

1 **Process Control of Charging and Discharging of Magnetically Suspended** 2 **Flywheel Energy Storage System**

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6 **Abstract**

7 Flywheel energy storage system (FESS) is an energy conversion device designed for energy transmission
8 between mechanical energy and electrical energy. There are high requirements on the power capacity, the
9 charging efficiency and the output precision of FESS. Active magnetic bearings are used to suspend the
10 flywheel (FW) rotor of the FESS in air to eliminate friction. A high rotating speed of the flywheel can increase
11 the power capacity but it also increases the disturbance load torque on the FW rotor. An observation control
12 model of load torque is therefore proposed to mitigate the disturbance load torque acting on magnetically
13 suspended FESS (MS-FESS) during the charging process. Moreover, for the discharging process of MS-FESS,
14 a compound control model combing the sliding model control and the extend state observer is proposed to
15 improve the response speed and the output voltage precision. Simulations and experiments are conducted to
16 testify the control performances of proposed control models during the charging and discharging processes of
17 a MS-FESS. The charging efficiency is improved by 17.6% with 46.6% reduction of the output voltage error
18 after using the proposed control models for the charging and discharging process control. The proposed control
19 method has high potential to be applied for process control of charging and discharging of practical MS-FESS.

20 **Keywords**—flywheel energy storage system; charging process; discharging process; observation control model;
21 **compound control.**

23 **1 Introduction**

24 Flywheel energy storage system (FESS) [1-4] is a complicate energy storage and conversion device [5, 6].
25 The FESS could convert electrical energy to mechanical energy by increasing the rotating speed of flywheel
26 (FW) rotor, so the FESS can be regarded as a motor during the charging process. On the other hand, the FESS
27 could release the stored mechanical energy by decelerating the rotating speed and therefore the FESS could be
28 considered as a generator during the discharging process. As an energy storage equipment, it has a series of
29 advantages on long life span, high conversion efficiency, great energy density and tiny environmental impact.
30 Therefore, the FESS has been widely utilized in the uninterruptable power supply system [7, 8], the power
31 transmission system from offshore wind farm to onshore grid system [9, 10], the isolated hybrid system [11-
32 14], the power supply system of satellite [15, 16] and the energy harvesting system of vehicle [17-19].
33

34 In order to maximize the storage capacity of FESS with constant moment of inertia and to reduce the
35 energy loss, magnetic suspension technique is used to levitate the FW rotor to avoid the contact between the
36 FW rotor and the stator. This kind of FESS could be classified as the magnetically suspended flywheel energy
37 storage system (MS-FESS) [20, 21]. The friction between the FW rotor and the stator could be eliminated by
38 levitating the FW rotor to the balanced position in air, and the position of FW rotor could be controllable in five
39 degrees of freedom (DOFs) by regulating the magnetic forces of the magnetic suspension system [22].

40 In the MS-FESS, the control of charging process could affect its conversion efficiency from electrical
41 energy to mechanical energy, and the control of discharging process determine its steady-state precision of
42 output voltage. Therefore, a good control method for the charging and discharging processes of MS-FESS is
43 critical for its enhancement of storage capacity and energy conversion efficiency.

44 A nonlinear control model based on model predictive control [23] was proposed to a FESS in presence of
45 model uncertainties and external disturbances. Simulations were conducted to verify that the power quality of
46 direct current (DC) link voltage was enhanced. A battery/flywheel hybrid energy storage system was used to
47 mitigate load fluctuations in a shipboard microgrid [24]. An optimization approach and a lookup-table-based
48 approach were proposed to outperform the baseline control in terms of power fluctuation mitigation and power
49 loss reduction. Both the advantages and disadvantages of each strategy were analyzed. The results showed that
50 the optimization-based method could achieve a 2% to 5% reduction in power loss in comparison to the same
51 tracking error at nominal and high sea states. A periodic event-triggered control was designed for a FESS matrix
52 farm in the wind power plant [25]. Simulation results indicated that the number of triggers during the power
53 conversion process was greatly reduced and the convergence time is shortened to 0.7s. A robust discharging
54 strategy was proposed to incorporate the speed variation to the DC-link voltage control system [26]. A speed-
55 independent extended state observer was designed to enhance the robustness during the discharging process.
56 Experimental results proved that the control model could accurately track the DC-link voltage from 0V to 300V.
57 A direct voltage control strategy without intermediate current loop was designed for FESS to ensure the fast
58 dynamic performance within its wide operation range [27]. An extended state observer was used to estimate the
59 disturbances acting on the FESS. The experimental results implied that the maximum tracking error was reduced
60 from 7V to 4V, and the recovery time was reduced from 0.25s to 0.1s when the FESS worked at 12000rpm. A
61 three-layer control system including a frequency control model, a mode coordination model and a speed control
62 mode was designed to reduce DC-link voltage variation of FESS in microgrid [28]. The settling time was
63 shortened to 1.6s and maximum overshoot was reduced to 11V. A fuzzy proportional plus derivative control
64 method was proposed for frequency regulation of a 2MW wind-power farm with 400kW flywheel energy
65 storage unit [29]. The simulation results showed that the maximum frequency variation was reduced to 0.04Hz
66 which is better than the traditional proportional plus derivative control method.

67 The control methods of FESS are investigated to improve the charging efficiency and the discharging
 68 precision in those above-mentioned papers, but most of them are designed for the hybrid energy storage system
 69 based on FESS. In addition, the FW rotor used in those hybrid energy storage systems are supported by
 70 mechanical bearings, and the charging and discharging processes of FESS with external disturbances are also
 71 neglected. Therefore, the charging and discharging processes of MS-FESS are worthy of being researched to
 72 improve charging efficiency and discharging precision. This article would be focused on two parts as following,

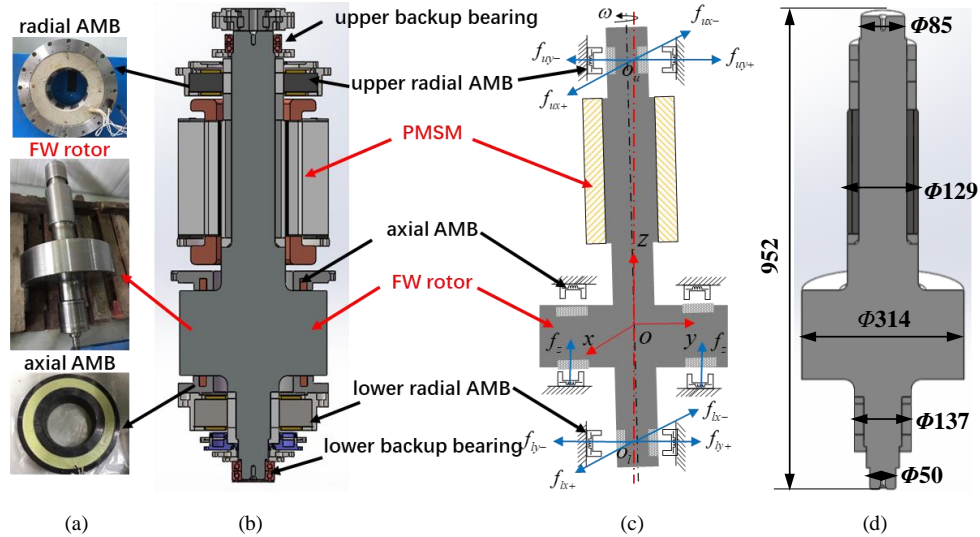
73 1. The charging process of the MS-FESS is investigated. A sliding mode control (SMC) method based on
 74 extended state observation (ESO) model is designed to improve the charging efficiency and robustness of the
 75 MS-FESS. The charging period is reduced from 85s to 70s through enhancing the stability of charging current.

76 2. The discharging process of the MS-FESS is also researched with focus on the process stability and the
 77 precision improvement of the output DC bus voltage of the MS-FESS. The error between the actual DC bus
 78 voltage and reference value is reduced from 3.28V to 1.75V when the load resistance varies.

79 This article is organized as follows. The structure and operational principle of the MS-FESS are introduced
 80 in section 2. A SMC based on the ESO model are designed for the charging and the discharging processes.
 81 Simulation results of the charging and discharging processes of MS-FESS with the ESO model are presented
 82 in section 3. Experimental testing results of the control performance of charging and discharging processes are
 83 presented in section 4. Finally, conclusions are drawn on the charging and discharging control of the MS-FESS.

84 2 The Brief Introduction about MS-FESS

85 2.1 The Mechanical Structure of MS-FESS



86 Fig. 1. The prototype of MS-FESS, (a) the components of MS-FESS, (b) the prototype of MS-FESS, (c) the suspension state of MS-
 87 FESS, (d) mechanical dimensions of flywheel rotor.

88 The prototype of MS-FESS is shown in Fig. 1, and the main components have a magnetic suspension
 89 system and a motor/generator system. As shown in Fig. 1(a) and (b), the magnetic suspension system including

90 two radial active magnetic bearing (AMB) units and an axial AMB unit. The FW rotor with a permanent magnet
 91 synchronous motor (PMSM) is suspended at the equilibrium positions in radial and axial directions. Fig. 1(c)
 92 shows the forces acting on the FW rotor. The upper and lower radial AMB generate magnetic forces to levitate
 93 the FW rotor at the radial balanced position, and the axial AMB generates magnetic forces to suspend the FW
 94 rotor at the axial equilibrium position. Moreover, the upper and lower backup bearings, as the protective devices,
 95 can restrain any excessive displacement runout of FW rotor in case of failure of the magnetic suspension system.
 96 The motor/generator system is consisted of a PMSM and a FW rotor with a large equatorial moment of inertia.
 97 The PMSM turns the rotating FW rotor around the axial principal axis during the charging process.

98 The MS-FESS could convert electrical energy input to mechanical energy by increasing the rotating speed
 99 of FW rotor during the charging process, and the stored energy can be written as

$$E = \frac{1}{2} J_e \omega_r^2 \quad (1)$$

101 where J_e is the moment of inertia of FW rotor around the axial principal axis, and ω_r is the angular velocity of
 102 the FW rotor around the axial principal axis.

103 The mechanical energy of the MS-FESS can be converted back to electrical energy by reducing the rotating
 104 speed during the discharging process, and the released energy could be expressed as

$$\Delta E = \frac{1}{2} J_e (\omega_{ri}^2 - \omega_{rt}^2) \quad (2)$$

106 where ω_{ri} is initial angular velocity of FW rotor, and ω_{rt} is terminal angular velocity of FW rotor.

107 2.2 The Charging Process Control of MS-FESS

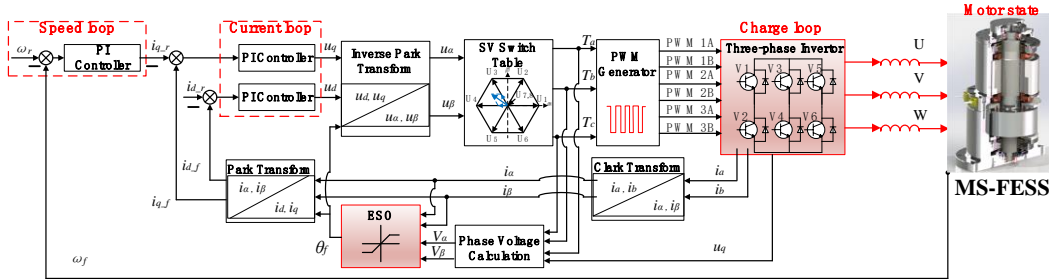


Fig. 2. The control diagram of MS-FESS during the charging process.

110 The MS-FESS could work at two statuses including the charging and discharging processes by sharing the
 111 same three-phase inverter with different control commands. The energy of the MS-FESS is stored as mechanical
 112 energy by accelerating the FW rotor during the charging process at when the MS-FESS is working at the motor
 113 state. During the discharging process, the stored energy of the MS-FESS would be released by decelerating the
 114 rotating speed of the FW rotor, and the MS-FESS is working at the generator state.

115 The control diagram of the MS-FESS during the charging process is shown in Fig. 2, the energy storage is
 116 accomplished by increasing the rotating speed of FW rotor. The control models of the MS-FESS during the
 117 charging process include the speed control model and the current control model. In the speed control, a
 118 proportional integral (PI) feedback control model is used to regulate the q -axis current using the reference

119 rotating speed and the feedback speed. For the current control, a PI feedback control model is used to generate
 120 the q -axis control voltage combining the feedback term and the input term of q -axis current. Moreover, another
 121 PI feedback control is used to realize the closed-loop control of the d -axis current based on the feedback current.
 122 Furthermore, the space vector (SV) switch table would generate the switch signals depending on the α -axis and
 123 β -axis voltages, and the pulse width modulation (PWM) signals are then generated to control the switching
 124 sequence and the duty cycle of three-phase inverter. Finally, the three-phase voltages are output to accelerate
 125 the FW rotor in order to convert the input electrical energy to mechanical energy during the charging process.

126 The d -axis voltage and the q -axis voltage can be respectively expressed as

$$127 \quad \begin{cases} u_d = R_s i_d + L_d \frac{di_d}{dt} - \omega_s L_q i_q \\ u_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_s (L_d i_d + \psi_s) \end{cases} \quad (3)$$

128 where u_d is the d -axis control voltage, and u_q is the q -axis control voltage. R_s is the stator resistance. L_d is the d -
 129 axis equivalent inductance, and L_q is the q -axis equivalent inductance. i_d is the d -axis control current, and i_q is
 130 the q -axis control current. ω_s is the electromagnet angular velocity, and ψ_s is the rotor flux linkage.

131 The electromagnetic torque function of MS-FESS could be written as

$$132 \quad \begin{cases} T_e = \frac{3P_n[\psi_s i_q + (L_d - L_q)i_d i_q]}{2} \\ T_e = T_l + B\omega_r + J_e \frac{d\omega_r}{dt} \end{cases} \quad (4)$$

133 where T_e is the electromagnetic torque. P_n is the number of pole pairs. T_l is the load torque, and B is the damping
 134 coefficient. The relationship between the angular velocity and the electromagnet angular velocity is written as

$$135 \quad \omega_s = P_n \omega_r \quad (5)$$

136 In the charging process of the MS-FESS, the reference value of the d -axis current is defined as $i_{d_r}=0$. the
 137 differential equation of the q -axis control current and the angular velocity could be written as

$$138 \quad \begin{cases} \frac{di_q}{dt} = \frac{-R_s}{L_q} i_q + \frac{1}{L_q} u_q - \frac{\psi_s}{L_q} \omega_s \\ \frac{d\omega_r}{dt} = \frac{-B}{J_e} \omega_r + \frac{1}{J_e} T_e - \frac{1}{J_e} T_l \\ T_e = \frac{3}{2} P_n \psi_s i_q \end{cases} \quad (6)$$

139 When the load torque of the MS-FESS is constant, the space function of the MS-FESS during the charging
 140 process could be expressed as

$$141 \quad \begin{cases} \dot{\mathbf{x}}_c = A_c \mathbf{x}_c + B_c \mathbf{u}_c \\ \mathbf{y}_c = C_c \mathbf{x}_c \end{cases} \quad (7)$$

142 where the space vector $\mathbf{x}_c = [q_r \ \omega_r \ T_l]^T$ (q_r is the angle position).

143 The input vector is $\mathbf{u}_c = i_q$, and the output vector is $\mathbf{y}_c = \omega_r$. The state matrix, the input and output matrices
 144 could be expressed respectively as

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-B}{J_e} & \frac{-1}{J_e} \\ 0 & 0 & 0 \end{bmatrix}; \quad B_c = \begin{bmatrix} 0 \\ \frac{k_t}{J_e} \\ 0 \end{bmatrix}; \quad C_c = [0 \quad 1 \quad 0] \quad (8)$$

The state space function in (7) could be rewritten in augmented form as

$$\begin{cases} \dot{\mathbf{x}}_{c1} = A_{c1}\mathbf{x}_{c1} + B_{c1}\mathbf{u}_c - B_{c2}T_l \\ \mathbf{y}_{c1} = C_{c1}\mathbf{x}_{c1} \end{cases} \quad (9)$$

where $\mathbf{x}_{c1} = [q_r \quad \omega_r]^T$, $A_{c1} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-B}{J_e} \end{bmatrix}$, $B_{c1} = \begin{bmatrix} 0 & \frac{k_t}{J_e} \end{bmatrix}^T$, $B_{c2} = \begin{bmatrix} 0 & \frac{1}{J_e} \end{bmatrix}^T$, and $C_{c1} = [0 \quad 1]$.

The load torque is regarded as a disturbance in the charging process of the MS-FESS. The proposed ESO for the load torque could be written as

$$\dot{\tilde{\mathbf{x}}}_{c1} = A_{c1}\tilde{\mathbf{x}}_{c1} + B_{c1}\mathbf{u}_c - B_{c2}\tilde{T}_l + H_{c1}(C_{c1}\mathbf{x}_{c1} - C_{c1}\tilde{\mathbf{x}}_{c1}) \quad (10)$$

where those terms with the sign ‘~’ represent the observed value of corresponding terms, and $H_{c1}=[h_1 \ h_2]^T$.

The error function between the observer function in (10) and the state space function in (9) could be derived and written as

$$\dot{\mathbf{e}}_{c1} = \dot{\mathbf{x}}_{c1} - \dot{\tilde{\mathbf{x}}}_{c1} = (A_{c1} - H_{c1}C_{c1})\mathbf{e}_{c1} - B_{c2}(T_l - \tilde{T}_l) = A_e\mathbf{e}_{c1} - W \quad (11)$$

where $A_e = (A_{c1} - H_{c1}C_{c1})$, $W = B_{c2}(T_l - \tilde{T}_l)$ and $\mathbf{e}_{c1} = \mathbf{x}_{c1} - \tilde{\mathbf{x}}_{c1}$.

In order to realize an accurate observation for the load torque, the error function in (11) must be a convergence equation. According to Popov hyper-stability theory, the following two conditions must be satisfied to guarantee the convergence of the error function.

(i) $H(s)=(sI-A_e)^{-1}$ is a positive real matrix.

(ii) with $\mathbf{e}_y = \mathbf{y} - \tilde{\mathbf{y}}$, there is $\int_0^t \mathbf{e}_y^T W dt > -\gamma^2$.

Therefore, in order to ensure the error function can meet the two conditions above, the observation function of the load torque is designed and written as

$$\tilde{T}_l = \left(K_p + \frac{K_I}{s} \right) \mathbf{e}^T B_{c2} \quad (12)$$

where K_p is gain coefficient of estimating load torque, and K_I is integral coefficient of estimating load torque.

By expanding (11) into two equations, we have

$$s\tilde{q}_r = \tilde{\omega}_r + h_1(q_r - \tilde{q}_r) \quad (13)$$

$$s\tilde{\omega}_r = -\frac{B}{J_e}\tilde{\omega}_r - \frac{1}{J_e}\tilde{T}_l + \frac{1}{J_e}T_e + h_2(q_r - \tilde{q}_r) \quad (14)$$

Considering that $s q_r = \omega_r$, the equation (13) could be rewritten as

$$(q_r - \tilde{q}_r) = \frac{1}{s+h_1}(\omega_r - \tilde{\omega}_r) \quad (15)$$

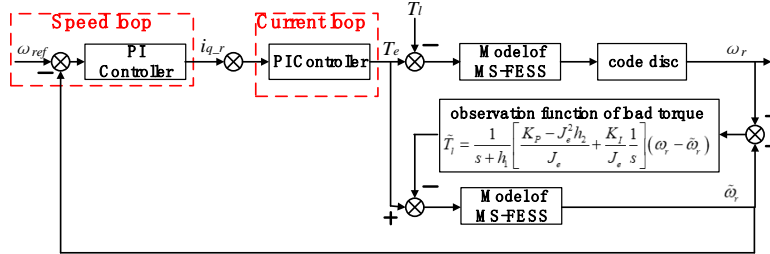
Substituting (12) and (15) into (14), the observation function for rotating speed could be obtained and written as

$$\tilde{\omega}_r = \frac{1}{J_e+B}T_e - \frac{1}{J_e+B} \frac{1}{s+h_1} \left(\frac{K_p - J_e^2 h_2}{J_e} + \frac{K_I}{J_e} \frac{1}{s} \right) (\omega_r - \tilde{\omega}_r) \quad (16)$$

174 Compare (16) to (4), the observation function of load torque could be written as

$$175 \quad \tilde{T}_l = \frac{1}{s+h_1} \left(\frac{K_P - J_e^2 h_2}{J_e} + \frac{K_I}{J_e s} \right) (\omega_r - \tilde{\omega}_r) \quad (17)$$

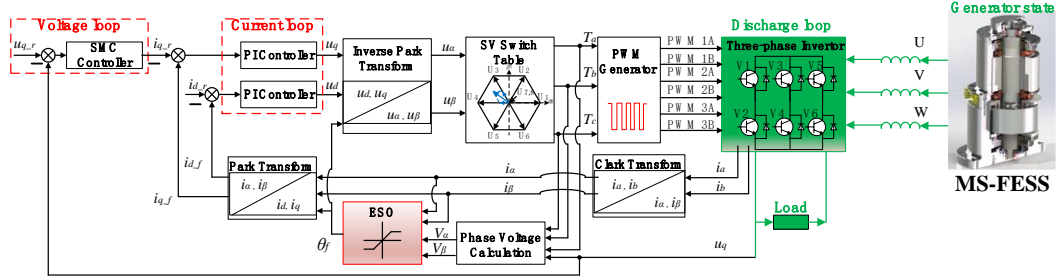
176 Therefore, the load torque caused by the speed error of FW rotor could be obtained.



177
178 Fig. 3. The control model of MS-FESS during the charging process.

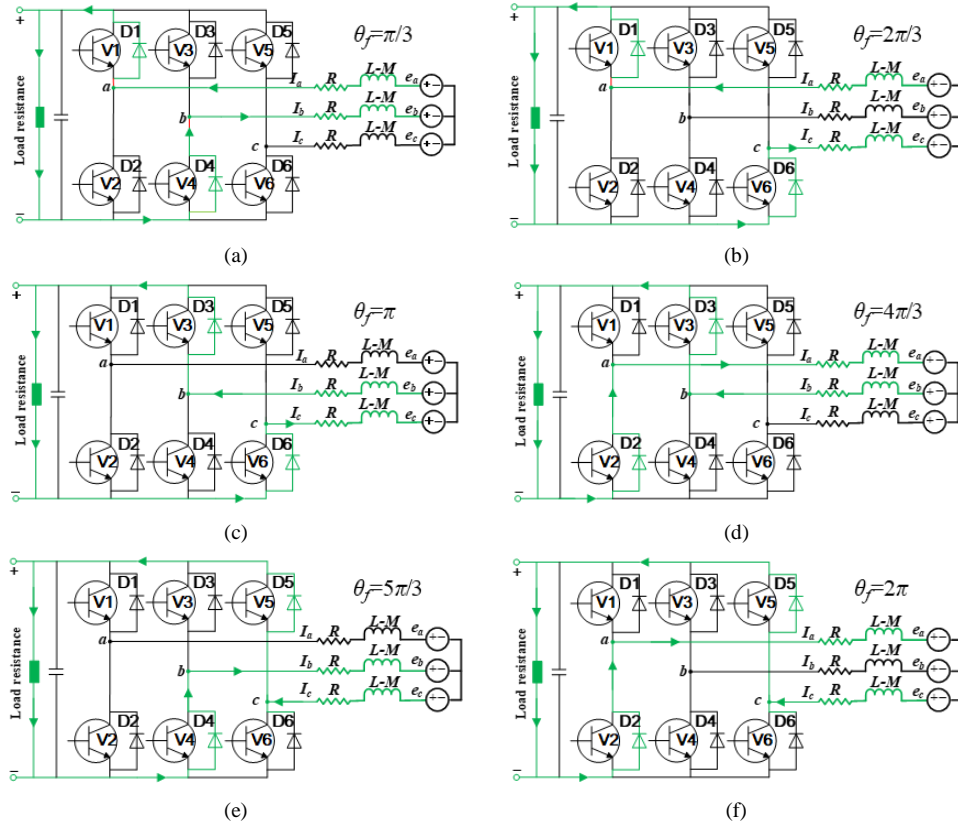
179 According to the above observation function design of load torque, the equivalent control model of the
180 MS-FESS during the charging process is shown in Fig. 3. Given that the reference d -axis input current $i_{d,r}=0$,
181 only the speed control and the q -axis current control are used to regulate the drive torque of MS-FESS, and
182 ESO model of load torque is proposed to mitigate the disturbance due to load torque.

183 2.3 The Discharging Process Control of MS-FESS



184
185 Fig. 4. The control diagram of MS-FESS during the discharging process.

186 When the MS-FESS is switched from the charging process to the discharging process, the reference input
187 would be switched from the rotating speed to the reference DC bus voltage. As shown in Fig. 4, the stored
188 mechanical energy could be converted to the electrical energy by decelerating the rotating speed of FW rotor.
189 The back electromotive force (EMF) in three-phase windings of the MS-FESS is converted to the reference DC
190 bus voltage through the three-phase inverter. In the discharging process of MS-FESS, a sliding mode controller
191 (SMC) is designed for the voltage control loop with the output DC bus voltage. Furthermore, a PI controller is
192 designed for the control loop of q -axis current and d -axis current. The PMW signals are generated to regulate
193 the duty cycle and the on-off sequence of the discharging loop. Therefore, the discharging process of the MS-
194 FESS would be realized.



195 Fig. 5. The rectification models of the MS-FESS during the discharging process, (a) the electrical angle of MS-FESS is $\pi/3$, (b) the electrical
 196 angle of MS-FESS is $2\pi/3$, (c) the electrical angle of MS-FESS is π , (d) the electrical angle of MS-FESS is $4\pi/3$, (e) the electrical angle of
 197 MS-FESS is $5\pi/3$, (f) the electrical angle of MS-FESS is 2π .

198 For the discharging process of the MS-FESS, there are two operational modes including the passive
 199 discharging mode and the active discharging mode. For the passive discharging mode, the back EMF of the
 200 MS-FESS could be passively converted to the DC bus voltage which varies with the discharging process. The
 201 detailed discharging process with six recertification steps is shown in Fig. 5, and the current pathway is marked
 202 by the green line. During the passive discharging mode, the insulated gate bipolar transistors (IGBTs) of the
 203 three-phase inverter are switched off. The current rectification is accomplished through regulating the switch
 204 sequence of diodes. For instance, the a -node voltage is highest in the rectification process as shown in Fig. 5(a)
 205 with the electrical angle at $\pi/3$, and the b -node voltage is lowest during the first stage of passive discharging
 206 process with diode D1 and diode D4 switched on. Moreover, the current rectifications from Fig. 5(a) to (f)
 207 would be sequentially conducted when the electrical angle is increased from $2\pi/3$ to 2π in steps as shown in
 208 Fig. 5. Therefore, the current pathway would vary with the electrical angle during the rectification process.

209 For the active control of discharging process, the DC bus voltage acting on the load resistance is measured
 210 by the voltage sensors, and then fed back to the voltage control loop for the determination of the q -axis control
 211 current. Moreover, the designed ESO model could estimate the electrical angle based on phase voltages and

212 phase currents. Through the Clark transform and the Park transform, the phase currents i_a and i_b could be
 213 transferred to the d -axis feedback current i_{d_f} and the q -axis feedback current i_{q_f} . Through the PI controller of
 214 current control loop, the d -axis voltage u_d and the q -axis voltage u_q could be determined. The d -axis control
 215 voltage u_d and the q -axis control voltage u_q are then transferred to the α -axis control voltage u_α and the β -axis
 216 control voltage u_β through the inverse Park transform. Therefore, based on the space vectors T_a , T_b and T_c output
 217 from the switch table, the equivalent PMW sequence signals could be generated to control the switch sequence
 218 of transistors for the active discharging process.

219 In the feedback loop of discharging process, the feedback terms of d -axis and q -axis currents are

$$220 \begin{cases} i_{d_f} = i_\alpha \cdot \cos\theta_f + i_\beta \cdot \sin\theta_f \\ i_{q_f} = -i_\alpha \cdot \sin\theta_f + i_\beta \cdot \cos\theta_f \end{cases} \quad (14)$$

221 where θ_f is the electrical angle estimated by the SMC model.

222 To design the SMC model of voltage control loop, the equivalent α -axis and β -axis back EMFs of MS-
 223 FESS are defined as

$$224 \begin{cases} V_\alpha = -k_e \omega_r \sin\theta_f \\ V_\beta = -k_e \omega_r \cos\theta_f \end{cases} \quad (15)$$

225 where k_e is the back EMF coefficient. The differential functions of equivalent α -axis control current i_α and
 226 equivalent β -axis control current i_β are written as

$$227 \begin{cases} \frac{di_\alpha}{dt} = -\frac{R_s \cdot i_\alpha}{L_s} + \frac{V_\alpha}{L_s} + \frac{u_\alpha}{L_s} \\ \frac{di_\beta}{dt} = -\frac{R_s \cdot i_\beta}{L_s} + \frac{V_\beta}{L_s} + \frac{u_\beta}{L_s} \end{cases} \quad (16)$$

228 When the MS-FESS works at transition process at the rated rotating speed, there is $\frac{d\omega_r}{dt} = 0$.

229 The derivative functions of back EMF are

$$230 \begin{cases} \frac{dV_\alpha}{dt} = -\omega_r \cdot V_\beta \\ \frac{dV_\beta}{dt} = -\omega_r \cdot V_\alpha \end{cases} \quad (17)$$

231 So the differential functions of equivalent α -axis control current i_α and β -axis control current i_β in (16) can
 232 be rewritten as

$$233 \begin{cases} \frac{di_\alpha^0}{dt} = -\frac{R \cdot i_\alpha^0}{L_s} + \frac{u_\alpha - K_f z_{e\alpha} - z_\alpha}{L_s} \\ \frac{di_\beta^0}{dt} = -\frac{R \cdot i_\beta^0}{L_s} + \frac{u_\beta - K_f z_{e\beta} - z_\beta}{L_s} \end{cases} \quad (18)$$

234 where K_f (greater than 0) is the feedback parameter, z_α and z_β are defined as the switch functions of SMC model.

235 The switch functions of SMC model are expressed as

$$236 \begin{cases} S_\alpha = i_\alpha^0 - i_\alpha \\ S_\beta = i_\beta^0 - i_\beta \end{cases} \quad (19)$$

237 The switch functions of SMC model could be written as

$$\begin{bmatrix} z_\alpha \\ z_\beta \end{bmatrix} = k \cdot \text{sat}(S_{\alpha\beta}) = \begin{cases} k, & S_{\alpha\beta} > \Delta \\ \frac{kS}{\Delta}, & -\Delta < S_{\alpha\beta} < \Delta \\ -k, & S_{\alpha\beta} < -\Delta \end{cases} \quad (20)$$

239 where k is the control coefficient of the switch function.

240 Based on the Lyapunov stability, the switch function should satisfy the following condition.

$$241 \quad \dot{e}_s = \dot{S}_{\alpha\beta} S_{\alpha\beta} \Rightarrow k > \max(|V_\alpha| \quad |V_\beta|) \quad (21)$$

242 The control functions of SMC model could be designed as following

$$243 \quad \begin{cases} z_{e\alpha} = \frac{z_\alpha \cdot \omega_c}{(s + \omega_c)} \\ z_{e\beta} = \frac{z_\beta \cdot \omega_c}{(s + \omega_c)} \end{cases} \quad (22)$$

244 where ω_c is the cutoff frequency of the low-pass filter.

245 The derivative equations of switch function are

$$246 \quad \begin{cases} \frac{dS_\alpha}{dt} = -\frac{R \cdot S_\alpha}{L_s} + \frac{u_\alpha - K_f z_{e\alpha} - z_\alpha}{L_s} \\ \frac{dS_\beta}{dt} = -\frac{R \cdot S_\beta}{L_s} + \frac{u_\beta - K_f z_{e\beta} - z_\beta}{L_s} \end{cases} \quad (23)$$

247 For the stable SMC model of MS-FESS, the state lies on the sliding surface, so we get

$$248 \quad \begin{cases} S_\alpha = i_\alpha^0 - i_\alpha = 0 \\ S_\beta = i_\beta^0 - i_\beta = 0 \end{cases} \quad (24)$$

249 Furthermore, we get

$$250 \quad \begin{cases} e_\alpha = K_f z_{e\alpha} + z_\alpha \\ e_\beta = K_f z_{e\beta} + z_\beta \end{cases} \quad (25)$$

251 Finally, the electrical angle of FW rotor could be obtained and written as

$$252 \quad \theta_f = -\tan^{-1} \left(\frac{e_\alpha}{e_\beta} \right) = -\tan^{-1} \left(\frac{K_f z_{e\alpha} + z_\alpha}{K_f z_{e\beta} + z_\beta} \right) \quad (26)$$

253 For the voltage control loop of charging process, the error function between reference voltage u_{q_r} and
254 output voltage u_q could be expressed as

$$255 \quad e_{q_u} = u_{q_r} - u_q \quad (27)$$

256 For the current control loop of charging process, the q -axis control current with PI control model could be
257 obtained as following

$$258 \quad i_{q_r} = e_{q_u} \cdot \left(K_{qu_p} + K_{qu_i} \frac{1}{s} \right) \quad (28)$$

259 The error function between q -axis control current and q -axis feedback current during charging process is

$$260 \quad e_{q_i} = i_{q_r} - i_{q_f} \quad (29)$$

261 The q -axis voltage u_q based on the PI control model could be expressed as

$$262 \quad u_q = e_{q_i} \cdot \left(K_{qi_p} + K_{qi_i} \frac{1}{s} \right) \quad (30)$$

263 The d -axis voltage u_d based on the PI control model is written as

264
$$u_d = e_{d_i} \cdot \left(K_{di_p} + K_{di_i} \frac{1}{s} \right); \text{ s. t. } e_{d_i} = i_{d_r} - e_{d_f} \quad (31)$$

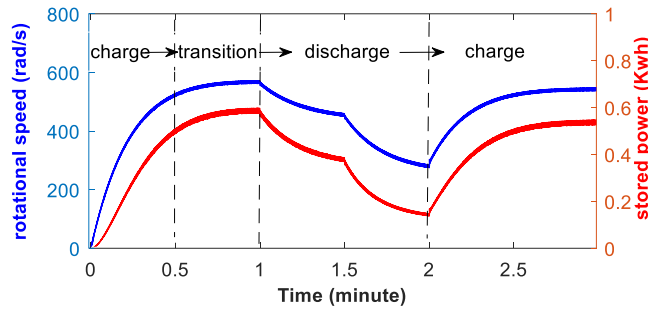
265 In the practical control during the active discharging process of MS-FESS, the reference d -axis control
 266 current is defined as $i_{d_r}=0$. Through on the inverse Park transform about the d -axis and q -axis control voltages,
 267 the α -axis and β -axis voltages during the discharging process can be written as

268
$$\begin{cases} u_\alpha = u_d \cdot \cos\theta_f - u_q \cdot \sin\theta_f \\ u_\beta = -u_d \cdot \sin\theta_f + u_q \cdot \cos\theta_f \end{cases} \quad (32)$$

269 Therefore, the active charging process of the MS-FESS would be realized.

270 3 Simulation about Charging and Discharging Process of MS-FESS

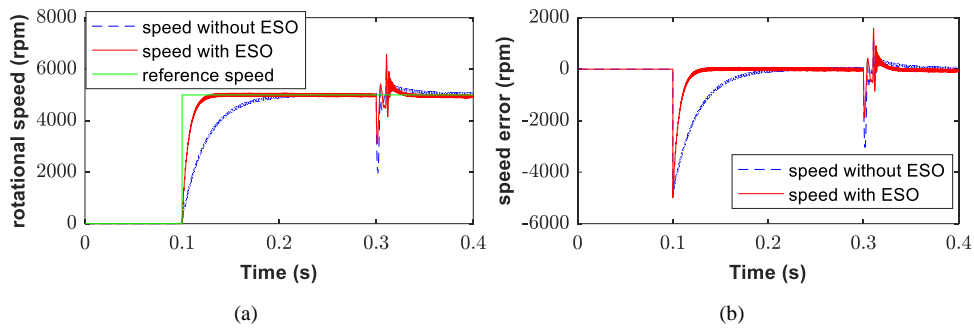
271 3.1 The Charging and Discharging Power Curve of MS-FESS



272
 273 Fig. 6. The charging and discharging power curve of MS-FESS with the rotating speed.

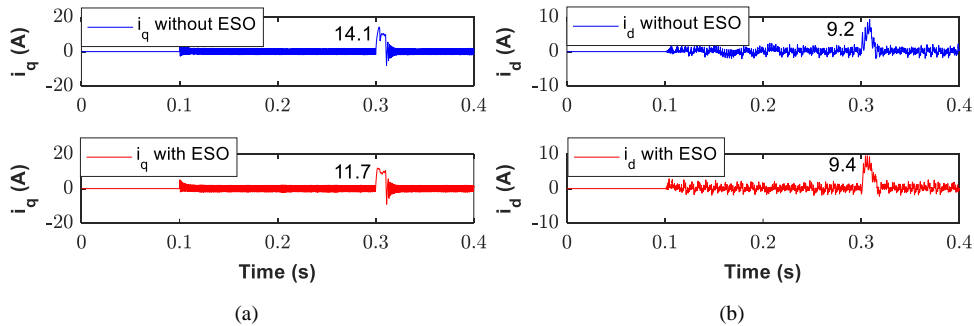
274 The charging and discharging processes of MS-FESS are simulated to compare the control performances
 275 of different control models, and the relationship between the stored energy and the rotating speed during the
 276 charging process and discharging process are illustrated in Fig. 6. The stored energy is improved with the
 277 increase of rotating speed of FW rotor during the charging process. The operational status of the MS-FESS
 278 would be switched from the charging process to the transition process to maintain the rated power storage 0.5
 279 kW·hr by keeping the rotating speed of FW rotor constant. In the discharging process, according to the reference
 280 DC bus voltage, the mechanical energy of the MS-FESS is converted to electrical energy through decelerating
 281 the rotating speed of FW rotor. When the discharging process of MS-FESS is over, the stored energy would be
 282 recharged by increasing the rotating speed of FW rotor again, and the operational mode of the MS-FESS is
 283 switched to charging mode again.

284 **3.2 The Charging Process of MS-FESS Using ESO Model**



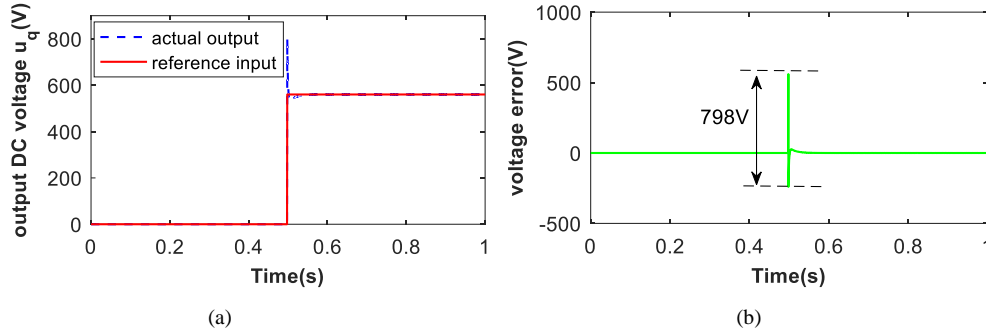
285 Fig. 7. The speed curve of FW rotor during the charging process of MS-FESS, (a) the speed curve of FW rotor, (b) the speed error
 286 between the actual speed curve and the reference speed curve.

287 The speed curves and the control currents of the MS-FESS are recorded during the charging process in the
 288 simulation. The error between the actual speed and the reference speed is analyzed to verify the effectiveness
 289 of the ESO model used in the charging process. The speed curves of the MS-FESS during the charging process
 290 are plotted in Fig. 7 when the reference rotating speed is 5000 rpm. In Fig. 7(a), the actual speed of FW rotor
 291 could track the reference speed with a short time-delay. For the speed curve of the MS-FESS without using the
 292 ESO model, the time-delay is about 0.1s. However, the time delay is reduced to 0.02s after applying the ESO
 293 model. An impulse disturbance is imposed to the speed when the simulation time reaches to 0.3s. The speed
 294 error response of the FW rotor is shown in Fig. 7(b). The maximum speed error without using the ESO model
 295 is about 3000rpm at the response point, and it is reduced to 2000rpm after the ESO model is applied.
 296 Simultaneously, the control currents i_q and i_d of the MS-FESS during the charging process are recorded and
 297 they are plotted in Fig. 8. For the q -axis control current i_q as shown by the blue line in Fig. 8(a), the maximum
 298 variation of the q -axis current i_q reaches to 14.1A without the ESO model, and it is reduced to 11.7A after
 299 applying the ESO model which is about 17% difference between the two cases. Fig. 8(b) shows the curves of
 300 the d -axis current i_d , the obvious variation of the d -axis current i_d occurs when the speed curve of MS-FESS
 301 varied from the rated value, but the deflection magnitudes of d -axis current i_d without and with the ESO model
 302 are quite close.



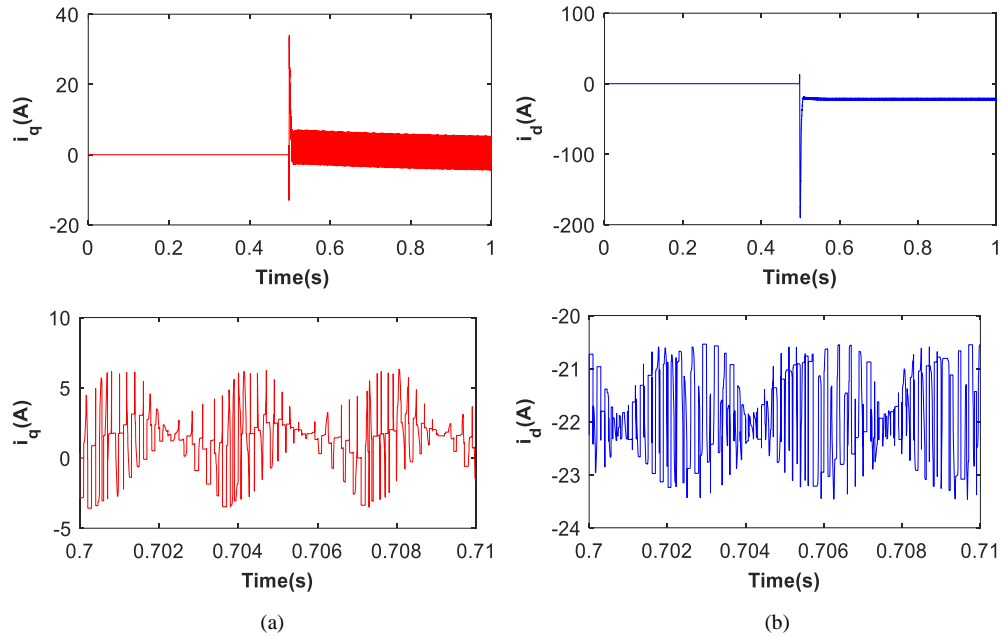
303 Fig. 8. The control current curves during the charging process of MS-FESS, (a) the q -axis current i_q curve of MS-FESS, (b) the d -axis
 304 current i_d curve of MS-FESS.

305 3.3 The Discharging Process of MS-FESS Using SMC Model



306 Fig. 9. (a) The DC bus voltage curve during the discharging process of MS-FESS, (b) the voltage error between the actual output DC bus
 307 voltage and the reference input DC bus voltage during the discharging process of MS-FESS.

308 The discharging process of MS-FESS is also investigated by simulation. The DC bus voltage and the
 309 control currents of MS-FESS are compared and analyzed. Firstly, the DC bus voltages during the discharging
 310 process are recorded and plotted in Fig. 9(a) with the reference DC bus voltage sets at 560V. There is a short
 311 time-delay between the reference voltage and the actual output voltage. The DC bus voltage is hold at the stable
 312 amplitude 560V during the discharging process. As shown in Fig. 9(b), the maximum error between the
 313 reference input DC bus voltage and the actual output DC bus voltage is about 798V due to the time delay of the
 314 speed control loop and the current control loop. The d -axis and q -axis current curves of the ME-FESS during
 315 the discharging process are recorded and plotted in Fig. 10 to verify the tracking performance of the SMC model.
 316 As shown in Fig. 10(a), although the q -axis current i_q has an obvious oscillation at the initial discharging moment,
 317 the q -axis current i_q of the MS-FESS varies within the stable range [-5A 5A] during the discharging process.
 318 On the other hand, the d -axis current i_d during the discharging process jumps to a negative value -22A when
 319 the MS-FESS is switched to the discharging process as shown in Fig. 10(b). The MS-FESS is then working at
 320 the field-weakening operation process. The d -axis current i_d oscillates stably with a small amplitude during the
 321 discharging process of the MS-FESS.



322 Fig. 10. The d -axis and q -axis current curves during the discharging process of MS-FESS, (a) the q -axis current curve during the discharging
 323 process of MS-FESS, (b) the d -axis current curve during the discharging process of MS-FESS.

324 Above all, for the charging process of the MS-FESS, the designed ESO model could accurately estimate
 325 the disturbance acting on the speed regulation process, and the variations of the control current and the rotating
 326 speed of the FW rotor could be effectively suppressed. For the discharging process of the MS-FESS, the control
 327 model could exactly track the reference input DC bus voltage, and the control currents of the ME-FESS are
 328 maintained within stable ranges.

329 4 Charging and Discharging Experiment of MS-FESS

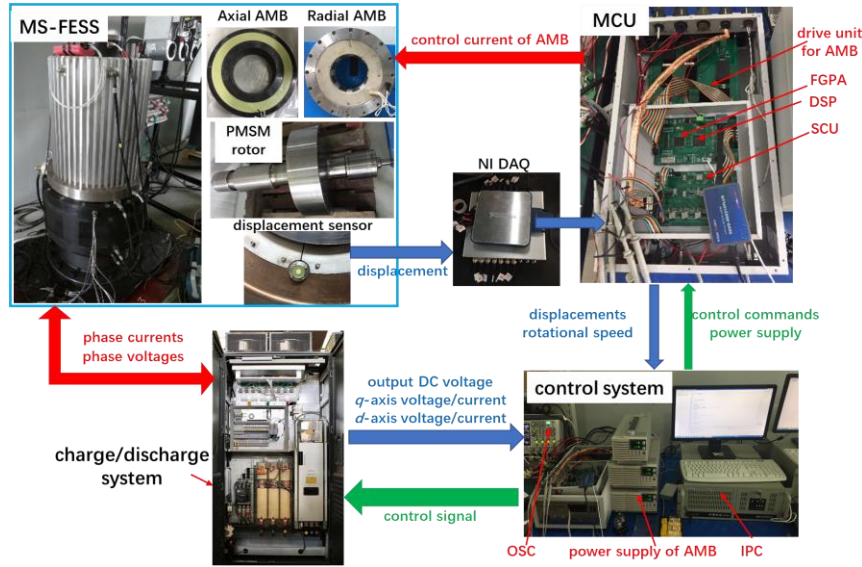
330 4.1 Experimental Setup of MS-FESS

331 The experimental setup of MS-FESS is shown in Fig. 11. The major components include a PMSM rotor
 332 system, a magnetic suspension system, a charging/discharging system, a control system and a measurement
 333 system. The PMSM could govern the rotating speed of FW rotor to realize operational switching of the charging
 334 and the discharging processes. A pump is used to provide vacuum inside to mitigate the wind resistance at high
 335 rotating speed. The magnetic suspension system has two couples of radial AMB units at upper end and lower
 336 end are used to control the radial translations and tilting of FW rotor. The axial translation of FW rotor is
 337 controlled by two couples of axial AMB units at upper side and lower side of FW rotor's disc, so the friction
 338 between the FW rotor and the stator could be avoided. In the charging and discharging system of the MS-FESS,
 339 the three-phase inverting and rectifying system is composed of three couples of IGBT units, three couples of
 340 rectifier diodes and a voltage conversion module, and the charging/discharging process of the MS-FESS could
 341 be realized by the inverting/rectifying system based on the proposed control methods as described before. The
 342 control system is based on a DSP+FPGA control chip for signal processing and code programming. In addition,

343 a data acquisition (DAQ) module and the displacement sensors form the measurement system to collect the
 344 current and voltage signals of the MS-FESS and the dynamic displacements of FW rotor during the charging
 345 and discharging processes. The industrial PC (IPC) and the oscilloscope (OSC) monitor the system status of the
 346 MS-FESS. The detailed system parameters of the MS-FESS are listed in

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350 TABLE. I, and the control parameters of the MS-FESS during the charging and discharging processes are
 351 listed in TABLE II.



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Fig. 11. The whole experimental setup of MS-FESS.

TABLE. I. The system parameters of MS-FESS.

Setup	Attributes
PMSM/FW rotor system	mass $m=150\text{Kg}$ torque coefficient $k_t=0.54$ back-EMF coefficient $k_e=0.065$ rotor flux linkage $\psi_s=0.18\text{Wb}$ stator resistance $R=0.24\Omega$ stator flux inductance $L_d=L_q=0.08\text{mH}$ equatorial moment of inertia $J_e=1.459\text{Kg}\cdot\text{m}^2$ rotating speed $\omega=5000\text{rpm}$ pole pairs $p_p=2$ material of FW rotor 35CrMnSiA
Magnetic suspension system	radial displacement stiffness -2800N/mm axial displacement stiffness -1700N/mm radial current stiffness 620N/A axial current stiffness 470N/A power consumption at steady-state 147W

	power consumption during suspension 188W
Charging/Discharging system	Self-designed system IGBT: CM35MXA-24S Diode: RM600DY-34S
Control system	DSP: TMS320F28335 FPGA: Altera EPF10K30RC208
Measurement system	power supply system: Tektronix Keithley sensitivity of displacement sensor 3.3V/mm DAQ Board: NI PCI 6355 Oscilloscope: Keysight 2000 X-Series

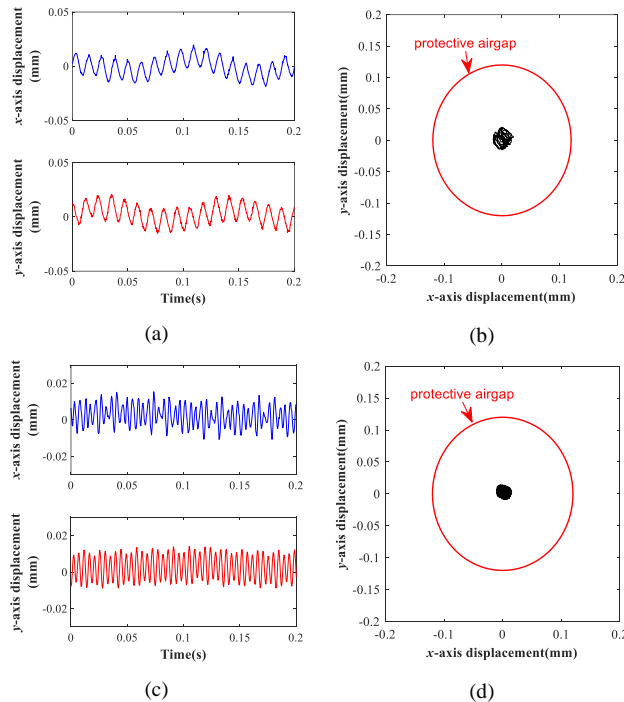
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TABLE. II. The control parameters of MS-FESS during charging and discharging processes.

Symbol	Quantity	Value
h_1	parameter of ESO	0
h_2	parameter of ESO	1.5×10^3
K_p	gain coefficient of ESO	1.1
K_I	integral coefficient of ESO	0.02
k	control parameter of SMC model	65
K_f	feedback coefficient of SMC model	1
ω_c	cutoff frequency of low-pass filter	1000Hz
$K_{qu,p}$	proportional coefficient of q -axis voltage	0.8
$K_{qu,i}$	integral coefficient of q -axis voltage	22
$K_{qi,p}$	proportional coefficient of d -axis voltage	9.6
$K_{qi,i}$	integral coefficient of d -axis voltage	24

361 4.2 The Control Performance of Magnetic Suspension System



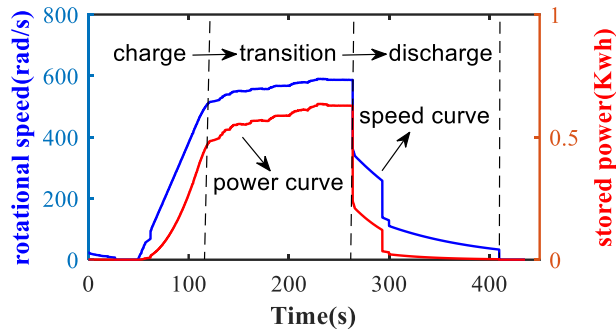
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Fig. 12. The suspension performances of magnetic suspension system, (a) dynamic displacements of flywheel rotor at 5000rpm, (b) axis orbit of flywheel rotor at 5000rpm, (c) dynamic displacements of flywheel rotor at 12000rpm, (d) axis orbit of flywheel rotor at 12000rpm.

364 The magnetic suspension system is the critical unit of MS-FESS, the suspension performance is tested first.
 365 When the rotational speed of flywheel rotor is 5000rpm, the dynamic displacements of flywheel in x and y axis
 366 are shown in Fig. 12(a), the max displacement is 0.02mm which is 16.7% of the 0.12mm protective airgap. The
 367 axis orbit of flywheel rotor at 5000rpm is plotted in Fig. 12(b), and it shows that the flywheel rotor can be
 368 stably suspended within the protective airgap. Moreover, when the rotational speed of flywheel rotor is
 369 12000rpm, the dynamic displacements in x and y axes are illustrated in Fig. 12(c). The max displacement is
 370 0.01mm which is only 8.4% of the protective airgap. The axis orbit of flywheel rotor is plotted in Fig. 12(d).
 371 It shows that the flywheel rotor is stably suspended within the protective airgap. Moreover, the power of
 372 magnetic suspension system during the suspension process is 187.54W, and the power at steady suspension
 373 state is 146.88W.

374 4.3 The Energy Storage Curves of MS-FESS During Charging and Discharging Processes



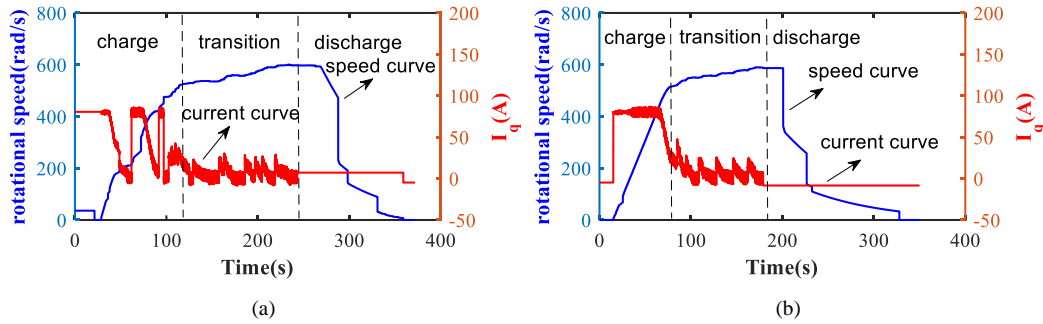
375 Fig. 13. The relationship between the stored energy and the rotating speed during the charging and discharging processes of MS-FESS.
 376

377 In this part of experiment, the power storage curves of the MS-FESS during the charging and discharging
 378 processes are recorded and plotted in Fig. 13. The rotating speed of the MS-FESS is shown by the blue line
 379 while the stored power of the MS-FESS during the charging and discharging processes is displayed by the red
 380 line. The working process of the MS-FESS could be divided into three processes including the charging process,
 381 the transition process and the discharging process. When the MS-FESS is working in the charging mode, the
 382 stored energy raises with the increase of rotating speed, and its rated value is 0.5kW·h at 5000rpm. Furthermore,
 383 the operation of the MS-FESS would be switched from the charging mode to the transition or the holding mode
 384 when the rotating speed kept at the rated value. During the discharging process of the MS-FESS, the stored
 385 energy would rapidly fall to the set point by decelerating the rotating speed, so the stored energy can be released
 386 as electrical energy. Therefore, the fast switches among different operational modes could be realized, and the
 387 charging and discharging process of the MS-FESS could be accomplished by storing of the mechanical energy
 388 and releasing to electrical energy.

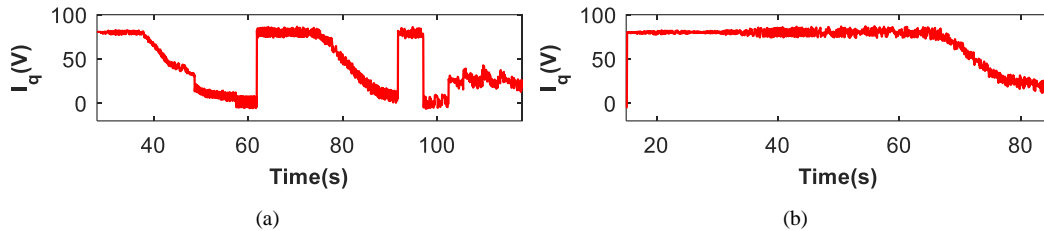
389 4.4 The Charging Control Process of MS-FESS Using ESO Model

390 The charging performance of the MS-FESS is tested on the rotor acceleration and the control current
 391 variations during the charging process. The charging period and the stability of control current are shown to be

392 the critical parameters of the charging process. The control currents of the MS-FESS during the charging and
 393 discharging processes with and without the proposed ESO control model are plotted in Fig. 14(a) and (b),
 394 respectively. In both cases, the q -axis control current i_q of the MS-FESS is regulated to 80A to accelerate the
 395 rotating speed of FW rotor during the charging process to convert the electrical energy to mechanical energy.
 396 The control current shows much bigger fluctuations in Fig. 14(a). The control currents are varied in a stable
 397 range in both cases after the rotating speed of FW rotor reaches the rated value and the control mode is switched
 398 to maintain the speed of FW rotor in the transition process.



399 Fig. 14. The control currents of MS-FESS during the charging and discharging process, (a) the control currents of the MS-FESS without
 400 the ESO model, (b) the control currents of the MS-FESS with the ESO model.



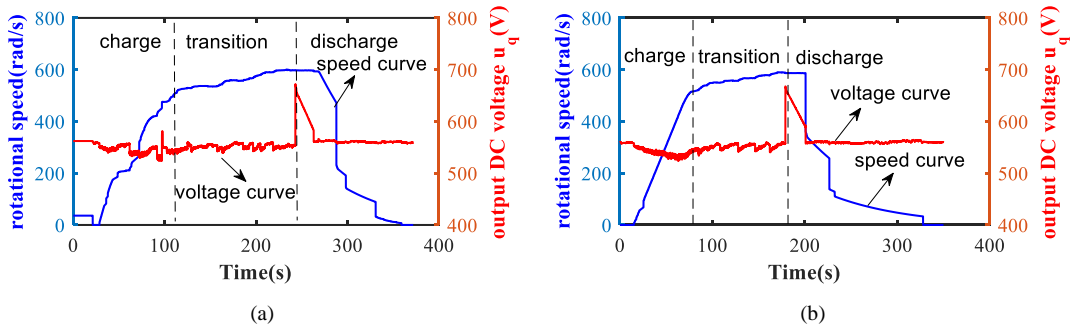
401 Fig. 15. The control current curve during the charging process of the MS-FESS, (a) the control current curve of the MS-FESS without the
 402 ESO model, (b) the control current curve of the MS-FESS with the ESO model.

403 The q -axis control currents of the MS-FESS during the charging process are shown in Fig. 15 to compare
 404 the charging performance of different control models. For the q -axis current curve without the ESO control
 405 model in Fig. 15(a), there are large fluctuations of the control current about the reference current 80A during
 406 the charging process, and therefore the acceleration of the FW rotor during the charging process is affected and
 407 it takes a longer acceleration period during charging as shown in Fig. 14(a). On the other hand, the q -axis
 408 current curve with the ESO model as shown in Fig. 15(b) can be maintained at the set value 80A with just very
 409 small variations. As a result, the charging period of the MS-FESS with the ESO model is greatly shortened from
 410 85s into 70s.

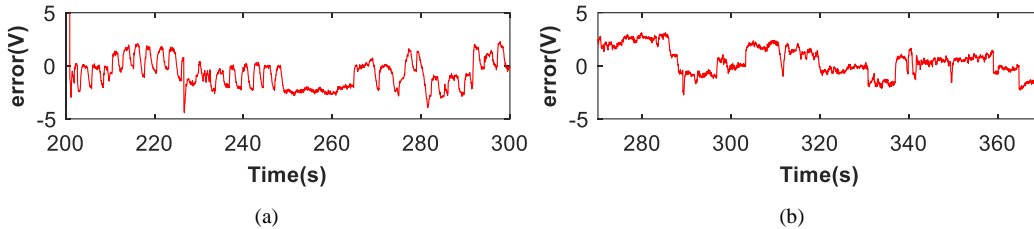
411 4.5 The Discharging Control Process of MS-FESS Using SMC Model

412 Moreover, the precision of output DC bus voltage is the most important parameter to the discharging
 413 process, and the robustness of discharging process is also critical when the load resistance varies. The speed
 414 curve and the output DC bus voltage of the MS-FESS during the discharging process are measured for testing

415 of the proposed control method for discharging. The measured output DC bus voltage curves with and without
 416 using the SMC model are plotted in Fig. 16(a) and (b), respectively. The output DC bus voltages of the MS-
 417 FESS in both cases vary with the rotating speed during the charging process and the transition process, and it
 418 is maintained at the steady-state amplitude by tracking the reference value during the discharging process. The
 419 error between the actual output DC bus voltage and the reference input DC voltage (560V) during the
 420 discharging process of the ME-FESS with and without using the SMC model are plotted in Fig. 17(a) and (b),
 421 respectively. The root mean square (RMS) value of voltage error is used as an index to evaluate the control
 422 performance of the MS-FESS during the discharging process. For the voltage error without the SMC model in
 423 Fig. 17(a), the RMS of output DC bus voltage during the discharging process is 3.28V while it is reduced to
 424 1.75V after using the SMC model as shown in Fig. 17(b). The relative reduction is 46.6% after applying the
 425 SMC model in the control of the discharging process of the MS-FESS.



426 Fig. 16. The output DC bus voltage during the charging and discharging processes of the MS-FESS, (a) the output DC bus voltage of the
 427 MS-FESS without the SMC model, (b) the output DC bus voltage of the MS-FESS with the SMC model.



428 Fig. 17. The output voltage error during the discharging process of MS-FESS, (a) the voltage error of MS-FESS without the SMC model,
 429 (b) the voltage error of the MS-FESS with the SMC model.

430 Above all, the experimental tests show that the proposed ESO control model can improve the charging
 431 performance of the MS-FESS by accurately tracking the control current for accelerating the rotating speed of
 432 the FW rotor. Moreover, the proposed SMC control model can reduce the voltage error between the actual
 433 output DC bus voltage and the reference voltage during the discharging process of the MS-FESS.

434 5 Conclusion and Discussion

435 The charging and discharging processes of MS-FESS are analyzed in this article. An ESO control model
 436 is proposed to improve the charging speed of the MS-FESS and a SMC control model is proposed to reduce the
 437 voltage error during the discharging process of the MS-FESS. In the charging process of the MS-FESS with the

438 proposed ESO model, the reference speed curve of the MS-FESS is more accurately tracked. The acceleration
439 period of charging process is reduced from 85s to 70s, so the ESO model could significantly improve the
440 efficiency and robustness of the MS-FESS during the charging process. Moreover, for the discharging process
441 of the MS-FESS, the voltage error between the actual value and the reference voltage is reduced by 46.6% with
442 the designed SMC model. The test results show that the proposed SMC model can significantly improve the
443 output precision of DC bus voltage.

444 In the future work, the charging and discharging process of MS-FESS with higher rotational speed will be
445 investigated, and the combination of several MS-FESS will be considered to improve the energy storage.
446 Moreover, other control methods would be designed for the MS-FESS to improve the overall system
447 performance.

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451 **Conflict of Interest Statement**

452 The authors declare no conflict of interest in preparing this article.

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