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Optimal operating strategy of short turning lines for the battery electric bus system

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ABSTRACT

Operating short turning line is an efficient strategy to satisfy the unevenly distributed demand during peak periods while reducing operational cost. However, for the battery electric bus (BEB) system, the application of the strategy is challenging due to the disadvantages of BEBs, such as limited driving mileage and long charging time. Improper vehicle configuration and charging scheduling may dramatically increase the operational cost and cut the benefits of these strategies. In this work, we propose a general framework to design an effective short turning strategy for the BEB system at a tactical planning level. First, the trade-off relationship between the battery capacity and the average trip time is identified by modeling the BEBs operations. Second, a microeconomic model is formulated to jointly optimize the frequencies and charging schedules of the whole bus line and the short turning line, to effectively minimize passengers' waiting time and operational cost. Finally, numerical experiments have been carried out for an illustrative linear line to demonstrate the potential benefits of the sub-line operating strategy compared with the normal operation.

1. Introduction

The intensity of urban traffic demand makes it necessary to develop an efficient, convenient, and high-capacity public transport system. However, the existing urban transit system has several disadvantages, such as bus exhaust emissions and fossil fuel dependence. In 2016, a study by Climate Action European Commission reported that the transport sector contributes up to one quarter of the greenhouse gas (GHG) emissions in Europe, resulting in air pollution in European cities (European Commission, 2016). Globally, emissions from heavy-duty vehicles like buses and trucks have grown 2.6% annually since 2000 (Teter, 2020). To reduce GHG emissions generated from public transit operation, one promising approach is the electrification of city buses, which has been promoted rapidly in recent years. For example, all diesel buses in Beijing, China, will be replaced with new energy vehicles by the end of 2021, and among the new energy vehicles, over 90% of them are battery electric buses (BEBs) (Zhang et al., 2021). Due to the high energy efficiency and zero tailpipe emissions, BEBs are expected to become mainstream in urban cities in the foreseeable future (Mahmoud et al., 2016; Pelletier et al., 2019).

Despite the fast development, there are challenges in BEB operations that need to be resolved by operators, such as charging infrastructure location problem, BEBs charging problem, etc. Among all these problems, the BEB relocation problem receives insufficient attention in the literature. To be more specific, reallocating buses to cater to the passenger demand variations is a tactical planning problem for the public transit system (Tirachini et al., 2011; Gkiotsalitis et el., 2019). The efficiency of urban transit operations is negatively affected by uneven passenger load along the bus lines in the peak period. To address this problem, the bus operators often increase the bus frequency to better cater to the spatial cluster of transit demand. Although modifying the frequency of the entire bus line is a direct approach to reducing the waiting time of passengers in the line during the morning and evening rush hour, buses may significantly be underutilized in most sections of the bus line, and the service quality may not be guaranteed given the limited vehicle resources. The inefficiency of normal operation optimization based on frequency adjustment for the entire line imposes the need to reallocate and circulate buses, particularly on the most demanded segments of bus network during peak periods. Acting as the vehicle reallocation strategies, sub-lines like short turning lines and interlining lines can readily increase the service frequency of highly-demanded segments by operating buses to serve only the targeted segments (Furth and Day, 1985; Ceder and Stern, 1981; Ceder, 1989; Cortés et al., 2011; Delle Site and Filippi, 1998). Furth (1987) examined the efficiency of the short turning strategy

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Nomenclature

s_0	Start stop of the short turning line
s_1	End stop of the short turning line
N	Number of bus stops on the linear bus line
β	Battery performance in terms of kilometer driven in each unit of battery energy
L	Length of the linear bus line
θ	State of charge range
α	Charging efficiency
P	Charging power of charging station
t_0	Delay which results from entering and leaving charging station
v	Average operating speed considering dwelling times at bus stops
v'	Deadheading speed
P_W	Waiting time value
T	The operation period
q_{max}	Expected demand in the most loaded section under the normal operation
θ	State of charge range
β	Battery performance in terms of kilometer driven in each unit of battery energy
α	Charging efficiency
P	Charging power of charging station
t_0	Delay which results from entering and leaving charging station
v	Average operating speed considering dwelling times at bus stops
v'	Deadheading speed
P_W	Waiting time value
λ_{kl}	Trip rate between stops k and l
$\lambda_k^+(l_1, l_2)$	Boarding rate at stop k of passengers whose destinations range from stop l_1 to stop l_2 inclusive
λ_k^{1+}	Boarding rate at stop k in direction 1
λ_k^{2+}	Boarding rate at stop k in direction 2
n	An integer variable indicating the frequency of the short turning line is a multiple n of the frequency of the whole line
f'	Frequency of the bus line under the normal operation
f_A	Frequency of the whole line under the short turning strategy
f_B	Frequency of the short turning line under the short turning strategy
E_A	Battery capacity of the fleet A
E_B	Battery capacity of the fleet B
E'	Battery capacity of BEBs under the normal operation
t_A	Average trip time for one circulation of the whole line under the short turning strategy
t_B	Average trip time for one circulation of the short turning line under the short turning strategy
t'	Average trip time for one circulation under the normal operation
r_A	Maximum number of circulations that BEBs from the whole line can run under the short turning strategy
r_B	Maximum number of circulations that BEBs from the short turning line can run under the short turning strategy
r'	Maximum number of circulations that BEBs can run under the short turning strategy
L_B	Average circulation length of fleet B
K_A	Passenger capacity of fleet A
K_B	Passenger capacity of fleet B
K'	Passenger capacity of BEBs under the normal operation
Q	Fleet size
C_W'	Passengers' waiting time cost under the normal operation
C_o	Operational cost under the normal operation
C_W^s	Passengers' waiting time cost under the short turning operation
C_o^s	Operational cost under short turning operation
C_{Total}'	Total cost under the normal operation
C_{Total}^s	Total cost under the short turning operation

in balancing passenger load for a single line system based on the assumption that the frequency of a short turning line is a multiple of the frequency of a full line. Three optimal coordination operation schemes for minimizing the fleet size, passengers' waiting time, and both were proposed. Tirachini et al. (2011) proposed a microeconomic model to quantify the benefits of short turning strategy in a single line system. A number of numerical experiments were conducted to figure out the typical patterns to which a short turning strategy can better be applied. Cortés et al. (2011) extended the microeconomic model to consider an integrated strategy combining deadheading and short turning in the single line system. The integrated strategy could increase the frequencies of two highly-demanded segments even if they are spatially separated, but an extra cost associated with deadheading should be weighed against the benefits of the waiting time reduction of passengers. Gkiotsalitis et al. (2019) implemented the short turning and interlining strategies in a bus network to deal with the highly unevenly-distributed demand. First, potential short turning lines and interlining lines that could connect high-loaded segments were generated through a rule-based approach. Then a cost minimization model was developed to allocate available vehicles to existing bus lines and candidate sub-lines. Numerical experiments based on a mid-sized bus network in Hague showed that adding a few short turning and interlining lines can significantly reduce the passengers' waiting time and operational cost.

To the best of our knowledge, the bus reallocation problem for the BEB system has not been studied in the existing literature. The time-consuming charging and limited driving mileage of BEB make it challenging to design effective sub-line operating strategies. On the one hand, to fully utilize battery capacity, operators need to arrange BEBs operating on fixed bus lines (Chen et al., 2018). If a BEB from an existing line is dispatched to serve a sub-line in a peak period, its battery capacity and passenger capacity may not be fully utilized. On the other hand, the costs of operating sub-lines may be significantly high without careful charging scheduling. Hence charging scheduling of BEBs should be considered to achieve the efficient operation of existing bus lines and sub-lines during peak periods.

Motivated by the above consideration, this study proposes a general framework to design the effective operating strategy for short turning lines of a BEB system at a tactical planning level. By analyzing the impact of the charging schedule on the BEB operation, the costs of operating the whole line and short turning line with given frequencies are explicitly established. Then the short turning line design problem is modeled as an unconstrained optimization problem to obtain the analytical expressions for optimal frequencies. Numerical experiments are conducted to explore the benefit of the proposed strategy. The contributions of this study are summarized as follows:

(1) We introduce the short turning strategy in the BEB system to adjust the vehicle supply during peak periods, and establish passengers' waiting cost and operational cost to evaluate the effect of the strategy on both sides.

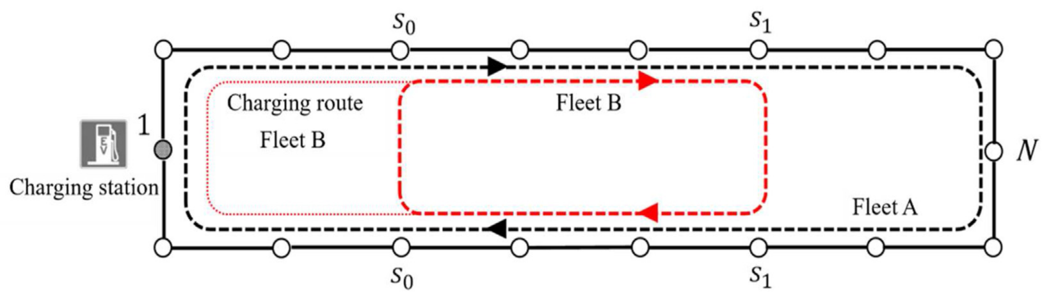


Fig. 1. Illustration of short turning strategy.

(2) The short turning line design problem for the BEB system is formulated as a microeconomic model, which allows us to find the analytical expressions for optimal frequencies under normal operation and short turning strategy,
(3) Numerical experiments on a linear bus line are conducted to identify the typical demand patterns and bus network configurations that work well under the proposed strategy.

In the remainder of this paper, Section 2 elaborates on the assumptions, notations, and problem description. A cost minimization model for frequency setting and charging scheduling of the short turning line and the whole line is developed in Section 3. Section 4 carries out numerical experiments based on a linear bus line. Section 5 presents the conclusions and discussions.

2. Assumptions, notations, and problem description

2.1. Short turning line

We consider a linear bus line that uses BEBs to serve passengers. Since the passenger demand exhibits a significant imbalance along a bus line during peak periods, conventional BEB operation is inadequate to meet the needs of passengers. In this study, two different operation strategies, i.e., normal operation and short turning strategies, are adopted to deal with this problem. The normal operation increases the frequency of the whole line by using a larger BEB fleet, which acts as a benchmark to quantify the benefits of the short turning strategy. In contrast, the latter adds a short turning line to increase the frequency of the highly-demanded segments, and there will be two BEB fleets, one is intended for serving the whole line (fleet A), while the other is designed for serving the short turning line (fleet B).

Specifically, a short turning line serves a segment of the bus line with a dedicated bus fleet during the peak period as shown in Fig. 1. The BEB operating the short turning line starts serving at stop s_0 and ends at stop s_1 in one direction, then it will turn and start service again from stop s_1 to stop s_0 in the other direction. On the other hand, the BEB may need to charge its battery at the charging station that is usually deployed at the initial stop of the bus line during the operation period. In order to minimize the impact on passengers, the BEB will run without passengers as soon as possible from stop s_0 to the charging station (deadheading). After fully charging the battery, the BEB will deadhead back to stop s_0 and start the next circulation.

To implement the short turning strategy in the BEB system, operators should simultaneously optimize the start and end stops associated with the short turning line, frequencies of the short turning line and the whole line, passenger and battery capacity of two BEB fleets. For the normal operation, in particular, the decision variables are the frequency of the bus line and passenger and battery capacity of the BEB fleet.

2.2. Characterization of BEB operation

For the BEB system, the operation of BEB is limited by the driving mileage, which is mainly dependent on its battery capacity. BEBs always need to charge their batteries after executing several circulations, and the charging time cannot be negligible. To ensure the formulated model is mathematically tractable, the following assumptions are made:

i. Charging station is only located at the start stop of the bus line.
ii. The BEB fleet will be fully charged at the beginning of the operation period.
iii. The average operating speed of BEBs is constant along the transit line.
iv. BEB drivers will fully charge batteries at the charging station.
v. BEBs will run without passengers to charge their batteries (deadheading).
vi. The delay in entering and leaving the charging station is set to be fixed.
vii. All BEBs are identical, and their electric consumption is distance-dependent.

2.2.1. Normal operation

Based on these assumptions, we proceed to investigate the characteristics of BEBs operation and the impact of charging during the operation period on battery capacity selection and the average trip time of completing one full circulation. Considering a linear bus line that contains N evenly distributed bus stops with corridor length L. Fleet A operates along the whole line to maintain a service frequency f_A during the operation period T. Let r_A denote the maximum number of circulations that a BEB can run with a fully charged battery; it follows that

$$r_A = \frac{\theta E_A \beta}{2L} \tag{1}$$

where β is the battery performance in terms of kilometer driven in each unit of battery energy, θ is the state of charge range, and E_A represents the battery capacity.

According to Chen et al. (2018), each BEB will fully charge the battery at the charging station precisely after completing r_A circulations to reduce delay caused by entering and leaving the charging station, then the minimum battery capacity that satisfies the operating requirement is given by

$$E_A(r_A) = \frac{2Lr_A}{\theta\beta} \tag{2}$$

Let t_A be the average trip time for BEB completing one circulation. During the operation period T, the total number of times of charging that each BEB requires is $\frac{T}{t_A r_A} - 1$, and the time spent is $\frac{2Lr_A}{\beta}/\alpha P$ per time, where α denotes charging efficiency, and P represents the charging power. Then the average charging time for each circulation is calculated by

$$\left(\frac{T}{t_A r_A} - 1\right)\frac{2Lr_A}{\beta\alpha P}\Big/\left(\frac{T}{t_A}\right) \tag{3}$$

Let m denote the number of chargers at the charging station, to avoid vehicle queuing at the charging station; it should not be less than the number of vehicles arriving within the average charging time as shown in Fig. 2, that is

$$m \geq f\left(\frac{T}{tr}-1\right)\left(\frac{2Lr}{\alpha P\beta}\right) \Big/ \frac{T}{t} \tag{4}$$

The average trip time for BEB completing one circulation includes the service time along the bus line and the average charging time and delay per circulation. It can be formulated by

$$t_A(r_A)=\begin{cases}\dfrac{\dfrac{2L}{v}+\dfrac{1}{r_A}\left(\dfrac{2Lr_A}{\alpha P\beta}+t_0\right)}{1+\dfrac{1}{T}\left(\dfrac{2Lr_A}{\alpha P\beta}+t_0\right)} & 1\leq r_A<\dfrac{vT}{2L}\\ \dfrac{2L}{v} & r_A=\dfrac{vT}{2L}\end{cases} \tag{5}$$

where v denotes the operating speed and t_0 represents the delay in entering and leaving the charging station. We have the following proposition regarding the average trip time.

Proposition 1. *The average trip time for completing one circulation is a strictly monotonically decreasing function of r_A.*

Proof. *Since*

$$\frac{\partial t_A(r_A)}{\partial r_A}=\frac{-\frac{t_0}{r_A{}^2}\left[1+\frac{1}{T}\left(\frac{2Lr_A}{\alpha P\beta}+t_0\right)\right]-\frac{2L}{\alpha P\beta T}\left[\frac{2L}{v}+\frac{1}{r_A}\left(\frac{2Lr_A}{\alpha P\beta}+t_0\right)\right]}{\left[1+\frac{1}{T}\left(\frac{2Lr_A}{\alpha P\beta}+t_0\right)\right]^2}<0, \tag{6}$$

We can conclude that $t_A(r_A)$ is a strictly monotonically decreasing function with respect to r_A. This completes the proof. □

Let Q_A denote the size of fleet A. Given the service frequency of the bus line and the relationship $Q_A = f_A t_A$, Eq. (2) *and* Proposition 1 *imply that a reduction in the battery capacity of BEBs will lead to an increase in fleet size.*

2.2.2. Short turning operation

We then extend the above formulations of battery capacity and average trip time to the short turning line. According to assumptions i and v, BEBs of fleet B should deadhead to the start stop to charge their batteries. Therefore, the charging trips between the charging station and the start stop of the short turning line should be taken into account in the formulations of battery capacity and average trip time. Let r_B denote the maximum number of circulations that a BEB from the short turning line can run. Then the battery capacity and average trip time for one circulation, namely E_B and t_B, can be calculated by

$$E_B(r_B,s_0,s_1)=\begin{cases}\dfrac{2L}{\theta\beta}\left(\dfrac{s_1-s_0}{N-1}r_B+\dfrac{s_0-1}{N-1}\right) & 1\leq r_B<\dfrac{vT}{2L}\left(\dfrac{N-1}{s_1-s_0}\right)\\ \dfrac{2L}{\theta\beta}\left(\dfrac{s_1-s_0}{N-1}\right)r_B & r_B=\dfrac{vT}{2L}\left(\dfrac{N-1}{s_1-s_0}\right)\end{cases} \tag{7}$$

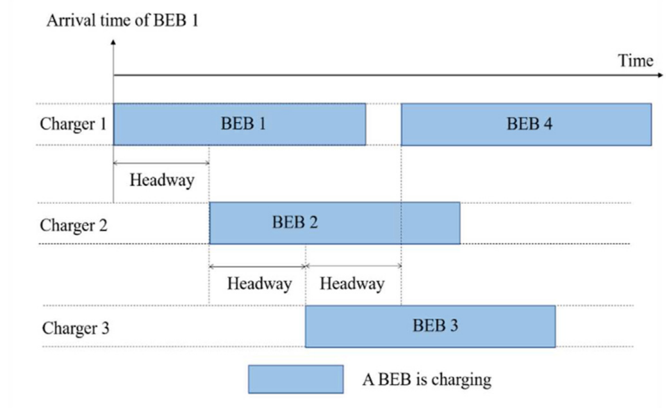


Fig. 2. The chargers' utilization over time.

where v' denotes the deadheading speed when BEBs of fleet B run without passengers from the stop s_0 to charging station.

3. Model formulation

Similar to the transit network design problem, the objective function of the short turning line design problem is to minimize the passengers' waiting time cost and the operational cost. It is assumed that the passenger demand is known and fixed over the studied period. Let λ_{kl} denote the trip rate between stops k and l (pax/h). Then the boarding rate at stop k of passengers whose destinations range from stop l_1 to stop l_2 (inclusive), denoted by $\lambda_k^+(l_1,l_2)$, is equal to $\sum_{l=l_1}^{l_2}\lambda_{kl}$; the total boarding rate at stop k in direction 1, denoted by λ_k^{1+}, is equal to $\sum_{l=k+1}^{N}\lambda_{kl}$; the total boarding rate at stop k in direction 2, namely λ_k^{2+}, is equal to $\sum_{l=1}^{k-1}\lambda_{kl}$. Kindly note that direction 1 represents BEBs moving from stop 1 to stop N and direction 2 represents BEBs moving back to stop 1 from stop N.

3.1. Waiting time cost

For the normal operation, let C_W' denote the waiting time cost; it could be calculated by

$$C_W'=\frac{TP_W}{2f'}\left\{\sum_{k=1}^{N-1}\lambda_k^{1+}+\sum_{k=2}^{N}\lambda_k^{2+}\right\} \tag{9}$$

where P_W is the waiting time value and f' is the frequency of the bus line under the normal operation.

$$t_B(r_B,s_0,s_1)=\begin{cases}\dfrac{\dfrac{2L}{v}\left(\dfrac{s_1-s_0}{N-1}\right)+\dfrac{1}{r_B}\left[\dfrac{2L}{\alpha P\beta}\left(\dfrac{s_1-s_0}{N-1}r_B+\dfrac{s_0-1}{N-1}\right)+t_0+\dfrac{2L}{v'}\left(\dfrac{s_0-1}{N-1}\right)\right]}{1+\dfrac{1}{T}\left[\dfrac{2L}{\alpha P\beta}\left(\dfrac{s_1-s_0}{N-1}r_B+\dfrac{s_0-1}{N-1}\right)+t_0+\dfrac{2L}{v'}\left(\dfrac{s_0-1}{N-1}\right)\right]} & 1\leq r_B<\dfrac{vT}{2L}\left(\dfrac{N-1}{s_1-s_0}\right)\\ \dfrac{2L}{v}\left(\dfrac{s_1-s_0}{N-1}\right) & r_B=\dfrac{vT}{2L}\left(\dfrac{N-1}{s_1-s_0}\right)\end{cases} \tag{8}$$

For the short turning strategy, we assume that passengers make their travel plans based on the existing bus lines, and the presence of the short turning line will not motivate passengers to change their travel routes. In other words, passengers will take the short turning line only when their origins and destinations are covered by the short turning line. Then the passengers' waiting time cost under short turning operation, namely C_W^s, could be calculated by

$$C_W^s = \frac{TP_W}{2}\left\{\sum_{k=1}^{s_0-1}\frac{\lambda_k^{1+}}{f_A} + \sum_{k=s_0}^{s_1-1}\left(\frac{\lambda_k^{+}(k+1,s_1)}{f_A+f_B} + \frac{\lambda_k^{+}(s_1+1,N)}{f_A}\right) + \sum_{k=s_1}^{N-1}\frac{\lambda_k^{1+}}{f_A} + \sum_{k=s_1+1}^{N}\frac{\lambda_k^{2+}}{f_A} + \sum_{k=s_0+1}^{s_1}\left(\frac{\lambda_k^{+}(1,\ s_0-1)}{f_A} + \frac{\lambda_k^{+}(s_0,k-1)}{f_A+f_B}\right) + \sum_{k=2}^{s_0}\frac{\lambda_k^{2+}}{f_A}\right\} \tag{10}$$

3.2. Operational cost (normal operation)

Regarding the operational cost, Jansson (1980) proposed a function of passenger capacity and fleet size to express the operational cost for a single transit line system, which consists of crew costs, fuel consumption, bus maintenance, and so on. In this study, we extend the function to express the cost of operating BEBs. Let C_o denote the operational cost, which is composed of vehicle-hour cost c and vehicle-kilometer cost c'. Both of them are linear functions of the passenger capacity K and battery size E. Specifically, we have

$$c(K,E) = c_0 + c_1K + c_2E,\ c'(K,E) = c_0' + c_1'K + c_2'E \tag{11}$$

$$C_o = c(K,E)QT + c'(K,E)vQT \tag{12}$$

For the normal operation, the passenger capacity of BEBs, namely K' should accommodate the expected demand in the most loaded section, namely q_{max}; then we have

$$q_{max} = \max_{k\in\{1,\cdots,N-1\}}\left\{\sum_{i=1}^{k}\sum_{j=k+1}^{N}\lambda_{ij}, \sum_{i=k+1}^{N}\sum_{j=1}^{k}\lambda_{ij}\right\} \tag{13}$$

$$K' = \frac{q_{max}}{\eta f'} \tag{14}$$

where η denotes the maximum occupancy rate.

Then total cost C_{Total} is the sum of passenger cost and operational cost, i.e.,

$$C_{Total}' = \frac{TP_W}{2}\left\{\sum_{k=1}^{N-1}\frac{\lambda_k^{1+}}{f'} + \sum_{k=2}^{N}\frac{\lambda_k^{2+}}{f'}\right\} + \left(c_0 + c_1\frac{q_{max}}{\eta f'} + c_2E'\right)f'Tt' + \left(c_0' + c_1'\frac{q_{max}}{\eta f'} + c_2'E'\right)f'T2L \tag{15}$$

where E' and t' denote the battery capacity and average trip time of BEBs under normal operation calculated by Eqs. (2) and (5), respectively.

Let r' be the maximum number of circulations that a BEB could run; we can obtain the optimal value of frequency by means of first-order optimality conditions as follows:

$$f^*(r') = \begin{cases} \sqrt{\dfrac{\frac{P_W}{2}\left\{\sum_{k=1}^{N-1}\lambda_k^{1+} + \sum_{k=2}^{N}\lambda_k^{2+}\right\}}{(c_0+c_2E')t' + (c_0'+c_2'E')2L}} & 1 \le r' < \dfrac{vT}{2L} \\ \sqrt{\dfrac{\frac{P_W}{2}\left\{\sum_{k=1}^{N-1}\lambda_k^{1+} + \sum_{k=2}^{N}\lambda_k^{2+}\right\}}{\left(c_0 + c_2\dfrac{2L\frac{vT}{2L}}{\theta\beta}\right)\dfrac{2L}{v} + \left(c_0' + c_2'\dfrac{2L\frac{vT}{2L}}{\theta\beta}\right)2L}} & r' = \dfrac{vT}{2L} \end{cases} \tag{16}$$

To find the optimal solution for the normal operation, we could enumerate r' as an integer from 1 to $\frac{vT}{2L}$, and then obtain E', t', f^*, and C_{Total}' from Eqs. ((2), (5), (15) and (16) accordingly. Then the values of E', t', f^* with the minimum C_{Total}' will be the optimal solution. Since the enumeration size $\left[1, \frac{vT}{2L}\right]$ is generally not large, the search process will not be time-consuming.

3.3. Operational cost (short turning operation)

For the short turning strategy. Let K_A represent the passenger capacity of fleet A and K_B represent the passenger capacity of fleet B; they can be expressed by

$$K_A(f_A,f_B,s_0,s_1) = \frac{1}{\eta}max\left\{\varphi_{max}^{in,A}(f_A,f_B),\ \frac{q_{max}^{out}}{f_A}\right\} \tag{17}$$

$$K_B = \frac{q_{max}^{in,AB}}{\eta(f_A+f_B)} \tag{18}$$

where functions $\varphi_{max}^{in,A}(f_A,f_B)$, q_{max}^{out}, and $q_{max}^{in,AB}$ are defined in the appendix.

The operational cost under short turning operation, denoted by C_o^s, is the sum of the costs associated with fleets A and B, i.e.,

$$C_o^s = (c_0 + c_1K_A + c_2E_A)f_ATt_A + (c_0' + c_1'K_A + c_2'E_A)f_AT2L + (c_0 + c_1K_B + c_2E_B)f_BTt_B + (c_0' + c_1'K_B + c_2'E_B)f_BTL_B(r_B,s_0,s_1) \tag{19}$$

where $L_B(r_B,s_0,s_1)$ denotes the average circulation length of fleet B. Specifically, we have

$$L_B(r_B,s_0,s_1) = \begin{cases} \left(\dfrac{s_1-s_0}{N-1}\right)2L + \left(\dfrac{1}{r_B} - \dfrac{t_B(r_B,s_0,s_1)}{T}\right)\left(\dfrac{s_0-1}{N-1}\right)2L & 1 \le r_B < \dfrac{vT}{2L}\left(\dfrac{N-1}{s_1-s_0}\right) \\ \left(\dfrac{s_1-s_0}{N-1}\right)2L & r_B = \dfrac{vT}{2L}\left(\dfrac{N-1}{s_1-s_0}\right) \end{cases} \tag{20}$$

The total cost under short turning operation, namely, C_{Total}^s, is the sum of C_W^s and C_o^s as provided in Eqs. (10) and (19). Since the total cost depends on s_0, s_1, r_A, r_B, f_A, and f_B, the problem could be solved in two stages. First, we treat s_0, s_1, r_A, and r_B as parameters and apply the first-order optimality conditions to find f_A, f_B, and C_{Total}^s. In the second stage, the feasible limit parameters could be explored to find the solution that minimizes C_{Total}^s.

Because the derivative of K_A with respect to f_A could not be obtained analytically, the first-order conditions cannot yield analytical forms for the variables f_A and f_B, and thus the problem must be solved numerically. Nevertheless, if the operational costs do not depend on capacity K, i.e.,

$c_1 = 0$ and $c_1^{'} = 0$ (see Eq. (11)), then the optimal frequencies can be derived as follows:

$$f_A^*(r_A, r_B, s_0, s_1) = \sqrt{\frac{\frac{P_W}{2} g_1(s_0, s_1)}{(c_0 + c_2E_A)t_A + (c_0^{'} + c_2^{'}E_A)2L - (c_0 + c_2E_B)t_B - (c_0^{'} + c_2^{'}E_B)2L_B}} \quad (21)$$

$$f_B^*(r_A, r_B, s_0, s_1) = \sqrt{\frac{\frac{P_W}{2} g_2(s_0, s_1)}{(c_0 + c_2E_B)t_B + (c_0^{'} + c_2^{'}E_B)2L_B}} - f_A^*(r_A, r_B, s_0, s_1) \quad (22)$$

where $g_1(s_0, s_1)$ denotes the total number of passengers whose origins or destinations are not covered by the short turning line, while $g_2(s_0, s_1)$ denotes the total number of passengers whose origins and destinations are covered by the short turning line. They are calculated as follows:

$$g_1(s_0, s_1) = \sum_{k=1}^{s_0-1} \lambda_k^{1+} + \sum_{k=s_0}^{s_1-1} \lambda_k^{+}(s_1+1, N) + \sum_{k=s_1}^{N-1} \lambda_k^{1+} + \sum_{k=s_1+1}^{N} \lambda_k^{2+} + \sum_{k=s_0+1}^{s_1} \lambda_k^{+}(1,\ s_0-1) + \sum_{k=2}^{s_0} \lambda_k^{2+} \quad (23)$$

$$g_2(s_0, s_1) = \sum_{k=s_0}^{s_1-1} \lambda_k^{+}(k+1, s_1) + \sum_{k=s_0+1}^{s_1} \lambda_k^{+}(s_0, k-1) \quad (24)$$

Note that if $f_B = 0$, then C_{Total}^s is equivalent to $C_{Total}^{'}$. So, if f_B^* is positive, the short turning strategy could be beneficial.

For the situation of a scheduled service as introduced by Furth (1987), i.e., $f_B = nf_A$, the passenger capacity of the two types of fleets could be expressed by

$$K_A = \frac{1}{\eta f_A} max\{q_{max}^{in,A}(n, s_0, s_1),\ q_{max}^{out}\} \quad (25)$$

$$K_B = \frac{q_{max}^{in,AB}(s_0, s_1)}{\eta(1+n)f_A} \quad (26)$$

where n is the "scheduling mode" and

$$q_{max}^{in,A}(n, s_0, s_1) = \max_{k\in\{s_0,\cdots,s_1-1\}} \left\{ \frac{q_k^{AB,1}}{1+n} + q_k^{A,1},\ \frac{q_k^{AB,2}}{1+n} + q_k^{A,2} \right\}.$$

By employing Eqs. (25) and (26) to calculate the total cost, the optimal frequency could be derived as follows:

$$f_A^*(n, r_A, r_B, s_0, s_1) = \sqrt{\frac{\frac{P_W}{2}\left(g_1(s_0, s_1) + \frac{g_2(s_0, s_1)}{1+n}\right)}{(c_0 + c_2E_A)t_A + (c_0^{'} + c_2^{'}E_A)2L + n(c_0 + c_2E_B)t_B + n(c_0^{'} + c_2^{'}E_B)2L_B}} \quad (27)$$

Then the values of n, s_0, s_1, r_A, and r_B are enumerated for minimizing the total cost. As pointed out previously, the time consumption of the procedure could be acceptable.

4. Numerical experiments

Consider a single linear bus corridor with 10 bus stops and a charging station located at the terminal stop. Table 1 presents the parameter setting in the numerical experiments.

4.1. Radial corridor pattern and cross-town route pattern

Two different passenger flow concentration pattern scenarios from Tirachini et al. (2011) are considered as shown in Fig. 3, where Scenario 1 describes a radial corridor pattern in the morning rush hours, while Scenario 2 represents a cross-town activity pattern. The digit at row i and column j in the OD matrix denotes the trip rate from stop i to stop j (pax/h).

Table 2 shows the optimal results under the normal operation and the short turning strategy for Scenario 1. The optimal short turning line will be established between stop $s_0 = 7$ and $s_1 = 10$, which coincides with the most loaded sections. Under the short turning strategy, frequency is improved from 18.3 veh/h to 27.2 veh/h at the segment covered by the short turning line and declines from 18.3 veh/h to 13.6 veh/h at the rest of the line. The strategy achieves a saving in operational cost but a loss in user cost, with a total saving of 3.7%. The waiting time saving for passengers covered by the short turning line is offset by the prolonged waiting time for passengers whose origins or destinations are outside the short turning line. For the charging plans of two fleets, the fleet A will charge the battery at the charging station precisely after completing 2 circulations, while the fleet B will run continuously during the entire rush hours. For the former, although frequent charging will increase the average trip time, the low-capacity batteries will lower the cost of battery investment. In the contrary, the operator prefers to equip high-capacity batteries for the fleet B, because charging at stop 1 will result in a long delay in running an empty vehicle between the charging station and the sub-line.

The results obtained in Scenario 2 are shown in Table 3. The optimal short turning line is the segment between stop $s_0 = 5$ and $s_1 = 8$ where 73% of the demand occurs. The strategy achieves total cost savings higher than 11%, much better than Scenario 1. Both passengers and operator will benefit from the short turning strategy because the waiting time loss by passengers whose origin or destinations are outside the short turning line could be compensated by the waiting time saving of passengers covered by the short turning line. Second, a relatively small BEB fleet is able to provide services along the short turning line and the whole line. In both scenarios, the battery capacity of fleet A will remain unchanged after applying the strategy.

Table 1
Parameter definitions and values in the linear bus line.

Notation	Definition	Value
T	Operation period	3 h
L	Corridor length	8 km
v	Operating speed	21.33 km/h
v'	Deadheading speed	30 km/h
θ	State of charge range	0.6
β	Energy consumption rate	0.8 km/kWh
α	Charging efficiency	0.9
P	Charging power of charging station	120 kW
t_0	Delay in entering and leaving charging station	0.083 h
η	Maximum design occupancy rate	0.9
P_W	Waiting time value	5.4 $/h
c_0	Coefficient in vehicle-hour cost	3.6
c_1	Coefficient associated with passenger capacity in vehicle-hour cost	0.06
c_2	Coefficient associated with battery size in vehicle-hour cost	0.0125
c_0'	Coefficient in vehicle-kilometer cost	0.8
c_1'	Coefficient associated with passenger capacity in vehicle-kilometer cost	0.002
c_2'	Coefficient associated with battery size in vehicle-kilometer cost	0.0005

O\D	1	2	3	4	5	6	7	8	9	10
1		29	14	64	4	3	3	1	1	25
2	14		15	70	4	4	3	1	1	27
3	5	5		49	3	3	2	1	0	19
4	8	7	4		18	15	12	4	3	111
5	74	63	35	0		5	4	1	1	37
6	4	4	2	0	0		5	2	1	50
7	1	1	0	0	0	3		20	16	636
8	8	6	3	0	0	26	5		7	262
9	16	14	7	0	0	58	11	0		77
10	13	11	6	0	0	47	9	0	10	

O\D	1	2	3	4	5	6	7	8	9	10
1		5	8	9	13	10	17	41	4	2
2	7		7	8	11	8	15	35	4	2
3	40	35		13	17	13	23	55	6	3
4	20	18	22		24	19	33	77	8	4
5	17	15	19	34		150	265	624	67	35
6	15	13	16	29	348		267	630	67	36
7	20	18	23	40	484	131		529	56	30
8	21	18	23	40	494	134	130		52	28
9	3	3	3	6	72	19	19	5		5
10	1	1	1	2	23	6	6	2	3	

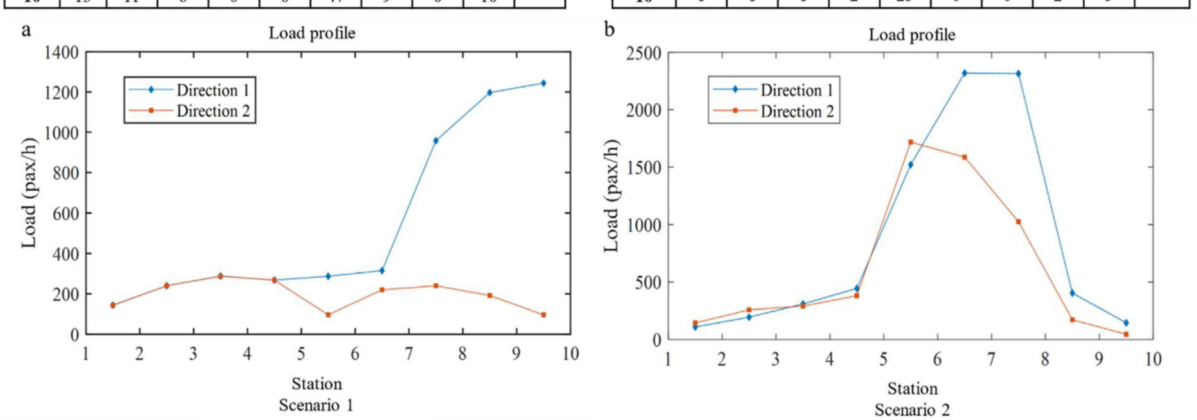


Fig. 3. Origin-destination matrices and load profiles for Scenarios 1 and 2.

4.2. *Influence of operation period*

As the battery capacity is upper-bounded by the operation period T, it is important to understand how the duration of the operation period affects the results of strategy in terms of cost savings. Two additional passenger load scenarios are considered as shown in Fig. 4. Scenario 3 shows a peak load of 500 pax/h in segments between stops 5 and 8. As expected, the optimal short turning line starts at stop 5 and ends at stop 8. Fig. 5 depicts the tendency of change in cost savings and the numbers of circulations that the fully charged batteries can support with respect to the operation period T for both fleet A and B. Based on Fig. 5a, we can see that the longer the operation period, the less benefit the strategy could result in. The reason is that when the operation period is relatively short, e.g., $T \leq 1.5$ h, a longer operation period T requires a larger battery for fleet B. As for fleet A, although the battery size remains unchanged, the charging delay will increase. When the operation period is relatively long, e.g., $T > 1.5$ h, both battery sizes of fleet A and B will not change with the increase of T, but the charging delay may increase. Fig. 5b depicts the tradeoff between the charging delay and battery size. When $T \leq 1.5$ h, since the travel distance to the charging station for fleet B is long and the total number of circulations that fleet B should run is relatively small, it is worth using larger batteries to make the BEBs run continuously during the whole operation period. When $T > 1.5$ h, since there are more circulations that fleet B should run, a smaller-capacity

Table 2
Solutions for scenario 1.

Operation	Normal	Strategy
f_A (veh/h)	18.3	13.6
f_B (veh/h)		13.6
E_A (kWh)	67	67
E_B (kWh)		134
K_A (pax)	76	62
K_B (pax)		40
Q (veh)	16	15
C_{Total} (\$)	2217.6	2135.5
C_W (\$)	936.9	942.4
C_o (\$)	1280.7	1193.2
ΔC_{Total} (%)	−3.7	
ΔC_W (%)	0.6	
ΔC_o (%)		
$s_0 = 7$, $s_1 = 10$, $n = 1$, $r_A = 2$, $r_B = 12$		

Table 3
Solutions for Scenario 2.

Operation	Normal	Strategy
f_A (veh/h)	30.1	16.6
f_B (veh/h)		33.2
E_A (kWh)	67	67
E_B (kWh)		134
K_A (pax)	86	76
K_B (pax)		40
Q (veh)	26	23
C_{Total} (\$)	3728	3293.1
C_W (\$)	1543.4	1439.5
C_o (\$)	2184.6	1853.6
ΔC_{Total} (%)	−11.7	
ΔC_W (%)	−6.7	
ΔC_o (%)	−15.2	
$s_0 = 5$, $s_1 = 8$, $n = 2$, $r_A = 2$, $r_B = 12$		

battery could be better. Fig. 5c shows that an increase of T may not affect the optimal battery size for fleet A. Because fleet A serves the entire route, it is not economical to decrease the charging delay considering the increased cost of a large battery.

Scenario 3 suggests that the benefit of the strategy becomes less for a prolonged operation period. It naturally raises the question of whether there exists a peak demand pattern under which the benefit of the strategy shifts from positive to negative as the operation period is extended. To answer this question, we design Scenario 4 to capture the unaccustomed feature. Different from Scenario 3, the peak passenger load of Scenario 4 is reduced to 170 pax/h. Fig. 6 describes the change tendency in cost savings and the numbers of circulations that the fully charged batteries can support with respect to the operation period T for both fleets A and B.

Based on Fig. 6 we can see that the strategy is beneficial when $T \leq 1.2$ h. The total cost saving at $T = 1$ h is better than the total cost saving at $T = 0.8$ h. This is because the battery sizes and the total numbers of circulations that two fleets should run remain unchanged with the operation period. So, more passengers could benefit from the strategy at $T = 1$ h. When $T \geq 1.4$ h, the strategy is no longer beneficial due to the decrease in operating efficiency of the fleet B caused by battery charging during the operation period.

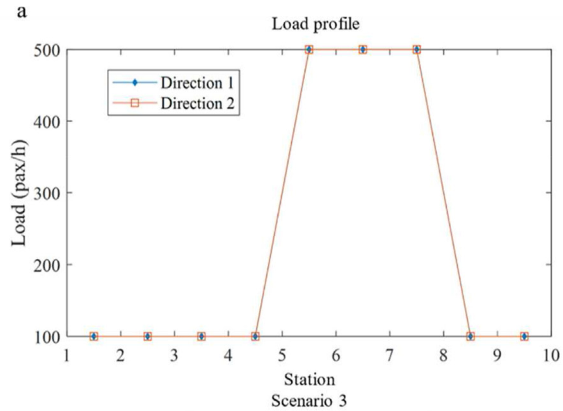
5. Conclusions

This study has addressed the periodical frequency and bus reallocation problem for the BEB system, considering the limited driving mileage and long charging time of BEBs. We leverage the flexibility of short turning lines to deal with the unevenly distributed demand during peak periods. The proposed frequency and bus reallocation problem is formulated as an optimization problem by a microeconomic approach, considering the frequency and charging plan of the short turning line and the whole line as decision variables. Numerical experiments based on a

O\D	1	2	3	4	5	6	7	8	9	10
1		10	0	0	0	0	0	0	0	90
2	10		10	0	0	0	0	0	0	0
3	0	10		10	0	0	0	0	0	0
4	0	0	10		10	0	0	0	0	0
5	0	0	0	10		100	0	310	0	0
6	0	0	0	0	100		100	0	0	0
7	0	0	0	0	0	100		100	0	0
8	0	0	0	0	310	0	100		10	0
9	0	0	0	0	0	0	0	10		10
10	90	0	0	0	0	0	0	0	10	

O\D	1	2	3	4	5	6	7	8	9	10
1		10	0	0	0	0	0	0	0	90
2	10		10	0	0	0	0	0	0	0
3	0	10		10	0	0	0	0	0	0
4	0	0	10		10	0	0	0	0	0
5	0	0	0	10		70	0	10	0	0
6	0	0	0	0	70		70	0	0	0
7	0	0	0	0	0	70		70	0	0
8	0	0	0	0	10	0	70		10	0
9	0	0	0	0	0	0	0	10		10
10	90	0	0	0	0	0	0	0	10	

a



b

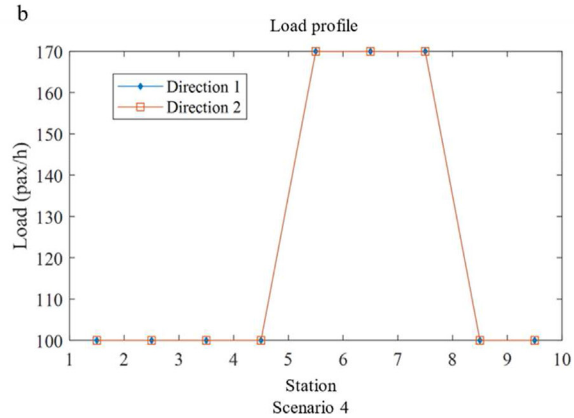


Fig. 4. Origin-destination and load profiles for Scenarios 3 and 4.

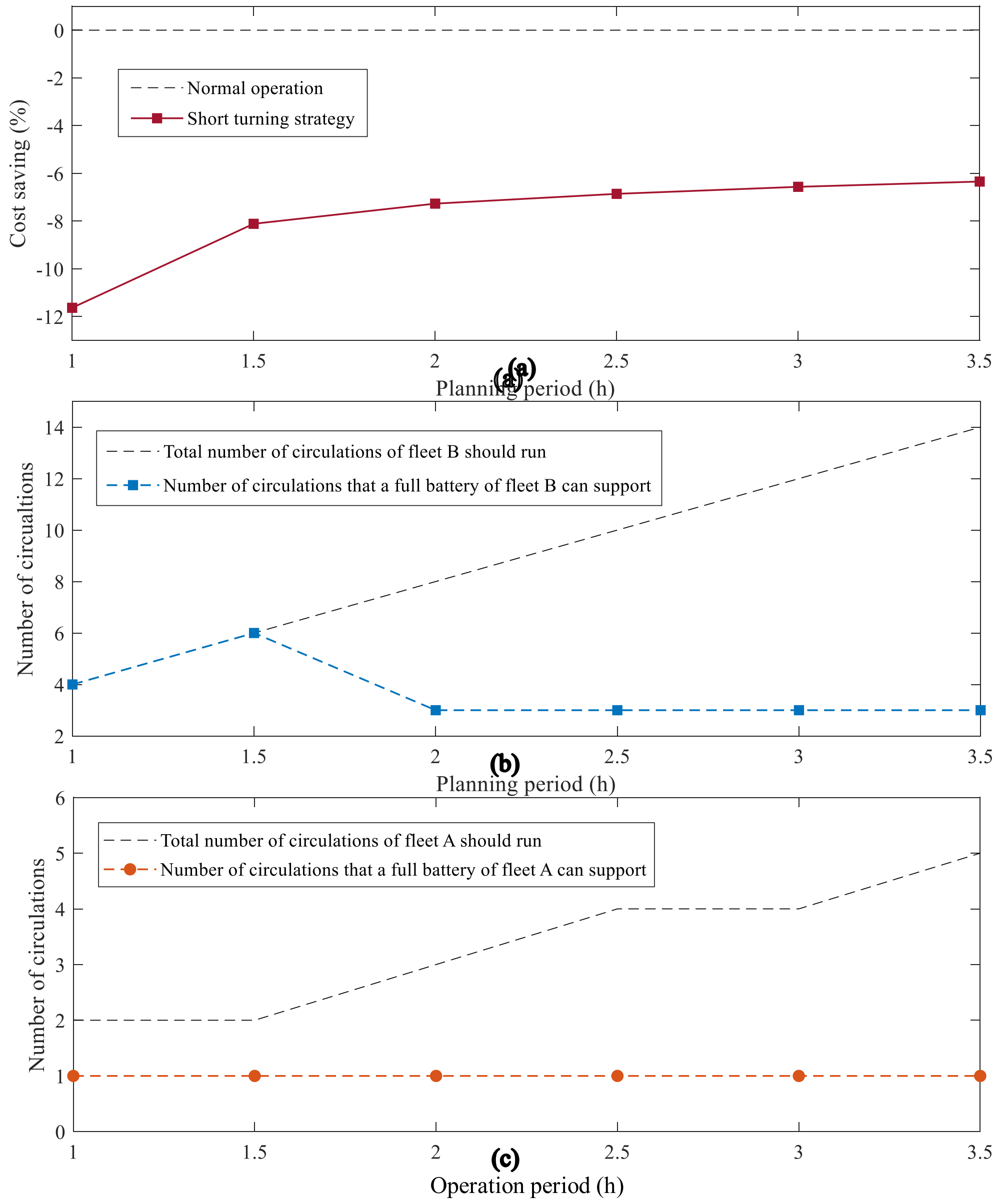


Fig. 5. Sensitivity analysis of operation period T under Scenario 3.

linear bus line with 10 stops are conducted to show the effectiveness of the model and the benefits of the short turning strategy. The key findings of this work are highlighted below:

- Both passengers and operators are likely to benefit from the short turning strategy during the peak period.
- The reduction of passengers' waiting time is attributable to the increase of frequency in high demanded segments, which means the benefits of the strategy will be more apparent if the demand imbalance is more prominent.
- The decrease in operational cost is resulted from the use of a fleet that is smaller in size and passenger capacity. It implies that the benefits of strategy would be underestimated if the operational cost is supposed to be independent of passenger capacity.
- To ensure that the strategy is beneficial to the BEB system, the batteries of BEB fleets serving the short turning line should be large enough to support the BEBs operating continuously during the whole operation period. This is because the large charging trip distance and the long charging time incurred by battery charging during BEB operation will significantly reduce the efficiency of operating a short turning line.
- As the operation period becomes longer, less benefit could be achieved from the strategy. The reason is that for the BEB fleet serving the short turning line, the required battery capacity to support BEBs operating continuously will significantly increase in a long operation period. This may also be attributed to the increased total number of times of charging for the BEB fleet serving the whole line. Viable ways to alleviate this problem are to increase the charging power of the charging station and construct additional charging stations so as to shorten the charging trips of the BEB fleet for the short turning line.

The methodology proposed in this paper provides operators with effective tools for adopting proper electric bus operating strategies according to the present demand distribution. Several directions could be

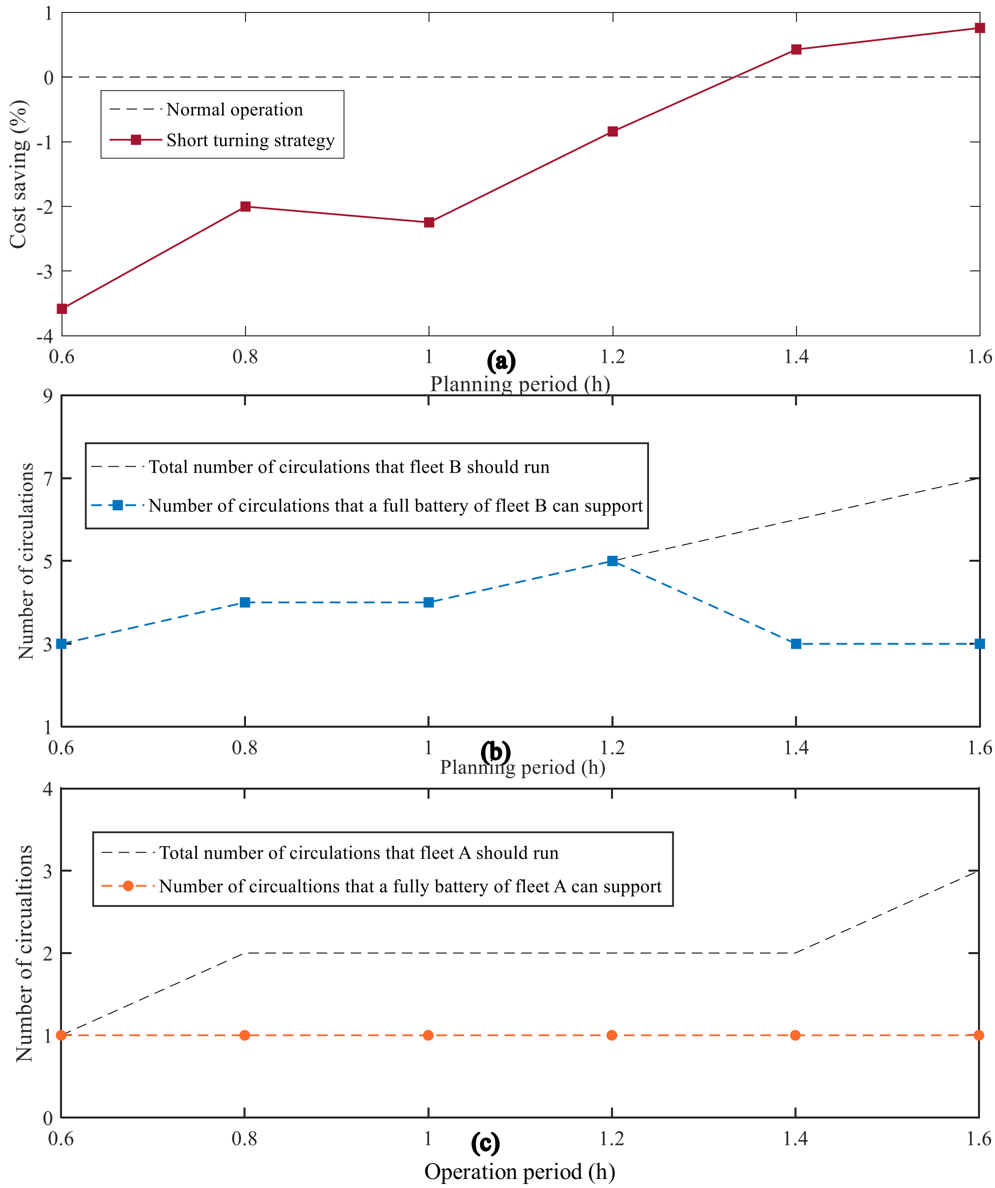


Fig. 6. Sensitivity analysis of operation period T under Scenario 4.

extended for future research. One promising direction is to extend the application of sub-lines, including asymmetric short turning line and interlining line in the BEB system, where deadheading trips are not only applicable for charging the battery but also provide a link between two different highly-demanded segments. Furthermore, since the charging cost is a important component of the operational cost of the BEB fleet and it will be significantly high if BEBs are charged in the power peak hours under time-of-use electricity tariffs. How to create charging schedules with the aim to reduce charging costs considering the whole day's activity of BEBs is a challenging but interesting problem to be investigated in the context of the bus reallocation problem.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Expected demand of the most loaded section for fleet A and fleet B

Since fleet B serves the short turning line, the maximum load of fleet B occurs only in the short turning line. Let $q_{max}^{in,AB}$ denote the expected demand of the most loaded section covered by the short turning line; then we have

$$q_{max}^{in,AB} = \max_{k\in\{s_0,\cdots,s_1-1\}}\left\{\sum_{i=s_0}^{k}\sum_{j=k+1}^{s_1}\lambda_{ij}, \sum_{i=k+1}^{s_1}\sum_{j=s_0}^{k}\lambda_{ij}\right\}$$

$$K_B = \frac{q_{max}^{in,AB}}{\eta(f_A+f_B)}$$

The analysis for fleet A may be more complicated because the maximum load may occur on any section along the transit line. Let q_{max}^{out} denote the expected demand of the most loaded section outside the short turning line; then we have

$$q_{max}^{out} = \max_{k\in\{1,\cdots,s_0-1,s_1,\cdots,N-1\}}\left\{\sum_{i=1}^{k}\sum_{j=k+1}^{N}\lambda_{ij}, \sum_{i=k+1}^{N}\sum_{j=1}^{k}\lambda_{ij}\right\}$$

For the section covered by the short turning line, let $q_k^{A,1}$ and $q_k^{A,2}$ denote the expected demand of passengers whose origin or destination are outside the short turning line between stop k and stop $k+1$ along directions 1 and 2, respectively. It follows that

$$q_k^{A,1} = \sum_{i=1}^{s_0-1}\sum_{j=k+1}^{N}\lambda_{ij} + \sum_{i=s_0}^{k}\sum_{j=s_1+1}^{N}\lambda_{ij},\ \forall k = s_0,\cdots,s_1-1$$

$$q_k^{A,2} = \sum_{i=k+1}^{N}\sum_{j=1}^{s_0-1}\lambda_{ij} + \sum_{i=s_1+1}^{N}\sum_{j=s_0}^{k}\lambda_{ij},\ \forall k = s_0,\cdots,s_1-1$$

Additionally, let $q_k^{AB,1}$ and $q_k^{AB,2}$ denote the expected demand of passengers whose origins or destinations are covered by the short turning line between stop k and stop $k+1$ along directions 1 and 2, respectively. We have

$$q_k^{AB,1} = \sum_{i=s_0}^{k}\sum_{j=k+1}^{s_1}\lambda_{ij},\ \forall k = s_0,\cdots,s_1-1$$

$$q_k^{AB,2} = \sum_{i=k+1}^{s_1}\sum_{j=s_0}^{k}\lambda_{ij},\ \forall k = s_0,\cdots,s_1-1$$

Then the maximum expected load covered by the short turning line for fleet A, namely, $\varphi_{max}^{in,A}$, can be obtained as follows:

$$\varphi_{max}^{in,A}(f_A,f_B) = \max_{k\in\{s_0,\cdots,s_1-1\}}\left\{\frac{q_k^{AB,1}}{f_A+f_B}+\frac{q_k^{A,1}}{f_A}, \frac{q_k^{AB,2}}{f_A+f_B}+\frac{q_k^{A,2}}{f_A}\right\}$$

Therefore, the relationship between passenger capacity in fleet A and the frequencies f_A and f_B can be expressed by $K_A(f_A,f_B,s_0,s_1) = \frac{1}{\eta}max\left\{\varphi_{max}^{in,A}(f_A,f_B), \frac{q_{max}^{out}}{f_A}\right\}$.

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