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29 **Abstract:** Flood control operation (FCO) of a reservoir is a complex optimization  
30 problem with a large number of constraints. With the rapid development of optimization  
31 techniques in recent years, more and more research efforts have been devoted to  
32 optimizing FCO problems. However, for solving large-scale reservoir group  
33 optimization problem, this is still a challenging task. In this work, a reservoir group  
34 FCO model is established with minimum flood volume stored in each reservoir and  
35 minimum peak flow of downstream control point during the dispatch process. At the  
36 same time, a flood forecast model for FCO of a reservoir group is developed by  
37 coupling Yin-Yang firefly algorithm (YYFA) with  $\varepsilon$  constrained method. As a case  
38 study, the proposed model is applied to a three-reservoir flood control system in Luanhe  
39 River Basin consisting of reservoirs, river channels, and downstream control points.  
40 Results show that optimal operation of three reservoirs systems can efficiently reduce  
41 the occupied storage capacity for flood control and flood peaks at downstream control  
42 point of the basin. The proposed method can be extended to FCO of other reservoir  
43 groups with similar conditions.

44 **Key words:** Flood control operation; reservoir group; swarm intelligence; YYFA  
45 algorithm;  $\varepsilon$  constrained method

46

## 47 **1 Introduction**

48 Reservoir operation plays an important area of research in flood management,  
49 which helps to reduce flood damages, minify flood peaks, control flood and reserve  
50 flood during flood seasons (Hlavinek 2009; Luo et al. 2015; Rahimi et al. 2020). Flood  
51 control operation (FCO) of complex reservoir system is a significant non-engineering  
52 measure to effectively alleviate flood disasters by the complementarity of reservoirs  
53 (Zhu et al. 2016). A typical flood control system includes reservoirs, levee, river channel,  
54 flood-relief area, flood diversion and downstream control points, and has characteristics  
55 of large scale and nonlinearity (Chen et al., 2017). This renders it extremely difficult to  
56 attain an optimal FCO strategy. An optimization solution is considered to be one of  
57 major challenges of flood control optimal operation model.

58 During the past decades, many techniques have been developed to solve reservoir  
59 operation problem (Yu et al., 2019), such as linear programming (Needham et al., 2000),  
60 non-linear programming (Unver and Mays, 1990), and dynamic programming  
61 (Yakowitz, 1982; Zhao et al., 2017). Non-linear programming methods have limitations  
62 of slow convergence speed, long computation time whereas dynamic programming  
63 methods face the curse of dimensionality (Bai et al., 2015; Yeh, 1985). **Recently**, some  
64 nature-inspired optimization methods have been widely applied to solve multi-  
65 constrained optimization models of large-scale reservoirs (Hossain and El-shafie, 2013).  
66 Genetic algorithms (GAs) are taken as a representative approach of this type, and they  
67 have been widely applied to solve water resources system optimization (Ahmad et al.,  
68 2014; Malekmohammadi et al., 2010). These works have demonstrated that GA is

69 superior to traditional methods in terms of computational requirements in water  
70 resources management (Luo et al., 2015). Afshar (2013) presented three constrained  
71 versions of PSO algorithm for efficient optimal operation of multi-reservoir systems.  
72 Luo et al. (2015) developed a combined PSO and estimation of distribution algorithm  
73 for solving reservoir FCO. Most recently, Chen et al. (2020) used PSO with adaptive  
74 random inertia weights for multi-objective reservoir operation. Guvengir et al. (2021)  
75 used an improved PSO for short-term flood control and long-term energy maximization  
76 in multi-reservoir systems. Although some techniques have been proposed to optimize  
77 a FCO model, solving multi-reservoir FCO problem is more difficult than a single  
78 reservoir (Li and Ouyang, 2015; Qi et al., 2017). Therefore, more investigations on  
79 FCO models and new techniques are still required to obtain an effective FCO strategy.

80 As a swarm intelligence algorithm, Firefly algorithm (FA) was proposed by Yang  
81 (2008), which is based on grouping behavior of fireflies (Yang, 2014). FA has been  
82 shown to perform better than GA or PSO over several numerical benchmarks (Zhou et  
83 al., 2019). Due to its simplicity, flexibility, robustness and effectiveness, it has been  
84 widely used in many fields to solve optimization problems (Altabeeb et al., 2021;  
85 Danandeh Mehr et al., 2019; Garousi-Nejad et al., 2016; Kaveh et al., 2019; Mosavvar  
86 and Ghaffari, 2019). Although these works have shown that FA is an effective  
87 optimization technique in many optimization problems, its role in exploration will be  
88 greatly weakened if the brightest firefly falls into local optimization. In order to reduce  
89 the number of times of good balance between exploration and exploitation functions,  
90 Wang et al. (2020) presented a Yin-Yang firefly algorithm (YYFA) based on

91 dimensionally Cauchy mutation.

92 The objectives of this study are therefore to develop a flood forecast model for  
93 FCO of a reservoir group coupling YYFA with  $\varepsilon$  constrained method. Its practicability  
94 is then verified through a case study of complex flood control system operation in  
95 Luanhe River Basin. This research has some novelties and contributions as mentioned  
96 below. Firstly, a generalized FCO model of three reservoirs is established for flood  
97 control benefits of each reservoir and downstream flood control point safety, which can  
98 be extended to more reservoirs. Secondly, the good nodes set (GNS) strategy, self-  
99 learning strategy and randomly attraction model in YYFA can provide inspirations for  
100 solving FCO model of three reservoirs. Finally, the  $\varepsilon$  constrained method can use  
101 available information from infeasible region by relaxing constraint conditions of the  
102 optimization model and thus helping enhance global optimization ability.

103 The paper is organized as follows. Section 2 presents rules and mathematical  
104 model for FCO of three reservoirs. Section 3 gives a brief introduction of YYFA.  
105 Section 4 introduces the case study including the studied area, used data, forecasted  
106 inflow flood, joint FCO modelling, results and discussions. Finally, Section 5 provides  
107 the conclusions of this work.

## 108 **2. FCO of multi-reservoir**

### 109 **2.1 Rules of FCO**

110 During flood season, rules of FCO are usually used to guide reservoir operation  
111 according to the current storage state of reservoir group and the forecasted inflow.  
112 According to characteristics and experiences of the reservoir flood control system, three

113 aspects need to be considered. Firstly, in the process of regulating a flood in a reservoir,  
114 the ratio of the storage capacity of the reservoir to the maximum flood control storage  
115 capacity should be as small as possible. Secondly, at the end of a flood regulation, the  
116 water storage should be close to the ideal storage capacity of the reservoir as far as  
117 possible. Finally, during the flood adjustment process, the maximum flow of the  
118 reservoir or the maximum combined flow through the downstream control point should  
119 be as small as possible. The first aspect reflects the flood safety of the reservoir itself;  
120 the second aspect considers the connection of two floods and the benefits of the  
121 reservoir; the third aspect denotes flood safety of the downstream control point.

## 122 **2.2 Mathematical model**

123 In general, a multi-reservoir flood control system in a river basin includes  
124 reservoirs, levee, river channel, flood-relief area, flood diversion and downstream  
125 control points. For particularly large basins, there are also flood storage and detention  
126 areas. This paper does not consider any flood storage and detention areas. The  
127 generalized network of a flood control system is shown in Figure 1.

128

129

Insert Figure 1

130 The purposes of FCO of a reservoir group are to minimize the peak flow of  
131 reservoirs and mitigate flood disasters in downstream protection areas. In the process  
132 of flood operation, the initial water level of each reservoir for flood operation is the  
133 flood limit level. In order to meet the next flood, the reservoir water level should drop  
134 back to the flood limit level at the end of flood operation. In this paper, the objectives

135 are to control the flood volume stored in the reservoirs as small as possible and  
 136 minimize the peak flow of downstream control point during the dispatch process. A  
 137 typical flood process is taken as an example. The objective function of the optimization  
 138 model is:

$$139 \quad Ob = \min(\omega_1 \cdot V_1' + \omega_2 \cdot V_2' + \omega_3 \cdot V_3' + \omega_4 \cdot Q') \quad (1)$$

140 where  $V_1'$ ,  $V_2'$  and  $V_3'$  are the normalized maximum storage capacities of three reservoirs  
 141 after treatment during FCO, respectively.  $Q'$  represents the normalized peak flow at  
 142 downstream control point.  $\omega_1$ ,  $\omega_2$ ,  $\omega_3$  and  $\omega_4$ , are weights of objectives 1 to 4,  
 143 respectively. In order to eliminate the influence of different units, the raw values are  
 144 first normalized by Eqs. (2) and (3) as follows:

$$145 \quad V_i' = \frac{V_{z,i} - V_{l,i}}{V_{z,i} - V_{l,i}} \quad (i = 1, 2, 3) \quad (2)$$

$$146 \quad Q' = \frac{Q}{Q_{max}} \quad (3)$$

147 where  $V_i$  denotes the maximum storage capacity of the  $i$ -th reservoir during flood  
 148 control operations;  $V_{z,i}$  is the total flood control capacity of the  $i$ -th reservoir;  $V_{l,i}$  is the  
 149 storage capacity corresponding to the flood control limit water level of the  $i$ -th reservoir;  
 150  $Q$  is the peak flow of downstream control point during FCO;  $Q_{max}$  is the maximum  
 151 discharge volume without disasters at the downstream control point on the basis of  
 152 historical data.  $V_{z,i} - V_{l,i}$  represents the storage capacity of the  $i$ -th reservoir between the  
 153 flood control limit water level and the maximum water level, which is termed the  
 154 maximum flood control storage capacity. Thus,  $V_i'$  indicates the proportion of flood  
 155 storage capacity occupied during the dispatching process of the  $i$ -th reservoir. For the

156 *i*-th reservoir, the smaller is  $V_i'$ , the safer is the reservoir. Besides, for the downstream  
 157 control point, the smaller is  $Q'$ , the less likely the river channel is affected by the flood.

158 The constraints of the optimization model are given as follows (Luo et al. 2015):

159 (1) Water balance constraint:

$$160 \quad V_{t+1,i} - V_{t,i} = (Q_{t+1,i} - q_{t+1,i}) \cdot \Delta t \quad (4)$$

161 where  $V_{t+1,i}$  is the *i*-th reservoir storage at the end of the period ( $10^8\text{m}^3$ );  $V_{t,i}$  is the  
 162 *i*-th reservoir storage at the beginning of the period ( $10^8\text{m}^3$ );  $Q_{t+1,i}$  is the *i*-th reservoir  
 163 inflow in the *t*-th period ( $\text{m}^3/\text{s}$ );  $q_{t+1,i}$  is the *i*-th reservoir discharge flow in the *t*-th period  
 164 ( $\text{m}^3/\text{s}$ );  $\Delta t$  is the length of the computational period.

165 (2) Reservoir water level constraint:

$$166 \quad Z_{\min,i} \leq Z_i \leq Z_{\max,i} \quad (i = 1, 2, 3) \quad (5)$$

167 where  $Z_i$  means the water level of the *i*-th reservoir during flood control operations (m);  
 168  $Z_{\min,i}$  denotes the lowest allowable water level during flood control operations, which  
 169 denotes the flood limit water level (m);  $Z_{\max,i}$  means the allowable highest water level  
 170 during flood control operations, which denotes the check water level (m).

171 (3) Discharge flow limit constraint:

$$172 \quad 0 \leq q_i \leq q_{\max,i} \quad (i = 1, 2, 3) \quad (6)$$

173 where  $q_i$  is the discharge of the *i*-th reservoir;  $q_{\max,i}$  is the discharge capacity of each  
 174 period for the *i*-th reservoir ( $10^8\text{m}^3$ ).

175 (4) Terminal water level constraint:

$$176 \quad Z_{\text{end},i} = Z_{e,i} \quad (i = 1, 2, 3) \quad (7)$$

177 where  $Z_{\text{end},i}$  and  $Z_{e,i}$  are the terminal and targeted terminal water levels for *i*-th reservoir

178 (m), respectively.

### 179 **3. Yin-Yang Firefly Algorithm**

180 In order to improve the performance of FA, Wang et al. (2020) presented a YYFA  
181 algorithm based dimensional Cauchy mutation. A detailed introduction of YYFA and  
182 its theoretical background can be found in Wang et al. (2020). On the basis of the  
183 effectiveness of YYFA to solve continuous unconstrained optimization problems, we  
184 make modifications on the attraction model and the mutation mode in the YYFA for a  
185 more powerful search capability. The main procedure of YYFA for FCO is stated as  
186 follows.

#### 187 **3.1 GNS strategy**

188 In YYFA, if the initial population is able to respond to the spatial characteristic of  
189 the search space, the population will obtain better information. This can improve the  
190 optimization quality. In YYFA, the GNS strategy proposed by Xiao et al. (2007), is  
191 employed to initialize locations of fireflies. Hua and Wang (1978) proved that  
192 deviations of GNS strategy generation points were much smaller than those of random  
193 generation points.

#### 194 **3.2 Attraction model**

195 In YYFA, a random attraction model (RAM) proposed by Wang et al. (2016) was  
196 originally adopted to meet the exploration function of YYFA. Considering the complex  
197 inequality constraints in flood control process, a more effective attraction and search  
198 model inspired by Pan et al. (2019) is incorporated into YYFA instead of RAM. Firstly,  
199 the full attraction model is adopted with all equations in the random attraction model

200 retained. If the chosen firefly is not better than the current firefly, the current firefly will  
 201 move according to Eq. (8). Finally, the firefly will cancel the move if it gets worse.  
 202 Details about the search strategy can be referred to Pan et al. (2019).

$$203 \quad x_{current,d}^* = x_{current,d} + \varphi(x_{current,d} - x_{chosen,d}) \quad (8)$$

204 where  $x_{current,d}$ ,  $x_{chosen,d}$  are the  $d$ th dimension positions of the current firefly and its  
 205 chosen objective, respectively and  $\varphi$  is a random value generated uniformly in the range  
 206  $[-1, 1]$ .

### 207 **3.3. Yin-Yang firefly self-learning strategy (YYFSS)**

208 YYFA explores the search space through YYFSS and performs high-level data  
 209 mining to obtain the best fireflies by Cauchy mutation (Wang et al., 2020). After the  
 210 population position is updated, YYFA chooses the firefly  $x_{p,d}$  with the best fitness as the  
 211 "Yang firefly" and then it is given a certain self-learning time. Next, a new firefly  $x_{o,d}$   
 212 is randomly created as "Yin firefly" in the search space. In this paper, we replace  
 213 Cauchy mutation with a more advantageous technique of Lévy flight. Lévy flight has  
 214 been used widely to enhance the performance of optimization algorithms (Dinkar and  
 215 Deep 2018; Ingle and Jatoth 2020; Liu et al. 2020; Zhang et al. 2020). The position of  
 216  $x_{o,d}$  is corrected and updated according to the following equation:

$$217 \quad x_{o,d} = x_{p,d} + Lévy \cdot (x_{r1,d} - x_{r2,d}) \quad (9)$$

218 where  $x_{o,d}$  is the position of Yin firefly in  $d$ th dimension,  $x_{p,d}$  the position of Yang  
 219 firefly in  $d$ th dimension,  $Lévy$  is a random value generated by the Lévy distribution.  
 220  $x_{r1,d}$ ,  $x_{r2,d}$  represent the position in  $D$ -dimension of two fireflies randomly chosen from  
 221 the population, respectively; The mathematical form of Lévy distribution and its index

222 settings can be found in Zhang et al. (2020).

### 223 3.4. YYFA

224 The main computation steps and equations of YYFA are given as follows (Wang et  
225 al., 2020):

226 Let  $D$  be the dimension of the search space. According to the movement  
227 characteristics of fireflies, the location update equation is given as:

$$228 \quad x_{i,d}(t+1) = x_{i,d}(t) + \alpha(x_{j,d}(t) - x_{i,d}(t)) + \beta(t)\varepsilon(i) \quad (10)$$

229 where  $x_{i,d}$  and  $x_{j,d}$  denote the position in  $D$ -dimension of fireflies  $i$  and  $j$ ,  
230 respectively,  $\alpha$  is the attractiveness,  $t$  is the iteration number,  $\beta$  is the step factor, and  $\varepsilon$   
231 is within a uniform distribution between  $[-0.5, 0.5]$ . The step factor  $\alpha$  and attractiveness  
232  $\beta$  in the proposed approach are updated by Eqs. (11) and (12), respectively.

$$233 \quad \alpha = \alpha_{\min} + (\alpha_0 - \alpha_{\min})e^{-\gamma r_{ij}^2} \quad (11)$$

$$234 \quad \beta(t+1) = \beta(t) \left(1 - \frac{t}{T}\right) \quad (12)$$

235 where  $\alpha_{\min}$  denotes the lower bound of attractiveness,  $T$  is the upper limit of iteration.

236 In this paper, for YYFA, we set  $\beta(0) = 0.5$ ,  $\alpha_{\min} = 0.1$ ,  $\alpha_0 = 1$  and  $\gamma = 1$ .  $r_{ij}$  is the distance  
237 between two fireflies as defined in Wang et al. (2020):

$$238 \quad r_{ij} = \|X_i - X_j\| = \sqrt{\sum_{d=1}^D (x_{i,d} - x_{j,d})^2} \quad (13)$$

## 239 4. Case Study

### 240 4.1 Study area and data

241 Luan River originates from foothills of Bayantugur in Fengning County, Hebei  
242 Province, and flows through 27 cities, counties, and districts in Hebei Province, Inner

243 Mongolia Autonomous Region, and Liaoning Province. Luanhe River is connected with  
244 Bohai Sea in Leting County, Hebei Province. The river has a total length of 888km and  
245 controls a drainage area of 44600 km<sup>2</sup>, of which 98% are mountainous areas and 2%  
246 are plains. The Luanhe River system is shown in [Figure 2](#). The average annual  
247 precipitation of the basin is 595mm, and the precipitation is mainly concentrated in the  
248 summer, accounting for about 67-76% of the total precipitation of the whole year. The  
249 average annual runoff is 46.94 10<sup>8</sup>m<sup>3</sup>. The maximum peak discharge mainly appears in  
250 July or August.

251

252 Insert Figure 2.

253 There are three large reservoirs in the middle and lower reaches of Luanhe River,  
254 namely Panjiakou Reservoir, Daheiting Reservoir and Taolinkou Reservoir, which  
255 jointly undertake the flood control task in lower reaches. Panjiakou Reservoir is located  
256 in middle reaches of Luanhe River in Hebei Province, China. It is the source of Luanhe  
257 River diversion project. It controls a drainage area of 33,700 km<sup>2</sup>, accounting for 75%  
258 of Luone River drainage area. The total storage capacity of the reservoir is 2.93 billion  
259 m<sup>3</sup>. Its main functions are flood control, water supply and power generation. Daheiting  
260 reservoir is located on the mainstream of Luan River in Tangshan City, Hebei Province,  
261 30km away from Panjiakou reservoir and controls a drainage area of 35300 km<sup>2</sup>.  
262 Daheiting reservoir is an annual regulating reservoir with a total storage capacity of  
263 3.37 billion m<sup>3</sup>. An important role of Daheiting Reservoir is to undertake the water  
264 regulation of Panjiakou Reservoir, and undertake tasks of water supply, flood control

265 and power generation. Taolinkou Reservoir is located on Qinglong River, the main  
266 tributary of Luanhe river, with a controlled drainage area of 5060 km<sup>2</sup> and a total storage  
267 capacity of 8.59 billion m<sup>3</sup>. The reservoir mainly provides agricultural production and  
268 urban water for Tangshan and Qinhuangdao cities. It is a modern large-scale water  
269 conservancy project with comprehensive functions such as water supply, power  
270 generation, tourism and aquaculture. Characteristics of the above three reservoirs are  
271 shown in Table 1.

272 Insert Table 1

## 273 **4.2 Flood routing**

274 Xinanjiang Model was presented by Zhao et al. (1980) and has been successfully  
275 and widely applied to flood forecasting (Wang et al. 2012; Xu et al. 2013; Zhao 1992).  
276 The model divides the whole watershed into many sub-units, and computes runoff and  
277 confluence of each sub-unit to obtain the outlet flood process of the sub-unit, and then  
278 performs river flood computation below the outlet to obtain the sub-unit outflow  
279 process for all sub-units. The watershed outflow processes are added together to obtain  
280 the total watershed outflow process. Details regarding theoretical background and  
281 parameter optimization of Xinanjiang model can be found in [Xu et al. \(2013\)](#) and [Zhao](#)  
282 [\(1992\)](#).

283 In the downstream of Daheiting Reservoir, Luanhe River flows through five  
284 counties including Qianxi, Qian'an, Luanxian, Luannan, Leting, etc. in Tangshan City.  
285 The reservoir group, composed of the above three reservoirs, jointly undertakes flood  
286 control tasks of lower reaches of Luanhe River. Taking Luanxian as the control point of

287 lower reaches of Luanhe River, the upstream can be divided into 5 sub-regions  
288 according to the location of the three reservoirs. Hence, there are five regional flood  
289 processes including: flood in the upstream of Panjiakou Reservoir, interval flood  
290 between Panjiakou and Daheiting Reservoir, interval flood from Daheiting Reservoir  
291 to Luanxian, flood in the upstream of Taolinkou Reservoir, and interval flood from  
292 Taolinkou Reservoir to Luanxian. Assuming that the inflow of Panjiakou Reservoir is  
293  $Q_1$ , the interval flood from Panjiakou Reservoir to Daheiting Reservoir is  $Q_2$ , the  
294 interval flood from Daheiting Reservoir to Luanxian control point is  $Q_3$ , the flood in  
295 the upstream of Taolinkou Reservoir is  $Q_4$ , and the interval flood from Taolinkou  
296 Reservoir to Luan County is  $Q_5$ . A generalized flood routing process in Luanhe River  
297 Basin is shown in [Figure 3](#).

298 Insert Figure 3.

299 In this work, Xinanjiang model is applied to forecast the flood process of the above  
300 five sub-regions using meteorological data. [Figure 4](#) gives the flood forecast of a typical  
301 rainfall in five sub-regions. The flood duration of the predicted flood is 144 hours, and  
302 the computation period is 3 hours.

303 Insert Figure 4.

304 The discharge flows of Daheiting Reservoir and Taolinkou Reservoir evolve to  
305 Luanxian through the river channel. The flood routing process is computed by linear  
306 Muskingum flood routing method, which is expressed by Eq (14) (Gill 1978; Wang et  
307 al. 2010).

$$308 \quad S(t) = k[xI(t) + (1-x)O(t)] \quad (14)$$

309 where  $S(t)$  represents the channel storage at time  $t$ ;  $K$  represents the storage-time

310 constant for the river reach and  $x$  represents weighting factor;  $I(t)$  and  $O(t)$  represent  
311 inflow and outflow rates at time  $t$ , respectively. The parameters of the Muskingum  
312 model in the study area are determined. The joint FCO process of the three reservoirs  
313 is described as follows:

314 For Panjiakou Reservoir, the reservoir inflow is  $Q_1$ , and after FCO of Panjiakou  
315 reservoir, the reservoir discharge flow is  $q_1$ .

316 For Daheiting Reservoir, the reservoir inflow is composed of two parts, the local  
317 inflow  $Q_2$  and Panjiakou reservoir discharge flow  $q_1$ . Because Panjiakou Reservoir  
318 and Daheiting Reservoir are close to each other,  $q_1$  does not need to be adjusted and  
319 computed, and can be used directly as a component of the inflow of Daheiting reservoir.  
320 After FCO of Daheiting reservoir, the reservoir discharge flow is  $q_2$ .

321 For Taolinkou Reservoir, the reservoir inflow is  $Q_4$ , and after FCO of discharge  
322 reservoir, the discharge flow is  $q_3$ .

323 For Luanxian control point, the flow  $Q$  is composed of four parts: the discharge  
324 flow  $q_4$  of Daheiting Reservoir, which is evolved from  $q_2$ ; the discharge flow  $q_5$  of  
325 Taolinkou Reservoir, which is evolved from  $q_3$ ; the interval inflow  $Q_3$  from Daheiting  
326 reservoir to LuanXian control point; the interval inflow  $Q_5$  from Taolinkou reservoir to  
327 LuanXian control point.

### 328 **4.3 Objective weight by analytic hierarchy process**

329 In this paper, a three-scale method is used to determine the weights of the three  
330 reservoirs and LuanXian control point. Details regarding the construction of the  
331 judgment matrix by the three-scale method is as follows:

332 Firstly, the experts compare the importance of each object, and give values in the  
 333 three-scale comparison matrix  $A=(a_{ij})_{m \times m}$ . Among them: if  $i$ -th object is more important  
 334 than  $j$ -th object,  $a_{ij}$  is 2; if  $i$ -th object is as important as  $j$ -th object,  $a_{ij}$  is 1; if  $i$ -th object  
 335 is not as important as  $j$ -th object,  $a_{ij}$  is 0.

336 Secondly, the sum of the objects of the three-scale comparison matrix in the same  
 337 row is computed. The maximum value is recorded as  $r_{max}$ , and the minimum value is  
 338 recorded as  $r_{min}$ .

339 Finally, the direct comparison matrix is transformed into an indirect judgment  
 340 matrix by the following formula:

$$341 \quad d_{ij} = \begin{cases} \frac{r_i - r_j}{r_{max} - r_{min}}(b_m - 1) + 1 & r_i - r_j \geq 0 \\ 1 & r_i - r_j < 0 \\ \frac{r_i - r_j}{r_{max} - r_{min}}(b_m - 1) + 1 & \end{cases} \quad (14)$$

342 where  $b_m$  is the base point comparison scale, ranging between 4 to 9;  $r_i$  and  $r_j$  are  
 343 the sum of the elements of  $i$ -th and  $j$ -th row of the three-scale comparison matrix,  
 344 respectively.

345 Finally, the eigenvalues and eigenvectors of the indirect judgment matrix are  
 346 computed. The vector corresponding to the largest eigenvalue is used as the weight of  
 347 each object.

348 The weights of Panjiakou Reservoir, Daheiting Reservoir, Taolinkou Reservoir  
 349 and LuanXian control point are computed as 0.2, 0.3, 0.2 and 0.3, respectively.

#### 350 4.4 Constraint handling techniques

351 In order to address the constraints of water level and flow limit,  $\varepsilon$  constrained

352 method proposed by Zheng et al. (2012) is adopted to guide the firefly population to  
 353 move to a feasible region. The idea of  $\varepsilon$  constrained method is that the fireflies can make  
 354 full use of the available information from the infeasible region by relaxing the constraint  
 355 conditions, and then the relaxation shrinks to find the global optimal solution by an  
 356 effective comparison rule. The implementation of  $\varepsilon$  constrained method can be  
 357 illustrated with the following minimization problem containing only inequality  
 358 constraints.

$$359 \quad \begin{aligned} & \text{minimize} && f(X) \\ & \text{s.t.} && g_i(X) \leq 0, i = 1:q \end{aligned} \quad (15)$$

360 where  $X = (x_1, x_2, \dots, x_n)$  is an  $n$  dimensional vector and  $g_q(X) \leq 0$  is the  $q^{\text{th}}$  inequality  
 361 constraint. A candidate solution, which is a firefly, whose constraint violation is defined  
 362 as:

$$363 \quad G(X) = \sum_{i=1}^q \max(g_i(X), 0) \quad (16)$$

364 The comparison between two fireflies  $X_1$  and  $X_2$  in YYFA is based on Eq. (17):

$$365 \quad X_2 \text{ is better than } X_1 \Leftarrow \begin{cases} f(X_2) < f(X_1), & \text{if } G(X_1) = G(X_2) = 0 \\ f(X_2) < f(X_1), & \text{if } G(X_1) = 0 \cap 0 < G(X_2) \leq \varepsilon \\ f(X_2) < f(X_1), & \text{if } G(X_2) = 0 \cap 0 < G(X_1) \leq \varepsilon \\ G(X_2) < G(X_1), & \text{if } G(X_1) > 0 \cap G(X_2) > 0 \\ G(X_2) = 0, & \text{if } G(X_1) > \varepsilon \end{cases} \quad (17)$$

366 The parameter  $\varepsilon$  is computed from generation to generation by Eq. (18):

$$367 \quad \varepsilon(t+1) = \begin{cases} \varepsilon(t) / 1.035, & \varepsilon > 10^{-6} \\ 0, & \varepsilon \leq 10^{-6} \end{cases} \quad (18)$$

#### 368 **4.5 Optimal operation of joint flood control using YYFA**

369 With the help of  $\varepsilon$  constrained method, specific steps for attaining optimal FCO of  
 370 a reservoir group using Yin-Yang Firefly Algorithm are as follows:

371 Step 1: Set the parameters. The maximum number of iterations is set as  $T$ , and the  
 372 number of populations is set as  $M$ . The self-learning times of Yang firefly ( $SL$ ) and the  
 373 initial constraint relaxation  $\varepsilon(0)$  are also needed. The optimal  $T$  and  $M$  can be  
 374 determined through multiple computations.

375 Step 2: Initialization of fireflies. In accordance with the given computation period  
 376 of the forecasted flood inflow, the joint operation process of the reservoir group is set.  
 377 Fireflies are constructed with the discharge flow of each reservoir at the end of each  
 378 period as the control variable. In this paper, the flood period is 48, the number of  
 379 reservoirs is 3, so the vector dimension is 144. With the defined population number  $M$ ,  
 380 the firefly population can be expressed in the following matrix form:

$$381 \quad Y(0) = [X_1(0), X_2(0), \dots, X_M(0)] = \begin{bmatrix} x_{1,1}(0) & x_{1,2}(0) & \dots & x_{1,M}(0) \\ x_{2,1}(0) & x_{2,2}(0) & \dots & x_{2,M}(0) \\ \dots & \dots & \dots & \dots \\ x_{144,1}(0) & x_{144,2}(0) & \dots & x_{144,M}(0) \end{bmatrix} \quad (19)$$

382 where  $X_i(0)$  is the  $i$ th initial firefly, which contains 144 elements, of which the 1st  
 383 to 48-th elements represent discharge flow of Panjiakou Reservoir, and the 49-th to  
 384 96-th elements represent discharge flow of Daheiting Reservoir, and the 97-th to 144-  
 385 th elements represent discharge flow of Taolinkou Reservoir at the end of each period.  
 386 After the size of the fireflies' matrix is determined, GNS strategy is used to generate  
 387 new data to improve the representativeness of the initial data.

388 Step 3: Computation of the fitness degree. Using the given objective function  
 389 constructed by Eq. (1), the fitness of each firefly, that is, the brightness of the firefly is  
 390 computed, and the best firefly is determined using  $\varepsilon$  constrained method with its

391 position in the indexed swarm. After  $t$ -th iterations, the rearranged population is  
392 recorded as  $[X_1(t), X_2(t), \dots, X_M(t)]$

393 Step 4: Yin-Yang firefly self-learning. The best firefly  $X_p(t)$  is selected as the  
394 "Yang firefly", that is  $X_l(t)$ , and a firefly in the population is randomly selected as "Yin  
395 firefly". The "Yang firefly" and the "Yin firefly" are searched according to the strategy  
396 to obtain the current global optimal firefly. They are then returned to the corresponding  
397 positions in the swarm.

398 Step 5: Firefly population evolution. The firefly swarm is updated according to Eq.  
399 (1) to Eq. (4).

400 Step 6: Termination criteria. Steps 3 to 5 are repeated until the pre-set iteration  
401 maximum is reached.

402 Step 7: The vector corresponding to the finally obtained brightest firefly denotes  
403 the discharge flow process after the optimal operation.

404 The proposed YYFA approach for flood operation is shown in Fig. 5.

405

## 406 **4.6 Results and Discussions**

407 The main objective is to control the flood volume stored in each reservoir to be as  
408 small as possible, and to eliminate the peak flow as far as possible at the downstream  
409 control point. According to the above description, YYFA coupled with  $\epsilon$  constrained  
410 method is adopted to solve the real-world multi-reservoir FCO problems of Panjiakou  
411 Reservoir, Daheiting Reservoir and Taolinkou Reservoir. The determination of some  
412 important parameters of YYFA algorithm, for example population size and maximum

413 iteration times, has a great impact on the model. The maximum number of iterations is  
414 set to 100000. For the purpose of comparison, the algorithm is run 10 times  
415 independently using different population sizes. 'Std', 'Min', 'Mean', 'Median', 'Max,  
416 represent standard deviation, minimum value, average value, Median value and  
417 maximum value of the proposed models, respectively. Statistical results of optimal joint  
418 flood control operation of the reservoir group are shown in [Table 2](#). It can be observed  
419 that the best objective value can be obtained when the population size is 200. [Figures 6](#)  
420 [to 8](#) give the operation process of the three reservoirs. The combined flow process after  
421 optimal joint operation, original combined flow process, routing discharge flow of each  
422 reservoir and two sub-region inflows at Luanxian control point are shown in [Figure 9](#).  
423 From results of [Figures 6 to 9](#) and [Table 3](#), we can see that the occupied flood control  
424 capacities of Panjiakou Reservoir, Daheiting Reservoir and Taolinkou Reservoir are  
425 81.13%, 89.65% and 88.63%, respectively. This provides operational space for flood  
426 forecasting uncertainty. The maximum flood peak flow is clipped 26.68% at Luanxian  
427 control point. Hence, the proposed YYFA coupled with  $\varepsilon$  constrained method well  
428 achieves the objective of joint FCO. **In order to validate the feasibility of the proposed**  
429 **method and make benchmark comparison, original PSO and FA are employed to solve**  
430 **this problem under the same conditions. Unfortunately, they cannot find a feasible**  
431 **solution. As an illustration, Figures 10 to 11 give operation results of Panjiakou**  
432 **reservoir only.**

433  
434 Insert Table 2.  
435

436 Insert Table 3.

437

438 Insert Figure 6.

439

440 Insert Figure 7.

441

442 Insert Figure 8.

443 Insert Figure 9.

444 Insert Figure 10.

445 Insert Figure 11.

446

## 447 **5. Conclusions**

448 Reservoirs are among the most effective tools for flood management in flood  
449 control system although their optimal scheduling solution is considered to be  
450 challenging. In this paper, a new optimization technique named YYFA is developed as  
451 a new tool to address this challenge. A  $\epsilon$  constrained method is used to address complex  
452 constraints of FCO. This paper establishes an optimization model with the goal of  
453 minimizing the occupation of flood control capacity and joint minimization of flood  
454 peak at the downstream flood control points for multi-reservoir system. Taking three  
455 reservoir group systems in Luanhe River Basin in China as the research object, the  
456 proposed model and method are verified by using 3-hour predicted inflow time series.  
457 Results show that the occupied flood control capacities of Panjiakou Reservoir,  
458 Daheiting Reservoir and Taolinkou Reservoir are 81.13%, 89.65% and 88.63%,  
459 respectively, and the flood peak at Luanxian control point station can be clipped 26.68%.  
460 The results show that the presented model is effective for multi-reservoir FCO system.

461 Furthermore, YYFA algorithm can also be used as an effective tool to solve other  
462 practical engineering optimization problems.

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467

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469

470

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607

608 **Table captions**

609 **Table 1.** Characteristics of the reservoirs

610 **Table 2.** Operation results using different population sizes

611 **Table 3.** Statistical results of the joint flood control optimal operation

612

613 **Figure captions**

614 **Figure 1.** The generalized network of a flood control system

615 **Figure 2.** The location of Luanhe River reservoir group

616 **Figure 3.** Generalized flood routing process in Luanhe River Basin

617 **Figure 4.** Flood forecast of a typical rainfall in five sub-regions

618 **Figure 5.** Flood control operation using YYFA approach

619 **Figure 6.** Operation process of Panjiakou Reservoir

620 **Figure 7.** Operation process of Daheiting Reservoir

621 **Figure 8.** Operation process of Taolinkou Reservoir

622 **Figure 9.** Flood Routing processes of Luanxian control point

623 **Figure 10.** Operation process of Panjiakou Reservoir using PSO

624 **Figure 11.** Operation process of Panjiakou Reservoir using FA

625