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Robustness evaluation for multi-subnet composited complex network of urban public transport

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Abstract Drawing on complex network theory, this paper combines bus network and subway network into a multi-subnet composited complex network (MSCCN) of urban public transport (UPT). Then, the cascading failure of the MSCCN nodes and edges was modelled, and the passenger flow transfer rules were established under node and edge failures. Considering the impact of extreme weather on the UPT, a synthetic operator was created to measure the robustness of UPT MSCCN, in the light of network topology and passenger flow, and used to quantify the robustness of UPT MSCCN under extreme weather. Finally, a UPT MSCCN was set up for Qingdao, China, and subject to cascading failure simulation. The simulation results reveal how MSCCN robustness changes with different variables. The research findings help to optimize the response of the MSCCN to extreme weather.

1. Introduction

In urban areas, public transport is the primary means to meet the daily travel demand of numerous urban residents. However, urban public transport (UPT) might operate abnormally under emergencies, and even get paralyzed in severe cases. For instance, UPT stations and lines could fail continuously, when a huge number of passengers transfer simultaneously. Since the UPT is a typical complex network [1,2], the continuous failure can be described well with the cascading failure model [3] of complex network. The cascading failure seriously hinders the normal travel of residents, exposing the vulnerability of the UPT to violent attacks and severe damages. Therefore, it is important to evaluate the robustness of the UPT based on the complex network theory.

Many scholars have explored the robustness of the UPT. Some [4,5] held that the robustness only depends on the failure consequence of certain nodes, and has nothing to do with the probability of failure. Some [6–8] argued that the robustness is related to risk, and regarded UPT robustness as the product of failure probability and failure consequence. However, most studies on the UPT only consider a single public transport network (e.g. bus network or subway network). Even if the two networks are both considered, the researchers rarely distinguish between them, but treat the stations and lines of the two networks as homogenous. This is obviously in conflict with the actual situation. In the real world, the bus network and subway network dif-

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fer greatly in station capacity, load capacity, line length, etc. The robustness of urban public transport system is the specific application of the concept of robustness in the field of urban public transport. Extreme weather will have an impact on all stations and lines of urban transportation system, and this impact will spread through the conversion of system passenger flow between different stations and lines, which will lead to the occurrence of cascading failures of stations and lines in urban public transport system.

Inspired by the composited complex network of multiple subsets [9], this paper composites the bus network and subway network of UPT into a multi-subnet composited complex network (MSCCN). Then, the passenger flow was imported to the UPT MSCCN to simulate the functional change of the actual UPT under cascading failure. The consequences and influencing factors of UPT cascading failure were discussed in details. The research findings provide theoretical guidance for the UPT to deal with extreme situations.

2. Literature review

Many scholars have evaluated the robustness of various real-world systems [10–12], using the feature indices of complex networks. When it comes to the robustness of transport networks, the evaluation aims to assess the effects of node and edge attack strategies on the performance of the entire network. Dakic et al. [13] analyzed the attack vulnerability of UPT in L space and P space. By scanning the entire network, Sui et al. [14] examined the impact of partial failure of UPT functions on social cost. Yap et al. [15] determined the most vulnerable edge of multi-level UPT, and quantified the social cost of that edge, revealing the extreme fragility of the congestion connection in light rail or subway. Dong et al. [16] evaluated UPT robustness with complex network indices: node size, mean shortest path, and network efficiency. Zhang et al. [17] quantified the structural vulnerability of urban rail transit network under random and deliberate attacks. Taking the relative size of the most connected subgraph and network efficiency as indices, Han et al. [18] measured the invulnerability of UPT under different attack modes, and attributed the UPT reliability to the stability of key bus hubs.

The effect of UPT under extreme conditions has also attracted much attention. Rowan et al. [19] studied the UPT changes induced by extreme weather and climate change. Based on the data on smart bus card and weather conditions, Miao et al. [20] investigated the impact of weather variables on UPT, and found that temperature has the greatest impact on the travel of urban residents. Xu et al. [21] expounded the impact of rainstorm and other extreme weathers on urban road operation, from the angles of travel demand, travel mode, road operation, and traffic safety. Starting with measured data on traffic flow, Vlahogianni et al. [22] summarized the micro- and macro-scale features of traffic flow on expressways under rainstorm and other extreme weathers. Zhao et al. [23] analyzed the influence of rain and snow over traffic congestions on urban expressways, predicted the state of traffic congestion with the Markov model, and verified the effectiveness of the model against the actual congestion situation.

3. Construction of UPT MSCCN

Bus and subway are the main transport modes of UPT. If any emergency or extreme weather occurs, traffic congestion and accident will ensue, making some UPT stations and lines fail (i.e. unable to transport passengers). In this case, the passengers need to be transferred from the failed stations and lines to other normal stations and lines. The transfer might overload the normal stations and lines, resulting in functional failure. This process will continue until all UPT stations and lines resume the normal state or enter the failure state. In other words, the UPT will end up in cascading failure.

In most of the relevant studies, the UPT cascading failure is abstracted into a single complex network, and the network change under random or deliberate attack is simulated with various cascading failure models. However, a single complex network only contains one kind of nodes and edges, failing to reflect the heterogeneity of nodes and edges in different UPT subnetworks. This defect can be solved by the MSCCN, a network of heterogenous nodes and edges that illustrates the relationship between various subnetworks and their elements. Compared with signal complex network, the MSCCN can fully demonstrate the heterogeneity of different UPT subnetworks, and provide a full picture of the UPT.

In this paper, the bus network and subway network are composited into the UPT MSCCN under the following hypotheses:

(1) Each station in the bus network is a network node. If two stations are adjacent to each other on a bus line, the corresponding network nodes must be connected.
(2) Each station in the subway network is a network node. If two stations are adjacent to each other on a subway line, the corresponding network nodes must be connected.
(3) If the distance between a bus station and a subway station is shorter than 200 m, the stations are transfer stations, and the corresponding network nodes must be connected. The edge between these nodes represents the transfer path between bus and subway in the UPT.
(4) If more than one bus lines or subway lines pass between two stations, the corresponding network nodes should be connected by only one edge, in order to prevent repeated connections; these lines are differentiated by the passenger flow distribution of the edge.
(5) The MSCCN is unidirectional, without considering the direction of bus or subway train.
(6) If a station fails, all the passengers at the station will be transferred to its adjacent nodes. The number of passengers at each station depends on the passenger flow on the edge between the station and the failed station as a proportion of the passenger volume of the failed station.

The MSCCN can be defined as a four-tuple $G = (V, E, R, F)$, where V is the set of various nodes, E is the set of edges, $R = R_1 \times \ldots \times R_i \times \ldots \times R_n$ is the set of relationships, and F: $E \xrightarrow{\varphi} R$ is the function that maps each relationship to specified edges. During the composition of the MSCCN, the relationship between bus network and subway network was established based on transfer stations. The sketch map of UPT MSCCN is shown in Fig. 1.

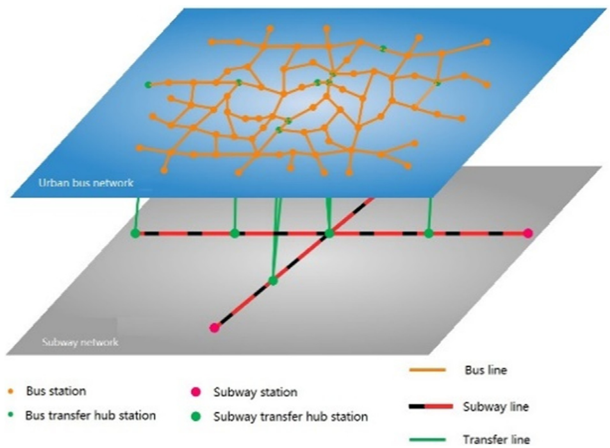

Fig. 1 The sketch map of UPT MSCCN.

The adjacency matrix A of the UPT MSCCN consists of an adjacency matrix of bus network A_b, an adjacency matrix of subway network A_s, and an adjacency matrix between the two networks $A_{b,s}$. Among them, the adjacency matrix of bus network A_b can be defined as:

$$A_b = \begin{bmatrix} a_{11,b} & \cdots & a_{1n,b} \\ \vdots & \ddots & \vdots \\ a_{n1,b} & \cdots & a_{nn,b} \end{bmatrix}$$

where, n is the number of nodes in the bus network; $a_{ij,b}$ is an indicator of the adjacency between two nodes $n_{i,b}$ and $n_{j,b}$ in the bus network:

$$a_{ij,b} = \begin{cases} 0, & when\ node\ n_{i,b}\ and\ node\ n_{j,b}\ are\ not\ directly\ connected \\ 1, & when\ node\ n_{i,b}\ and\ node\ n_{j,b}\ are\ directly\ connected \end{cases}$$

The adjacency matrix of subway network A_s can be defined as:

$$A_s = \begin{bmatrix} a_{11,s} & \cdots & a_{1m,s} \\ \vdots & \ddots & \vdots \\ a_{m1,s} & \cdots & a_{mm,s} \end{bmatrix}$$

where, m is the number of nodes in the subway network; $a_{ij,s}$ is an indicator of the adjacency between two nodes $n_{i,s}$ and $n_{j,s}$ in the subway network:

$$a_{ij,s} = \begin{cases} 0, when\ node\ n_{i,s}\ and\ node\ n_{j,s}\ are\ not\ directly\ connected \\ 1, when\ node\ n_{i,s}\ and\ node\ n_{j,s}\ are\ directly\ connected \end{cases}$$

The adjacency matrix between the two networks $A_{b,s}$ can be defined as:

$$A_{b,s} = \begin{bmatrix} a_{11,bs} & \cdots & a_{1m,bs} \\ \vdots & \ddots & \vdots \\ a_{n1,bs} & \cdots & a_{nm,bs} \end{bmatrix}$$

where, n and m are the number of nodes in the bus network and subway network, respectively; $a_{ij,bs}$ is an indicator of the distance between a node $n_{i,b}$ in the bus network and a node $n_{j,s}$ in the subway network:

$$a_{ij,bs} = \begin{cases} 0, when\ the\ distance\ between\ node\ n_{i,b}\ and\ node\ n_{j,s}\ is\ less\ than\ 200m \\ 1, when\ the\ distance\ between\ node\ n_{i,b}\ and\ node\ n_{j,s}\ is\ not\ less\ than\ 200m \end{cases}$$

$a_{ij,bs}$ becomes the outer edge connecting the two subnets. The number of outer edges determines the quality of connectivity between the two subnets.

In the MSCCN, the weight of an edge is the passenger flow on that edge as a proportion of the total passenger flow in the network. Then, the weight matrix W of the MSCCN can be defined as:

$$W = \begin{bmatrix} w_{11} & \cdots & w_{1|N|} \\ \vdots & \ddots & \vdots \\ w_{|N|1} & \cdots & w_{|N||N|} \end{bmatrix}$$

where, $|N|$ is the total number of nodes in the MSCCN; $|N| = n + m$, w_{ij} is the weight of edge e_{ij}:

$$w_{ij} = f_{ij}/f_{total}$$

where, f_{ij} is the passenger flow on edge e_{ij}; f_{total} is the total passenger flow of the MSCCN.From the perspective of passenger transport function, the importance of each node n_i in the MSCCN was measured by the passenger flow through the node per unit time, a.k.a. the passenger flow intensity S_i:

$$S_i = \sum_i f_{ij}$$

The capacity of nodes and edges was measured by the maximum passenger volume that can be transported by them per unit time. Hence, the edge capacity matrix C of the MSCCN can be defined as:

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1|N|} \\ \vdots & \ddots & \vdots \\ c_{|N|1} & \cdots & c_{|N||N|} \end{bmatrix}$$

where, c_{ij} is the capacity of edge e_{ij}, i.e. the maximum passenger volume that can be transported by edge e_{ij} per unit time.

The capacity c_i of a node n_i in the MSCCN can be determined by that of its directly connected edges:

$$c_i = \sum_i c_{ij}$$

According to the classical load capacity model [24], the capacity of an edge in the network is proportional to its initial load. Therefore, the capacity of edge e_{ij} can be expressed as:

$$c_{ij} = (1 + \beta)I_{ij}$$

where, β is the tolerance coefficient, reflecting the proportion of additional flow that can be accommodated by edge e_{ij}; I_{ij} is the initial load on edge e_{ij}.

4. Cascading failure model of UPT MSCCN

In the actual UPT, the failure of any node or edge will propagate to other nodes or edges, as all nodes and edges are coupled with each other. The failure propagation continues until the entire network is restabilized or collapses. This chain reaction is called cascading failure.

The proposed UPT MSCCN is an unidirectional weighted network, in which each edge is weighted by the passenger flow on the edge as a proportion of the total passenger flow. Under extreme conditions, the cascading failure of the MSCCN will unfold as follows: At first, the MSCCN belongs to normal state, where all nodes and edges can transport passengers normally. Once an extreme condition occurs, some nodes or edges will fail directly. Then, the passengers transported by the failed nodes or edges will be transferred to adjacent nodes and edges. The transport might overload the adjacent nodes and edges, causing them to fail. The node or edge failure will continue to spread until the entire network ceases to have any node or edge failure, or eventually collapses.

To sum up, the initial failure of nodes and edges is induced by the extreme condition, but the subsequent cascading failure process is driven by the transfer of passenger flow from the failed nodes and edges.

4.1. Flow-based cascading failure model

Here, a flow-based cascading failure model is designed for UPT MSCCN. The model consists of two parts: a model of node failure and a model of edge failure.

(1) Node failure model

In the model of node failure, the state variable v_i of node i can be expressed as:

$$v_i(t+1) = \left|(1-\vartheta_1)f(v_i(t)) + \vartheta_1 \sum_{j=1,j\neq i}^{|N|_M} f_{ij}f(v_i(t))/S_i\right|$$

where, t is time; $v_i(t+1)$ and $v_i(t)$ are the state variables of node i at time $t+1$ and time t, respectively (if $0 \le v_i(t+1) \le 1$, the node operates normally; if and $v_i(t+1) > 1$, the node fails); ϑ_1 is the coupling strength of node i, reflecting the degree of interaction between node i and its adjacent node (in the real world, ϑ_1 represents the impact of the state of a bus or subway station on the state of its adjacent bus or subway station); $|N|_M$ is the number of nodes in the most connected subgraph of the MSCCN (with chaotic features, traffic flow can be represented by chaotic Logistic map [25,26]); f is the function indicating that each MSCCN node is a chaotic dynamic system, which simulates the local dynamic behavior of a bus or subway station in the actual UPT.

Next, the external disturbance variable *edv* was introduced to the model of node failure. The variable was generated under extreme conditions, which will affect all nodes in the MSCCN. Thus, the state variable of node i under extreme conditions can be expressed as:

$$v_i(t) = \frac{1}{1-\sigma edv^{\tau}}\left|(1-\vartheta_1)f(v_i(t-1)) + \vartheta_1 \sum_{j=1,j\neq i}^{|N|_M} f_{ij}f(v_i(t-1))/S_i\right|$$

The state variable of node i multiplied by $1/1-\sigma edv^{\tau}$ indicates that extreme conditions will increase the failure probability of MSCCN nodes. Besides, the growth of *edv* will increase the probability of $v_i(t) > 1$, that is, cause more nodes to fail.

Once node i fails, the node and its directly connected edges will be removed. The state variables of the removed nodes and edges will always be infinitely positive, and the passengers transported by these nodes and edges will be transferred to their adjacent nodes and edges.

The model of node failure can accurately describe the cascading failure process of MSCCN nodes under extreme conditions. In the beginning, the UPT MSCCN operates normally, where the state variables of all nodes fall within [0, 1]. After the occurrence of extreme conditions, some nodes might fail and their state variables will change. The state change of a failed node will propagate to its adjacent nodes through the transfer of passenger flow, kicking off a new round of node failure. The propagation will continue until the network resumes normal operation or collapse as a whole.

(2) Edge failure model

In the model of edge failure, the state variable of an edge in the MSCCN is not solely dependent on the edge itself, but also affected by the two nodes on its ends. The state variable u_{ij} of edge e_{ij} can be expressed as:

$$u_{ij}(t+1) = \max\left\{\left|(1-\vartheta_2)f(u_{ij}(t)) + \frac{\vartheta_2}{2}\left(\sum_{w=1,w\neq i,j}^{|N|_M} f_{iw}f(u_{iw}(t))/S_i + \sum_{z=1,z\neq i,j}^{|N|_M} f_{jz}f(u_{jz}(t))/S_j\right)\right|, v_i(t), v_j(t)\right\}$$

where, $v_i(t)$ and $v_j(t)$ are the state variables of node i and node j at time t, respectively (if $v_i(t) > 1$ or $v_j(t) > 1$, then $u_{ij}(t+1) > 1$; that is, edge e_{ij} will fail as long as one of the two nodes on its ends fails); $u_{ij}(t+1)$ and $u_{ij}(t)$ are state variables of edge e_{ij} at time t and time $t+1$, respectively (if $0 \le u_{ij}(t+1) \le 1$, edge e_{ij} operates normally; if $u_{ij}(t+1) > 1$, edge e_{ij} fails); ϑ_2 is the coupling strength of edge e_{ij}, reflecting how much the state change of the edge is affected by that of its adjacent edges. The above equation indicates that the state of the edge e_{ij} is affected not only by its own disturbance, but also by node i and node j.

Similar to the model of node failure, an external disturbance variable *edv* was also introduced to the model of edge failure. The variable was generated under extreme conditions, which will affect all edges in the MSCCN. Thus, the state variable of edge e_{ij} under extreme conditions can be expressed as:

$$u_{ij}(t) = \max\left\{\frac{1}{1-\sigma edv^{\tau}}\left|(1-\vartheta_2)f(u_{ij}(t-1)) + \frac{\vartheta_2}{2}\left(\sum_{w=1,w\neq i,j}^{|N|_M} f_{iw}f(u_{iw}(t-1))/S_i + \sum_{z=1,z\neq i,j}^{|N|_M} f_{jz}f(u_{jz}(t-1))/S_j\right)\right|, \frac{1}{1-\sigma edv^{\tau}}v_i(t-1), \frac{1}{1-\sigma edv^{\tau}}v_j(t-1)\right\}$$

The state variable of edge e_{ij} at time t is affected by two factors: (1) the state and passenger flow of adjacent edges e_{iw} and e_{jz} as well as nodes i and j at time $t-1$; (2) the intensity of external disturbance variable *edv*. If $u_{ij}(t-1) > 1$, then $u_{ij}(t) > 1$, and the state variables of the edge e_{ij} will remain infinitely positive after time t. If $0 \le u_{ij}(t-1) \le 1$, then the $u_{ij}(t)$ value will be calculated by the failure model.

According to the model of edge failure, the growth of external disturbance variable *edv* will increase the failure probability of edges; the more intense the disturbance, the greater the probability for bus and subway lines to fail. Once fails, edge e_{ij} will be removed from the network, and its passenger flow will be transferred to its adjacent edges.

The model of edge failure can accurately describe the cascading failure process of MSCCN nodes under extreme conditions. In the beginning, the UPT MSCCN operates normally, where the state variables of all edges fall within [0, 1]. After the occurrence of extreme conditions, some edges might fail and their state variables will change. The state change of a failed node will propagate to its adjacent edges through the transfer of passenger flow, kicking off a new round of node failure. The propagation will continue until the network resumes normal operation or collapse as a whole.

4.2. Transfer rules of passenger flow

In UPT MSCCN, passenger flow transfer takes place under three conditions: passenger overload, node failure and edge failure. The transfer occurs under the assumption that the passenger flow of overloaded or failed nodes and edges will be transferred to the normal nodes or edges directly connected with them. After all, passengers in real-world UPT tend to move from the failed station or line to the nearest station or line to shorten the travel time.

The proposed flow-based cascading failure model focuses on the passenger flow transfer in the most connected subgraph (M) of the MSCCN. The isolated failed nodes and edges are removed from the MSCCN, along with their passenger flows. The transfer rules of passenger flow were defined as follows in each of the three conditions:

(1) Transfer rules of passenger flow under passenger overload

Step 1. Search for overloaded edge e_{ij} in the most connected subgraph of the MSCCN.

Step 2. Remove edge e_{ij} from the MSCCN.

Step 3. Transfer the passenger flow f_{ij} of edge e_{ij} to its adjacent normal edges e_{iw} and e_{jz}. The transfer from edge e_{ij} to edge e_{iw} can be described as $f_{iw} \rightarrow f_{iw} + \Delta f_{iw}$, where $\Delta f_{iw} = f_{ij} \cdot (f_{iw} - c_{ij})/(S_i - f_{ij})$. The transfer from edge e_{ij} to edge e_{jz} is similar to that from edge e_{ij} to edge e_{iw}.

Step 4. Judge whether all edges have been traversed. If yes, terminate the search; otherwise, repeat Steps 1–3.

(2) Transfer rules of passenger flow under node failure

Step 1. Search for failed node i in the most connected subgraph of the MSCCN. Since the failed node cannot operate normally, no bus or subway could pass through this node, not to mention transporting passengers via the node.

Step 2. Remove node i and all edges connected to it.

Step 3. Transfer the passenger flow S_i of node i to the normal nodes j adjacent to it.

Step 4. Change the passenger flow of edge e_{jz} connected with node j by $f_{jz} \rightarrow f_{jz} + \Delta f_{jz}$, where $\Delta f_{jz} = f_{ij} \cdot f_{jz}/(S_i - f_{ij})$.

Step 5. Judge whether all nodes have been traversed. If yes, terminate the search; otherwise, repeat Steps 1–4.

(3) Transfer rules of passenger flow under edge failure

Step 1. Search for failed edge i in the most connected subgraph of the MSCCN. Since the failed edge cannot operate normally, no bus or subway could pass through this edge, not to mention transporting passengers via the edge.

Step 2. Remove edge e_{ij} from the MSCCN.

Step 3. Transfer the passenger flow of edge e_{ij} to its adjacent normal edges e_{iw} and e_{jz}. The transfer from edge e_{ij} to edge e_{iw} can be described as $f_{iw} \rightarrow f_{iw} + \Delta f_{iw}$, where $\Delta f_{iw} = f_{ij} \cdot (f_{iw} - c_{ij})/(S_i - f_{ij})$. The transfer from edge e_{ij} to edge e_{jz} is similar to that from edge e_{ij} to edge e_{iw}.

Step 4. Judge whether all edges have been traversed. If yes, terminate the search; otherwise, repeat Steps 1–3.

The above three passenger flow transfer processes can be expressed by the flow chart shown in Fig. 2.

5. Simulation and results analysis

5.1. Robustness indices of UPT MSCCN

The network robustness under different attack strategies is usually measured by the change of node size and network efficiency in the most connected subgraph [27–29]. This paper considers how the topology and functions of UPT MSCCN change under cascading failure. Since the UPT MSCCN mainly serves to transport passengers, the change of network function could be measured by the change of network passenger flow. Therefore, the following indices were selected to evaluate the robustness of UPT MSCCN.

(1) The change of node size

Initially, all the nodes of the UPT MSCCN are concentrated in one connected subgraph. The number of nodes in this graph is denoted as $|N|_A$. When node and edge failures cease, the network may contain multiple connected subgraphs. The number of nodes in this subgraph is denoted as $|N|_M$. Then, the change of the node size of the most connected subgraph can be expressed as:

$$\rho(|N|) = \frac{|N|_A - |N|_M}{|N|_A}$$

where, $\rho(|N|) \in [0, 1]$ is the node loss rate of the most connected subgraph. Cascading failure propagation is usually regarded as a branching process. The initial failure (the root node of the tree) will cause more failures (the leaf node of the tree) to form a tree structure. Therefore, the change of the node size of the most connected subgraph can be expressed linearly.

(2) The change of network efficiency

The convenience of connection or communication between a pair of nodes can be measured by efficiency, which depends on the shortest path. If node i and node j are disconnected, the shortest path d_{ij} between them is infinitely positive. Then, the connectivity efficiency E_{ij} between the two nodes can be expressed as: $E_{ij} = 1/d_{ij}$. The network efficiency E is the mean connectivity efficiency of all node pairs:

$$E = \frac{1}{|N|_A(|N|_A - 1)} \sum_{i \neq j} \frac{1}{d_{ij}}$$

Let E_I and E_C be the efficiencies of UPT MSCCN in the initial state and after cascading failure, respectively. Then, the change of network efficiency $\rho(E) \in [0, 1]$ can be expressed as:

$$\rho(E) = \frac{E_I - E_C}{E_I}$$

(3) The change of total passenger flow

Passenger flow mirrors functional integrity of UPT MSCCN. In this paper, only the change of total passenger flow in the most connected subgraph is considered, while that in

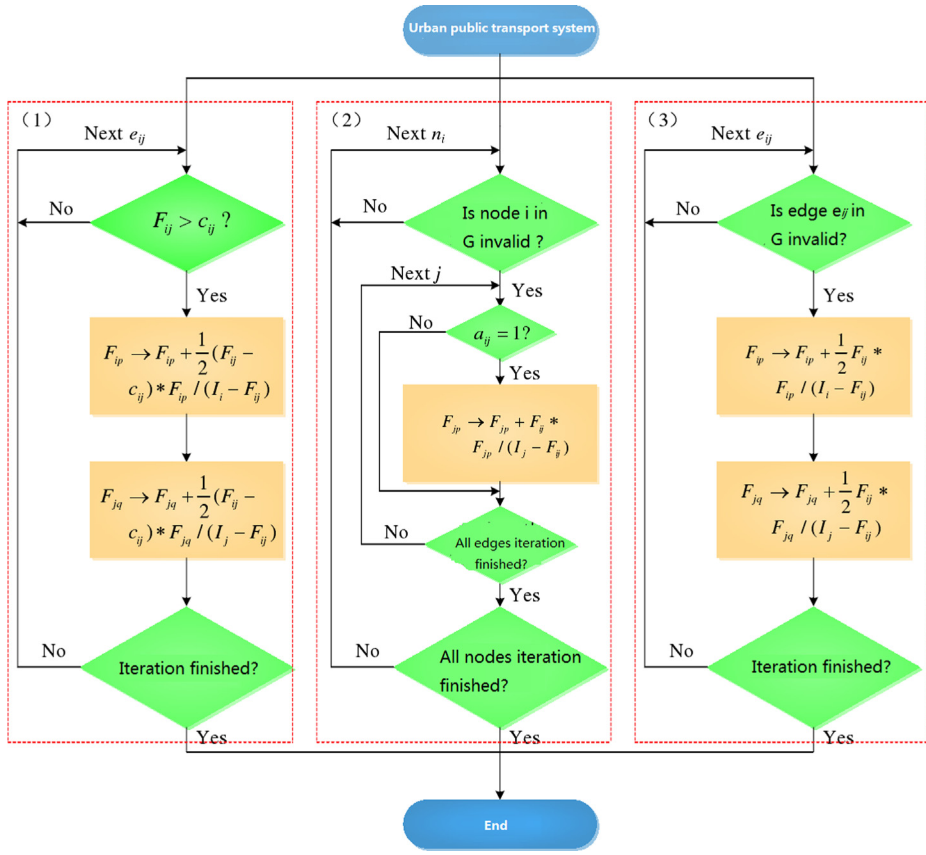

Fig. 2 The flow chart of three passenger flow transfer processes.

other isolated parts is directly removed. The total passenger flow f_{total} of the most connected subgraph can be obtained by adding up the passenger flows of the edges in the subgraph:

$$f_{total} = \sum_{i \in M}^{|N|_M} \sum_{j \in M, j>i}^{|N|_M} f_{ij}$$

where, f_{ij} is the passenger flow on edge e_{ij}.

Let f_I and f_C be the total passenger flows of the most connected subgraph in the initial state and after cascading failure, respectively. Then, the change of the total passenger flow $\rho(f) \in [0, 1]$ of the subgraph can be expressed as:

$$\rho(f) = \frac{f_I - f_C}{f_I}$$

5.2. Synthetic operator

Based on the three robustness indices, a synthetic operator for robustness evaluation was designed by assigning weight to each index. The main index weighting methods include addition scoring method, weighted addition scoring method, modified weighted addition scoring method, and product scoring method. In this paper, the weighted addition scoring method [30] is adopted to synthesize the robustness operator R:

$$R = \delta_1 \rho(|N|) + \delta_2 \rho(E) + \delta_3 \rho(f)$$

where, δ_1, δ_2 and δ_3 are the effects of node size, network efficiency, and total passenger flow on network robustness ($\delta_1, \delta_2, \delta_3 \in (0, 1)$, and $\delta_1 + \delta_2 + \delta_3 = 1$). Obviously, the R value is negatively correlated with network robustness. In other words, a high value of the operator means the network topology and passenger transport function are greatly affected by extreme conditions of the same intensity.

5.3. Simulation of UPT MSCCN

Qingdao, a coastal city in northern China, was selected as an example to verify the robustness of PTS under cascading failure induced by extreme conditions. Figs. 3 and 4 present the subway lines and bus lines of Qingdao, respectively.

According to the statistics of Gaode Map, there are 56 regular bus stations, 23 subway stations, 97 bus lines, and 2 subway lines in Qingdao. The node degrees of the bus network, subway network, and the UPT MSCCN in Qingdao are displayed in Fig. 5. It can be seen that the node degrees of the three networks obey the power-law distribution to a certain extent. The distribution indicates that all three networks are scale free: most nodes have a few connected edges, and a few central nodes have many edges. Therefore, any small local change will be amplified and spread to the entire network, causing a total collapse.

According to Qingdao Municipal Transportation Committee, the bus network of Qingdao transports 2.74 million person·time of passengers each day, while the subway network carries 420,000 person time. The transfer rules of the bus network were defined as follows: first, the double track coefficient of each edge was counted; then, the total double line coefficient

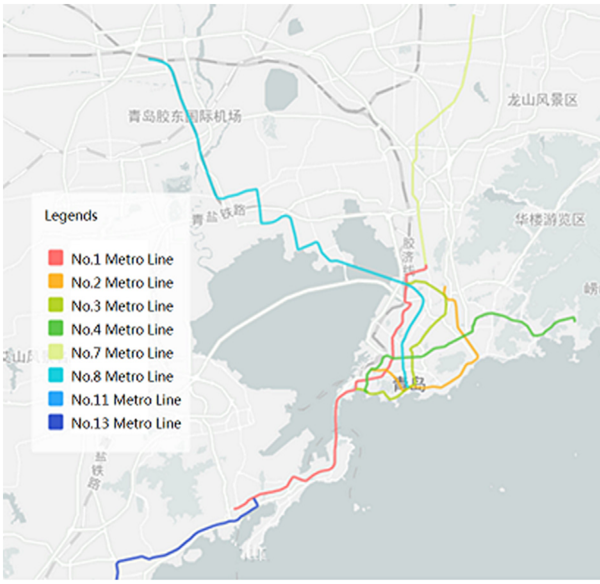

Fig. 3 The subway lines of Qingdao.

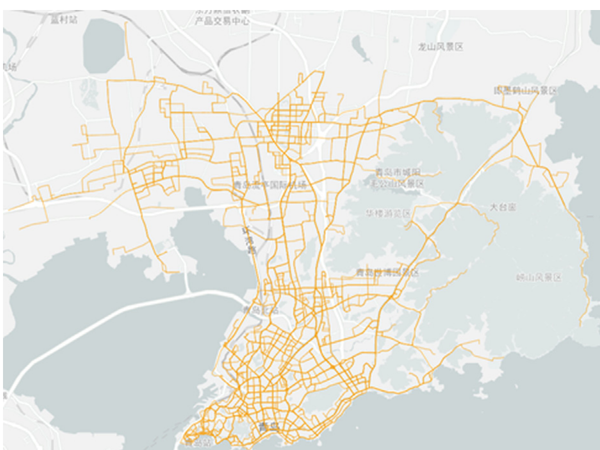

Fig. 4 The bus lines of Qingdao.

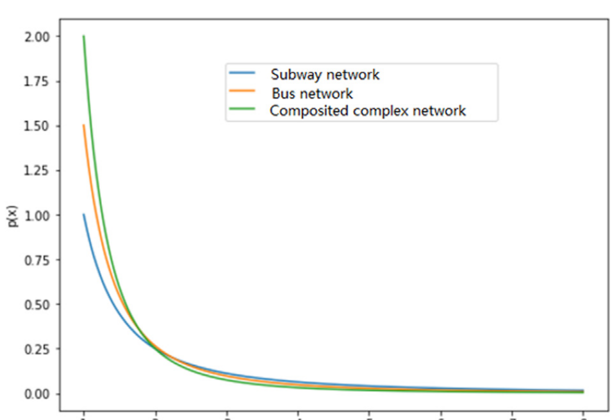

Fig. 5 Degree distribution of three networks.

of all edges, and the proportion of double line coefficient of each edge were calculated; after that, the passenger flow of each edge was obtained by multiplying the mean daily passenger volume of a bus with the proportion of double line coefficient of that edge. The transfer rules of the subway network were designed in a similar manner.

In this paper, the local sensitivity analysis method is mainly used to analyze the influence of the state changes of each node and edge on the network robustness in UPT MSCCN. Similar to the method of node sensitivity analysis, the local sensitivity analysis method is also used in the edge sensitivity analysis. Therefore, sensitivity is regarded as the feature of network robustness.

Next, the passenger flow was loaded into the MSCCN for simulation, which aims to (1) disclose the effect of external disturbance variable *edv* on cascading failure of MSCCN; (2) clarify the relationship between tolerance coefficient β of stations/lines and the robustness of the MSCCN; (3) reveal the influence of the coupling strength of nodes and edges ϑ_1 and ϑ_2 over cascading failure of MSCCN.

(1) Effect of external disturbance variable

In view of the above objectives, the control variates method [31] was introduced to analyze the influence of different variables on the model results. Firstly, the states of all nodes and edges were initialized in the range of [0,1], with $\beta = 0.3$, $\vartheta_1 = 0.2$, $\vartheta_2 = 0.2$, and $edv = \{1.1, 1.3, 1.5, 1.7, 1.9, 2.1\}$. On this basis, the cascading failure of the MSCCN in Qingdao was simulated by the proposed cascading failure model. The cascading failure process and robustness results of the MSCCN obtained through the simulation were recorded (Table 1).

As shown in Table 1, with the increase of *edv* value, the convergence time of the cascading failure of the MSCCN exhibited a reversed N-shaped curve. This means the propagation of station/line failure does not necessarily pick up speed with the growing intensity of extreme conditions. Rather, the failure propagation will speed up, only if the intensity reaches a certain level.

During the cascading failure of the UPT MSCCN, the curves of $\rho(|N|)$,$\rho(E)$, $\rho(f)$, and R were all on the rise under most conditions, indicating that the MSCCN only has one failure process. At $edv = 1.5$, the curves of $\rho(|N|)$,$\rho(E)$, $\rho(f)$, and R entered the second rising phase. This is because the passenger flow of the failed node or edge moves towards remote node or edge, which has smaller capacity than the node or edge connected to the failed node or edge.

(2) Effect of tolerance coefficient

Tolerance coefficient β can change the capacity of nodes and edges in UPT MSCCN, which in turn affect the failure of nodes and edges. Therefore, the robustness of the MSCCN of Qingdao was simulated under different tolerance coefficients. The convergence time of the cascading failure of the MSCCN was recorded in Table 2.

As shown in Table 2, with the increase of tolerance coefficient, the duration of cascading failure of the whole network increased at first and then decreased. In most cases, the MSCCN only had one cascading failure process. The only exception I s$\beta = 0.4$, where the network had two cascading failure processes. Hence, the cascading failure of the network

Table 1 The cascading failure results of the MSCCN with different external disturbance variables.

edv	Convergence time	$\rho(\|N\|)$	$\rho(E)$	$\rho(f)$	R
1.1	15	0.365	0.612	0.316	0.431
1.3	9	0.152	0.322	0.221	0.226
1.5	27	0.196	0.431	0.438	0.351
1.7	15	0.533	0.752	0.579	0.617
1.9	12	0.872	0.947	0.925	0.918
2.1	11	0.872	0.949	0.923	0.198

Table 2 The cascading failure results of the MSCCN with different tolerance coefficients.

Capacity tolerance coefficient β	Convergence time	$\rho(\|N\|)$	$\rho(E)$	$\rho(f)$	R
0.0	10	0.935	0.987	0.999	0.981
0.1	16	0.935	0.985	0.962	0.976
0.2	16	0.796	0.886	0.788	0.827
0.3	15	0.366	0.609	0.309	0.417
0.4	39	0.152	0.297	0.127	0.189
0.5	5	0.078	0.153	0.085	0.108

Table 3 The cascading failure results of the MSCCN with different node coupling strengths.

Node coupling strength ϑ_1	Convergence time	$\rho(\|N\|)$	$\rho(E)$	$\rho(f)$	R
0.1	15	0.357	0.625	0.316	0.431
0.3	15	0.568	0.758	0.507	0.607
0.5	15	0.568	0.762	0.546	0.622
0.7	15	0.568	0.762	0.546	0.622

Table 4 The cascading failure results of the MSCCN with different edge coupling strengths.

Edge coupling strength ϑ_2	Convergence time	$\rho(\|N\|)$	$\rho(E)$	$\rho(f)$	R
0.1	15	0.357	0.625	0.316	0.431
0.3	15	0.338	0.616	0.313	0.428
0.5	15	0.401	0.660	0.327	0.463
0.7	20	0.659	0.845	0.587	0.698

is essentially affected by the capacity change of nodes and edges. At $\beta = 0.0$ or $\beta = 0.1$, the values of $\rho(|N|)$,$\rho(E)$, $\rho(f)$, and R all approached 1, indicating that the network is least robust when its nodes and edges have small capacities.

(3) Effect of node coupling strength

Node coupling strength ϑ_1 is positively correlated with the degree of correlation between network nodes. The greater the coupling strength, the more prominent the impact of the state of a node on that of its adjacent nodes. To disclose the effect of node coupling strength on MSCCN robustness, the robustness of the MSCCN of Qingdao was simulated under different node coupling strengths.

As shown in Table 3, despite the change of node coupling strength, the convergence time of the MSCCN remained unchanged, indicating that the correlation between nodes has little to do with the duration of cascading failure. At $\vartheta_1 = 0.3$, the values of $\rho(|N|)$,$\rho(E)$, $\rho(f)$, and R were all relatively small; At $\vartheta_1 = 0.5$, the values of $\rho(|N|)$,$\rho(E)$, $\rho(f)$, and R were all relatively large and on roughly the same level. The results show that a small node coupling strength benefits network robustness. Moreover, at $\vartheta_1 > 0.5$, the increase of node coupling strength virtually had no impact on the robustness of the MSCCN.

(4) Effect of edge coupling strength

Like node coupling strength, edge coupling strength ϑ_2 is positively correlated with the degree of correlation between the edges in the MSCCN. The greater the coupling strength, the more prominent the impact of the state of an edge on that of its adjacent edges. To disclose the effect of edge coupling strength on MSCCN robustness, the robustness of the MSCCN of Qingdao was simulated under different edge coupling strengths.

As shown in Table 4, at $\vartheta_2 \leq 0.5$, the convergence time of cascading failure of the MSCCN remained the same; at $\vartheta_2 = 0.7$, the convergence time became longer. Therefore, the cascading failure of the network is only greatly affected by the change of edge coupling strength, when the edges are strongly correlated with each other. At $\vartheta_2 = 0.1$, $\vartheta_2 = 0.3$, and $\vartheta_2 = 0.5$, the curves of $\rho(|N|)$,$\rho(E)$, $\rho(f)$, and R were very close, and lower than those of $\vartheta_2 = 0.7$. Therefore, a small edge coupling strength is conducive to network robustness, and the growing edge coupling strength has a limited effect on how robust the network is.

6. Conclusions

This paper combines the bus network and subway network of the UPT into an MSCCN, and then proposes a flow-based cascading failure model of UPT MSCCN. Next, the transfer rules of passenger flow were put forward under the failure of nodes and edges and passenger overload, respectively. According to the meaning of system robustness, three robustness indices were selected from the aspects of network topology and network function: the node size in the most connected subgraph, the network efficiency, and the change of passenger flow in the most connected subgraph. The selected indices were integrated into a synthetic operator for MSCCN robustness. Finally, the effects of different variables on MSCCN robustness can be proved through simulations. Although this paper discusses the cascading failure of public transport network under extreme weather conditions, the cascading failure caused by accidental traffic accidents and other factors are similar.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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