

Harnessing impact-induced cracking via stiffness heterogeneity

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Abstract

Mechanical heterogeneity refers to the spatial inhomogeneity of the mechanical properties in materials, which is a common feature of composites consisting of multiple distinct phases. Generally, the effects of mechanical heterogeneity on the overall properties of the composites, such as stiffness and strength, are thought to follow the rule of mixture. Here, we investigate the cracking behavior of composite plates under impact and found that the rule of mixture may break down in describing the cracking resistance of composites with high stiffness heterogeneity. Our results show that the resistance of a composite plate, which consists of two phases of distinct stiffnesses, against dynamic cracking strongly depends on the hybridizing manner of the two phases. When the stiff phase is dispersed in the compliant matrix, the resulting composite plate exhibits superior cracking resistance compared to the monolithic plates made of either phase. In contrast, if the compliant phase is dispersed in the stiff matrix, the resulting composite plate displays reduced cracking resistance and thus higher absorption of the impact energy as compared to the monolithic controls. Our work provides an approach to harnessing the dynamic fracture by controlling the stiffness heterogeneity, which would be of great value to the design and fabrication of the protective armors and energy-absorbing shields.

Keywords: Mechanical heterogeneity, Ballistic cracking, Crack-inclusion interactions, Structure-property relationship, Protective armors

33 **1. Introduction**

34 The mechanical properties and behavior of solid materials are determined not only
35 by their chemical compositions but also by the way of how these compositions are
36 bonded together in space, namely the so-called structure. Revealing the structure-
37 property relations and then applying them to direct the design and manufacturing of
38 new materials is an everlasting topic in materials science and engineering. As a typical
39 structural attribute, mechanical heterogeneity refers to the spatial variation and
40 inhomogeneity of the mechanical properties (*e.g.*, elasticity and plasticity) in materials.
41 It is commonly observed at multiple length scales in materials consisting of constituent
42 phases with distinct properties such as engineering composites [1-4] and natural
43 biological materials [5-9]. Mechanical heterogeneity has been shown to play an
44 important role in determining the overall mechanical properties of the composite
45 materials. Normally, materials with high heterogeneity are believed to have inferior
46 mechanical properties in comparison to the homogeneous counterparts made of similar
47 chemical compositions. This is because the mechanical mismatch between the distinct
48 building phases, upon external loading, tends to cause stress concentration and strain
49 localization near the phase interfaces. Nevertheless, sometimes heterogeneity was also
50 found to benefit the mechanical properties of the materials. For example, the
51 heterogeneous-structured metallic materials achieved superior mechanical properties
52 that are not accessible to conventional homogeneous counterparts [10]; a “brick-and-
53 mortar” structure with high heterogeneity could effectively suppress the crack-induced
54 stress intensification and fortify the flaw tolerance of nacreous composites [11]; a 10-
55 15% micro- and nanomechanical heterogeneity was proved an optimal scheme to
56 promote the ductile behavior of bones in nano- and microscale [12]. Particularly, the
57 heterogeneity of plasticity at nano length scale in bones was demonstrated to promote
58 energy dissipation during plastic deformation [13]. Moreover, heterogeneity in yield
59 strength was found to benefit the strength-ductility synergy in metals [14].

60 In addition to plasticity, heterogeneity in stiffness, which is characterized by the
61 elastic modulus, was found to influence the fracture behavior of a material. Related

62 studies can be traced back to the investigations of the effect of an interface between two
63 dissimilar elastic materials on the propagation tendency of a crack. It is found that the
64 interface between dissimilar materials may enlarge or suppress the SIF of a crack in the
65 front of it, depending on the stiffness heterogeneity across the interface. When the crack
66 is situated on the compliant side of the interface and heads to the stiff side, the interface
67 would suppress the SIF at the crack tip; in contrast, if the crack is situated on the stiff
68 side of the interface and heads to the compliant side, the interface would enlarge the
69 SIF at the crack tip [15-17]. This conclusion still holds when a crack embedded in an
70 infinite elastic medium approaches to a circular inclusion with distinct elastic modulus,
71 which is a basic physical picture of fracture in composites with heterogeneous stiffness.
72 Tamate [18] and Atkinson [19] analytically solved the SIF at the crack tips under the
73 influence of inclusion with different moduli and distances away from the crack. For
74 more complicated cases, such as a crack under mixed-mode loading [20], the influence
75 of interfacial strength between the inclusion and matrix [21], finite element method
76 (FEM) was adopted to evaluate the SIF of a stationary crack in the presence of an
77 inclusion. With the aid of the extended finite element method (XFEM), Jiang et al. [22,
78 23] found a similar effect of inclusions on the SIF of a dynamic crack tip. Tran and
79 Truong [24] improved the smoothness of the stress and strain fields for the crack growth
80 problem in composite material by incorporating XFEM with a twice interpolation
81 method. The effect of inclusion on the SIF of a dynamic crack has been experimentally
82 verified by the photoelasticity technique [25]. As a special inclusion with zero stiffness,
83 a hole was also found to attract a crack propagating nearby [26], which is qualitatively
84 in accord with previous studies. Further extensions were made to the studies on the
85 effects of the heterogeneous interface on crack deflection and kinking [27, 28] and out-
86 of-plane excursions of cracks [29]. Utilizing stiffness heterogeneity, researchers have
87 effectively enhanced the fracture toughness of composites [30-32].

88 Although the loading and the geometries of the models in the above works were
89 simplified, the revealed phenomena implied that stiffness heterogeneity might have a
90 similar effect on impact-induced cracking and therefore can be applied to harness

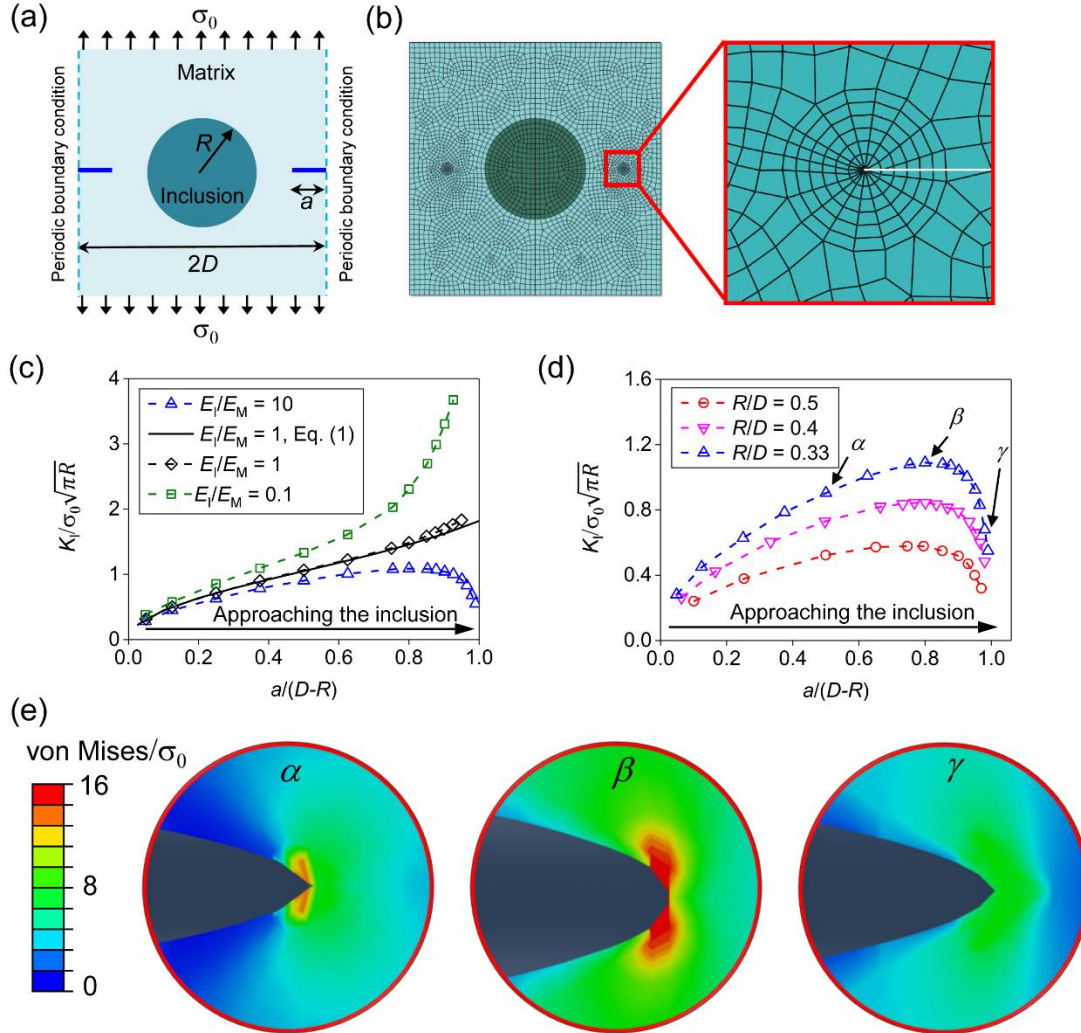
91 impact fracture. To verify this hypothesis, in this work we firstly carry out a systematic
92 computational study on the effects of stiffness heterogeneity on the cracking caused by
93 impact. Our study starts from the effect of circular inclusions on the growing tendency
94 of a pre-existing crack under a static load. Then, the discussion is extended to the
95 dynamic cracking in inclusion-matrix composites caused by ballistic impact. Two types
96 of composites with complementary hybridizing schemes are studied: stiff inclusions
97 embedded in a compliant matrix (S@C) and compliant inclusions embedded in a stiff
98 matrix (C@S). Parametric studies on the effects of inclusion size, stiffness, volume
99 fraction, inter-inclusion spacing, and distribution pattern are carried out, followed by
100 the experimental verification of the effect of stiffness heterogeneity on cracking
101 resistance. Finally, the paper is concluded after discussing the synergetic effect of the
102 S@C and C@S composites under ballistic impact. The results obtained in the present
103 study are believed to serve as a general guide for the development of anti-impact
104 materials and protective shields.

105

106 **2. Interference of stiffness heterogeneity with SIF of a static crack**

107 To quantify the effect of stiffness heterogeneity on the propagation tendency of a
108 crack, an idealized finite element (FE) model is constructed (ABAQUS/Standard,
109 Dassault Systèmes), in which a square plate (edge length: $2D$) with pre-existing cracks
110 contains a circular inclusion of radius R at the center (see Fig. 1(a)). Periodic boundary
111 conditions are applied on both lateral sides of the model. That is, the model depicts a
112 representative volume element (RVE) of a periodic structure with a period of $2D$. The
113 thickness and the period of the model are taken as 5 mm and 100 mm, respectively. The
114 inclusion and the matrix are assumed perfectly bonded. Young's modulus and Poisson's
115 ratio of the matrix are prescribed as $E_M = 10$ GPa and $\nu_M = 0.3$, respectively. Eight-
116 node linear brick elements with reduced integration (C3D8R in ABAQUS) are
117 employed except in the region around the crack tip, where six-node linear triangular
118 prism elements are applied to improve the accuracy in describing the singular stress
119 field with square-root singularity. A uniform tensile stress σ_0 is applied along the
120 direction perpendicular to the surface of the pre-existing cracks. The SIF (K_I) is derived

121 from $K_I = \sqrt{J E_M}$, where J is the calculated J-integral around the crack tip [33]. Due
 122 to the symmetry of the model, we do not distinguish the two cracks in the model when
 123 discussing the SIF in the following.



124

125 Figure 1. (a) The RVE model applied in finite element analysis; (b) Meshing of the
 126 model and enlarged view at the crack tip. Computed variation of the stress intensity
 127 factor (normalized) at the crack tip with the crack length (normalized) for inclusion-
 128 matrix composites with inclusions of (c) different stiffnesses, and (d) different sizes.
 129 The inclusion radius in (c) is taken as $R = 0.33D$ and the inclusion modulus in (d) is
 130 taken as $E_I = 10 E_M$; (e) Snapshots of the von Mises stress field (normalized by the load
 131 σ_0) at three selected moments as indicated in (d). α : $a / (D - R) = 0.48$; β : $a / (D -$
 132 $R) = 0.75$; γ : $a / (D - R) = 0.98$.

133

134 Fig. 1(c) shows the calculated SIF as a function of crack length for three cases

135 with inclusions of different stiffnesses: $E_I = 10E_M$, $E_I = E_M$ and $E_I = 0.1E_M$. For
136 the homogeneous case with $E_I = E_M$, the analytical solution to the SIF exists and is
137 given by [34]

$$138 \quad K_I = \sqrt{\frac{2D}{\pi a} \tan \frac{\pi a}{2D} \sigma_0 \sqrt{\pi a}} \quad (1)$$

139 The consistency between such an analytical solution and our calculated results shown
140 in Fig. 1(c) validates the calculated SIF in other cases. Comparison between the three
141 cases shown in Fig. 1(c) confirms that stiffness heterogeneity affects the SIF, which
142 determines the growing tendency of a crack. Particularly, stiff inclusion ($E_I > E_M$)
143 suppresses the SIF and therefore resists the crack propagation, while compliant
144 inclusion ($E_I < E_M$) enhances the SIF and therefore facilitates the crack propagation.

145 We further studied the size effect of the inclusions. Fig. 1(d) shows the variation of
146 the calculated SIF with the crack length for cases with inclusions of different sizes and
147 given modulus $E_I = 10E_M$. In these three cases, the SIF exhibits a similar variation
148 trend with the increasing crack size. That is, it increases initially and then decreases
149 with the increase of the crack size. Such variation of SIF can be further visualized from
150 the stress (von Mises) field near the crack tip at three representative moments, as shown
151 in Fig. 1(e). Comparison between these three studied cases indicates that larger
152 inclusion imposes higher suppression on the SIF.

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154 **3. Effect of stiffness heterogeneity on ballistic cracking**

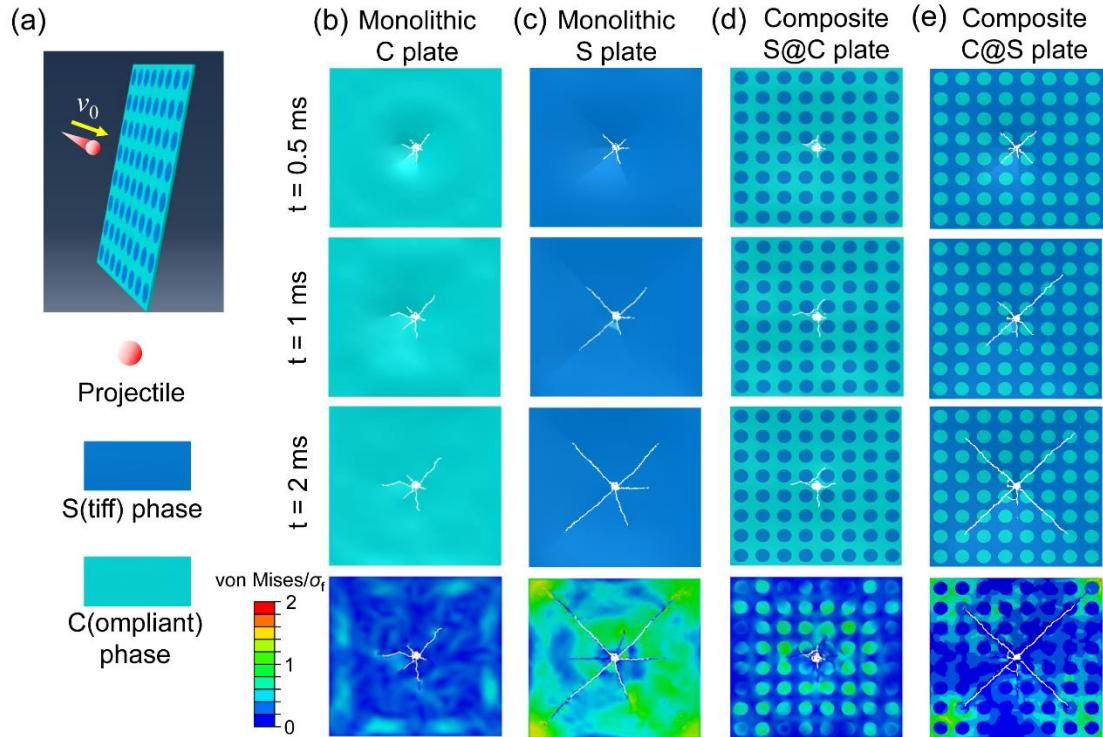
155 **3.1 Simulation of ballistic impact**

156 To extend our study from static cracks to dynamic fracture, we simulated the
157 ballistic impact process of a spherical projectile (radius: 2 mm) on composite plates (96
158 mm \times 96 mm \times 1 mm) composed of a stiff (S) phase and a compliant (C) phase, as
159 shown in Fig. 2(a). The modulus of the stiff phase is taken as 10 times that of the
160 compliant phase. To mask off the possible influence of the factors other than stiffness
161 on the cracking behavior, the other properties of these two phases, such as density,
162 Poisson's ratio, fracture strength, and fracture energy are assumed the same. **Both**

163 compliant phase and stiff phase are assumed as brittle materials with fracture energy
164 $\Gamma = 10 \text{ J} \cdot \text{m}^{-2}$, which is close to the fracture energy of glass (Fig. S1). As we are
165 concerned about the cracking behavior in the composite plates, the material of the
166 projectile is simply assumed as steel with linear elasticity. The detailed material
167 properties of each phase and the projectile are summarized in Table 1. Two
168 complementary hybridizing schemes for the composite plates are considered. One is to
169 embed stiff inclusions into the compliant matrix (denoted as S@C), and the other is to
170 embed compliant inclusions into the stiff matrix (denoted as C@S). For comparison,
171 monolithic plates of the same dimensions composed of compliant phase or stiff phase
172 only are applied as the control cases. Both plates and projectiles are modelled with four-
173 node shell elements (S4R in ABAQUS) with a thickness of 1 mm. Simply-supported
174 boundary conditions are applied on four vertexes of the plate. The initial velocity of the
175 projectile is taken as $8 \text{ m} \cdot \text{s}^{-1}$ perpendicularly towards the plate center. The friction
176 coefficient between the projectile and plate is set as 0.2. The crack initiation and
177 propagation in the plate upon the ballistic impact by the projectile are simulated by the
178 element deletion technique (brittle cracking material model in ABAQUS/Explicit) [35],
179 whereby an element is “deleted” by gradually reducing its stiffness to zero when its
180 maximum principal stress reaches the prescribed fracture strength. During the crack
181 opening, linear stress reduction is introduced to describe the stress variation of the
182 element, which consumes the fracture energy of the material.

183 **Table 1.** Material properties adopted in FE simulations

Properties	Stiff Phase	Compliant Phase	Projectile
Density, ρ (g cm ⁻³)	1.3	1.3	7.8
Young's modulus, E (GPa)	2	0.2	210
Poisson's ratio, ν	0.3	0.3	0.3
Fracture strength, σ_f (MPa)	2	2	-
Fracture energy, Γ (J m ⁻²)	10	10	-



184

185 Figure 2. (a) Schematic showing the simulation model of a projectile perpendicularly
 186 impacting on a plate. (b-e) Snapshots of the calculated cracking process at time $t = 0.5$,
 187 1, and 2 ms: (b) monolithic compliant plate, (c) monolithic stiff plate, (d) S@C
 188 composite plate, and (e) C@S composite plate. Von Mises stress distributions at $t = 2$
 189 ms shown at the bottom row is normalized by the fracture strength (σ_f) of the materials.
 190 The radius of the inclusions in (d) and (e) is 4 mm, and the inter-inclusion spacing
 191 (center-to-center) is 12 mm.

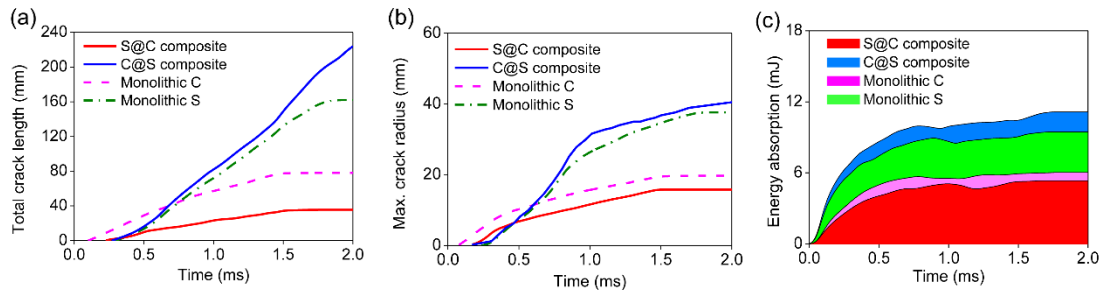
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193 3.2 Results and discussions

194

The snapshots of the cracking process of four plates upon ballistic impact are
 195 displayed in Fig. 2. For the two monolithic cases, it can be seen that cracks propagate
 196 faster and longer in the pure stiff plate (Fig. 2(c)) than in the pure compliant plate (Fig.
 197 2(b)), although the fracture strength (σ_f) and fracture energy (Γ) of both materials are
 198 taken the same. This can be attributed to the lower toughness ($\frac{1}{2}\sigma_f^2/E$) of the stiff
 199 material. If the fracture strength of the stiff material is increased to such a value that its
 200 toughness would be equal to that of the compliant material, both monolithic plates will
 201 exhibit similar cracking configurations under impact (see Fig. S2(a, b)). In the S@C
 202 composite plate (Fig. 2(d)), the ballistic cracks emitted from the impact point
 203 (compliant region) are deflected or blocked by the inclusions (stiff phase), and thereby

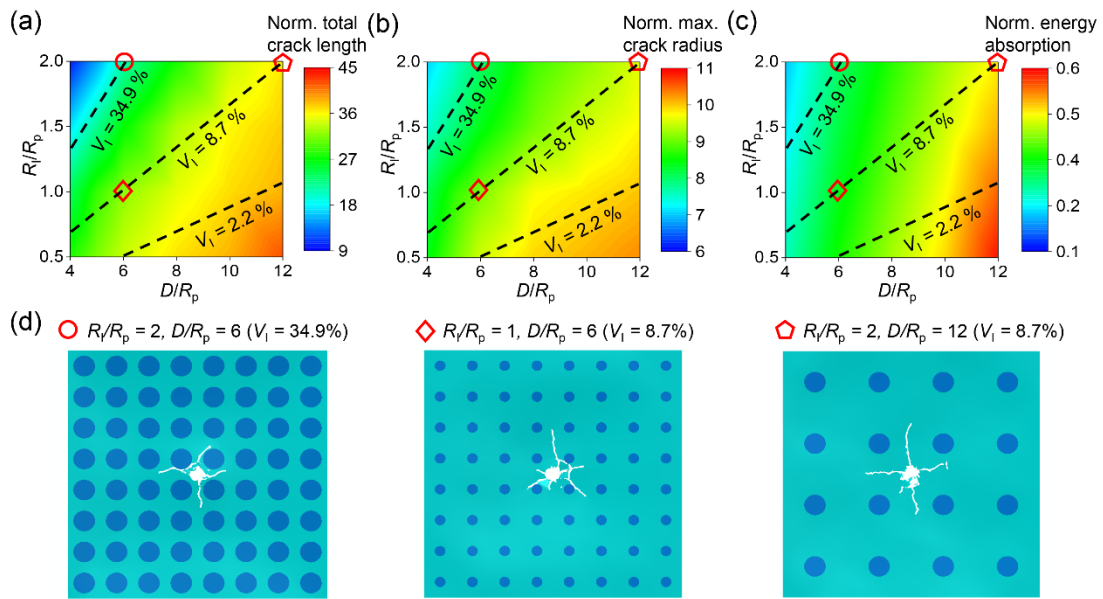
204 crack propagation are constrained in a limited region; in contrast, in the C@S plate (Fig.
 205 2(e)), the ballistic cracks initiating from the impact point (stiff region) penetrate and
 206 pass through the inclusions (compliant phase). Consequently, crack propagation is
 207 highly exacerbated in the C@S plate as compared to the S@C composite plate and two
 208 monolithic controls. Similar result is observed when the toughness ($\frac{1}{2} \sigma_f^2 / E$) of both
 209 phasic materials are taken as the same (see Fig. S2(c, d)). The contrast of different plates
 210 in resisting dynamic cracking shown in Fig. 2 (S@C > monolithic > C@S) also agrees
 211 with the effect of the heterogeneous interface on the SIF of a stationary crack as shown
 212 in Fig. 1. In both composite plates, the stress level in the compliant phase is lower than
 213 that in the stiff phase, implying that the ballistic cracks are prone to propagate into the
 214 regions with lower stress level.



215
 216 Figure 3. Calculated evolutions of (a) the total crack length, (b) the maximum crack
 217 radius, and (c) energy absorption with the time in the impact process. Here the total
 218 crack length and maximum crack radius are normalized by the radius of the projectile
 219 (R_p) and energy absorption is normalized by the initial kinetic energy of the projectile.
 220 This normalization scheme is applied throughout this paper.

221 To make a quantitative comparison between the results from different plates, we
 222 examined the total crack length and the maximum crack radius in the four simulated
 223 cases, as shown in Fig. 3(a) and Fig. 3(b) respectively. Here, the total crack length is
 224 calculated by summing up all the cracks developed in the plate, and the maximum crack
 225 radius is the length of the longest radial crack generated by the impact. Based on either
 226 the total crack length or the maximum crack radius as calculated, the cracking resistance
 227 of the four plates can be ranked in the following sequence: S@C composite >
 228 monolithic C > monolithic S > C@S composite. Moreover, we examined the energy

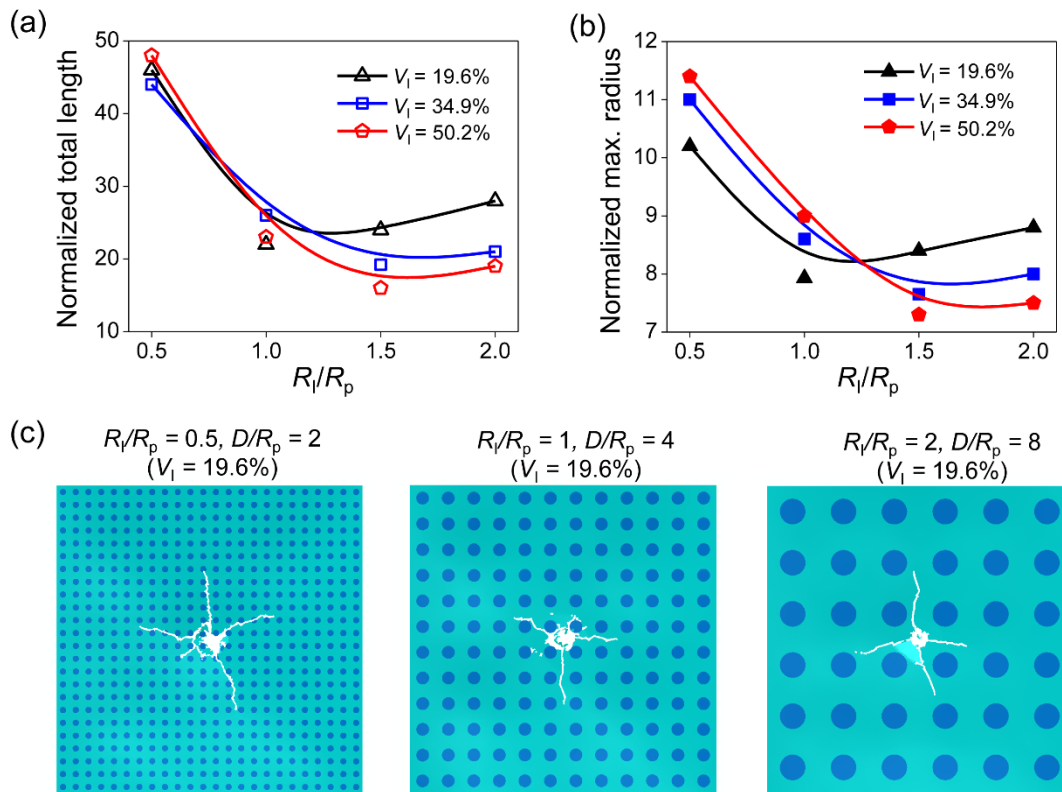
229 absorption, which is defined as the loss of the kinetic energy of the projectile during
 230 impact (Fig. 3(c)). These four plates exhibit an opposite sequence in energy absorption:
 231 S@C composite < monolithic C < monolithic S < C@S composite. This makes sense
 232 because the absorbed energy is proportional to the total length of cracks generated in
 233 the plate given that the fracture energy (Γ) of both phases are presumably identical in
 234 our simulations.



235
 236 Figure 4. Contour maps of (a) total crack length, (b) maximum crack radius, and (c)
 237 energy absorption as functions of the normalized inclusion radius (R_1/R_p) and the
 238 normalized inter-inclusion spacing (D/R_p). Total crack length and maximum crack
 239 radius are normalized by the radius of the projectile (R_p) and the energy absorption is
 240 normalized by the initial kinetic energy of the projectile. (d) Post-impact configurations
 241 of three selected cases as marked by symbols of a circle, a diamond, and a pentagon in
 242 (a)-(c).

243 To further explore the potential of the S@C composite in resisting crack
 244 propagation, more FE simulations are conducted on a series of S@C composite plates
 245 with different inclusion radius (R_1) and inter-inclusion spacing (D). The mechanical
 246 properties of each phase are kept unchanged, as well as the properties and initial
 247 velocity of the projectile. The simulation results including the total crack length,

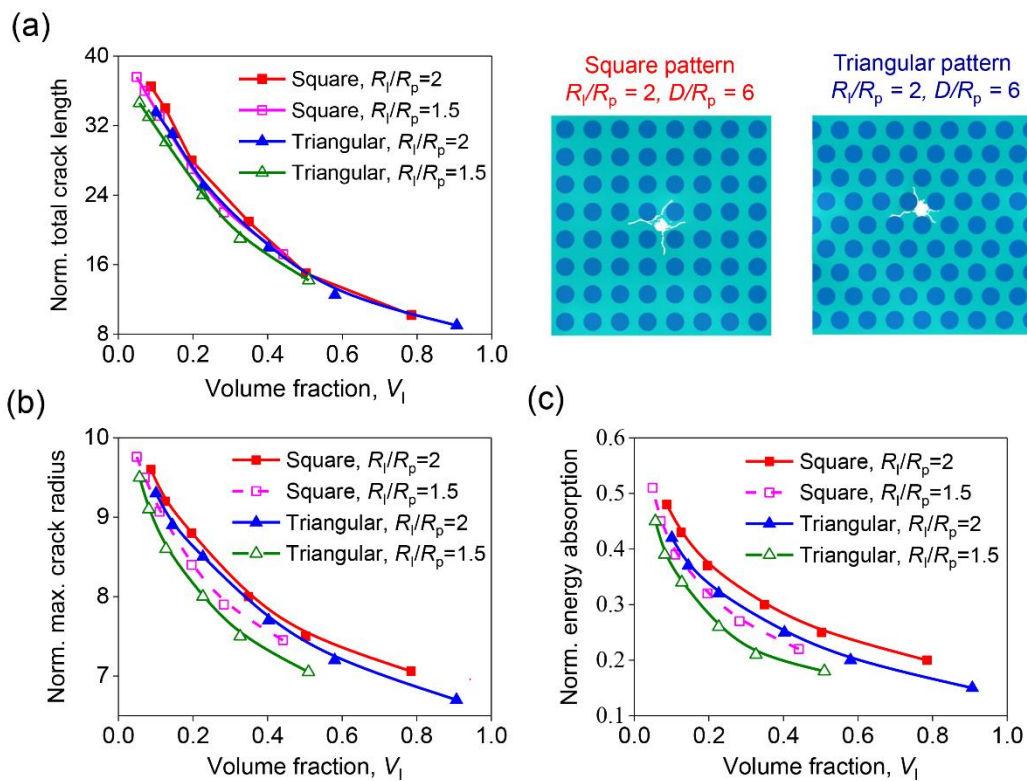
248 maximum crack radius, and energy absorption by the end of the simulation are plotted
 249 in terms of R_I/R_p and D/R_p in Fig. 4. It can be seen that crack propagation can be
 250 restrained further by reducing the inter-inclusion spacing (D) or enlarging the inclusion
 251 radius (R_I). This is because reducing D (with fixed R_I) or increasing R_I (with fixed D)
 252 shortens the distance between the impact point and the nearest inclusions, which will
 253 effectively enhance the suppression effect of the stiff inclusion on the crack propagation.



254 Figure 5. Variations of (a) total length and (b) maximum radius with the normalized
 255 radius of inclusions (R_I) for the given volume fraction of the inclusion (V_I). Here, both
 256 total length and maximum radius are normalized by the radius of projectile (R_p). (c)
 257 The post-impact cracking configuration of the S@C composite plates with the same
 258 volume fraction ($V_I = 19.6\%$) but different R_I/R_p .
 259

260 To visualize the effects of R_I and D on cracking, the post-impact configurations
 261 are shown in Fig. 4(d) for three selected cases which are marked with a circle, a
 262 diamond, and a pentagon symbols in Figs. 4(a-c). Comparison between them
 263 reconfirms that larger inclusions (R_I) or shorter inter-inclusion spacing (D) results
 264 in higher resistance to ballistic cracking. Given the ratio of R_I/D , namely the volume
 265 fraction of the inclusion phase, the variations of the total crack length and maximum

266 crack radius with R_I/R_p are shown in Fig. 5(a) and (b), respectively. It can be seen
 267 that the cracking resistance of the S@C composite plates still depends on the size of the
 268 stiff inclusion when the volume fraction (V_I) is given. The best performance (minimum
 269 cracking) occurs when the inclusion size is around $1\sim 1.5R_p$. For instance, given
 270 volume fraction $V_I = 19.6\%$, the S@C composite plate exhibits the best performance
 271 in resisting cracking when the inclusion size is taken as a moderate value of $R_I \cong R_p$,
 272 as shown by Fig. 5. The existence of such an optimal inclusion size resulting in the best
 273 crack resistance could be interpreted as follows. Given volume fraction (V_I), larger
 274 inclusion size implies greater distance between the impact point of the projectile and
 275 nearest the stiff inclusions, therefore weaker suppression effect upon the garnered
 276 cracks; while if the inclusions are excessively small, the generated dynamic cracks can
 277 easily bypass the stiff inclusions by a small deflection when growing, resulting in longer
 278 cracks.

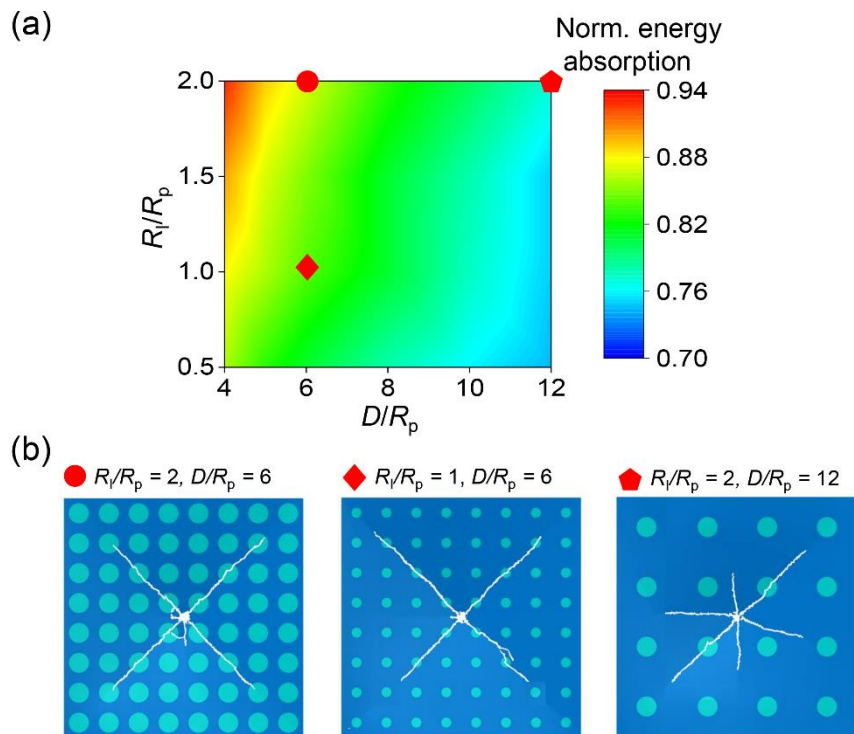


279

280 Figure 6. Variations of the normalized (a) total crack length, (b) maximum crack radius,

281 and (c) energy absorption as functions of volume fraction of stiff inclusion in S@C
 282 composite plates with different distribution patterns.

283 In the above discussion, the stiff inclusions in the S@C composite plates are
 284 presumably distributed in a square pattern. To reveal the effect of distribution pattern
 285 on the resistance to cracking, we further considered the S@C plates with stiffer
 286 inclusions distributed in an equilateral triangular pattern. Figs. 6(a-c) compare the
 287 calculated total crack length, maximum crack radius, and energy absorption between
 288 the S@C plates with inclusions distributed in different patterns. Given inclusion size,
 289 the resistance to ballistic cracking in the S@C plates increases as the volume fraction
 290 of inclusions increases. Given both size and volume fraction of inclusion, the triangular
 291 pattern results in better resistance to ballistic cracking as compared to the square
 292 counterparts. **This is mainly due to the smaller distance between the impact point and**
 293 **the nearest stiff inclusions, therefore stronger suppression effect to the cracking, in the**
 294 **triangular pattern as compared to the square pattern with the same inclusion size and**
 295 **volume fraction (Fig. S3).**



296
 297 Figure 7. (a) Contour map of energy absorption as a function of inclusion radius (R_i)
 298 and inter-inclusion spacing. Here the compliant inclusions are distributed in a pattern.
 299 Here energy absorption is normalized by the initial kinetic energy of the projectile. (b)

300 The cracking configurations of the C@S composite plates after impact by a projectile.

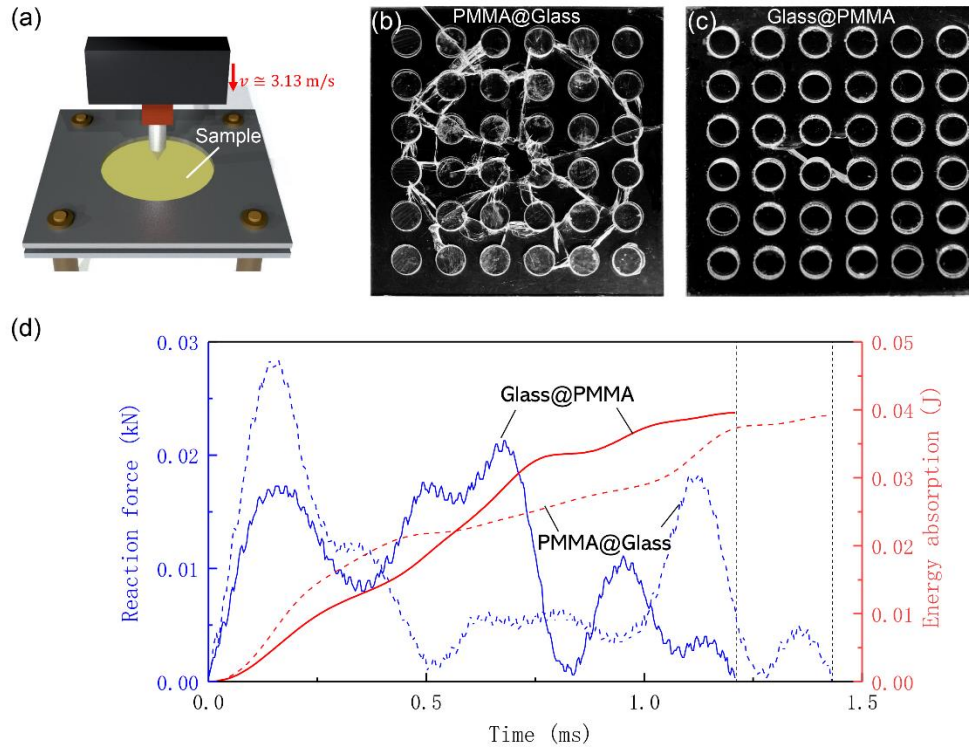
301 On the other hand, for the complementary C@S composites which exhibit superior
302 energy absorption potential, we investigated the dependence of energy absorption on
303 the size of the compliant inclusions and inter-inclusion spacing, as shown in Fig. 7(a).
304 It can be seen that larger inclusions and smaller inter-inclusion spacing lead to higher
305 energy absorption of C@S composites. The cracking configurations of three selected
306 cases are shown in Fig. 7(b). Similarly, for C@S plates with given volume fraction (V_I),
307 the one with triangular patterned inclusions exhibits relative higher energy absorption
308 in comparison with the one with inclusions in square pattern (Fig. S4).

309

310 3.3 Experimental verification

311 To experimentally verify the effect of stiffness heterogeneity on the resistance to
312 impact cracking, we carried out drop weight tests (Dynatup 9250HV, Instron) on the
313 S@C and C@S plate samples synthesized with polymethyl methacrylate (PMMA) and
314 soda-lime glass. Here, PMMA serves as the compliant (C) phase, while glass serves as
315 the stiff (S) phase. The elastic moduli of PMMA and glass at ambient temperature are
316 2-3 GPa and ~60 GPa, respectively. Fig. 8(a) shows the schematics of the experimental
317 setup and Figs. 8(b, c) display the images of two types of the composite after the drop
318 weight impact tests. As expected, severe cracking is observed in the C@S
319 (PMMA@Glass) sample, and some cracks initiated from the impact point even
320 penetrate and pass through the PMMA inclusions (Fig. 8(b)). In contrast, in the S@C
321 (Glass@PMMA) sample, fewer and shorter cracks are generated and no clear crack
322 penetration into the glass inclusions is observed (Fig. 8(c)), which is consistent with the
323 prediction of the numerical simulations above. Fig. 8(d) shows the variations of the
324 reaction force exerted on the projectile and the energy absorption (calculated from the
325 loss of kinetic energy of the projectile) with the time. It can be seen that PMMA@Glass
326 plate imposes higher resistant force to the projectile as compared to the Glass@PMMA
327 plate. However, both plates exhibit similar energy absorption even though the cracking
328 in the PMMA@Glass plate is apparently severer than that in the Glass@PMMA plate

329 (Fig. 8(b, c)). This is basically due to the much higher fracture energies (Γ) of PMMA
 330 in comparison to that of the glass, as has been demonstrated by FE simulation (Table
 331 S1, Fig. S5).



332
 333 Figure 8. (a) Schematic illustration showing the experiment the drop weight impact test.
 334 (b-c) Photos of PMMA@Glass and Glass@PMMA composite plates after drop weight
 335 tests. The dimensions of the plates are $100 \text{ mm} \times 100 \text{ mm} \times 2 \text{ mm}$. The diameter of the
 336 inclusions is 10 mm , and the inter-inclusion spacing (center-to-center) is 15 mm . To
 337 enhance the interfacial strength between the glass and PMMA, the glass surfaces were
 338 functionalized with γ -MPS [30]. The drop weight applied was 17.34 kg and the drop
 339 height was 0.5 m corresponding to an impact velocity of $3.13 \text{ m} \cdot \text{s}^{-1}$. The tup used is
 340 a stainless steel cylinder (diameter: 5 mm) with a conical end (included angle $\approx 85^\circ$).
 341 (d) Variations of reaction force applied on the projectile (left axial) and energy
 342 absorption (right axial) as functions of time. Solid lines and dash lines stand for the
 343 drop weight test on Glass@PMMA plate and PMMA@Glass plate, respectively.

344 4. Discussion and conclusions

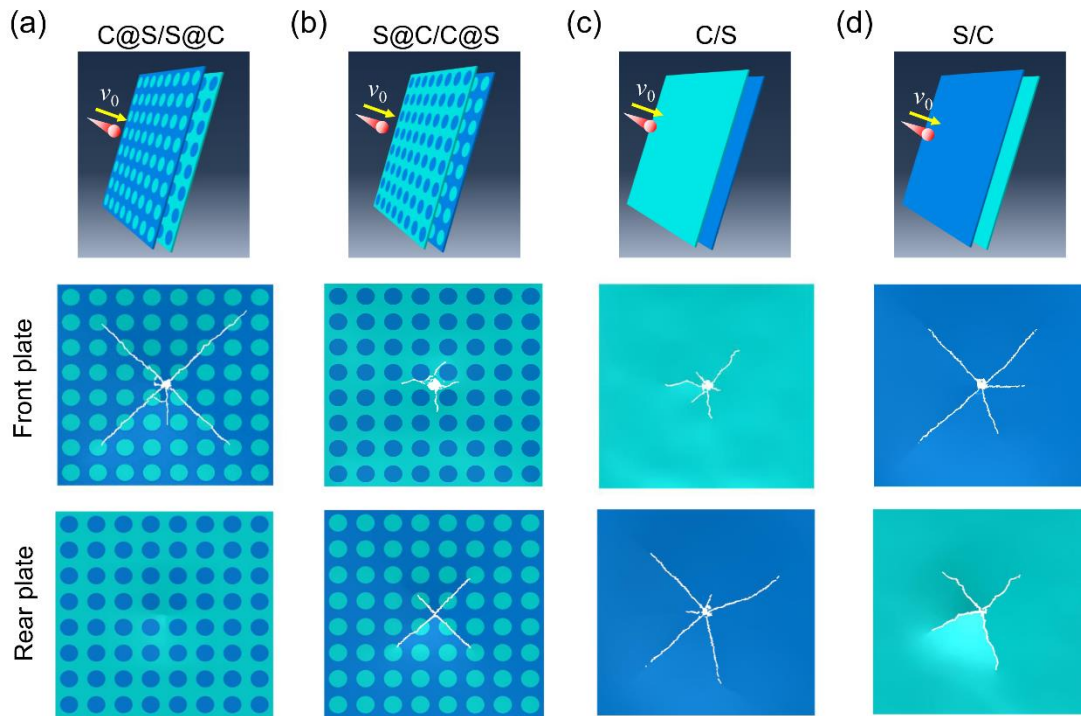
345 In this work, we systematically studied the effects of stiffness heterogeneity on the
 346 resistance of composites to impact cracking. It is revealed that impact cracking can be
 347 significantly prohibited in S@C composites produced by hybridizing stiff inclusions
 348 into a compliant matrix. The performance of S@C composites in prohibiting cracking

349 can be optimized by tuning the size, volume fraction, and distribution pattern of the stiff
350 inclusions. Moreover, the performance of the S@C composites in resisting impact
351 cracking also depends on other practical factors such as the interfacial strength between
352 the S and C phases and the impacting position of the projectile. FE simulations reveal
353 that weak inclusion/matrix bonding would diminish the cracking resistance of the S@C
354 composites especially for the cases with smaller inter-inclusion spacing (Fig. S6);
355 similarly, the cracking resistance of S@C composites will be weakened to some extent
356 if the projectile impacts on the inclusion phase instead (Fig. S7).

357 On the other hand, for the complementary C@S composites, which are the
358 counterparts produced by hybridizing compliant inclusions into a stiff matrix, are found
359 to facilitate impact cracking and therefore exhibit superior competence in energy
360 absorption. Such distinct mechanical behaviors of these two types of composites in
361 response to the impact loading have also been demonstrated in the experiment. The
362 mechanism behind this phenomenon might be attributed to the inverse proportionality
363 of energy release rate, which can be deemed as the driving force of crack propagation,
364 to the stiffness of material under a given loading or stress intensity factor.

365 The distinct behaviors of S@C and C@S composites under impact loading endow
366 them with different functionalities in application. For the materials whose structural
367 integrity is crucial, applying the S@C hybridizing scheme could enhance their
368 resistance to cracking. For the energy-absorption materials, which are often used as
369 disposable shields for protection, applying the C@S hybridizing scheme could promote
370 the competence of energy absorption. Moreover, these two types of hybridizing
371 strategies can be used together to exert their synergic effects. For example, we can stack
372 them in tandem to form a double-layer assembly, as shown in Fig. 9. Similar FE
373 simulations under the same ballistic impact as described above show that the cracking
374 configurations of the double-layer assembly depend on the stacking sequence of the
375 plates. If the C@S plate is placed in the front of the S@C plate, the front C@S plate is
376 cracked severely after impact while the rear S@C plate is almost intact (Fig. 9(a)). In
377 contrast, if the S@C plate is placed ahead of the C@S plate, the projectile can penetrate

378 both plates (Fig. 9(b)). A similar phenomenon is also observed in the control double-
 379 layer assemblies composed of the monolithic stiff and compliant plates, as shown in
 380 Fig. 9(c) and (d). In summary, our work provides a theoretical guideline for harnessing
 381 impact cracking by tuning stiffness heterogeneity. This strategy could be applied further
 382 to controllably deflect and guide the crack propagation trajectory, and finally constrain
 383 the fracture within a limited region [36, 37]. Furthermore, our hybridizing schemes can
 384 be applied in combination with other approaches such as applying T-stress [38] to obtain
 385 more efficient crack-controlling strategies. These technologies are believed of great
 386 value to the design and manufacture of anti-impact materials such as windshields and
 387 shields for space stations and satellites [39].



388
 389 Figure 9. Simulated cracking configurations of double-layer assemblies composed of
 390 (a) a C@S plate backed by an S@C composite plate, (b) an S@C plate backed by a
 391 C@S plate, (c) a monolithic compliant plate backed by a monolithic stiff plate, (d) a
 392 monolithic stiff plate backed by a monolithic compliant plate. In all cases, the two plates
 393 are parallel and separated by 1 cm. The radius and spacing between inclusions, if
 394 available, are $R_1 = 4$ mm and $D = 12$ mm, respectively.

395

396 **Declaration of Competing Interest**

397 The authors declare that they have no known competing financial interests or personal
398 relationships that could have appeared to influence the work reported in this paper.

399

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