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Two-sided pricing strategies for a parking sharing platform: reselling or commissioning?

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Abstract

A parking sharing platform plays the role of a matchmaker between parking ‘slot owners/sharers’ and ‘slot renters’ who demand parking. The platform can act as either a reseller or a commissioner, and can have different economic objectives when setting prices for owners and renters. This paper is the first to address a platform operator’s pricing strategy in a two-sided parking sharing market under different business formats (reselling and commissioning) and different economic objectives (profit-maximization and social welfare-maximization). The number of parking slots rented out is assumed to be an increasing function of the number of suppliers (owners) who offer a slot for rent, and the number of demanders (parkers) who request a slot. We derive the supply-demand matching equilibrium and the platform’s two-sided pricing strategies for the alternative business formats and objectives. We also analytically and numerically examine how the buying and selling prices, and numbers of participating owners, renters, and transactions vary with parameter values in each business format. We find that a welfare-maximizing platform earns a positive profit if the matching function has decreasing returns to scale, and incurs a deficit if the matching function has increasing returns to scale. If the cost of inconvenience to slot owners from renting a slot equals their cost of disappointment from failing to rent it out, then the reselling and commissioning formats can replicate each other (i.e, they can yield identical platform profit and social welfare). If the cost of inconvenience is greater then the commissioning format is superior, whereas if the disappointment is greater the reselling format is preferred.

Keywords: Parking sharing; two-sided market; platform pricing; reselling; commissioning.

1 Introduction

Parking downtown can be a headache for commuters and businesses, and cruising for parking contributes significantly to traffic congestion (Shoup, 2006). Substantial efforts have been made to developing both pricing and quantity control strategies for parking management. For instance, Arnott and Inci (2006) and Zheng and Geroliminis (2016) modeled cruising for parking that is an increasing function of the parking occupancy rate, and examined parking pricing strategies to reduce cruising time and improve system efficiency. He et al. (2015), Inci and Lindsey (2015), Du and Gong (2016), and Liu et al. (2021) examined pricing strategies with spatially distributed parking and/or different types of parking. Another branch of studies focused on the integrated modeling of parking and departure-time choices, and analyzed the effectiveness of time-dependent or location-dependent parking pricing as a substitute for road congestion pricing (Zhang et al., 2008; Qian et al., 2011, 2012; Liu and Geroliminis, 2016).

Some other studies have adopted models with rigid parking capacity constraints, and examined various control strategies including reservation schemes (Zhang et al., 2011; Yang et al., 2013; Liu et al., 2014; Chen et al., 2015), slot allocation mechanisms (Zou et al., 2015), and parking navigation systems (Chen et al., 2019). In addition, Geng and Cassandras (2013) proposed a 'smart parking' system comprising a smartphone-based app to help individuals reserve or find both on-street and off-street parking slots. Such real-time parking guidance systems are now in service. For example, the SFpark program (http://sfpark.org/) combines parking and traffic information, and sets demand-dependent parking fees in order to alleviate parking and traffic congestion in sectors of San Francisco. Demand-based pricing has been implemented in other cities, too, including Los Angeles, New York, Seattle, Washington, D.C., and Calgary.

This paper concerns shared parking, which is emerging as a new tool to make more efficient use of parking space by renting it out when the owners are not using it. The market potential for shared parking arises from predictable variations in daily, weekly, and seasonal demand for parking space at different locations (e.g., Institute for Transportation and Development Policy, 2014; Ma, 2016). Residential parking spaces are in high demand in the evenings and on weekends, but often unused on weekdays if the owners drive to work. Since demand for parking at workplaces, offices, universities, schools, and banks exhibits the opposite pattern, residents who live near such businesses and other institutions can rent their parking out to them, or directly to individuals such as commuters. Businesses that experience peak demands at different times can also share parking spaces. For example, restaurants, bars, and cinemas with demands that peak in the evenings can share spaces with businesses that have weekday peaks. By making private parking space publicly available for rent, shared parking helps alleviate any shortage of parking spaces while also allowing owners to make money. It has the greatest market potential in areas with mixed office, commercial, and residential land-use developments, where residential parking is located close to work, shopping, and other destinations. Such mixed land uses exist in a number of cities. For instance, Lai et al. (2021) identify 91 large clusters in Guangzhou City with business and residential areas within walking distance of each other.

Shared parking is an example of a two-sided market in which two groups of agents interact through an intermediary or platform. Some two-sided markets have operated successfully for decades, such as credit card services that connect cardholders and affiliated merchants, and operating systems that connect application developers and clients. Rochet and Tirole (2003) were the first to identify and analyze the commonality across these diverse businesses and markets. They noted how participation on each side of the market creates positive network externalities, and how the volume of transactions depends on the prices set by the platform to the two sides. Other early contributions include Caillaud and Jullien (2003) who derived the equilibrium properties and allocative efficiency of a particular two-sided market. Armstrong (2006) explored different modes of price competition among multiple platforms. A substantial body of work on two-side markets now exists. Roson (2005) provided an early literature review. More recently, Kenney and Zysman (2016) offered a general review on the rising platform economy. Nevertheless, parking demand and supply generally differ from other commodity sharing businesses in that they involve spatio-temporal heterogeneity that results in supply-demand matching frictions.

Thanks to the development of mobile apps and online platforms, various forms of shared mobility services have grown rapidly: notably the smartphone-based taxi e-hailing systems and ride-sourcing platforms such as Uber and Lyft. A number of studies investigated the pricing strategies associated with these platforms based on the aggregate matching model introduced by Yang and Yang (2011), which captures the matching friction between ride demanders (customers) and ride suppliers (drivers). For instance, He and Shen (2015) and Wang et al. (2016) modeled the taxi markets with e-hailing, and examined the optimal pricing strategies. Zha et al. (2016) examined the ride-sourcing market, and modeled the matching between riders and drivers with an exogenous matching function. Wang et al. (2018) studied the cost-sharing strategies for driver-rider in a carpooling or ridesharing program.

Shared parking is a recent and still embryonic example of a two-sided market in transportation. Shared parking operates by temporarily repurchasing or renting private parking slots, and making them available to public users during certain times of day. A prominent example is Moby (https://www.mobypark.com), a company that operates in The Netherlands, France, Belgium, and other European countries. Its platform accepts parking space offers from companies and individuals, and matches them with customers through its website. Owners indicate when a space is available, and set the price and minimum duration for renting it out. Moby charges a commission of 20 percent plus Value Added Tax (VAT). Similar parking sharing platforms are operating in Sydney, Melbourne, and Brisbane (OSCAR: https://www.sharewithoscar.com.au/) and several Chinese cities (Yan et al., 2020).

A few studies have analyzed several aspects of shared parking markets. Guo et al. (2016) developed a simulation-based decision method to determine the optimal parking repurchase strategy. They adopted a Gaussian mixture model to describe the time-varying arrival and departure rates of drivers, and estimated the expected repurchase amounts and parking time via simulation optimization. Shao et al. (2016) investigated advanced booking and allocation of shared parking slots. They proposed a binary integer linear programming model to allocate parking requests to specific parking slots in order to maximize the parking slot utilization or accommodate as many requests as possible given parking space and time constraints. Xu et al. (2016) examined the revenue flow from parking sharing, and demonstrated the potential for a sharing system to generate substantial social welfare gains in large cities. Xiao and Xu (2018) and Xiao et al. (2020) studied a double auction mechanism for allocating parking slots that is designed to boost participation. Xiao and Xu (2019) extended the model by assuming that suppliers and demanders can experience regret from either winning or losing in the auction. Yan et al. (2020) find survey evidence that residential parking space owners are loss averse with respect to uncertain revenues from sharing their space.

In another branch of studies, Liu et al. (2022) examined the pricing strategy of a parking sharing platform when shared parking and curbside parking are both available. Other studies consider the spatial distribution of parking (Zhang et al., 2020), the integration of parking sharing and carsharing (Jian et al., 2020), and the joint modeling of destination and parking choices (Liu et al., 2021).

This paper extends the literature by exploring the pricing strategy of a shared parking platform subject to matching frictions between parking suppliers and demanders. Two types of business formats are considered in which the shared parking platform serves as either a reseller or a commissioner. As a reseller, the platform takes full liability for repurchased parking slots. Slot owners receive a guaranteed payment, and no longer have access to their parking slots during the contract period regardless of how much their slots are utilized. By contrast, if the platform serves as a commissioner, the owner has to relinquish usage of the space only if it is actually rented. The owner's revenue depends on the utilization rate and the payment set by the platform per unit occupied time.

For each business format, we derive the parking owners' and renters' decisions and the supply-demand matching equilibrium. We also analyze how the supply-demand equilibrium varies with parameter values in the two business formats. Some studies in the operations management field have examined the reselling and commissioning business formats for the e-commerce platform (e.g., Abhishek et al., 2016; Zhang and Zhang, 2020; Xu et al., 2021). However, the reselling and commissioning formats for the parking sharing platform have not yet been studied. The key difference from the typical e-commerce platform is the existence of supply and demand matching friction. In contrast to Shao et al. (2016) and Xu et al. (2016), we do not model the allocation of slots between individual owners and renters. Instead, the allocation is determined by supply and demand using the matching function in Yang and Yang (2011), mentioned above. As Shao et al. (2016) show, the overall time-of-day, one-to-one matching outcomes (where individual requests are allocated to specific shared lots under parking space and time constraints) can be accurately characterized by this matching function. The matching function allows for tractable mathematical model formulation and economic analysis to help generate managerial insights.

This paper also investigates the two-sided pricing strategies of reselling and commissioning platforms with two alternative economic objectives: profit-maximization or social welfare-maximization. The optimal buying and selling prices and the associated platform profit are derived for each objective and format. We consider two costs for parking slot sharers: "inconvenience" and "disappointment". Sharers incur an opportunity cost since they cannot use the slots during the contract period. We call this cost "inconvenience". If owners offer their parking slots, but no one rents them, they receive no money and may regret the decision to join the platform. We call this regret "disappointment".

Disappointment can take two forms. First, an owner who expects to rent out a slot may plan accordingly, and may have less use for a slot if they have to adjust their plans on short notice. This sort of adjustment cost is analogous to the scheduling costs identified empirically by Peer et al. (2015), and studied theoretically by Verhoef (2020). These two studies address how the costs of planning are higher, and the values of time spent at activities are lower, when individuals have little time to adapt or reorganize in response to shocks or other unanticipated developments. The second form of disappointment is largely psychological in nature, and corresponds to frustration, letdown, or disillusionment from not renting a slot. It may be appreciable for someone who is participating in a parking sharing program for the first time. After gaining experience, the intensity of disappointment may fade as individuals become acclimated to the uncertain prospects of renting a slot out.

In summary, we develop a static, aggregate model to elucidate the market equilibrium among parkers, parking slot owners, and a parking sharing platform operator. The model features matching frictions between parkers and owners, and two business formats: reselling and commissioning. The major contributions of this paper are as follows. (i) It is the first to develop a two-sided parking sharing market equilibrium model that explicitly incorporates matching frictions between parkers and parking slot owners. (ii) It is the first to compare two business formats, reselling and commissioning, for the parking sharing platform. Conditions under which one business format is superior to the other in terms of profit or social welfare are identified and analyzed. (iii) It examines analytically the optimal pricing strategy of the parking sharing platform for the two business formats and two administrative objectives (profit and social welfare maximization). The results yield insights into how parking sharing should be operated. (iv) Numerical examples are used to illustrate how the market equilibrium depends on parameter values.

The rest of the paper is organized as follows. Section 2 formulates the demand and supply of the two-sided parking sharing market, and introduces the demand and supply matching function. Section 3 derives the two-sided pricing strategies of a reselling platform for both profit-maximizing and welfare-maximizing objectives. Section 4 does the same for a commissioning platform. Section 5 examines how equilibrium quantities vary with parameter values, and compares the two business formats. Section 6 illustrates the results using numerical examples, and Section 7 concludes.

2 Problem setting and formulation

Following Caillaud and Jullien (2003) and Rochet and Tirole (2003), we consider a two-sided market bridged by a single parking sharing platform on which private slot owners (hereinafter “owners”) offer spare parking slots to potential users (hereinafter “parkers”). Table 1 lists the main variables of the model.

Table 1: Notational glossary

Symbol	Definition
B_d	Gross benefit of a parker from renting a space
G	Matching function
g	Amount of time a shared slot is available
h	Parking duration of a parker
k	Platform’s operating cost per slot rented per unit time
N_a	Number of accepted parking requests
N_o	Actual supply of shared parking = Number of owners who offer their parking slots on the platform
$\hat{N}_o$	Potential supply of shared parking = Total number of owners

N_d	Actual demand for shared parking = Number of parkers who request a slot on the platform
$\hat{N}_d$	Potential demand for shared parking = Number of individuals who want a parking slot
p_o	Price paid per unit time to an owner by the platform (buying price)
p_o^c	Buying price of a commissioning platform
p_o^r	Buying price of a reselling platform
p_d	Price charged per unit time to a parker by the platform (selling price)
p_d^c	Selling price of a commissioning platform
p_d^r	Selling price of a reselling platform
R_o	Disappointment of owner from not renting a slot
R_d	Disappointment of parker from not getting a slot
U_o	Net utility of an owner
$\hat{U}_o$	Reservation utility of an owner (idiosyncratic)
U_d	Expected net utility of a parker from requesting a slot
$\hat{U}_d$	Reservation utility of a parker (idiosyncratic)
W_o	Inconvenience to an owner when a slot is rented
$\varepsilon_{p_o}^{N_o}$	Elasticity of number of offered slots w.r.t. buying price
$\varepsilon_{p_d}^{N_o}$	Elasticity of number of offered slots w.r.t. selling price
$\eta_{p_o}^{N_a}$	Elasticity of number of accepted requests w.r.t. buying price
$\eta_{p_d}^{N_a}$	Elasticity of number of accepted requests w.r.t. selling price
θ	Utilization rate of shared slots
φ	Acceptance rate experienced by parkers
$\lambda_{p_o}^{N_d}$	Elasticity of parking demand w.r.t. buying price
$\lambda_{p_d}^{N_d}$	Elasticity of parking demand w.r.t. selling price
$\sigma_{N_o}^{N_a}$	Elasticity of number of accepted requests w.r.t. number of owners
$\sigma_{N_d}^{N_a}$	Elasticity of number of accepted requests w.r.t. number of parkers

2.1 Demand side (parkers)

Consider a mass $\hat{N}_d$ of parkers who decide whether to request a shared parking slot through the platform, or choose an alternative (e.g., searching for a public parking slot, or using another transport mode). A parker who rents a space on the platform occupies it for a duration h, pays a price of p_d per unit time, and receives a gross utility or benefit of B_d. A parker can rent a space only if their request is fulfilled.

Let N_d denote the number of parkers who request a space, and N_a the number whose requests are accepted. The acceptance rate is then $\varphi = N_a/N_d$, which is also the probability that a given parker's request is granted. A parker whose request is rejected incurs a disappointment cost R_d. Similar to owners (as discussed in the Introduction), the disappointment for a parker from not getting a parking spot can take two forms. The parker may have to quickly adjust their plans by finding an alternative parking spot, postponing the trip, or taking another travel mode. Disappointment may also be psychological in nature. It is analogous to the regret that a demander experiences from losing in the auction as modeled by Xiao and Xu (2019). Their model also features regret from paying more than the market-clearing price. There is no analogue to this in our model.

A parker who requests a shared parking slot thus has an expected net utility of

$$U_d = (B_d - p_d h)\,\varphi - (1-\varphi)\,R_d, \tag{1}$$

On the right-hand side (RHS) of Eq. (1), $(B_d - p_d h)$ in the first term is a parker's utility if his/her parking request is accepted, which equals the gross benefit minus the rental cost; φ is the acceptance probability. In the second term, R_d is the disappointment cost of an unfulfilled request and $(1-\varphi)$ is the probability a request gets rejected. To maintain analytical tractability, we assume identical parking duration h, gross benefit B_d, and disappointment cost R_d for all parkers. However, parkers' parking requests may differ by time slot or location, and so may the shared parking supply. Thus, there is matching friction between the demand and supply in the shared parking market, which is modeled by an aggregate matching function in Section 2.4.

While parkers are assumed to derive the same gross benefit from using shared parking, they differ in the utility they derive from using a public slot or taking another transport mode. The differences may arise due to differences in spatial proximity to other options, values of time, or intrinsic preferences. The utility from the alternative, denoted by $\hat{U}_d$, serves as a reservation utility for choosing the shared parking option. If parkers are indexed in order of increasing reservation utility, the n^{th} parker has a reservation utility of $\hat{U}_d(n)$, which is an increasing function of n. The number of parkers who request a space, N_d, is determined by the condition $U_d = \hat{U}_d(N_d)$ with U_d given by Eq. (1).

2.2 Supply side (owners)

On the supply side there is a mass $\hat{N}_o$ of slot owners, each of whom has a parking slot available for a duration of time, g. Each owner decides whether to make a slot available for rent through the platform. If the owner offers the slot, and the platform succeeds in renting it, the platform pays the owner at a rate of p_o per unit time. For brevity, the buying price, p_o, will sometimes be called the fee. The owner cannot use the slot while it is rented, and incurs an inconvenience cost of W_o (the inconvenience cost may also include the communication/coordination effort to allow the parker to use the rented space). Consequently, an owner who rents a slot gains a net utility of $U_o = p_o g - W_o$. Similar to the specification for parkers, we assume that all owners can rent their slot for the same duration, g, and incur the same inconvenience cost, W_o. However, the available parking slots may differ in location and when they are available, which creates matching friction with demand.

Owners are assumed to have idiosyncratic reservation utilities, $\hat{U}_o$, for participating in the shared parking market.[1] If owners are indexed in order of increasing reservation utility, then the n^{th} owner has a reservation utility of $\hat{U}_o(n)$ which is an increasing function of n. The number of owners who participate on the platform depends on the business format (reselling or commissioning), which is discussed next.

2.3 Business formats of the platform

The platform operator can adopt one of two business models or formats: it can act as either a reseller (r) or a commissioner/agent (c). The two formats differ in how the incomes of owners are determined.

2.3.1 Reselling format

As a reseller, the platform takes full responsibility for renting slots. Once an owner commits a slot to the platform, the platform pays the owner a guaranteed payment p_o^r per unit time regardless of whether the slot gets rented or not. The owner receives a net utility of

$$U_o^r = p_o^r g - W_o. \tag{2}$$

The first term on the RHS of Eq. (2), $p_o^r g$, is the fee paid by the platform to the slot owner, which equals the rent per unit time p_o^r multiplied by the renting duration g. The second term is the inconvenience cost associated with sharing the slot. The number of parkers who rent a space, N_o^r, is determined by the condition $U_o^r = \hat{U}_o(N_o^r)$, where $\hat{U}_o(\cdot)$ is the reservation utility function given in Section 2.2.

2.3.2 Commissioning format

If the platform serves as a commissioner, owners are paid only if their slots are rented. Since each owner makes their slot available for a period g, total supply is

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[1] Yan et al. (2020) find that younger parking slot owners and owners with low or medium salaries are more willing to participate in shared parking programs than their older and higher-salary counterparts.

gN_o. The number of parkers who are granted a space is N_a, and each occupies a space for a period h. Total demand is therefore hN_a, and the average utilization rate of slots is:

$$\theta = \frac{hN_a}{gN_o}.$$

With probability θ, an owner succeeds in renting his slot for the full g hours. He is paid at a rate p_o^c, and receives a net utility of $p_o^c g - W_o$. With probability $1-\theta$, the owner fails to rent his slot. He receives no payment, and incurs a disappointment cost of R_o. As noted in the introduction, and similar to the case of parkers who fail to secure a spot, disappointment can take two forms: one arising from the cost of having to change plans on short notice, and the other psychological. The psychological disappointment for an owner is analogous to the regret from losing in the auction model proposed by Xiao and Xu (2019). Again, there is no analogue here to the regret from winning in their model. The owner's expected utility U_o^c is thus

$$U_o^c = \theta \left(p_o^c g - W_o\right) - \left(1-\theta\right) R_o, \tag{3}$$

On the RHS of Eq. (3), $(p_o^c g - W_o)$ is an owner' utility if his/her slot is rented and θ is the probability of successfully renting out the slot. R_o is the disappointment cost associated with failing to rent out a slot, and $(1-\theta)$ is the probability a slot is not rented out.

The number of owners who rent a space, N_o^c, is determined by the condition $U_o^c = \hat{U}_o\left(N_o^c\right)$, which can be written as follows:

$$\theta \left(p_o^c g - W_o + R_o\right) = \hat{U}_o + R_o. \tag{4}$$

Table 2: Summary of reselling and commissioning formats

Business format	Owner's income	Owner's expected utility
Reseller	A guaranteed constant	$U_o^r = p_o^r g - W_o$
Commissioner	Probabilistic	$U_o^c = \theta \left(p_o^c g - W_o\right) - \left(1-\theta\right) R_o$

Table 2 summarizes the features of the reselling and commissioning formats. In the reselling case, slot utilization has no direct impact on owners' incomes although the equilibrium value of p_o^r depends on the average utilization rate, θ. In contrast, with commissioning an owner is paid only if his individual slot is rented out. With either format, owners join the platform if and only if their expected utility exceeds their reservation utility.

2.4 Matching function

We now introduce an aggregate supply-demand matching function to specify the number of accepted requests. A similar approach has been adopted to model matching of passengers and drivers in traditional taxi markets (Yang and Yang, 2011), taxi e-hailing markets (He and Shen, 2015; Wang et al., 2016) and ride-sourcing markets (Zha et al., 2016; Wang et al., 2018). The aggregate matching function reflects the frictions in the parking sharing supply-demand matching, which are due to the spatio-temporal heterogeneity in terms of parking availability and parking demand. As Shao et al. (2016) show, the overall time-of-day one-to-one matching outcomes can be accurately characterized by an aggregate function of demand and supply intensity. An aggregate matching function treats individual heterogeneity implicitly. It allows for tractable mathematical model formulation and economic analysis, and it is conducive to managerial insights. It should be calibrated with real-world data to fit local market conditions in a particular area. The number of matches or transactions, N_a, is specified by the function:

$$N_a = G\left(N_o, N_d\right), \tag{5}$$

where N_o and N_d denote supply and demand for shared parking, respectively, and $\partial G/\partial N_o > 0$ and $\partial G/\partial N_d > 0$. Let $\sigma_{N_o}^{N_a}$ and $\sigma_{N_d}^{N_a}$ denote the elasticities of N_a with respect to N_o and N_d, respectively:

$$\sigma_{N_o}^{N_a} = \frac{\partial N_a}{\partial N_o}\frac{N_o}{N_a}; \quad \sigma_{N_d}^{N_a} = \frac{\partial N_a}{\partial N_d}\frac{N_d}{N_a}. \tag{6}$$

Elasticities $\sigma_{N_o}^{N_a}$ and $\sigma_{N_d}^{N_a}$ denote the percentage changes in matches, N_a, induced by a one-percent change of demand N_d and supply N_o, respectively. Shao et al. (2016) found that these elasticities are positive and less than one. This is consistent with the common sense that, under normal demand and supply conditions, an increase in either supply or demand alone (with the other held constant) yields a less-than-proportional increase in successful matches (Yang and Yang, 2011; Yang et al., 2014; Wang et al., 2016, 2018). Thus, it is assumed that $0 < \sigma_{N_o}^{N_a}, \sigma_{N_d}^{N_a} \leq 1$. The matching function exhibits locally increasing returns to scale if $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} > 1$, locally decreasing returns to scale if $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} < 1$, and locally constant returns to scale if $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} = 1$. Following the specific parking allocation model in Shao et al. (2016), in Section 5.3 we adopt a Cobb-Douglas matching function for which the elasticities $\sigma_{N_o}^{N_a}$ and $\sigma_{N_d}^{N_a}$ are constants. In the numerical studies in Section 6, we also investigate another specific matching function used in the literature.

The equilibrium values of the shared parking supply N_o^r and N_o^c (in the reselling and commissioning formats, respectively) and the demand N_d can be written as functions of the buying and selling prices (p_o, p_d):

$$N_o^r = N_o^r(p_o), N_o^c = N_o^c(p_o, p_d), N_d = N_d(p_o, p_d). \tag{7}$$

Note that N_o^r does not depend on p_d because owners are guaranteed a fixed income of $p_o^r g$ in the reselling format. Substituting Eq. (7) into Eq. (5), and rearranging terms, the transaction volume N_a can be rewritten as a function of the prices:

$$N_a = N_a(p_o, p_d). \tag{8}$$

Throughout the paper, it is assumed that the platform operates with positive transaction volume so that $N_d \geq N_a > 0$, and $gN_o \geq hN_a > 0$. It is also assumed that demand and supply are both price sensitive so that $\partial N_o^c/\partial p_d \leq 0$ (with $\partial N_o^r/\partial p_d = 0$), $\partial N_o/\partial p_o \geq 0$, $\partial N_d/\partial p_d \leq 0$, $\partial N_d/\partial p_o \geq 0$. Section 5 describes in more detail how the equilibrium values of N_o and N_d change with p_o and p_d.

2.5 Elasticities

We now introduce the elasticities of N_a, N_o, and N_d with respect to the buying and selling prices. Let $\eta_{p_o}^{N_a}$ and $\eta_{p_d}^{N_a}$ denote the elasticities of the number of accepted requests with respect to prices p_o and p_d, respectively:

$$\eta_{p_o}^{N_a} = \frac{\partial N_a}{\partial p_o}\frac{p_o}{N_a};\ \eta_{p_d}^{N_a} = \frac{\partial N_a}{\partial p_d}\frac{p_d}{N_a}. \tag{9}$$

As the fee p_o increases, supply N_o and transactions volume N_a increase. As the selling price p_d increases, demand N_d and transactions volume N_a decrease. Thus, $\eta_{p_o}^{N_a} \geq 0$ and $\eta_{p_d}^{N_a} \leq 0$.

Let $\varepsilon_{p_o}^{N_o}$ and $\varepsilon_{p_d}^{N_o}$ denote the elasticities of N_o with respect to p_o and p_d, respectively:

$$\varepsilon_{p_o}^{N_o} = \frac{\partial N_o}{\partial p_o}\frac{p_o}{N_o};\quad \varepsilon_{p_d}^{N_o} = \frac{\partial N_o}{\partial p_d}\frac{p_d}{N_o}. \tag{10}$$

For both business formats, $\varepsilon_{p_o}^{N_o} > 0$. For the commissioning format, $\varepsilon_{p_d}^{N_o} < 0$, whereas for the reselling format, $\varepsilon_{p_d}^{N_o} = 0$ (these results are proved in Section 5).

Let $\lambda_{p_o}^{N_d}$ and $\lambda_{p_d}^{N_d}$ denote the elasticities of N_d with respect to p_o and p_d, respectively:

$$\lambda_{p_o}^{N_d} = \frac{\partial N_d}{\partial p_o}\frac{p_o}{N_d};\quad \lambda_{p_d}^{N_d} = \frac{\partial N_d}{\partial p_d}\frac{p_d}{N_d}. \tag{11}$$

For both business formats, $\lambda_{p_o}^{N_d} > 0$, and $\lambda_{p_d}^{N_d} < 0$. It is straightforward to show that the elasticities for the number of accepted requests can be expressed as functions of the other three pairs of elasticities:

$$\eta_{p_o}^{N_a} = \sigma_{N_o}^{N_a}\varepsilon_{p_o}^{N_o} + \sigma_{N_d}^{N_a}\lambda_{p_o}^{N_d}; \quad \eta_{p_d}^{N_a} = \sigma_{N_o}^{N_a}\varepsilon_{p_d}^{N_o} + \sigma_{N_d}^{N_a}\lambda_{p_d}^{N_d}. \tag{12}$$

Based on the definition of the average utilization rate $\theta = \frac{hN_a}{gN_o}$ and the conditions in Eqs. (5) and (7) (i.e., $N_a = G(N_o, N_d)$ and $N_o^c = N_o^c(p_o, p_d)$), the average utilization rate can be rewritten as a function of the prices: $\theta = \theta(p_o, p_d)$. The partial derivatives of θ with respect to the prices are:

$$\frac{\partial\theta}{\partial p_o} = \left(\eta_{p_o}^{N_a} - \varepsilon_{p_o}^{N_o}\right)\frac{\theta}{p_o}; \quad \frac{\partial\theta}{\partial p_d} = \left(\eta_{p_d}^{N_a} - \varepsilon_{p_d}^{N_o}\right)\frac{\theta}{p_d}. \tag{13}$$

Since $\eta_{p_o}^{N_a} \geq 0$ and $\varepsilon_{p_o}^{N_o} > 0$, the sign of $\eta_{p_o}^{N_a} - \varepsilon_{p_o}^{N_o}$ is a priori indeterminate. Hence, the utilization rate can either increase or decrease with respect to p_o. Similarly, given $\eta_{p_d}^{N_a} \leq 0$ and $\varepsilon_{p_d}^{N_o} \leq 0$, the sign of $\eta_{p_d}^{N_a} - \varepsilon_{p_d}^{N_o}$ is also a priori indeterminate. The utilization rate can either increase or decrease with respect to p_d.

3 The two-sided pricing strategy for a reselling platform

This section analyzes the two-sided pricing strategy of a reselling platform operator that maximizes either profit or social welfare. To economize on notation, superscript 'r' is omitted except when reporting the main results.

3.1 Profit maximization (reselling format)

In the profit maximization regime, the platform operator sets the fee p_o and the buying price p_d to maximize platform profit. This problem can be formulated as:

$$\max_{p_o \geq 0, p_d \geq 0} \Pi = p_d h N_a - p_o g N_o - k h N_a. \tag{14}$$

The first term on the RHS of Eq. (14) is the revenue collected from parkers. As defined earlier, h is the parking duration, p_d is the selling price per unit time, and N_a is the number of accepted requests. The second term is payment to owners, where g is the slot time provided by an owner, p_o is the fee per unit time, and N_o is the number of owners who make their slot available. The third term denotes the platform operating cost associated with the transactions, where k is the unit operating cost (marginal cost) incurred per slot per unit time. The operating cost is assumed to be proportional to parking duration.[2] We ignore any overall fixed

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[2] The care and associated labor allocated to a parker is related to the parking duration. For example, there might be some agreements/contracts between the platform operator and the local parking manager (e.g., property management companies in China). The local parking manager will check the parked cars regularly (often for safety reasons in China) and the cost or fee for this

operating cost for the platform since it does not affect results of interest.

Note that the profit function in Eq. (14) is not necessarily concave, and the optimal solution might be non-unique. It is assumed that a single interior solution obtains, and the necessary first-order optimality conditions are used to derive analytical results. The first-order conditions (FOCs) for profit maximization, $\partial\Pi/\partial p_o = 0$ and $\partial\Pi/\partial p_d = 0$, can be written as follows

$$(p_d - k)\, hN_a \eta_{p_o}^{N_a} = p_o g N_o \left(1 + \varepsilon_{p_o}^{N_o}\right), \tag{15}$$

$$-\,(p_d - k)\, hN_a \eta_{p_d}^{N_a} = p_d h N_a. \tag{16}$$

Rearranging Eq. (16) yields

$$\frac{p_d^{PM} - k}{p_d^{PM}} = -\frac{1}{\eta_{p_d}^{N_a}}, \tag{17}$$

where superscript 'PM' denotes profit maximization. The left-hand side of Eq. (17) is the Lerner index which measures the platform's market power. When the selling price is chosen to maximize profit, the Lerner index is equal to the negative inverse of the price elasticity of demand facing the platform, as per the well-known inverse elasticity rule. Since $0 \leq \left(p_d^{PM} - k\right)/p_d^{PM} \leq 1$, it follows that $\eta_{p_d}^{N_a} \leq -1$, i.e., the platform operates on the elastic portion of the demand curve.

The FOC in Eq. (15) can be written as

$$\frac{\eta_{p_o}^{N_a}}{1 + \varepsilon_{p_o}^{N_o}} = \frac{p_o g N_o}{(p_d - k)\, hN_a}. \tag{18}$$

If the profit is positive, the RHS of Eq. (18) is less than one so that the LHS must be less than one, too. Hence, $\eta_{p_o}^{N_a} < 1 + \varepsilon_{p_o}^{N_o}$. Elasticity $\eta_{p_o}^{N_a}$ is the proportional rate at which transactions volume, N_a, increases with the fee, p_o. The term $1 + \varepsilon_{p_o}^{N_o}$ is the proportional rate at which payments to owners increase with p_o. To maximize profit, the growth rate in transactions volume must be less than the growth rate in payments. Otherwise, the platform could increase profit by raising p_o.

Based on the FOCs in Eqs. (15) and (16), we can obtain the following proposition regarding the optimal prices and profit.

Proposition 1. *When the parking platform serves as a profit-maximizing reseller, and $k > 0$, the optimal prices are*

$$p_o^{PM,r} = -\frac{\eta_{p_o}^{N_a}}{\left(1 + \eta_{p_d}^{N_a}\right)\left(1 + \varepsilon_{p_o}^{N_o}\right)} \frac{hN_a}{gN_o} k, \tag{19}$$

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service is often duration-dependent. It may also be necessary to move a car to another slot if the time available at the initial slot runs out.

$$p_d^{PM,r} = \frac{\eta_{p_d}^{N_a}}{1+\eta_{p_d}^{N_a}}k. \tag{20}$$

Platform profit is

$$\Pi^{PM,r} = \frac{\eta_{p_o}^{N_a} - 1 - \varepsilon_{p_o}^{N_o}}{\left(1+\eta_{p_d}^{N_a}\right)\left(1+\varepsilon_{p_o}^{N_o}\right)} hN_a k. \tag{21}$$

Proof. Eqs. (19) and (20) are derived by simultaneously solving Eqs. (15) and (16), and Eq. (21) is derived by substituting Eqs. (19) and (20) into Eq. (14). ☐

If $k = 0$, $\eta_{p_d}^{N_a} = -1$, and variations of Eqs. (19)-(21) apply. Prices and profit are continuous functions of k even in the limit as $k \to 0$. As noted above, the profit is positive if $\eta_{p_o}^{N_a} < 1 + \varepsilon_{p_o}^{N_o}$. Using Eq. (19) and Eq. (20), the fee can be written as a fraction of the selling price:

$$\frac{p_o^{PM}}{p_d^{PM}} = \left(-\frac{1}{\eta_{p_d}^{N_a}}\right)\left(\frac{\eta_{p_o}^{N_a}}{1+\varepsilon_{p_o}^{N_o}}\right)\left(\frac{hN_a}{gN_o}\right). \tag{22}$$

The RHS of Eq. (22) is the product of three terms, enclosed in brackets, each of which is less than one. The first term is less than one because $\eta_{p_d}^{N_a} < -1$ when $k > 0$, as per Eq. (17). (If $k = 0$, Eq. (22) still holds with $\eta_{p_d}^{N_a} = -1$.) The second term is less than one if the profit is positive, and it is smaller the larger is the profit. The third term is the average utilization rate of slots, θ. The lower the utilization rate, the less profitable an additional owner is, ceteris paribus, and hence the less the platform operator is willing to pay owners.

3.2 Social welfare maximization (reselling format)

We now assume that the platform operator seeks to maximize social welfare without any profit constraint. To derive the solution, it is necessary to account for the reservation utilities of owners and parkers, and the cost of disappointment to parkers, R_d. (The cost of rejection to owners, R_o, matters only for the commissioning format.) A parker who joins the platform gets an expected utility given in Eq. (1). A parker who does not join receives their idiosyncratic reservation utility, $\hat{U}_d$. An owner who joins the platform gets an expected utility given in Eq. (2). An owner who does not join receives their idiosyncratic reservation utility, $\hat{U}_o$. The profit is given by Eq. (14). Multiplying each component term by the number of individuals in question, adding the products, and cancelling revenues which are transfers between owners/parkers and the platform operator, gives expected social welfare:

$$SW = \left(B_d + R_d - kh\right) N_a - R_d N_d - W_o N_o + \int_{N_d}^{\hat{N}_d} \hat{U}_d\left(x\right) dx + \int_{N_o}^{\hat{N}_o} \hat{U}_o\left(y\right) dy. \tag{23}$$

The number of parkers who request a space, N_d, is determined by the condition that the marginal parker is indifferent about joining the platform. Given Eq. (1), the condition is

$$\left(B_d + R_d - p_d h\right) \frac{N_a}{N_d} - R_d = \hat{U}_d \left(N_d\right). \tag{24}$$

The number of owners who offer a space, N_o, is determined by the same condition as in Section 2.3 that the marginal owner is indifferent about joining the platform:

$$p_o^r g - W_o = \hat{U}_o \left(N_o\right). \tag{25}$$

The planner's problem is to maximize SW in Eq. (23) subject to Eqs. (24) and (25) and the matching function defined in Eq. (8). The necessary optimality conditions (for an interior optimum) are

$$\frac{\partial SW}{\partial p_o} = \left(B_d + R_d - kh\right) \frac{\partial N_a}{\partial p_o} - \left(B_d + R_d - p_d h\right) \frac{N_a}{N_d} \frac{\partial N_d}{\partial p_o} - p_o g \frac{\partial N_o}{\partial p_o} = 0, \tag{26}$$

$$\frac{\partial SW}{\partial p_d} = \left(B_d + R_d - kh\right) \frac{\partial N_a}{\partial p_d} - \left(B_d + R_d - p_d h\right) \frac{N_a}{N_d} \frac{\partial N_d}{\partial p_d} - p_o g \frac{\partial N_o}{\partial p_d} = 0. \tag{27}$$

Given Eq. (8),

$$\frac{\partial N_a}{\partial p_o} = \frac{\partial N_a}{\partial N_d} \frac{\partial N_d}{\partial p_o} + \frac{\partial N_a}{\partial N_o} \frac{\partial N_o}{\partial p_o}, \tag{28}$$

$$\frac{\partial N_a}{\partial p_d} = \frac{\partial N_a}{\partial N_d} \frac{\partial N_d}{\partial p_d} + \frac{\partial N_a}{\partial N_o} \frac{\partial N_o}{\partial p_d}. \tag{29}$$

For the reselling format, $\partial N_o / \partial p_d = 0$. The second term on the RHS of Eq. (29) is thus zero. Based on the optimality conditions, we obtain the following proposition.

Proposition 2. *When the parking platform serves as a social-welfare-maximizing reseller, the optimal prices are*

$$p_d^{FB,r} = \sigma_{N_d}^{N_a} k + \left(1 - \sigma_{N_d}^{N_a}\right) \frac{B_d + R_d}{h}, \tag{30}$$

$$p_o^{FB,r} = \frac{1}{g} \left(B_d + R_d - kh\right) \frac{\partial N_a}{\partial N_o}, \tag{31}$$

where FB denotes the first-best optimum.

Proof. Substituting Eq. (29) into Eq. (27), and using the elasticity $\sigma_{N_d}^{N_a}$ of the matching function, yields Eq. (30). Eq. (31) is derived by substituting Eq. (28) into Eq. (26), and replacing p_d by Eq. (30). □

In Proposition 2, Eq. (30) prescribes that the optimal selling price is a weighted average of the marginal operating cost, k, and the expression $\left(B_d + R_d\right)/h$. To understand Eq. (30), suppose, contrary to what has been assumed, that $\sigma_{N_d}^{N_a} = 1$.

Adding a parker to the platform then does not affect the probability, φ, that other parkers secure a slot. The additional parker imposes no externality on other parkers. The only cost to the system is the transactions cost that is incurred only if the parker obtains a space. Hence, $p_d^{FB,r} = k$. Now suppose that $\sigma_{N_d}^{N_a} = 0$. Adding a parker to the platform has no effect on how many in total get a slot. In effect, the new parker displaces one of the others with probability φ. This denies the other parker the benefit from renting a space, B_d, and also imposes on them a disappointment cost of R_d. Consequently, $p_d^{FB} = (B_d + R_d)/h$. The selling price is scaled down by h because the price is paid over this period. The selling price does not depend on the operating cost because the number of spots that are rented does not change.

Eq. (31) indicates that the optimal fee is proportional to the rate of increase in the number of parkers who can rent a spot, $\partial N_a/\partial N_o$. If a new parker does get a spot, he receives the benefit, B_d, and avoids the cost of disappointment, R_d. A transactions cost of k is incurred per unit of time. The net benefit is thus $B_d + R_d - kh$. The fee is scaled down by g because it is paid over this period.

Regarding the platform profit under first-best pricing, we have the following proposition.

Proposition 3. *When the parking platform serves as a social-welfare-maximizing reseller, the platform profit is*

$$\begin{aligned}\Pi\left(p_o^{FB,r}, p_d^{FB,r}\right) &= \left(1 - \frac{\partial N_a}{\partial N_o}\frac{N_o}{N_a} - \frac{\partial N_a}{\partial N_d}\frac{N_d}{N_a}\right)(B_d + R_d - kh)\,N_a \\ &= \left(1 - \sigma_{N_o}^{N_a} - \sigma_{N_d}^{N_a}\right)(B_d + R_d - kh)\,N_a,\end{aligned} \tag{32}$$

where $\Pi\left(p_o^{FB,r}, p_d^{FB,r}\right) > 0$ *if* $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} < 1$*;* $\Pi\left(p_o^{FB,r}, p_d^{FB,r}\right) < 0$ *if* $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} > 1$*; and* $\Pi\left(p_o^{FB,r}, p_d^{FB,r}\right) = 0$ *if* $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} = 1$.

Proof. Eq. (32) is derived by substituting Eqs. (30) and (31) into Eq. (14). ☐

Proposition 3 indicates that the platform earns a positive profit if the matching function has decreasing returns to scale (i.e., $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} < 1$), a negative profit under increasing returns to scale (i.e., $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} > 1$), and zero profit under constant returns to scale (i.e., $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} = 1$).

4 The two-sided pricing strategy for a commissioning platform

Section 3 derived the pricing strategy of a platform that resells private parking slots and provides owners with a guaranteed payment. This section undertakes a parallel treatment of the commissioning format in which owners are paid only if their individual slots are rented. For each accepted request, the platform operator charges a commission equal to the difference between the selling price and the buying price. To economize on notation, superscript 'c' denoting the commission format is omitted except when reporting the main results.

4.1 Profit maximization (commissioning format)

The profit-maximizing platform operator's problem can be formulated as:

$$\max_{p_o \geq 0, p_d \geq 0} \Pi = p_d h N_a - p_o g \theta N_o - k h N_a. \tag{33}$$

The first term in Eq. (33) is the revenue collected from parkers whose requests are fulfilled. The second term is payment to owners which depends on the utilization rate of the slots, θ. The third term is the platform's cost of processing accepted requests. Except for the factor θ in the second term, Eq. (33) is the same as Eq. (14) for the reselling format.

Given $\theta = hN_a/(gN_o)$, Eq. (33) can be rewritten as:

$$\max_{p_o \geq 0, p_d \geq 0} \Pi = \left(p_d - p_o - k\right) h N_a. \tag{34}$$

The first-order necessary conditions for profit maximization can be written as

$$p_o = \eta_{p_o}^{N_a} \left(p_d - p_o - k\right), \tag{35}$$

$$p_d = -\eta_{p_d}^{N_a} \left(p_d - p_o - k\right). \tag{36}$$

Based on the optimality conditions, we obtain the following proposition.

Proposition 4. *When the parking platform serves as a profit-maximizing commissioner, and $k > 0$, the optimal prices are*

$$p_o^{PM,c} = -\frac{\eta_{p_o}^{N_a}}{1 + \eta_{p_o}^{N_a} + \eta_{p_d}^{N_a}} k, \tag{37}$$

$$p_d^{PM,c} = \frac{\eta_{p_d}^{N_a}}{1 + \eta_{p_o}^{N_a} + \eta_{p_d}^{N_a}} k. \tag{38}$$

Platform profit is

$$\Pi^{PM,c} = -\frac{khN_a}{1 + \eta_{p_o}^{N_a} + \eta_{p_d}^{N_a}}. \tag{39}$$

Proof. Eqs. (37) and (38) are derived by solving Eqs. (35) and (36) simultaneously. Eq. (39) is derived by substituting Eqs. (37) and (38) into Eq. (33). □

Rearranging Eq. (36) yields a counterpart to Eq. (17) for the reselling format:

$$\frac{p_d^{PM} - p_o^{PM} - k}{p_d^{PM}} = -\frac{1}{\eta_{p_d}^{N_a}}. \tag{40}$$

The Lerner index is calculated using the combined cost of selling a slot, $p_o^{PM} + k$. Eq. (37) and Eq. (38) yield a counterpart to Eq. (22) for the reselling format:

$$\frac{p_o^{PM}}{p_d^{PM}} = -\frac{\eta_{p_o}^{N_a}}{\eta_{p_d}^{N_a}}.$$

As a fraction of the selling price, the fee is larger the more elastic are transactions with respect to the fee and the less elastic they are with respect to the selling price.

4.2 Social welfare maximization (commissioning format)

The welfare function for a commissioning platform differs from the reselling platform in two respects. First, an owner who joins the platform receives an expected utility given in Eq. (3) instead of Eq. (2). An owner whose slot is not rented out incurs a disappointment cost. Second, the platform profit is given in Eq. (33) instead of Eq. (14). Payments to owners depend on the utilization rate of slots. After cancelling terms, the welfare function is:

$$SW = \left(B_d + R_d - kh + \frac{h}{g}\left(R_o - W_o\right)\right) N_a - R_d N_d - R_o N_o + \int_{N_d}^{\hat{N}_d} \hat{U}_d\left(x\right) dx + \int_{N_o}^{\hat{N}_o} \hat{U}_o\left(y\right) dy. \tag{41}$$

The number of parkers is still determined by the condition in Eq. (24):

$$\left(B_d + R_d - p_d h\right) \frac{N_a}{N_d} - R_d = \hat{U}_d\left(N_d\right). \tag{42}$$

The number of owners is determined by the condition

$$\theta\left(p_o^c g + R_o - W_o\right) - R_o = \hat{U}_o\left(N_o\right). \tag{43}$$

The planner's problem is to maximize SW in Eq. (41) subject to Eq. (42), Eq. (43) and the matching function. Using Eq. (28) and Eq. (29), the first-order necessary conditions can be written as

$$\begin{aligned}\frac{\partial SW}{\partial p_o} = &\left(\left(B_d + R_d - kh + \frac{h}{g}\left(R_o - W_o\right)\right)\frac{\partial N_a}{\partial N_d} - \left(B_d + R_d - p_d h\right)\frac{N_a}{N_d}\right)\frac{\partial N_d}{\partial p_o} \\ &+ \left(\left(B_d + R_d - kh + \frac{h}{g}\left(R_o - W_o\right)\right)\frac{\partial N_a}{\partial N_o} - \theta\left(R_o - W_o + p_o g\right)\right)\frac{\partial N_o}{\partial p_o} = 0\end{aligned}, \tag{44}$$

$$\begin{aligned}\frac{\partial SW}{\partial p_d} = &\left(\left(B_d + R_d - kh + \frac{h}{g}\left(R_o - W_o\right)\right)\frac{\partial N_a}{\partial N_d} - \left(B_d + R_d - p_d h\right)\frac{N_a}{N_d}\right)\frac{\partial N_d}{\partial p_d} \\ &+ \left(\left(B_d + R_d - kh + \frac{h}{g}\left(R_o - W_o\right)\right)\frac{\partial N_a}{\partial N_o} - \theta\left(R_o - W_o + p_o g\right)\right)\frac{\partial N_o}{\partial p_d} = 0\end{aligned}. \tag{45}$$

Based on the optimality conditions, we obtain the following proposition.

Proposition 5. *When the parking platform serves as a social-welfare-maximizing commissioner, the optimal prices are*

$$p_o^{FB,c} = \sigma_{N_o}^{N_a}\left(\frac{B_d + R_d}{h} - k\right) + \left(1 - \sigma_{N_o}^{N_a}\right)\frac{W_o - R_o}{g}, \tag{46}$$

$$p_d^{FB,c} = \sigma_{N_d}^{N_a} k + \left(1 - \sigma_{N_d}^{N_a}\right)\frac{B_d + R_d}{h} + \sigma_{N_d}^{N_a}\frac{W_o - R_o}{g}. \tag{47}$$

Proof. Eqs. (46) and (47) are derived by simultaneously solving Eqs. (44) and (45), which are linear equations with respect to p_o and p_d. □

In Proposition 5, the parking price governed by Eq. (47) matches Eq. (30) for the reselling format except for addition of the last term on the RHS. Unlike for the reselling format, with commissioning the choice of p_d affects the supply of owners. Given $\sigma_{N_d}^{N_a} > 0$, an increase in the number of parkers raises the number of transactions and hence the number of owners who succeed in renting their slots. An owner forgoes the benefit, W_o, from using a slot for other purposes, but avoids the disappointment, R_o, from failing to rent it. The net cost, $W_o - R_o$, appears in the last term of Eq. (47) so that parkers face the full cost they create if they decide to participate in the platform.

To compare Eq. (46) with Eq. (31) for the socially optimal fee in the reselling format, we rewrite Eq. (31) as follows:

$$p_o^{FB,r} = \sigma_{N_o}^{N_a}\left(\frac{B_d + R_d}{h} - k\right)\theta. \tag{48}$$

Eq. (46) differs from Eq. (48) in two ways. First, it is inflated by a factor $1/\theta > 1$ to offset the lower probability with the commissioning format that an owner can rent their slot. Second, it includes the term $\left(1-\sigma_{N_o}^{N_a}\right)(W_o - R_o)/g$. The reasoning for this component is similar to that for the third term in Eq. (47) as well as the term $\left(1-\sigma_{N_d}^{N_a}\right)(B_d + R_d)/h$ in Eq. (30) for $p_d^{FB,r}$. If $\sigma_{N_o}^{N_a} < 1$, an additional owner reduces the probability that other owners can rent their slots. This saves them the opportunity cost of renting their slots, W_o, but imposes on them a disappointment cost of R_o.

Regarding the platform profit under social welfare maximization, we have the following proposition.

Proposition 6. *When the parking platform serves as a social-welfare-maximizing commissioner, the platform profit is*

$$\Pi\left(p_o^{FB,c}, p_d^{FB,c}\right) = \left(1-\sigma_{N_o}^{N_a}-\sigma_{N_d}^{N_a}\right)\left(B_d + R_d - kh - \frac{h}{g}(W_o - R_o)\right)N_a, \quad (49)$$

where $\Pi\left(p_o^{FB,c}, p_d^{FB,c}\right) > 0$ *if* $\sigma_{N_o}^{N_a}+\sigma_{N_d}^{N_a} < 1$; $\Pi\left(p_o^{FB,c}, p_d^{FB,c}\right) < 0$ *if* $\sigma_{N_o}^{N_a}+\sigma_{N_d}^{N_a} > 1$; *and* $\Pi\left(p_o^{FB,c}, p_d^{FB,c}\right) = 0$ *if* $\sigma_{N_o}^{N_a}+\sigma_{N_d}^{N_a} = 1$.

Proof. Eq. (49) is derived by substituting Eqs. (46) and (47) into Eq. (34). ☐

Proposition 6 indicates that, similar to the reselling format, the platform earns a positive profit if the matching function has decreasing returns to scale (i.e., $\sigma_{N_o}^{N_a} + \sigma_{N_d}^{N_a} < 1$), a negative profit under increasing returns to scale, and zero profit under constant returns to scale. Compared to platform profit in the commissioning format given by Eq. (32), the profit in Eq. (49) includes the additional term $-\frac{h}{g}(W_o - R_o)$. This term reflects the opportunity cost and disappointment that owners incur in the commissioning format.

5 Supply-demand equilibrium: comparative statics

In this section we examine how parameter values affect the supply-demand equilibrium in the reselling and commissioning formats. Each format has three endogenous quantities, N_o, N_d, and N_a, and two endogenous prices, p_o, and p_d. Deriving comparative statics results for all five variables is analytically intractable. So, we proceed in two steps. In the first step (Section 5.1), we treat prices p_o and p_d parametrically and derive comparative statics results for N_o, N_d, and N_a with respect to the two prices as well as other parameter values. This step can be viewed as applying in the short run when either prices are slow to adjust to market conditions, or prices are regulated for some reason. The comparative static results apply regardless of how prices are set, and not only to the profit-maximizing and welfare-maximizing regimes.

In the second step (Section 5.2), we revert to the ‘long run’ in which all five variables are endogenous. We assume that the matching function has a Cobb-Douglas form, and derive comparative statics results analytically (using symbolic software). The resulting expressions are extremely long, and cannot be signed analytically. So we solve the functions numerically for a range of parameter values, and compare the signs with those derived in step one. To economize on space, we perform this second step only for the profit-maximization regime.

5.1 Comparative statics with fixed prices

Comparative statics are derived for each format by differentiating the equilibrium conditions.

Reselling format: As discussed in Section 2, equilibrium in the reselling format is governed by Eqs. (24), (25) and (5), which are repeated here:

$$(B_d + R_d - p_d h)\, N_a = \left(\hat{U}_d\,(N_d) + R_d\right) N_d, \tag{50a}$$

$$p_o g - W_o = \hat{U}_o\,(N_o)\,, \tag{50b}$$

$$N_a = G\,(N_o, N_d)\,. \tag{50c}$$

Recall that Eq. (50a) is the participation condition for the marginal parker, Eq. (50b) is the participation condition for the marginal owner, and Eq. (50c) is the matching function.

Commissioning format: Equilibrium in the commissioning format is governed by the following conditions

$$(B_d + R_d - p_d h)\, N_a = \left(\hat{U}_d\,(N_d) + R_d\right) N_d, \tag{51a}$$

$$hN_a\,(p_o g + R_o - W_o) = gN_o \left(\hat{U}_o\,(N_o) + R_o\right), \tag{51b}$$

$$N_a = G\,(N_o, N_d)\,. \tag{51c}$$

Eqs. (51a) and (51c) are identical to their counterparts in Eq. (50). The only difference is Eq. (51b), which accounts for the possibility that owners fail to rent their spots and incur a disappointment cost.

Based on the equilibrium conditions, the derivatives of N_d, N_o, and N_a with respect to W_o, B_d, g, h, k, R_o, R_d, p_o, and p_d are derived and the signs are reported in Table 3. Detailed derivations are relegated to Appendix A and Appendix B.

Effect of W_o. A higher inconvenience to owners from renting a slot reduces the equilibrium numbers of owners, parkers, and matches on the platform in both formats. These effects are intuitively obvious.

Effect of B_d. An increase in utility from renting a slot induces more parkers to use

Table 3: Comparative statics with fixed prices ('+': positive; '0': zero; '$-$': negative; '•': sign indeterminate)

Format	Reselling			Commissioning		
	N_o	N_d	N_a	N_o	N_d	N_a
W_o	$-$	$-$	$-$	$-$	$-$	-
B_d	0	+	+	+	+	+
g	+	+	+	•	•	•
h	0	$-$	$-$	•	•	•
k	0	0	0	0	0	0
R_o	0	0	0	$-$	$-$	$-$
R_d	0	$-$	$-$	$-$	$-$	$-$
p_o	+	+	+	+	+	+
p_d	0	$-$	$-$	$-$	$-$	$-$

the platform and secures more matches in both formats. It has no effect on owners in the reselling format because their income is guaranteed. However, because there are more parkers the number of matches still grows.

Effect of g. In the reselling format, an increase in the amount of time that a slot is available increases all three quantities. With p_o fixed, the increase in duration raises the income that owners can earn. This attracts more owners, which raises the matching rate for parkers and encourages more of them to join the platform. Differently, in the commissioning format, the effects of g cannot be signed as it marginally reduces the utilization rate on the one hand (discourages owners) and increases the payment to owners on the other hand (attracts owners).

Effect of h. In the reselling format, an increase in the duration of parking time demanded does not affect owners because their incomes are guaranteed. However, it reduces the number of parkers who wish to participate because the cost of renting space for a longer stay rises, while both the benefit they derive from parking, B_d, and the rental cost per hour, p_d, are held fixed. Similar to the effect of g, the effect of h with the commissioning format is a priori ambiguous.

Effect of k. In both formats, the platform's operating cost has no effect on equilibrium since it does not affect owners or parkers directly, and prices are held constant.

Effect of R_o. In the reselling format, the cost of disappointment for owners has no effect on equilibrium because owners face no risk. In the commissioning format, an increase in the disappointment cost of owners discourages owners and thus reduces the overall number of matches and parkers.

Effect of R_d. With both formats, an increase in the disappointment cost of parkers discourages parkers, and hence the number of matches drops. It also reduces the number of owners in the commissioning format. However, in the reselling format, it has no effect on owners because their income is guaranteed.

Effect of p_o. In both formats, an increase in the fee attracts more owners to participate. The matching rate rises, which encourages more parkers to participate as well.

Effect of p_d. In both formats, parking demand and the number of matches falls when the price of renting increases. It has no effect on owners in the reselling format because their income is guaranteed, but reduces the number of owners in the commissioning format.

In summary, many comparative statics effects are the same in both formats. Two differences are worth noting. First, in the reselling format the number of owners is not affected by many parameters because owners' incomes are guaranteed. Second, the effects of the two parking durations, g and h, are a priori ambiguous in the commissioning format. Unlike in the reselling format, both parameters affect the participation condition for owners in Eq. (51b). So does the disappointment cost, R_o, and, as explained below in Section 5.3, the size of R_o relative to the inconvenience of renting a slot, W_o, determines the efficiency of the commissioning format.

5.2 Comparative statics with endogenous prices

We now turn to the long run in which p_o and p_d are endogenous (i.e., they will be optimized for either profit maximization or social welfare maximization). As mentioned, the long-run effects cannot be signed analytically as they involve both the equilibrium conditions and optimality conditions in relation to p_o and p_d. Thus, we solve the effects numerically for a range of parameter variables and compare them with the short-run effects (Table 3). To economize on space, we only analyze the profit-maximization regime.

We first introduce the functional forms and parameter values that are used as the base case in the numerical studies in Section 6. We assume that the matching function has a Cobb-Douglas form: $N_a = A(N_d)^{\sigma_{N_d}^{N_a}}(N_o)^{\sigma_{N_o}^{N_a}}$, where parameter A ($A > 0$) depends on the size and characteristics of the market, and $\sigma_{N_d}^{N_a}$ and $\sigma_{N_o}^{N_a}$ are the (constant) elasticities, as defined in Eq. (6). According to the shared parking supply-demand matching algorithm developed in Shao et al. (2016), the best-fitting Cobb-Douglas matching function given parking durations $g = 7$ and $h = 3$ is:

$$N_a = 0.9(N_d)^{0.46}(N_o)^{0.62}, \tag{52}$$

with a goodness of fit of $R^2 = 0.9772$. We thus set $A = 0.9$, $\sigma_{N_d}^{N_a} = 0.46$ and

$\sigma_{N_o}^{N_a} = 0.62$. Since $\sigma_{N_d}^{N_a} + \sigma_{N_o}^{N_a} > 1$, the function exhibits increasing returns to scale. It should be noted that the Cobb-Douglas function adopted only provides a local approximation to the "true" matching function. Clearly, Eq. (52) is not applicable if the result is $N_a > N_d$ or $hN_a > gN_o$. In the numerical studies, we will compare the results with an alternative matching function.

Other base-case parameter values are listed in Table 4. The average parking duration h and average renting duration g are set according to Shao et al. (2016). Other parameters are chosen so that the solution is comparable with real-world data (e.g., the parking price p_d resembles the hourly public parking prices obtained from the Transport Department of Hong Kong).

Table 4: Base-case parameter values

W_o	B_d	g	h	k	R_o	R_d	$\hat{N}_o$	$\hat{U}_o(n)$	$\hat{N}_d$	$\hat{U}_d(n)$
35	60	7	3	1/3	15	15	500	$25+0.15n$	1000	$0.025n$

We present the comparative statics results for the profit maximization regime in Table 5. To ease comparison, the short-run effect is listed to the right in each column wherever the sign differs from the counterpart in Table 3. As shown in Table 5, in the reselling format, nine of the 21 signs for N_o, N_d, and N_a differ from the short run when prices are fixed. In the commissioning format, no less than twelve of the 21 signs for the quantities differ from the short run with prices fixed. In four cases, the signs of the derivatives depend on parameter values and are denoted by '$-/+$' or '$+/-$'. For example, the sign of derivative $\partial N_a/\partial R_d$ is negative when evaluated at the base-case parameter values, but can be positive with other values. Several effects that are zero in the short run become positive or negative in the long run when the platform operator adjusts prices. Explanations follow, with a focus mainly on the effects on prices and differences compared with the short-run effects.

Table 5: Comparative statics for profit maximization in two formats with endogenous prices ('+': positive; '0': zero; '−': negative; '●': sign indeterminate)

Format	Reselling								Commissioning							
	N_o		N_d		N_a		p_o	p_d	N_o		N_d		N_a		p_o	p_d
W_o	$-$		$-$		$-$		$+$	$-$	$-$		$-$		$-$		$+$	$+$
B_d	$+$	0	$+$		$+$		$+$	$+$	$+$		$+$		$+$		$+$	$+$
g	0	$+$	0	$+$	0	$+$	$-$	0	$+$	●	$+$	●	$+$	●	$-$	$-$
h	$-$	0	$-$		$-$		$-$	$-$	$-$	●	$-$	●	$-$	●	$-$	$-$
k	$-$	0	$-$	0	$-$	0	$-$	$+$	$-$	0	$-$	0	$-$	0	$-$	$+$
R_o	0		0		0		0	0	$-$		$+/-$	$-$	$-$		$-/+$	$-$
R_d	$+$	0	$-$		$-/+$		$+$	$+$	$+$	$-$	$-$		$-/+$	$-$	$+$	$+$

Effect of W_o. In both formats, a higher inconvenience to owners from renting a slot induces the operator to support the market by raising the fee paid to owners and reducing the price paid by parkers. Nevertheless, as in the short run the equilibrium numbers of owners, parkers, and matches still fall.

Effect of B_d. In both formats, an increase in utility for parkers from renting a slot induces the operator to raise the price for parkers. The operator also raises the fee paid to owners to accommodate the greater demand. In the new equilibrium, all quantities increase.

Effect of g. In the reselling format, if owners make their slots available for a longer time, the operator reduces p_o, and unlike in the short run none of the quantities change. To see why, note that parameter g does not affect the operator's profit directly and it only appears as $p_o g$ in equilibrium condition Eq. (50b) governing participation by owners. The operator can attract the same number of owners by varying p_o inversely with g to maintain owner's income, $p_o g$. In the commissioning format, the operator reduces both prices if g increases. This differs from the reselling format in which p_d does not change. All quantities also increase, unlike in the short run where the effects are ambiguous.

Effect of h. If parkers require more time to park with no corresponding increase in the benefit, willingness to pay for parking per hour falls in both formats. The operator accommodates the drop by reducing p_d. It also reduces p_o because less parking space is needed. Unlike in the short run, all quantities decrease.

Effect of k. An increase in the transactions cost makes the platform less profitable in both formats. The operator protects its profit margin by increasing p_d and reducing p_o; the opposite to how it responds to an increase in W_o. All quantities fall.

Effect of R_o. In the reselling format, the cost of disappointment for owners has no effect on equilibrium because owners face no risk, similar to the short run. In the commissioning format, an increase in R_o gives the operator a stronger incentive to increase the probability that space will be rented out. It does so by reducing p_d to boost demand and may also reduce p_o to decrease supply. Supply duly drops, while demand may rise unlike in the short run.

Effect of R_d. If the cost of disappointment for parkers increases, in both formats the operator has a stronger incentive to increase the probability that space will be available. It does so by raising both p_o and p_d. Unlike in the short run, the number of parkers falls and the number of participating owners rises.

In summary, many of the comparative statics effects in both the reselling and commissioning formats change sign in the long run when prices adjust (in order to achieve profit maximization). This highlights the importance of considering how the platform operator will respond if supply or demand conditions change.

5.3 Platform profit and social welfare

We now compare profits and welfare in the reselling and commissioning formats. The rankings turn out to depend on the relative size of two cost parameters for owners: the cost of inconvenience, W_o, and the cost of disappointment, R_o. The main results for both the profit-maximization and welfare-maximization regimes are summarized in three propositions.

Proposition 7. *Assume that owners experience the same disutility from inconvenience and disappointment (i.e., $W_o = R_o$). Then the platform operator can: (a) replicate any commissioning policy by choosing a suitable reselling policy, and (b) replicate any reselling policy by choosing a suitable commissioning policy.*

Proof. See Appendix C. □

According to Proposition 7, if $W_o = R_o$, the reselling and commissioning formats are equally effective in maximizing either profit or welfare. If prices in each format are set optimally, the platform hosts the same numbers of owners and parkers, and supports the same number of matches. To understand this result, note that the equilibrium conditions for the reselling and commissioning formats differ only in the conditions for owners, Eqs. (50b) and Eq. (51b), respectively. If $W_o = R_o$, condition Eq. (51b) simplifies to $\theta p_o g - W_o = \hat{U}_o(N_o)$. Except for the factor θ, this is the same as Eq. (50b). Setting the price for owners equal to θp_o in the reselling format is therefore equivalent to setting a price of p_o in the commissioning format.

Propositions 8 and 9 establish a ranking for the two formats when $W_o \neq R_o$.

Proposition 8. *Assume that owners incur greater cost from inconvenience than disappointment (i.e., $W_o > R_o$). Then a profit-maximizing platform operator can earn a higher profit, and a social-welfare-maximizing operator can generate a higher surplus, with the commissioning format than the reselling format.*

Proof. See Appendix D. □

Proposition 9. *Assume that owners incur greater cost from disappointment than inconvenience (i.e., $W_o < R_o$). Then a profit-maximizing platform operator can earn a higher profit, and a social-welfare-maximizing operator can generate a higher surplus, with the reselling format than the commissioning format.*

Proof. The proof of Proposition 9 is similar to that of Proposition 8, and is thus omitted. □

According to Propositions 7-9, the commissioning format is superior to the reselling format if $W_o > R_o$, and the reselling format is superior if $W_o < R_o$. The relative size of W_o and R_o depends on an owner's characteristics and circumstances. Parameter W_o depends mainly on the opportunity cost of not having a parking space available while it is being rented. It may be small for a person who lives alone, drives to work, and rarely needs to use the parking space while at work. The cost may be significant for a multi-person household with only one parking spot, and members who stay at home. It may also be substantial for a business that experiences substantial and unpredictable fluctuations in demand such that all parking spaces sometimes fill up. The disappointment cost R_o can have both a planning and psychological element. The planning cost may be high if the owner cannot adapt by quickly finding an alternative use of the parking space. The psychological cost may be appreciable for someone who is participating in a parking sharing program for the first time. After gaining experience, the intensity of disappointment may fade as individuals become acclimated to the uncertain prospects of renting a slot out. This is consistent with the theory of reference-dependent preferences, "gain–loss" utility, and empirical studies of experience and acclimatization (see, for example, List (2003) and Kőszegi and Rabin (2006)).

On balance, it seems plausible that $W_o > R_o$. If so, the commissioning format dominates the reselling format regardless of the platform operator's objectives. The advantage of the commissioning format derives from the assumption that the owner of a parking space can use it unless it is actually rented. By contrast, with the reselling format the owner can no longer access it when it is offered for rent regardless of whether a renter is found. As noted in the introduction, Moby uses a commissioning format although individual owners are free to set their own fees.

6 Numerical analysis

This section explores numerical examples to illustrate the optimal operation decisions and resulting system performance under different administrative objectives and business formats. We first analyze the base-case results, and then compare the results with those with a different matching function. We also conduct sensitivity analysis to examine the system equilibrium under alternative parameter values.

6.1 Base-case analysis and comparison with an alternative matching function

The base-case setting was introduced in Section 5.2. As mentioned, the matching function and the associated parameters are set according to the results of Shao et al. (2016). Other parameter values are listed in Table 4. In addition to the base-case setting, an alternative matching function is adopted for comparison, which is given by:

$$N_a = \frac{C_1 N_o N_d}{N_o + N_d + C_2}, \tag{53}$$

This matching function is used in Wang et al. (2018) to characterize the supply-demand matching in a ridesharing program. Similar to the Cobb-Douglas matching function in Eq. (52), it exhibits increasing returns to scale. Parameter values are assumed to be $C_1 = 2.5$ and $C_2 = 0.5$, under which the parking sharing prices are of comparable magnitude to the real-world data provided by the Transport Department of Hong Kong.

The system equilibrium will be assessed not only by platform profit and social welfare, but also the gross profit margin of the platform. Net profit is given by Eqs. (14) and (33) in the two formats, respectively. Gross profit is $p_d h N_a - p_o g N_o$ in the reselling format, and $p_d h N_a - p_o g \theta N_o$ in the commissioning format. The gross profit margin is defined to be gross profit divided by revenue:

$$\text{Margin} = \begin{cases} 1 - \dfrac{p_o}{\theta p_d} \text{ (in the reselling format)}, \\ 1 - \dfrac{p_o}{p_d} \text{ (in the commissioning format)}. \end{cases} \tag{54}$$

Table 6 presents the results with the base-case setting and the alternative matching function. For each setting, we examine results in four scenarios: (a) profit-maximization with reselling format (PM-R); (b) social welfare-maximization with reselling (SW-R); (c) profit-maximization with commissioning (PM-C); and (d) social welfare-maximization with commissioning (SW-C).

The results in the base-case setting are shown in the left panel of Table 6. In the PM-R regime (Column 2), the acceptance rate of parking demands is $\varphi = 0.80$, and the occupancy rate is $\theta = 0.84$. The profit margin of 0.21 is close to the 20 percent commission rate charged by Moby. Parking demand N_d and matches N_a respond strongly to the price charged to parkers p_d because their respective elasticities are all relatively large, i.e., $\varepsilon_{p_d}^{N_d}$ and $\varepsilon_{p_d}^{N_a}$. Parking supply N_o is unaffected by price p_d because owners are guaranteed income at rate p_o. In the SW-R regime, the buying price for owners p_o is higher and selling price for parkers p_d is lower than those in the PM-R regime, leading to a higher utilization rate $\theta = 0.90$, higher acceptance rate $\varphi = 0.87$, and larger social welfare. However, a platform operating in the SW-R regime requires a subsidy to sustain the service.

The corresponding results for the commissioning format are presented in Columns 4 & 5 in Table 6. In the PM-C regime, compared to the reselling format (PM-R), prices p_d and p_o, supply N_o, and matches N_a are higher, while demand N_d is slightly lower. The acceptance rate of parking demands $\varphi = 0.89$ is higher, but the occupancy rate $\theta = 0.79$ is lower. Parking supply, demand, and matches are considerably more responsive to the selling price p_d. Consistent with Proposition 8, since $W_o > R_o$ the platform earns a higher profit in the commissioning format than reselling. The profit difference is nearly 21%, suggesting that the platform would strongly prefer commissioning to reselling if the parameter values in the example are representative of real market conditions. Both prices are higher in the SW-C regime than the SW-R regime, but demand is lower. Consistent with Proposition 8, the commissioning format is superior as both platform profit and social welfare are larger than in the reselling format.

The results with the alternative matching function in Eq. (53) are presented in the right panel of Table 6 (Columns 6-9). Optimal prices and the resulting system equilibrium are comparable to those in the base case. Prices, supply, demand, and matches are higher in the commissioning format than reselling for both the PM and SW regimes. The commissioning format is preferable to the reselling format as it generates a larger profit in the profit-maximization regime, and a larger social welfare in the social welfare-maximization regime.

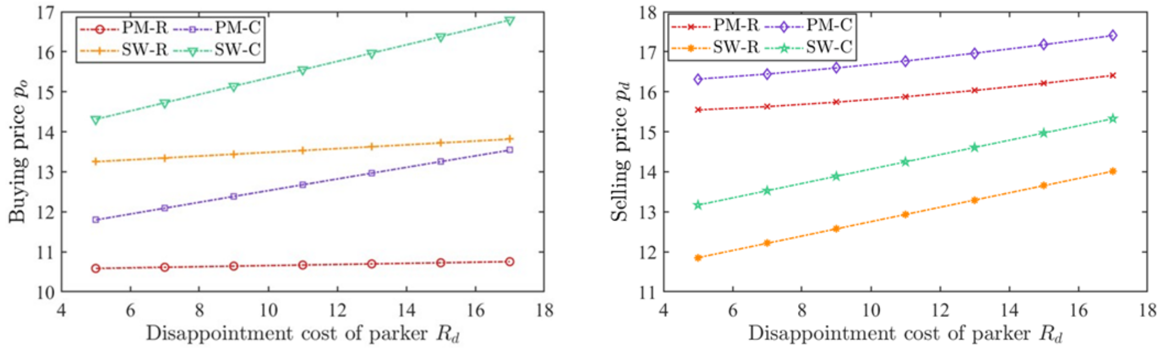
Table 6: Optimal solutions under different administrative regimes and matching functions

Matching function	$N_a = 0.9(N_o)^{0.62}(N_d)^{0.46}$				$N_a = \frac{2.5\cdot N_o N_d}{N_o+N_d+0.5}$			
	PM-R	SW-R	PM-C	SW-C	PM-R	SW-R	PM-C	SW-C
N_o	101	240	115	264	68	135	84	168
N_d	246	580	236	545	131	261	144	288
p_o	10.73	13.72	13.25	16.38	10.02	11.47	13.99	16.64
p_d	16.21	13.65	17.18	14.97	17.83	16.57	18.25	16.95
N_a	198	503	210	518	111	223	133	265
φ	0.80	0.87	0.89	0.95	0.85	0.85	0.92	0.92
θ	0.84	0.90	0.79	0.84	0.70	0.71	0.68	0.68
$\varepsilon_{p_d}^{N_o}$	0.00	0.00	-3.68	-1.13	0.00	0.00	-3.75	-1.59
$\varepsilon_{p_d}^{N_d}$	-2.22	-1.17	-5.45	-2.61	-2.98	-2.04	-6.15	-3.26
$\varepsilon_{p_d}^{N_a}$	-1.02	-0.54	-4.79	-1.70	-1.02	-0.70	-4.64	-2.21
Profit(10^3)	1.86	-2.98	2.26	-2.71	1.09	-0.02	1.56	-0.02
$SW(10^5)$	0.47	0.49	0.48	0.50	0.45	0.46	0.46	0.47
Margin	0.21	-0.12	0.23	-0.09	0.20	0.02	0.23	0.02

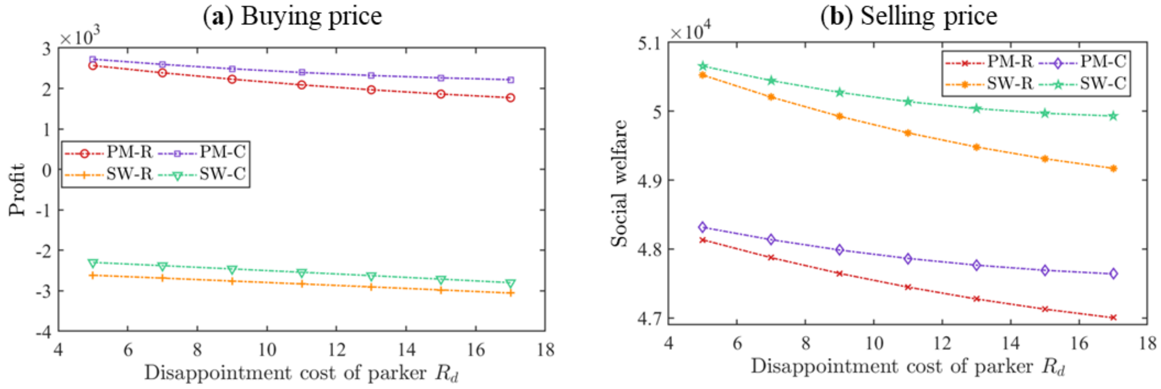
6.2 Varying the disappointment costs

In this subsection, we examine the optimal pricing strategies and the resulting system efficiency with varying disappointment costs for parkers and owners, while holding other parameters fixed at their base-case values.

The effects of varying the disappointment cost for parkers (R_d) are presented in Figure 1. Consistent with Table 5, with the increase of parker's disappointment cost, the platform operator raises the buying and selling prices in all regimes (Figure 1(a) and Figure 1(b)). Compared to the reselling format, the commissioning operator raises the buying price more quickly in each administrative regime. This is because, when the parker's disappointment cost is larger, being rejected is more costly for the parker. To attract parkers, the operator has to raise the buying price to attract more owners to participate in the program. Moreover, since the commissioning operator only pays for successful matches, he/she concerns less on supply wastage, and hence can be more aggressive in raising the buying price to attract owners. As shown in Figure 1(c) and Figure 1(d), profit and social welfare decrease with parker's disappointment, R_d, in all regimes. While the trend of profit is almost parallel, social welfare drops more quickly in the reselling format than with commissioning in both administrative regimes.



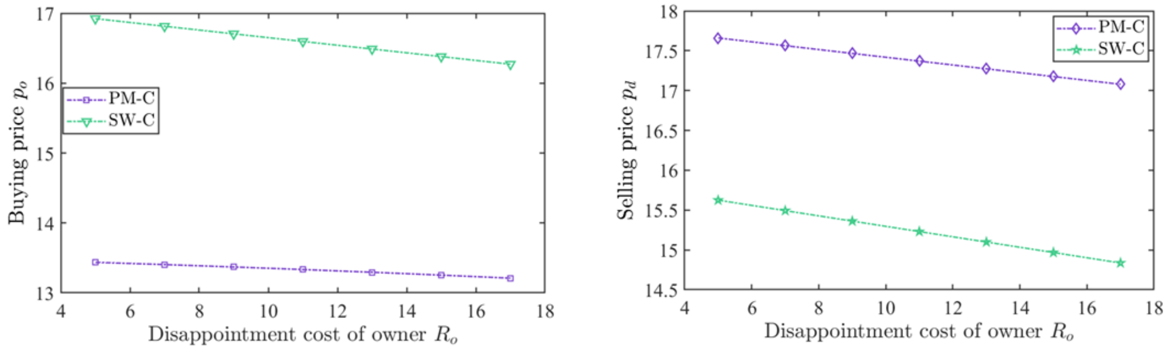
(a) Buying price **(b)** Selling price



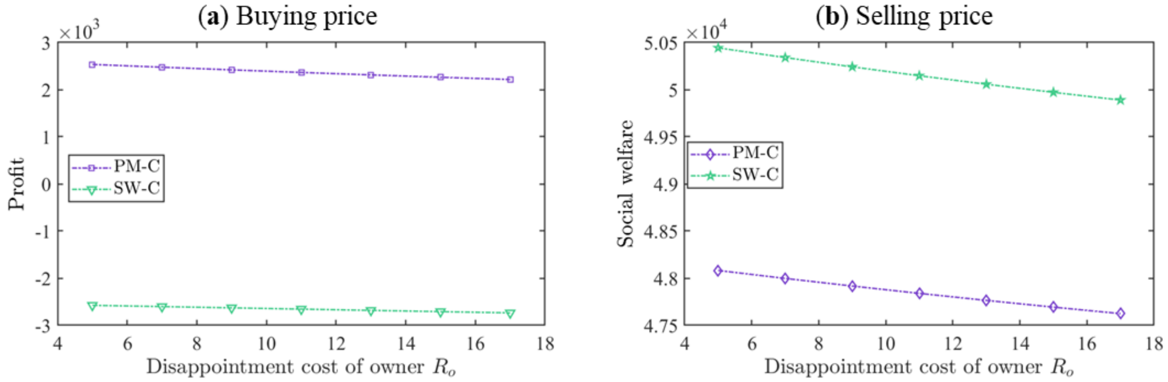
(c) Profit **(d)** Social welfare

Figure 1: Optimal pricing strategy, profit and social welfare as functions of the disappointment cost of parkers, R_d

Figure 2 presents the effects of varying the disappointment cost for owners, R_o, which matters only for the commissioning format. The owners' fee decreases with R_o in both regimes. The operator limits the loss of business by reducing the selling price.



(a) Buying price **(b)** Selling price



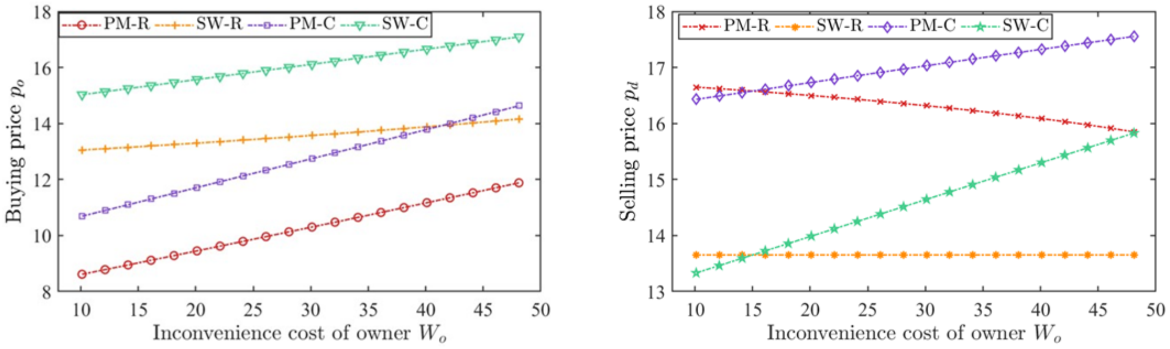
(c) Profit **(d)** Social welfare

Figure 2: Optimal pricing strategy, profit and social welfare as functions of owners' disappointment cost, R_o, with the commissioning format

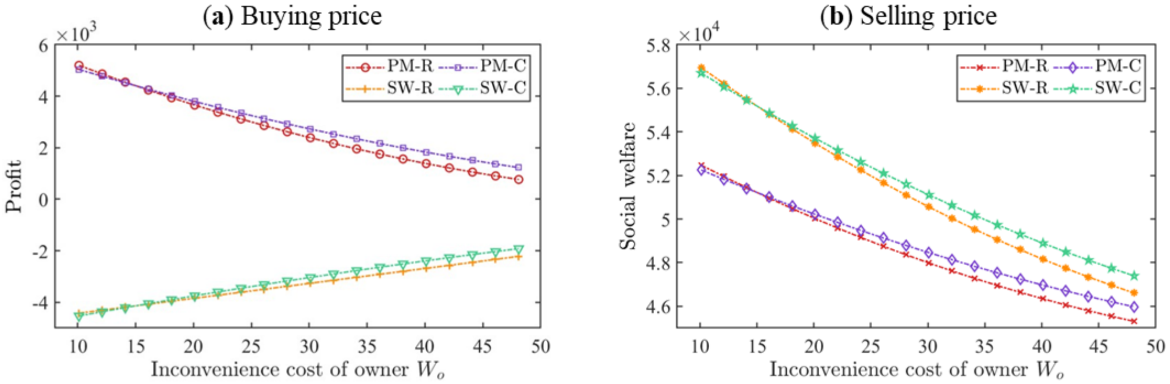
6.3 Varying the inconvenience cost

Figure 3 shows the effects of varying the inconvenience cost for owners. As W_o rises, the operator increases the buying price p_o in all regimes (Figure 3(a)) to limit the loss of supply. Consistent with Table 5, the selling price p_d (Figure 3(b)) follows the same trend in the commissioning format, but in the PM-R regime the operator reduces p_d. The inconvenience cost does not affect p_d in the SW-R regime.

Figure 3(c) and Figure 3(d) depict how platform profit and social welfare vary with the inconvenience cost. Consistent with Propositions 8 and 9, if the inconvenience cost is smaller than the disappointment cost (i.e., $W_o < R_o = 15$), profit is higher in the PM-R regime than the PM-C regime, and social welfare is higher in the SW-R regime than the SW-C regime. The converse is true if $W_o > R_o$. Profits and social welfare decrease with inconvenience cost in the profit-maximization regime, as does social welfare in the social-welfare-maximizing regime. However, profits in the social-welfare-maximizing regime increase with W_o. This is because the operator prices slots below cost to exploit economies of scale, and the losses incurred diminish as the volume of parking sales declines.



(a) Buying price **(b)** Selling price

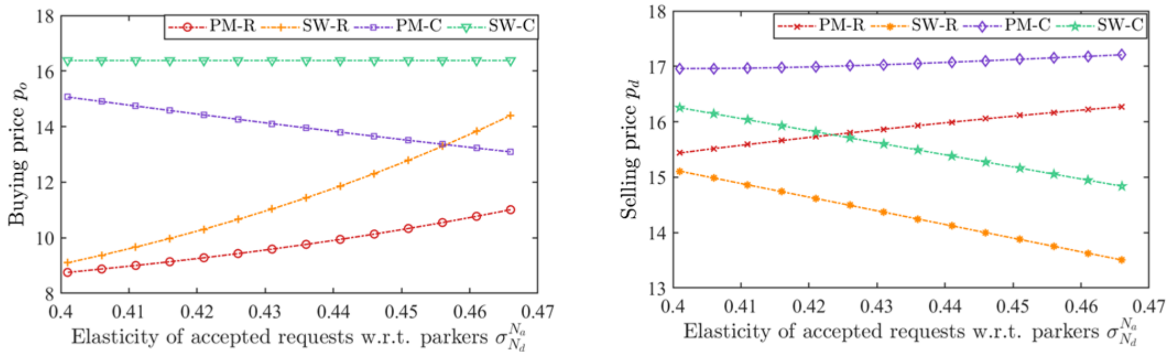


(c) Profit **(d)** Social welfare

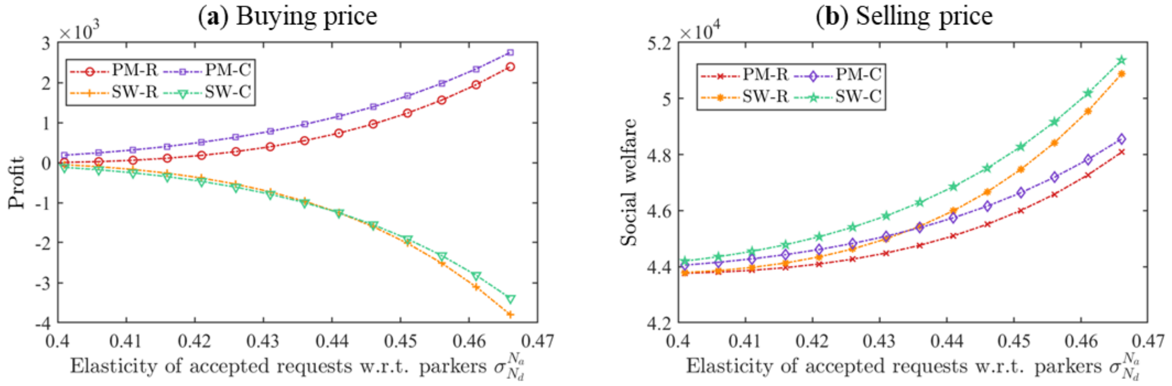
Figure 3: Optimal pricing strategy, profit and social welfare as functions of inconvenience cost of owner W_o

6.4 Varying the matching function elasticity

In this subsection, we vary the elasticity $\sigma_{N_d}^{N_a}$ of the number of accepted requests with respect to the number of parkers, using the Cobb-Douglas type matching function in the base case. The results are presented in Figure 4. As $\sigma_{N_d}^{N_a}$ rises, the reselling operator raises the buying fee to attract more supply. Consequently, a slot can be more easily matched to a request. In contrast, the profit-driven commissioning operator (PM-C) reduces the buying price and slightly increase the selling price. This is because the platform charges only for successful matches, and even with fewer owners and parkers the platform can achieve matches more easily. At the same time, when matching is more efficient, the operator earns more profit in the profit-maximization regimes and generates more surplus in the social welfare-maximization regimes.



(a) Buying price **(b)** Selling price



(c) Profit **(d)** Social welfare

Figure 4: Optimal pricing strategy, profit and social welfare as functions of matching elasticity $\sigma_{N_d}^{N_a}$

In summary, we find that in the base case, where the owners' cost of disappointment is less than the inconvenience cost, the commissioning format is more profitable than the reselling format for profit-maximization, and yields a higher welfare for social welfare-maximization. The reselling format is preferred if the owner's disappointment cost is greater than inconvenience cost.

7 Conclusion

This study makes the first attempt to investigate the two-sided pricing problem faced by a parking sharing platform as either a reseller or a commissioner under matching frictions between parking suppliers and demanders. As a reseller, the platform operator purchases the right to use parking slots and rents them to parking customers. The reselling platform takes full responsibility for renting the slots and pays the owners a fixed amount regardless of whether the slots are rented. By contrast, in the commissioning format the operator pays the owners only if the unit is rented. If it is not rented, the owner can continue to use it.

We model the behaviors of parking owners and renters for the two business formats, and derive the supply-demand matching equilibrium. Rather than adopting the operational-level approach of Shao et al. (2016) and Xu et al. (2016), whereby slots are allocated between individual owners and renters, we use a matching function that determines the total number of slots rented as a function of the total numbers offered and demanded. We examine analytically how system parameter values affect the supply-demand matching equilibrium in the parking sharing market.

Given the behavior of owners and parkers, the two-sided pricing strategies of the parking sharing platform are investigated for the reselling and commissioning formats, and for two administrative objectives: profit maximization and social welfare maximization. For each combination of format and objective function, the optimal buying and selling prices and resulting platform profit are derived analytically and compared. In the case of social welfare maximization, the platform earns positive profits if the matching function has decreasing returns to scale, and incurs a deficit if the matching function has increasing returns to scale. If owners happen to experience the same disutility from inconvenience and disappointment, the reselling and commissioning formats yield identical selling prices, platform profit, and social welfare. If the disutility from inconvenience is greater, the commissioning format yields higher profit and welfare. If disutility from disappointment is greater, the reselling format is superior on both counts. The latter case becomes less likely as time goes by, and owners acclimate to the possibility that their parking slots do not always get rented out.

This study takes a static equilibrium approach aimed at deriving analytical insights. By adopting an aggregate matching function, the model accommodates some degree of spatial and temporal heterogeneity in parking slot availability. To maintain analytical tractability, we assume the parkers' parking duration, owners' inconvenience cost, and disappointment cost are homogeneous, while permitting heterogeneity in parkers' reservation utilities. The aggregate, static model considers a long-run equilibrium. It does not explicitly address such operational features as the one-to-one matching process, the adjustment and evolution of users' behavior over time, and the real-time interaction or bidding process involving parkers, owners, and the platform operator. Future research could look into many operational level aspects of parking sharing platforms by incorporating system dynamics and more spatio-temporal heterogeneity.

Other extensions are also possible. Yield management strategies involving basic rewards and sales commissions can be considered that effectively combine the reselling and commissioning formats. Competition between shared parking platforms (Rochet and Tirole, 2003; Armstrong, 2006; Economides and Tåg, 2012) can be entertained. Coexistence of shared parking (involving private slots) and conventional public parking is another institutional setting of interest. Introducing a spatial dimension to the model, parking search, cruising time (Liu and Geroliminis, 2016), walking distance, and parking information systems are yet further extensions worth exploring. Last but not least, the possibility that parkers will need to cancel their parking reservations could be added to the model. A pricing or penalty mechanism for reservation cancellation could be studied, similar to order cancellations in ride-sourcing systems (Wang et al., 2020).

Author Statement.

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Appendix A: Comparative statics of the reselling format

Table 3 lists the comparative statics effects of parameters on the equilibrium values of N_d, N_o, and N_a. This appendix establishes the formulas and signs of the derivatives with respect to p_o and p_d in the short run when the prices are treated as parameters rather than endogenous variables. To economize on space, formulas and signs for the other derivatives are listed but not derived.

Differentiating Eq. (50) with respect to p_o, we have

$$\left(B_d + R_d - hp_d\right)\frac{\partial N_a}{\partial p_o} = \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right)\frac{\partial N_d}{\partial p_o}, \tag{A.1a}$$

$$g = \hat{U}_o' \frac{\partial N_o}{\partial p_o}, \tag{A.1b}$$

$$\frac{\partial N_a}{\partial p_o} = G_d \frac{\partial N_d}{\partial p_o} + G_o \frac{\partial N_o}{\partial p_o}, \tag{A.1c}$$

where $\hat{U}_d' = \frac{d\hat{U}_d}{dN_d}$, $\hat{U}_o' = \frac{d\hat{U}_o}{dN_o}$, $G_d = \frac{\partial G}{\partial N_d}$, and $G_o = \frac{\partial G}{\partial N_o}$.

Solving Eq. (A.1) as a system of linear equations for $\frac{\partial N_o}{\partial p_o}$, $\frac{\partial N_d}{\partial p_o}$, and $\frac{\partial N_a}{\partial p_o}$, we obtain

$$\frac{\partial N_o}{\partial p_o} = \frac{g}{\hat{U}_o'}, \quad \frac{\partial N_d}{\partial p_o} = \frac{g\left(B_d + R_d - hp_d\right)G_o}{\hat{U}_o'\Delta}, \quad \frac{\partial N_a}{\partial p_o} = \frac{g\left(\hat{U}_d + \hat{U}_d' N_d + R_d\right)G_o}{\hat{U}_o'\Delta}, \tag{A.2}$$

where

$$\Delta \equiv \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right) - \left(B_d + R_d - hp_d\right)G_d. \tag{A.3}$$

Given Eq. (50a), Δ can be written:

$$\Delta = \hat{U}_d' N_d + \left(1 - \sigma_{N_d}^{N_a}\right)\left(\hat{U}_d + R_d\right) > 0,$$

where the inequality follows from the assumption $\sigma_{N_d}^{N_a} < 1$. It then follows that

$$\frac{\partial N_o}{\partial p_o} > 0, \quad \frac{\partial N_d}{\partial p_o} > 0, \quad \frac{\partial N_a}{\partial p_o} > 0. \tag{A.4}$$

Differentiating Eq. (50) with respect to p_d, we have

$$-hN_a + \left(B_d + R_d - hp_d\right)\frac{\partial N_a}{\partial p_d} = \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right)\frac{\partial N_d}{\partial p_d}, \tag{A.5a}$$

$$0 = \hat{U}_o' \frac{\partial N_o}{\partial p_d}, \tag{A.5b}$$

$$\frac{\partial N_a}{\partial p_d} = G_d \frac{\partial N_d}{\partial p_d} + G_o \frac{\partial N_o}{\partial p_d}. \tag{A.5c}$$

Solving Eq. (A.5) as a system of linear equations yields

$$\frac{\partial N_o}{\partial p_d} = 0, \quad \frac{\partial N_d}{\partial p_d} = -\frac{hN_a}{\Delta} < 0, \quad \frac{\partial N_a}{\partial p_d} = -\frac{hN_a G_d}{\Delta} < 0. \tag{A.6}$$

Derivatives for the other parameters are derived in a similar way.

For W_o:

$$\frac{\partial N_o}{\partial W_o} = -\frac{1}{\hat{U}_o'} < 0, \quad \frac{\partial N_d}{\partial W_o} = -\frac{\left(B_d + R_d - hp_d\right)G_o}{\hat{U}_o'\Delta} < 0, \quad \frac{\partial N_a}{\partial W_o} = -\frac{\left(\hat{U}_d + \hat{U}_d' N_d + R_d\right)G_o}{\hat{U}_o'\Delta} < 0. \tag{A.7}$$

For B_d:

$$\frac{\partial N_o}{\partial B_d} = 0, \quad \frac{\partial N_d}{\partial B_d} = \frac{N_a}{\Delta} > 0, \quad \frac{\partial N_a}{\partial B_d} = \frac{N_a G_d}{\Delta} > 0. \tag{A.8}$$

For g:

$$\frac{\partial N_o}{\partial g}=\frac{p_o}{\hat{U}_o'}>0,\ \ \frac{\partial N_d}{\partial g}=\frac{p_o\left(B_d+R_d-hp_d\right)G_o}{\hat{U}_o'\Delta}>0,\ \ \frac{\partial N_a}{\partial g}=\frac{p_o\left(\hat{U}_d+\hat{U}_d'N_d+R_d\right)G_o}{\hat{U}_o'\Delta}>0. \tag{A.9}$$

For h:

$$\frac{\partial N_o}{\partial h}=0,\ \ \frac{\partial N_d}{\partial h}=-\frac{p_dN_a}{\Delta}<0,\ \ \frac{\partial N_a}{\partial h}=-\frac{p_dN_aG_d}{\Delta}<0. \tag{A.10}$$

For k:

$$\frac{\partial N_o}{\partial k}=0,\ \ \frac{\partial N_d}{\partial k}=0,\ \ \frac{\partial N_a}{\partial k}=0. \tag{A.11}$$

For R_o:

$$\frac{\partial N_o}{\partial R_o}=0,\ \ \frac{\partial N_d}{\partial R_o}=0,\ \ \frac{\partial N_a}{\partial R_o}=0. \tag{A.12}$$

For R_d:

$$\frac{\partial N_o}{\partial R_d}=0,\ \ \frac{\partial N_d}{\partial R_d}=\frac{N_a-N_d}{\Delta}<0,\ \ \frac{\partial N_a}{\partial R_d}=\frac{\left(N_a-N_d\right)G_d}{\Delta}<0. \tag{A.13}$$

Appendix B: Comparative statics of the commissioning format

As in Appendix A, we derive the comparative statics effects for changes in p_o and p_d, and only list results for the other parameters.

Differentiating Eq. (51) with respect to p_o, we have

$$\left(B_d+R_d-hp_d\right)\frac{\partial N_a}{\partial p_o}=\left(\hat{U}_d+\hat{U}_d'N_d+R_d\right)\frac{\partial N_d}{\partial p_o}, \tag{A.14a}$$

$$hN_ag+h\left(p_og-W_o+R_o\right)\frac{\partial N_a}{\partial p_o}=g\left(\hat{U}_o+\hat{U}_o'N_o+R_o\right)\frac{\partial N_o}{\partial p_o}, \tag{A.14b}$$

$$\frac{\partial N_a}{\partial p_o}=G_d\frac{\partial N_d}{\partial p_o}+G_o\frac{\partial N_o}{\partial p_o}. \tag{A.14c}$$

Solving Eq. (A.14) as a system of linear equations yields

$$\frac{\partial N_o}{\partial p_o}=\frac{hN_ag\Delta}{\Gamma},\ \frac{\partial N_d}{\partial p_o}=\frac{hN_ag\left(B_d+R_d-hp_d\right)G_o}{\Gamma},\ \frac{\partial N_a}{\partial p_o}=\frac{hN_ag\left(\hat{U}_d+\hat{U}_d'N_d+R_d\right)G_o}{\Gamma}, \tag{A.15}$$

where

$$\Gamma=g\left(\hat{U}_o+\hat{U}_o'N_o+R_o\right)\Delta-h\left(p_og-W_o+R_o\right)\left(\hat{U}_d+\hat{U}_d'N_d+R_d\right)G_o. \tag{A.16}$$

Given Eq. (51), Γ can be written:

$$\Gamma = g\hat{U}_o' N_o \Delta + g\left(\hat{U}_o + R_o\right)\left[\left(1-\sigma_{N_o}^{N_a}-\sigma_{N_d}^{N_a}\right)\left(\hat{U}_d + R_d\right) + \left(1-\sigma_{N_o}^{N_a}\right)\hat{U}_d' N_d\right],$$

where $\Delta = \hat{U}_d' N_d + \left(1-\sigma_{N_d}^{N_a}\right)\left(\hat{U}_d + R_d\right) > 0$ as in Appendix A. The expression Γ is positive unless the term $\left(1-\sigma_{N_o}^{N_a}-\sigma_{N_d}^{N_a}\right)$ is sufficiently small (e.g., approaches the minimum of -1), $\hat{U}_o' N_o$ is sufficiently smaller than $\hat{U}_o + R_o$, and $\hat{U}_d' N_d$ is sufficiently smaller than $\hat{U}_d + R_d$. In what follows we assume $\Gamma > 0$. It then follows from Eq. (A.15) that

$$\frac{\partial N_o}{\partial p_o} > 0, \frac{\partial N_d}{\partial p_o} > 0, \frac{\partial N_a}{\partial p_o} > 0. \tag{A.17}$$

Differentiating Eq. (51) with respect to p_d, we have

$$-hN_a + \left(B_d + R_d - hp_d\right)\frac{\partial N_a}{\partial p_d} = \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right)\frac{\partial N_d}{\partial p_d}, \tag{A.18a}$$

$$h\left(p_o g - W_o + R_o\right)\frac{\partial N_a}{\partial p_d} = g\left(\hat{U}_o + \hat{U}_o' N_o + R_o\right)\frac{\partial N_o}{\partial p_d}, \tag{A.18b}$$

$$\frac{\partial N_a}{\partial p_d} = G_d\frac{\partial N_d}{\partial p_d} + G_o\frac{\partial N_o}{\partial p_d}. \tag{A.18c}$$

Solving Eq. (A.18) as a system of linear equations yields

$$\begin{aligned}
\frac{\partial N_o}{\partial p_d} &= -\frac{h^2 N_a\left(p_o g - W_o + R_o\right)G_d}{\Gamma} < 0,\\
\frac{\partial N_d}{\partial p_d} &= \frac{hN_a\left(h\left(p_o g - W_o + R_o\right)G_o - g\left(\hat{U}_o + \hat{U}_o' N_o + R_o\right)\right)}{\Gamma} < 0,\\
\frac{\partial N_a}{\partial p_d} &= -\frac{hN_a g\left(\hat{U}_o + \hat{U}_o' N_o + R_o\right)G_d}{\Gamma} < 0.
\end{aligned} \tag{A.19}$$

Results for the other parameters are as follows.

For W_o:

$$\begin{aligned}
\frac{\partial N_o}{\partial W_o} &= -\frac{hN_a\Delta}{\Gamma} < 0, \quad \frac{\partial N_d}{\partial W_o} = -\frac{hN_a\left(B_d + R_d - hp_d\right)G_o}{\Gamma} < 0,\\
\frac{\partial N_a}{\partial W_o} &= -\frac{hN_a\left(\hat{U}_d + \hat{U}_d' N_d + R_d\right)G_o}{\Gamma} < 0.
\end{aligned} \tag{A.20}$$

For B_d:

$$\begin{aligned} \frac{\partial N_o}{\partial B_d} &= \frac{N_a h \left(p_o g - W_o + R_o\right) G_d}{\Gamma} > 0, \\ \frac{\partial N_d}{\partial B_d} &= -\frac{N_a \left(h \left(p_o g - W_o + R_o\right) G_o - g \left(\hat{U}_o + \hat{U}_o' N_o + R_o\right)\right)}{\Gamma} > 0, \\ \frac{\partial N_a}{\partial B_d} &= \frac{N_a g \left(\hat{U}_o + \hat{U}_o' N_o + R_o\right) G_d}{\Gamma} > 0. \end{aligned} \tag{A.21}$$

For g:

$$\begin{aligned} \frac{\partial N_o}{\partial g} &= \frac{\left(h N_a \left(W_o - R_o\right)\right) \Delta}{\Gamma}, \\ \frac{\partial N_d}{\partial g} &= \frac{\left(h N_a \left(W_o - R_o\right)\right) \left(B_d + R_d - h p_d\right) G_o}{\Gamma}, \\ \frac{\partial N_a}{\partial g} &= \frac{\left(h N_a \left(W_o - R_o\right)\right) \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right) G_o}{\Gamma}, \end{aligned} \tag{A.22}$$

where $\frac{\partial N_o}{\partial g}, \frac{\partial N_d}{\partial g}, \frac{\partial N_a}{\partial g} > 0 \text{ if } W_o > R_o$, $\frac{\partial N_o}{\partial g}, \frac{\partial N_d}{\partial g}, \frac{\partial N_a}{\partial g} = 0 \text{ if } W_o = R_o$, and $\frac{\partial N_o}{\partial g}, \frac{\partial N_d}{\partial g}, \frac{\partial N_a}{\partial g} < 0 \text{ if } W_o < R_o$.

For h:

$$\begin{aligned} \frac{\partial N_o}{\partial h} &= \frac{N_a \left(p_o g - W_o + R_o\right) \left(\Delta - p_d h G_d\right)}{\Gamma}, \\ \frac{\partial N_d}{\partial h} &= \frac{N_a \left(B_d + R_d\right) \left(p_o g - W_o + R_o\right) G_o - p_d N_a g \left(\hat{U}_o + \hat{U}_o' N_o + R_o\right)}{\Gamma}, \\ \frac{\partial N_a}{\partial h} &= \frac{-p_d N_a g \left(\hat{U}_o + \hat{U}_o' N_o + R_o\right) G_d + N_a \left(p_o g - W_o + R_o\right) \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right) G_o}{\Gamma}, \end{aligned} \tag{A.23}$$

where the signs of $\frac{\partial N_o}{\partial h}$, $\frac{\partial N_d}{\partial h}$, $\frac{\partial N_a}{\partial h}$ are, a priori, indeterminate.

For k:

$$\frac{\partial N_o}{\partial k} = 0, \frac{\partial N_d}{\partial k} = 0, \frac{\partial N_a}{\partial k} = 0. \tag{A.24}$$

For R_o:

$$\begin{aligned} \frac{\partial N_o}{\partial R_o} = \frac{\left(h N_a - g N_o\right) \Delta}{\Gamma} < 0, \frac{\partial N_d}{\partial R_o} &= \frac{\left(h N_a - g N_o\right) \left(B_d + R_d - h p_d\right) G_o}{\Gamma} < 0, \\ \frac{\partial N_a}{\partial R_o} &= \frac{\left(h N_a - g N_o\right) \left(\hat{U}_d + \hat{U}_d' N_d + R_d\right) G_o}{\Gamma} < 0. \end{aligned} \tag{A.25}$$

For R_d:

$$\frac{\partial N_o}{\partial R_d} = \frac{(N_a - N_d)\, h\, (p_o g - W_o + R_o)\, G_d}{\Gamma} < 0,$$

$$\frac{\partial N_d}{\partial R_d} = -\frac{(N_a - N_d)\left(h\,(p_o g - W_o + R_o)\, G_o - g\left(\hat{U}_o + \hat{U}_o' N_o + R_o\right)\right)}{\Gamma} < 0, \quad \text{(A.26)}$$

$$\frac{\partial N_a}{\partial R_d} = \frac{(N_a - N_d)\, g\left(\hat{U}_o + \hat{U}_o' N_o + R_o\right) G_d}{\Gamma} < 0.$$

Appendix C: Proof of Proposition 7

Assume $W_o = R_o$, and call the common value C. Owners receive expected utility $U_o^r = p_o^r g - C$ in the reselling format, and $U_o^c = p_o^c g \theta - C$ in the commissioning format. Owners are risk neutral in the sense that they care only about the expected price they receive for offering a slot for rent.

From Eq. (1), the number of parkers depends on p_d and φ where $\varphi = N_a/N_d$. Combined with the matching function, the number of parkers who join the platform can be written as a function of p_d and N_o: $N_d = \hat{N}_d\,(p_d, N_o)$.

Similarly, given Eq. (2), Eq. (4), and the matching function the numbers of owners who join the platform in the reselling and commissioning format are $N_o^r\,(p_o)$ and $\hat{N}_o^c\,(p_o, N_d)$, respectively.

Proof of (a): Any commissioning policy can be replicated by a reselling policy

Let (p_o^c, p_d^c) be a commissioning policy and θ^c be the resulting average utilization rate of slots. This commissioning policy is replicated in the reselling format by setting $p_d^r = p_d^c$ and $p_o^r = \theta^c p_o^c$. Owners receive the same expected price in the two formats, and thus offer the same supply. Hence $N_o^r = N_o^c$. Since $p_d^r = p_d^c$, $\hat{N}_d\,(p_d^r, N_o^r) = \hat{N}_d\,(p_d^c, N_o^c)$ and $N_d^r = N_d^c$. Thus, in both formats the platform operator pays owners the same expected fee and charges parkers the same price. It also attracts the same numbers of owners and parkers. Profit and social welfare are thus the same as well.

Proof of (b): Any reselling policy can be replicated by a commissioning policy

Let (p_o^r, p_d^r) be a reselling policy and θ^r be the resulting average utilization rate of slots. To replicate this with the commissioning format, set $p_d^c = p_d^r$. Then vary p_o^c until $\theta^c = \theta^r$ so that $hN_a^c/\,(gN_o^c) = hN_a^r/\,(gN_o^r)$. Using the matching function, this can be written as $G\,(N_d^c, N_o^c)/N_o^c = G\,(N_d^r, N_o^r)/N_o^r$. The two functions are identical in form, and given $\sigma_{N_o}^{N_a} < 1$ they are monotonically decreasing functions of N_o. Hence, if $N_d^c = N_d^r$, then $N_o^c = N_o^r$.

Now $N_d^c = \hat{N}_d\,(p_d^c, N_o^c)$ and $N_d^r = \hat{N}_d\,(p_d^r, N_o^r)$. Given $p_d^c = p_d^r$, the first arguments in each function are the same. If $N_o^c = N_o^r$, the second arguments are also the same and $N_d^c = N_d^r$. Hence, if $N_o^c = N_o^r$ then $N_d^c = N_d^r$. It follows that the numbers of owners and parkers are the same for the two formats. The platform operator pays owners the same expected fee, and charges parkers the same price. The operator's profit is thus the same for the two formats, and so is social welfare.

Appendix D: Proof of Proposition 8

The proof entails showing that, for any reselling policy, a commissioning policy can be chosen that outperforms it in profit and social welfare.

With the reselling format, owners receive a utility $U_o^r = p_o^r g - C_1$ where $C_1 = W_o$. With the commissioning format, their utility is $U_o^c = p_o^c g\theta - C_2$ where $C_2 = \theta W_o + (1-\theta) R_o$. Given $W_o > R_o$, $C_2 < C_1$: owners incur a lower expected cost with the commissioning format.

Let (p_o^r, p_d^r) be a reselling policy and θ^r be the resulting average utilization rate of slots. To prove the commissioning format outperforms the reselling format, we set $p_d^c = p_d^r$. Then we vary p_o^c until $\theta^c = \theta^r$ so that $hN_a^c / (gN_o^c) = hN_a^r / (gN_o^r)$. Owners earn the same expected income from the platform with the commissioning format, and incur a lower expected cost. Hence, $N_o^c > N_o^r$. The number of participating parkers is $N_d^r = N_d(p_d^r, N_o^r)$ in the reselling format, and $N_d^c = N_d(p_d^c, N_o^c)$ in the commissioning format. With $p_d^c = p_d^r$ and $N_o^c > N_o^r$, $N_d^c > N_d^r$. Consequently, $N_a^c > N_a^r$: the number of transactions is larger in the commissioning format. A profit-maximizing operator therefore earns higher profit, and a welfare-maximizing operator generates a higher social surplus. By adjusting p_d^c and p_o^c, the operator can do better yet.

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