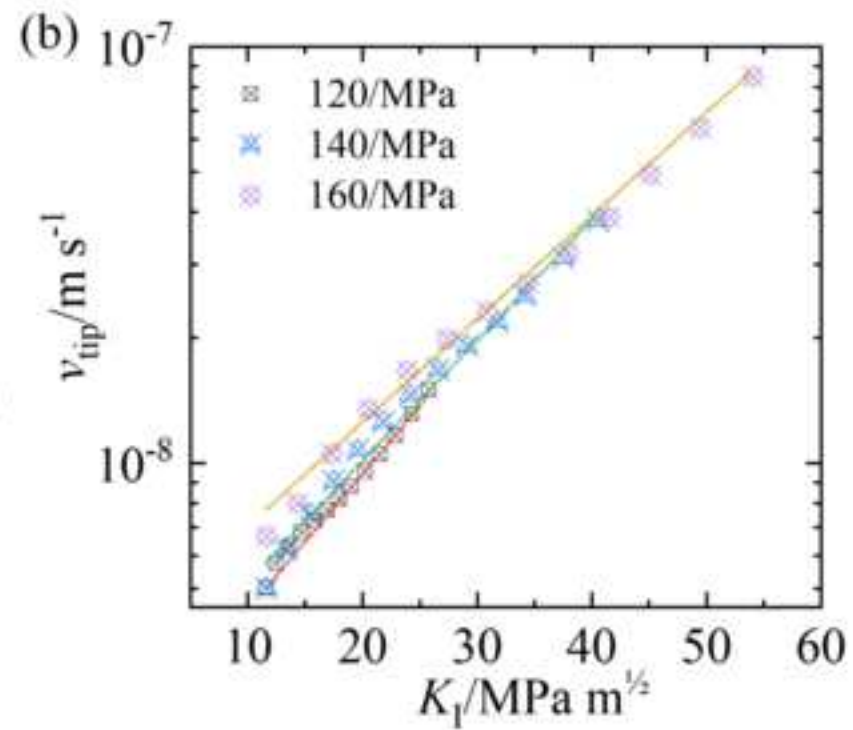
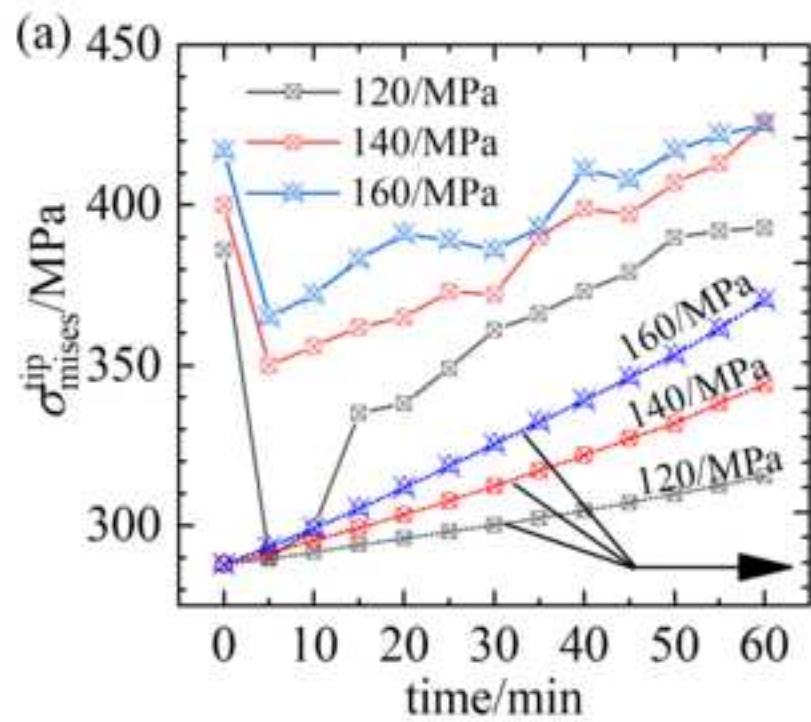


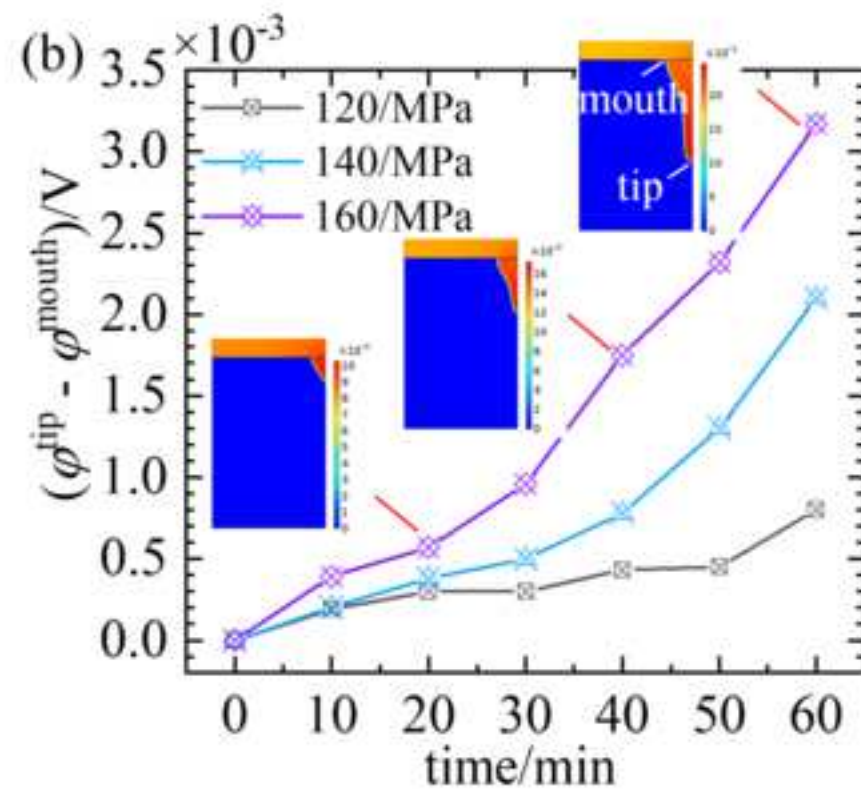
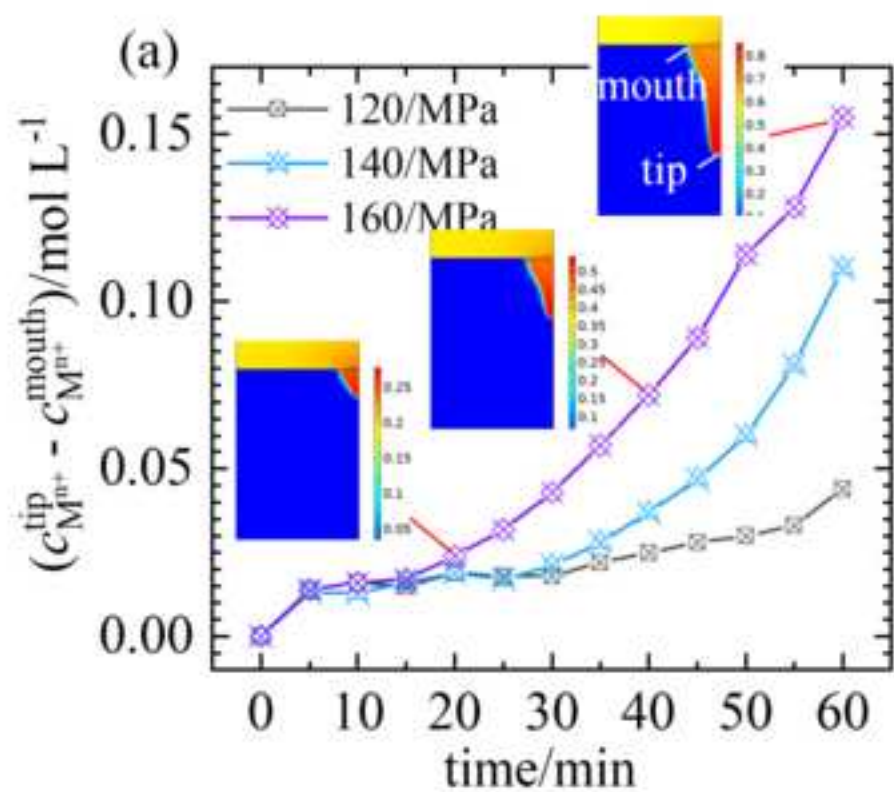


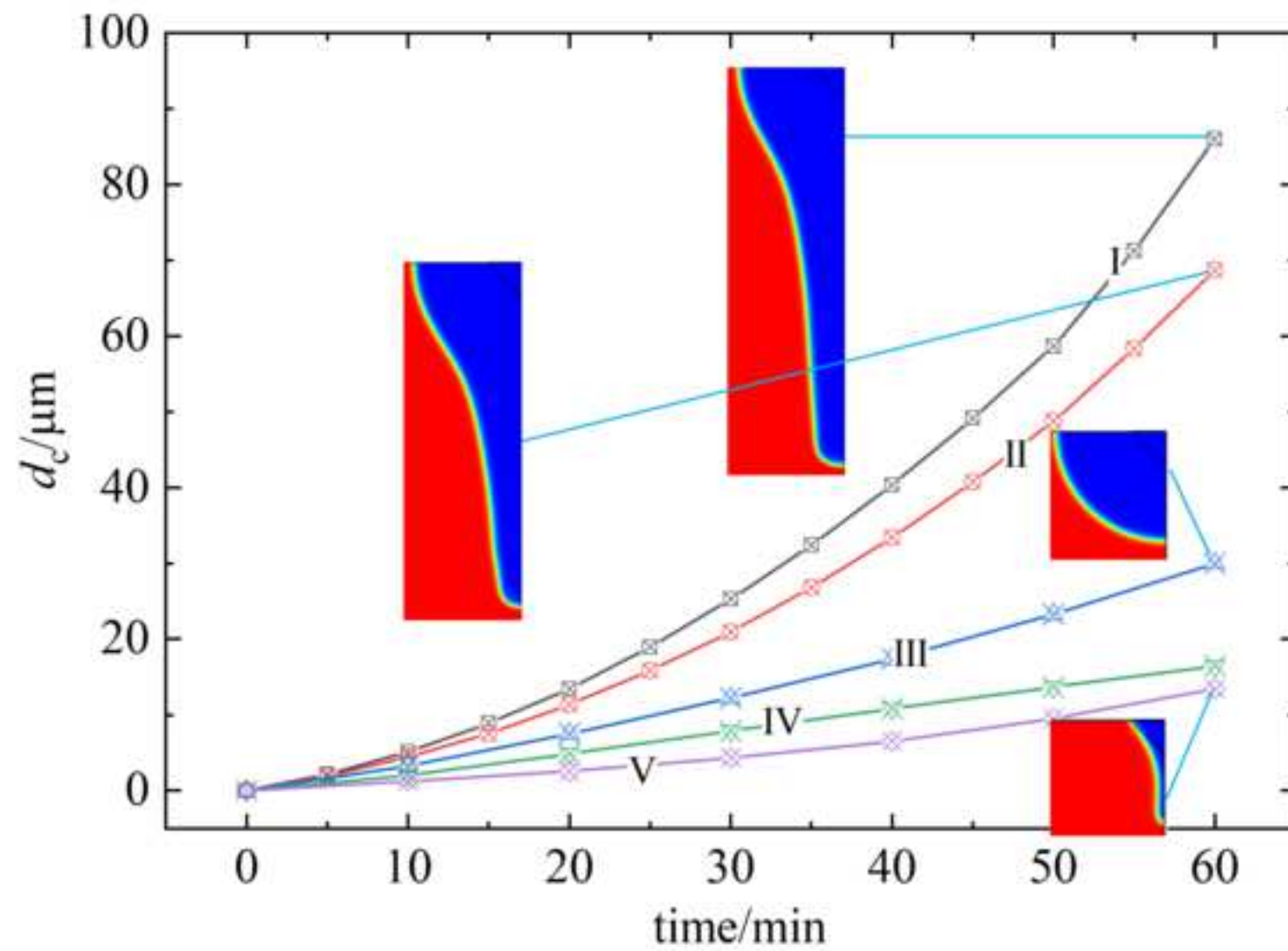


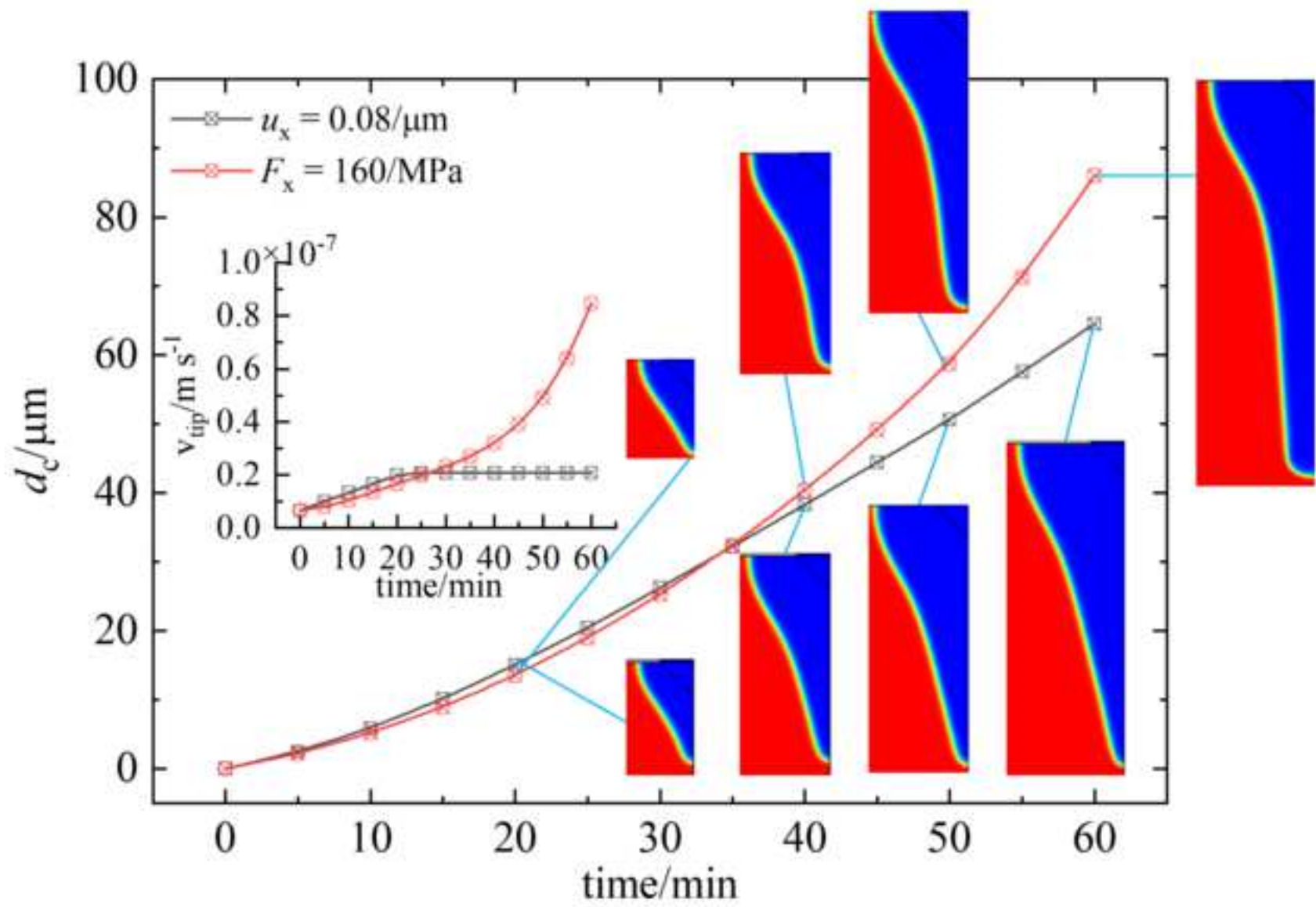


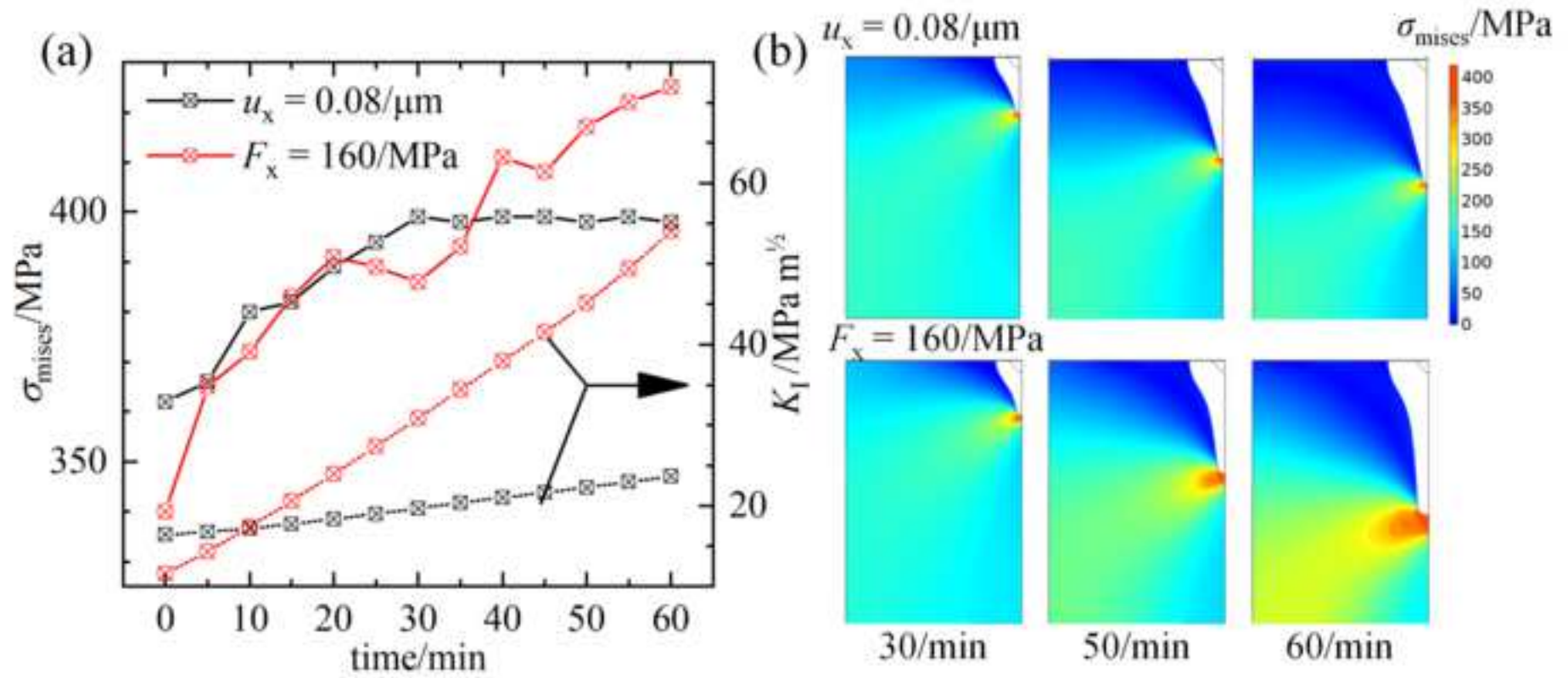


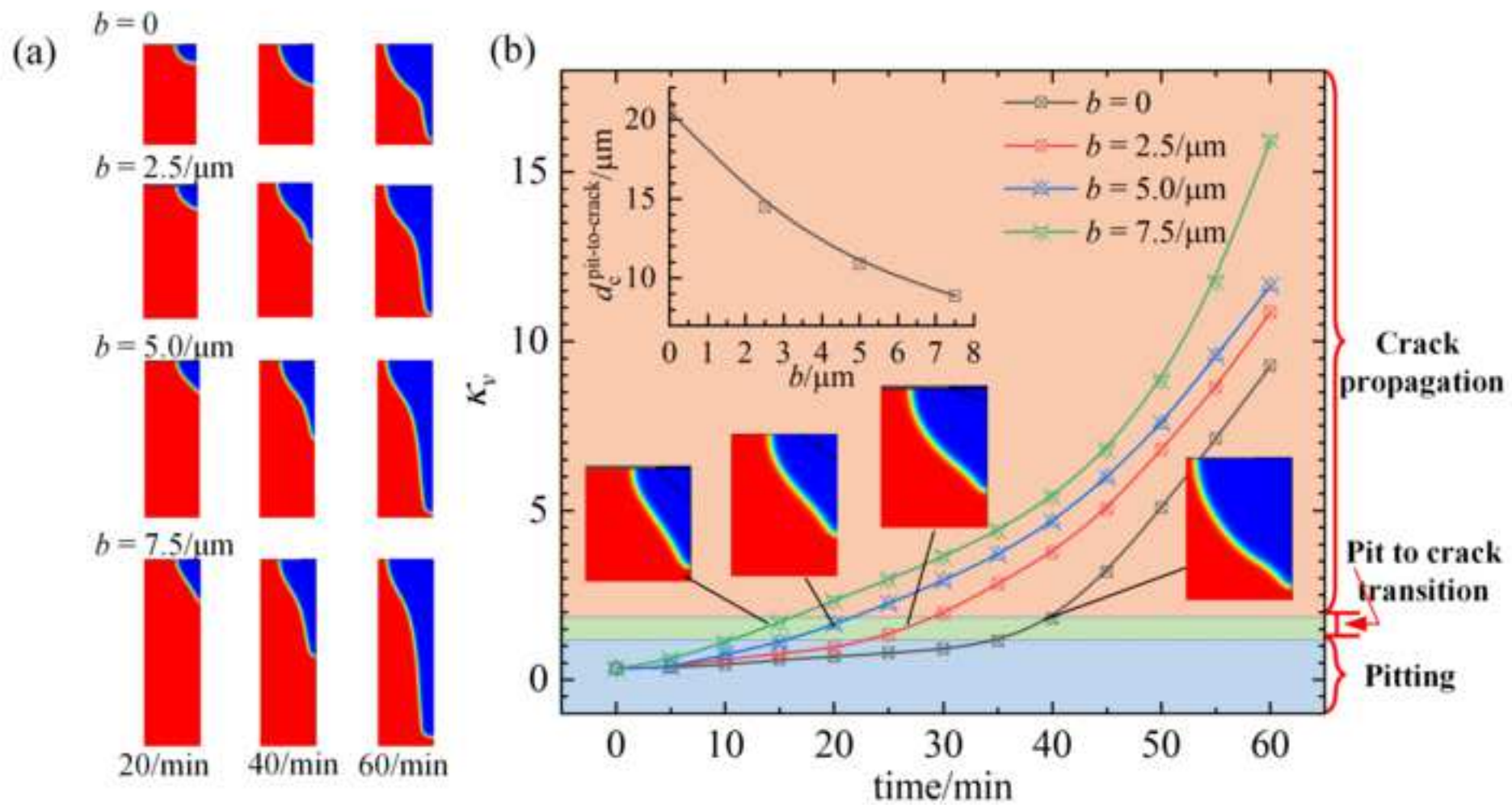


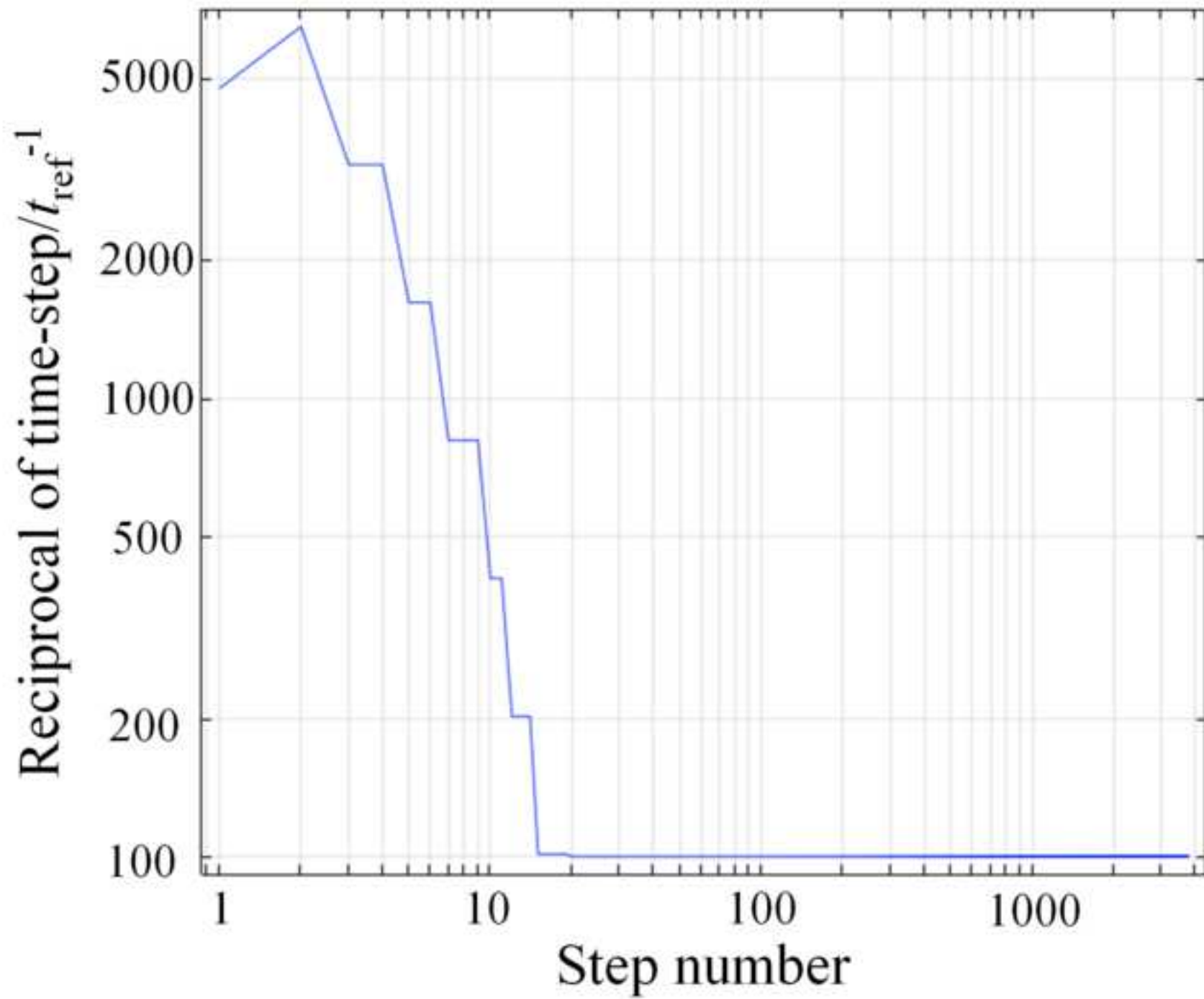












Highlights

- A mechano-chemical coupling model is proposed to study SCC
- The critical condition of pit-to-crack transition is unveiled
- The accelerated cracking due to the interplay of stress and corrosion is predicted
- The variation of electrochemical environment near a crack tip is predicted
- The effects of initial stress state and pit geometry are evaluated



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