https://dx.doi.org/10.1016/j.tre.2021.102465

Emergency Facility Location Problems in Logistics: Status and Perspectives

Wei Wang, Shining Wu, Shuaian Wang, Lu Zhen, Xiaobo Qu

Abstract

Emergencies that pose potential threats to our health, life, and properties can happen anywhere and anytime and may result in huge losses if they are not handled timely and effectively. An immediate response to emergencies is the key to mitigate these threats and losses. As the response time is largely dependent on the number and location of emergency facilities, the problem of how to determine the optimal number of emergency facilities and their best locations is of great strategic importance and of great interest to researchers. One of the most common approaches for researchers to address the emergency facility location problem is to model it as a discrete coverage-based emergency facility location problem. This paper provides a comprehensive overview of this problem, including mathematical models and their extensions and applications. In addition, the commonly used solution methods and some promising future research questions based on covering models are discussed.

Keywords: Emergency facility location, Emergency service, Covering problem, Mathematical modeling

1. Introduction

The term emergency refers to an unexpected and dangerous situation that poses risks to health, life, and properties and that must be dealt with immediately before it gets worse. Depending on the frequency and the scale, emergencies can range from incidents that affect a single person (such as some routine emergencies like heart attack, car crash, and residential fire) to events that affect a large group of people (such as natural disasters like floods, earthquake, hurricanes and human-caused disasters such as explosions and wars). For all kinds of emergency situations, saving life and properties as much as possible is always the most important goal of emergency rescue. How well such a goal can be attained is significantly influenced by the response time of rescue. While the response time of a routine emergency is largely and directly affected by the number and location of facilities in a strategic level, that of a large-scale emergency is affected also by many other factors such as coordination in a large network, the multiplicity of objectives of different parties, etc. In this paper, to keep our focus we mainly survey research on facility location problem for routine emergency.

Responses to routine emergencies usually involve police, fire services, and emergency medical services (EMS) departments (Zeng et al., 2021) and effective decisions require multi-disciplinary research efforts (Qu and Wang, 2021). When an emergency call comes, mobile servers (various types of vehicles) must depart from their bases and arrive at the emergency site as soon as possible or within certain time limits. The time interval between the reception of the call and the arrival at the incident is referred to as the response time, which, in a strategic level, is directly affected by the number and location of facilities. Not having enough number of facilities or placing them in wrong locations can significantly increase the response time, resulting in serious loss of life and properties. Thus, identifying the optimal number of emergency facilities and their best locations is the main topic of this research.

In general, the emergency facility location problem can be divided into continuous and discrete location problem. For the former, facilities can be located anywhere in a continuous region, which involves integral when formulating the problem. This is computationally challenging and it is further complicated when the non-convexity of regions has to be considered in the model. Because of the limit of computation techniques, the continuous location problem has not been extensively explored. With the development of computer processing ability and geographic information systems (GIS), this problem gradually gains attention (Love, 1972; Okabe and Suzuki, 1997; Murray et al., 2008; Matisziw and Murray, 2009; Yao and Murray, 2014; Berman et al., 2016; Blanquero et al., 2016; Fröhlich et al., 2020). For the discrete problem, facilities can be built only at a finite number of candidate locations. This could not only reduce the workload for data collection and processing, but also reduce the dimension of the problem. Therefore, the majority of location problems studied in the literature fall into the latter category. The discrete location problems can be divided into three groups: (i) p-center problem (also called 'min-max' problem), which aims to minimize the maximum service distance between candidate facilities and demand nodes (the traffic dynamics, e.g., Gu et al., 2020, Wu et al., 2020, Shi et al., 2021, are usually not considered); (ii) p-median problem (also called 'min-sum' problem), which aims to minimize the weighted total or average service distance. Both these two types of problems are first proposed by Hakimi (1964, 1965); (iii) covering problem, which seeks for optimal coverage of demand within response distance or time requirement. The covering problem is the most widely studied among the three groups and thus we focus on covering problem in this research. There are studies that use key performance indicators (KPIs) in different categories to evaluate the solutions obtained by discrete location models and interested readers can refer to Fadda et al. (2020, 2021).

This research serves three main purposes: first, to provide a comprehensive overview of the discrete coverage-based location models for routine emergency services as well as their extensions and applications; second, to outline the commonly used solution techniques; third, to propose some promising future research questions. Noting that there is already some review literature for discrete coverage-based location models, we first comment on these articles, pointing out the research gap, and then delineate the contributions of this research. ReVelle et al. (1970) analyses the similarities and differences between the private and public sector location models and discuss their applications and solution methods, respectively. ReVelle et al. (1977) first introduces simulation and queuing models used to address location and allocation problems in the background of EMS and then discusses three context-free models that could be applied to solve EMS problems, especially maximal covering location problem (MCLP) which could satisfy the demand of EMS policy makers. Daskin et al. (1988) reviews covering models considering the situation when the nearest vehicles are busy. ReVelle (1989) analyses several classical deterministic and probabilistic covering models for public facility location, proposes two models not covered by analyzed research, and predicts potential models that might be constructed in the future. Green and Kolesar (2004) provides comprehensive background information and history of the development of emergency response system since 1960s. It also analyses and summarizes all the relevant articles published in *Management Science* over the past few decades as well as identifies the potential challenges in implementing modeling and analysis to deal with emergency problems. Li et al. (2011) reviews covering problem for emergency response based on model types, which include classical models, their extensions, and prevailing models at that time, and introduces solving techniques. Simpson and Hancock (2017) describes the focuses of operational research in emergency response in different time periods and how they have changed over time. Bélanger et al. (2019) discusses recent advances in location, allocation, and vehicle dispatching in EMS. Most of the above articles are organized by "*models*", which provide a clear reference for researchers to learn the distinctions between different models, while our paper is organized around "*problems*". We review how new problems in practice leads to new models and how the consideration of new features in existing problems results in extensions of models. We use this problem-driven structure in hope of providing a review by which researchers can better know the evolution (and connection) of research in the topic and how the need of solving practical problems drives research development.

Another contribution is that we summarize the commonly used solution methods for covering problem. The third contribution is that we propose several valuable promising future research questions that have been widely accepted as important and have been receiving increasing attention, but remain open questions until now. We have to mention that although this research focuses on routine emergencies, the models and solution methods discussed could also be applied to other topics, such as the location of charging stations for electric vehicles under deterministic or uncertain demand (Frade et al., 2011; Wang and Lin, 2013; Faridimehr et al., 2018; MirHassani et al., 2020).

At the end of the section, we briefly discuss the differences in planning locational decisions when emergencies are large scale and introduce some related research. Large-scale emergencies, because of the low frequency and the tremendous resources demand in a short time, have distinctive characteristics that cannot be satisfactorily handled by the network of routine emergencies. For example, the suddenly overwhelm resources demand results in a more complicated network for large-scale emergencies, which have to simultaneously consider pre- and post-disaster processes. As a result, a multi-level network, including distribution centers used for pre-stocking and post-distributing relief resources, shelters used as transit stations for resource distribution and victim evacuation, medical centers, and emergency sites, have to be designed (Huang et al., 2021). In contrast, routine emergencies, which are usually habitual requests, can be handled with a simpler network with less layers. In addition, the attributes associated with each demand point in the large-scale emergency are distinct, thus the definition for coverage involves different facility quantity and quality requirements for each demand point. This is just to name a few. Because of the different characteristics of these two types of emergencies and large number of studies on each type, they can be reviewed separately. Readers interested in large-scale emergencies can refer to Liu et al. (2019b), Zhang et al. (2019, 2021), Li et al. (2020), Zhong et al. (2020), Azizi et al. (2021), Monemi et al. (2021), Shu et al. (2021), Uichanco (2021), and Wang et al. (2021a, 2021b).

The remainder of this paper is organized as follows. In Section 2, we introduce the literature search strategy and result. In Section 3, we thoroughly explain the deterministic problems and their extensions and applications. The probabilistic and stochastic counterparts are illustrated in Section 4. In Section 5, we briefly review commonly used solution techniques for covering problem. The promising future research questions for covering problem are outlined in Section 6. Section 7 concludes this review. The commonly used abbreviations are listed in Table 1.

Table 1. Commonly used abbreviation

Abbreviation	Meaning
ALS	advanced life support
BLS	basic life support
DCM	double coverage model
DRO	distributionally robust optimization
EMS	emergency medical services
GIS	geographic information systems
HOSC	hierarchical objective set covering model
LSCP	location set covering problem
MALP	maximum availability location problem
MCLP	maximal covering location problem
MEXCLP	maximum expected covering location problem
RO	robust optimization
SP	stochastic programming

2. Survey Scope and Method

We used a computerized search. To avoid bias in the coverage, we searched three databases: Scopus, Google Scholar, and Science Citation Index. For each database, we searched for studies based on search keys related to emergency facility location, such as emergency (service) facility location, EMS location, ambulance location, fire station location, etc., and model type, such as LSCP, MCLP, queuing, SP, RO, DRO, etc., in title, abstract, and keywords. After initial search, we obtained literature that mainly explores discrete coverage-based models for emergency facility location. Then, we retrieved the studies cited in these papers and the studies citing them. In addition to the above-mentioned criteria, we used three other criteria. First, we confined the review to work published in English in refereed journals or edited volumes, except three conference proceedings that include studies having fundamental and significant influences on emergency facility location problem. Second, we excluded the papers that do not involve modeling approaches as this paper mainly focuses on mathematical models for emergency facility location. Third, we also dropped literature that advances in the aspect of solution algorithm but leaves the model unchanged as algorithm is not the main focus in this article. Finally, we found 87 related papers, of which 60 papers discuss classic coverage-based models and their extensions and applications, and the other 27 papers that consider richer realistic features and constraints are used for future research questions for covering problem. The number of publications is summed up every ten years and the results are shown in Table 2. We can see from this table that the number of publications is relatively stable. The largest number of fundamental studies were done during 1980s, as at that time, the success of the first manned lunar landing encouraged the implementation of computer models and mathematical analysis to tackle fundamental social problems. For example, the fire department, police department, and health services in New York City were keen to collaborate with research institutions and universities to tackle daily problems and find out the way to make daily operation more efficient (Green and Kolesar, 2004). Thus, a lot of research on location of fire stations, police stations and ambulances came out during that time. Another feature is that the research problems and models become more complicated over time, considering more realistic features and constraints, as the advances in computing techniques make the previously complex problems easier to solve. In the following, the development of covering problem of routine emergencies will be thoroughly discussed.

Table 2. The number of publications

	Deterministic Location Problem			Probabilistic and Stochastic Location Problem					Future	Total
	Resource Minimization	Coverage Maximization	Multi-performance	Probabilistic Optimization	Queuing Theory	Chance Constraint	Stochastic Programming	Robust Optimization	Research Questions	
1970s	4	2			2				2	10
1980s	1	4	3	8	3					19
1990s		2		2	3				3	10
2000s		5		1	3	1	1		5	16
2010s		2		3	1	2	2	2	13	25
2020s							2	1	4	7
Total	5	15	3	14	12	3	5	3	27	

3. Deterministic Location Problem

The emergency facility location problem has been studied since 1970s. In the very beginning when not much research had been conducted, it is urgent for people to understand the first-moment performance (e.g., average resources, coverage, etc.) of a system. Therefore, various deterministic models based on simple settings were developed at that time. According to the performance measures, the problem could be divided into three categories: resource minimization, coverage maximization, and multi-performance problem. In the following subsections, these three types will be explained in detail.

3.1. Resource Minimization Problem

Table 3. A brief overview of research on resource minimization problem

Model	Objectives	Decisions	Solution techniques	References
LSCP	Minimize the number of fire stations	Location	Linear Programming with Cut Constraint	Toregas et al. (1971)
LSCP	Maximize the number of tower ladder	Location	Heuristic Algorithm	Walker (1974)
LSCP	Minimize the number of fire stations Maximize the number of existing stations	Location	Cutting Plane Integer Programming	Plane and Hendrick (1977)
Four-stage model	Minimize demands uncovered	Location Assignment	Heuristic Algorithm	Kolesar and Walker (1974)
LSCP	Minimize the number of fire stations	Location Assignment	Lagrangian Relaxation Boolean Method	Schreuder (1981)

Note: (i) Location means whether to locate the station in the potential facility site and the decision variable can only be 0 or 1. (ii) Assignment means which site should the vehicle be assigned and the decision variable usually is 0 or 1.

Table 4. Notations for resource minimization problem

Notations	Explanation of notations
Sets	
I	the set of demand nodes (indexed by i)
J	the set of candidate facility sites (indexed by j)
N_i	the coverage set of demand node i, i.e., the set of facility sites that can cover demand node i ($N_i = \{j \in J: d_{ij} \leq S\}$)
Deterministic parameters	
d_{ij}	the shortest distance between demand node i and facility site j
S	the response distance limit for coverage, e.g., a demand node i is covered if and only if $d_{ij} \leq S$
Decision variables	
x_j	1 if a facility is located at site j, 0 otherwise

The resource minimization problem mainly focuses on satisfying demands with the minimum number of resources. All the references and notations are briefly summarized in Table 3 and Table 4, respectively. The very first study in this area is done by Toregas et al. (1971), which proposes location set covering problem (LSCP). It identifies the minimum number of fire stations under the constraints that all demands have to be covered. The LSCP model is as follows:

$$\min \sum_{j \in J} x_j \tag{1}$$

subject to

$$\sum_{j \in N_i} x_j \geq 1, \quad \forall i \in I \tag{2}$$

$$x_j \in \{0,1\}, \quad \forall j \in J. \tag{3}$$

The objective (1) is to find the minimum number of total facilities required. Constraints (2) require that each demand is covered by at least one facility within distance requirement. Constraints (3) define the domains of x_j. Walker (1974) deals with the situation where a number of existing conventional aerial ladders in the fire stations have to be replaced by new tower ladders. This replacement problem is formulated as an LSCP, aiming at minimizing the number of fire stations needed to be filled with a tower ladder when each pair of neighboring fire stations has to be assigned one tower ladder, and then solved in a very little computation time by a heuristic method that contains three reduction procedures reducing the problem size. Plane and Hendrick (1977) helps fire department to reduce the number of fire stations while maintaining current service level, which is done in two steps: first, to identify the minimum number of fire stations that cover all focal points by closing mis-located stations and opening new ones; second, to keep as many existing stations as possible. A hierarchical objective set covering model with a weight factor to differentiate between the new stations and the existing ones is formulated with the objective function shown as follows:

$$\min \sum_{j=1}^{l} x_j + \sum_{j=l+1}^{n} (1+w) x_j \tag{4}$$

where $w \in [0, 1/n]$ and l represents the number of exiting fire stations. The number of fire stations satisfying coverage requirement is first calculated and then the solution with maximum number of existing stations is chosen because of establishing costs of new fire companies. This model, coupled with empirical judgement from fire chief and officers, saves fire service company 2.8 million dollar over a 6-year period. Kolesar and Walker (1974) helps design response network for fire department, using LSCP to minimize the relocation of fire ladders and engines separately from outside the region when all three closest engines or two closest ladders to a fire alarm box are busy fighting other fires. Schreuder (1981) determines the minimum number of fire stations for fire department in Rotterdam. The region is divided into two categories: I_1 denotes a set of districts covered only once within standard response time, I_2 the districts where double coverage is required. The objective is to establish the minimum number of fire stations that could satisfy the coverage requirements.

3.2. Coverage Maximization Problem

Table 5. A brief overview of research on coverage maximization problem

Model	Objectives	Decisions	Solution techniques	References
MCLP	Maximize coverage	Coverage	Greedy Adding Greedy Adding with Substitution Linear Programming Branch and Bound	Church and ReVelle (1974)
Weighted MCLP	Maximize population and property coverage	Location Coverage	-	Schilling et al. (1980)
Weighted LSCP MCLP	Minimize the number of fire stations Maximize coverage	Location	ArcGIS CPLEX	Aktaş et al. (2013)
FLEET	Maximize coverage by two types of equipment	Location Coverage	Linear Programming	Schilling et al. (1979)
Weighted MCLP	Maximize the weighted sum of eight demand surrogates	Location Coverage	Linear Programming	Eaton et al. (1985)
Extension of MCLP	Maximize once coverage and multiple coverage	Location Once coverage Additional coverage	Goal Programming	Storbeck (1982)
DCM	Maximize the demand covered by two ambulances within smaller radius	Location Once coverage Double coverage	Tabu Search	Gendreau et al. (1997)
Extension of DCM	Minimize total cost	Location Once coverage Double coverage	Ant Colony Optimization	Su et al. (2015)
Generalized MCLP	Maximizes total weighted coverage	Location Coverage	Branch and Bound	Church and Roberts (1983)
Generalized MCLP	Maximizes total weighted coverage	Location Coverage	Greedy Heuristic	Berman and Krass (2002)
Capacitated MCLP	Maximizes total weighted coverage	Location Coverage	Lagrangian Relaxation	Pirkul and Schilling (1991)
Gradual covering problem	Minimize noncoverage cost related to distance	Location Coverage	Branch and Bound	Drezner et al. (2004)
Generalized MCLP	Maximizes total weighted coverage	Location Coverage	Greedy Heuristic Linear Programming Branch and Bound	Berman et al. (2003)
MCLP	Maximizes the coverage level within the maximum critical distance	Location Coverage	Lagrangian Relaxation	Karasakal and Karasakal (2004)
Cooperative LSCP Cooperative	Minimize the number of facilities Maximizes total weighted	Location	Big-Triangle Small-Triangle global optimization technique	Berman et al. (2009)

MCLP	coverage

Note: Coverage means whether the demand node is covered within the required response standards.

Table 6. Notations for coverage maximization problem

Notations	Explanation of notations
Sets	
N_i^p	the coverage set of demand node i for primary equipment
N_i^s	the coverage set of demand node i for special equipment
J_N	the set of potential new facility locations
N_i^s	$N_i^s = \{j \in J: d_{ij} \leq s\}$
N_i^t	$N_i^t = \{j \in J: s < d_{ij} \leq t\}$
N_i^r	$N_i^r = \{j \in J: t < d_{ij} \leq r\}$
Deterministic parameters	
a_i	the population to be served at demand node i
p	the number of facilities to be located
p^p	the number of primary equipment units available
p^s	the number of special equipment units available
p^z	the number of new facilities to be built
M	a non-Archimedean weight
η	the proportion of the total demand that must be covered by an ambulance within r_1
γ_{ij}	1 if demand node i is covered by j within the small radius r_1, 0 otherwise
δ_{ij}	1 if demand node i is covered by j within the large radius r_2, 0 otherwise
$r_1(r_2)$	two response distance limits for coverage where $r_1 < r_2$
$s/t/r$	the response distance limit for gradual coverage, where $s \leq t \leq r$
w^s	the weight attached to the coverage between distance $0 – s$
w^t	the weight attached to the coverage between distance $s – t$
w^r	the weight attached to the coverage between distance $t – r$
Decision variables	
x_j^p	1 if primary equipment is located at node j, 0 otherwise
x_j^s	1 if special equipment is located at node j, 0 otherwise
y_i	1 if a demand point i is covered, 0 otherwise
y_i^+	the number of additional coverage for demand node i within S
y_i^-	1 if demand node i is not covered within distance S, 0 otherwise
y_i^k	1 if demand node i is covered k $(k = 1$ or $2)$ times within the small radius r_1, 0 otherwise
y_i^s	1 if a demand point i is covered at a distance between $0 – s$, 0 otherwise
y_i^t	1 if a demand point i is covered at a distance between $s – t$, 0 otherwise
y_i^r	1 if a demand point i is covered at a distance between $t – r$, 0 otherwise

Coverage maximization problem aims at maximizing the people, property, or other surrogate measures covered under the limited resources. All the references and additional notations are briefly summarized in Table 5 and Table 6, respectively. The pioneering research in this area is Church and ReVelle (1974), which proposes maximal covering location problem (MCLP) as follows:

$$\max \sum_{i \in I} a_i y_i \tag{5}$$

subject to

$$\sum_{j \in N_i} x_j \geq y_i, \quad \forall i \in I \tag{6}$$

$$\sum_{j \in N_i} x_j = p \tag{7}$$

$$x_j \in \{0,1\}, \quad \forall i \in I \tag{8}$$

$$y_i \in \{0,1\}, \quad \forall j \in J. \tag{9}$$

The objective (5) maximizes the population covered. Constraints (6) state that demand node i is covered only if there is at least one facility in the coverage set N_i. Constraint (7) sets the limitation on the total number of facilities that can be built. Constraints (8) and (9) define the domains of x_j and y_i. This problem is solved by heuristic methods and linear programming relaxation (i.e., relax the zero-one restriction on x_j and y_i and only require nonnegativity on two variables), the fractional results of which are tackled by branch and bound. Schilling et al. (1980) replaces population in MCLP with other surrogate measures when dealing with real-world decision-making situation. Aktaş et al. (2013) helps Istanbul Metropolitan Municipality in configuring fire stations to respond to residences and historic sites as quickly as possible. It formulates 10 what-if scenarios, such as constructing new fire stations on the basis of the existing ones, building all fire stations from scratch, or configurating fire stations under budget restriction. These scenarios are calculated in CPLEX and the solution finally chosen increases the coverage by more than 27%.

The above problems only require single coverage. There are situations that multiple vehicle types are required or the closest emergency resource is unavailable. To address these problems, multiple coverage is proposed. Schilling et al. (1979) is the first to consider two different types of equipment (i.e., pumper and ladder vehicles) when design fire protection system. The definition of coverage changes a little bit under this situation. Demand is covered only when it is simultaneously within distance standard of two types of equipment, resulting in a model called FLEET, which deals with a general problem setting where a fixed number of facilities with limited capacities for two equipment types are located to maximize the demand covered by both types within their respective distance. The FLEET model is given by:

$$\max \sum_{i \in I} a_i y_i \tag{10}$$

subject to

$$\sum_{j \in N_i^p} x_j^p \geq y_i, \quad \forall i \in I \tag{11}$$

$$\sum_{j \in N_i^s} x_j^s \geq y_i, \quad \forall i \in I \tag{12}$$

$$\sum_{j \in J} x_j^p = p^p \tag{13}$$

$$\sum_{j \in J} x_j^s = p^s \tag{14}$$

$$\sum_{j \in J_N} x_j = p^z \tag{15}$$

$$x_j^p \leq x_j, \quad \forall j \in J_N \tag{16}$$

$$x_j^s \leq x_j, \quad \forall j \in J_N \tag{17}$$

$$x_j^p, x_j^s \in \{0,1\}, \quad \forall j \in J \tag{18}$$

$$y_i \in \{0,1\}, \quad \forall i \in I \tag{19}$$

$$x_j \in \{0,1\}, \quad \forall j \in J_N. \tag{20}$$

Constraints (11) and (12) require that the demand should be covered. Limitations on the number of equipment and new facilities are stated by constraints (13)–(15). Constraints (16) and (17) guarantee the placement of equipment at nodes where facility is located. Eaton et al. (1985) designs a two-tiered (advanced and basic life support) EMS response system to maximize the total weighted coverage of eight demand surrogates (i.e., critical, noncritical and total calls, total population, Black, Hispanic, and Anglo population, and elderly citizens). Storbeck (1982) uses goal programming method to formulate multiple coverage within the framework of maximal coverage as follows:

$$\min \sum_{i \in I} (-y_i^+ + M a_i y_i^-) \tag{21}$$

subject to

$$\sum_{j \in N_i} x_j - y_i^+ + y_i^- = 1, \quad \forall i \in I \tag{22}$$

$$\sum_{j \in J} x_j = p \tag{23}$$

$$x_j, y_i^- \in \{0,1\}, \quad \forall i \in I, j \in J \tag{24}$$

$$y_i^+ \text{ integer}, \ \forall i \in I. \tag{25}$$

Objective (21) is to optimize the problem in two-step process: it first minimizes uncovered demand and then maximizes multiple coverage. Constraints (22) put both possible deviations above (additional coverage) and under goal (first coverage) into the coverage expression of each demand node. Gendreau et al. (1997) proposes double coverage model (DCM) where all demands are required to be covered within the larger radius r_2, which ensures that there is at least one ambulance located within r_2 of each demand, while a proportion, denoted by η, of demands are covered within the smaller radius r_1. The objective is to maximize the demand doubly covered within the shorter distance r_1 under the limited number of ambulances. The model can be formulated as:

$$\max \sum_{i=1}^{n} a_i y_i^2 \tag{26}$$

subject to

$$\sum_{j=1}^{m} \delta_{ij} y_j \geq 1, \quad \forall i \in I \tag{27}$$

$$\sum_{i=1}^{n} a_i y_i^1 \geq \eta \sum_{i=1}^{n} a_i \tag{28}$$

$$\sum_{j=1}^{m} \gamma_{ij} y_j \geq y_i^1 + y_i^2, \quad \forall i \in I \tag{29}$$

$$y_i^2 \leq y_i^1 \tag{30}$$

$$\sum_{j=1}^{m} y_j = p \tag{31}$$

$$y_j \leq p_j, \quad \forall j \in J \tag{32}$$

$$y_i^1, y_i^2 \in \{0,1\}, \quad \forall i \in I \tag{33}$$

$$y_j \text{ integer}, \ \forall j \in J. \tag{34}$$

In this model, objective (26) is to maximize total demand doubly covered within r_1. Constraints (27) require a mandatory coverage within r_2. Constraint (28) requires that at least a proportion of α demand is covered at least once within r_1. Constraints (29) state that the number of times the demand is covered within r_1 cannot exceed the total number of ambulances located within the same distance. Constraint (30) requires that demand has to be at least covered once before it is doubly covered. Constraint (31) limits total number of ambulances and Constraints (32) limit the number of ambulances at each facility site. Constraints (33) and (34) set domains of y_i^k and y_j respectively.

Su et al. (2015) extends DCM by replacing the maximal coverage with minimal total cost in the objective function. Total cost is composed of expected cost for delayed emergency service and operational cost for ambulances and stations. The expected delay cost is the product of delay time and average delay cost per minute. Delay time is calculated according to the following rule: (i) demand node doubly covered within shorter distance r_1 can be viewed as fully covered and thus does not incur a delay cost; (ii) since DCM requires a mandatory coverage within a larger distance r_2, even if the ambulance within r_1 fails to respond to demand immediately, the demand can at least be covered by ambulance within r_2, which means the expected delay time could be approximated as the difference between response time of r_2 and r_1 (i.e., $t_{r_2} - t_{r_1}$). If the demand node is covered only once within r_1, a probability p_{out} that the nearest ambulance is out should be considered in delay time calculation (i.e., $p_{out}(t_{r_2} - t_{r_1})$). (iii) if demand node is not covered within r_1, the expected delay time is $t_{r_2} - t_{r_1}$. A trade-off between delay cost and operational cost is considered when optimizing the location.

The coverage defined in single and multiple coverage models is binary, that is, a certain demand is either fully covered if there is at least one facility located within the coverage distance standard or not covered at all if the facility is located slightly further than requirement. This assumption may not be reasonable in some situations. Therefore, the gradual coverage models which formulate the coverage level by functions of distance come into being to relax the assumption. There are mainly three types of gradual coverage models based on the coverage function: the first one is the stepwise function of coverage; the second one is the decreasing linear function of the distance; the third one is neither convex nor concave function (other than stepwise function). The first research to consider the stepwise function is Church and Roberts (1983) which models the decay of the coverage over distance with a stepwise function and replaces "all or nothing" coverage treatment in original MCLP model with this new coverage function. The model is given as follows:

$$\max \sum_{i\in I} (w^s a_i y_i^s + w^t a_i y_i^t + w^r a_i y_i^r) \quad (35)$$

subject to

$$\sum_{j\in N_i^s} x_j \geq y_i^s, \quad \forall i \in I \quad (36)$$

$$\sum_{j\in N_i^t} x_j \geq y_i^t, \quad \forall i \in I \quad (37)$$

$$\sum_{j\in N_i^r} x_j \geq y_i^r, \quad \forall i \in I \quad (38)$$

$$y_i^s + y_i^t + y_i^r \leq 1, \quad \forall i \in I \quad (39)$$

$$\sum_{j\in J} x_j = p \quad (40)$$

$$x_j \in \{0,1\}, \quad \forall j \in J \quad (41)$$

$$y_i^s, y_i^t, y_i^r \in \{0,1\}, \quad \forall i \in I \quad (42)$$

Objective (35) maximizes total weighted coverage, which is a function of distance. Constraints (36)–(38) define the coverage under different distances. Constraints (39) ensure that the demand is only covered by one distance range. Constraint (40) limits the total number of facilities. Berman and Krass (2002) follows the footstep of Church and Roberts (1983) and proposes generalized maximal cover location problem where a decreasing step function of the distance to the closest facility is used to describe the coverage level with each level corresponding to a coverage radius. Pirkul and Schilling (1991) is the early research that uses linear function to express coverage level. The purpose is to simultaneously maximize coverage and improve service level of uncovered nodes, which is measured by a linear function of the distances between facilities and uncovered demands: the smaller the value, the better the result. The linear function is also used by Drezner et al. (2004) to model the coverage when the distance is between the upper and lower bounds. When the distance is below the lower bounds or beyond the upper bound, the demand point is fully or not covered, respectively. Berman et al. (2003) and Karasakal and Karasakal (2004) introduce coverage decay functions which could be neither convex or concave between lower and upper distance standards. The coverage level of a demand node in the above models is assumed to depend only on its distance to the closet facility. Relaxing this assumption, Berman et al. (2009) assumes that every facility could provide a certain degree of coverage (the effect of which decays with distance) for a demand node. The coverage effects from all (nearby) locations could be aggregated. The coverage status of each demand node depends on whether the summation exceeds a certain threshold.

The idea of gradual coverage not only is applicable for emergency response system, but also has practical value for problems in other areas. Drezner and Drezner (2014) locates multiple cell phone towers to maximize the minimum cover of demand nodes. Karatas and Eriskin (2021) investigates the location, number, and size of undesirable facilities. Fadda et al. (2020) discusses multipath traveling salesman problem under the assumption that the path cost is an increasing function of the actual flow of traffic.

3.3. Multi-performance Problem

Table 7. A brief overview of research on multi-performance problem

Model	Objectives	Decisions	Solution techniques	References
HOSC	Minimize the number of ambulances Maximize the number of additional ambulances for multiple coverage	Location Number of additional ambulances	Linear Programming	Daskin and Stern (1981)
Weighted combination of LSCP and MCLP	Minimize the number of facilities Maximize multiple coverage	Location Number of additional ambulances	Heuristic Algorithm	Eaton et al. (1986)
BACOP1 BACOP2	Minimize the number of facilities Maximize backup coverage Maximize both once and double coverage	Location Once coverage Double coverage	Linear Programming	Hogan and ReVelle (1986)

Even though single-performance models can solve many problems, in some cases, multiple performance measures must be considered together. Therefore, multi-performance problem, which integrates several performance measures into a single model, comes into focus. All the references are briefly summarized in Table 7. Daskin and Stern (1981) proposes a hierarchical objective set covering model (HOSC) where it first identifies the minimum number of ambulances needed for a complete coverage (i.e., each demand zone is covered by at least one ambulance) and then selects from alternative optimum that maximizes the number of additional ambulances for multiple coverage of each zone. The model is formulated as follows:

$$\min W \sum_{j\in J} x_j - \sum_{i\in I} A_i \tag{43}$$

subject to

$$\sum_{j\in N_i} x_j - A_i \geq 1, \quad \forall i \in I \tag{44}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{45}$$

$$A_i \geq 0, \quad \forall i \in I. \tag{46}$$

where W is some positive weight and A_i is the number of additional ambulances that could respond to demand i within distance requirement. Eaton et al. (1986) extends

Daskin and Stern (1981) by multiplying demand to additional coverage when designing EMS system in urban area, which simultaneously minimizes the number of facilities and maximizes multiple demand coverage. Due to limitation of computational capacity, this paper develops a multi-objective heuristic to solve the problem. Hogan and ReVelle (1986) proposes two hierarchical models, BACOP1 and BACOP2, to model backup coverage in different settings. BACOP1 is a combination of LSCP and MCLP, which means mandatory coverage is still required. The model is given as follows:

$$\min \sum_{j \in J} x_j \tag{47}$$

$$\max \sum_{i \in I} a_i y_i^2 \tag{48}$$

subject to

$$\sum_{j \in E_i} x_j \geq 1, \quad \forall i \in I \tag{49}$$

$$\sum_{j \in N_i} x_j - y_i^2 \geq 1, \quad \forall i \in I \tag{50}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{51}$$

$$y_i^2 \in \{0,1\}, \quad \forall i \in I. \tag{52}$$

It uses two response distances, S and S' where $S > S'$. The full coverage is achieved in a stricter distance standard S', indicating higher priority for the first coverage, and then the maximum demand doubly covered within a larger distance S can be used to distinguish between a set of alternate optima. BACOP2 simultaneously optimizes first and backup coverage without a mandatory coverage requirement. An instance with 30 demand nodes is used to test the model and the results suggest that with the same number of facilities, more backup coverage can be achieved.

4. Probabilistic and Stochastic Location Problem

The above deterministic location models can obtain optimality or near-optimality in simplified assumptions of the real-life practices. As researchers have deeper understanding about the problems under these simple settings, more complicated problems with secondary features (e.g., covering probabilities, service level, etc.) are taken into account, leading to the development of models considering uncertainties, such as uncertain resource availability, demand, traffic condition, cost, etc. Among these uncertainties, availability and demand are the most frequently discussed and thus in this section, we mainly focus on these two types of uncertainties.

4.1. Location Problem with Uncertain Availability

To simplify the problem, most research assumes that closest emergency resource will always be available to emergency calls and will never be busy. However, in practice, we have the situation that resource is engaged in other calls at a time a new call arrives and this newly generated call has to be served by a more remoted resource or wait in queue. Probabilistic optimization and queuing theory are the most commonly used methods to address this problem. The former considers the assumption of independence between servers, while the latter relaxes this assumption.

4.1.1. Probabilistic Optimization

Probabilistic optimization usually defines event probability and puts it either in the objective function to obtain the expected value or in the constraints to satisfy the required reliability level. All the references and additional notations are briefly summarized in Table 8 and Table 9, respectively.

Table 8. A brief overview of research on probabilistic optimization

Model	Objectives	Decisions	Solution techniques	References
MEXCLP	Maximize expected coverage	Deployment Coverage	Linear Programming	Daskin (1982, 1983)
Extension of MEXCLP	Maximize expected coverage	Deployment Coverage	Linear Programming	Saydam and McKnew (1985)
MEXCLP	Formulate a new demand deployment policy	Deployment Coverage	Heuristic Algorithm Simulation	Fujiwara et al. (1987)
A combination of MEXCLP and FLEET	Minimize the number of people not covered within response standard	Location Deployment The number of vehicles needed for full coverage	LINDO	Bianchi and Church (1988)
PLSCP	Minimize the number of vehicles under the reliability requirement	Location	Linear programming with a branch-and-bound	ReVelle and Hogan (1988)
MALP	Maximize coverage with reliability	Location Coverage	Linear programming with a branch-and-bound	ReVelle and Hogan (1989a)
MALP α-reliable p-center	Maximize coverage with reliability Minimize the maximum time	Location Coverage	-	ReVelle and Hogan (1989b)
A combination of MALP and FLEET	Maximize coverage with reliability	Location (considering vehicle type) Coverage (considering vehicle type)	MPSX	ReVelle and Marianov (1991)
Probabilistic DSM	Maximize coverage with reliability within tighter response standard	Deployment (considering vehicle type) Coverage (considering vehicle type, response standards)	Genetic Algorithm	Liu et al. (2016)

A combination of MEXCLP and MALP	Maximize coverage with reliability	Location Coverage	CPLEX	Sorensen and Church (2010)
A combination of MEXCLP and MALP	Maximize expected coverage Minimize expected cost	Coverage Deployment	-	El Itani et al. (2019)
Integer program with reliability constraints	Minimize total cost	Deployment	Linear programming with a branch-and-bound	Ball and Lin (1993)
LSCP with reliability constraints	Minimize the number of vehicles	Deployment	Simulation	Borrás and Pastor (2002)

Note: (i) Allocation means whether the demand point is allocated to this facility and the decision variable usually is binary. (ii) Deployment means how many vehicles are deployed at each facility to serve the assigned demands.

Table 9. Additional notations for probabilistic optimization

Notations	Explanation of notations
Sets	
M_i	the set of demand nodes whose distances to demand node i do not exceed S ($M_i = \{k \in I: d_{ki} \leq S\}$)
B_i	the set of demand nodes whose distances to facility site j do not exceed S ($B_i = \{i: d_{ij} \leq S, \forall j \in N_i\}$)
Deterministic parameters	
q	the busy fraction of each server, i.e., the probability that the facility is not working
n_i	the number of facility sites in N_i
W_{jk}	the cost of housing k vehicles at station j
$D(j)$	the number of calls that has to be served by station j
α	service reliability level
L	the maximum number of vehicles available for any station
Decision variables	
x_{jk}	1 if there are k vehicles at station j
y_i^k	1 if a demand point i is covered by at least k facilities, 0 otherwise
z_j	the number of facilities located at j
m_j	the number of vehicles the station j holds

The pioneering works that consider probability in the objective function are Daskin (1982, 1983), which adopt the concept of system-wide busy fraction, i.e., the probability that the server cannot respond to the demand as it is serving other demand, to original MCLP to maximize the expected number of demands covered and this model is called the maximum expected covering location problem (MEXCLP). The model can be presented as:

$$\max \sum_{i \in I} \sum_{k=1}^{n_i} (1-q) q^{k-1} a_i y_i^k \tag{53}$$

subject to

$$\sum_{k=1}^{n_i} y_i^k \leq \sum_{j \in N_i} z_j, \quad \forall i \in I \tag{54}$$

$$\sum_{j \in J} z_j \leq p \tag{55}$$

$$y_i^k \in \{0,1\}, \quad \forall i \in I, k \in \{1,...,n_i\} \tag{56}$$

$$z_j \text{ integer}, \quad \forall j \in J. \tag{57}$$

Constraints (54) guarantee that the sum of coverage cannot exceed the number of facilities located in N_i. Constraint (55) restricts that facilities to be built cannot exceed p. Constraints (56) and (57) define the domains of y_i^k and z_j. This large integer programming problem is solved by a heuristic algorithm called single node substitutions. To solve this problem to optimality, Saydam and McKnew (1985) rewrite MEXCLP to a separable programming problem as follows:

$$\max \sum_{i \in I} (1 - \sum_{v=1}^{p+1} q^{(v-1)} w_{iv}) a_i \tag{58}$$

subject to

$$\sum_{j \in J} z_j \leq p \tag{59}$$

$$\sum_{j \in N_i} z_j - \sum_{v=1}^{p+1} (v-1) w_{iv} = 0, \quad \forall i \in I \tag{60}$$

$$\sum_{v=1}^{p+1} w_{iv} = 1, \quad \forall i \in I \tag{61}$$

$$w_{iv} \in \{0,1\}, \quad \forall i \in I, v \in \{1,2,...,p+1\} \tag{62}$$

$$z_j \text{ integer}, \quad \forall j \in J. \tag{63}$$

where $\sum_{v=1}^{p+1} (v-1) w_{iv}$ represents the number of facilities in the coverage set of demand node i. Constraints (61) and (62) state that only one weighting factor w_{iv} equals 1 for each demand node i and the rest is 0. This new model guarantees an optimal solution and generates solutions fast even in large size problems. More importantly, this model is flexible to be extended to generate optimal solutions under different requirements. For example, by adjusting the lower bound of v, i.e., $\sum_{v=c+1}^{p+1} w_{iv} = 1$, this model could guarantee that at least c facilities could cover each demand node. Fujiwara et al. (1987) applies MEXCLP to explore the best way to deploy ambulances in Bangkok considering demand increasing after advertising for free ambulance services. The deployment policies generated are put into a simulation model, the results of which show that the recommended plan can save 29% of the ambulance needed while maintaining the same service level. Bianchi and Church (1988) combines MEXCLP with FLEET to simultaneously locate stations and allocate ambulances. The results of a numerical example show that the coverage level could still remain high even with a 40% reduction in facilities.

Another type of probabilistic optimization puts probability in the constraints, resulting in a problem called chance constrained facility location problem. Chapman and White (1974) defines a system-wide busy fraction q and puts it in the chance constraints to calculate the lower bound for the number of servers needed to reach a certain service level. Chance constraint is the requirement that the probability of the demand being responded by at least one server is no less than a reliability level α, which can be expressed as:

$$1 - q^{\sum_{j \in N_i} x_j} \geq \alpha \tag{64}$$

After transformation, it can be written as:

$$\sum_{j \in N_i} x_j \geq b \tag{65}$$

where $b = \lceil \frac{\log(1-\alpha)}{\log q} \rceil$, $\lceil a \rceil$ denotes the smallest integer greater than or equal to a. This constraint is incorporated into LSCP to get the minimum number of emergency vehicles. ReVelle and Hogan (1988) extends the busy fraction to be area-specific, which means the busy fraction of servers is identical in the same demand area, but could vary from area to area. It can be expressed as:

$$q_i = \frac{\bar{t} \sum_{k \in M_i} a_k}{24 \sum_{j \in N_i} x_j}, \quad \forall i \in I \tag{66}$$

where M_i represents the set of demand nodes whose distances to demand node i do not exceed S ($M_i = \{k \in I : d_{ki} \leq S\}$); a_k is the number of calls per day originated from demand node k; $\bar{t}$ is the average duration of a single call (in hours). The numerator and denominator represent the total daily service time needed and available in the area around demand node i, respectively. The chance constraints can be rewritten as:

$$\sum_{j \in N_i} x_j \geq b_i, \quad \forall i \in I \tag{67}$$

where b_i is the smallest integer satisfying:

$$1 - (\frac{F_i}{b_i})^{b_i} \geq \alpha, \quad \forall i \in I \tag{68}$$

where $F_i = \frac{\bar{t} \sum_{k \in M_i} a_k}{24}$. ReVelle and Hogan (1988) incorporates the chance constraints into LSCP to get the minimum number of vehicles under the reliability requirement, resulting in a model called probabilistic location set covering problem (PLSCP). ReVelle and Hogan (1989a, 1989b) incorporate both system-wide and area-specific busy fraction into MCLP and propose maximum availability location problem (MALP). Two models, MALP I and MALP II, which have the same objective to maximize the population covered with α reliability, but use different types of busy fraction are proposed. MALP I utilizes a system-wide busy fraction:

$$q = \frac{\bar{t} \sum_{i \in I} a_i}{24 \sum_{j \in J} x_j} \tag{69}$$

where $\bar{t}$ is the average duration of a call. The numerator and denominator represent the total daily service time needed and available, respectively. MALP II uses an area-specific busy fraction, the same as Eq. (66). The chance constraint for system-wide and area-specific busy fraction is the same as expression (65) and (67) respectively. The general formulation for both MALP I and MALP II can be expressed as:

$$\max \sum_{i \in I} a_i y_i^B \tag{70}$$

subject to

$$\sum_{k=1}^{B} y_i^k \leq \sum_{j \in N_i} x_j, \quad \forall i \in I \tag{71}$$

$$y_i^k \leq y_i^{k-1}, \quad \forall i \in I, k \in (2, \ldots, B) \tag{72}$$

$$\sum_{j \in J} x_j = p \tag{73}$$

where B can be replaced by b and b_i, respectively. y_i^B equals 1 if demand node i is covered by B servers with reliability α. y_i^k is 1 if demand node i is covered k times within distance standard. The second constraints require sequence of coverage. When applying MALP to fire department where more than one type of equipment is needed, ReVelle and Marianov (1991) extends the model to consider two types of equipment, which formulates a combined model of MALP and FLEET by introducing busy fraction and chance constraint for each equipment separately. Both MALP and this combined model only maximize the coverage of demand nodes with sufficient number of servers available and the coverage that does not meet reliability requirement is not counted in the objective function, which leads to some demand nodes uncovered. This drawback is addressed by Liu et al. (2016) which combines local reliability with double standard model. Like DSM, two distance standards are considered: a tighter radius (or a primary distance standard) r_1 for a proportion of coverage that reaches guaranteed service reliability and a looser radius (or a secondary distance standard) r_2 for full coverage. This extended model maximizes coverage of demand at guaranteed service reliability in a primary distance standard and at the same time ensures a full coverage in a secondary distance standard. Sorensen and Church (2010) combines the expression of local reliability of MALP and maximum expected coverage of MEXCLP in one model and compares this new model with the two original models in a range of test problems and simulation model. The results show that the incorporation of local reliability into MEXCLP can improve the total coverage and MEXCLP is more suitable than MALP when the goal is to maximize total coverage instead of the coverage above a required service level. El Itani et al. (2019) also considers the combination of MEXCLP and MALP and proposes a bi-objective model to simultaneously maximize expected coverage and minimize expected cost when paying for the external ambulances is allowed. Ball and Lin (1993) extends the busy fraction to a more realistic site-specific one, which means the busy fraction could be different for each server. This research aims to minimize the total cost incurred when holding vehicles at stations, which can be expressed as:

$$\min \sum_{j \in J} \sum_{k=1}^{L} W_{jk} x_{jk} \tag{74}$$

subject to

$$\prod_{j \in N_i} \prod_{k=1}^{L} P(D(j) \geq k)^{x_{jk}} \leq 1 - \alpha, \quad \forall i \in I \tag{75}$$

$$\sum_{k=1}^{L} x_{jk} \leq 1, \quad \forall j \in J \tag{76}$$

$$x_{jk} \in \{0,1\}, \quad \forall j \in J, k \in \{1,...,L\} \tag{77}$$

where W_{jk} is the cost of housing k vehicles at station j; x_{jk} equals 1 if there are k vehicles at station j; $D(j)$ is the number of calls that has to be served by station j; L is the maximum number of vehicles available for any station. The expression $\prod_{k=1}^{L} P(D(j) \geq k)^{x_{jk}}$ in constraints (75) represents busy fraction of station j, which is the probability that total demand in the coverage area of the station j is no less than the vehicles supplied at this station and the left side of constraint (75) is the probability that demand originated from node i cannot be responded by any vehicle located within distance requirement of demand node i. Borrás and Pastor (2002) keeps the multiplication structure of chance constraint in Ball and Lin (1993) but replace the busy fraction of a station with the expression below:

$$r_j = \frac{\bar{t} \sum_{i \in B_j} a_i}{24 m_j} \tag{78}$$

where $B_j = \{i : d_{ij} \leq S, \forall j \in N_i\}$ and m_j is the number of vehicles the station j holds.

The numerator and the denominator of (78) represent the total daily service time required by the demand sites assigned to j and the daily available service time at j, respectively.

4.1.2. Queuing Theory

In practice, different facilities sometimes cooperate with each other when there is a shortage of servers responding to demand in an area. The most commonly used technique to capture this feature is queuing theory. All the references are briefly summarized in Table 10.

Table 10. A brief overview of research on queuing theory

Model	Objectives	Decisions	Solution techniques	References

Hypercube queuing model	Develop tools to evaluate system performance measures	-	-	Larson (1974)
Approximation procedure	Approximation procedure for hypercube queuing model	-	-	Larson (1975), Jarvis (1985), Brandeau and Larson (1986)
A combination of MEXCLP with hypercube queuing model	Maximize expected coverage	Coverage	Heuristic Algorithm	Batta et al. (1989)
MEXCLP	Maximize expected coverage	Coverage	Fixed Point Iterative Method	Goldberg et al. (1990)
MEXCLP	Maximize expected coverage	Deployment Coverage	Hypercube-embedded Genetic Algorithm	Saydam and Aytuğ (2003)
MEXCLP2	Maximize the expected number of Priority 1 calls	Deployment Coverage	CPLEX	McLay (2009)
Queuing PLSCP	Minimize the number of servers	Location	Linear programming with a branch-and-bound	Marianov and ReVelle (1994)
Queuing PLSCP	Minimize the total number of allocated ambulances	Location Deployment Coverage Allocation	Iterative Optimization Algorithm	Kim and Lee (2016)
Queuing MALP	Maximize coverage with reliability	Location Coverage	Linear programming with a branch-and-bound	Marianov and ReVelle (1996)
A combination of MEXCLP with MALP	Maximize coverage with reliability	Location Coverage	Simulated Annealing	Galvão et al. (2005)

The earliest research applying queuing theory in emergency service planning is Larson (1974) which proposes the hypercube queuing model to evaluate performance measures, such as busy fraction and mean travel time, of a multi-server system with distinguishable servers under various strategies influencing the deployment of resources. As the model will become computationally intensive with the increase of servers, Larson (1975) proposes an approximation procedure, which assumes: (i) there is a preferred sequence of servers dispatched to the emergency call in each region and the most preferred one is dispatched if available; (ii) probability of dispatching jth preferred unit to a call is proportional to multiplication of busy fraction of $(j-1)$th server and probability of availability of jth server; (iii) when server identity is not considered, j servers could be regarded as the result of random selection without replacement from an $M/M/p$ system. By making these assumptions, the probability that first available server is the kth server is given by:

$$P_k = Q(p, q, k-1) q^{k-1} (1-q) \quad (79)$$

where

$$Q(p,q,k-1) = \frac{\sum_{j=k-1}^{p-1}\{(p-k)!(p-j)/(j-k+1)!\}(p^j/p!)q^{j-k+1}}{(1-q)(\sum_{i=0}^{p-1} p^i q^i/i!) + p^p q^p/p!}, \quad k = 1,\cdots,p \quad (80)$$

If servers are independent, the probability that first available server is the kth server selected is $q^{k-1}(1-q)$. $Q(p,q,k-1)$ is called the correction factor that indicates how much deviation of the result of independence assumption from exact results should be corrected. Jarvis (1985) extends the approximation procedure to a generalized form where the service time distribution depends on both server and customer. Brandeau and Larson (1986) improves the approximation of the hypercube model by incorporating three travel time patterns and applies the improved models to deploy the ambulance in Boston, resulting in remarkable cost savings.

The hypercube queuing model and approximation procedure have been incorporated into many types of location models to account for the dependency between servers. Batta et al. (1989) relaxes the assumptions of MEXCLP by embedding a hypercube queuing model in a single node substitution heuristic method to compute expected demand coverage iteratively. During each iteration, the problem could be solved more efficiently by an approximation procedure. This paper also proposes an adjusted MEXCLP model that uses correction factor proposed by Larson (1975) to account for the dependency between facilities. Three models, MEXCLP, hypercube queuing model, and adjusted MEXCLP, are compared using a test problem and a gap between expected coverage obtained by MEXCLP and hypercube queuing model is observed, especially when q goes bigger, because MEXCLP ignores facility cooperation. The adjusted MEXCLP model yields better agreement on expected coverage with hypercube queuing model and still keep the same overall quality for location as those generated by MEXCLP. Goldberg et al. (1990) extends MEXCLP using the approximation procedure proposed by Jarvis (1985). The goal is to maximize the expected number of customers responded within eight minutes. Saydam and Aytuğ (2003) also combines MEXCLP with hypercube queuing model. To increase the accuracy of the estimated coverage, a genetic algorithm (GA) is proposed and proved to yield better solution than corrected MEXCLP. McLay (2009) extends MEXCLP into considering two types of vehicles, multiple customer types, and dependency between servers, resulting in a model called MEXCLP2. The dependency is formulated based on the model proposed by Jarvis (1985). The goal is to maximize the expected number of Priority 1 calls, which is life-threatening and the survivalbility is highly related to the repsonse time, served in a given amount of time.

Another type of model that is frequently combined with queuing theory is chance constrained model where the busy fraction is expressed by the steady state probability. Marianov and ReVelle (1994) proposes queuing probabilistic location set covering problem (Q-PLSCP), which extends the PLSCP proposed by ReVelle and Hogan (1988) by incorporating queuing theory. It divides the whole region into several neighborhoods and in each neighborhood, the service system is regarded as an M/M/s/s queuing system. The minimum number of servers b_i needed to reach a reliability level α in Eq. (67) is derived through the steady state probability. The probability of all servers s being busy at steady state can be expressed as:

$$q_s = \frac{(\frac{1}{s!})\rho_i^s}{1+\rho_i+(\frac{1}{2!})\rho_i^2+\cdots+(\frac{1}{s!})\rho_i^s} \tag{81}$$

where $\rho_i = \frac{\lambda_i}{\mu_i}$ is the utilization ratio; λ_i and μ_i are the arrival rate and service rate in the neighborhood. The chance constraint in this problem can be reformulated as $1 - q_s \geq \alpha$. As q_s is decreasing function of s, we can always find the minimum number of servers b_i ($b_i \leq s$), which satisfies the following constraint:

$$\frac{(\frac{1}{b_i!})\rho_i^{b_i}}{1+\rho_i+(\frac{1}{2!})\rho_i^2+\cdots+(\frac{1}{b_i!})\rho_i^{b_i}} \leq 1-\alpha \tag{82}$$

Kim and Lee (2016) uses hypercube queuing model to calculate the steady state probability used in the chance constraint of PLSCP. Transition is only allowed between 'neighboring' states (i.e., states can only differ in one server while the states of the other servers are the same). Marianov and ReVelle (1996) treats the call-to-service system in a more general way that the service time is not exponentially distributed but generally distributed. Thus, the service system in each neighborhood is regarded as an M/G/s/s queueing system. The whole region is divided into several neighborhoods under the assumption that the demand rate in different neighborhoods does not differ significantly. Objective function and constraints stay the same as those in MALP except the calculation of busy fraction, which is the same as Eq. (81). Galvão et al. (2005) applies hypercube model to account for dependency in MALP, which uses busy fraction for each server and define y_{jk} such that $y_{jk} = 1$ if server k is located at j, $y_{jk} = 0$ otherwise to identify exact location of each server. The chance constraint in the original model is replaced by one incorporating Larson's correction factor as follows:

$$\left[\{1-\prod_{k=1}^{p} q_k^{\sum_{j\in N_i} y_{jk}} Q(p,q,\sum_{j\in N_i}\sum_{k=1}^{p} y_{jk}-1)\}-\alpha\right] y_i \geq 0, \quad \forall i \in I \tag{83}$$

where y_i equals 1 if demand i is covered with α reliability, 0 otherwise; q_k denotes busy fraction for server k. This problem is solved by simulated annealing.

4.2. Location Problem with Uncertain Demand

Emergency demand changes every day. The early models usually use average demand, while last twenty years, researchers focus more on models addressing the uncertain nature of the demand. The most commonly used methods to formulate the problem are chance constraint, stochastic programming (SP), and robust optimization (RO). We will introduce the research using these methods in this section. Readers that are interested in other methods could refer to Yang et al. (2019). All the references and additional notations are briefly summarized in Table 11 and Table 12, respectively.

Table 11. A brief overview of research on location problem with uncertain demand

Model	Objectives	Decisions	Solution techniques	References
SP with joint chance	Minimizes total cost	Location	CPLEX	Beraldi et al. (2004)

constraints		Deployment		
SP with integrated chance constraints	Minimizes total cost	Location Deployment	Heuristic Algorithm	Noyan (2010)
SP with stochastic dominance constraints				
SP with individual chance constraints	Minimizes total cost	Location Deployment Allocation	Branch-and-Cut	Zhang and Li (2015)
Two-stage SP with joint chance constraints	Minimizes total cost	Location Deployment Allocation	Branch-and-Cut Heuristic Algorithm	Beraldi and Bruni (2009)
Scenario-based two-stage SP	Minimizes total cost	Location Deployment Allocation	Sampling Approach	Nickel et al. (2016)
Two-stage SP considering vehicle and demand type	Minimizes total cost	Location Deployment Allocation Number of unsatisfied demand	Sample Average Approximation	Boujemaa et al. (2017)
Two-stage SP considering vehicle type	Maximize expected coverage	Location Deployment Allocation Coverage	Sample Average Approximation	Nelas and Dias (2020)
Two-stage SP considering vehicle and demand type	Maximize coverage Minimize lost calls	Location Coverage Allocation Lost calls	Branch-and-Benders-Cut	Yoon et al. (2021)
RO	Minimizes total cost Maximize coverage	Location Deployment Allocation	Branch-and-Cut	Zhang and Jiang (2014)
Two-stage DRO	Minimizes total cost	Location Deployment Allocation	Outer Approximation Algorithm	Liu et al. (2019a)
Two-stage RO	Minimize weighted travel time	Edge flow location	Regression Machine Learning	Boutilier and Chan (2020)

Table 12. Additional notations for location problem with uncertain demand

Notations	Explanation of notations
Deterministic parameters	
g_j	the per-unit capacity cost of facility site j
c_{ij}	the service fulfilment cost of facility site j dealing with the request at demand node i
f_j	the fixed cost of opening a facility at node j

q_j		the capacity constraint of facility site j
Random parameters		
ξ_i		the random service request generated at the demand node i
m_i		the maximum number of concurrent demands at the demand node i
Decision variables		
x_{ij}		the number of vehicles located at j that are used to cover the service requests at the demand node i
y_{ij}		1 if the demand node i is assigned to facility site j
z_{ij}		the fraction of demand at demand node i served by facility site j

4.2.1. Chanced Constrained Location Problem

Due to the varying demand, chance constrained problem allows one or several constraints to be violated. Beraldi et al. (2004) is one of the first studies that use joint probabilistic constraints to formulate demand uncertainty when dealing with facility location and sizing problem for EMS. The joint probabilistic constraints are made on the entire geographical area rather than on individual demand points, ensuring that all demand points can be covered above a certain reliability level. The constraint is nonconvex and to make it more computationally tractable, it is reformulated to a linear inequality with one side being the corresponding p-efficient point of the marginal distribution of demand. The reformulated model is solved by the commercial solver CPLEX. The model is as follows:

$$\min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j x_j \tag{84}$$

subject to

$$\mathbb{P}\left(\sum_{j \in N_i} x_{ij} \geq \xi_i, \quad \forall i = 1,\ldots,I \right) \geq \alpha \tag{85}$$

$$\sum_{i \in M_j} x_{ij} \leq q_j x_j, \quad \forall j \in J \tag{86}$$

$$x_j \in \{0,1\}, x_{ij} \text{ integer}, \quad \forall i \in I, j \in J. \tag{87}$$

Objective (84) is to minimize total cost. Constraint (85) is a joint chance constraint that guarantees the reliability level of the entire geographical area is above a certain value. Constraints (86) limit the number of vehicles that can be hosted at each facility site. Noyan (2010) replaces computationally intractable probabilistic chance constraints used in the stochastic model with two alternatives. The purpose is to construct models that could solve problems with a large set of scenarios because the ability to handle a large set of scenarios is of great importance for modeling uncertainty in practice. The first one is integrated chance constraints which directly keep the expected number of unmet demand below certain target service level simultaneously for individual demand point and the whole system. The other is the stochastic dominance constraints, which compare total unmet demand with a reference random outcome based on the increasing convex order rule. Zhang and Li (2015) also uses probabilistic chance constraint to model demand uncertainty. It proposes a new random parameter, i.e., the maximum number of concurrent demands, and embeds it into the chance constraint, which sets a lower bound to the probability that the concurrent demands served by each station cannot exceed the number of vehicles stationed at this facility. To deal with the nonconvexity, Zhang and Li (2015) approximates it by second-order convex cone constraint and the original model is transformed into a conic quadratic mixed-integer program, which is solved efficiently by branch-and-cut method.

4.2.2. Stochastic Programming

When dealing with uncertainties in the facility location problem, a lot of research usually assumes that the decision makers have complete knowledge about the uncertain parameters because of the given probabilistic distribution, which is based on historical information. Under this assumption, a growing number of studies use stochastic programming to deal with the uncertainties, especially the two-stage stochastic programming, which formulates uncertainties in a two-step process: In the first stage, the nonanticipative decisions have to be made without knowing the realization of uncertain parameters; In the second stage, recourse decisions are made based on the realization of uncertain parameters and are conditional on the first-stage decisions.

The two-stage stochastic programming used to solve routine emergencies begins with the work by Beraldi and Bruni (2009), which extends Beraldi et al. (2004) by innovatively incorporating joint probabilistic chance constraints into the traditional two-stage stochastic programming model to explore base station location, fleet size, and ambulance allocation problem for EMS under demand uncertainty. The objective function is the sum of three costs: opening cost of stations, the cost of housing vehicles, and the expected cost of service fulfillment. In the first stage, the location and capacity of stations are determined. In the second stage, the uncertain demand is represented by a finite number of scenarios and the ambulances are allocated to demand accordingly. The stochastic system congestion constraint in the original model is replaced by a joint probabilistic counterpart that requires certain reliability level has to be satisfied for the entire demand area. This replacement allows the exploration of the influences of different reliability levels on solutions. The problem is solved by branch-and-cut and three heuristic approaches. The model is as follows:

$$\min \sum_{j \in J} (g_j y_j + f_j x_j) + \mathbb{E}_{\xi}[Q(x, y, \xi)] \tag{88}$$

subject to

$$y_j \le q_j x_j, \quad \forall j \in J \tag{89}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{90}$$

$$y_j \text{ integer}, \quad \forall j \in J \tag{91}$$

where

$$Q(x, y, \xi) = \min \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij}(\xi_i) \tag{92}$$

$$\mathbb{P}\left(\sum_{i \in M_j} \xi_i y_{ij}(\xi_i) \leq y_j, j = 1,\ldots,J \right) \geq \alpha \tag{93}$$

$$\mathbb{P}\left(\sum_{j \in N_i} y_{ij}(\xi_i) \geq 1, i = 1,\ldots,I \right) \geq \alpha \tag{94}$$

$$y_{ij}(\xi_i) \leq y_j, \quad \forall i \in I, j \in J \tag{95}$$

$$y_{ij}(\xi_i) \in \{0,1\}, \quad \forall i \in I, j \in J \tag{96}$$

Objective (88) is to minimize two-stage expected total cost: opening cost of stations and the cost of housing vehicles of the first stage, and the expected cost of service fulfillment of the second stage. Constraints (89) require that the number of vehicles hosted in each station cannot exceed the capacity. Constraints (90) and (91) set domains for decision variables. Expression (92) defines the second-stage recourse cost. Constraints (93) and (94) are two joint probabilistic constraints. Constraints (95) ensure that the assignment can be successful only when there are vehicles at that facility. Nickel et al. (2016) applies scenario-based two-stage stochastic model to optimize the location and number of ambulances and their bases under demand uncertainty. Each scenario is a complete realization of demands of each demand node. With the probability associated to each scenario, the expected total cost can be calculated. As the number of scenarios would grow prohibitively large, making the problem intractable, a sampling approach, which approximates the optimal solution, is proposed to solve the problem efficiently. Boujemaa et al. (2017) extends the previous two-stage stochastic model by adding vehicle and demand type, each associated with different response standard. The objective is to minimize the sum of the first-stage opening and capacity cost of the ambulance stations and the second-stage expected traveling cost and penalty cost incurred by unmet demand. A sample average approximation (SAA) algorithm that makes a balance between tractability and the quality of solution is used to obtain solutions. Research also considering multi-type vehicles includes Nelas and Dias (2020) and Yoon et al. (2021). Nelas and Dias (2020) allows vehicle substitution among different vehicle types when the closest vehicles needed cannot respond. It also considers the episode overlapping and vehicle availability, which are characterized by incompatibility and availability matrix, respectively. The uncertainty about the occurrence of emergency episodes is formulated as a two-stage stochastic program with scenarios generated by Monte Carlo simulation. This problem is solved by sample average approximation method. Yoon et al. (2021) uses two-stage stochastic programming model to optimize the location and allocation of two types of ambulances considering the uncertain prioritized demand. The objective is to maximize the expected coverage, which is expressed by the gains obtained when demand is responded by the right ambulance type within the response standard minus penalty cost for lost demands. The model is then extended to consider nontransport vehicles, which can respond to demands faster but cannot transport patients to hospitals. These two models are relaxed to a more general model that the "all or nothing" coverage criteria is replaced by gradual coverage and the response by inappropriate ambulances is allowed. A match utility is added to measure the degree of match between demand and ambulance type. The problem of small size could be solved by SAA while the one of large size is solved by Branch-and-Benders-Cut method.

4.2.3. Robust Optimization

The RO is relatively new research area, receiving a lot of attention only in the last decade. Therefore, only limited number of studies can be found in this area. The RO usually assumes that no distributional information is known about the uncertain parameters except for the support. To find a solution that performs well in all possible realization, RO optimizes the worst-case scenarios, resulting in over-conservative results. To make the results less conservative, another type of model called distributionally robust optimization (DRO) comes into being, which assumes that decision makers have partial knowledge about probability distribution of uncertainty.

Zhang and Jiang (2014) is the first to use RO to explore the bi-objective EMS system design under uncertain demand and maximal concurrent demand. The first objective is to minimize total cost comprised of station construction cost, vehicle operating cost, and transportation cost. The second objective is to maximize demand coverage. The decisions involved are station location, demand assignment, and vehicle placement. Due to the lack of the probabilistic information of uncertain parameters, a robust optimization model is used to formulate the problem where all the uncertain parameters are assumed to belong to ellipsoidal uncertainty sets. The formulated model is a conic quadratic mixed-integer program, which can be solved efficiently by branch-and-cut method in CPLEX. The robust optimization model is as follows:

$$\min \sum_{j\in J} f_j x_j + \sum_{j\in J} g_j y_j + \sum_{j\in J} c\, t_j \tag{97}$$

$$\min \sum_{j\in J} f_j x_j + \sum_{j\in J} g_j y_j + W \sum_{j\in J} \tilde{t}_j \tag{98}$$

subject to

$$\sum_{j\in J} z_{ij} = 1, \quad \forall i \in I \tag{99}$$

$$z_{ij} \le x_j, \quad \forall i \in I, j \in J \tag{100}$$

$$\sum_{i\in I} d_{ij}\bar{\xi}_i z_{ij} + \theta \sqrt{\sum_{i\in I}\sum_{k\in I} \sigma_{ik} d_{ij} d_{kj} z_{ij} z_{kj}} \le t_j, \quad \forall j \in J \tag{101}$$

$$\sum_{i\in I} \overline{m}_i z_{ij} + \gamma \sqrt{\sum_{i\in I}\sum_{k\in I} \psi_{ik} z_{ij} z_{kj}} \le y_j, \quad \forall j \in J \tag{102}$$

$$\sum_{i\in I\setminus M_j} \bar{\xi}_i z_{ij} + \delta \sqrt{\sum_{i\in I\setminus M_j}\sum_{k\in I\setminus M_j} \sigma_{ik} z_{ij} z_{kj}} \le \tilde{t}_j, \quad \forall j \in J \tag{103}$$

$$0 \leq z_{ij} \leq 1, \quad \forall i \in I, j \in J \tag{104}$$

$$x_j \in \{0,1\}, y_j \in \mathbb{Z}^+, t_j, \tilde{t}_j \in \mathbb{R}^+, \quad \forall j \in J \tag{105}$$

where θ, γ, and δ are safety parameters; σ_{ik} and ψ_{ik} are covariance between demands at node i and k and between the maximum number of concurrent demands at node i and k, respectively. Expressions (97) and (98) are the bi-objective of this research. Objective (97) is to minimize the sum of station construction cost, vehicle operating cost, and expected transportation cost between demand nodes and facility sites. The third term in objective (98) is the weighted cost for demands not served in time. Constraints (99) ensure that all demands are served. Constraints (100) guarantee that demands can only be assigned to open facilities. Constraints (101)–(103) are robust expressions of the total distance to serve demand, of the constraint that the number of emergency vehicles at site j is no less than the maximum number of concurrent demands assigned to it, and of the penalty cost for not serving demands in time, respectively. To decrease the overconservatism caused by RO, Liu et al. (2019a) proposes a two-stage distributionally robust model with joint chance constraints to location and sizing problem of EMS taking into consideration the same random parameters as those in Zhang and Jiang (2014). The objective is to minimize the supremum of the expected total cost when random demand and the maximal concurrent demand are restricted to certain distributional sets. A data-driven approach is used to derive the first moment of uncertain demand, with which the distributional set of demand can be constructed. The distribution function of the random maximal concurrent demand is constrained by ellipsoid set with known mean and covariance. With these distributional sets, the model is transformed into a parametric second-order cone program, which could be solved efficiently by an outer approximation algorithm. The model is presented as follows:

$$\min \left(\sum_{j \in J} f_j x_j + \sum_{j \in J} g_j y_j + \sup_{\xi \in \Xi, m \in \mathscr{M}} \mathbb{E}[g(x, y, \xi, m] \right) \tag{106}$$

subject to

$$y_j \leq M x_j, \quad \forall j \in J \tag{107}$$

$$x_j \in \{0,1\}, \quad \forall j \in J \tag{108}$$

$$y_j \in \mathbb{Z}^+, \quad \forall j \in J \tag{109}$$

where

$$g(x, y, \xi, m) = \min c \sum_{i \in I} \left(\xi_i \sum_{j \in J} d_{ij} z_{ij} \right) \tag{110}$$

$$\sum_{j \in N_i} z_{ij} = 1, \quad \forall i \in I \tag{111}$$

$$z_{ij} \leq x_j, \quad \forall i \in I, j \in J \tag{112}$$

$$\mathbb{P}\left(\sum_{i\in M_j} m_i z_{ij} \le y_j, \quad \forall j \in J\right) \ge \alpha \tag{113}$$

$$0 \le z_{ij} \le 1, \quad \forall i \in I, j \in J \tag{114}$$

where $\Xi = \{\xi : (\mathbb{E}(\xi) - \mu)^{\mathrm{T}}\Sigma^{-1}(\mathbb{E}(\xi) - \mu) \le \epsilon^2\}$ is the distributional set of uncertain demand with estimated mean $\mu \in \mathbb{R}^+$ and covariance matrix $\Sigma \ge 0$; $\mathscr{M} = \{m : (m-u)^{\mathrm{T}}\Gamma^{-1}(m-u) \le \eta^2, \mathbb{E}(m) = u, \mathbb{E}(m^2) = u^{\mathrm{T}}u + \Gamma\}$ is the distributional set of uncertain maximal concurrent demand with known mean and covariance matrix. Objective (106) is to minimize the supremum of the expected total cost comprised of station construction cost, vehicle operating cost, and transportation cost. Constraints (107) guarantee that ambulances can only be assigned to open facilities. Objective function (110) minimizes total transportation cost. Constraints (111) ensure that all demands are served. Constraints (112) illustrate that demands can only be assigned to open facilities. Constraint (113) is a joint chance constraint that ensures that the demands in the entire geographical area can be covered above a certain reliability level. Boutilier and Chan (2020) proposes two-stage robust optimization model to optimize location and routing of ambulances in low- and middle-income countries taking into consideration uncertain demand and travel time. The deterministic counterpart is equivalent to the shortest path problem where the length of the path is edge based and the location of ambulance stations depends on the route choice from station to demand points. As the lack of the historical data in low- and middle-income countries, the uncertainty set of demand is simulated based on the results obtained from census data and regression model. Travel time is modeled by interdiction-based uncertainty set where the baseline travel time for each edge is estimated by random forest model.

5. Solution Techniques

In this section, we outline the commonly used solution techniques for covering problem, which for ease of reference are listed in Table 13. There are mainly three types of techniques: exact methods, heuristic (approximation) algorithms, and simulation. Before the twenty-first century, the computing capacity was limited and the scale of the research problem was usually small. A lot of research solves the location problem by relaxing the integer constraints and for the non-integer results, the Branch-and-Bound and Cutting Plane are quite popular during that time period. As computing capacity improves, research on how to solve large problems in a reasonable amount of time gains popularity, giving rise to many heuristic algorithms, such as Tabu Search (TS), Genetic Algorithm (GA), Lagrangian Relaxation (LR), etc. TS is a local search method that iteratively replaces the current solution with the best non-visited solution in the neighborhood. To avoid being stuck in suboptimal regions, it uses tabu list to keep track of solutions that have already been visited. Another widely used method is GA, which is a random search algorithm that imitates the process of natural selection. Specifically, in the beginning, a set of solutions, which are called individuals, are evaluated for their fitness scores. The individuals with higher scores are more likely to be selected to breed a new generation after a series of genetic operations: crossover and mutation. Keep selecting and mating until the termination condition has been reached. LR is also a very popular method which relaxes some constraints that complicate the problem with Lagrangian multipliers, resulting in a simplified version of the problem. As the problem becomes more complicated with random parameters, heuristic algorithms still maintain their popularity though with a little sacrifice of the accuracy. However, to make the problem tractable without losing the accuracy, some exact methods, such as Branch-and-Cut and Benders Decomposition, come into focus. Branch-and-Cut is a combination of Branch-and-Bound and Cutting Plane to tighten the linear programming. Benders Decomposition divides the original problem into a master problem and subproblems. By iteratively solving the master and sub problem, the algorithm terminates when the difference between upper and lower bounds is below a certain value. Besides exact methods and heuristic algorithms, simulation is also used. But most of the time, it works as a tool to evaluate the results obtained from other methods.

Table 13. The solution techniques used to solve covering problem

Solution Techniques	References
Linear Programming	Toregas et al. (1971), Church and ReVelle (1974), Schilling et al. (1979), Daskin and Stern (1981), Daskin (1982, 1983), Saydam and McKnew (1985), Hogan and ReVelle (1986), Berman et al. (2003)
Branch-and-Bound	Church and ReVelle (1974), Church and Roberts (1983), ReVelle and Hogan (1988), ReVelle and Hogan (1989a), Ball and Lin (1993), Marianov and ReVelle (1994), Marianov and ReVelle (1996), Berman et al. (2003), Drezner et al. (2004)
Cutting Plane	Plane and Hendrick (1977)
Greedy Heuristic	Berman and Krass (2002), Berman et al. (2003)
Lagrangian Relaxation	Schreuder (1981), Pirkul and Schilling (1991), Karasakal and Karasakal (2004)
Tabu Search	Gendreau et al. (1997)
Ant Colony	Su et al. (2015)
Genetic Algorithm	Saydam and Aytuğ (2003), Liu et al. (2016)
Simulated Annealing	Galvão et al. (2005)
Branch-and-Cut	Beraldi and Bruni (2009), Zhang and Jiang (2014), Zhang and Li (2015), Yoon et al. (2021)
Approximation	Nickel et al. (2016), Boujemaa et al. (2017), Liu et al. (2019a), Nelas and Dias (2020), Yoon et al. (2021)
Benders Decomposition	Yoon et al. (2021)
Machine Learning	Boutilier and Chan (2020)
Simulation	Fujiwara et al. (1987), Borrás and Pastor (2002), Yoon et al. (2021)

6. Future Research Questions

Most of the research presented so far explores classic location models as well as their extensions and applications for emergency services. In this section, we outline some promising future research questions for covering problems.

1) Maximal Survival Models. Although the ultimate goal of EMS is to save properties

and lives as much as possible, few studies directly formulate the objective as a function of the properties and lives that have been saved. Instead, most studies regard response time or distance as the approximate alternative to evaluate the effectiveness and efficiency of the EMS network. One reason is that many regulations explicitly set response distance or time limits and to comply with the requirements, it is reasonable to utilize the limits in the models. Another reason is that it is not easy to formulate the general survival rates function for all EMS responses as different causes, such as cardiac arrest, fires, and traffic accident, may lead to different consequences. However, there still exist studies that use survival probability as the objective and are proved to be better suited for EMS location planning than those that only consider response time and distance as objectives (Erkut et al., 2008; McLay and Mayorga, 2010; Knight et al., 2012; Amorim et al., 2020).

2) Geometric Representation and GIS. The influence of geometric representation of the area where the candidate facilities and demands are located is often neglected in the facility location research. To avoid the curse of dimensionality and reduce the complexity, most of the covering models use the centroids of an area to represent the whole area, which could result in potential measurement errors (Hillsman and Rhoda, 1978; Goodchild, 1979; Current and Schilling, 1990; Miller, 1996; Murray and O'Kelly, 2002). A growing number of research is exploring ways to reduce the influence of these errors on location. GIS is a very prevalent and helpful tool to achieve this goal, which is of significant help in capturing, organizing, analyzing and visualizing geographic data (Murray, 2005; Alexandris and Giannikos, 2010; Yang et al., 2020).
3) Multi-period Facility Location Problem. Most of the facility location problem is studied in a static environment. However, there are situations that the characteristics of the system vary significantly over a period, sometimes even over a day. Therefore, some research begins to explore the models that could capture the changing state of the system, resulting in multi-period models which could formulate the location problem dynamically (Repede and Bernardo,1994; Rajagopalan, Saydam and Xiao, 2008; Schmid and Doerner, 2010; Başar, Çatay, and Ünlüyurt, 2011; Van den Berg and Aardal, 2015; Degel et al., 2015; Peng, 2020).
4) Location and Dispatch. Location decisions are usually strategic while the dispatch decisions are operational. Most of the time, these two types of decisions are determined separately. However, as the dispatch decisions based on different dispatching policies would influence the availability of vehicles and thus the coverage results, more and more research combines these decisions together (Toro-Díaz et al., 2013; Sung and Lee, 2018; Enayati et al., 2019; Bélanger et al., 2020).
5) Location and Real-time Redeployment. The emergency response systems are designed to respond to demands within certain standards. With the increase of service demand, operational cost, and traffic burdens, relocation is regarded as an effective method to achieve the goal without too many side effects. Although redeployment differs from location in many aspects, the combination is proved to effectively improve the planning results (Gendreau et al., 2001; Naoum-Sawaya and

Elhedhli, 2013; Saydam et al., 2013; Van Barneveld et al., 2018).

7. Concluding Remarks

Facility location models for emergency service planning have been evolving with increasingly sophisticated features to address different emerging practical problems with complicated constraints. We review in this paper discrete coverage-based facility location models for emergency services, their extensions and applications, and the commonly used solution techniques. We also summarize some possible future research directions.

Acknowledgments

This study is supported by the Sino-Sweden bilateral project via the National Key R&D Program of China (Project No. 2018YFE0102700) and Vinnova/FFI. All authors contribute equally to the paper and are co-first authors.

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