https://doi.org/10.1016/j.tre.2021.102306

Investment Competition on Dedicated Terminals under Demand Ambiguity

Shiyuan Zheng a, Changmin Jiang b,*, Xiaowen Fu c

a *School of Transportation, Shanghai Maritime University, Shanghai, China* b *School of International Trade and Economics, University of International Business and Economics, Beijing, China* c *Department of Industrial and Systems Engineering, the Hong Kong Polytechnic University, Hong Kong, China*

Abstract: This paper establishes real option game models to investigate two competing shipping lines' timing decisions on their respective dedicated terminal (DT) investments in the presence of demand ambiguity. In addition, we compare the shipping lines' DT investment timing with the choice under the social optimum, and discuss the subsidy policies with which a government can regulate the shipping lines' investment timing. The results indicate that (1) if the shipping lines are symmetric (i.e., have the same DT capacities, investment costs, and operation costs while having symmetric demands), their DT investment game could reach sequential investment equilibrium, where the leader makes a preemptive investment while the follower's investment is delayed compared to the case without ambiguity; (2) when the government has complete information on the shipping lines' ambiguity level, the regulation rule exhibits a stepwise structure where the social optimum (or the mixed timing between the social optimum and the shipping lines' optimum, respectively) are implemented, if the social optimal timing is late (or earlier, respectively) than the shipping lines' break-even timing; (3) when the government has incomplete information on the shipping lines' ambiguity level, the stepwise structure still holds but the shipping lines can obtain extra subsidies due to their information advantages compared to the complete information case; and (4) compared to the complete information case, the incomplete information may promote (or delay, respectively) the regulated DT investment timing, if the increasing ambiguity has positive (or negative, respectively) marginal contribution of the investment timing on the shipping lines' profits.

Keywords: Dedicated terminal; Investment competition; Knightian uncertainty; Real option game; Multiple-priors expected utility (MEU)

1. Introduction

In the past decades, the traditional competition between shipping lines and between ports has been evolving into the competition between supply chains (Notteboom, 2007). To offer more competitive services, shipping lines are entering the area of terminal management worldwide. In particular, a dedicated terminal (DT) is a common form of vertical integration between shipping lines and terminals (Alvarez-SanJaime et al., 2013). For instance, Orient Overseas Container Line (OOCL) has two DTs located in Kaohsiung and Long Beach.[1] Mediterranean Shipping Company (MSC) launched its container terminal operator company, Terminal Investment Limited (TIL) in 2000, which now operates 34 terminals in 22 countries, including terminals in the cities of Antwerp, Le Havre, Bremerhaven, and Marseille. Its company profile describes the company's goal as "(TIL) invests in, develops and manages container terminals around the world. It was founded in 2000 to secure berths and terminal capacity in the major ports used by Mediterranean Shipping Company (MSC)."[2] Maersk line and A.P. Moller Terminal, though legally two independent companies, belong to the same group. These two companies have established a close partnership whereby Maersk line is secured dedicated capacity in certain strategically important ports.[3]

Although DTs offer shipping lines many benefits, such as greater flexibility, reliability, short turnaround time, and increased efficiency, shipping lines also experience substantial costs and risks to construct and operate DTs (Haralambides et al., 2002), due to the irreversibility of the terminal investment and the uncertainty of the future demand[4]. Shipping lines' investments in DTs may expand their potential market because of the better services provided by the DTs. However, in addition to better services, market expansion can be affected by many other factors, including the economic development in the hinterland, the competition from the other ports, and the regional transport and logistics infrastructure (Song, 2003; Talley, 2009, 2014; Lam and Yap 2011; Marquez - Ramos, 2014; Zhuang et al., 2014; Homsombat et al., 2016; Tu et al. 2018). These complicated, interdependent factors make the benefits of DT investment uncertain and dynamic. Furthermore, many of these factors are not controlled and even not clearly recognized by the investors (either the shipping lines or the government). They create not only risks but also ambiguity regarding the prospects of the DT investment. In many studies (e.g., Dixit and Pindyck, 1994; Azevedo and Paxson, 2014), risk/uncertainty is characterized by a specific probability measure, whereas ambiguity cannot be described by a single probability measure and should be characterized by a set of probability measures, which is also called Knightian uncertainty (Knight, 1921). Such demand ambiguity makes DT investment timing a more realistic albeit difficult decision to make.

Besides the demand ambiguity, shipping lines' competition complicates their DT investment timing decisions further. Some studies indicate that DTs are popular among the world's top container shipping companies and have become one battleground in their competition (e.g., Table

[1]http://www.oocl.com/eng/aboutoocl/companyprofile/containerterminals/Pages/default.aspx.
[2]https://www.tilgroup.com/about
[3]https://shippingwatch.com/Ports/article8568451.ece
[4]Although terminals can be sold generally, as a dedicated facility for a specific shipping line, the DT may not fit for others. For example, the Bao Steel Company has its DT (Majishan Terminal) in the Port of Zhoushan to facilitate its metal ore transport (http://www.bulkcn.com/bulkmachi/majishanport.htm). Due to its specialty on function and location, and the huge investment costs, it may be difficult for other shipping companies to buy it. Therefore, DTs are useful to their investors but may not be suitable for others. We propose that the DT investment is irreversible, at least partially irreversible.

3 in Van de Voorde and Vanelslander, 2008; Table 1 in Alvarez-SanJaime et al., 2013).[5] The shipping lines face the tradeoff between making a preemptive DT investment to gain the first mover advantage, or waiting for better opportunities in the future (because of the increasing but uncertain demand). On the other side, the shipping lines' competition on their DT investments affects the social welfare and thereby attracts attentions from the government. The government may wish to use the regulation tools (e.g., subsidy) to align the shipping lines' DT investment timing to the social optimum. Indeed, it has been well recognized that DT investments can have important yet complex implications to ports, shipping lines, competition and social welfare. OECD (2011) noted "*Some shipping lines operate or own terminals within ports. This level of integration between the companies can provide them with incentives to restrict access to their facilities only to their own downstream operations…can have welfare benefits if it creates incentives for the upstream operator to invest in facilities that it would not have invested in if it had to allow downstream competitors to access them. For example, the terminal operator may purchase specialized modern unloading equipment that creates efficiencies in unloading time. However, if some of the benefit of this new equipment were shared with a downstream competitor, it may no longer be a viable investment.*" It should be noted that there has been no consensus on the effects of such vertical investments, which have significant practical implications. For example, in the privatization of the Turkish Port of Ismir and the Port of Mersin, it was decided that the two ports should be separately managed, and the operating rights cannot be transferred to liner transport or ship brokers. These decisions were made by the government to safeguard competition (OECD 2011). The problem under investigation is of significant theoretical and practical values.

In our paper, we analytically investigate two shipping lines' investment competition regarding two DTs under market ambiguity. Specifically, we aim to answer the following research questions: (1) When facing demand ambiguity, how do the competing shipping lines determine their timing of DT investment? [6] Will they engage in a simultaneous investment game or a sequential investment game? (2) What are the impacts of the competition between shipping lines and the degree of ambiguity on the shipping lines' DT investment timing decisions? (3) Compared to the social optimal level, are the shipping lines' DT investment timing expedited or delayed? If the shipping lines' DT investment timing is not consistent with the social optimum, could the government use regulation tools to induce social optimum, especially when the government has incomplete information on the shipping lines' ambiguity level?

Within these research questions, there exist two issues: terminal investment competition under dynamically stochastic and ambiguous environment, and the government's regulation under incomplete information, which have important implications both in theory and in practice. The issue of ambiguity has a realistic meaning because the different decision makers (even from the same shipping line) may have different perceptions because of the differences in their attitudes, confidence, or information of such a new project (DT investment). Other reasons include factors that may influence the project prospect, such as hinterland development, competition from rivals, and regional logistics infrastructure. These differences in the decision makers' subjective measures

---

[5]As reported in Table 1 in Alvarez-SanJaime et al. (2013), MSC and CMA-CGM have their DTs in the Port of Le Havre. In addition, MSC has its DTs in the Port of Antwerp, where COSCO Pacific, CMA-CGM, P&O Nedlloyd have their DTs too. These examples show the competing shipping lines' DT investment strategies in recently years.

[6]Shipping lines' decision on which port to invest is affected by many economic and political factors, and prior decisions of shipping networks. The current paper focuses on when they should make such investments, given the DT plans.

determine the ambiguity. The issue of incomplete information also has a realistic meaning because shipping lines' subjective ambiguity levels cannot be obtained easily by the government. To formulate an effective regulation, the government must design the incentive mechanism to elicit the shipping lines to reveal their private information (their ambiguity levels) truthfully. The regulation outcomes under incomplete information may deviate from the outcomes under complete information. Therefore, investigating these two issues can help shipping lines and the government to make proper decisions in a realistic setting.

To answer these research questions, we use the multiple-priors expected utility form to describe the ambiguity-averse shipping lines' preferences. By applying the approach of a real option game, the optimal DT investment timing rules under different scenarios (simultaneous investment and sequential investment) are obtained and compared to identify the optimal equilibrium decisions for the two shipping lines. In addition, the shipping lines' DT investment timings are compared to the socially optimal timing. We discuss the possibility for the government to use the subsidy to regulate the shipping lines' DT investment timing decisions.

Our work contributes to the related literature in the following ways.

(1) We analytically investigate competing shipping lines' DT investment timing decisions under demand ambiguity. Although some studies have investigated DT or exclusive transport infrastructure investment under demand ambiguity (see the detailed literature review in Section 2), none has explored DT investment competition under ambiguity. Given that competition is an important and nonnegligible feature in the maritime industry, our study provides a more realistic picture. Moreover, our conclusion on the sequential investment equilibrium reveals insights into shipping lines' DT investment that cannot be revealed without considering competition.

(2) We examine the government's regulation on the shipping lines' DT investment behavior, especially when it has incomplete information on the shipping lines' ambiguity level. We obtain the stepwise structure of the subsidy regulation policy (on both complete information and incomplete information), which depends on the comparisons of three thresholds: the social optimum, the shipping lines' break-even timing, and the mixed timing between the social optimum and the shipping lines' optimum (i.e., profit maximization). Moreover, we use a principal–agent framework to transfer the government's DT investment regulation under incomplete information into a bi-level programming problem. The conclusions state that compared to complete information, whether incomplete information delays or promotes the regulated DT investment timing depends on the sign of the "incentive correction term". If the shipping lines have the lowest ambiguity, the regulation outcomes under incomplete information are the same as under complete information. The shipping lines can obtain extra "information subsidies" under incomplete information. Our study provides useful policy implications for governments to align the shipping lines' DT investment, because shipping lines' ambiguity level is difficult to assess in practice.

The remainder of this paper is organized as follows. Section 2 reviews the literature, and Section 3 investigates two competing shipping lines' DT investment timing decisions. Section 4 examines the social optimum for the DT investment timing of competing shipping lines and how the government should regulate the sipping lines' DT investment under both complete and incomplete information. The last section concludes the paper and identifies areas for future investigation. Appendix A presents the logic of the paper.

2. Literature Review

Three strands of literature—terminal investment, dedicated terminals, and transportation investment decision under ambiguity—are related to this study.

The literature on port/terminal investment is rich. Most studies have discussed this issue under certainty, for example, Musso et al. (2006), Anderson et al. (2008), Luo et al. (2012), Zheng and Negenborn (2014), Simkins and Stewart (2015), Tan et al. (2015), Cheng and Yang (2017), Zheng et al. (2017), Dong et al. (2018), and Zhu et al. (2019). Specifically, some literature has addressed port/terminal investment in a supply chain setting, for example, Kaysi and Nehmeb (2016), Song et al. (2018), and Asadabadi, and Elise Miller-Hooks (2018, 2020). We confine our discussions in a simplified chain comprising a government, two shipping lines, and three terminals. We focus on two issues: terminal investment under a dynamically stochastic and ambiguous environment, and the government's regulation under incomplete information. According to our review of the literature, few studies have investigated these two issues (especially in the transportation area).

Next, we focus on port/terminal investments under uncertainty. Meersman (2005) uses the real options model to find the optimal port investment law under revenue uncertainty. Allahviranloo and Afandizadeh (2008) use fuzzy integer programming to find the port investment criteria under the uncertainty of a cargo forecast. Zheng and Negenborn (2017) use the real options method and the least squares Monte Carlo simulation algorithm to examine the terminal investment timing decisions in a competitive, uncertain market. Li and Cai (2017) use the real options model to investigate the government's incentive policies on private investment under uncertainty. Balliauw et al. (2019) use real option game models to explore the capacity investment of two competing ports under uncertainty.

Here we focus on the comparison of our paper to that of Balliauw et al. (2019). Although our paper discusses a similar topic, that is, terminal investment competition in a dynamically stochastic setting, and uses a similar approach (the real option game), we have two new characteristics—ambiguity and the government's regulation under incomplete information—that are absent in their paper. These two issues lead to very different conclusions (Section 1). Moreover, some conclusions in Balliauw et. al. (2019) are from numerical studies, whereas most of our conclusions are obtained analytically.

Vertical integration in the shipping industry has led to the prevalence of DTs over the past decades. Some literature discusses the reasons for and the benefits of DTs. For instance, Alvarez-SanJaime et al. (2013) demonstrate that the existence of DTs is mainly due to the differences between the objectives of ports and shipping lines. Haralambides et al. (2002) offer a detailed analysis of the costs (e.g., diseconomies of scale in ports) and the benefits (e.g., flexibility, reliability, short turnaround time, and high efficiency) of DTs. Kaselimi et al. (2011) suggest some advantages that a shipping line can exploit from DT, including value-added services to customers and increased profit. Hsu et al. (2015) use a revised importance-performance analysis model to evaluate the service attributes of DTs from users' perspectives.

Another topic in the DT literature is the competition between DTs and public terminals (PTs). Kaselimi et al. (2011) analyze the impacts of DT facilities on the competition between the existing multi-user terminals. Alvarez-SanJaime et al. (2013) examine the competition between a DT and a PT under the scenarios of exclusive use (the DT is used only by its shipping line investor and the

PT is used only by the investor's rival), mixed use (the DT is used only by its investor, and the PT can be used by both shipping lines), and non-exclusive use (the DT and the PT can be used by both shipping lines). They conclude that the DT investor is better to allow other shipping lines, including her rivals, to use her DT. Although these studies discuss the aspects of DT, none has investigated DT investment under ambiguity. Due to market complexity, there are many factors that DT investors do not know exactly. Therefore, our discussion on DT investment under ambiguity should fit reality better and can provide more insights into DT management.

The studies of transportation investment and planning under ambiguity have been limited (with a few notable examples such as Gao and Driouchi, 2013; Wang and Zhang, 2018; Randrianarisoa and Zhang, 2019 and Zheng et al., 2020). The major difference between this paper and the literature is that we analyze shipping lines' DT investment competition in a dynamic setting and the government's regulation on DT investment competition, especially when it has incomplete information on shipping lines' ambiguity degree. To be more specific, Gao and Driouchi (2013) assume that the investor needs to invest immediately or abandon the investment opportunity forever. In other words, they do not consider the possible flexibility of investment timing, or the value of delayed investment, which is an important feature in our model. Wang and Zhang (2018) and Randrianarisoa and Zhang (2019) have investigated seaport disaster adaptation investment under ambiguity and competition. However, their analysis is in a static ambiguity setting where the disaster occurrence probability is uncertain, and our study is in a dynamic ambiguity setting where the market expansion caused by the DT investment follows a range of geometric Brown motions (GBMs). Technically, our paper is most relevant to Zheng et al. (2020), with differences in the following aspects: (i) Zheng et al. (2020) model one investor (one airline investing an exclusive terminal), and our study considers the investment competition between two investors (two competing shipping lines) under the dynamic ambiguity setting. The investment game results in the challenging analysis of the equilibrium selection (simultaneous equilibrium or sequential equilibrium) and provides descriptions closer to the industrial dyanmics. (ii) Zheng et al. (2020) investigate the regulation on the airport exclusive terminal investment timing while assuming that the government knows the airline's ambiguity level exactly. We relax this assumption and explore the regulation when the government has incomplete information on the shipping lines' ambiguity level.

3. DT investment competition under ambiguity

In this section, we first present the basic framework of our models and the formulation of ambiguity. Second, we analyze the shipping lines' optimal DT investment decisions under the scenarios of simultaneous investments and sequential investment. We also compare the outcomes of these two scenarios and obtain the equilibrium of the DT investment competition.

3.1 Model basics

We consider two shipping lines, 1 and 2 (SL1 and SL2, respectively), that offer differentiated and substitutable freight services in a particular port. To improve the service, each shipping line plans to build its DT (DT1 or DT2), which only it can use. Additionally, there is a PT in the port, which can be used by both shipping lines. We assume that the demand functions of the two shipping lines have the following linear forms:

$$p_{11} = 1 + v - q_{11} - b(q_{13} + q_{23} + q_{22}) \quad (1)$$

$$p_{13} = 1 - q_{13} - b(q_{11} + q_{23} + q_{22}) \quad (2)$$

$$p_{23} = 1 - q_{23} - b(q_{11} + q_{13} + q_{22}) \quad (3)$$

$$p_{22} = 1 + v - q_{22} - b(q_{11} + q_{13} + q_{23}) \quad (4)$$

where p_{11} and p_{13} denote the fare of SL1 through DT1 and the PT, respectively; q_{11} and q_{13} denote the freight service volumes of SL1 through DT1 and the PT, respectively; p_{22} and p_{23} denote the freight rate/fare of SL2 through DT2 and the PT, respectively; q_{22} and q_{23} denote the freight service volumes of SL2 through DT2 and the PT, respectively. $b \in (0,1]$ is the substitution degree of the freight services provided by the two shipping lines, which represents the competition intensity, with a higher b indicating a more intensified competition. To simplify the problem, we assume that SL1 and SL2 are symmetric. We normalize the market scale to 1 and assume that the DT can promote its investor's market scale to $1+v$ by improving services, where $v > 0$. A similar assumption has been common in the DT literature (e.g., Alvarez-SanJaime et al., 2013). All notations used in this paper are summarized in Table B-1 in Appendix B. The market structure is presented in Figure 1.

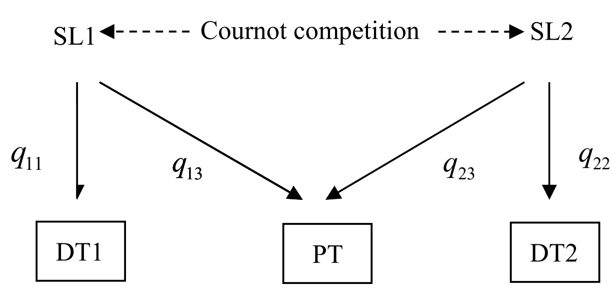


Figure 1 Market structure

We propose the following two-stage game to analyze the shipping line's DT investment decisions. Two scenarios for their investment behaviors, simultaneous and sequential, are considered. In Stage 1, SL1 and SL2 determine whether and when to invest in their DTs simultaneously to maximize their total future profits under the simultaneous scenario. Under the sequential scenario, one shipping line first makes the investment decision as the leader, and the other chooses its investment after it observes the leader's investment outcomes. To simplify the problem, we assume that the shipping lines' DTs are symmetric and their investments are lumpy and equal to I. In Stage 2, given the construction of one DT or two DTs, the two shipping lines engage in a simultaneous Cournot competition. The shipping lines choose their service volumes through the DT and the PT, that is, q_{11}, q_{13} and q_{22}, q_{23}, to maximize their instantaneous profits. The profit functions of the two shipping lines are presented as follows:

$$\pi_1 = p_{11}q_{11} + (p_{13} - f)q_{13} \tag{5}$$

$$\pi_2 = p_{22}q_{22} + (p_{23} - f)q_{23} \tag{6}$$

where π_1 and π_2 are their profits, and f is the terminal charge of the PT. To simplify the problem, we normalize the operation costs of the two shipping lines, the DTs, and the PT to 0.

Here we assume that the two shipping lines are symmetric, which means that they have the same service levels and the same marginal operation costs (which are normalized to 0). Their DTs have the same capacities and thereby their investments are the same. The symmetry assumption is commonly used to model competitors in the literature of economics, game theory (e.g., Jehle and Reny, 2001) and real option game (e.g., Dixit and Pindyck, 1994). The main reason for such simplification is to better reveal the underlying economic intuition by focusing on the effects of key factors/variables. For example, if the demand for shipping line is modeled as slightly better than the other, even such a difference is very small to the extent that it will not change (real comparison) results, it would be difficult to identify mathematically the impacts on market equilibrium results when the analytical solutions are already complex (which is the case of our study). Because in this study we would like to focus on the implications of DT (instead of the differences in shipping lines per se), we have considered the symmetry case. Of course, if we want to more closely examine the effects of the differences between shipping lines, then it is necessary to model asymmetric shipping lines (and thereby their demand promotions follow different evolution patterns). Note in such a case obtaining closed form solution is only the first step, since no conclusions can be obtained if no analytical comparison results or comparative statics can be obtained. This can be a useful further investigation. Given the complexity of the problem and solutions, some alternative simplifying assumptions may have to be made to make the model and comparisons mathematically tractable. In our model, the two shipping lines compete in the same market. Each of their DT has a promotion on the market demand. Because the shipping lines are symmetric, their DT promotions are also symmetric, with the demand evolution in the same pattern. In practice, DTs are used by the world's leading container shipping companies, whose scales are of similar magnitudes, with comparable operations in key markets and ports. The assumption of symmetry should be reasonable in practice for such markets. As indicated in (1), this allows us to better reveal and highlight the effects of DT. It would of course be a useful extension to consider asymmetric cases, which would be a even more realistic characterization of many other markets. Combined and compared to the analytical results obtained in this paper, we would be in a better position to evaluate the individual and combined effects of the factors related to shipping lines, ports, DTs, and government policy etc. The current study offers fresh insights on this important issue, but is not capable of answering all the related questions.

Furthermore, we investigate the optimal investment decisions of the DTs from a social perspective, where the government regulates the DT investments in Stage 1 to maximize social welfare[7], which is the sum of the (net) consumer surplus (the consumer surplus minus their

[7]According to the microeconomics (Jehle and Reny, 2001), we have the following relationships: Social welfare = consumers' net surplus (i.e., consumers' surplus - their payments to the firms) + the firms' profits (i.e., the payments they received from the consumers - their cost). Therefore, social welfare = consumers' surplus – firms'

payments to the shipping lines), the profits of the shipping lines, and the profit of the PT.[8]

$$u(q_{11}, q_{13}, q_{22}, q_{23}) = (1+v)(q_{11}+q_{22}) + q_{13} + q_{23} - (q_{11}^2 + q_{13}^2 + q_{22}^2 + q_{23}^2)/2$$
$$-b(q_{11}q_{13} + q_{11}q_{23} + q_{11}q_{22} + q_{13}q_{23} + q_{13}q_{22} + q_{22}q_{23}) \tag{7}$$

3.2 Expression of ambiguity

Ambiguity refers to the probability distribution over states when the event is unknown or uncertain (Ellsberg, 1961). Ambiguity may result from the difference in the decision makers' subjective measures of the probabilities of the events, which are described by their ambiguity levels. In our paper, we assume the demand expansion caused by the DT construction, v_t, follows the GBM:

$$dv_t = \mu v_t dt + \sigma v_t dB_t \tag{8}$$

where dB_t is a Wiener process, and μ and σ are the expected growth rate and the volatility of v_t, respectively. We assume that $v_0 > 0$, $r > 2\mu + \sigma^2$ and $\sigma > 0$, where r is the riskless discount rate.[9] To incorporate demand ambiguity, v_t is expanded into a set of equations through a range of GBMs such that

$$dv_t = (\mu - \sigma\theta_t)v_t dt + \sigma v_t dB_t^\theta \tag{9}$$

where the density generators θ_t are restricted to the non-stochastic interval $\mathrm{K} = [-\kappa, \kappa]$ that defines the level of ambiguity, with a bigger κ representing a higher level of ambiguity. Equation (9) is commonly used in the literature that has examined investment under ambiguity, for example, Chen and Epstein (2002) and Nishimura and Ozaki (2007). Because SL1 and SL2 are symmetric, we can reasonably assume their ambiguity levels are the same, that is, $\kappa_1 = \kappa_2 = \kappa_S$, where the subscript S is used to indicate the SLs and distinguish the government's ambiguity level κ_G.

---

costs. In our paper, the shipping lines' operation costs are normalized to 0. Thus, in our paper, social welfare = consumers' surplus. Moreover, the derivation of the consumers' surplus with respect to their consumption quantity = the products' prices. Therefore, we can obtain the consumers' surplus from the demand functions (1)–(3). We can verify that $\partial u/\partial q_{11} = p_{11}$, $\partial u/\partial q_{13} = p_{13}$, $\partial u/\partial q_{23} = p_{23}$, $\partial u/\partial q_{22} = p_{22}$. These are consistent with the properties of the social welfare function.

[8]For simplicity we assume that the shipping lines can invest and hold 100% of a DT without revenue sharing with the port (e.g., APM holds a 100% share in some of its DTs in, e.g., Rotterdam, Algeciras, Aarhus, and Genoa,. See Table 1 in Alvarez-SanJaime et al., 2013).

[9]First, we need $r > \mu$; otherwise, the expected value of the DTs (either to the shipping lines or to the government) could become infinitely large as time goes on infinitely, making the discussion senseless. Moreover, to gurantee the risk-adjusted discount rates are positive (the detailed explanations can be found in Section 3.3.3), we need $r > 2\mu + \sigma^2$, a standard assumption in the literature of real options (e.g., Dixit and Pindyck, 1994, p. 197).

In our model, the market scale is normalized to 1 and the demand promotion v_t can also be treated as the difference in the market potentials between the DT and the PT. Compared to the mature PT, which has a long operation history in the market, the relative advantage of the newly constructed DT is not very certain for its investor, due to a lack of historical data regarding the DT's impact on specific markets. Therefore, we assume that only the DTs cause the demand promotion stochastically.

3.3 Simultaneous investment scenario

DT projects result in extra profits for the shipping lines and extra social welfare for society. To make the DT investment, the shipping lines (or the government) must assess the profit increment (or the social welfare increment) from the DT construction. Therefore, in the following subsection, we first calculate the shipping lines' instantaneous profits and the instantaneous social welfare without DTs.

3.3.1 Shipping lines' instantaneous profits and the instantaneous social welfare without DTs

Without DTs, the demand system (1)–(4) becomes

$$p_{13} = 1 - q_{13} - bq_{23} \tag{10}$$

$$p_{23} = 1 - q_{23} - bq_{13} \tag{11}$$

The two SLs' profit functions become

$$\pi_1 = (p_{13} - f)q_{13} \tag{12}$$

$$\pi_2 = (p_{23} - f)q_{23} \tag{13}$$

We easily obtain the SLs' optimal output decisions and profits without DT as follows:

$$q_{13N} = q_{23N} = \frac{1-f}{2+b} \tag{14}$$

$$\pi_{1N} = \pi_{2N} = \frac{(1-f)^2}{(2+b)^2} \tag{15}$$

where the "N" in the subscript denotes without DT. The social welfare function under the case without DT becomes

$$u(q_{13}, q_{23}) = q_{13} + q_{23} - (q_{13}^2 + q_{23}^2 + 2bq_{13}q_{23})/2 \tag{16}$$

Substituting (14) into (16), we can obtain the instantaneous social welfare without DT as

$$u_N = \frac{(1-f)[(1+f)b + 3 + f]}{(2+b)^2} \tag{17}$$

3.3.2 Shipping lines' instantaneous profits after investing in DTs

To investigate the shipping lines' decisions under simultaneous DT investment, we first analyze their output decisions in Stage 2. Substituting (1)–(4) into (5) and (6), we find that the equilibrium outputs of the two shipping lines in each period are

$$q_{11} = q_{22} = \frac{(2+b)v + 3bf + 2 - 2b}{4(1-b)(1+2b)} \tag{18}$$

$$q_{13} = q_{23} = \frac{-3bv - (2+b)f + 2 - 2b}{4(1-b)(1+2b)} \tag{19}$$

Substituting (18)–(19) into (5) and (6), we obtain the two shipping lines' instantaneous profit increment after the DT constructions as follows:

$$\Delta\pi_{1t} = \Delta\pi_{2t} = \omega_{II,S} v_t^2 + \omega_{I,S} v_t + \omega_{0,S} \tag{20}$$

where $\omega_{II,S} = \frac{3b^2 + 4b + 2}{8(1-b)(1+2b)^2}$, $\omega_{I,S} = \frac{(5f-2)b^2 + 4fb + 2}{4(1-b)(1+2b)^2}$, and

$$\omega_{0,S} = \frac{(3f-2)(f+2)b^4 + (48f^2 - 48f + 16)b^3 + (30f^2 + 12f - 12)b^2 + (32f - 8)b + 8}{8(1-b)(2+b)^2(1+2b)^2}$$

We easily observe that $\omega_{II,S} > 0$. In addition, to keep the shipping lines' fares and outputs positive, we need $0 < f < 1$, which leads to $\omega_{I,S} > 0$. The signs of the coefficients in the shipping lines' instantaneous profit increment functions satisfy the condition that the profit flow should be a convex function of the stochastic variable (see Dixit and Pindyck, 1994, pp. 197).

3.3.3 Shipping line's DT investment decisions

From (20), we observe that the shipping lines' profit increments are related to the stochastic process v_t. With market uncertainty, SL1 and SL2 experience the tradeoff between making their investments immediately and waiting for better opportunities; in other words, their investment decisions are real options investment problems. Many preference models have been used to describe a decision maker's utility under ambiguity (e.g., see Klibanoff et al., 2005; Skiadas, 2014). An infrastructure investor (e.g., a DT investor), will probably be conservative regarding the investment outcomes and is inclined to make decisions extremely cautiously due to the large investment cost and the irreversible investment. It can be reasonably assumed that the shipping lines consider only the "worst" situation; hence, we use the Multiple Prior Expected Utility framework in continuous time (Chen and Epstein, 2002; Nishimura and Ozaki, 2007). Thus, the ambiguity-averse shipping lines' present values of future profit increment after the DT construction is defined as follows:

$$\Pi_i = \inf_{\theta \in \mathrm{K}} E^{Q^\theta}\left[\int_0^\infty e^{-rt} \Delta\pi_{it} dt \mid F_t\right], \forall i = 1, 2 \tag{21}$$

where Π_i is SL i's expected present value of the future profit increment at time 0. From (13) we observe that Π_i is SL i's expected present value of its future profit increment in the worst case. Next, a lemma provides the value of Π_i. All proofs are provided in the Appendix.

Lemma 1. *Given that the ambiguity-averse shipping line i's preferences are (21) and the rectangular structure of beliefs* P *, its expected present value of the future profit increment is given by*

$$\Pi_i = \Phi_{II,S}\omega_{II,S}v_t^2 + \Phi_{I,S}\omega_{I,S}v_t + \frac{\omega_{0,S}}{r} \tag{22}$$

where $\Phi_{II,S} = \dfrac{1}{r-2(\mu-\kappa_S\sigma)-\sigma^2}$, $\Phi_{I,S} = \dfrac{1}{r-(\mu-\kappa_S\sigma)}$.

Next, we examine the impact of SL i's ambiguity degree κ_S on its expected present value of future profit increment Π_i. We call the terms $\Phi_{II,S}$, $\Phi_{I,S}$, and $1/r$ the "risk-adjusted discount rate under ambiguity (RDRA)" (Zheng et al., 2020). Therefore, Π_i is the expected present value of SL i's profit increment calculated with RDRA. We easily find that $\partial\Phi_{II,S}/\partial\kappa_S < 0$ and $\partial\Phi_{I,S}/\partial\kappa_S < 0$. Because $\omega_{II,S} > 0$, $\partial\omega_{II,S}/\partial\kappa_S = 0$, $\omega_{I,S} > 0$, $\partial\omega_{I,S}/\partial\kappa_S = 0$, and $\partial\omega_{0,S}/\partial\kappa_S = 0$, we have $\partial\Pi_i/\partial\kappa_S < 0$; thus, an increase in ambiguity has a negative impact on the expected present value of the shipping line's future profit increment.

Now, we analyze SL i's investment decision in Stage 1 to maximize its opportunity value; in other words, SL i experiences the following optimal stopping problem:

$$Z_i = \max_{\tau_i \ge 0}[\inf_{\theta\in[-\varphi,\varphi]} E^{Q^\theta}(\int_{\tau_i}^{\infty} e^{-rt}\Delta\pi_{it}dt \mid F_t) - e^{-r\tau_i}I] \tag{23}$$

by choosing an (F_t)-stopping time, that is, $\{\tau_i \ge 0\} \in F_t$, where Z_i is SL i's option value of its DT investment and τ_i is its investment timing. Equation (23) denotes that SL i attempts to determine its DT investment timing to maximize its net present value, which is its expected present value of the future profit increment minus the present value of DT investment. Because SL1 and SL2 are symmetric, their optimal DT investment timings are the same in the simultaneous investment scenario, which we denote as v_F^*. By solving (23), we can obtain SL i's DT investment rule in the simultaneous scenario as Proposition 1.

Proposition 1. *If* $I \le \omega_{0,S}/r$ *, both shipping lines invest immediately, that is,* $v_F^* = v_0$ *; otherwise, their optimal DT investment timing, that is,* v_F^**, is the positive root of the following* *quadratic equation:*

$$\Phi_{II,S}(1-\frac{2}{\beta_S})\omega_{II,S}v^2+\Phi_{I,S}(1-\frac{1}{\beta_S})\omega_{I,S}v+\frac{\omega_{0,S}}{r}=I \qquad (24)$$

where $\beta_S=\frac{1}{2}-\frac{\gamma_S}{\sigma^2}+\sqrt{(\frac{\gamma_S}{\sigma^2}-\frac{1}{2})^2+\frac{2r}{\sigma^2}}>2$ and $\gamma_S=\mu-\kappa_S\sigma$.

Proposition 1 provides the optimal DT investment rule under the simultaneous investment scenario as follows: once the market scale expansion v_t reaches or exceeds the threshold v_F^*, that is, $v_t \ge v_F^*$, both shipping lines should make the DT investment immediately. In the LHS of (16), the term $1-2/\beta_S$ and $1-1/\beta_S$ are the inverses of the "option value multipliers." Because $\beta_S>2$, these two inverses are both less than 1. The optimal DT investment rule (24) indicates that under ambiguity each shipping line's expected present value of the future profit increment must exceed its investment cost by the option value multipliers. This result is consistent with standard real options theory.

Next, we investigate the impact of the competition between the shipping lines on their DT investment timing. Corollary 1 summarizes the conclusions.

Corollary 1. *Define the parameter areas with respect to* b *and* f *as follows:* $M_1=\{0\le b<\hat{b},0\le f\le 1\}$, $M_2=\{\hat{b}\le b<1,0\le f\le \hat{f}_1(b)\}$, $M_3=\{\hat{b}\le b<1,\hat{f}_1(b)\le f\le \hat{f}_2(b)\}$, *and* $M_4=\{\hat{b}\le b<1,\hat{f}_2(b)\le f\le 1\}$, *where* $\hat{b}$ *is the root of the following equation that is between 0 and 1:* $6b^3+13b^2+10b-2=0$, $\hat{f}_1(b)=\frac{4b^3-2b^2-8b+6}{10b^3+11b^2+2b+4}$, *and*

$$\hat{f}_2(b)=-\frac{2(16-28b-46b^2+63b^3+46b^4-53b^5+2b^6)}{3b(40+76b+34b^2+32b^3+59b^4+2b^5)}$$

$$+\frac{2}{3}\frac{\sqrt{256+1504b+2432b^2-1144b^3-6060b^4-3120b^5+3399b^6+3774b^7+267b^8-988b^9-352b^{10}+16b^{11}+16b^{12}}}{b(40+76b+34b^2+32b^3+59b^4+2b^5)}$$

We also have the following regulation results:

(i) If $(b,f)\in M_1\bigcup M_2$, *an increase in competition between the shipping lines delays their DT investment timing, that is,* $\partial v_F^*/\partial b>0$;

(ii) if $(b,f)\in M_4$, *an increase in competition between the shipping lines promotes their DT investment timing, that is,* $\partial v_F^*/\partial b<0$;

(ii) if $(b,f)\in M_3$, *the impact of increasing competition between the shipping lines on their DT*

investment timing is uncertain.

From Corollary 1 we know that in the simultaneous investment scenario, the marginal impact of the competition on the shipping lines' DT investment timing depends on two factors: the competition between the shipping lines and the PT charge. As indicated by Proposition 1, the expected present value of future profit increment determines the DT investment timing. Increasing competition decreases the shipping lines' instantaneous profits both before and after their DT construction. The final outcome depends on the impacts of competition on the decreasing comparison. When the competition between the shipping lines or the PT charge is very low (area M_1 and M_2), the decrease of the shipping lines' instantaneous profits after their DT construction dominates those before their DT construction; thereby, the DT investments are delayed. When one or both of these two factors are sufficiently high (area M_4), the decrease in their instantaneous profits before their DT construction dominates; thereby, the DT investments are promoted. In the intermediate area (area M_3), how the DT investment is affected is uncertain. Figure 2 illustrates the parameter areas which have different impacts on the shipping lines' DT investment timing

For the impact of ambiguity on the shipping lines' DT investment timing, our conclusions are similar to those of Zheng et al. (2020), which indicates that the ambiguity delays the DT investment, that is, $\partial v_F^* / \partial \kappa_S > 0$, if the shipping lines are ambiguity averse.

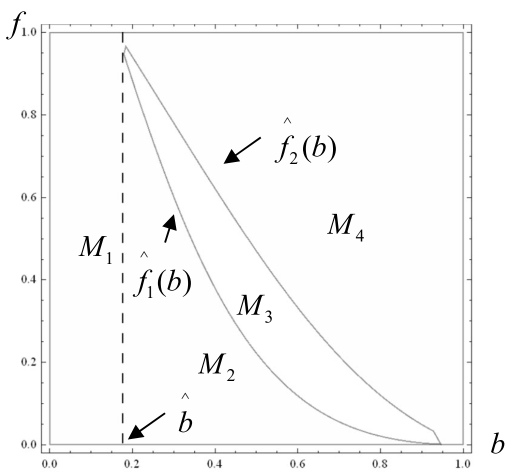


Figure 2 The parameter areas which have different impacts on the shipping lines' DT investment timing

3.4 Sequential investment scenario

3.4.1 Shipping lines' outputs decisions when only one shipping line makes the DT investment

Without loss of generality, we assume that SL 1 is the leader (i.e., it invests in DT ahead of SL 2). After its DT investment, the demand system becomes

$$p_{11} = 1 + v - q_{11} - b(q_{13} + q_{23}) \tag{25}$$

$$p_{13} = 1 - q_{13} - b(q_{11} + q_{23}) \tag{26}$$

$$p_{23} = 1 - q_{23} - b(q_{11} + q_{13}) \tag{27}$$

The equilibrium outputs of the two shipping lines after SL 1's DT construction are

$$q_{11} = \frac{(2-b)[(2+b)v + 3bf - 2b + 2]}{4(1-b)(2+2b-b^2)} \tag{28}$$

$$q_{13} = \frac{b(b-4)v + (2-f)b^2 + (2f-6)b + 4 - 4f}{4(1-b)(2+2b-b^2)} \tag{29}$$

$$q_{23} = \frac{-bv - (2+b)f + 2}{4 + 4b - 2b^2} \tag{30}$$

Then we obtain the leader's instantaneous profit increment as follows:

$$\Delta\pi_{Lt} = \omega_{II,SL} v_t^2 + \omega_{I,SL} v_t + \omega_{0,SL} \tag{31}$$

where the added subscript "L" represents the leader, and

$$\omega_{II,SL} = \frac{b^4 - 4b^3 - 4b^2 + 8b + 8}{8(1-b)(2+2b-b^2)^2}, \quad \omega_{I,SL} = \frac{fb^4 + (4-8f)b^3 + (4f-8)b^2 + (12f-4)b + 8}{4(1-b)(2+2b-b^2)^2}$$

$$\omega_{0,SL} = \frac{\begin{array}{c}(2-f)(3f-2)b^6 + (8-8f)b^5 - (36f^2 - 24f + 4)b^4 + \\ (48f^2 - 104f + 32)b^3 + (72f^2 - 16f - 32)b^2 + (96f - 32)b + 32\end{array}}{8(1-b)(4+6b-b^3)^2}$$

It is straightforward to show that $\omega_{II,SL} > 0$ and $\omega_{I,SL} > 0$ when $b \in [0,1)$ and $f \in [0,1]$.

3.4.2 Shipping line's DT investment decisions

Using a similar approach as in Section 3.3.3, the leader's expected present value of the future profit increment can be expressed as follows when there is only one DT:

$$\Pi_L = \Phi_{II,S}\omega_{II,SL} v_t^2 + \Phi_{I,S}\omega_{I,SL} v_t + \frac{\omega_{0,SL}}{r} \tag{32}$$

When the follower, that is, SL 2, makes the DT investment too, the two shipping lines' outputs decisions and their instantaneous profit increment are the same as in Section 3.3.2. Thus, the follower's option value can be expressed as

$$Z_F = \begin{cases} \Pi_F(v_t) - I & if \quad v_t \geq v_F^* \\ (v_t / v_F^*)^{\beta_S}[\Pi_F(v_F^*) - I] & if \quad v_t < v_F^* \end{cases} \tag{33}$$

where $v_F^* = v_S^*$ is the follower's DT investment timing and the added subscript "F" represents the follower. In this equation, the follower's expected present value of the future profit increment Π_F has the same form as (22). Using the method proposed by Dixit and Pindyck (1994), the leader's option value can be expressed as Proposition 2.

Proposition 2. *The option value of the leader can be expressed as*

$$Z_L = \begin{cases} \Pi_F(v_t) - I & if \quad v_t \geq v_F^* \\ \Pi_L(v_t) - I + (v_t / v_F^*)^{\beta_S} [\Pi_F(v_F^*) - \Pi_L(v_F^*)] & if \quad v_t < v_F^* \end{cases} \tag{34}$$

(34) suggests that the leader's option value shows a stepwise structure depending on v_t. If v_t is later than the follower's DT investment time v_F^*, the leader's option value is the same as the follower's because the two symmetric shipping lines divide the market. If v_t is earlier than v_F^*, the leading investor's profit increment is different at different times. Before v_F^*, the leader's present value of the future profit increment is Π_L. After v_F^*, the leader loses some profit increment because of the entry of the follower. The present value of this future profit increment loss is indicated by the term $(v_t / v_F^*)^{\beta_S} [\Pi_L(v_F^*) - \Pi_F(v_F^*)]$. Therefore, the present value of the leader's profit increment is represented by the term $\Pi_L(v_t) + (v_t / v_F^*)^{\beta_S} [\Pi_F(v_F^*) - \Pi_L(v_F^*)]$. The final option value of the leader is the difference between this term and the DT investment cost I, if $v_t < v_F^*$.

If $I \leq \omega_{0,SL} / r$, from the analysis in Section 3.4.1 we know that both shipping lines make an immediate investment and the sequential investment scenario will never occur. Therefore, we now focus on the case when $I > \omega_{0,SL} / r$. With Proposition 2, we can obtain the optimal DT investment timing of the two shipping lines in the sequential investment scenario as follows.

Proposition 3. *When* $I > \omega_{0,SL} / r$ *and* the *two shipping lines make DT investments sequentially, the follower's optimal timing is* v_F^* *(which is the positive root of the equation in [24]), and the leader's optimal timing* v_L^* *is the positive root of the equation in (35):*

$$\Phi_{II,S}\omega_{II,SL}[v^2 - v^{\beta_S}(v_F^*)^{2-\beta_S}] + \Phi_{I,S}\omega_{I,SL}[v - v^{\beta_S}(v_F^*)^{1-\beta_S}] + (\frac{\omega_{0,SL}}{r} - I)[1 - v^{\beta_S}(v_F^*)^{-\beta_S}] = 0 \tag{35}$$

When there is no DT, the shipping line has two choices: invest in DT now as a leader or invest in the future as a follower. Comparing its option value under these two choices, (35) provides the leader's threshold timing v_L^*, under which the two choices generate equal option values (see the proof of Proposition 4). When the time is earlier (or later, respectively) than v_L^*, the option value of waiting as a follower is more (or less, respectively) than the option value of investing as a leader and the shipping line chooses the waiting (or investing, respectively) strategy. After the follower invests his DT, both shipping lines have their DTs, and the situation is the same as that in Section 3.3. Thus, the follower's DT investment timing is the same as the shipping lines' DT investment timing in the simultaneous scenario v_F^*.

3.5 Which will occur: simultaneous investment or sequential investment?

To assess whether simultaneous investment or sequential investment might occur, we first calculate v_M^*, which is the leader's optimal timing if he is the only investor while the other shipping line never invests. After comparing v_L^*, v_F^*, and v_M^*, we have Corollary 2.

Corollary 2. *If* $I > \omega_{0,SL}/r$, v_M^* *is the positive root of the equation in (36):*

$$\Phi_{II,S}(1-\frac{2}{\beta_S})\omega_{II,SL}v^2 + \Phi_{I,S}(1-\frac{1}{\beta_S})\omega_{I,SL}v + \frac{\omega_{0,SL}}{r} = I \tag{36}$$

Moreover, we have $v_L^* < v_M^* < v_F^*$.

Corollary 2 shows that the leader makes an earlier DT investment than his optimal timing as a monopoly investor. Without threats from SL 2, SL 1 would invest at v_M^*. Under the pressure of a possible investment from the competitor SL2, SL1 has to invest in advance; otherwise, SL2 would invest at v_L^* when the leader's option value is larger than that of the follower's. In other words, SL1 makes a preemptive investment (Chevalier-Roignant et al., 2011). If the two shipping lines are symmetric, each has a 50% probability to be the leader and invests at v_L^*. After observing the leader's investment decision, the follower makes his DT investment at v_F^*. Notably, the rules of the game do not preclude that SL2 invests first; however, after observing SL1's investment, SL2's investment is prevented automatically because it would suffer a loss by doing so. Thus, we present Proposition 4 to summarize the conclusion on the assessment of the investment scenarios. Moreover, the shipping lines' DT investment timing sequence is presented by Figure 3.

Proposition 4. *If* $I > \omega_{0,SL}/r$, *the sequential investment occurs in the DT competition game between two symmetric shipping lines.*

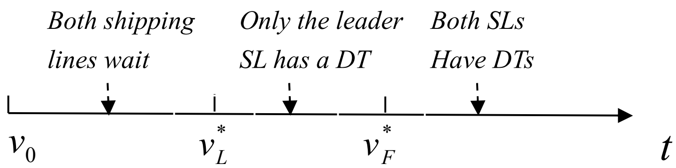


Figure 3 The DT market in different time

4. Regulation on shipping lines' DT investment timing

In this section, we first demonstrate that the sequential investment dominates the simultaneous investment in terms of social optimum. Moreover, the social optimal DT investment timing in the sequential investment scenario is obtained. Next, we show that the social optimal DT investment timing is mostly not consistent with the shipping lines' preferred timing, even with the government having the same ambiguity level with the shipping lines. The inconsistency raises the necessity for the government's regulation on the shipping lines' investment behavior. We investigate the possibility for the government to use the lump-sum subsidy to align the shipping lines' DT investment timing with the social optimum, especially when it has incomplete information on the shipping lines' ambiguity levels.

4.1 Social optimal timing for the DT investment

Substituting (28)–(30) into social welfare function (7), we obtain the instantaneous social welfare increment Δu_{1t} after the construction of the leader's DT as follows:

$$\Delta u_{1t} = \omega_{II,GL} v_t^2 + \omega_{I,GL} v_t + \omega_{0,GL} \tag{37}$$

where $\omega_{II,GL} = \dfrac{3b^4 - 10b^3 - 14b^2 + 24b + 24}{16(1-b)(2+2b-b^2)^2}$,

$$\omega_{I,GL} = \frac{(f-4)b^4 + (24-10f)b^3 + (2f-28)b^2 + (16f-16)b + 24}{8(1-b)(2+2b-b^2)^2}$$

$$\omega_{0,GL} = \frac{\begin{array}{c}(2-3f)(2+7f)b^6 + (42f^2 - 24f - 16)b^5 + (18f^2 + 112f - 52)b^4 \\ -(96f^2 + 128f - 200)b^3 - (24f^2 + 96f + 104)b^2 + (128f - 128)b + 96\end{array}}{16(1-b)(4+6b-b^3)^2}$$

We know that $\omega_{II,GL} > 0$ and $\omega_{I,GL} > 0$.

Substituting (18)–(19) into the social welfare function (7), we can obtain the instantaneous social welfare increment Δu_{2t} after the follower makes the DT investment as follows:

$$\Delta u_{2t} = \omega_{II,G} v_t^2 + \omega_{I,G} v_t + \omega_{0,G} \tag{38}$$

where $\omega_{II,G} = \dfrac{7b^2 + 14b + 6}{8(1-b)(1+2b)^2}$, $\omega_{I,G} = \dfrac{(5f-10)b^2 + 4(f+1)b + 6}{4(1-b)(1+2b)^2}$,

$$\omega_{0,G} = \frac{(2-3f)(11f+6)b^4 - (42f^2+48f-24)b^3 - (6f^2-12f+60)b^3 + 32fb + 24}{8(1-b)(2+b)^2(1+2b)^2}$$

We easily obtain that $\omega_{II,G} > 0$ and $\omega_{I,G} > 0$.

Similar to Section 3.3.2, we can define the present value of the future social welfare increment after the first and the second DT construction as follows:

$$U_i = \inf_{\theta \in K} E^{Q^\theta}[\int_0^\infty e^{-rt} \Delta u_{it} dt \mid F_t], i = L, F \tag{39}$$

Where U_i is the expected present value of the future social welfare increment at time 0 after the *i*th DT construction. We assume that the government is ambiguity averse too. Using similar approach as in Section 3.3, we obtain the social optimal timing of the first and the second DT investment as follows:

Proposition 5. *If* $I \leq \omega_{0,GL} / r$, *it is better to construct two DTs immediately for the social optimum, that is,* $v_{LG}^* = v_{FG}^* = v_0$. *If* $I > \omega_{0,GL} / r$, *the social optimal investment timing for the first DT, that is,* v_{LG}^*, *is the positive root of the quadratic equation in (40):*

$$\Phi_{II,G}(1-\frac{2}{\beta_G})\omega_{II,GL}v^2 + \Phi_{I,G}(1-\frac{1}{\beta_G})\omega_{I,GL}v + \frac{\omega_{0,GL}}{r} = I \tag{40}$$

where $\beta_G = \frac{1}{2} - \frac{\gamma_G}{\sigma^2} + \sqrt{(\frac{\gamma_G}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} > 2$ and $\gamma_G = \mu - \kappa_G \sigma$.

The social optimal investment timing of the second DT, that is, v_{FG}^*, *is the positive root of the quadratic equation in (41):*

$$\Phi_{II,G}(1-\frac{2}{\beta_G})\omega_{II,G}v^2 + \Phi_{I,G}(1-\frac{1}{\beta_G})\omega_{I,G}v + \frac{\omega_{0,G}}{r} = 2I \tag{41}$$

Moreover, $v_{LG}^* < v_{FG}^*$.

From Proposition 5, we know that when $I > \min(\frac{\omega_{0,S}}{r}, \frac{\omega_{0,G}}{2r})$, there exist two optimal investment timings for the social welfare, that is, an earlier timing v_{LG}^* when only one DT is the best choice for the social welfare, and a later timing v_{FG}^* when the second DT can be constructed for the social optimum. In other words, if a simultaneous game is considered, it is worse than the sequential game in terms of the social welfare optimum. Simultaneously investing in two DTs at v_{LG}^* is too early because the social welfare increment is insufficiently large, and simultaneous investing in two DTs at v_{FG}^* is too late because the possible social welfare increment during the period from v_{LG}^* to v_{FG}^* (if the first DT is constructed at v_{LG}^*) is lost. Therefore, the sequential investment dominates the simultaneous investment when $I > \min(\frac{\omega_{0,S}}{r}, \frac{\omega_{0,G}}{2r})$.

Next, we explore whether the shipping lines' DT investment timing and the corresponding social optimal timing are equal. We easily find that when $I \le \min(\frac{\omega_{0,S}}{r}, \frac{\omega_{0,G}}{2r})$, the DT investment timing preferred by both the leader and follower always coincides with that of the government. Therefore, we focus on the regulation when $I > \min(\frac{\omega_{0,S}}{r}, \frac{\omega_{0,G}}{2r})$ hereinafter. We first compare the follower's investment timing and the social optimum. We find that they are mostly different, even under some strict assumptions, for example, the government having the same ambiguity level as the shipping lines, $\kappa_S = \kappa_G$. Corollary 3 summarizes the comparison results.

Corollary 3. *Suppose that the government has the same ambiguity level with the shipping lines, that is,* $\kappa_S = \kappa_G = \kappa$*, we have* $\Phi_{II,S} = \Phi_{II,G} = \Phi_{II} = \frac{1}{r - 2(\mu - \kappa\sigma) - \sigma^2}$*,* $\Phi_{I,S} = \Phi_{I,G} = \Phi_I = \frac{1}{r - (\mu - \kappa\sigma)}$*,* $\gamma_S = \gamma_G = \gamma = \mu - \kappa\sigma$*, and* $\beta_S = \beta_G = \beta = \frac{1}{2} - \frac{\gamma}{\sigma^2} + \sqrt{(\frac{\gamma}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}$. *Given that the ambiguity degree* κ *is fixed, we define the parameter areas with respect to* b *and* f *as follows:* $\Gamma_1 = \{0 \le b < (\sqrt{55} - 3)/23, 0 \le f \le 1\}$*,* $\Gamma_2 = \{(\sqrt{55} - 3)/23 \le b < 1, 0 \le f \le \frac{4 + 12b - 6b^2 - 10b^3}{22b + 46b^2 + 13b^3}\}$*,* $\Gamma_3 = \{(\sqrt{55} - 3)/23 \le b < 1, \frac{4 + 12b - 6b^2 - 10b^3}{22b + 46b^2 + 13b^3} \le f \le 1\}$*, we find that:*

(i) if $(b, f) \in \Gamma_1 \cup \Gamma_2$*, the follower's optimal DT investment timing is always later than the*

social optimum, and they can never coincide, that is, $v_F^* > v_{FG}^*$.

(ii) If $(b,f)\in\Gamma_3$, *the follower's optimal DT investment timing may be or may not be the same as the social optimum. Specifically, when the following equality holds, the follower's optimal DT investment timing is the same as the social optimum; otherwise, they are not the same.*

$$\frac{\sqrt{[\Phi_I(1-\frac{1}{\beta})\frac{\omega_{I,G}}{2}]^2-\Phi_{II}(1-\frac{2}{\beta})\frac{\omega_{II,G}\omega_{0,G}}{r}}-\Phi_I(1-\frac{1}{\beta})\frac{\omega_{I,G}}{2}}{\omega_{II,G}}$$

$$=\frac{\sqrt{[\Phi_I(1-\frac{1}{\beta})\omega_{I,S}]^2-4\Phi_{II}(1-\frac{2}{\beta})\frac{\omega_{II,S}\omega_{0,S}}{r}}-\Phi_I(1-\frac{1}{\beta})\omega_{I,S}}{\omega_{II,S}}$$

The parameter scopes mentioned in Corollary 3 are represented in Figure 4.

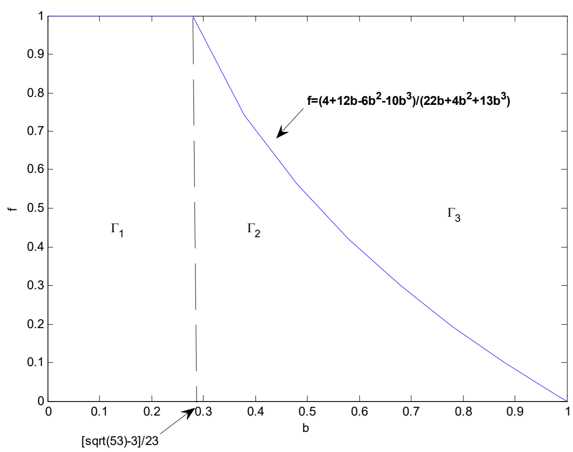


Figure 4 The parameter scopes which have different comparison results

Corollary 3 provides the following insights:

(i) If the DT investment cost is very low such that it can be covered in the beginning (for both the government and the shipping lines), it will be invested immediately (by both the government and the shipping lines) and there is no need to regulate.

(ii) If the DT investment cost cannot be covered at the beginning, both the government and the shipping lines will wait for better opportunities. The comparison of the follower's optimal timing and the social optimal timing depends on the shipping lines' competition and the charge of the PT, when the government has the same ambiguity level as the shipping lines. If the competition between the shipping lines is weak (i.e., $0\le b<(\sqrt{55}-3)/23$), the shipping lines' incentives to construct the DTs are weak because their relatively strong monopoly powers lead to smaller incremental profits contributed by the DTs. Therefore, the follower's optimal timing is always later than the social optimal timing. If the competition between the shipping lines is strong (i.e.,

$(\sqrt{55}-3)/23 \le b < 1$) and the charge of the PT is low (i.e., $0 \le f \le \dfrac{4+12b-6b^2-10b^3}{22b+46b^2+13b^3}$), the shipping lines' incentives to construct the DTs are still weak because the low PT charge leads to smaller advantage of DT. This also makes the follower's optimal timing later than the social optimal timing.

(iii) If both the competition intensity between the shipping lines and the charge of the PT are high, the follower's optimal timing may be earlier than the social optimal timing, because the shipping line is eager to gain competitive advantage through its DT and the high PT charge pushes more users to use its DT.

Using similar logic, we can find that the leader's timing decision is mostly not consistent with the social optimum. Therefore, government regulation is necessary.

4.2 Using subsidies to regulate shipping lines' DT investments

Subsidies are common in the port industry and can be offered in different manners, for example, paying for terminal construction directly (e.g., in the construction of Yangshan Port in China), giving ports preferential policies (e.g., loan guarantees and tax incentives for port construction investment in the United Kingdom), or reducing lease payments in concessions. These subsidies are important for terminal investors to cover their budget deficits and thereby change their investment behavior. Now, we investigate the possibility for the government to use the lump-sum subsidy, which is allocated after the shipping lines' DT investment, to regulate their investment decisions.

When the government regulates the shipping lines' investment behavior, it needs to know their ambiguity levels. Because the shipping lines' ambiguity levels are their private information, it is difficult for the government to know them exactly. Therefore, we must discuss the regulation when the government has incomplete information on the shipping lines' ambiguity levels.

We explore the regulation step by step. First, we discuss the case where the government knows the shipping lines' ambiguity levels exactly, that is, it has the complete information on κ_S. Next, we extend our discussion to the incomplete information setting, where the government does not know κ_S exactly, that is, only its distribution. Finally, we compare the regulation rules under these two cases and indicate the impacts of incomplete information on the DT investment regulation.

4.2.1 Regulation under complete information

Because the investment timing rules of the leader and the follower differ, and their corresponding social optimal timings differ too, their subsidies differ after the construction of the first and the second DTs. From (35), we know that the leader's optimal timing decision depends on the follower's timing. Therefore, according to backward induction, we first discuss the regulation on the follower's DT timing, which can be used subsequently to investigate the leader's investment timing regulation. The government's regulation on the follower can be described as the following optimization problem[10]:

[10] According to the standard assumption in the regulation literature, e.g., Laffont and Tirole (1993), in this study,

$$\max_{v_{FR},P_F}\{(\frac{v_t}{v_{FR}})^{\beta_G}[\frac{U_F(v_{FR})}{2}-I]-P_F\} \tag{42a}$$

$$\text{s.t.}\quad Z_F=(\frac{v_t}{v_{FR}})^{\beta_S}[\Pi_F(v_{FR})-I]+P_F\geq 0 \tag{42b}$$

$$P_F\geq 0 \tag{42c}$$

where $U_F=\Phi_{II,G}\omega_{II,G}v_t^2+\Phi_{I,G}\omega_{I,G}v_t+\frac{\omega_{0,G}}{r}$ is the government's present value of the future social welfare increment after the follower's DT is constructed. v_{FR} and P_F are the regulated DT investment timing and the subsidy to the follower, respectively. The objective function (42a) means that the government determines the regulated investment timing v_{FR} and the subsidy P_F to maximize its option value of the social welfare increment[11]. Constraint (42b) is the Participation Constraint (PC), which means that the follower's option value under the regulation should not be lesser than his reserved value 0. Constraint (42b) requires that the subsidy needs to be non-negative. Solving (42a)–(42b), we have Proposition 6.

Proposition 6. *Let* v_{F1} *and* v_{F2} *be the (minimum) positive root of the following equations*

$$\begin{aligned}&[(\beta_G-2)\Phi_{II,G}\omega_{II,G}+2(\beta_S-2)v^{\beta_G-\beta_S}\Phi_{II,S}\omega_{II,S}]v^2+[(\beta_G-1)\Phi_{I,G}\omega_{I,G}\\&+2(\beta_S-1)v^{\beta_G-\beta_S}\Phi_{I,S}\omega_{I,S}]v+\frac{\beta_G\omega_{0,G}+2\beta_S v^{\beta_G-\beta_S}\omega_{0,S}}{r}=2(\beta_G+\beta_S v^{\beta_G-\beta_S})I\end{aligned} \tag{43}$$

$$\Phi_{II,S}\omega_{II,S}v^2+\Phi_{I,S}\omega_{I,S}v+\frac{\omega_{0,S}}{r}=I \tag{44}$$

respectively. Then, under complete information, the government's regulated investment timing for the follower's DT, that is, v_{FR}^* *, can be expressed as follows: (i) if* $v_{FG}^*\geq v_{F2}$*, then* $v_{FR}^*=v_{FG}^*$*,* $P_F=0$*; (ii) if* $v_{FG}^*<v_{F2}$ *and* $v_{F1}\geq v_{F2}$*, then* $v_{FR}^*=v_{F1}$*,* $P_F=0$*; (iii) if* $v_{FG}^*<v_{F2}$ *and* $v_{F1}<v_{F2}$*, then* $v_{FR}^*=v_{F1}$*,* $P_F=(\frac{v_t}{v_{FR}^*})^{\beta_S}(I-\Phi_{II,S}\omega_{II,S}v_{FR}^{*2}-\Phi_{I,S}\omega_{I,S}v_{FR}^*-\frac{\omega_{0,S}}{r})$.

Similarly, the government's regulation on the leader can be described as the following: optimization problem:

---

we assume that the government has all the bargaining power and offers a take-it-or-leave-it subsidy contract to the shipping lines.

[11] The form of U_F can be obtained from (39) by using an approach similar to that in Section 3.3.3.

$$\max_{v_{LR},P_L}\{(\frac{v_t}{v_{LR}})^{\beta_G}[U_L(v_{LR})-I]-P_L\} \tag{45a}$$

$$\text{s.t.}\quad Z_L=(\frac{v_t}{v_{LR}})^{\beta_S}\{\Pi_L(v_{LR})-I+(\frac{v_{LR}}{v_{FR}^*})^{\beta_S}[\Pi_F(v_{FR}^*)-\Pi_L(v_{FR}^*)]\}+P_L\geq 0 \tag{45b}$$

$$P_L\geq 0 \tag{45c}$$

where $U_L=\Phi_{II,G}\omega_{II,GL}v_t^2+\Phi_{I,G}\omega_{I,GL}v_t+\frac{\omega_{0,GL}}{r}$ is the government's present value of the future social welfare increment after the leader's DT is constructed. v_{LR} and P_L are the regulated DT investment timing and the subsidy to the leader, respectively. Solving (45a)–(45c), we obtain the regulation outcomes on the leader's timing decision as follows.

Proposition 7. *Let* v_{L1} *and* v_{L2} *be the (minimum) positive root of the following equations*

$$\begin{aligned}&[(\beta_G-2)\Phi_{II,G}\omega_{II,GL}+(\beta_S-2)v^{\beta_G-\beta_S}\Phi_{II,S}\omega_{II,SL}]v^2+[(\beta_G-1)\Phi_{I,G}\omega_{I,GL}\\&+(\beta_S-1)v^{\beta_G-\beta_S}\Phi_{I,S}\omega_{I,SL}]v+\frac{\beta_G\omega_{0,GL}+\beta_S v^{\beta_G-\beta_S}\omega_{0,SL}}{r}=(\beta_G+\beta_S v^{\beta_G-\beta_S})I\end{aligned} \tag{46}$$

$$\Pi_L(v)-I+(\frac{v}{v_{FR}^*})^{\beta_S}[\Pi_F(v_{FR}^*)-\Pi_L(v_{FR}^*)]=0 \tag{47}$$

respectively. Then, under complete information, the government's regulated investment timing for the leader's DT, that is, v_{LR}^**, can be expressed as follows: (i) if* $v_{LG}^*\geq v_{L2}$*, then* $v_{LR}^*=v_{LG}^*$*,* $P_L=0$*; (ii) if* $v_{LG}^*<v_{L2}$ *and* $v_{L1}\geq v_{L2}$*, then* $v_{LR}^*=v_{L1}$*,* $P_L=0$*; (iii) if* $v_{LG}^*<v_{L2}$ *and* $v_{L1}<v_{L2}$*, then* $v_{LR}^*=v_{L1}$*,* $P_L=(\frac{v_t}{v_{LR}^*})^{\beta_S}\{I-\Pi_L(v_{LR}^*)-(\frac{v_{LR}^*}{v_{FR}^*})^{\beta_S}[\Pi_F(v_{FR}^*)-\Pi_L(v_{FR}^*)]\}$.

From Propositions 6 and 7, we find that the regulated DT investment rules have similar structures for both the leader and the follower: a stepwise structure depending on the comparison of three thresholds—the social optimum (v_{FG}^* or v_{LG}^*), the shipping lines' break-even timing (v_{F2} *or* v_{L2}), and the mixed timing between the social optimum and the shipping lines' optimum (v_{F2} *or* v_{L2}). The government faces a tradeoff between the social welfare increment (arising from earlier DT construction) and a higher subsidy. When the social optimal timing cannot be implemented voluntarily, the government must compare the following two options:

subsidizing the shipping lines to encourage early DT investment (Point [iii] in Propositions 6 and 7) or wait until their PC Constraints are satisfied (Point [ii] in Propositions 6 and 7). The explanations on Propositions 6 and 7 are similar to those of Zheng et al. (2020).

4.2.2 Regulation under incomplete information

Now, we assume that the government does not know the SL's ambiguity level κ_S exactly and knows only that it follows a certain distribution in $[\kappa_0, \kappa_1]$ and its cumulative distribution function (c.d.f.) and probability density function (p.d.f.) are $H(\kappa)$ and $h(\kappa)$, respectively. Moreover, we assume that its distribution satisfies the Monotone Likelihood Ratio Property (MLRP), i.e., $\frac{d}{d\kappa}[\frac{H(\kappa)}{h(\kappa)}] \geq 0$.[12] To elicit the SLs to reveal their private information on κ_S truthfully, the government should use the following incentive mechanism. The government should ask SL1 and SL2 simultaneously to reveal their ambiguity level $\hat{\kappa}_S$ and then determine the investment timing $v_{LRI}(\hat{\kappa}_S)$, $v_{FRI}(\hat{\kappa}_S)$ and the related subsidy $P_{LI}(\hat{\kappa}_S)$, $P_{FI}(\hat{\kappa}_S)$, respectively, based on their report $\hat{\kappa}_S$. In this case, the added subscript "*I*" indicates the incomplete information case. Then, the government's regulation problem for the follower can be expressed as the following bi-level programming:

$$\max_{v_{FRI}, P_{FI}} E_{\kappa_S}(U_F) = E_{\kappa_S}\{(\frac{v_t}{v_{FRI}})^{\beta_G}[\frac{U_F(v_{FRI})}{2} - I] - P_{FI}\} \tag{48a}$$

$$\text{s.t. } Z_F = \max_{\hat{\kappa}_S}(\frac{v_t}{v_{FRI}})^{\beta_S}[\Pi_F(\hat{\kappa}_S) - I] + P_{FI} \geq 0 \tag{48b}$$

$$P_{FI} \geq 0 \tag{48c}$$

The objective function (48a) means that the government determines the follower's regulated investment timing v_{FRI} and subsidy P_{FI} to maximize its expected option value of the social welfare increment. Constraint (48b) is the Incentive Compatibility constraint, which means that the follower chooses his report $\hat{\kappa}_S$ to maximize his option value of the profit increment under the regulation rule. Constraint (48b) also contains the PC, which indicates that the follower's optimal option value of the profit increment is non-negative. Solving the bi-level programming (48a)–(48c), we present Proposition 8 to describe the regulation rule on the follower under incomplete information.

Proposition 8. *Suppose that the government does not know the exact value of the follower's ambiguity levels and knows only its c.d.f. and p.d.f. as* $H(\kappa)$ *and* $h(\kappa)$*, respectively. Let*

[12]The MLRP assumption is common in the principal–agent literature (Fudenberg and Tirole, 1991).

$Y_F(v,\kappa)=v^{-\beta}[\Pi_F-I]$, $P_{F0}=\int_{\kappa_S}^{\kappa_1}\frac{\partial Y_F(v_{FRI}^*,\kappa)}{\partial\kappa}d\kappa$ and v_{F3} *be the (minimum) positive root of the following equations:*

$$[(\beta_G-2)\Phi_{II,G}\omega_{II,G}+2(\beta_S-2)v^{\beta_G-\beta_S}\Phi_{II,S}\omega_{II,S}]v^2+[(\beta_G-1)\Phi_{I,G}\omega_{I,G}$$
$$+2(\beta_S-1)v^{\beta_G-\beta_S}\Phi_{I,S}\omega_{I,S}]v+\frac{\beta_G\omega_{0,G}+2\beta_S v^{\beta_G-\beta_S}\omega_{0,S}}{r}+\frac{H}{h}\cdot\frac{\partial^2 Y_F(v,\kappa_S)}{\partial v\partial\kappa}=2(\beta_G+\beta_S v^{\beta_G-\beta_S})I \tag{49}$$

Then, under incomplete information, the government's regulated investment timing for the follower's DT, that is, v_{FRI}^* *can be expressed as follows: (i) if* $v_{FG}^*\geq v_{F2}$, *then* $v_{FRI}^*=v_{FG}$, $P_{FI}=P_{F0}$*; (ii) if* $v_{FG}^*<v_{F2}$ *and* $v_{F3}\geq v_{F2}$, *then* $v_{FRI}^*=v_{F3}$, $P_{FI}=P_{F0}$*; (iii) if* $v_{FG}^*<v_{F2}$ *and* $v_{F3}<v_{F2}$, *then* $v_{FRI}^*=v_{F3}$, $P_{FI}=(\frac{v_t}{v_{FRI}^*})^{\beta_S}(I-\Phi_{II,S}\omega_{II,S}v_{FRI}^{*2}-\Phi_{I,S}\omega_{I,S}v_{FRI}^*-\frac{\omega_{0,S}}{r})+P_{F0}$.

Moreover, compared to the complete information case, the government's regulated investment timing to the follower under incomplete information is delayed (or promoted, respectively), that is, $v_{FRI}^*\geq v_{FR}^*$ *(or* $v_{FRI}^*\leq v_{FR}^*$, *respectively), if* $\frac{\partial^2 Y_F(v_{FRI}^*,\kappa_S)}{\partial v\partial\kappa}\leq 0$ *(or* $\frac{\partial^2 Y_F(v_{FRI}^*,\kappa_S)}{\partial v\partial\kappa}\geq 0$, *respectively). Specifically, when* $\kappa_S=\kappa_0$, *incomplete information has no impact on the government's regulated investment timing, that is,* $v_{FRI}^*=v_{FR}^*$.

Similarly, the government's regulation on the leader under incomplete information can be described as follows:

$$\max_{v_{LRI},P_{LI}} E_{\kappa_S}(U_L)=E_{\kappa_S}\{(\frac{v_t}{v_{LRI}})^{\beta_G}[U_L(v_{LRI})-I]-P_{LI}\} \tag{50a}$$

$$\text{s.t. } Z_L=\max_{\hat{\kappa}_S}(\frac{v_t}{v_{LRI}})^{\beta_S}\{\Pi_L(v_{LRI})-I+(\frac{v_{LRI}}{v_{FRI}^*})^{\beta_S}[\Pi_F(v_{FRI}^*)-\Pi_L(v_{FRI}^*)]\}+P_{LI}\geq 0 \tag{50b}$$

$$P_{LI}\geq 0 \tag{50c}$$

Solving (50a)–(50c), we propose Proposition 9 to describe the regulation rule on the leader under incomplete information.

Proposition 9. *Suppose that the government does not know the exact value of the leader's ambiguity levels and knows only its c.d.f. and p.d.f. as* $H(\kappa)$ *and* $h(\kappa)$, *respectively. Let* $Y_L(v,\kappa)=v^{-\beta}(\Pi_L-I)$, $P_{L0}=\int_{\kappa_S}^{\kappa_1}\frac{\partial Y_L(v_{LRI}^*,\kappa)}{\partial\kappa}d\kappa$ and v_{L3} *be the (minimum) positive root* *of the following equations:*

$$[(\beta_G-2)\Phi_{II,G}\omega_{II,GL}+(\beta_S-2)v^{\beta_G-\beta_S}\Phi_{II,S}\omega_{II,SL}]v^2+[(\beta_G-1)\Phi_{I,G}\omega_{I,GL} +(\beta_S-1)v^{\beta_G-\beta_S}\Phi_{I,S}\omega_{I,SL}]v+\frac{\beta_G\omega_{0,GL}+\beta_S v^{\beta_G-\beta_S}\omega_{0,SL}}{r}+\frac{H}{h}\cdot\frac{\partial^2 Y_L(v,\kappa_S)}{\partial v\partial\kappa}=(\beta_G+\beta_S v^{\beta_G-\beta_S})I \quad (51)$$

Then, under incomplete information, the government's regulated investment timing for the leader's DT, that is, v^*_{LRI} *can be expressed as follows: (i) if* $v^*_{LG}\geq v_{L2}$*, then* $v^*_{LRI}=v^*_{LG}$*,* $P_{LI}=P_{L0}$*; (ii) if* $v^*_{LG}<v_{L2}$ *and* $v_{L3}\geq v_{L2}$*, then* $v^*_{LRI}=v_{L3}$*,* $P_{LI}=P_{L0}$*; (iii) if* $v^*_{LG}<v_{L2}$ *and* $v_{L3}<v_{L2}$*, then* $v^*_{LRI}=v_{L3}$*,* $P_{LI}=(\frac{v_t}{v^*_{LR}})^{\beta_S}\{I-\Pi_L(v^*_{LR})-(\frac{v^*_{LR}}{v^*_{FR}})^{\beta_S}[\Pi_F(v^*_{FR})-\Pi_L(v^*_{FR})]\}+P_{L0}$*.*

Moreover, compared to the complete information case, the government's regulated investment timing to the leader under incomplete information is delayed (or promoted, respectively), that is, $v^*_{LRI}\geq v^*_{LR}$ *(or* $v^*_{LRI}\leq v^*_{LR}$*, respectively), if* $\frac{\partial^2 Y_L(v^*_{LRI},\kappa_S)}{\partial v\partial\kappa}\leq 0$ *(or* $\frac{\partial^2 Y_L(v^*_{LRI},\kappa_S)}{\partial v\partial\kappa}\geq 0$*, respectively). Specifically, when* $\kappa_S=\kappa_0$*, incomplete information has no impact on the government's regulated investment timing, that is,* $v^*_{LRI}=v^*_{LR}$*.*

Based on Propositions 8 and 9, our insights are as follows:
(i) Compared to the complete information case, the mixed timing determination equations, that is, (49) and (51), have the extra "incentive correction terms" ($\frac{H}{h}\cdot\frac{\partial^2 Y_F(v,\kappa_S)}{\partial v\partial\kappa}$ and $\frac{H}{h}\cdot\frac{\partial^2 Y_L(v,\kappa_S)}{\partial v\partial\kappa}$) compared to the complete information cases, that is, (43) and (46). This is common in principal–agent models. These terms may cause the distortions in the regulation results under incomplete information. If $\frac{\partial^2 Y_F(v,\kappa_S)}{\partial v\partial\kappa}$ is positive (or negative, respectively), it satisfies the CS^+ (or CS^-, respectively) [13] condition, and the regulated investment timing expedites (or delays, respectively) as the shipping lines' ambiguity level increases, that is, $\partial v^*_{FRI}/\partial\kappa_S<0$ (or $\partial v^*_{FRI}/\partial\kappa_S>0$, respectively). $\frac{\partial^2 Y_F(v,\kappa_S)}{\partial v\partial\kappa}$ indicates the impact of increasing ambiguity to the marginal contribution of the investment timing on the follower's benefit. If this term is positive (or negative, respectively), increasing ambiguity results in larger (or smaller, respectively) marginal

[13] CS indicates the single-crossing condition (Fudenberg and Tirole, 1991).

contributions and thereby earlier (or later, respectively) investment timing for the follower ($\partial Y_F / \partial v < 0$ because Y_F is concave to v). Because mixed timing must compromise the benefits of both the government and the follower, the regulated investment timing is thereby expedited (or delayed, respectively). Similar explanations can be applied to the regulated timing for the leader.

(ii) When $\kappa_S = \kappa_0$, the shipping lines have the lowest ambiguity and the regulated timing under incomplete information is the same as under complete information. This finding is consistent with the principle of "no distortions at the top" in principle-agent theory (Fudenberg and Tirole, 1991).

(iii) Compared to complete information, both the leader and the follower have the extra "information subsidy," namely, P_{L0} and P_{F0}, under incomplete information. Unlike the complete information scenario where the subsidies may not be necessary, the subsidies are always necessary because of the shipping lines' information advantages. Therefore, the government needs more subsidies under incomplete information.

4.3 Regulation outcome summary under different scenarios

In this section, we aim to answer important questions related to the regulation outcome comparisons under different scenarios based on Propositions 6 to 9. The answers to the following questions are useful in the policy implications.

First, can the social optimum be realized? The answer is yes. Under both complete information and incomplete information, when the social optimum timing for the first DT (or the second DT, respectively) is later than the leader's (or the follower's, respectively) break-even timing, the first best can be achieved voluntarily or costless. When the social optimum timing for the first DT (or the second DT, respectively) is earlier than the leader's (or the follower's, respectively) break-even timing, the first best cannot be achieved, and the regulation leads to a mixed outcome between the social optimum and the shipping lines' optimum.

Second, is a subsidy necessary to reach the first-best solution? Does a subsidy accomplish the first-best solution? The answer is no. The first best is costless if it can be achieved, under both complete information and incomplete information. The subsidy is used to remedy the DT investment timing deviation from the social optimum partially if the social optimum cannot be realized.

Third, if the first-best solution cannot be reached, what is the welfare loss, and how much does a subsidy reduce the welfare loss? Under complete information, the regulation leads to the mixed timing between the social optimum and the shipping lines' optimum (for both the leader and the follower), which is later than the social optimal timing (but earlier than the shipping lines' optimum) if the latter cannot be achieved. With the subsidy, the welfare loss is $U_L(v_{LG}^*) - U_L(v_{L1})$ for the first DT construction, and $U_F(v_{FG}^*) - U_F(v_{F1})$ for the second DT construction, where $U_L = \Phi_{II,G}\omega_{II,GL}v_t^2 + \Phi_{I,G}\omega_{I,GL}v_t + \frac{\omega_{0,GL}}{r}$ and

$U_F = \Phi_{II,G}\omega_{II,GF}v_t^2 + \Phi_{I,G}\omega_{I,GF}v_t + \frac{\omega_{0,GF}}{r}$. Without the subsidy, the welfare loss is $U_L(v_{LG}^*) - U_L(v_L^*)$ for the first DT construction and $U_F(v_{FG}^*) - U_F(v_F^*)$ for the second DT construction. Because the mixed timing is earlier than the shipping lines' optimum, the social welfare loss is rescued by $U_L(v_L^*) - U_L(v_{L1})$ for the first DT construction, and $U_F(v_F^*) - U_F(v_{F1})$ for the second DT construction with the subsidy.

Fourth, what is the extra welfare loss due to incomplete information? If the first best can be achieved, incomplete information does not lead to welfare loss. If it cannot be achieved but the shipping lines' ambiguity level is the lowest, that is, $\kappa_S = \kappa_0$, incomplete information does not lead to extra welfare loss, compared to the complete information case. If it cannot be achieved, and the shipping lines' ambiguity level is not the lowest, that is, $\kappa_S > \kappa_0$, incomplete information may lead to extra and less welfare loss, depending on the signs of $\frac{\partial^2 Y_F(v_{FRI}^*, \kappa_S)}{\partial v \partial \kappa}$ (for the follower) and $\frac{\partial^2 Y_L(v_{LRI}^*, \kappa_S)}{\partial v \partial \kappa}$ (for the leader). Specifically, when these signs are negative (or positive, respectively), incomplete information leads to a more delayed (or less delayed, respectively) regulation timing and extra (or less, respectively) welfare loss, compared to the complete information case.

4.4 A numerical example

To better understand the previous conclusions, we use a numerical example to illustrate them. The parameters are as follows: $r = 0.1$, $\mu = 0.03$, $\sigma = 0.01$, $b = 0.5$, $I = 50$, $f = 0.45$, $\kappa_S = \kappa_G = \kappa \in [0,5]$.[14] In the incomplete information case, κ follows the uniform distribution, $\kappa \sim U(0,5)$. The numerical results are illustrated in Figures 5, 6, and 7. Among these figures, Figure 5 relates to Propositions 1, 3, and 5 and Corollary 3, which indicate the preferred DT investment timing of SLs and the government. Figure 6 relates to Propositions 6 and 8, which indicate the regulated policies for the follower under complete information and incomplete information. Figure 7 relates to Propositions 7 and 9, which indicate the regulated policies for the leader under complete information and incomplete information. In our numerical example, $b = 0.5 > (\sqrt{55} - 3)/23 \approx 0.186$, $f = 0.45 < \frac{4 + 12b - 6b^2 - 10b^3}{22b + 4b^2 + 13b^3} \approx 0.514$, that is, $(b, f) \in \Gamma_2$; we find that in Figure 5 the social optimum of the follower investment timing v_{FG} is always earlier

---

[14] According to Schröder (2011), κ should be restricted to $\kappa < (r - \mu)/\sigma$. Moreover, although the values of the parameters are assumed, they do not affect the results qualitatively because the results have been proved analytically.

than SL's preferred v_F, which coincides with Corollary 3. Figure 5 also illustrates that the leader's preferred timing is later than the social optimum too. The differences (between the social optimum and the SL's optimum for both the leader and the follower) increase as the ambiguity level κ increases. Figures 6 and 7 show the regulated DT investment timing and the related subsidy for both the leader and the follower under complete information and incomplete information. For the follower (Figure 6), when $\kappa \le 1.3$, $v_{FG} \ge v_{F2}$, and the regulated timing is v_{FG} (under both complete information and incomplete information). The subsidy is 0 under complete information and positive under incomplete information (the information subsidy). Because $\kappa > 1.3$, $v_{FG} < v_{F2}$, and the regulated timing becomes v_{F1} (under complete information) and v_{F3} (under incomplete information). When $1.3 < \kappa < 2$, $v_{F1} > v_{F2}$, and the subsidy under complete information is still 0 and positive under incomplete information. Because $\kappa > 2$, $v_{F1} < v_{F2}$, and the subsidy under complete information now becomes positive and increases as κ increases because the difference between the regulated timing v_{F1} and the follower's break-even timing v_{F2} widens and he needs more subsidies to cover his investment deficit. The information subsidy is still positive in order to induce the follower's truthful reporting of his ambiguity level. These results coincide with Proposition 6. A similar stepwise structure can be found in the regulated timing and the subsidy to the leader in Figure 7, under complete information and incomplete information.

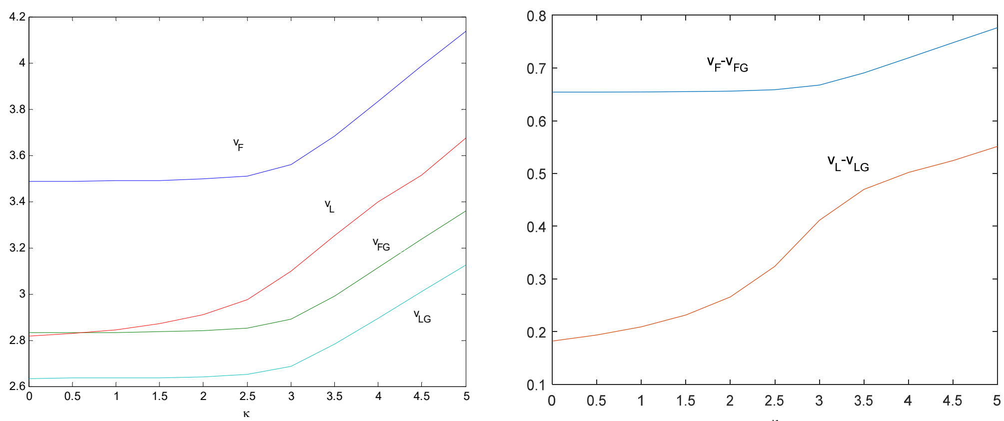
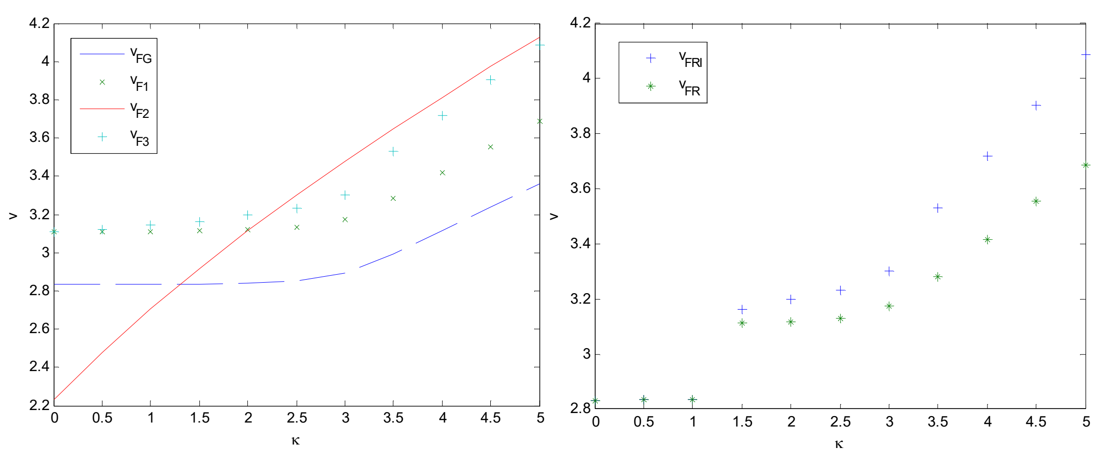


Figure 5. Comparisons of the preferred DT investment timing of SLs and the government



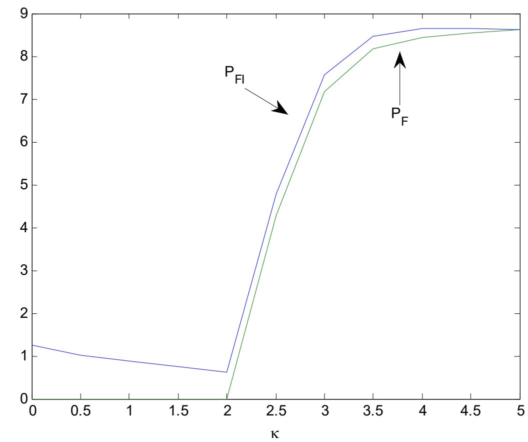
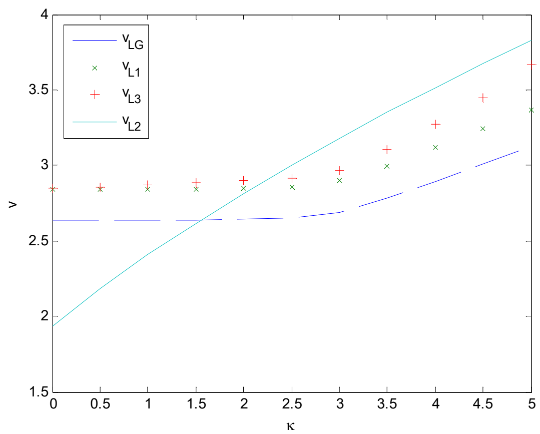
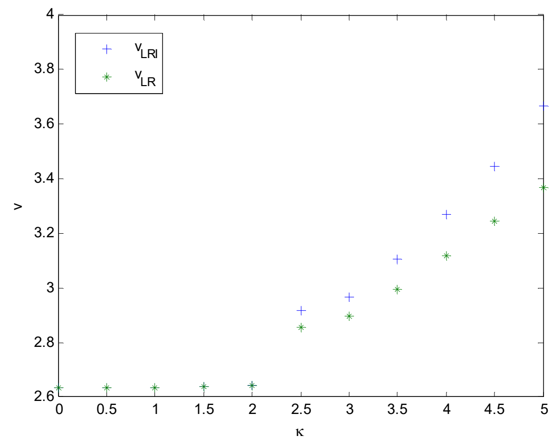


Figure 6. The government's DT regulation policy on the follower





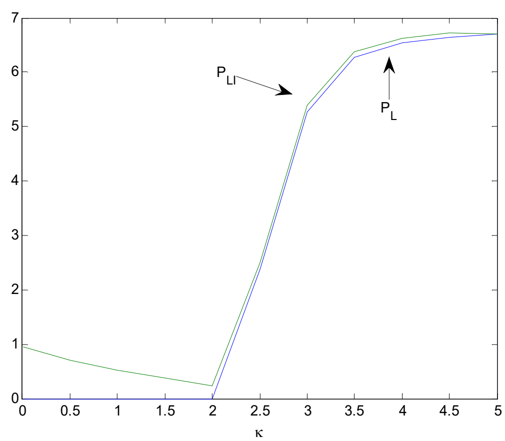


Figure 7. The government's DT regulation policy on the leader

5. Discussion and Conclusions

In this paper, we use real option game models to obtain two symmetric competing shipping lines' optimal DT investment timing rules. We conclude that the sequential DT investment could occur with each shipping line having a 50% probability of being the leader and making a preemptive investment. In addition, we discuss the design of the subsidy policies, especially when the government does not know the shipping lines' ambiguity level exactly. Next, we summarize our main results as follows (corresponding to our research questions identified in Section 1):

(1) The sequential equilibrium occurs (if the DT investment cost is not very low) in the

shipping lines' DT investment competition, with the leader making a preemptive DT investment earlier than its investment timing as the monopoly investor. The follower makes its DT investment decision only when its expected present value of the future profit exceeds its investment cost by the option value multipliers.

(2) Both the competition between the shipping lines and the PT charge affect the follower's instantaneous profit increment and thereby his DT investment timing. When these two factors are very low, its DT investments are delayed as the shipping lines' competition intensifies. When these two factors are sufficiently high, his DT investments are promoted as the shipping lines' competition intensifies. In the intermediate area, how the DT investment is affected by the increasing competition is uncertain. The shipping lines' increasing ambiguity always delays the DT investment timing.

(3) The shipping lines' DT investment timing may not be consistent with the social optimum, which makes the government's regulation based on the subsidy policies necessary. The regulation rule (under both complete information and incomplete information) exhibits a stepwise structure, depending on the comparisons of three thresholds: the social optimum, the shipping lines' break-even timing, and the mixed timing between the social optimum and the shipping lines' optimum. Specifically, when the government has incomplete information on the shipping lines' ambiguity level, the shipping lines can obtain extra subsidies because of their information advantages.

These theoretical results provide the following policy implications:

(1) Market uncertainty (e.g., the ambiguity) causes the possible delayed investment in DT compared to the social optimum if the investor is private and the competition between the shipping lines is low. Port privatization has been pursued by quite many ports in recent years and might improve port efficiency. However, this phenomenon is also controversial because private ports may abuse their monopoly power and harm social welfare. Our paper reiterates this opinion from the perspective of DT investment under demand ambiguity. Our conclusions indicate that private investors are inclined to delay their DT investment when experiencing market ambiguity and lower competition.

(2) The regulation on shipping lines' DT investment is necessary in the competing DT market. The government can use subsidies to align the shipping lines' DT investment with the social optimum. When the ambiguity level is high, subsidies are necessary to promote the shipping lines' DT investment.

(3) The incomplete information on shipping lines' ambiguity level may distort the regulation results, and the government needs more subsidies, compared to the complete information case.

Suggested topics for further research are as follows. In Section 3.1, we assume that each DT can be used by its investor only. However, in practice, some DTs are also used by rival shipping lines (the non-exclusive DT, see Alvarez-SanJaime et al., 2013). Therefore, examining the investment timing of the non-exclusive DT under ambiguity would be a valuable extension to the work in this paper. Also the strategic berth template decisions (e.g., Imai et al., 2014; Iris et al., 2018) to partition port capacity among the DT investors and their competing shipping lines are worth investigating, especially when the future demand is stochastic and ambiguous. Furthermore, shipping alliances are popular in today's shipping industry. Considering the possible alliance between the shipping lines and its DT construction strategies under ambiguity can provide more policy implications on the governance of DTs in a shipping alliance era. Another possible extension is to relax the assumption of the symmetry between the shipping lines, and investigate their DT competition when their demand promotions follow the different evolutions.

References

Allahviranloo, M., Afandizadeh, S., 2008. Investment optimization on port's development by fuzzy integer programming. European Journal of Operational Research 186 (1), 423-434.

Alvarez-SanJaime, O., Cantos-Sanchez, P., Moner-Colonques, R., Sempere-Monerris, J.J., 2013. Vertical integration and exclusivities in maritime freight transport. Transportation Research Part E 51, 50-61.

Anderson, C.M., Park, Y.A., Chang, Y.T., Yang, C.H., Lee, T.W., Luo, M., 2008. A game-theoretic analysis of competition among container port hubs: the case of Busan and Shanghai. Maritime Policy and Management 35 (1), 5-26.

Asadabadi, A., Elise Miller-Hooks, E., 2018. Co-opetition in enhancing global port network resiliency: A multi-leader, common-follower game theoretic approach. Transportation Research Part B 108, 281-298.

Asadabadi, A., Elise Miller-Hooks, E., 2020. Maritime port network resiliency and reliability through co-opetition. Transportation Research Part E 137.

Azevedo, A., Paxson, D., 2014. Developing real option game models. European Journal of Operational Research 237, 909-920.

Balliauw, M., Kort, P.M., Zhang, A., 2019. Capacity investment decisions of two competing ports under uncertainty: A strategic real options approach. Transportation Research Part B 122, 249-264.

Cheng, J., Yang, Z., 2017. The equilibria of port investment in a multi-port region in China. Transportation Research Part E 108, 36-51.

Chen, Z., Epstein, L., 2002. Ambiguity, risk, and asset returns in continuous time. Econometrica 70 (4), 1403-1443.

Chevalier-Roignanta, B., Flath, C.M., Huchzermeiera, A., Trigeorgisc, A., 2011. Strategic investment under uncertainty: A synthesis. European Journal of Operational Research 215 (3), 639-650.

Dixit, A.K., Pindyck, R.S., 1994. *Investment under Uncertainty*. Princeton University Press, Princeton, NJ.

Dong, G., Zheng, S., Lee, P.T.W., 2018. The effects of regional port integration: The case of Ningbo-Zhoushan Port. Transportation Research Part E 120, 1-15.

Ellsberg, D., 1961. Risk, ambiguity and the savage axioms. The Quarterly Journal of Economics 75 (4), 643-669.

Fudenberg, D., Tirole. J., 1991. *Game Theory*. MIT Press, Cambridge, MA.

Gao, Y., Driouchi, T., 2013. Incorporating Knightian uncertainty into real options analysis: Using multiple-priors in the case of rail transit investment. Transportation Research Part B 55, 23-40.

Haralambides, H., Cariou, P., Benacchio, M., 2002. Costs, benefits, and pricing of dedicated container terminals. International Journal of Maritime Economics 4 (1), 21-34.

Homsombat, W., Ng, A., Fu, X., 2016. Regional transformation and port cluster competition: The case of the Pearl river delta in South China, Growth and Change, 47(3), 349-62.

Hsu, W.K.K., Yu, H.F., Huang, H.S.H., 2015. Evaluating the service requirements of dedicated container terminals: a revised IPA model with fuzzy AHP. Maritime Policy and Management 42 (8), 789-805.

Imai, A., Yamakawa, Y., Huang, K., 2014. The strategic berth template problem. Transportation Research Part E 72, 77-100

Iris, Ç., Lalla-Ruiz, E., Lam, J.S.L., Voß, S., 2018. Mathematical programming formulations for the strategic berth template problem. Computers & Industrial Engineering 124, 167-179.

Jehle, G.A., Reny, P.J., 2001. *Advanced Microeconomic Theory*. Addison-Wesley.

Kaselimi, E.N., Notteboom, T.E., De Borger, B., 2011. A game theoretical approach to competition between multi-user terminals: the impact of dedicated terminals. Maritime Policy and Management 38 (4), 395-414.

Kaysi, I., Nehmeb, N., 2016. Optimal investment strategy in a container terminal: A game theoretic approach. Maritime Economics & Logistics 18 (3), 250–263.

Klibanoff, P., Marinacci, M., Mukerji, S., 2005. A smooth model of decision making under ambiguity, Econometrica 73 (6), 1849-1892.

Knight, F.H., 1921. *Risk, Uncertainty and Profit*. Houghton Mifflin Company, Boston, MA.

Laffont, J.J., Tirole, J., 1993. *A Theory of Incentives in Procurement and Regulation*. MIT Press, Boston, MA.

Lam, J.S.L., Yap, W.Y. 2011. Container port competition and complementarity in supply chain systems: Evidence from the Pearl River Delta. Maritime Economics & Logistics, 13, 102-120.

Li, S., Cai, H., 2017. Government incentive impacts on private investment behaviors under demand uncertainty. Transportation Research Part E 101, 115-129.

Luo, M., L. Liu., Gao, F., 2012. Post-entry container port capacity expansion. Transportation Research Part B 46, 120-138.

Marquez‐Ramos, L., 2014. Port facilities, regional spillovers and exports: Empirical evidence from Spain. Papers in Regional Science, 95(2), 329-351.

Meersman, H.M.A., 2005. Port investment in an uncertain environment. Research in Transportation Economics 13: 279-298.

Musso, E., Ferrari, C., Benacchio, M., 2006. Port investment: profitability, economic impact and financing. Research in Transportation Economics 16, 171-218.

Nishimura, K.G., Ozaki, H., 2007. Irreversible investment and Knightian uncertainty. Journal of Economic Theory 136, 668-694.

Notteboom, T., 2007. Strategic challenges to container ports in a changing market environment. Research in Transportation Economics 17, 29-52.

Randrianarisoa, L.M, Zhang, A., 2019. Adaptation to Climate Change Effects and Competition Between Ports: Invest Now or Later? Transportation Research Part B 123, 279-322.

Skiadas, C., 2014. Smooth ambiguity aversion toward small risks and continuous-time recursive utility. Journal of Political Economy 12 (4), 775-792.

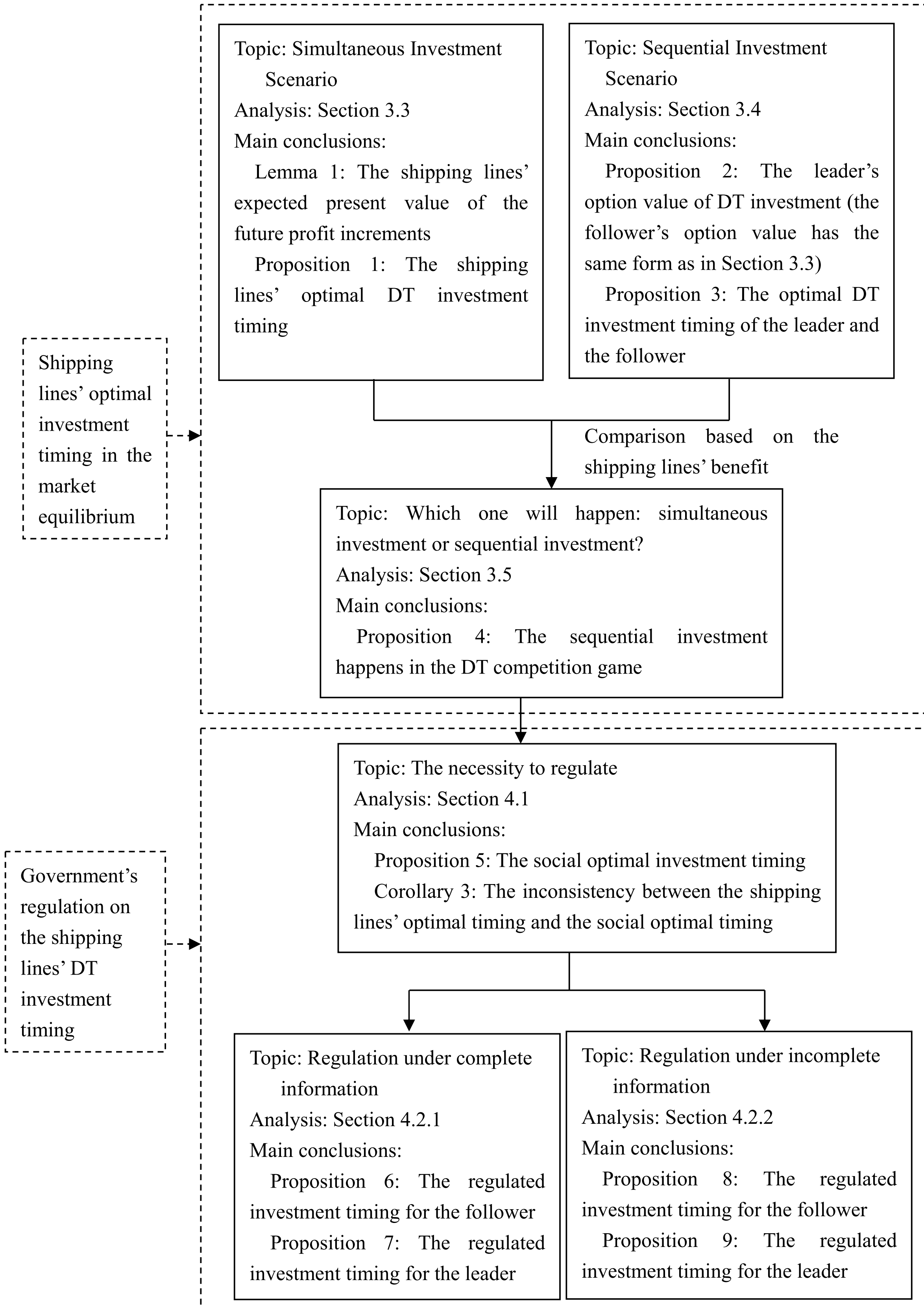
Schröder, D., 2011.Investment under ambiguity with the best and worst in mind. Mathematics and Financial Economics 4 (2), 107-133.

Simkins, Z.S., Stewart, R.D., 2015. Factoring a real value of cargo in port investment and funding decisions. Maritime Policy and Management 42 (7), 712-728.

Slack, B., Fremont, A., 2005. The transformation of port terminal operations - from the local to the

global. Transport Reviews 25 (1), 117-130.
Song, D.W., 2003, Port Co-opetition in concept and practice, Maritime Policy and Management, 30(1), 29-44.Song, Z., Tang, W., Zhao, R., 2018. Cooperation mode for a liner company with heterogeneous ports: Business cooperation vs. port investment. Transportation Research Part E 118, 513-533.
Talley, W.K., 2009. Port Economics. Routledge
Talley, W.K., 2014. Maritime transport chains: Carrier, port and shipper choice effects. International Journal of Production Economics 151, 174-179.
Tan, Z., Li, W., Zhang, X., Yang, H., 2015. Service charge and capacity selection of an inland river port with location-dependent shipping cost and service congestion. Transportation Research Part E 76, 13-33.
Tu, N., Adiputranto, D., Fu, X., Li, Z.C., 2018. Shipping network design in a growth market: The case of Indonesia, Transportation Research Part E. 117, 108-125.
Van de Voorde, E., Vanelslander, T., 2008. Market power and vertical and horizontal integration in the maritime shipping and port industry. No. 2009-2. OECD/ITF Joint Transport Research Centre Discussion Paper.
Wang, K., Zhang, A., 2018. Climate change, natural disasters and adaptation investments: Inter-and intra-port competition and cooperation. Transportation Research Part B: Methodological, 117, 158-189.
Zheng, S., Fu, X., Jiang, C., Ge, Y.E., 2020. Investment timing decisions on airport exclusive terminals under demand ambiguity. Transportation Research Part B 139, 343-363.
Zheng, S., Ge, Y.E., Fu, X., Nie, Y., Xie, C., 2017. Modeling collusion-proof port emission regulation of cargo-handling activities under incomplete information. Transportation Research Part B 104, 543-567.
Zheng, S., Negenborn.R., 2014. Centralization or decentralization: A comparative analysis of port regulation modes. Transportation Research Part E 69, 21-40.
Zheng, S., Negenborn, R., 2017. Terminal investment timing decisions in a competitive setting with uncertainty using a real option approach. Maritime Policy and Management 44 (3), 392-411.
Zhu, S., Zheng, S., Ge, Y.E., Fu, X., Sampaio, B., Jiang, C., 2019. Vertical integration and its implications to port expansion. Maritime Policy & Management 46, 920-938.
Zhuang, W., Fu X., Luo, M., 2014. A game theory analysis of port specialization – Implications to the Chinese port industry, Maritime Policy & Management, 41(3), 268–287.

Appendix A Main logic of the paper



Appendix B Notations

Table B-1 Notations

Parameters and variables	
p_{11}, p_{22}, p_{13}, p_{23}	The shipping lines' fare through the DTs and the PT
q_{11}, q_{22}, q_{13}, q_{23}	The shipping lines' freight service volumes through the DTs and the PT
b	The shipping lines' service substitution degree
f	Terminal charge of the PT
I	DT investment
π_1, π_2	Shipping lines' instantaneous profits
v_t	Demand promotion caused by the DT at time *t*
μ, σ	The expected growth rate and the volatility of v_t
θ_t	Density generator
dB_t, dB_t^θ	Wiener process and the Wiener process under density generator
κ_G, κ_S	Ambiguity level of the government and the shipping lines
r	Riskless discount rate
u	Social welfare
Π_i, Π_L, Π_F	SL *i*'s , the leader's and the follower's expected present value of the future profit increment at time 0
U_L ,U_F	The expected present value of social welfare increment after the first and the second DT at time 0
Z_i, Z_L, Z_F	SL *i*'s, the leader's and the follower's option value of their DT investments
β_S, β_G	The positive roots of the equations in the real option models in Proposition 1 and Proposition 5
$\Phi_{II,S}$, $\Phi_{I,S}$	The shipping lines' risk-adjusted discount rates under ambiguity
$\Phi_{II,G}$, $\Phi_{I,G}$	The government's risk-adjusted discount rates under ambiguity
$\omega_{II,S}$, $\omega_{I,S}$, $\omega_{0,S}$	The coefficients of the shipping line's expected present value of the future profit increment
$\omega_{II,SL}$, $\omega_{I,SL}$, $\omega_{0,SL}$	The coefficients of the leader's expected present value of the future profit increment

$\omega_{II,G}$, $\omega_{I,G}$, $\omega_{0,G}$	The coefficients of the expected present value of social welfare increment caused by the second DT
$\omega_{II,GL}$, $\omega_{I,GL}$, $\omega_{0,GL}$	The coefficients of the expected present value of social welfare increment caused by the first DT
v_{F1}, v_{F2}, v_{F3}, v_{L1}, v_{L2}, v_{L3}	The (minimum) positive roots of Equations (43), (44), (49), (46), (47), and (51)
v_S^*, v_F^*, v_L^*, v_M^*	The shipping lines' optimal DT investment timing in the simultaneous investment scenario, and the follower and the leader. The positive roots of Equation (36)
v_{LG}^*, v_{FG}^*	The social optimal DT investment timing for the first and the second DT
v_{LR}^*, v_{FR}^*	The regulated DT investment timing for the leader and the follower under complete information
P_L, P_F	The subsidies to the leader and the follower under complete information
v_{LRI}^*, v_{FRI}^*	The regulated DT investment timing for the leader and the follower under incomplete information
P_{LI}, P_{FI}	The subsidies to the leader and the follower under incomplete information
H, h	The c.d.f. and p.d.f. of κ_S
M_1, M_2, M_3, M_4	Different parameter areas in Corollary 1
Γ_1, Γ_2, Γ_3	Different parameter areas in Corollary 3
Subscripts	
1, 2, 3	1 and 2 represent the shipping lines and their DTs; 3 represents the PT
G	Government
S	Shipping lines
N	Without DT
R	Regulation
I	Incomplete information
t	Time
F	Follower
L	Leader
0, I, II	The constants, the coefficients of the linear terms and the coefficients of the quadratic terms in the functions

Appendix C Proofs of the propositions and corollaries

Proof of Lemma 1:

The proof can be obtained by using an approach similar to that of Proposition 1a in Zheng et al. (2020). Here, we present it briefly as follows:

Let $\Delta\pi_{it} = \Delta\pi_{II,St} + \Delta\pi_{I,St} + \omega_{0,S}$ (52)

where $\Delta\pi_{II,St} = \omega_{II,S} v_t^2$ and $\Delta\pi_{I,St} = \omega_{I,S} v_t$. We apply the Ito lemma to $\Delta\pi_{II,St}$ and obtain

$$d\Delta\pi_{II,St} = \omega_{II,S} v_t^2 [(2\mu - 2\sigma\theta_t + \sigma^2)dt + 2\sigma dB_t^\theta].$$

Therefore, $\Delta\pi_{II,St} = \Delta\pi_{II,S0} \exp[(2\mu - \sigma^2)t - 2\sigma\int_0^t \theta_s ds + 2\sigma B_t^\theta)]$ (53)

We also apply the Ito lemma to $\Delta\pi_{I,St}$ and obtain

$$d\Delta\pi_{I,St} = \omega_{I,S} v_t [(\mu - \sigma\theta_t)dt + \sigma dB_t^\theta].$$

Therefore, $\Delta\pi_{I,St} = \Delta\pi_{I,S0} \exp[(\mu - \frac{1}{2}\sigma^2)t - \sigma\int_0^t \theta_s ds + \sigma B_t^\theta]$. (54)

We plug (52), (53), and (54) into (21) and obtain

$$\inf_{\theta\in K_S} E^{Q^\theta}[\int_0^\infty e^{-rt}\Delta\pi_{it} dt \mid F_t] = \inf_{\theta\in K_S} \int_0^\infty E_0^\theta [e^{-rt}(\Delta\pi_{II,S0}\exp(2\mu t - \sigma^2 t - 2\sigma\int_0^t \theta_s ds + 2\sigma B_t^\theta)$$

$$+\Delta\pi_{I,S0}\exp((\mu - \frac{1}{2}\sigma^2)t - \sigma\int_0^t \theta_s ds + \sigma B_t^\theta) + \omega_{0,S})]dt$$

$$= \int_0^\infty [\Delta\pi_{II,S0}\exp((2\mu - r - \sigma^2)t)\cdot \inf_{\theta\in K_S} E_0^\theta(\exp(2\sigma(B_t^\theta - \int_0^t \theta_s ds)))$$

$$+\Delta\pi_{I,S0}\exp((\mu - r - \frac{1}{2}\sigma^2)t)\cdot \inf_{\theta\in K_S} E_0^\theta(\exp(\sigma(B_t^\theta - \int_0^t \theta_s ds))) + \omega_{0,S}\exp(-rt)]dt \quad (55)$$

Notably, $\theta_t \in K_S = [-\kappa_S, \kappa_S]$; thus, we know that

$$E_0^\theta[\exp(2\sigma(B_t^\theta - \int_0^t \theta_s ds))] \ge E_0^\theta[\exp(2\sigma(B_t^\theta - \int_0^t \kappa_S ds))] = E_0^\theta[\exp(2\sigma(B_t^\theta - \kappa_S t))]$$

$$= \exp(2\sigma^2 t - 2\sigma\kappa_S t)$$

$$E_0^\theta[\exp(\sigma(B_t^\theta - \int_0^t \theta_s ds))] \ge E_0^\theta[\exp(\sigma(B_t^\theta - \int_0^t \kappa_S ds))] = E_0^\theta[\exp(\sigma(B_t^\theta - \kappa_S t))]$$

$$= \exp(\frac{1}{2}\sigma^2 t - \sigma\kappa_S t)$$

Therefore, $\inf_{\theta\in K_S} E^\theta[\exp(2\sigma(B_t^\theta - \int_0^t \theta_s ds))] = \exp(2\sigma^2 t - 2\sigma\kappa_S t)$ (56)

and $\inf_{\theta\in K_S} E^\theta[\exp((B_t^\theta - \int_0^t \theta_s ds)\sigma)] = \exp(\frac{1}{2}\sigma^2 t - \sigma\kappa_S t)$. (57)

Plugging (56) and (57) into (55), we obtain

$$\inf_{\theta\in K_S} E^{Q^\theta}[\int_0^\infty e^{-rt}\Delta\pi_{it} dt \mid F_t] = \Delta\pi_{II,S0}\int_0^\infty [\exp((2\mu - r - \sigma^2)t)\cdot\exp(2\sigma^2 t - 2\sigma\kappa_S t)]dt +$$

$$\Delta\pi_{I,S0}\int_0^\infty [\exp((\mu - r - \frac{1}{2}\sigma^2)t)\cdot\exp(\frac{1}{2}\sigma^2 t - \sigma\kappa_S t)]dt + \frac{\omega_{0,S}}{r}$$

$$= \Delta\pi_{II,S0}\int_0^\infty \exp((2\mu - 2\kappa_S\sigma + \sigma^2 - r)t) + \Delta\pi_{I,S0}\int_0^\infty \exp((\mu - \kappa_S\sigma - r)t)dt + \frac{\omega_{0,S}}{r} \quad (58)$$

$$= \frac{\omega_{II,S} v_0^2}{r - 2(\mu - \kappa_S\sigma) - \sigma^2} + \frac{\omega_{I,S} v_0}{r - (\mu - \kappa_S\sigma)} + \frac{\omega_{0,S}}{r}$$

which is (22). □

Proof of Proposition 1:

The proof can be obtained by using an approach similar to that of Proposition 1b in Zheng et al. (2020). Here, we present the proof of $\beta_S > 2$ as follows:

We know that β_S is the positive solutions of the quadratic equation (59):

$$1/2\sigma^2\beta(\beta-1)+\gamma\beta-r=0 \tag{59}$$

where $\gamma = \mu - \kappa_S\sigma$.

Notably, the risk-adjusted discount rates under ambiguity are both positive, leading to $r-2(\mu-\kappa_S\sigma)-\sigma^2>0$. This inequality is equivalent to the inequality in equation (60):

$$1/2\sigma^2\phi(\phi-1)+(\mu-\kappa_S\sigma)\phi-r<0 \tag{60}$$

when $\phi = 2$.

Comparing (59) and (60), we obtain that $\beta_S > 2$. □

Proof of Corollary 1:

By calculating, we have $\dfrac{\partial\omega_{II,S}}{\partial b}=\dfrac{6b^3+13b^2+10b-2}{8(1-b)^2(1+2b)^3}$

$\dfrac{\partial\omega_{I,S}}{\partial b}=\dfrac{2(5f-2)b^3+(11f+2)b^2+2(4+f)b+4f-6}{4(1-b)^2(1+2b)^3}$ and

$$\frac{\partial\omega_{0,S}}{\partial b}=\frac{\begin{array}{c}(6f^2+8f-8)b^6+(177f^2-212f+84)b^5+(96f^2+184f-120)b^4+(102f^2\\+252f-92)b^3+(228f^2-184f+168)b^2+(120f^2-112f+48)b+64f-80\end{array}}{8(1-b)^2(2+b)^2(1+2b)^3}.$$

When $(b,f)\in\Delta_1\bigcup\Delta_2$, $\dfrac{\partial\omega_{II,S}}{\partial b}<0$, $\dfrac{\partial\omega_{I,S}}{\partial b}<0$, and $\dfrac{\partial\omega_{0,S}}{\partial b}<0$. From (24), we know that $\dfrac{\partial v_S^*}{\partial b}>0$. When $(b,f)\in\Delta_4$, $\dfrac{\partial\omega_{II,S}}{\partial b}>0$, $\dfrac{\partial\omega_{I,S}}{\partial b}>0$, and $\dfrac{\partial\omega_{0,S}}{\partial b}>0$. From (24), we know that $\dfrac{\partial v_S^*}{\partial b}<0$. When $(b,f)\in\Delta_3$, $\dfrac{\partial\omega_{II,S}}{\partial b}>0$, $\dfrac{\partial\omega_{I,S}}{\partial b}>0$, and $\dfrac{\partial\omega_{0,S}}{\partial b}<0$. From (24), we know that the sign of $\dfrac{\partial v_S^*}{\partial b}$ is uncertain. □

Proof of Proposition 2:

If $v_t \ge v_F^*$, the follower has invested, and the two symmetric shipping lines divide the market equally and have the same option value as $Z_L = Z_F = \Pi_F(v_t) - I$. If $v_t < v_F^*$, the leader (here, we suppose SL1 without loss of generosity because the two shipping lines are symmetric) would be the only DT investor and his option value would equal to his present value of the future profit increment minus his DT investment, that is, $\Pi_L(v_t)-I$, if the follower (SL 2) does not invest forever. However, because SL2 would invest at v_F^*, SL 1's present value of the future profit increment is "exchanged" from $\Pi_L(v_F^*)$ to $\Pi_F(v_F^*)$ after SL2's investment. Therefore, SL1's present value of the profit increment loss caused by the DT investment of SL 2 at time v_t is $(v_t/v_F^*)^\beta[\Pi_L(v_F^*)-\Pi_F(v_F^*)]$, where the term $(v_t/v_F^*)^\beta$ is the "expected discount factor" (Dixit and Pindyck, 1994). In summary, SL 1's option value before the time v_F^* is $\Pi_L(v_t)-I-\{(v_t/v_F^*)^\beta[\Pi_L(v_F^*)-\Pi_F(v_F^*)]\}=\Pi_L(v_t)-I+(v_t/v_F^*)^\beta[\Pi_F(v_F^*)-\Pi_L(v_F^*)]$. □

Proof of Proposition 3:

Before the time of the follower's DT construction, that is, v_F^*, the leader, SL 1's option value is $(v_t/v_F^*)^\beta[\Pi_F(v_F^*)-I]$ if it chooses waiting. If SL 1 chooses investing, its option value is $\Pi_L(v_t)-I+(v_t/v_F^*)^\beta[\Pi_F(v_F^*)-\Pi_L(v_F^*)]$. When these two values are equal, that is, the following equation holds, SL1 is indifferent between a leader and a follower.

$$\Pi_L(v_t)-I+(v_t/v_F^*)^\beta[\Pi_F(v_F^*)-\Pi_L(v_F^*)]=(v_t/v_F^*)^\beta[\Pi_F(v_F^*)-I] \tag{61}$$

According to the rent equalization principle (Fudenberg and Tirole, 1991), the preemptive strategy is no longer profitable at v_L^*, which is the positive root of Equation (61). In other words, v_L^* is SL 1's optimal DT investment timing as the leader. Substituting (22) and (32) into (61), and performing a rearrangement, we obtain (35). □

Proof of Corollary 2:

First, using a similar approach as in Proposition 1, we can prove that v_M^* is the positive root of Equation (36). Second, by calculating, we have $\omega_{II,S}<\omega_{II,SL}$, $\omega_{I,S}<\omega_{I,SL}$, and $\omega_{0,S}<\omega_{0,SL}$. Comparing (24) and (36), we know that $v_M^*<v_F^*$. Examining (35), we find that when $v_F^*\to\infty$, (35) becomes

$$\Phi_{II,S}\omega_{II,SL}v^2+\Phi_{I,S}\omega_{I,SL}v+\frac{\omega_{0,SL}}{r}=I \tag{62}$$

Comparing (62) and (36), we know that $v_L^* < v_M^*$ when $v_F^* \to \infty$. Moreover, examining (35), we know that $\partial v_L^* / \partial v_F^* > 0$ because $\beta_S > 2$. Therefore, we have $v_L^* < v_M^*$ given all v_F^*. Combining the above proof results, we have $v_L^* < v_M^* < v_F^*$. □

Proof of Proposition 4:

The proof can be directly obtained from Proposition 3 and Corollary 2. □

Proof of Proposition 5:

Use a similar approach as that in Proposition 1. □

Proof of Corollary 3:

For Part (i), the proof of can be directly obtained in a manner similar to that used in Propositions 1 and 5.

It can be analytically shown that $\frac{\omega_{II,G}}{2} - \omega_{II,S} > 0$ and $\frac{\omega_{I,G}}{2} - \omega_{I,S} > 0$ for all $0 < b < 1$ and $0 < f < 1$. Thus, investigating (24) and (41), we can conclude that the comparison between v_F^* and v_{FG}^* depends on the sign of $\frac{\omega_{0,G}}{2} - \omega_{0,S}$.

$$\frac{\omega_{0,G}}{2} - \omega_{0,S} = \frac{-(2-2b+3bf)[(13b^3+46b^2+22b)f+10b^3+6b^2-12b-4]}{16(1-b)(2+b)^2(1+2b)^2}.$$

It is obvious that the denominator of $\frac{\omega_{0,G}}{2} - \omega_{0,S}$ is positive. In the numerator, because $2-2b+3bf > 0$, it can be proved that $(13b^3+46b^2+22b)f+10b^3+6b^2-12b-4 < 0$, when $0 \le b < (\sqrt{55}-3)/23$ or $0 \le f \le \frac{4+12b-6b^2-10b^3}{22b+46b^2+13b^3}$, which analytically illustrates the DT investment threshold between (i), (ii) and (iii) in Corollary 3. □

Proof of Proposition 6:

The proof can be obtained by using an approach similar to that in Proposition 4 in Zheng et al. (2020). Here, we present it briefly as follows:

If $v_{FR} \ge v_{F2}$, we easily verify that v_{FR} satisfies Constraint (42b) when $P_F = 0$. Problem (42a)–(42c) becomes an unconstraint optimization problem as follows:

$$\max_{v_{FR}} \{ (\frac{v_t}{v_{FR}})^{\beta_G} [\frac{U_F(v_{FR})}{2} - I] \} \tag{63}$$

Solving (63) leads to $v_{FR}^{*}=v_{FG}^{*}$, which proves Part (i) of Proposition 6.

If $v_{FR}<v_{F2}$, Constraint (42b) cannot be satisfied with $P_F=0$ and $v_{FR}^{*}=v_{FG}^{*}$. In objective function (42a), we observe that the coefficient of the positive decision variable P_F is negative. Therefore, to maximize (42a), Constraint (42b) is binding, and the following equation must hold at the optimum:

$$P_F=(\frac{v_0}{v_{FR}})^{\beta_S}(I-\Phi_{II,S}\omega_{II,S}v_R^2-\Phi_{I,S}\omega_{I,S}v_{FR}-\frac{\omega_{0,S}}{r}) \quad (64)$$

We substitute (64) into (42a) and find that the first-order condition of v_{FR} is now (43). Therefore, $v_{FR}^{*}=v_{F1}$. If $v_{F1}\geq v_{F2}$, we easily verify that v_{F1} satisfies Constraint (22b) when $P_F=0$, which proves Part (ii) of Proposition 6. If $v_{F1}<v_{F2}$, P_F must be positive and $P_F=(\frac{v_t}{v_{FR}^{*}})^{\beta_S}(I-\Phi_{II,S}\omega_{II,S}v_{FR}^{*2}-\Phi_{I,S}\omega_{I,S}v_{FR}^{*}-\frac{\omega_{0,S}}{r})$ to satisfy (42b) when $v_{FR}^{*}=v_{F1}$, which proves Part (iii) of Proposition 6.
Moreover, we must prove the existence of the positive roots for Equation (43). Let

$$\mathrm{M}(v)=[(\beta_G-2)\Phi_{II,G}\omega_{II,G}+2(\beta_S-2)v^{\beta_G-\beta_S}\Phi_{II,S}\omega_{II,S}]v^2+[(\beta_G-1)\Phi_{I,G}\omega_{I,G}$$

$$+2(\beta_S-1)v^{\beta_G-\beta_S}\Phi_{I,S}\omega_{I,S}]v+\frac{\beta_G\omega_{0,G}+2\beta_S v^{\beta_G-\beta_S}\omega_{0,S}}{r}-2(\beta_G+\beta_S v^{\beta_G-\beta_S})I$$

When $v=0$, $\mathrm{M}(v)=\frac{\beta_G\omega_{0,G}}{r}-2\beta_G I<0$, because $\beta_G>0$ and $I>\min(\frac{\omega_{0,S}}{r},\frac{\omega_{0,G}}{2r})$.

Moreover, $\mathrm{M}(v)$ is a continuous polynomial function with respect to v, and its highest order is greater than or equal to 2. Because $v\geq 0$, $\lim_{v\to\infty}\mathrm{M}(v)=\infty$. Therefore, Equation (43) has at least one positive root. □

Proof of Proposition 7:

Use an approach similar to that of Proposition 6. □

Proof of Proposition 8:

From the Revelation Principle (Fudenberg and Tirole, 1991), we know that the government can limit the search of the mechanism to the class of direct mechanisms, where the follower truthfully reveals his ambiguity level to maximize his profit. Applying the envelop theorem to (48b) implies that

$$\frac{\partial Z_F(\hat{\kappa}_S)}{\partial\kappa_S}=\frac{\partial Y_F(v_{FRI}^{*},\kappa_S)}{\partial\kappa} \quad (65)$$

Therefore, giving the follower the "information subsidy" $P_{F0}=\int_{\kappa_S}^{\kappa_1}\frac{\partial Y_F(v_{FRI}^{*},\kappa)}{\partial\kappa}d\kappa$ can always force him to truthfully reveal his κ_S. Rearranging Constraint (48b) we have

$$P_{FI} = Z_F - (\frac{v_t}{v_{FRI}})^{\beta_S}[\Pi_F(\hat{\kappa_S}) - I] \tag{66}$$

Substituting (65) and (66) into (48a)–(48c), we transform (48a)–(48c) into the following optimal control problem:

$$\max_{v_{FRI}} \int_{\kappa_0}^{\kappa_1} [(\frac{v_t}{v_{FRI}})^{\beta_G}(\frac{U_F}{2} - I) - Z_F + (\frac{v_t}{v_{FRI}})^{\beta_S}(\Pi_F - I)]h(\kappa_S)d\kappa_S \tag{67a}$$

s.t. $(\frac{v_t}{v_{FRI}})^{\beta_S}[\Pi_F - I] + P_{FI} \geq 0$ (67b)

(65)

where v_{FRI} is the control variable, and Z_F is the state variable.

We set up the following Hamiltonian function:

$$H = \int_{\kappa_0}^{\kappa_1} [(\frac{v_t}{v_{FRI}})^{\beta_G}(\frac{U_F}{2} - I) - Z_F + (\frac{v_t}{v_{FRI}})^{\beta_S}(\Pi_F - I)]h(\kappa_S) + \lambda \frac{\partial Y_F(v_{FRI}, \kappa_S)}{\partial \kappa} \tag{68}$$

where λ is the co-state variable of κ_S. The first-order conditions of this Hamiltonian function with respect to v_{FRI} and Z_F are

$$\frac{\partial H}{\partial v_{FGI}} = [(\beta_G - 2)\Phi_{II,G}\omega_{II,G} + 2(\beta_S - 2)v^{\beta_G - \beta_S}\Phi_{II,S}\omega_{II,S}]v^2 + [(\beta_G - 1)\Phi_{I,G}\omega_{I,G} + 2(\beta_S - 1)v^{\beta_G - \beta_S}$$
$$\Phi_{I,S}\omega_{I,S}]v + \frac{\beta_G\omega_{0,G} + 2\beta_S v^{\beta_G - \beta_S}\omega_{0,S}}{r} + \frac{H}{h} \cdot \frac{\partial^2 Y_F(v, \kappa_S)}{\partial v \partial \kappa} - 2(\beta_G + \beta_S v^{\beta_G - \beta_S})I + \lambda \frac{\partial^2 Y_F(v, \kappa_S)}{\partial v \partial \kappa} = 0 \tag{69}$$

$$\frac{\partial H}{\partial Z_F} = -h(\kappa_S) = -\frac{d\lambda}{d\kappa_S} \tag{70}$$

From (70), we have

$$\lambda = H(\kappa_S) \tag{71}$$

Substituting (71) into (69) and making simplification, we obtain (49).

If $v_{FG}^* \geq v_{F2}$, the social optimum v_{FG} can be implemented. However, to induce the follower to report his κ_S truthfully, the subsidy P_{F0} is necessary to guarantee Constraint (48b). If $v_{FG}^* < v_{F2}$, the social optimum v_{FG} cannot be implemented and the mixed optimum v_{F3} can be implemented. Because $v_{F3} \geq v_{F2}$, the follower's PC Constraint (67b) can still be satisfied. Therefore, only the information subsidy P_{F0} is necessary. If $v_{FG}^* < v_{F2}$ and $v_{F3} < v_{F2}$, not only the information subsidy P_{F0} but also the participation subsidy $(\frac{v_t}{v_{FRI}^*})^{\beta_S}(I - \Phi_{II,S}\omega_{II,S}v_{FRI}^{*2} - \Phi_{I,S}\omega_{I,S}v_{FRI}^* - \frac{\omega_{0,S}}{r})$ is necessary to implement the mixed optimum v_{F3}.

Comparing (43) and (49), we easily know the relationship between v_{FRI}^* and v_{FR}^*. When $\kappa_S = \kappa_0$, we know that $H(\kappa_S) = 0$, which leads to $v_{FRI}^* = v_{FR}^*$.

The proof of the existence of the positive roots for Equation (49) can be made using logic similar to that in the proof of Proposition 6. □

Proof of Proposition 9:

Use an approach similar to that in Proposition 8. □