Received: 17 March 2021 | Revised: 28 April 2021 | Accepted: 11 May 2021

DOI: 10.1049/cth2.12157

IET Control Theory & Applications

The Institution of Engineering and Technology WILEY

ORIGINAL RESEARCH PAPER

Saturated adaptive barrier sliding mode control with state-dependent uncertainty limit

Haifeng Ma[1,2] | **Weixiang Liu**[1,2] | **Yangmin Li**[3] | **Zhanqiang Liu**[1,2] | **Sukun Tian**[1,2]

[1] School of Mechanical Engineering, Shandong University, Jinan, China

[2] Key Laboratory of High Efficiency and Clean Mechanical Manufacture of MOE/Key National Demonstration Center for Experimental Mechanical Engineering Education, Jinan, China

[3] Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

Correspondence
Sukun Tian, School of Mechanical Engineering, Shandong University; Key Laboratory of High Efficiency and Clean Mechanical Manufacture of MOE/Key National Demonstration Center for Experimental Mechanical Engineering Education, Jinan, China.
Email: sukhum169@sdu.edu.cn

Abstract

This paper proposed a saturated adaptive barrier sliding mode control (SABSMC) for disturbed systems with parameter uncertainty, external disturbance, and input saturation. One distinctive feature of the proposed method is that even under the influence of input saturation, it is able to drive the system trajectory into a prespecified neighbourhood of the sliding surface, independent of the upper limit of lumped disturbance. Moreover, attributing to the designed adaptive laws, the switching gain of the proposed method can be adaptively adjusted in a wise manner. Another attractive feature of the proposed method is that it removes any presumption of constant upper limit on the lumped disturbance. Instead, SABSMC utilises a more rational state-dependent upper limit assumption. The system dynamics of the closed-loop system is analysed via the Lyapunov method in the presence of parameter uncertainty, external disturbance, and input saturation. The effectiveness of the proposed method is confirmed through numerical simulations.

1 | INTRODUCTION

As an efficient control method, sliding mode control (SMC) has developed greatly owing to its strong robustness to parameter uncertainty and external disturbance [1–3], and has been broadly implemented in various actual situations [4, 5]. In SMC construction, the switching gain is one of the key parameters that affect the controller performance. The choice of the switching gain, nevertheless, is a tough task. To guarantee system stability, the switching gain should be adjusted to be greater than the upper limit of the overall parameter uncertainty and disturbance (or we called the lumped disturbance) [6]. However, due to the complicacy of the lumped disturbance, such a limit is time-varying and, furthermore, commonly it is hard to know in numerous real scenarios [7]. Therefore, the chosen switching gain should be sufficiently large to deal with different types of lumped disturbances, which will eventually lead to parameter overestimation, undesired chattering, and control accuracy deterioration [8].

In order to improve the robustness and adaptability of controlled systems, the SMC strategy is hoped to be adaptive to various disturbances. The pursuit of this objective has led to the development of adaptive SMC (ASMC), where the switching gain is automatically tuned for the establishment of a sliding mode (SM) regardless of the upper limit of the lumped disturbance [9, 10]. Various approaches have been presented for constructing ASMC. These approaches roughly fall into the next three cases: (a) ASMC on the basis of the equivalent control [11–13]. This method is realised by adopting low-pass-filtered equivalent control to approximate the value of disturbance. Nevertheless, the use of a low-pass filter requires the information on the upper limit of disturbance for the purpose of appropriately selecting the filter parameter; (b) ASMC on the basis of monotonically increasing the gain [14–16]. The main idea of this method is to increase the gain until reaching the upper limit of disturbance and SM and then keeping the gain fixed. The main drawback of this method is that it does not prevent the gain from increasing even if the disturbance or control error is reduced, resulting in the overestimation problem; (c) ASMC on the basis of increasing and decreasing the gain [17–34]. To tackle the disadvantages in method (b), method (c) has been presented, where the gain increases (resp. decreases) until SM is achieved (resp. lost). This method avoids the big overestimation of the gain in method (b).

It is worth noting that the aforementioned ASMC methods [11–34] may provide conservative results since the lumped disturbance is supposed to be upper limited by a constant a priori. As parameter uncertainty, which is part of the lumped disturbance and is commonly coupled with system states, consideration of this constant limit restricts the system states a priori (i.e. before performing closed-loop stability), which is a restrictive assumption. Recently presented ASMC strategies reported in [35, 36] do not require the unreasonable constant upper limit assumption on the lumped disturbance. The main drawback of the strategies is that the ultimate bounds in [35, 36] are greatly affected by the unknown lumped disturbance. Therefore the control performance is difficult to be pre-evaluated and prespecified, and the lumped disturbance makes a great impact on the control accuracy. Furthermore, the ASMC strategies in [11–36] have ignored a very important issue, that is, input saturation, which always results in performance deterioration and even system instability if it is ignored. Although ASMC with input saturation issue has been studied in [37–40], they employ monotonically increasing adaptive gains, which lead to the overestimation problem. In addition, the lumped disturbance in [37–40] is assumed to have a priori constant upper limit, which is a pretty restrictive assumption. To the author's knowledge, no work has been carried out on the design of an ASMC controller, which can solve the above-mentioned problems simultaneously.

To solve the above-mentioned problems, a new ASMC strategy termed saturated adaptive barrier SMC (SABSMC) is formulated for a class of disturbed systems with parameter uncertainty, external disturbance, and input saturation. One distinguishing characteristic of SABSMC is that it is capable of driving the system trajectory into a prespecified neighbourhood of the sliding surface and remaining inside it even under the influence of input saturation. This prespecified neighbourhood is not affected by the lumped disturbance, which allows predesigning and minishing the ultimate bound and tracking error ahead of time. Hence, the proposed method is promising for actual applications. The switching gain can be adaptively adjusted in a wise way by utilising the barrier function. Another distinctive feature of SABSMC is that it utilises a more proper state-dependent structure of the upper limit on the lumped disturbance and does not presume the lumped disturbance to be upper limited by a constant a priori. The effectiveness of the controlled system is proved in theory by considering parameter uncertainty, external disturbance, and input saturation. Numerical simulations are conducted to test the superiority of SABSMC.

2 | PROBLEM DESCRIPTION

Consider a class of continuous-time systems with parameter uncertainty, external disturbance, and input saturation

$$\dot{x}(t) = \Phi x(t) + \Gamma \mathrm{sat}(\nu(t)) + f(t), \tag{1}$$

where $x^n \in \mathbb{R}^n$, $v \in \mathbb{R}$, and $f \in \mathbb{R}^n$ denotes the state vector, control input, and external disturbance, respectively. f is assumed to satisfy the matching condition. The system matrix Φ is supposed to be unknown. In actual applications, input saturation will inevitably occur as a result of the physical limitation of the control capacity. Input saturation is represented by $\mathrm{sat}(\nu(t))$

$$\mathrm{sat}(\nu(t)) = \begin{cases} \nu_M, & \text{if } \nu(t) > \nu_M \\ \nu(t), & \text{if } -\nu_M \le \nu(t) \le \nu_M \\ -\nu_M, & \text{if } \nu(t) < -\nu_M \end{cases}, \tag{2}$$

where $\nu_M > 0$ stands for the maximum value of the control input. In addition, the saturation function $\mathrm{sat}(\nu(t))$ is further expressed as $\mathrm{sat}(\nu(t)) = \Theta(\nu(t))\cdot\nu(t)$ with

$$\Theta(\nu(t)) = \begin{cases} \dfrac{\nu_M}{\nu(t)}, & \text{if } \nu(t) > \nu_M \\ 1, & \text{if } -\nu_M \le \nu(t) \le \nu_M \\ -\dfrac{\nu_M}{\nu(t)}, & \text{if } \nu(t) < -\nu_M \end{cases}. \tag{3}$$

$\Theta(\nu(t)) \in (0, 1]$ can be regarded as an index for the saturation level of the control input [37]. There exists a constant δ_1 satisfying $0 < \delta_1 \le \min(\Theta(\nu(t))) \le 1$ [37, 38].

The switching function is defined as follows:

$$\eta(t) = Gx(t), \tag{4}$$

where G is a constant gain vector.

For the convenience of analysis, the parameter uncertainty Φ along with the external disturbance $f(t)$ is treated as a lumped disturbance $\xi(t)$

$$\xi(t) = \Phi x(t) + f(t). \tag{5}$$

3 | SABSMC

In this part, a SABSMC scheme is first designed. Then the controlled system performance is theoretically analysed.

3.1 | Design of SABSMC

The following assumption of the lumped disturbance is introduced.

Assumption 1 [35]: The lumped disturbance $\xi(t)$ is upper limited by

$$\begin{aligned} |G\xi(t)| &= \left|G\left(\Phi x(t) + f(t)\right)\right| \\ &< \vartheta_1^* + \vartheta_2^* \|x(t)\| \end{aligned}, \tag{6}$$

where ϑ_1^* and ϑ_2^* are unknown positive constants.

***Remark* 1:** Previous ASMC methods assume that $\xi(t)$ or $\dot{\xi}(t)$ is upper limited by a constant a priori. It can be observed that

Assumption 1 is more general as the constant upper limit is substituted by a state-dependent upper limit. ■

In view of the system (1) and *Assumption 1*, the new SABSMC is designed

$$\nu(t) = -(G\Gamma)^{-1}\left[\tau\eta(t) + u_s(t,\eta(t))\,\mathrm{sign}(\eta(t))\right], \quad (7)$$

where the adaptive switching gain $u_s(t, \eta(t))$ is defined as

$$u_s(t,\eta(t)) = \begin{cases} u_{s1}(t,\eta(t)), & \text{if } |\eta(t)| \geq \varepsilon \\ u_{s2}(t,\eta(t)), & \text{if } |\eta(t)| < \varepsilon \end{cases}, \quad (8)$$

with

$$u_{s1}(t,\eta(t)) = \mu\hat{\chi}_1(t)\left(\hat{\vartheta}_1(t) + \hat{\vartheta}_2(t)\|x(t)\|\right), \quad (9)$$

$$u_{s2}(t,\eta(t)) = \mu\hat{\chi}_2(t)\kappa_a(\eta(t)), \quad (10)$$

and the update laws are chosen as

$$\dot{\hat{\vartheta}}_1(t) = \alpha_1|\eta(t)|, \quad (11)$$

$$\dot{\hat{\vartheta}}_2(t) = \alpha_2|\eta(t)|\|x(t)\|, \quad (12)$$

$$\dot{\hat{\chi}}_1(t) = \mu\hat{\chi}_1^3(t)\left(\hat{\vartheta}_1(t) + \hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)|, \quad (13)$$

$$\dot{\hat{\chi}}_2(t) = \mu\hat{\chi}_2^3(t)\kappa_a(\eta(t))|\eta(t)|, \quad (14)$$

with the barrier function

$$\kappa_a(\eta(t)) = \frac{\alpha_3|\eta(t)|}{\varepsilon - |\eta(t)|}, \quad (15)$$

where $\tau, \varepsilon, \alpha_1, \alpha_2, \alpha_3 > 0, \mu > 1$ are the control gains, and

$$\hat{\vartheta}_1(0) > 0,\ \hat{\vartheta}_2(0) > 0,\ \hat{\chi}_1(0) > 0,\ \hat{\chi}_2(0) > 0. \quad (16)$$

Recalling (15), it can be found that $\kappa_a(\eta(t)) : \eta(t) \in (-\varepsilon, \varepsilon) \rightarrow \kappa_a(\eta(t)) \in [0, +\infty)$ is strictly increasing on $\eta(t) \in [0, \varepsilon)$ [25]. $\kappa_a(\eta(t))$ allows the switching gain to increase and decrease based on the current value of $\eta(t)$. If $\eta(t)$ reaches the sliding surface, $\kappa_a(\eta(t))$ and the switching gain will also become zero. Therefore, the switching gain of the reported method can be adaptively adjusted in an appropriate manner.

3.2 | Stability analysis

Theorem 1: Consider the system given in (1) with parameter uncertainty, external disturbance, and input saturation, and the lumped disturbance $\xi(t)$ satisfies Assumption 1, then the proposed SABSMC controller (7) with update laws (8)–(15) can drive the system trajectory into and remain always in a region $|\eta(t)| \leq \eta_1 < \varepsilon$

$$0 < \eta_1 = \varepsilon\frac{y}{y + \alpha_3} < \varepsilon, \quad (17)$$

with

$$y = \frac{\vartheta_1^* + \vartheta_2^*\|x(t)\|}{\delta_1\mu\hat{\chi}_2(t)}. \quad (18)$$

Proof: The stability of the controlled system is studied for the two feasible cases, that is, cases (i) and (ii).

Case (i): For $|\eta(t)| \geq \varepsilon$, (7) can be rewritten as

$$\nu(t) = -(G\Gamma)^{-1}\left[\tau\eta(t) + u_{s1}(t,\eta(t))\,\mathrm{sign}(\eta(t))\right]. \quad (19)$$

Consider the following Lyapunov function

$$V_1(t) = \frac{1}{2}\left[\eta^2(t) + \frac{1}{\alpha_1}\left(\hat{\vartheta}_1(t) - \vartheta_1^*\right)^2 + \frac{1}{\alpha_2}\left(\hat{\vartheta}_2(t) - \vartheta_2^*\right)^2 + \tilde{\chi}_1^2(t)\right], \quad (20)$$

where

$$\tilde{\chi}_1(t) = \delta_1 - \hat{\chi}_1^{-1}(t). \quad (21)$$

Taking the first derivative of $V_1(t)$ in (20) yields

$$\begin{aligned}\dot{V}_1(t) &= \eta(t)\dot{\eta}(t) + \frac{1}{\alpha_1}\left(\hat{\vartheta}_1(t) - \vartheta_1^*\right)\dot{\hat{\vartheta}}_1(t) \\ &+ \frac{1}{\alpha_2}\left(\hat{\vartheta}_2(t) - \vartheta_2^*\right)\dot{\hat{\vartheta}}_2(t) + \tilde{\chi}_1(t)\hat{\chi}_1^{-2}(t)\dot{\hat{\chi}}_1(t)\end{aligned}. \quad (22)$$

Substituting (1), (5), (11), (12) into (22) leads to

$$\begin{aligned}&\dot{V}_1(t) \\ &= \eta(t)\left[G\Gamma\Theta(\nu(t))\nu(t) + G\xi(t)\right] + \left(\hat{\vartheta}_1(t) - \vartheta_1^*\right)|\eta(t)| \\ &+ \left(\hat{\vartheta}_2(t) - \vartheta_2^*\right)|\eta(t)|\|x(t)\| + \tilde{\chi}_1(t)\hat{\chi}_1^{-2}(t)\dot{\hat{\chi}}_1(t)\end{aligned}. \quad (23)$$

Then, inserting (7), (9), and (13) into (23) yields

$$\begin{aligned}\dot{V}_1(t) &\leq |\eta(t)||G\xi(t)| \\ &- \eta(t)\Theta(\nu(t))\left[\begin{array}{l}\tau\eta(t) \\ +\mu\hat{\chi}_1(t)\left(\begin{array}{l}\hat{\vartheta}_1(t) \\ +\hat{\vartheta}_2(t)\|x(t)\|\end{array}\right)\mathrm{sign}(\eta(t))\end{array}\right] \\ &+ \left(\hat{\vartheta}_1(t) - \vartheta_1^*\right)|\eta(t)| + \left(\hat{\vartheta}_2(t) - \vartheta_2^*\right)|\eta(t)|\|x(t)\| \\ &+ \mu\tilde{\chi}_1(t)\hat{\chi}_1(t)\left(\hat{\vartheta}_1(t) + \hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)|\end{aligned}. \quad (24)$$

Taking into account Assumption 1, the following deduction can be generated

$$\begin{aligned}\dot{V}_1(t) &< |\eta(t)|\left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right) \\ &-\delta_1\left[\begin{array}{l}\tau\eta^2(t) \\ +\mu\hat{\chi}_1(t)\left(\begin{array}{l}\hat{\vartheta}_1(t) \\ +\hat{\vartheta}_2(t)\|x(t)\|\end{array}\right)|\eta(t)|\end{array}\right] \\ &+\left(\hat{\vartheta}_1(t)-\vartheta_1^*\right)|\eta(t)| + \left(\hat{\vartheta}_2(t)-\vartheta_2^*\right)|\eta(t)|\|x(t)\| \\ &+\mu\tilde{\chi}_1(t)\hat{\chi}_1(t)\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)|\end{aligned}. \quad (25)$$

Substituting (21) into (25) and noting (6) gives

$$\begin{aligned}\dot{V}_1(t) &< |\eta(t)|\left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right) \\ &-\delta_1\left[\begin{array}{l}\tau\eta^2(t) \\ +\mu\hat{\chi}_1(t)\left(\begin{array}{l}\hat{\vartheta}_1(t) \\ +\hat{\vartheta}_2(t)\|x(t)\|\end{array}\right)|\eta(t)|\end{array}\right] \\ &-|\eta(t)|\left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right) + |\eta(t)|\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right) \\ &+\mu\left(\delta_1-\hat{\chi}_1^{-1}(t)\right)\hat{\chi}_1(t)\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)|\end{aligned}. \quad (26)$$

A necessary algebra operation gives

$$\begin{aligned}\dot{V}_1(t) &< -\delta_1\tau\eta^2(t) - \delta_1\mu\hat{\chi}_1(t)\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)| \\ &\quad + |\eta(t)|\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right) \\ &\quad + \delta_1\mu\hat{\chi}_1(t)\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)| \\ &\quad - \mu\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)| \\ &= -\delta_1\tau\eta^2(t) + (1-\mu)\left(\hat{\vartheta}_1(t)+\hat{\vartheta}_2(t)\|x(t)\|\right)|\eta(t)|\end{aligned}. \quad (27)$$

By noting $\tau, \delta_1 > 0, \mu > 1$ along with $\hat{\vartheta}_1(0), \hat{\vartheta}_2(0) > 0$, it can be derived from (27) that $\dot{V}_1(t) < 0$. Hence, $\eta(t)$ is able to enter into the region $|\eta(t)| < \varepsilon$.

Case (ii): For $|\eta(t)| < \varepsilon$, (7) can be expressed as

$$\nu(t) = -(G\Gamma)^{-1}\left[\tau\eta(t) + u_{s2}(t,\eta(t))\,\mathrm{sign}(\eta(t))\right]. \quad (28)$$

Next, we will prove that if $|\eta(t)| > \eta_1$, then the proposed method (28) ensures $|\eta(t)| \le \eta_1 < \varepsilon$. Since $\eta(t)$ is bounded, $\nu(t)$ is also bounded and $\delta_1 > 0$ holds.

Let the Lyapunov function be of the form

$$V_2(t) = \frac{1}{2}\left[\eta^2(t) + \kappa_a^2(\eta(t))\right]. \quad (29)$$

Taking the derivative of $V_2(t)$ results in

$$\dot{V}_2(t) = \eta(t)\dot{\eta}(t) + \kappa_a(\eta(t))\dot{\kappa}_a(\eta(t)). \quad (30)$$

Substituting (1) and (4) into (30) leads to

$$\dot{V}_2(t) = \eta(t)\left[G\Gamma\Theta(\nu(t))\nu(t) + G\xi(t)\right] + \kappa_a(\eta(t))\dot{\kappa}_a(\eta(t)). \quad (31)$$

The first derivative of $\eta(t)$ can be represented as

$$\dot{\kappa}_a(\eta(t)) = \frac{\partial\kappa_a(\eta(t))}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial t}. \quad (32)$$

If $|\eta(t)| > \eta_1$, in view of (1), (4), (15), and (32), it can be derived that

$$\dot{\kappa}_a(\eta(t)) = \frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2}\left[G\Gamma\Theta(\nu(t))\nu(t) + G\xi(t)\right]. \quad (33)$$

Taking into account Assumption 1 and (28), the following deduction can be generated from (33)

$$\begin{aligned}\dot{\kappa}_a(\eta(t)) &< \frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2}\left\{\vartheta_2^*\|x(t)\| + \vartheta_1^*\right. \\ &\quad \left.-\delta_1\left[\tau\eta(t) + \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\right]\right\}.\end{aligned} \quad (34)$$

If $\eta(t) < -\eta_1$, similar to the treatment in (33) and (34), it is deduced that

$$\begin{aligned}\dot{\kappa}_a(\eta(t)) &= \frac{-\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2}\left[G\Gamma\Theta(\nu(t))\nu(t) + G\xi(t)\right] \\ &< \frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2}\left\{\vartheta_2^*\|x(t)\| + \vartheta_1^* - \delta_1\left[\tau|\eta(t)| + \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\right]\right\}\end{aligned}. \quad (35)$$

In consideration of (34) and (35), (31) becomes

$$\begin{aligned}\dot{V}_2(t) &< \eta(t)\left[G\Gamma\Theta(\nu(t))\nu(t) + G\xi(t)\right] \\ &+\kappa_a(\eta(t))\frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2} \\ &\times\left\{\vartheta_2^*\|x(t)\| + \vartheta_1^* - \delta_1\left[\tau|\eta(t)| + \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\right]\right\}\end{aligned}. \quad (36)$$

Inserting (6), (15), and (28) into (36) leads to

$$\begin{aligned}\dot{V}_2(t) &< \eta(t)\left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right) - \Theta(\nu(t))\left[\tau\eta(t)\right. \\ &\quad \left.+\ \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\,\mathrm{sign}(\eta(t))\right]\eta(t) \\ &+\kappa_a(\eta(t))\frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2}\left\{\vartheta_2^*\|x(t)\| + \vartheta_1^* - \delta_1\left[\tau|\eta(t)|\right.\right. \\ &\quad \left.\left.+\ \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\right]\right\} \\ &< |\eta(t)|\left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right) \\ &-\delta_1\left[\tau\eta^2(t) + \mu\hat{\chi}_2(t)\kappa_a(\eta(t))|\eta(t)|\right] \\ &+\kappa_a(\eta(t))\frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2} \\ &\times\left\{\vartheta_2^*\|x(t)\| + \vartheta_1^* - \delta_1\left[\tau|\eta(t)| + \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\right]\right\}\end{aligned}. \quad (37)$$

The right-hand side of (37) can be further expressed into the following form:

$$\begin{aligned}\dot{V}_2(t) &< -|\eta(t)|\left[\delta_1\mu\hat{\chi}_2(t)\kappa_a(\eta(t)) - \left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right)\right] \\ &-\delta_1\tau\eta^2(t) \\ &-\kappa_a(\eta(t))\frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2} \\ &\times\left\{\delta_1\left[\tau|\eta(t)| + \mu\hat{\chi}_2(t)\kappa_a(\eta(t))\right] - \left(\vartheta_2^*\|x(t)\| + \vartheta_1^*\right)\right\} \\ &= -|\eta(t)|\delta_1\mu\hat{\chi}_2(t)\left[\kappa_a(\eta(t)) - \frac{\vartheta_2^*\|x(t)\| + \vartheta_1^*}{\delta_1\mu\hat{\chi}_2(t)}\right] - \delta_1\tau\eta^2(t) \\ &-\kappa_a(\eta(t))\frac{\alpha_3\varepsilon}{[\varepsilon-|\eta(t)|]^2} \\ &\times\delta_1\mu\hat{\chi}_2(t)\left\{\kappa_a(\eta(t)) + \frac{\delta_1\tau|\eta(t)|}{\delta_1\mu\hat{\chi}_2(t)} - \frac{\vartheta_2^*\|x(t)\| + \vartheta_1^*}{\delta_1\mu\hat{\chi}_2(t)}\right\}\end{aligned}. \quad (38)$$

To facilitate the proof, the following variables are defined:

$$\partial = \kappa_a(\eta(t)) - \frac{\vartheta_2^*\|x(t)\| + \vartheta_1^*}{\delta_1\mu\hat{\chi}_2(t)}, \quad (39)$$

$$\varpi = \frac{\alpha_3 \varepsilon}{[\varepsilon - |\eta(t)|]^2}. \tag{40}$$

Substituting (39) and (40) into (38), a fundamental algebra yields

$$\begin{aligned} \dot{V}_2(t) &< -|\eta(t)|\,\delta_1 \mu \hat{\chi}_2(t)\,\partial - \kappa_a(\eta(t))\,\varpi \delta_1 \mu \hat{\chi}_2(t)\,\partial \\ &= -\delta_1 \mu \hat{\chi}_2(t)\,\partial\,(|\eta(t)| + \kappa_a(\eta(t))\,\varpi) \end{aligned}. \tag{41}$$

Noting that $\kappa_a(\eta(t))$ is strictly increasing when $\eta(t) \geq 0$ and strictly decreasing when $\eta(t) < 0$ [25] and $\eta_1 < |\eta(t)| < \varepsilon$, it can be derived that $\kappa_a(\eta(t)) > \kappa_a(\eta_1)$. Substituting the expression of η_1 (27) and (28) into (15) gives

$$\kappa_a(\eta_1) = \frac{\vartheta_1^* + \vartheta_2^* \|x(t)\|}{\delta_1 \mu \hat{\chi}_2(t)} > 0. \tag{42}$$

Therefore, $\partial > 0$ and $\varpi > 0$, the following deduction is obtained from (41)

$$\begin{aligned} \dot{V}_2(t) &< -\delta_1 \mu \hat{\chi}_2(t)\,\partial\,(|\eta(t)| + \kappa_a(\eta(t))\,\varpi) \\ &= -\sqrt{2}\delta_1 \mu \hat{\chi}_2(t)\,\partial \left(\frac{|\eta(t)|}{\sqrt{2}} + \frac{\kappa_a(\eta(t))\varpi}{\sqrt{2}} \right) \\ &\leq -\sqrt{2}\delta_1 \mu \hat{\chi}_2(t)\,\partial \min\{1, \varpi\} \left(\frac{|\eta(t)|}{\sqrt{2}} + \frac{\kappa_a(\eta(t))}{\sqrt{2}} \right) \\ &\leq -\sqrt{2}\delta_1 \mu \hat{\chi}_2(t)\,\partial \min\{1, \varpi\}\, V_2^{\frac{1}{2}}(t) \end{aligned}. \tag{43}$$

Now, consider the case $\eta(t) < \eta_1$, $\dot{V}_2$ will be sign indefinite and $\eta(t)$ may leave for η_1. Similar to the treatment in [25–27], [35], $\dot{V}_2 \leq 0$ and V_2 will be constant or decreasing again once $\eta(t)$ reaches η_1, which means that $|\eta(t)| \leq \eta_1$ holds for the rest of time.

It can be concluded from the above analyses that the system trajectory of (1) can enter into and ultimately bounded within a certain region $|\eta(t)| \leq \eta_1 < \varepsilon$ of the sliding surface.

Theorem 1 is proven.

***Remark* 2:** Compared to the state-of-the-art ASMC methods like [9–36], the most significant merits of the proposed SABSMC lie in the following two aspects: First, it forces the system trajectory into a prespecified neighbourhood ε of the sliding surface even under the influence of input saturation, uncertainty, and disturbance. The prespecified neighbourhood ε is independent of the lumped disturbance $\xi(t)$, which allows to design the ultimate bound of the system trajectory in advance and further improve the control accuracy. This is in contrast with previous methods whose upper limits depend on the lumped disturbance $\xi(t)$ or the derivative of lumped disturbance $\dot{\xi}(t)$. Second, the reported method does not impose a priori constant upper limit on the lumped disturbance $\xi(t)$. Instead, a more rational state-dependent assumption (6) is adopted in the proposed method. Moreover, the switching gain of SABSMC is adaptively regulated in a wise manner via the barrier function.

Remark 3: It should be pointed out that in previous methods [25–27], the convergence time $\bar{t}$ is designed as a control gain of the controller, which is difficult to find. In addition, $\eta(t)$ cannot converge to the ultimate bound again if an escape from the ultimate bound takes place at a certain time. In contrast, $\bar{t}$ is replaced by a more reasonable gain ε in the proposed method. Moreover, if an escape occurs at a certain time instant, the proposed method can drive $\eta(t)$ into the ultimate bound again as verified in *Theorem* 1. ■

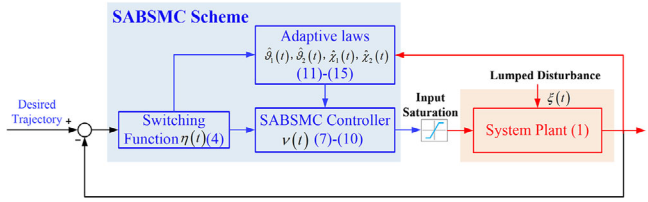

FIGURE 1 Block diagram of the proposed method

Based on the formulation in Section 3, the block diagram of the proposed method is illustrated in Figure 1.

4 | Simulation results

In this section, the effectiveness and virtue of SABSMC are demonstrated through a series of numerical simulations. Throughout simulations, the saturated system (1) with the following parameters [35] is employed: $\Phi = -1$, $\Gamma = 1$, $x(0) = -0.8$. The lumped disturbance is selected as $f(t) = 4.1x(t) + 0.2\sin(3\pi t)$. An adaptive SM controller presented in [35] and in the form of (44) is tested for comparison.

$$\nu_a(t) = -(G\Gamma)^{-1}\left[\tau_1 \eta(t) + \upsilon_1(t, \eta(t))\,\mathrm{sign}(\eta(t))\right], \tag{44}$$

where $\tau_1 > 0$, and

$$\upsilon_1(t, \eta(t)) = \hat{k}_0(t) + \hat{k}_1(t)\,\|x(t)\|, \tag{45}$$

with $\gamma_0, \gamma_1 > 0$

$$\dot{\hat{k}}_0(t) = |\eta(t)| - \gamma_0 \hat{k}_0(t), \tag{46}$$

$$\dot{\hat{k}}_1(t) = |\eta(t)|\,\|x(t)\| - \gamma_1 \hat{k}_1(t), \tag{47}$$

The impact of input saturation is also taken into account in simulation by assigning $\nu_M = 2.5$. To ensure a fair comparison, the initial conditions of the update laws of the two methods are assigned as the same set of values as $\hat{\vartheta}_1(0) = \hat{\vartheta}_2(0) = \hat{\chi}_1(0) = \hat{\chi}_2(0) = \hat{k}_0(0) = \hat{k}_1(0) = 0.5$. Besides, $\tau = \tau_1 = 4$. Other control gains of the two methods are adjusted in repeated trials to guarantee a compromise between convergence rate and control accuracy. These gains are selected as $G = 1$, $\varepsilon = 0.001$, $\alpha_1 = \alpha_2 = \gamma_0 = \gamma_1 = 1.5$, $\alpha_3 = 0.1$, $\mu = 1.1$.

The switching functions of the method in [35] and the proposed method are illustrated in Figures 2 and 3, respectively. It is seen that the method in [35] produces the width of the ultimate bound of 0.021. Concerning the proposed

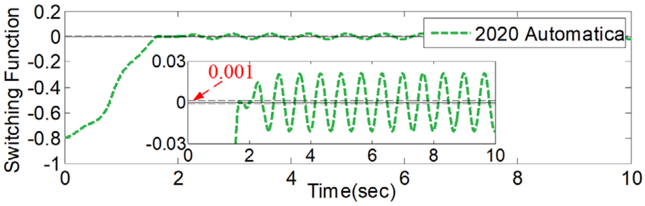

FIGURE 2 Switching function of the method in [35]

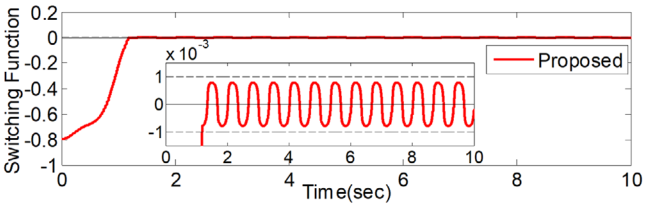

FIGURE 3 Switching function of the proposed method

method, the switching function can converge into and ultimately constrained by a prespecified bound $\varepsilon = 0.001$. As compared with the method in [35], the proposed method mitigates the width of the ultimate bound to 8.4×10^{-4}, which substantially reduces the width by 96%. The control inputs of the two methods are illustrated in Figures 4 and 5. It is observed that the control inputs satisfy the input constraints and are limited within ± 2.5 along the entire evolution. The behaviour of the adaptive switching gain $u_s(t, \eta(t))$ of the proposed method is depicted in Figure 6, which illuminates that $u_s(t, \eta(t))$ changes according to the variation of $\eta(t)$. Hence, the adaptive switching gain of SABSMC is regulated in a wise manner. The simulation results verify the superiority of the proposed method over the method in [35] under the influence of parameter uncertainty, external disturbance, and input saturation.

5 | CONCLUSION

A novel SABSMC method was developed in this paper dedicated to the robust control of saturated systems with respect to parameter uncertainty and external disturbance. In contrast to existing ASMC methods, the SABSMC drives the system trajectory into a prespecified, that is, independent of the lumped disturbance, region of the sliding surface in the presence of input saturation. Furthermore, SABSMC performs better control accuracy without imposing a priori constant upper limit assumption on the lumped disturbance. The property of SABSMC has been analysed in theory. Simulation results show that SABSMC outperforms the previous method under the influence of parameter uncertainty, external disturbance, and input saturation. In the future, the proposed method will be extended to the second-order SM case [41], and it will be applied to practical systems.

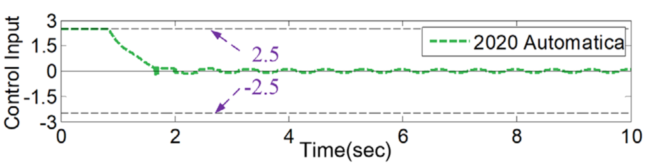

FIGURE 4 Control input of the method in [35]

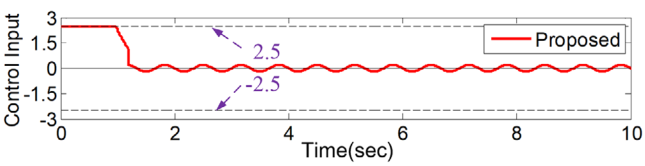

FIGURE 5 Control input of the proposed method

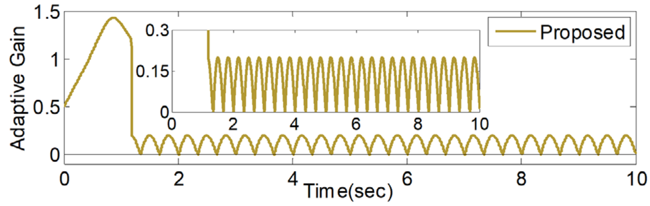

FIGURE 6 Adaptive switching gain $u_s(t, \eta(t))$ of the proposed method

REFERENCES

1. Han, Y., et al.: Non-fragile sliding mode control of discrete switched singular systems with time-varying delays. IET Control Theory Appl. 14(5), 726–737 (2020)
2. Li, J., Niu, Y.: Sliding mode control subject to rice channel fading. IET Control Theory Appl. 13(16), 2529–2537 (2019)
3. Ma, H., Li, Y., Xiong, Z.: Design of funnel function-based discrete-time sliding mode control. IET Control Theory Appl. 14(16), 2413–2418 (2020)
4. Wang, Z., Li, S., Li, Q.: Discrete-time fast terminal sliding mode control design for DC-DC buck converters with mismatched disturbances. IEEE Trans. Ind. Inf.. 16(2), 1204–1213 (2020)
5. Ma, H., et al.: An active control method for chatter suppression in thin plate turning. IEEE Trans. Ind. Inf. 16(3), 1742–1753 (2020)
6. Jiang, B., et al.: Observer-based adaptive sliding mode control for nonlinear stochastic markov jump systems via t-s fuzzy modeling: applications to robot arm model. IEEE Trans. Ind. Electron. 68(1), 466–477 (2021)
7. Kumari, K., et al.: Output feedback based event-triggered sliding mode control for delta operator systems. Automatica 103, 1–10 (2019)
8. Rubagotti, M., et al.: Constrained nonlinear discrete-time sliding mode control based on a receding horizon approach. IEEE Trans. Autom. Control. (2020). https://doi.org/10.1109/TAC.2020.3024349
9. Ferrara, A., Incremona, G., Regolin, E.: Optimization-based adaptive sliding mode control with application to vehicle dynamics control. Int. J. Robust Nonlinear Control 29(3), 550–564 (2019)
10. Shtessel, Y., Moreno, J., Fridman, L.: Twisting sliding mode control with adaptation: Lyapunov design, methodology and application. Automatica 75, 229–235 (2017)
11. Bartolini, G., et al.: Adaptive reduction of the control effort in chattering-free sliding-mode control of uncertain nonlinear systems. J. Appl. Math. Comp. 8(1), 51–71 (1998)
12. Utkin, V., Poznyak, A.: Adaptive sliding mode control with application to super-twist algorithm: equivalent control method. Automatica 49(1), 39–47 (2013)
13. Edwards, C., Shtessel, Y.: Adaptive continuous higher order sliding mode control. Automatica 65, 183–190 (2016)
14. Incremona, G., Cucuzzella, M., Ferrara, A.: Adaptive suboptimal second-order sliding mode control for microgrids. Int. J. Control 89(9), 1849–1867 (2016)

15. Chang, Y.: Adaptive sliding mode control of multi-input nonlinear systems with perturbations to achieve asymptotical stability. IEEE Trans. Autom. Control 54(12), 2863–2869 (2009)
16. Negrete-Chávez, D., Moreno, J.: Second-order sliding mode output feedback controller with adaptation. Int. J. Adapt. Control Signal Process. 30(8), (2016) 1523–1543
17. Edwards, C., Shtessel, Y.: Adaptive dual-layer super-twisting control and observation. Int. J. Control 89(9), 1759–1766 (2016)
18. Baek, J., Jin, M., Han, S.: A new adaptive sliding-mode control scheme for application to robot manipulators. IEEE Trans. Ind. Electron. 63(6), 3628–3637 (2016)
19. Xu, Q.: Precision motion control of piezoelectric nanopositioning stage with chattering-free adaptive sliding mode control. IEEE Trans. Autom. Sci. Eng. 14(1), 238–248 (2017)
20. Han, H., Wu, X., Qiao, J.: Design of robust sliding mode control with adaptive reaching law. IEEE Trans. Syst. Man Cybern.: Syst. 50(11), 4415–4424 (2020)
21. Van, M., Ge, S., Ren, H.: Robust fault-tolerant control for a class of second-order nonlinear systems using an adaptive third-order sliding mode control. IEEE Trans. Syst. Man Cybern.: Syst. 47(2), 221–228 (2017)
22. Fei, J., Chu, Y.: Double hidden layer output feedback neural adaptive global sliding mode control of active power filter. IEEE Trans. Power Electron. 35(3), 3069–3084 (2020)
23. Feng, Y., et al.: Integral-type sliding-mode control for a class of mechatronic systems with gain adaptation. IEEE Trans. Ind. Inf, 16(8), 5357–5368 (2018)
24. Plestan, F., et al.: New methodologies for adaptive sliding mode control. Int. J. Control 83(9), 1907–1919 (2010)
25. Obeid, H., et al.: Barrier function-based adaptive sliding mode control. Automatica 93, 540–544 (2018)
26. Obeid, H., et al.: Barrier function-based variable gain super-twisting controller. IEEE Trans. Autom. Control 65(11), 4928–4933 (2020)
27. Laghrouche, S., Harmouche, M., Chitour, Y., Obeid, H., Fridman, L.: Barrier function-based adaptive higher order sliding mode controllers. Automatica 123, 109355 (2021)
28. Ding, S., Liu, L., Park, J.: A novel adaptive nonsingular terminal sliding mode controller design and its application to active front steering system. Int. J. Robust Nonlinear Control 29(12), 4250–4269 (2019)
29. Bartolini, G., et al.: Adaptation of sliding modes. IMA J. Math. Control Inf. 30(3), 285–300 (2013)
30. Wang, G., Chadli, M., Basin, M.: Practical terminal sliding mode control of nonlinear uncertain active suspension systems with adaptive disturbance observer. IEEE/ASME Trans. Mechatron. 26, 789–797 (2020) https://doi.org/10.1109/TMECH.2020.3000122
31. Shtessel, Y., Taleb, M., Plestan, F.: A novel adaptive-gain supertwisting sliding mode controller: methodology and application. Automatica 48(5), 759–769 (2012)
32. Cong, B., Chen, Z., Liu, X.: On adaptive sliding mode control without switching gain overestimation. Int. J. Robust Nonlinear Control 24(3), 515–531 (2014)
33. Laghrouche, S., et al.: Control of PEMFC air-feed system using lyapunov-based robust and adaptive higher order sliding mode control. IEEE Trans. Control Syst. Technol. 23(4), 1594–1601 (2015)
34. Tran, D., Ba, D., Ahn, K.: Adaptive backstepping sliding mode control for equilibrium position tracking of an electrohydraulic elastic manipulator. IEEE Trans. Ind. Electron. 67(5), 3860–3869 (2020)
35. Roy, S., Baldi, S., Fridman, L.: On adaptive sliding mode control without a priori bounded uncertainty. Automatica 111, 108650
36. Roy, S., et al.: Adaptive-robust control of a class of EL systems with parametric variations using artificially delayed input and position feedback. IEEE Trans. Control Syst. Technol. 27(2), 603–615 (2019)
37. Zhu, Z., Xia, Y., Fu, M.: Adaptive sliding mode control for attitude stabilization with actuator saturation. IEEE Trans. Ind. Electron. 58(10), 4898–4907 (2011)
38. Xiao, B., Hu, Q., Zhang, Y.: Adaptive sliding mode fault tolerant attitude tracking control for flexible spacecraft under actuator saturation. IEEE Trans. Control Syst. Technol. 20(6), 1605–1612 (2012)
39. Aghababa, M.: Controlling fluctuated chaotic power systems with compensation of input saturation: application to electric direct current machines. ASME J. Dyn. Syst. Meas. Control 141(1), 011012 (2019)
40. Guo, Y., et al.: On adaptive sliding mode control without switching gain overestimation. Int. J. Robust Nonlinear Control 24(4), 1088–1100 (2019)
41. Mei, K., Ding, S.: Second-order sliding mode controller design subject to an upper-triangular structure. IEEE Trans. Syst. Man Cybern.: Syst. 51(1), 497–507 (2021)

How to cite this article: Ma H, Liu W, Li Y, Liu Z, Tian S. Saturated adaptive barrier sliding mode control with state-dependent uncertainty limit. IET Control Theory Appl. 2021;15:1762–1768.
https://doi.org/10.1049/cth2.12157