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Equilibrium analyses and operational designs of a coupled market with substitutive and complementary ride-sourcing services to public transits

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Abstract

The emerging on-demand ride-sourcing programs provided by transportation network companies (TNCs) have been reshaping the transportation industry. Research efforts in this area have covered topics such as supply-demand equilibrium, pricing, matching, dispatching, but have not fully spread to the potential impacts of ride-sourcing on public transit in multi-modal transportation systems. On the one hand, ride-sourcing services act as convenient feeders to solve first-mile/last-mile problems for public transit riders. On the other hand, direct origin-to-destination ride-sourcing services may also absorb passengers from public transit. In this paper, we propose a user equilibrium based mathematical model to analyze complement and substitution of ride-sourcing to public transit. Through both analytical and numerical discussions, we find that the fleet size of ride-sourcing vehicles can critically affect the complementary and substitutive relationship between ride-sourcing and public transit, and ride-sourcing service fares affect the market share between first-mile/last-mile (i.e., from origin to the transportation hub or from the hub to destination) and direct (i.e., from origin to destination) ride-sourcing services. We also examine the optimal strategies to maximize the TNC's profit and/or social welfare and find that the TNC can implement a Pareto-efficient strategy that makes both the two objectives better off.

Keywords: on-demand ride-sourcing, public transits, cooperation, complementation, substitution

1. Introduction

Urban transportation systems have experienced rapid changes in recent years with the popularity of dynamic (or on-demand) ride-sourcing services provided by transportation network companies (TNCs) such as Uber, Lyft and Didi Chuxing. Through an online platform or a smartphone app, ride-sourcing services provide an efficient approach to connect passengers and drivers for their mobility needs. Like other mobility components (e.g., public transit, taxi, and drive), ride-sourcing services are now playing an indispensable role in transportation systems. It was reported that Uber has operated services in 24 countries, and Didi Chuxing is now serving over 25 million trips every day in 400 cities in China. The emerging ride-sourcing services have refreshed the way people move as well as the pattern of urban mobility. It is therefore important to understand urban transportation systems with ride-sourcing markets in order to assist TNCs and the government in designing better operating strategies to achieve sustainable and efficient urban mobility.

Ride-sourcing services have attracted a lot of attention from researchers. Efforts thus far have been primarily directed toward the theoretical and empirical analysis of the properties of ride-sourcing markets (Zha et al., 2016; Ke et al., 2019a; Xu et al., 2018; Yang et al., 2019). From the system's perspective, the relation between ride-souring and other existing travel modes plays a determining role in the success of urban mobility. Due to the complexity, few theoretical framework have been proposed to comprehensively understand the potential implications of ride-sourcing services for a multi-modal transportation system. Although there is case-by-case empirical evidence (e.g., Li et al., 2016; Schaller, 2018, Hall et al., 2019), the research gap in analytical efforts still raises debates. Advocates argue that ride-sourcing services are complements to the existing travel modes. Moreover, efficient matching between drivers and passengers is generally expected to increase vehicle utilization rate, thereby achieving various socially beneficial objectives such as alleviating traffic congestion and air pollution. Critics, however, claim that by providing more convenient and comfortable ride services, TNCs add to vehicle traffic by attracting travelers from space-efficient modes like walking, transit, and biking. To this end, we aim to provide an analytical examination of the complement and substitution between ride-sourcing services and public transits in a multi-modal transportation market.

In a multi-modal urban transportation market, the two modes of ride-sourcing and public transit have distinctive features. On the one hand, ride-sourcing services accommodate on-demand door-to-door travel needs, which are generally costly; on the other hand, public transit services generally have fixed schedule and limited accessibility, but have low monetary costs (i.e., fares). The two modes are either substitutive or complementary. Passengers can take a ride-sourcing service or take a combination of ride-sourcing and public transit services to reach their destinations. In the latter case, ride-sourcing provides first-mile and (or last-mile) services as a feeder to public transit. Clearly, the demands for different travel modes depend on the competitiveness and complementarity of the two modes, which in turn depend on factors such as a passenger's total trip distance from origin to destination, the trip fares for the two modes, and the first-mile/last-mile distances. Cooperation between the government, the platform operator, and other service providers is essential for achieving a well-balanced multi-modal transportation system. In this paper, we develop a mathematical model to explain the intricate relationships among system decision variables (such as ride-sourcing fare rates, i.e., trip fare per unit distance), exogenous variables (such as network topologies, speeds of various modes) and endogenous variables (such as ride-sourcing waiting time) in a multi-modal transportation market with ride-sourcing and public transit services. Based on the modeling framework, we analytically and numerically investigate the effects of the decision variables (i.e., ride-sourcing fare rates and vehicle fleet size) on passengers' mode splits and examine the substitution and complement between ride-sourcing and public transit. Particularly, we discuss the optimal strategies for maximizing the TNC's profit in monopoly markets and for maximizing social welfare in social preferable markets. We note that the TNC can find a Pareto-efficient strategy that satisfies both the social welfare and its own profit. The contributions of this paper are summarized as follows:

- We develop a modeling framework to describe a market equilibrium where passengers choose among direct ride-sourcing services, public transit services, and the bundle of first-mile/last-mile ride-sourcing services and public transit services. It is different from most previous studies, e.g. Zha et al. (2016), Ke et al. (2020b), which only considered one type of ride-sourcing service (i.e., standard ride-sourcing service or ride-pooling service) and analyzed passengers' mode choices between a single ride-sourcing service and other non-ride-sourcing modes.
- We investigate the effects of the TNC's differentiated pricing strategy for direct and first-mile/last-mile ride-sourcing service, and vehicle fleet size, on the passengers' mode choices, the TNC's profit and social welfare. In particular, we analytically determine the optimal pricing formulas at monopoly optimum, social optimum.
- Our numerical and theoretical analyses offer guidance for the TNC to design appropriate operating strategies under different circumstances. Meanwhile, a Pareto-efficient frontier with the set of second-best solutions is sought out to well balance the trade-off between the TNC's profit and social welfare, which can help the government design efficient regulatory policies.

The remaining of this paper is organized as follows. Section 2 presents a comprehensive literature review on existing progresses on ride-sourcing research, as well as its impact on multi-modal transportation systems. Section 3 introduces a mathematical model that spells out the relation among the key variables in a transportation market with ride-souring and public transit. In Section 4, we propose optimization problems with respect to the profit of the TNC in monopoly markets and the social welfare in social preferable markets. To fully understand the complement and substitution of a ride-sourcing market to public transit as well as the impacts of decision variables on the objectives (i.e., the TNC's profit or social welfare) we illustrate numerical case studies in Section 5. Finally, Section 6 concludes this paper.

2. Literature review

Since the emergence in 2009, ride-sourcing services have been reshaping urban mobility and attracting heated discussions from researchers. A ride-souring market is similar to a taxi market in many ways, for example, their service fares are positively related to trip distance and time, and there are search and meeting frictions between passengers and drivers. Therefore, research on traditional taxi markets (Yang et al., 2010; Yang and Yang, 2011) and e-hailing taxi markets (He et al., 2015; He et al., 2018) have provided theoretical foundations for the analysis of ride-sourcing markets. Different from traditional taxi markets, there is no strict entry limitation in ride-sourcing markets; qualified registered private car owners have the flexibility to join or leave the market as dedicated drivers at any time (Rayle et al., 2014). Consequently, the balance between passenger demand and driver supply is an essential component in modeling ride-sourcing markets.

General research issues associated with ride-sourcing markets include the analysis of conditions and properties of the stationary supply-demand equilibrium (Bimpikis et al., 2019; Cachon et al., 2017; Xu et al., 2017; Zha et al., 2017; Ke et al., 2019a), temporal and spatial supply-demand predictions (Tong et al., 2017; Ke et al., 2017; Ke et al., 2019b), ride-sharing pattern analysis (Zhu et al., 2020b), implications on traditional taxi markets (Nie, 2017), competition between TNCs (Zha et al., 2016; Heilker and Sieg, 2018), pricing and surge pricing strategies (Castillo et al., 2017), government regulations and policies (Bai et al., 2018; Li et al., 2019), geometrical matching (Zha et al., 2018), order dispatching (Xu et al., 2018; Ke et al., 2020a), and even dynamic ride-sharing (Wang et al., 2018). However, these studies seldom consider the influence of ride-sourcing markets on existing travel modes (e.g., public transit, and driving) as well as the multi-modal urban transportation system.

The understanding of how ride-sourcing reshapes urban mobility can be biased without its interaction with public transit. As ride-sourcing becomes more and more preferable among passengers, its influence on public transit can be double-edged. On the one hand, ride-sourcing services act as feeders to solve first-mile or last-mile problems; on the other hand, they may also draw passengers away from public transit. There have been studies treating traditional modes (e.g., taxis, shuttles) and advanced modes (e.g., automated vehicles) as the complement for public transit (Yap et al., 2016; Wang, 2017; Chen and Wang, 2018). Compared with these traditional/advanced modes, ride-sourcing services provided by TNCs are more convenient/practical for passengers. In order to meet the needs of geographic coverage, some public transit operators have considered to integrate fixed route/point/zone ride-sourcing services into public transit systems (Li and Quadrifoglio, 2010; Chen and Nie, 2017; Maheo et al., 2017). Recently, there is collaboration between governments and TNCs to improve service quality (McCoy et al., 2018). However, these studies mainly focus on the development of efficient dispatching, scheduling and routing algorithms at a microscopic level. Equilibrium-based models have been proposed for system-level ride-sourcing and public transit ridership analysis, in which public transit was merely treated as a minor mode without a detailed formulation of its characteristics (Zha et al., 2016; Wang et al., 2018). Zhu et al. (2020a) examined the impacts of dynamic ride-sharing on public transit via a network model; they found that long-distance ride-sharing services provided by TNCs are absolute competitors for public transit, and TNCs can help to maintain a high level of public transit usage by implementing certain operating strategies.

There are also empirical studies showing the complement and substitution of ride-sourcing to public transit, as well as the influence of ride-sourcing services on traffic congestion. Based on urban mobility report and Uber ride-sourcing data, Li et al. (2016) found that Uber ride-sharing service has significantly reduced traffic congestion in urban areas in the United States. Using a difference-in-differences approach, Hall eta al. (2019) found that Uber is a complement for public transit on average, increasing public transit ridership by five percent after two years from its emergence. Particularly, the positive effects of Uber's entry on transit ridership are more significant in larger cities and for smaller transit agencies. However, Schaller (2018) claimed that ride-sourcing services increase vehicle usage by putting 2.8 new vehicle miles on the road for each mile in auto travel taken off. The conclusions from these empirical studies are mixed or inconsistent, partially due to the varying demand patterns, supply patterns, urban transportation network topologies across cases.

To summarize, comprehensive analytical modeling frameworks are emergently needed to help researchers to better understand the mechanism of how ride-sourcing markets affect the usage of public transit and the underlying impacts multi-modal transportation markets. This paper aims to fill this research gap by developing an equilibrium-based mathematical model that spells out passengers' choices in a transportation market with ride-sourcing and public transit. The model provides insights on the conditions and properties of the equilibrium among ride-sourcing services, public transit services, and the bundled services of the former two modes. Moreover, we investigate the operational designs (with the TNC's decision variables) in the multi-modal transportation market to maximize profit or social welfare.

It is worthy to mention the difference between our study and the seminal works in terms of methodology. First, most of the previous studies directed towards equilibrium analyses for ride-sourcing markets, such as Zha et al. (2016) and Ke et al. (2020b), only considered one type of ride-sourcing service in the market, for example, a regular ride-sourcing service or a ride-poling service. Then they used a simple demand function to characterize the declining trend of demand for ride-sourcing services with respect to its generalized cost. These models cannot be simply adopted to our research problem, since we need to describe passengers' mode choices between three modes, namely, public transit mode, direct ride-sourcing mode and bundled mode. To address this issue, we propose a model that can characterize a market equilibrium with three travel modes and analytically determine monopoly optimum, social optimum and second-best solutions. On the other hand, Zhu et al. (2020a) is the first to investigate the substitutive and complementary effects of ride-sourcing service to public transits in a theoretical way. However, their model is a network-based disaggregate model, which is hard for analytical investigations. In contrast, our model can determine the optimal pricing formulas at monopoly and social optimum, and investigate their properties.

3. Model description

Consider a continuous marketplace with a number of transportation hubs (i.e., public transit services are available in the hubs), where passengers' origins and destinations are also uniformly distributed over the space. Since the number of transportation hubs is finite, the distances between passengers' origins/destinations to the transportation hubs are not negligible. Thus passengers majorly have three travel modes:

- Mode ***b*** (bundled mode): he or she opts for a combination of two separate rides: one public transit service and ride-sourcing service, while the latter is used for addressing the first-mile/last-mile issues. For example, he or she takes an Uber to the closest transportation hub, then takes a bus to his/her destination.
- Mode ***p*** (public transit mode): he or she first walks (or bikes) to the closest transportation hub, then takes a public transit to his/her destination. Namely, he or she addresses the first-mile/last-mile issues by walking or biking;
- Mode ***r*** (direct ride-sourcing mode): he or she takes a direct ride-sourcing service from his/her origin to destination;

It is worth mentioning that we consider an aggregate model here without considering the road network. This aggregate model with three major travel options is to some degree representative for commuters who do not have their own private cars. In many cities, people's houses are sparsely distributed in suburb areas without easy accessibility to public transit services. Thus, for passengers without a private vehicle, the first-mile/last-mile issue of taking public transit is not negligible. A question faced by public transit riders is whether to pay a large trip fare by calling a Uber or spend a long time by walking/biking to/from the transportation hub. Alternatively, they can also call a ride-sourcing service directly from the origin to the destination, which is time-saving but expensive. Clearly, there are both substitutive and complementary effects between ride-sourcing services and public transit services: the first-mile/last-mile ride-sourcing service acts as a complement to public transit, but the direct ride-sourcing service is a substitution.

From the demand side, let $\bar{q}$ denote the total passenger demand rate (i.e., number of passengers per unit time), let q_b, q_p, and q_r represent demand rate for modes ***b***, ***p***, and ***r***, respectively. From the supply side, let N denote the vehicle fleet size of the ride-sourcing platform (i.e., the TNC). As for geographical topology, let l_{om} denote the average distance between passengers' origins/destinations to the nearest transportation hubs, let l_{md} denote average distance between the transportation hubs and the nearest origins/destinations of passengers, and let l_{od} denote the average distance from passengers' origins to destinations. We utilize average distances in this paper because they generally reflect the average accessibility of different service modes in a multi-modal transportation system. Instead, heterogeneous service distances could not bring more analytical insights but definitely cause complexity in model formulation. Let v_w, v_r, and v_p, respectively, indicate the average speeds of walking/biking, ride-sourcing vehicles, and public transits. For simplicity, we do not consider traffic congestion and simply assume the speeds are constant. Let w_{fr}, w_{dr}, and w_p, respectively, denote the average waiting times of first-mile/last-mile ride-sourcing services, direct ride-sourcing services, and public transit services. We assume the waiting time of public transit services is a constant, while the waiting time of first-mile/last-mile (or direct ride-sourcing) services is affected by availability of ride-sourcing vehicles, namely, the number of idle vehicles N^v over the marketplace. The N^v in turn depends on the fleet size N, and the demands for the two types of ride-sourcing services q_b and q_r. Without loss of generality, we assume passengers have a homogenous value of time (VOT), denoted by β. Let τ_b, τ_r, and τ_p indicate the trip fare rate (i.e., monetary trip fare per unit distance) of first-mile/last-mile ride-sourcing services, direct ride-sourcing services, and public transit services. In summary, variables l_{om}, l_{md}, l_{od}, v_w, v_r, v_p, w_p, τ_p, $\bar{q}$ and β are exogenous, variables q_b, q_p, q_r, N^v, w_{fr}, and w_{dr} are endogenous, and τ_b, τ_r, N are decision variables determined by the TNC.

Given the exogenous variables and decision variables, passengers' choices among the three modes form stationary demand rates q_b, q_p, and q_r and will affect ride-sourcing waiting times w_{fr} and w_{dr}. There are several ways to approximate the average waiting time according to previous studies. For example, Arnott, (1996) and Zha et al., (2017) assumed the customers' average waiting time to be inversely proportion to the square root of the number of idle vehicles, Bai et al. (2018) approximated the customers' average waiting time through a queuing model. In this paper, we assume the TNC implements a first-come-first-serve mechanism in which a passenger who raises a request (for either first-mile/last-mile service or direct ride-sourcing service) will be immediately matched with the closest idle driver. This implies the TNC does not discriminate either type of ride-sourcing services in the driver-passenger matching process. In this case, the matching time for both ride-sourcing services (the time from requesting the order to being matched online with a vehicle) is negligible, such that the waiting time is dominated by the picking-up time (the time from being matched online to being picked up by the vehicle) for both first-mile/last-mile services and direct ride-sourcing services. Moreover, due to the indiscrimination in dispatching, the average waiting time of a first-mile/last-mile ride-sourcing service is equal to the average waiting time of a direct ride-sourcing service, and given by:

$$w_{fr} = w(N^v) \tag{1}$$

$$w_{dr} = w(N^v) \tag{2}$$

where $w(\cdot)$ is a function of N^v with properties $w' < 0, w'' > 0$. In a stationary equilibrium state, each vehicle will be in one of the following disjoint status: (1) idle and waiting for a dispatch; (2) on the way of picking up a passenger who opts for first-mile/last-mile ride-sourcing service; (3) in the trip of delivering a passenger who opts for first-mile/last-mile ride-sourcing service; (4) on the way of picking up a passenger who opts for direct ride-sourcing service; (5) in the trip of delivering a passenger who opts for direct ride-sourcing service. Therefore, we shall have the following vehicle conservation equation:

$$N = N^v + q_b w_{fr} + q_b \frac{l_{om}}{v_r} + q_r w_{dr} + q_r \frac{l_{od}}{v_r} \tag{3}$$

where N^v indicates the number of idle vehicles in aforementioned status (1), $q_b w_{fr}$ indicates the number of vehicles in status (2), $q_b l_{om}/v_r$ indicates the number of vehicles in status (3), $q_r w_{dr}$ equals the number of vehicles in status (4), and $q_r l_{od}/v_r$ refers to the number of vehicles in status (5).

Table 1. Average trip fare and time cost of three modes

Mode	Average trip fare	Average trip time
b	$\tau_b l_{om} + \tau_p l_{md}$	$\left(\frac{l_{om}}{v_r}\right) + \left(\frac{l_{md}}{v_p}\right) + w_{fr} + w_p$
p	$\tau_p l_{md}$	$\left(\frac{l_{om}}{v_w}\right) + \left(\frac{l_{md}}{v_p}\right) + w_p$
r	$\tau_r l_{od}$	$\left(\frac{l_{od}}{v_r}\right) + w_{dr}$

Based on the performance measurements (i.e., waiting times, in-vehicle times, and trip fares), the average fares and trip times through the three modes (i.e., ***b***, ***p***, and ***r***) are given in Table 1. As aforementioned, the demands for the three modes q_b, q_p, and q_r are determined by the generalized cost of the three modes. With a homogenous VOT, the generalized cost of passengers opting for modes ***b***, ***p***, and ***r***, respectively, are given by,

$$C_b = \tau_b l_{om} + \tau_p l_{md} + \beta \left(\frac{l_{om}}{v_r} + \frac{l_{md}}{v_p} + w_{fr} + w_p \right) \tag{4}$$

$$C_p = \tau_p l_{md} + \beta \left(\frac{l_{om}}{v_w} + \frac{l_{md}}{v_p} + w_p \right) \tag{5}$$

$$C_r = \tau_r l_{od} + \beta \left(\frac{l_{od}}{v_r} + w_{dr} \right) \tag{6}$$

We let $F_b = \tau_b l_{om}$, which represents the average trip fare spent on the first-mile/last-mile ride-sourcing services by a passenger, let $F_r = \tau_r l_{od}$, which refers to the average trip fare spent for the direct ride-sourcing services by a passenger. Given the arrival rate of all passengers $\bar{q}$, the demands for the three modes are given by,

$$q_b = \Lambda_b\left(C_b, C_r, C_p\right) \tag{7}$$

$$q_r = \Lambda_r\left(C_b, C_r, C_p\right) \tag{8}$$

$$q_p = \bar{q} - q_b - q_r \tag{9}$$

where $\Lambda_b\left(C_b, C_r, C_p\right)$ is the demand function for mode ***b***, $\Lambda_r\left(C_b, C_r, C_p\right)$ is the demand function for mode ***r***. We assume $\Lambda_b^{(1)} < 0$, $\Lambda_b^{(2)} \geq 0$, $\Lambda_b^{(3)} \geq 0$, $\Lambda_r^{(1)} \geq 0$, $\Lambda_r^{(2)} < 0$, $\Lambda_b^{(3)} \geq 0$, where $\Lambda_b^{(i)}$ and $\Lambda_r^{(i)}$ refers to the first-order partial derivative of demand function $\Lambda_b\left(C_b, C_r, C_p\right)$ and $\Lambda_r\left(C_b, C_r, C_p\right)$ with respect to their i^{th} argument, respectively. This assumption indicates that the demand for one specific mode is negatively dependent on the generalized cost for this mode itself, but positively (or not) related to the generalized cost for its alternative modes. To sum up, the market equilibrium state can be solved by a system of nonlinear simultaneous equations consisting of Eqs. (1)-(9). The endogenous waiting times affect passengers' mode choices through the demand functions Eqs. (7)-(9) and the generalized cost functions Eq. (4)-(6), while the changes of passenger arrival rates will in turn influence the waiting times via the waiting time functions and the vehicle conservation equation, i.e. Eqs. (1)-(3). The interactions between the waiting times and the demand rates for the three modes constitute the market equilibrium. Given fixed exogenous settings, the TNC can influence this market equilibrium by leveraging the trip fare rates τ_b and τ_r (or alternatively, the average trip fares for the two services F_b and F_r), and the vehicle fleet size N.

4. Optimal strategy design

In this section, we investigate the optimal strategies (i.e., decision variables) under the equilibrium condition in the aforementioned multi-modal market. The following three market scenarios are considered and examined:

- Monopoly optimum (***MO***): the TNC freely chooses a strategy (i.e. a group of decision variables) to maximize its own profit, which is equal to the total trip fares charged from both first-mile/last-mile and direct ride-sourcing services less the total operating cost of its vehicles.
- Social optimum (***SO***): a strategy is chosen to maximize the total social welfare, which is defined as the summation of consumer surplus and TNC's profit. The social optimum is also called first-best (FB) solution, which may not be achievable since the TNC's profit at SO is generally negative.

- Second-best solution (***SB***): a strategy is chosen to maximize the total social welfare, while guaranteeing the TNC's profit to be larger than a certain level. The SB solution makes a trade-off between the MO and SO by balancing the two distinct objectives for maximizing the TNC's profit and social welfare.

4.1 Monopoly optimum

Consider a monopoly market where the TNC aims to maximize its profit by leveraging the three major decision variables: the trip fare rate for direct ride-sourcing service τ_r, the trip fare rate for first-mile/last-mile ride-sourcing service τ_b, the vehicle fleet size N. We understand that the ride-sourcing platforms (i.e., TNC) do not own vehicles and use wages to attract drivers to offer on-demand ride services. However, Zha et al. (2017) showed that a TNC behaves like a taxi platform which directly controls the fleet size, under the condition that rivers' reservation cost is homogenous and the potential driver supply is sufficient such that potential drivers will keep entering the ride-sourcing market unit the average net earnings of drivers reach zero. In other words, the TNC can control the fleet size through the wages paid to drivers, and an indirect control of fleet size is equivalent to a direct control of wages. In this section, we adopt Zha et al. (2017)'s assumption for simplicity, then the profit-maximization problem can be formulated as follows:

$$\text{(P1)} \max_{F_b, F_r, N} \Pi = q_b F_b + q_r F_r - cN \tag{10}$$

$$\text{s.t.} \quad \text{Eqs.}\,(1)-(9)$$

where Π is the profit per hour of the TNC, c is the operating cost per unit time of one ride-sourcing vehicle (or driver), demand for the two ride-sourcing modes q_b, q_r are endogenously determined by the market equilibrium depicted by Eq. (1)-(9). In Appendix A, we show that P1 is equivalent to a profit-maximizing problem with trip fares and wages as decision variables, under a driver supply assumption made by Zha et al. (2017). The average trip fare for bundled service F_b (i.e., $\tau_b l_{om}$), the average trip fare for direct ride-sourcing service F_r (i.e., $\tau_r l_{od}$) and the vehicle fleet size N are treated as the three decision variables. The first-order conditions of P1 are given by,

$$F_b = c\left(\frac{l_{om}}{v_r} + w_{fr}\right) + \frac{q_r \Lambda_r^{(1)} - q_b \Lambda_r^{(2)}}{\Lambda_r^{(2)} \Lambda_b^{(1)} - \Lambda_b^{(2)} \Lambda_r^{(1)}} \tag{11}$$

$$F_r = c\left(\frac{l_{od}}{v_r} + w_{dr}\right) + \frac{q_b \Lambda_b^{(2)} - q_r \Lambda_b^{(1)}}{\Lambda_r^{(2)} \Lambda_b^{(1)} - \Lambda_b^{(2)} \Lambda_r^{(1)}} \tag{12}$$

$$c = -(q_b + q_r)(\beta + c)w' \tag{13}$$

where Eq. (11) represents the pricing formula for the first-mile/last-mile ride-sourcing service, and takes the form of the Lerner formula (Lerner, 1934). The RHS of Eq. (11) consists of two terms: the first term represents the marginal cost of using a vehicle to serve a passenger in the trip phase (cl_{om}/v_r) and pick-up phase (cw_{fr}), while the second term is a monopoly mark-up that represents the market power of the monopoly platform to distort the trip fare from its efficient level. It is worth noting that the monopoly mark-up not only depends on the direct elasticities $\Lambda_r^{(2)}$ and $\Lambda_b^{(1)}$, but is also affected by the cross elasticities $\Lambda_r^{(1)}$ and $\Lambda_b^{(2)}$. Specifically, if the cross elasticities are equal to zero, namely, the demand for one mode only depends on the generalized cost of this mode instead of the generalized costs of other modes, then the monopoly mark-up reduces to $-q_b/\Lambda_b^{(1)}$, which is consistent with the MO pricing formula in Ke et al. (2020b) in a market with one type of ride-sourcing service.

Meanwhile, Eq. (12) describes the pricing formula for the direct ride-sourcing service, and also follows the Lerner formula. The RHS of Eq. (12) consists of two terms: the first term indicates the marginal cost of using a vehicle to serve a passenger in both the trip phase and pick-up phase, while the second term is a monopoly mark-up. Although this mark-up also contains the direct elasticities and cross elasticities, it has a different form with the monopoly mark-up in Eq. (11). Particularly, as the cross elasticities equals zero, the monopoly mark-up reduces to $-q_r/\Lambda_r^{(2)}$, which is only related to the demand and elasticity f the demand function of the direct ride-sourcing service.

4.2 Social optimum

We now consider an idealized scenario where a strategy is chosen to maximize the social welfare, which is defined as summation of the consumer surplus and the TNC's profit. Define $V(C_b, C_r)$ as the customers' surplus from completing a ride-sourcing trip (Zha et al., 2016). With certain regularity conditions (Sheffi, 1985), the following properties hold by construction:

$$\frac{\partial V}{\partial C_r} = -q_r \tag{14}$$

$$\frac{\partial V}{\partial C_b} = -q_b \tag{15}$$

The social-welfare maximization problem can then be formulated as:

$$\text{(P2)} \max_{F_b, F_r, N} S = V(C_b, C_r) + q_b F_b + q_r F_r - cN \tag{16}$$

$$\text{s.t.} \quad \text{Eqs.} (1) - (9)$$

where S is the social welfare, which is equal to the sum of consumer surplus $V(C_b, C_r)$ and the TNC's profit $q_b F_b + q_r F_r - cN$, while C_b, C_r, q_b, q_r are endogenously determined by the market equilibrium depicted by Eq. (1)-(9). It is shown that the first-order conditions of P2 are given by,

$$F_b = c\left(\frac{l_{om}}{v_r} + w_{fr}\right) \tag{17}$$

$$F_r = c\left(\frac{l_{od}}{v_r} + w_{dr}\right) \tag{18}$$

$$c = -(q_b + q_r)(\beta + c)w' \tag{19}$$

It can be shown that the SO trip fare for the first-mile/last-mile ride-sourcing service F_b is equal to the corresponding MO trip fare less the monopoly mark-up. Namely, F_b at social optimum is equal to the marginal cost of using one vehicle to serve a passenger in both the pick-up and delivery phase. Similarly, the trip fare for direct ride-sourcing service at social optimum F_r is also given by the marginal cost of employing one vehicle to serve a passenger caused by the pick-up (cw_{dr}) and delivery (cl_{od}/v_r). In addition, we can observe that Eq. (19) takes exactly the same form as Eq. (13), which indicates that this property holds at both MO and SO. At the SO condition, the TNC's profit is given by:

$$\Pi^{so} = q_b c\left(\frac{l_{om}}{v_r} + w_{fr}\right) + q_r c\left(\frac{l_{od}}{v_r} + w_{dr}\right) - cN = -cN^v < 0 \tag{20}$$

which shows that the TNC's profit at the SO solution is always negative. This implies that the SO is unattainable unless the government subsidizes the TNC.

4.3 Second-best solution

Since the TNC's profit may be in deficit in the SO (i.e., FB) scenario, we consider an SB scenario in which a strategy is chosen to maximize the social welfare while ensuring a certain level of profit:

$$\text{(P3)} \max_{F_b, F_r, N} S = V(C_b, C_r) + q_b F_b + q_r F_r - cN \tag{21}$$

$$\text{s.t.} \quad q_b F_b + q_r F_r - cN \geq \Pi^* \tag{22}$$

$$\& \quad \text{Eqs.} (1) - (9)$$

where objective (21) maximizes the social welfare, constraint (22) guarantees the TNC's profit is larger than a targeted level Π^*, while Eq. (1)-(9) ensures all the system endogenous variables meets the market equilibrium conditions. To solve this problem, we can formulate a Lagrangian as follows:

$$\text{(P3.1)} \max_{F_b, F_r, N} L = S + \xi(q_b F_b + q_r F_r - cN - \Pi^*) \tag{23}$$

$$\text{s.t.}\quad \text{Eqs.}\,(1)-(9)$$

where ξ is the Lagrange multiplier. The first-order conditions of P3.1 are given by,

$$F_b = c\left(\frac{l_{om}}{v_r} + w_{fr}\right) + \xi\,\frac{q_r\Lambda_r^{(1)} - q_b\Lambda_r^{(2)}}{\Lambda_r^{(2)}\Lambda_b^{(1)} - \Lambda_b^{(2)}\Lambda_r^{(1)}} \tag{24}$$

$$F_r = c\left(\frac{l_{od}}{v_r} + w_{dr}\right) + \xi\,\frac{q_b\Lambda_b^{(2)} - q_r\Lambda_b^{(1)}}{\Lambda_r^{(2)}\Lambda_b^{(1)} - \Lambda_b^{(2)}\Lambda_r^{(1)}} \tag{25}$$

$$c = -(q_b + q_r)(\beta + c)w' \tag{26}$$

Clearly, the pricing formulas at the SB solution, i.e. Eq. (24) and Eq. (25), are linear combinations of the pricing formulas at the monopoly optimum, i.e. Eq. (11) and Eq. (12), and the pricing formulas at the SO, i.e. Eq. (17) and Eq. (18). In the meantime, Eq. (26) has the same form as Eq. (13) and Eq. (19). This indicates that the formula in Eq. (26) holds for all SB solutions at the Pareto-efficient frontier that connects the monopoly optimum and social optimum (Yang and Yang, 2011). Since, no one can simultaneously increase both the TNC's profit and social welfare at any point of the Pareto-efficient frontier (namely, an SB solution), the Pareto-efficient frontier is generally regarded as an important measure for the performance of a specific regulation. A regulation/policy is said to be Pareto-efficient, if it can induce the TNC (which maximizes its own profit in the feasible space constrained by the regulation) to choose a targeted Pareto-efficient strategy (or second-best solution).

5. Numerical case study

The equilibrium-based model developed in Sections 3 and 4 characteristics the behavior of a transportation market with ride-sourcing and public transit. Optimal strategies can be implemented by the TNC to affect the equilibrium mode demands for achieving the MO situation (P1 in Section 4.1), the FB situation (P2 in Section 4.2), or the SB situation (P3 or P3.1 in Section 4.3). Due to the complexity of the model, the complementary and substitutive relationship between ride-sourcing and public transit cannot be fully spelled out through analytical derivations. In this section, numerical case studies are conducted to provide an intuitive sense of the equilibrium states and profit and/or social welfare maximization in the multi-modal market.

Without loss of generality, we utilize the logit model as the demand functions (Fisk, 1980), such that

$$q_b = \bar{q}\,\frac{\exp\left(-\kappa C_b\right)}{\exp(-\kappa C_b) + \exp(-\kappa C_r) + \exp\left(-\kappa C_p\right)} \tag{27}$$

$$q_p = \bar{q}\frac{\exp(-\kappa C_p)}{\exp(-\kappa C_b) + \exp(-\kappa C_r) + \exp(-\kappa C_p)} \tag{28}$$

$$q_r = \bar{q}\frac{\exp(-\kappa C_r)}{\exp(-\kappa C_b) + \exp(-\kappa C_r) + \exp(-\kappa C_p)} \tag{29}$$

where κ denotes the sensitivity coefficient of passengers' generalized costs. Based on the logit model, the consumers' surplus can be given by the following log-sum formula:

$$V(C_b, C_r) = \frac{\bar{p}}{\kappa}\log\left[\exp(-\kappa C_b) + \exp(-\kappa C_r) + \exp(-\kappa C_p)\right] + V_0 \tag{30}$$

which satisfies Eqs. (14)-(15).

Moreover, the waiting time of ride-sourcing services is estimated by the following formula, which has been used in previous studies (Ke et al., 2020b):

$$w_{fr} = w_{dr} = \frac{A}{N^{v\alpha}} \tag{31}$$

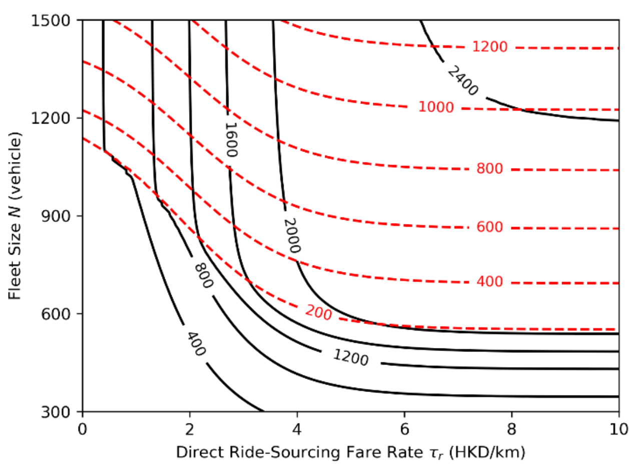
where A is a coefficient depending on area and network structures of examined city, vehicular speed, etc. while α is a dimensionless parameter representing the returns to scale. Normally, α is set to be 0.5, indicating that the average waiting time is inversely proportional to the square root of the number of idle vehicles.

We consider multi-modal travel scenarios in Hong Kong with the following exogenous parameters: $v_w = 5.6$ km/h, $v_r = 50.0$ km/h, $v_p = 45.0$ km/h, $\tau_p = 1.0$ HKD/km, $\beta = 80$ HKD/h, $c = 50$ HKD/vehicle·h, $\bar{q} = 3{,}000$ passenger/h, $A = 2.0$ h, $\alpha = 0.5$, $w_p = 0.10$ h, $\kappa = 0.1$ and $V_0 = 200$ k HKD/h. Let $l_{md} = 10.0$ km and $l_{od} = 10.0$ km, we examine three scenarios with different first-mile/last-mile distances: (1) $l_{om} = 1.0$ km represents a short (i.e., walking/biking accessible) first-mile/last-mile scenario; (2) $l_{om} = 1.5$ km represents a scenario with a medium first-mile/last-mile distance; and (3) $l_{om} = 2.0$ km denotes a long first-mile/last-mile distance scenario. Although numerical studies in one paper could be biased for all the real-world scenarios, the aforementioned numerical setting is represented in many large urban areas. That is, the first-mile/last-mile problem in the reality can be referred to as the "first-several-mile" or "last-several-mile" problem (Chen and Wang, 2018). We first discuss the changes of the equilibrium conditions with the TNC's decision variables (i.e., τ_b, τ_r, and N). Secondly, we analyze the TNC's strategies for profit and/or social welfare maximization (i.e., P1, P2, and P3).

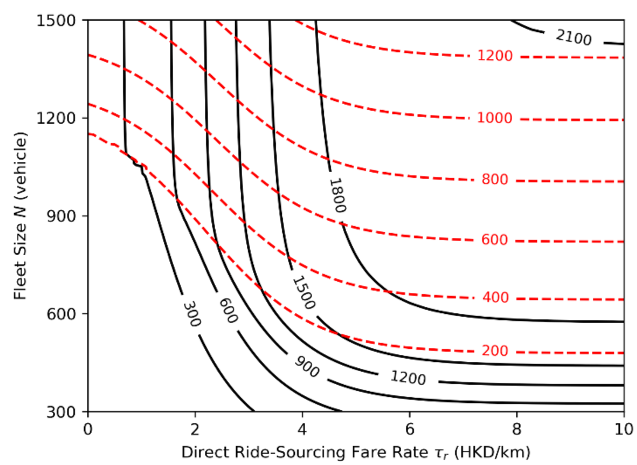
5.1 Analysis of equilibrium states

The numerical results below illustrate the equilibrium states with a medium first-mile/last-mile distance (i.e., $l_{om} = 1.5$ km). We allow the decision variables to vary within a large range to comprehensively examine the impact on passengers' choices.

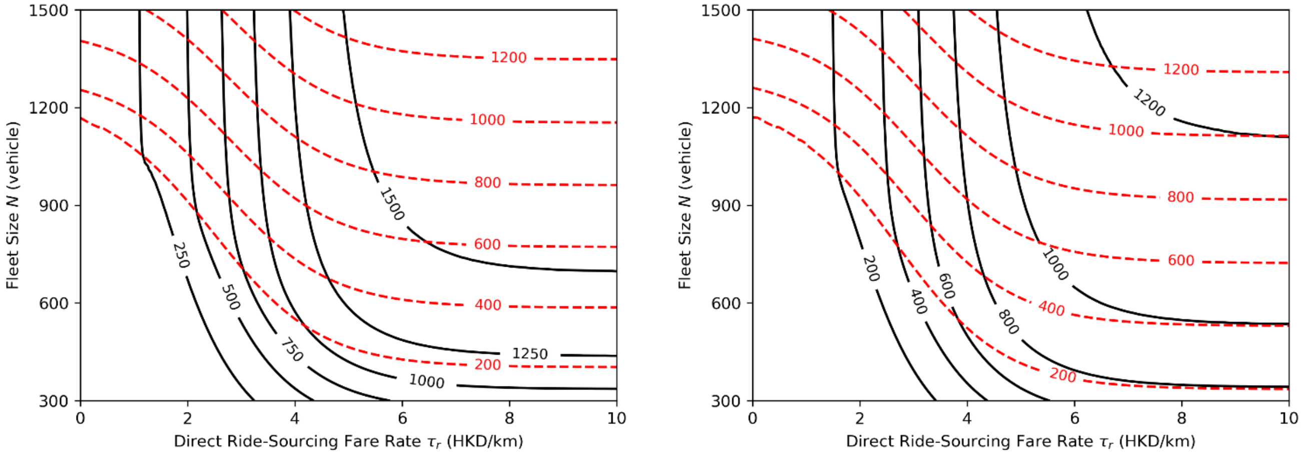
Figure 1 depicts the demand rate for mode $\boldsymbol{b}$ q_b under equilibrium states with different first-mile/last-mile ride-sourcing fare rates τ_b. In the subfigures, the horizontal and vertical axis denote the variation of direct ride-sourcing fare rate τ_r and vehicle fleet size N, respectively; solid black contour lines are drawn to show the changes of mode $\boldsymbol{b}$ demand rate q_b; dashed red contour lines represent the number of idle vehicles N^v obtained according to the equilibrium state. Under each τ_b, we note that N^v monotonously increases with N; once τ_r is too low or too high, N^v is insensitive to τ_r; while, if τ_r is within some medium interval, N^v will monotonously increase with τ_r (see Figure 1(a) when τ_r is between around 1.0 HKD/km and 4.0 HKD/km). We refer to such a direct ride-sourcing fare rate interval as the "idle vehicle shifting interval" (IVSI). The pattern of q_b is significantly affected by the IVSI. First, once τ_r is below the IVSI, there is only small demand for mode $\boldsymbol{b}$; this is because the fare for direct ride-sourcing services is low and mode $\boldsymbol{b}$ becomes unattractive to passengers. Second, if τ_r is within the IVSI, q_b will quickly increase with τ_r and become less sensitive to N; in other words, within the IVSI, fare rates of ride-sourcing services play more important roles than the fleet size in affecting passengers' choices. Third, once τ_r is above the IVSI, q_b will monotonously increase with N but will be insensitive to τ_r; the change of q_b with N under a high τ_r (i.e., above the IVSL) reflects the competition between mode $\boldsymbol{b}$ and mode $\boldsymbol{p}$: as N increases, the waiting time of ride-sourcing services becomes shorter and passengers are more inclined to use ride-sourcing for solving their first-mile/last-mile problems. Intuitively, we note that the lower/upper boundaries of the IVSI become higher as τ_b increases (see Figure 1(a)-(d)).



(a) $\tau_b = 0.0$ HKD/km



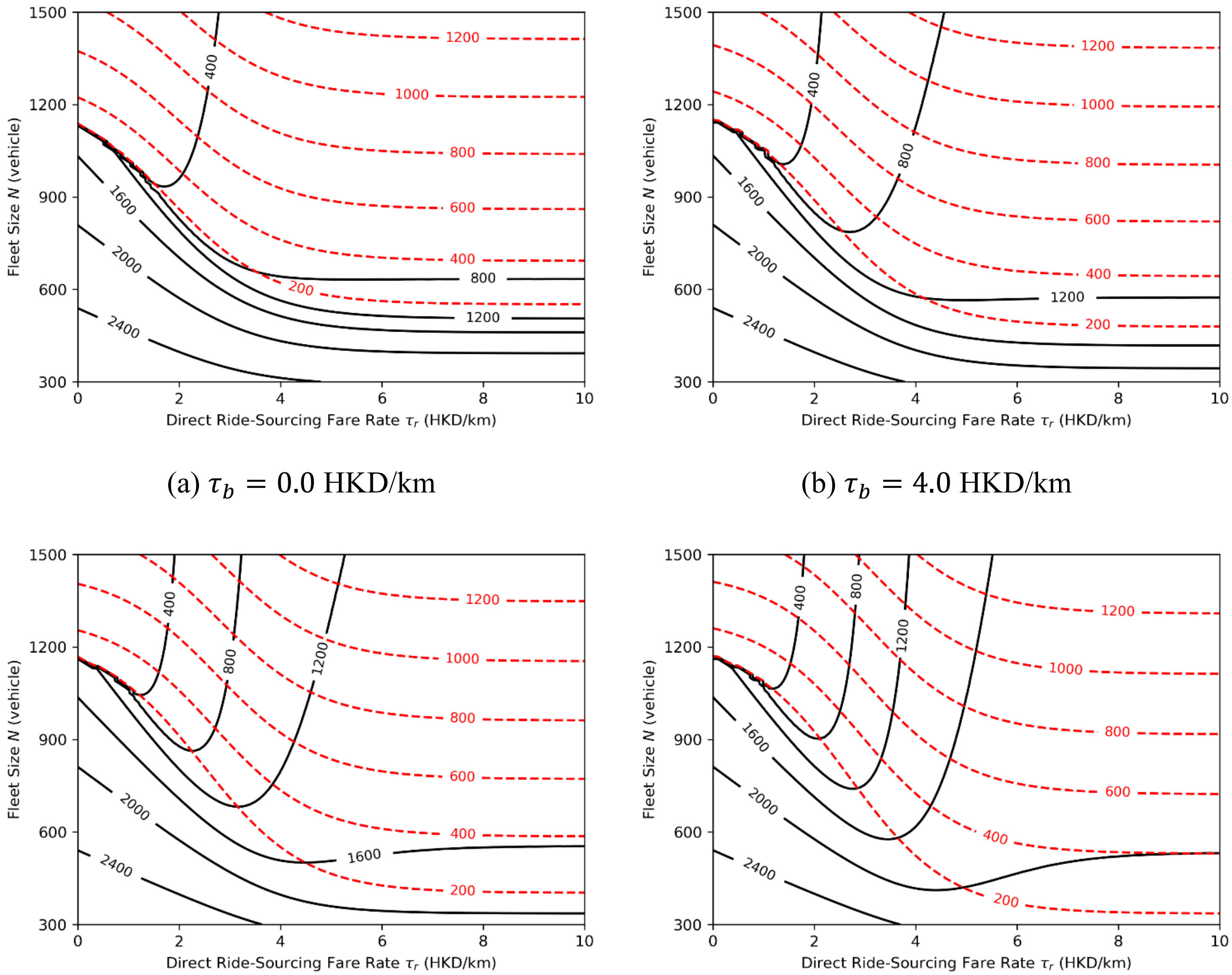
(b) $\tau_b = 4.0$ HKD/km



(c) $\tau_b = 8.0$ HKD/km (d) $\tau_b = 12.0$ HKD/km

Figure 1. Equilibrium demand rate for bundled mode q_b

Similar to Figure 1, Figure 2 shows the equilibrium demand rate for mode ***p*** q_p with different first-mile/last-mile ride-sourcing fare rates τ_b. The solid black contour lines denote q_p, and the dashed red contour lines still represent N^v. When both N and τ_r are small, we note that q_p decreases with τ_r. This is because passengers' preferences between ride-sourcing and public transit is dominated by N^v, which determines ride-sourcing waiting time; and according to the equilibrium condition, an increase in τ_r will lead to a larger N^v and enlarge both q_b (see Figure 1) and q_r (see Figure 3). Under a low τ_r, We not that q_p has a dramatic fall as N increases from around 1,100 to 1,200 (in Figure 2(a)–(d)). The fall of demand is mainly caused by the competitive demand pattern. That is, once the generalized cost of taking mode ***r*** is notably larger than the other two modes, a small shift in waiting time (due to the change of fleet size) will significantly affect the demand rates of the existing modes (i.e., modes ***b*** and ***p*** in this case), leading to an over acting demand shift phenomena. In Figure 2(d), we note that when both τ_b and τ_r are high, a notable amount of passengers will take mode ***p*** due to this low cost in trip fare.



(a) $\tau_b = 0.0$ HKD/km

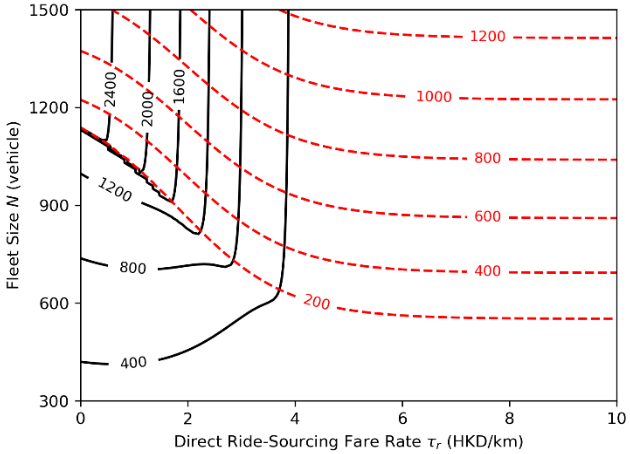
(b) $\tau_b = 4.0$ HKD/km

(c) $\tau_b = 8.0$ HKD/km

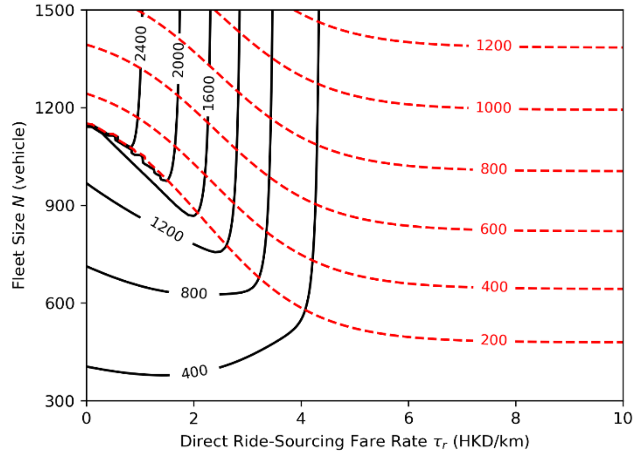
(d) $\tau_b = 12.0$ HKD/km

Figure 2. Equilibrium demand rate for public transit mode q_p

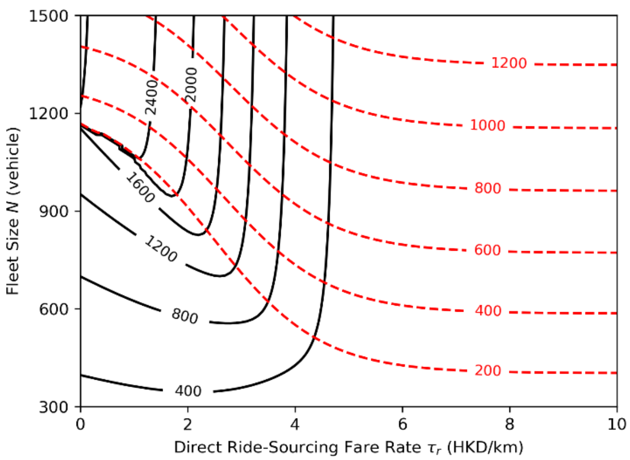
Finally, we utilize Figure 3 to illustrate Figure the equilibrium demand rate for mode ***r*** q_r with different τ_b. Similar to Figures 1 and 2, the solid black and dashed red contour lines represent the changes of q_r and N^v, respectively. Corresponding to the findings in Figure 2, q_r is significantly sensitive to (i.e., to monotonously decrease with) τ_r within the IVSI; while q_r is mainly determined by (i.e., to monotonously increase with) N when τ_r is below the IVSI, and it is small once τ_r is above the IVSI. Similar to the fall of q_p (see Figures 1 and 2), the quick demand shift between modes ***b*** and ***r*** with respect to τ_r within the IVSI is also caused by the competitive demand pattern.



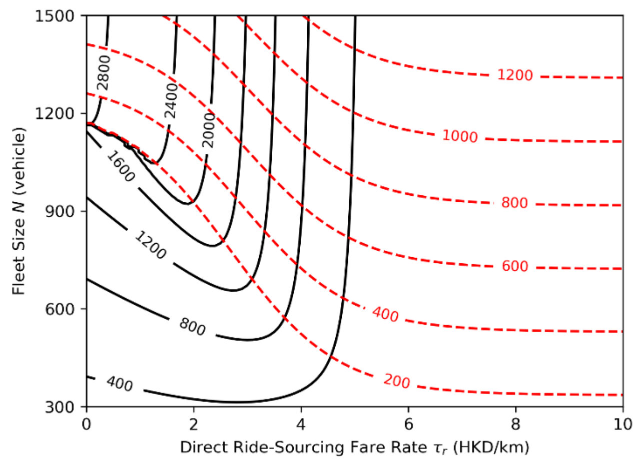
(a) $\tau_b = 0.0$ HKD/km



(b) $\tau_b = 4.0$ HKD/km



(c) $\tau_b = 8.0$ HKD/km



(d) $\tau_b = 12.0$ HKD/km

Figure 3. Equilibrium demand rate for direct ride-sourcing mode q_b

Based on the results of the medium first-mile/last-mile distance scenario in Figures 1, 2 and 3, we have the following conclusions. First, once there are sufficient ride-sourcing vehicles (i.e., fleet size is large) in the market, the demand for public transit mode (mode ***p***) is small, and the fare rates of ride-sourcing services play a decisive role in the competition between direct ride-sourcing services (mode ***r***) and the bundled mode with public transit and first-mile/last-mile ride-sourcing services (mode ***b***). With a fixed first-mile/last-mile ride-sourcing fare rate, there exists a range of direct service fare rate (i.e., the IVSI) such that passengers' preference between mode ***b*** and mode ***r*** is significantly sensitive to the direct service fare rate but insensitive to fleet size. Second, mode ***p*** will absorb large amount of passengers once the number of ride-sourcing vehicles is insufficient. In this manner, if the direct ride-sourcing fare rate is low, the demand for mode ***p*** is determined by both fare rates and fleet size (i.e., decrease with direct ride-sourcing fare rate and fleet size); while, it is mainly affected by fleet size if the direct ride-sourcing fare rate is high. The findings indicate that fleet size is critically important in the substitutive and complementary relationship between public transit and ride-sourcing, and fare rates affect the market share between first-mile/last-mile and direct ride-sourcing services. According to the findings, the government or decision-makers could develop regulation schemes to promote any of the three modes according to its potential social benefits.

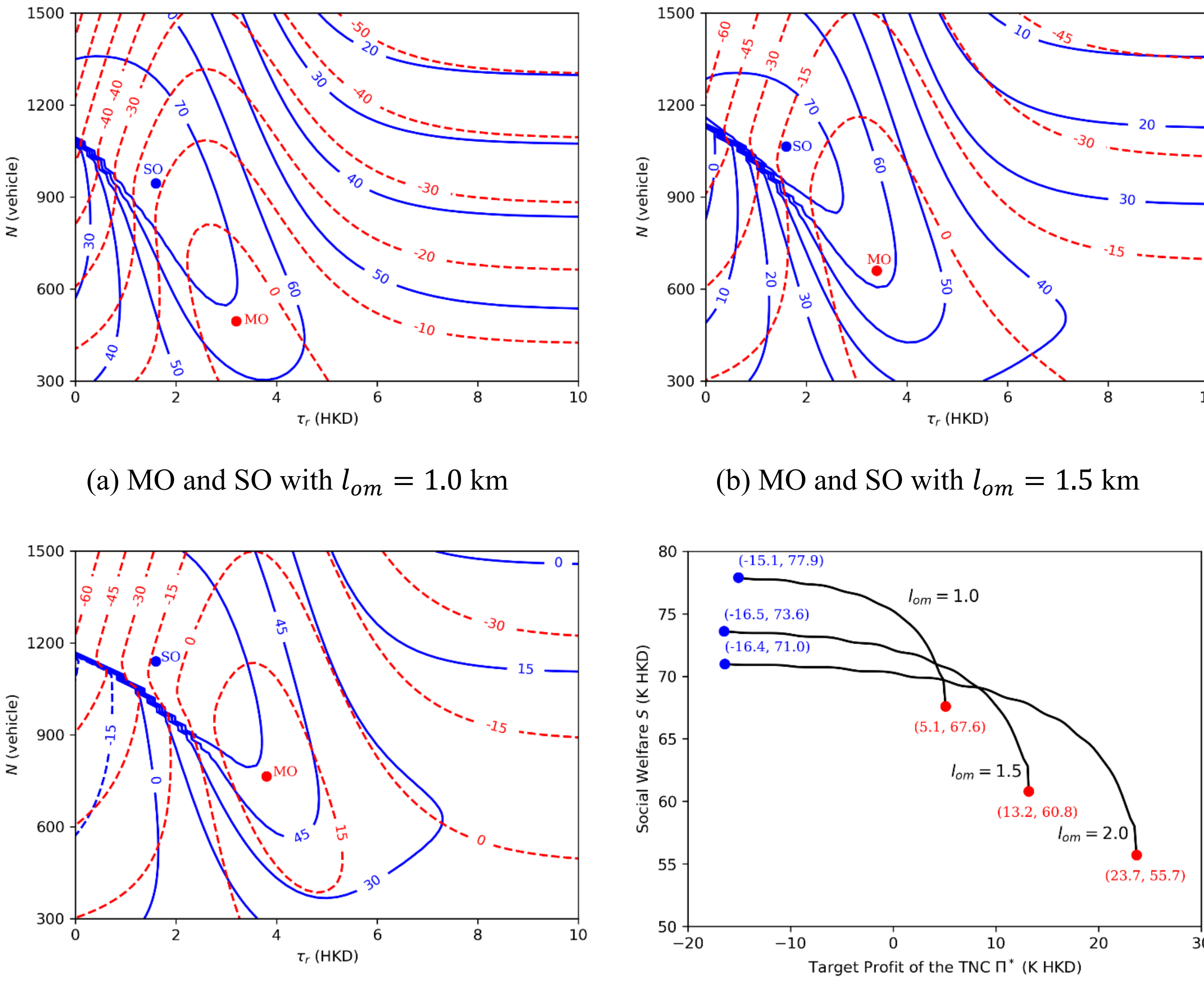
5.2 Analysis of profit and/or social welfare maximizing strategies

To better understand the optimal strategies of the TNC under different optimization scenarios, we numerically solve the aforementioned problems P1, P2, and P3 (P3.1) for the travel scenarios with three different first-mile/last-mile distances.

We first illustrate the impact of decision variables on the objective values of P1 and P2. Figure 4(a)-(c) depict the numerical results with l_{om} equal to 1.0 km, 1.5 km, and 2.0 km, respectively. The subfigures are drawn in the domain of $\tau_r \times N$. For each pair of (τ_r, N), we let τ_b to vary within a large range and numerically obtain the corresponding maximal social welfare S and the TNC's maximal profit Π.We use solid blue contour lines and dashed red contour lines to represent the obtained S and Π (measured in k HKD/h), respectively. Since the obtained S and Π under a fixed (τ_r, N) pair can be regarded as the local optimal objective values of P1 and P2, the global optimal (i.e., maximal) S and Π are found in the two-dimensional figure and are denoted by symbols "SO" and "MO", respectively. Note that for each pair of (τ_r, N), the corresponding S and Π could be under different values of τ_b due to distinctive first-order conditions of P1 and P2 (see Sections 4.1 and 4.2).

With a short first-mile/last-mile distance ($l_{om} = 1.0$ km), the optimal solution of P1 (i.e., the MO point) is located at $\tau_b = 20.6$ HKD/km, $\tau_r = 3.2$ and $N = 496$ vehicles that leads to a Π of 5.1 k HKD/h; and the optimal solution of P2 (i.e., the SO point) is at $\tau_b = 7.1$ HKD/km, $\tau_r = 1.6$ and $N = 942$ vehicles with the maximal S to be 77.9 k HKD/h (see Figure 4(a)). Once the first-mile/last-mile distance is short, passengers are inclined to mode $\boldsymbol{p}$, making ride-sourcing services uncompetitive. To maintain the market share of ride-sourcing services, the TNC might have to either increase N or reduce τ_r, in which the former strategy results in more operational costs and the latter one leads to less net revenue. As a result, we note that the TNC's profit is negative for most of the (τ_r, N) pairs (see the contour lines in Figure 4(a)). In the scenario with a medium first-mile/last-mile distance ($l_{om} = 1.5$ km), we obtain the MO point at $\tau_b = 17.5$ HKD/km, $\tau_r = 3.4$, $N = 659$ vehicles and $\Pi = 13.2$ k HKD/h and the SO point at $\tau_b = 4.6$ HKD/km, $\tau_r = 1.6$, $N = 1{,}066$ vehicles and $S = 73.6$ k HKD/h (see Figure 4(b)). Once we continue to increase the first-mile/last-mile distance (i.e., $l_{om} = 2.0$ km), the MO point becomes $\tau_b = 14.8$ HKD/km, $\tau_r = 3.8$, $N = 768$ vehicles and $\Pi = 23.7$ k HKD/h and the SO point becomes $\tau_b = 3.9$ HKD/km, $\tau_r = 1.6$, $N = 1{,}142$ vehicles and $S = 71.0$ k HKD/h (see Figure 4(c)). The results indicate that as the first-mile/last-mile distance becomes larger, the maximal profit of the TNC will increase; this is because mode

p is losing passengers due to the notable increase of walking/biking time from origins to the nearest transportation hubs (or from hubs to destinations). Based on Figure 4(a)-(c), we note that, under a fixed l_{om}, the ride-sourcing fare rates at MO are always higher than those at SO, while the fleet size at MO is lower than SO. The results are consistent with the analytical derivations in Sections 4.1 and 4.2. This is intuitive such that, when pursuing profit, the TNC needs higher fare rates to ensure high revenue and a small fleet size to reduce operational cost. Another interesting finding is that once the first-mile/last-mile distance shifts from short to long, the social welfare with a low direct ride-sourcing fare rate and a small fleet size (i.e., left-bottom corner in Figure 4(a)-(c)) will dramatically decrease from a high positive level to a negative level.



(a) MO and SO with $l_{om} = 1.0$ km

(b) MO and SO with $l_{om} = 1.5$ km

(c) MO and SO with $l_{om} = 2.0$ km

(d) SB with different l_{om}

Figure 4. Results of the TNC's profit and social welfare

Second, since the decision variables and the objective values (S and Π) at the SO point are notably different from those at the MO point, we illustrate the SB solution for problem P3 (P3.1) and examine whether there is Pareto-efficient condition for both social welfare and the TNC's profit. Pareto-efficiency is an economic state where resources cannot be reallocated to make one individual stakeholder better off without making the other worse off (Yang and Yang, 2011; Ke et al., 2020b). In Figure 4(d), we illustrate the SB conditions by plotting the curves between Π^* and its corresponding S (i.e., solution to P3), in which the blue and red points denote the SO and MO conditions, respectively. Since the curves with different first-mile/last-mile distances have a similar pattern, we take the scenario with $l_{om} = 2.0$ km as an example. Compared with the MO condition, in which the maximal profit is 23.7 k HKD/h and the social welfare is 55.7 k HKD/h, a slight reduction of profit in the SB condition can significantly improve social welfare. That is, we can regulate the target profit Π^* to be around 20.0 k HKD/h and maintain a high social welfare at round 65.0 k HKD. Since such a target profit is near the MO condition and the corresponding social welfare is close to the SO condition (i.e., 71.0k HKD), we claim that the TNC can find a Pareto-efficient strategy that satisfies both the social welfare and its own profit.

6. Conclusion

This study investigates a multi-modal transportation market, in which passengers can opt for one of the three modes: taking ride-sourcing service directly from origin to destination (mode ***r***), taking public transit at transportation hubs and walking/biking to/from hubs (mode ***p***), or taking public transit with ride-sourcing service for solving first-mile/last-mile issues (mode ***b***). The ride-sourcing services act as both complements (first-mile/last-mile service) and competitors (direct service) to public transits. Under this framework, we examine the impacts of the operating strategies, i.e., the fare rates for direct and first-mile/last-mile ride-sourcing services and vehicle fleet, on passengers' mode choices. We discuss the optimization problems for maximizing the TNC's total profit under an MO market, and for maximizing social welfare under an SO (i.e., FB) market and an SB market.

Our theoretical and numerical studies offer some valuable insights for the TNC and the government. It is found that the government will anticipate an optimal trip fare equal to the marginal cost of using a vehicle to serve a passenger in both trip and pick-up phase at SO. In contrast, the platform will set an optimal trip fare equal to the social optimum trip fare plus a monopoly mark-up, which depends on both the direct elasticities and cross elasticities. Moreover, we note that the fleet size of ride-sourcing vehicles can critically affect the complementary and substitutive relationship between ride-sourcing and public transit, and fare rates of direct and first-mile/last-mile ride-sourcing services affect the market share between the two types of ride-sourcing services. First, once the fleet size of ride-sourcing vehicles is large, passengers are inclined to modes with ride-sourcing services. As a result, public transit ridership is mainly affected by the competition between direct ride-sourcing and the bundled mode with public transit and first-mile/last-mile ride-sourcing services. Passengers' preference between the two aforementioned modes is mainly determined by the differentiated ride-sourcing fare rates. Second, the mode with walking/biking and public transit will absorb a large number of passengers once the number of ride-sourcing vehicles is insufficient. The sensitivity of such a public transit mode is notably affected by the direct ride-sourcing fare rate: if the fare rate is low, the mode demand will decrease with direct ride-sourcing fare rate and fleet size; while if the direct service fare rate is high, the mode demand is mainly affected by fleet size. For optimization problems, we find that the maximal profit of the TNC will increase with the first-mile/last-mile distance; under a fixed first-mile/last-mile distance, the ride-sourcing fare rates at the MO condition are always higher than those at the SO condition, while the fleet size at MO is lower than SO. Based on the results of the SB markets, we note that the TNC can find a Pareto-efficient operating strategy that satisfies both the social welfare and its own profit.

Our study opens up some new avenues for future extensions. To name a few, (1) modeling multi-modal transportation market with heterogeneous values of time of passengers; (2) incorporating considerations of traffic congestions on passengers' mode choices in a multi-modal transportation system; (3) studying effects of ride-splitting services (a special ride-sourcing service that allows one vehicle to service two or more riders each time) on transit usage and traffic congestion; (4) investigating the equilibrium games between ride-sourcing platforms and public transit operators, both of which aim to maximize their objectives; (5) ascertaining cooperative operational strategies between the platform and public transit operators, such as integrated trip fares and coordinated public transit schedules, to maximize the total profit or social welfare; (6) calibrating and validating the modeling results with actual multi-modal mobility data, and understanding passengers' multi-modal choice behavior via statistical or deep learning approaches.

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Appendix A

In this appendix, we demonstrate that the ride-sourcing platform which controls the trip fares and wages behaves like a taxi platform which leverages trip fares and fleet size, under the assumption made by Zha et al. (2017). Suppose the drivers' wage per km for first-mile and direct ride-sourcing services are σ_b and σ_r respectively, then the profit-maximization problem of the ride-sourcing platform can be formulated as:

$$\text{(PA1)} \max_{\tau_b,\tau_r,\sigma_b,\sigma_r} \Pi = q_b(\tau_b - \sigma_b)l_{om} + q_r(\tau_r - \sigma_r)l_{od} \tag{32}$$

$$\text{s.t. Eqs. (1)} - (9)$$

$$\frac{q_b\sigma_b l_{om} + q_r\sigma_r l_{od}}{N} - c = 0 \tag{33}$$

where Π is the profit per hour of the TNC, $q_b\sigma_b l_{om} + q_r\sigma_r l_{od}$ is the total earnings of all N drivers per unit time, c is the operating cost per unit time of one ride-sourcing vehicle (or driver). Clearly, Eq. (33) indicates that the net earnings of drivers reaches zero at the equilibrium. It is worth noting that other driver supply model can also be adopted; for example, by assuming drivers' reservation cost distributes over a range, such that the vehicle fleet size N will be given by an increasing function of drivers' average net earnings $(q_b\sigma_b l_{om} + q_r\sigma_r l_{od})/N - c$. In addition, the elasticity of drivers' willingness to provide ride services with respect to their net earnings can also be examined and incorporated into the supply function. For analytical tractability, this paper simply adopts Zha et al. (2017)'s assumption and use P1 to obtain the monopoly optimum solutions. By substituting Eq. (33) into Eq. (32), we can easily prove that problem PA1 is equivalent to problem P1. Similarly, we can prove that problems P2, P3 are equivalent to a welfare-maximization problem and a second-best maximization problem that regard trip fares and drivers' wages as decision variables, under the assumption made by Zha et al. (2017).

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