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Implications of Peer-to-Peer Product Sharing when a Firm Joins the Sharing Market

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Abstract

In peer-to-peer product sharing markets, the consumers, who own some products but do not fully utilize them, may share their products with some renters who do not own the products. In this paper, we consider a peer-to-peer product sharing problem, in which a firm that sells a product in a selling market may also directly share the product with the consumers. There are two types of consumers with high and low usage levels of the product. They may (1) buy the product to become an owner, and rent it out when they do not use it; (2) not buy the product but be a renter to rent the product for usage; or (3) neither buy nor rent the product. By analyzing a novel model, we study the effects of the firm's direct involvement in the product sharing and the firm's strategic decision of the product quality on equilibrium outcomes. We find that the firm will join the sharing market only when the proportion of high-usage consumers and the cost of joining are relatively low. When the firm can strategically decide the product quality, it is optimal for the firm to improve the product quality when it joins the sharing market. Moreover, we derive the results on how the selling price, selling quantity, sharing price, and the numbers of product owners and renters change when the firm joins the sharing market, and generate strategic and economic implications of the research findings.

***Keywords*:** sharing economy; peer-to-peer product sharing; pricing; quality

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1 Introduction

Peer-to-peer product sharing is widely adopted in various industries in the world, due to recent technological advances. Through the sharing platforms such as Uber, Turo and Airbnb, the product owners who own the products and do not fully utilize them, may share their products with the consumers who do not own the products. In traditional sharing economy, the product sharing is among the consumers.

However, the firm that sells the products in the market may also join the sharing market in order to gain more profits. For example, *car2go* is a car-sharing platform belonging to the *Daimler AG* company which owns the car brand of *Smart* and *Mercedes-Benz* in the selling market. It offers *Smart* and *Mercedes-Benz* vehicles and features one-way point-to-point rentals [1]. Meanwhile, consumers on the platform of *car2go* can not only share their own cars, but also rent the cars from the company or other owners [2].

In the literature, a few researches have studied the peer-to-peer product sharing (e.g., Benjaafar et al. (2018b), Jiang and Tian (2016) and Tian and Jiang (2018)). However, they consider that the product sharing is only among the consumers. The effects of sharing by the firm are still unknown. Therefore, in this paper, we study the product sharing when the firm that sells the product in the selling market also participates in the sharing economy. We seek to examine the following fundamental questions. First, what are the effects of the firm's direct involvement in the product sharing? Second, what are the effects of the firm's strategic decision of the product quality?

To address these questions, we consider a novel sharing economy model in which the firm not only sells products to the consumers, but also considers providing the product for rentals in the sharing market. The firm decides the selling price (and sharing quantity when it joins the sharing market) of the product. The sharing price is determined by the sharing market when the supply matches the demand in the market. We capture two typical characteristics of the consumers, i.e., the product valuation and the product usage level,

[1] https://www.cnbc.com/2017/12/05/car2go-adds-mercedes-benz-to-its-new-york-city-fleet.html
[2] https://www.car2go.co.il/en/mycar2go/

which play significant roles in consumers' product purchasing and renting decisions. The consumers may 1) buy the product to become an owner, and rent it out when they do not use it, 2) not buy the product but be a renter to rent the product for usage, or 3) neither buy nor rent the product. We first study the problem for a given quality level, and then extend our analysis to consider that the firm can strategically set the product quality.

To the best of our knowledge, this is the first study investigating the strategic and economic implications in sharing economy when a firm that sells the product in the selling market simultaneously shares the product in the sharing market. We highlight three major findings from our analysis. First, we identify the conditions for the firm to be better off when it joins the sharing market. We assume that there are two types of consumers with high and low usage levels of the product respectively. We find that the firm will join the sharing market when the proportion of the high-usage consumers is relatively low, which implies that there is relatively high demand of product renting in the market. The other condition for the firm to join the sharing market is that the cost of joining is relatively low.

Second, if the product quality is taken as given, then we find that the firm will increase the selling price so as to decrease the selling quantity, when it joins the sharing market. The sharing price in the market will be decreased, the number of owners will be decreased, and the number of renters will be increased, when the firm joins the sharing market. In other words, the firm will induce the decrease of the product sharing from the owners by increasing the selling price, so it can benefit from renting its own products in the market.

Third, if the firm can strategically decide the product quality, we find that the firm will increase the product quality level when it joins the sharing market. It is because a high product quality leads to a high selling quality and a high sharing quality. When the firm joins the sharing market, it can benefit from renting the products in the sharing market in addition to selling the products to the consumers. Thus, it can increase its product quality to increase the profit, although high product quality leads to high production cost. Similar to the case of a given product quality level, in this case the selling price and the number of renters will be increased, when the firm joins the sharing market. However, differing from the case of a given product quality level, in this case selling quantity and sharing price may not be decreased, due to the effects of the production cost of the product quality.

The rest of the paper is organized as follows. In Section 2, we present the literatures related to our paper. In Section 3, we present the model setup. In Section 4, we conduct the analysis when the firm cannot strategically decide its product quality. In Section 5, we study the problem when the firm can make strategic quality decision. In Section 6, we conclude our paper and provide some directions for future research. All proofs are given in the Appendix.

2 Literature Review

Our work is related to the literature stream of peer-to-peer sharing markets. As a growing stream within this literature, peer-to-peer product sharing has attracted the attention of researchers in recent years. Fradkin et al. (2015) studied the review bias in the setting of Airbnb and investigated whether the change in review system can reduce the bias. Zervas et al. (2015) compared the ratings of Airbnb and hotels on TripAdvisor, and found that there is only weak correlation in the ratings across the two platforms. Cullen and Farronato (2014) studied the matching of demand and supply when they are highly variable for peer-to-peer online marketplaces. Recently, Jiang and Tian (2016) and Benjaafar et al. (2018b) examined the strategic and economic impact of product sharing among the consumers. Fraiberger and Sundararajan (2017) developed a dynamic model of peer-to-peer rental market in which the consumers may also trade in the secondary market. Tian and Jiang (2018) studied how the product sharing affects the distribution channel, including the capacity of the manufacturer and the profit sharing between the manufacturer and the retailer.

Empirical studies have also been conducted in peer-to-peer product sharing. Clark et al. (2014) presented the results from the survey of British car-sharing members and showed that the car drivers reduce their personal usage but increase business journeys. Ballús-Armet et al. (2014) examined public perception of peer-to-peer car-sharing. They found that the awareness of car-sharing is low and approximately 25% of the car owners are willing to share their cars. Nijland et al. (2015) found that the car-sharing increases the car usage. Van der Linden (2016) investigated the differences of the growth of peer-to-peer car-sharing in different cities and concluded that cities where the regime of personal car ownership and use is less established lead to more shared cars.

Besides, we consider operations strategies for a firm in the sharing economy which has the feature of the two-sided market (see Rochet and Tirole (2006), Weyl (2010), and Weyl (2010)). In this research stream, Bai et al. (2018) developed a queueing model to study the coordination of supply and demand on a peer-to-peer platform. Benjaafar et al. (2018a) studied the labor welfare on a sharing platform where the labors decide whether and how much to work. Taylor (2018) developed a model with waiting-time-sensitive customers and independent service providers to investigate their impacts on the platform's optimal per-service price and wage.

Moreover, our work is also related to the concept of the 'servicization', which means that the firm retains the ownership of the product and charges the consumers for use. In this research stream, Agrawal and Bellos (2016) studied three models: a pure sales model, a pure servicizing model, and a model which involves both sales and servicizing, to investigate the economic and environmental potential of servicizing. Örsdemir et al. (2018) found that servicization can simultaneously increase the firm's profit and decrease the environmental impact compared to selling product. Bellos et al. (2017) considered that the manufacturer offers a car-sharing service in addition to selling cars, and designs its production line accordingly.

Furthermore, our work complements the research stream on the secondary used-goods market. Anderson and Ginsburgh (1994) concluded that a monopoly seller may gain or lose because of the existence of second-hand market but achieves a form of second-degree price discrimination. Hendel and Lizzeri (1999) presented a model in which the firm can interfere the secondary market in four ways: choosing the durability of the product, controlling the availability of used goods, changing the transaction costs, and influencing consumers' maintenance decisions. Johnson (2011) studied a scenario that the consumer valuations change over time. Chen et al. (2013) investigated the scenarios when the secondary market aids or harms the firm with durable goods.

Lastly, our research is also a complement to the literature of leasing. Desai and Purohit (1998) and Desai and Purohit (1999) studied the optimal marketing strategies with leasing and selling in the monopoly and competitive markets, respectively. Hendel and Lizzeri (2002) studied how the adverse selection in the leasing contract affects the market. Johnson and Waldman (2003) and Agrawal et al. (2012) investigated the relationship between new-car leasing and adverse selection. Agrawal et al. (2012) investigated when leasing can be environmentally worse or greener than selling.

3 The Model

We consider a model of the peer-to-peer product sharing where a monopolist firm is directly involved in the sharing market. On the one hand, the firm sells the product with unit selling price p in the market. The consumers may 1) buy the product and be an owner, and rent out the product when they do not use it, 2) not buy the product but be a renter to rent the product for usage, or 3) neither buy nor rent the product. Let q denote the selling quantity of the product in the market. On the other hand, the firm may also directly share the product with the renters in the sharing market. Let q_s denote the sharing quantity of the firm. The renters pay a sharing price p_s per product usage to the platform, and the platform keeps α proportion of the sharing price and give the remaining proportion $1-\alpha$ to the product owners. Let n_o and n_r denote the numbers of product owners and renters, respectively. Note that the number of product owners is defined only for the consumers, and the firm is excluded in it. The market structure is depicted in Figure 1.

The quality of the product is k. We assume that the total production cost of the product with quality k is ck^2. This quadratic form of the production cost implies an increasing marginal cost of the quality level, and is commonly used in the literature (see, e.g., Tagaras (1994), Lee et al. (2004), and Kaya and Özer (2009)). To avoid trivial outcomes, we assume that $c > \frac{(\theta-1)u_L(\alpha\theta(u_H-1)+u_L)}{\alpha^2\theta(u_H-1)^2+4(\theta-1)u_L^2}$. The firm aims to maximize its profit by optimally deciding

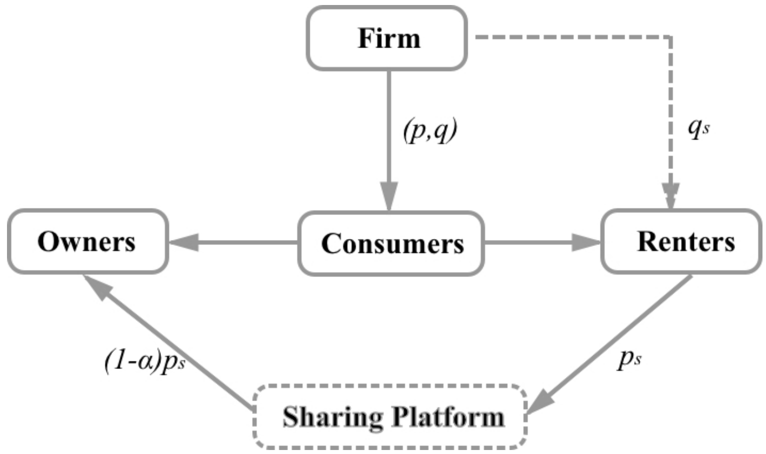

Figure 1: The Market Structure

the selling price (and sharing quantity if it shares products in the sharing market). We first study the problem for a given quality k, and then extend our analysis to consider that the firm strategically sets the product quality.

We capture consumers' two typical characteristics, i.e., the product valuation and the product usage level, which play decisive roles in product purchasing and renting decisions. The product valuation, denoted by v, is the valuation of a consumer to the product when she uses (i.e., buys or rents) the product. We assume that the consumers are heterogeneous in the valuation, which is uniformly distributed in $[0, 1]$. The product usage level is referred as the portion of usage that a consumer uses the product. For each product, the total usage level is 1. We consider that there are two types of consumers regarding the product usage level, i.e., the high usage type and low usage type. The usage levels of the high-type and low-type consumers are u_H and u_l, respectively, and we have $0 < u_L < u_H < 1$. We assume that the proportion of the high-usage consumers is θ ($0 < \theta < 1$), so the proportion of the low-usage consumers is $1 - \theta$. The consumers make purchasing and renting decisions to maximize their utilities.

Consumers' Utilities. Consumers will choose to buy, rent, or do not use the product based on their utilities. If a consumer with usage level u_i ($i = L, H$) buys the product and rents usage level $1 - u_i$ out when she does not use it, then she can receive a payment $(1 - u_i)(1 - \alpha)p_s$ from the sharing platform. Consequently, her utility of ownership can be expressed as $w_o^i = v + k + (1 - u_i)(1 - \alpha)p_s - p$. Note that the quality of the product is k. We assume that the consumers' utility is additively affected by the product quality. If the consumer chooses to rent the product, then she needs to pay $u_i p_s$ to the sharing platform. Thus, her utility from renting can be expressed as $w_r^i = v + k - u_i p_s$. The consumer compares w_o^i, w_r^i and 0, and chooses the one which benefits the most. However, if $w_o^i > w_r^i$ or $w_o^i < w_r^i$ for both $i = L$ and $i = H$, then all consumers will choose to own or rent the product, such that the equilibrium does not exist. Note that $w_o^i - w_r^i = (1 - \alpha + \alpha u_i)p_s - p$. Thus, one condition to guarantee the existence of the equilibrium is that $(1 - \alpha + \alpha u_H)p_s - p > 0 > (1 - \alpha + \alpha u_L)p_s - p$. It implies that the high-usage consumers will choose to buy and the low-usage consumers will choose to rent when their utilities are positive. We define a cutoff value $v_H = p - (1 - u_H)(1 - \alpha)p_s - k$ for the high-usage consumers. Then a high-usage consumer with valuation $v > v_H$ will buy the product and obtain the utility $w_o^H > 0$. Similarly, we define $v_L = u_L p_s - k$ as the cutoff value for the low-usage consumers. And then a low-usage consumer with valuation $v > v_L$ will choose to rent the product and obtain the utility $w_r^L > 0$. The consumers' choices are illustrated in Figure 2.

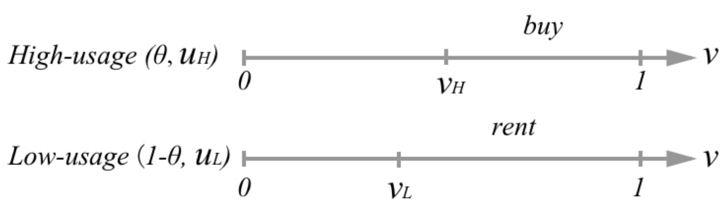

Figure 2: The Consumers' Choices

Market-clearing. In equilibrium, there will be a market-clearing sharing price p_s that makes the supply match the demand. In other words, in equilibrium, the total usage supply of the product (the aggregate of the non-used level of the product $1 - u_H$ from each owner) equals to the total usage demand (the aggregate of the need of usage level u_L from each renter) in the sharing market.

Timing of Events. The timing of events in the model is as follows. First, the firm chooses the selling price, and sharing quantity if it joins the sharing market. Second, the consumers decide whether to buy or rent the product. Third, the consumers who buy the product use the product with their usage levels and rent the product out when they do not use the product. The consumers who decide to be renters use the product and pay the sharing price. The equilibrium sharing price p_s will clear the market to make the supply match the demand.

We summarize the notations used in the paper in Table 1.

Table 1: Summary of Notations

Symbol	Description
p	Selling price of the product
q	Selling quantity of the product
p_s	Sharing price per unit product usage
α	Proportion of the sharing price that the sharing platform keeps
q_s	Sharing quantity of the firm if it joins the sharing market
c_s	Cost of the firm to join the sharing market
k	Quality of the product
ck^2	Total production cost of the products with quality k
v	Consumers' valuation to the product; $v \sim U[0, 1]$
u_i	Type i consumer's usage level of the product; $i \in \{L, H\}$ and $0 < u_L < u_H < 1$
w_r^i	Utility of type i consumer to buy the product; $i \in \{L, H\}$
w_o^i	Utility of type i consumer to rent the product; $i \in \{L, H\}$
θ	Proportion of high-usage consumers, so the proportion of low-usage consumers is $1 - \theta$
n_j	Number of product owners ($j = o$) and renters ($j = r$)

4 Analysis

To investigate the impact of the firm's involvement in the sharing market, we consider two cases: (1) No firm's involvement in the sharing market (N), in which the firm does not share the product in the sharing market, and (2) Firm's involvement in the sharing market (S), in which the firm directly shares the product in the sharing market. We use the general form of X^i to denote the optimal outcomes in equilibrium. Therein, the superscript $i, i \in \{N, S\}$ indicates the scenarios whether the firm is involved in the sharing market. X is the quantity of interest that can be selling price p, selling quantity q, sharing price p_s, number of product owners n_o, number of product renters n_r, and profit π. We first analyze the optimal equilibrium outcomes for the two cases, and then compare the two cases to analyze the impact of the firm's involvement in the sharing market. Note that in this section, we consider the case that the product quality is given, and in the next section, we will investigate the case that the firm will strategically decide its product quality.

4.1 No Firm's Involvement in the Sharing Market (N)

When the firm does not join the sharing market, its decision is the selling price p. The high-usage consumers with valuation $v > v_H$ buy the product and share the usage level of $1-u_H$ to the renters. The low-usage consumers with valuation $v > v_L$ rent the product with usage level u_L. When the supply of the product usage from the owners meets the demand of the product usage from the renters, we have

$$\theta(1-v_H)(1-u_H) = (1-\theta)(1-v_L)u_L.$$

By substituting $v_H = p-(1-u_H)(1-\alpha)p_s-k$ and $v_L = u_L p_s - k$ into the above equation, we can obtain the market-clearing sharing price, i.e., $p_s(p)$, for a given selling price, under which the supply matches the demand:

$$p_s(p) = \frac{(1+k)(1-\theta)u_L - (1+k-p)\theta(1-u_H)}{(1-\alpha)\theta(1-u_H)^2 + (1-\theta)u_L^2}. \tag{1}$$

Then, the corresponding market cutoff value for the high-usage consumers is $v_H(p) = p - (1-u_H)(1-\alpha)p_s(p) - k$, and the proportion of the consumers who buy the product is $\theta(1-v_H(p))$. Obviously, the firm that aims to maximize its profit will produce the product with the selling quantity q to fulfill the demand. Thus, we have

$$\begin{aligned} q(p) &= \theta(1-v_H(p)) \\ &= \theta\left(1-p+(1-u_H)(1-\alpha)p_s(p)+k\right). \end{aligned} \tag{2}$$

Then, the firm's profit function $\pi(p)$ can be expressed as

$$\begin{aligned}\pi(p) &= pq(p) - ck^2 \\ &= p\theta\left(1 - p + (1-u_H)(1-\alpha)\frac{(1+k)(1-\theta)u_L - (1+k-p)\theta(1-u_H)}{(1-\alpha)\theta(1-u_H)^2 + (1-\theta)u_L^2} + k\right) \\ &\quad -ck^2. \end{aligned} \tag{3}$$

It can be verify that the above objective function is concave in p. By considering the first-order condition, we obtain the optimal selling price p^N. Then, by substituting p^N into $p_s(p)$, $q(p)$, and $\pi(p)$ in Equations (1), (2), and (3), respectively, we obtain the optimal sharing price p_s^N, optimal selling quantity q^N, and firm's optimal profit π^N. In addition, we can also obtain the number of the owners and renters in the sharing market. Specifically, the number of owners equals to the selling quantity $n_o^N = q^N$. Given that the supply of the product usage from the owners equals to the demand of the product usage from the renters, then we can obtain the number of renters n_r^N. Next we summarize the equilibrium outcomes.

Lemma 1. *The equilibrium outcomes for the case of no firm's involvement in the sharing market are given as follows:*

$$\begin{aligned} p^N &= \frac{(1+k)[(1-\alpha)(1-u_H)+u_L]}{2u_L}, \\ n_o^N &= q^N = \frac{\theta(1-\theta)(1+k)u_L(1-u_H+u_L)}{2[\theta(1-u_H)^2+(1-\theta)u_L^2]}, \\ p_s^N &= \frac{(1+k)[2(1-\theta)u_L^2 + (1-\alpha)\theta(1-u_H)^2 - \theta(1-u_H)u_L)]}{2[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]}, \\ n_r^N &= \frac{\theta(1-\theta)(1+k)(1-u_H)(1-u_H+u_L)}{2[\theta(1-u_H)^2+(1-\theta)u_L^2]}, \\ \pi^N &= \frac{\theta(1-\theta)(1+k)^2[(1-\alpha)(1-u_H)+u_L]^2}{4[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]} - ck^2. \end{aligned}$$

4.2 Firm's Involvement in the Sharing Market (S)

When the firm joins the sharing market, its decisions are the selling price p and sharing quantity q_s. Compared to the case of no firm's involvement in the sharing market, in this case, the supply of the usage level is added by q_s^S. It is because the firm does not use the product itself, and provides the amount of q_s for rent with usage level 1. When the supply of the product usage from the owners meets the demand of the product usage from the renters, we have

$$\theta(1 - v_H)(1 - u_H) + q_s = (1 - \theta)(1 - v_L)u_L.$$

Similarly, by substituting $v_H = p - (1 - u_H)(1 - \alpha)p_s - k$ and $v_L = u_L p_s - k$ into the above equation, we can obtain the market-clearing sharing price, i.e., $p_s(p, q_s)$, for a given selling price and a given sharing quantity:

$$p_s(p, q_s) = \frac{(1 + k)(1 - \theta)u_L - (1 + k - p)\theta(1 - u_H) - q_s}{(1 - \alpha)\theta(1 - u_H)^2 + (1 - \theta)u_L^2} \tag{4}$$

Then, the corresponding market cutoff value for the high-usage consumers is $v_H(p, q_s) = p - (1 - u_H)(1 - \alpha)p_s(p, q_s) - k$, and the proportion of the consumers who buy the product is $\theta(1 - v_H(p, q_s))$. Given that the selling quantity equals to the consumers who buy the product, we have

$$\begin{aligned} q(p) &= \theta(1 - v_H(p, q_s)) \\ &= \theta\left(1 - p + (1 - u_H)(1 - \alpha)p_s(p, q_s) + k\right). \end{aligned} \tag{5}$$

Then, the firm's profit function $\pi(p, q_s)$ can be expressed as

$$\begin{aligned} \pi(p, q_s) &= pq(p, q_s) + p_s(p, q_s)q_s - ck^2 - c_s \\ &= p\theta\left(1 - p + (1 - u_H)(1 - \alpha)\frac{(1 + k)(1 - \theta)u_L - (1 + k - p)\theta(1 - u_H) - q_s}{(1 - \alpha)\theta(1 - u_H)^2 + (1 - \theta)u_L^2} + k\right) \\ &\quad + \frac{(1 + k)(1 - \theta)u_L - (1 + k - p)\theta(1 - u_H) - q_s}{(1 - \alpha)\theta(1 - u_H)^2 + (1 - \theta)u_L^2}q_s - ck^2 - c_s. \end{aligned} \tag{6}$$

Note that when the firm joins the sharing market, it may choose not to fulfill the demand, because fewer product owners in the market may increase the sharing price and its profit consequently. However, by assuming that the consumers with higher valuations will get the chance to own the product if the selling quantity cannot fulfill the demand, we can show that it is always better off for the firm to fulfill the demand. That is, profit increased from higher sharing price does not overweight the profit decreased from selling fewer products (see the proof in the Appendix).

Denote $\theta_1 = \frac{2u_L^2}{-\alpha(1-\alpha)(1-u_H)^2+(2-\alpha)(1-u_H)u_L+2u_L^2}$. Then, for the case of firm's involvement in the sharing market, we have the following results:

Lemma 2. *The optimal sharing quantity q_s^S decreases in θ. There exists a threshold θ_1 such that if $\theta \geq \theta_1$, then $q_s^S = 0$; otherwise, $q_s^S > 0$. Specifically, if $\theta \geq \theta_1$, then $q_s^S = 0$ and other equilibrium outcomes are the same with the case of no firm's involvement in the sharing market; otherwise, the equilibrium outcomes are as follows:*

$$
\begin{aligned}
q_s^S &= \frac{(1+k)(1-\theta)u_L[(\alpha-2)\theta u_L(1-u_H)+2(1-\theta)u_L^2+\alpha(1-\alpha)\theta(1-u_H)^2]}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}, \\
p^S &= \frac{(1+k)[-\alpha\theta(1-u_H)^2+(2-\alpha)(1-\theta)(1-u_H)u_L+2(1-\theta)u_L^2]}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}, \\
n_o^S &= q^S = \frac{(1+k)(1-\theta)u_L[2u_L-\alpha(1-u_H)]}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}, \\
p_s^S &= \frac{(1+k)[2(1-\theta)u_L-\alpha\theta(1-u_H)]}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}, \\
n_r^S &= \frac{(1+k)(1-\theta)[-\alpha^2\theta(1-u_H)^2+\alpha\theta(1-u_H)u_L+2(1-\theta)u_L^2]}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}, \\
\pi^S &= \frac{1}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}\{-k^2[(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2, \\
&\quad +\alpha\theta(1-\theta)u_L(1-u_H)]+k[-2\alpha\theta(1-\theta)u_L(1-u_H)+2(1-\theta)u_l^2]+ \\
&\quad [-\alpha\theta(1-\theta)u_L(1-u_H)+(1-\theta)u_L^2]\}-c_s.
\end{aligned}
$$

Lemma 2 indicates that when the proportion of the high-usage consumers is relatively high, the optimal sharing quantity of the firm is zero. In other words, it reveals that the firm will choose to join the sharing market only when the proportion of high-usage consumers is relatively small. The reason is that when the proportion of high-usage consumers is relatively large and the proportion of low-usage consumers is relatively small, the product usage supply is relatively high and the demand is relatively low. Consequently, the sharing price is low in the market. Then, for the firm, joining the sharing market with a low sharing price is worse off than just selling in the market with a large proportion of high-usage consumers who may be the product owners. In addition, the firm should increase the sharing quantity when the proportion of the high-usage consumers decreases.

4.3 Impact of the Firm's Involvement

We have obtained the optimal decisions of the firm in the two cases in which the firm joins or does not join the sharing market. Next we examine the impact of the firm's involvement in the sharing market. Note that, from Lemma 2, the firm will not join the sharing market if $\theta \geq \theta_1$. So in the following we only compare the results for the two cases when $\theta < \theta_1$.

We first define the differences between the equilibrium outcomes in the two cases. That is, we let $\Delta p = p^S - p^N$, $\Delta q = q^S - q^N$, $\Delta p_s = p_s^S - p_s^N$, $\Delta n_o = n_o^S - n_o^N$, $\Delta n_r = n_r^S - n_r^N$ and $\Delta\pi = \pi^S - \pi^N$ denote the differences between the selling prices, selling quantities, sharing prices, numbers of owners, numbers of renters and the profits, respectively, in the two cases. The impacts of the firm's involvement on the selling price and selling quantity are presented in the following proposition.

Proposition 1. *The difference between the selling prices is larger than zero, i.e., $\Delta p > 0$, and the difference between the selling quantities is less than zero, i.e., $\Delta q < 0$. As the proportion of high-usage consumers θ increases, Δp decreases and Δq increases.*

Proposition 1 indicates that the firm will increase its selling price and decrease its selling quantity when it joins the sharing market. Note that the condition for the firm to join the sharing market is that the proportion of high-usage consumer is relative small and the proportion of low-usage consumers is large. Under this condition, the sharing price in the market will be high enough to make the supply match the demand. Then the firm would like to benefit from the renting by joining the sharing market. Meanwhile, it will increase the selling price of the product so as to decrease the selling quantity. Consequently, the product usage supplied by the consumers will be decreased, and the firm can obtain more profit from the renting. Besides, the difference between the selling prices is decreasing and the difference between the selling quantities is increasing, when the proportion of high-usage consumers increases.

Note that the number of product owners equals to the selling quantity, so the impact of the firm's involvement on the number of product owners is the same as that on the selling quantity in Proposition 1. The following proposition presents the impact of the firm's involvement on the sharing price and the number of renters.

Proposition 2. *The difference between the sharing prices is less than zero, i.e.,* $\Delta p_s < 0$*, and the difference between the number of renters is larger than zero, i.e.,* $\Delta n_r > 0$*. As the proportion of high-usage consumers* θ *increases,* Δp_s *increases and* Δn_r *decreases.*

Proposition 2 indicates that the sharing price will be decreased and the number of renters will be increased when the firm joins the sharing market. The reason is that when the firm joins the sharing market, the product supply in the sharing market increases, leading to a lower sharing price. Then, more consumers will choose to rent the product to take the advantage of the low sharing price. Thus, the number of renters will increase. Besides, the difference between the sharing prices is increasing and the difference between the number of renters is decreasing, when the proportion of high-usage consumers increases.

The following proposition presents the impact of the firm's involvement on the firm's profit.

Proposition 3. *There exists a threshold* $\tilde{c}_s$ *such that* $\Delta\pi > 0$ *if* $c_s < \tilde{c}_s$*; and* $\Delta\pi \leq 0$ *if* $c_s \geq \tilde{c}_s$.

The firm will choose to join the sharing market when $\Delta\pi$ is positive. Proposition 3 indicates that the firm will not join the sharing market if the cost of joining is high. Note from Lemma 2 that the firm will also not join the sharing market if $\theta \geq \theta_1$. Combining these two results, we conclude that the firm will join the sharing market only when the proportion of high-usage consumers and the cost of joining are relatively low, i.e., $\theta < \theta_1$ and $c_s < \tilde{c}_s$.

5 Strategic Quality Decision

In this section, we consider that the firm can strategically decide the product quality. Similarly, we consider two cases: (1) No firm's involvement in the sharing market (N), and (2) Firm's involvement in the sharing market (S). We use p_k^i, selling quantity q_k^i, sharing price p_{sk}^i, number of product owners n_{ok}^i, number of product renters n_{rk}^i and profit π_k^i to denote the equilibrium outcomes of selling price, selling quantity, sharing price, number of product owners, number of product renters and profit, respectively. Therein, the superscript $i, i \in \{N, S\}$ indicates the scenarios whether the firm is involved in the sharing market. We first analyze the optimal equilibrium outcomes for the two cases, and then compare the two cases to analyze the impact of the firm's involvement in the sharing market.

5.1 No Firm's Involvement in the Sharing Market (N)

Note that if the firm can strategically decide the product quality, then at the beginning, it will first choose the product quality, which is followed by the other events. We can use the backward induction approach to solve the problem. In this way, except for the optimal decision of the quality, other optimal responses have already been given in Lemma 1. It can be verify that the profit function in Lemma 1 is concave in k. By considering the first-order condition, we obtain the optimal quality k^N. Then, by substituting k^N into the optimal responses, we obtain the equilibrium outcomes in the following lemma.

Lemma 3. *When the firm can strategically decide the product quality, the equilibrium out-*

comes for the case of no firm's involvement in the sharing market are given as follows:

$$
\begin{aligned}
k^N &= \frac{\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2}{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2},\\
p_k^N &= \frac{2[(1-\alpha)(1-u_H)+u_L][(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]c}{u_L\{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2\}},\\
n_{ok}^N &= q_k^N = \frac{2(1-\theta)[(1-\alpha)(1-u_H)+u_L]u_Lc}{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2},\\
p_{sk}^N &= \frac{2[(1-\alpha)\theta(1-u_H)^2-\theta(1-u_H)u_L+2(1-\theta)u_L^2]c}{u_L\{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2\}},\\
n_{rk}^N &= \frac{2\theta(1-u_H)[(1-\alpha)(1-u_H)+u_L]c}{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2},\\
\pi_k^N &= \frac{\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2c}{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2}.
\end{aligned}
$$

5.2 Firm's Involvement in the Sharing Market (S)

Similarly, the firm will first choose the product quality, which is followed by the other events. We use the backward induction approach to solve the problem. Except for the optimal decision of the quality, other optimal responses have already been given in Lemma 2. It can be verified that the profit function in Lemma 2 is concave in k. By considering the first-order condition, we obtain the optimal quality level k^S. Then, by substituting k^S into the optimal responses, we obtain the equilibrium outcomes in the following lemma.

Lemma 4. *When the firm can strategically decide the product quality, for the case of firm's involvement in the sharing market, we have the follow results: If $\theta \geq \theta_1$, then $q_s^S = 0$ and other equilibrium outcomes are the same as the case of no firm's involvement in the sharing* *market; otherwise, the equilibrium outcomes are as follows:*

$$\begin{aligned}
k^S &= \frac{(1-\theta)u_L[u_L-\alpha\theta(1-u_H)]}{(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)},\\
p_k^S &= \frac{[-\alpha\theta(1-u_H)^2+(2-\alpha)(1-\theta)(1-u_H)u_L+2(1-\theta)u_L^2]c}{(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)},\\
n_{ok}^S &= q_k^S = \frac{(1-\theta)[2u_L-\alpha(1-u_H)]u_L c}{(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)},\\
p_{sk}^S &= \frac{[2(1-\theta)u_L-\alpha\theta(1-u_H)]c}{(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)},\\
n_{rk}^S &= \frac{[-\alpha^2\theta(1-u_H)^2+\alpha\theta(1-u_H)u_L+2(1-\theta)u_L^2]c}{(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)},\\
\pi_k^S &= \frac{(1-\theta)u_L[u_L-\alpha\theta(1-u_H)]c}{(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)}-c_s.
\end{aligned}$$

5.3 Impact of the Firm's Involvement

We have obtained the optimal decisions of the firm in the two cases in which the firm joins or does not join the sharing market. Next we examine the impact of the firm's involvement in the sharing market. Note that, from Lemma 4, the firm will not join the sharing market if $\theta \geq \theta_1$. So in the following we only compare the results for the two cases when $\theta < \theta_1$.

We first define the differences between the equilibrium outcomes in the two cases. That is, we let $\Delta k = k^S - k^N$, $\Delta p_k = p_k^S - p_k^N$, $\Delta q_k = q_k^S - q_k^N$, $\Delta p_{sk} = p_{sk}^S - p_s^N$, $\Delta n_{ok} = n_{ok}^S - n_{ok}^N$, $\Delta n_{rk} = n_{rk}^S - n_{rk}^N$ and $\Delta \pi_k = \pi_k^S - \pi_k^N$ denote the differences between the product quality levels, selling prices, selling quantities, sharing prices, numbers of the owners, numbers of the renters and the profits, respectively, in the two cases. The impact of the firm's involvement on the quality level is presented in the following proposition.

Proposition 4. *The difference between the quality levels is larger than zero, i.e., $\Delta k > 0$.*

Proposition 4 indicates that the firm's optimal quality decision when it joins the sharing market is higher than that when it does not join. It implies that if the firm can make strategic quality decision, it will improve its product quality when it joins the sharing market. It is because a high product quality leads to a high selling quality and a high sharing quality. When the firm joins the sharing market, it can benefit from renting the products in the sharing market in addition to selling the products to consumers. Thus, it can increase its product quality to increase the profit, although a high quality level leads to a high production cost.

Next, we investigate the impact of the firm's involvement on the selling price and selling quantity. The results are presented in the following proposition.

Proposition 5. *The difference between the selling prices is larger than zero, i.e.,* $\Delta p_k > 0$*. For the selling quantity, there exists a threshold* c_q *such that* $\Delta q_k > 0$ *if* $c < c_q$*; and* $\Delta q_k \leq 0$ *if* $c \geq c_q$*.*

Proposition 5 indicates when the firm joins the sharing market, it will increase its selling price. This result is the same as that in the case if the product quality is given. However, differing from the case that the product quality is given, here, we find that the selling quantity may not be decreased when the firm joins the sharing market. Specifically, the selling quantity will be decreased if and only if (iff) $c \geq c_q$. It is because when the firm joins the sharing market, it will improve the product quality, which may lead to an increase in the selling quantity, although the selling price is increased.

The impact of the firm's involvement on the number of product owners is the same as that on the selling quantity in Proposition 5, because the number of product owners equals to the selling quantity. The following proposition presents the impact of the firm's involvement on the sharing price and the number of renters.

Proposition 6. *There exists a threshold* c_{ps} *such that* $\Delta p_{sk} < 0$ *if* $c > c_{ps}$*; and* $\Delta p_{sk} \geq 0$ *if* $c \leq c_{ps}$*. For the number of renters, the differences between the number of renters is larger than zero, i.e.,* $\Delta n_{rk} > 0$*.*

Recalling from Proposition 2 that if the product quality is given, the sharing price in the market will be decreased when the firm joins the sharing market. However, it may not be true if the firm can strategically chose the product quality. Proposition 6 indicates that when the firm can choose the product quality, the sharing price in the market may be increased when the firm joins the sharing market, due to the effects of the cost parameter. For the number of renters, we find that it is increased when the firm joins the sharing market. This result is the same as that in the case of given product quality.

The following proposition presents the impact of the firm's involvement on the firm's profit.

Proposition 7. *There exists a threshold $\hat{c}_s$ such that $\Delta\pi_k > 0$ if $c_s < \hat{c}_s$; and $\Delta\pi \leq 0$ if $c_s \geq \hat{c}_s$.*

Proposition 7 indicates that the firm will not join the sharing market if the cost of joining is high, i.e., $c_s \geq \hat{c}_s$. Note from Lemma 4 that the firm will also not join the sharing market if $\theta \geq \theta_1$. Combining these two results, we conclude that the firm will not join the sharing market when the proportion of high-usage consumers or the cost of joining are relatively high, i.e., $\theta > \theta_1$ or $c_s > \tilde{c}_s$. This result is similar to that when the firm cannot strategically decide the product quality.

6 Conclusions

In traditional sharing economy, the peer-to-peer product sharing is among consumers. In this paper, we develop a new sharing economy model in which a firm considers to join the sharing market. We envisage to find some strategic and economic implications when the firm is directly involved in the sharing market. In the model, we assume that there are two types of consumers with high and low usage levels of the product. The consumers decide to buy or rent the product or leave the market, based on the firm's decisions and the corresponding sharing price in the market.

We conclude that the firm will join the selling market only when the cost of joining and the proportion of high-usage consumers are relatively low. When the proportion of high-usage consumers is relatively low, the proportion of low-usage consumers is relatively high, such that the sharing supply is relatively low and the renting demand is relatively high in the market. We show how the selling price, selling quantity, sharing price, and the numbers of product owners and renters change when the firm joins the sharing market. We also conclude that when the firm can strategically decide the product quality, it will increase the product quality when it joins the sharing market.

We point out some directions for future research. First, we consider a monopoly market. It is interesting to study the product sharing in a competitive market, in which another firm may sell or rent products to the consumers and the sharing platform may also have competitors in the market. Second, we assume that the firm directly sells its product to the consumers. It is worth investigating how the channel structures influence the sharing market. Third, we assume that the fee received by the sharing platform is exogenously given. However, the platform may charge different proportion of the sharing price in different situations (e.g., when the firm joins or does not join the sharing market). So, in future research we may consider the setting with a different proportion of the sharing price. Finally, we do not model the uncertainty or moral hazard in the sharing market. In fact, the consumers may not be able to observe the product quality before she rents a product and the consumers who rent products may use the products carelessly. We may consider the effects of these uncertainties in future research.

Appendix

Proof of Lemma 1. The firm's profit function $\pi(p)$ is concave in p, that is, $\frac{\partial^2\pi(p)}{\partial p^2} = -\frac{2(1-\theta)u_L^2}{(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2} < 0$. We obtain the optimal selling quantity q^N with $\frac{\partial\pi(p)}{\partial p} = 0$. By substituting $p = p^N$ into $q(p), p_s(p)$ and $\pi(p)$, we can easily obtain q^N, p_s^N, and π^N. The number of renters n_r^N can also be obtained given the product usage supply equals the demand $q^N(1-u_H) = n_r^N u_L$. □

Proof of Lemma 2. With the firm's profit function $\pi(p, q_s)$, we solve the equation set of $\frac{\partial\pi(p,q_s)}{\partial p} = 0$ and $\frac{\partial\pi(p,q_s)}{\partial q_s} = 0$ to obtain p^S and q_s^S. p^S and q_s^S are the optimal solutions given that $(\frac{\partial^2\pi(p,q_s)}{\partial p\partial q_s})^2 - \frac{\partial^2\pi(p,q_s)}{\partial p^2}\frac{\partial^2\pi(p,q_s)}{\partial q_s^2} = \frac{\theta(4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2)}{((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)^2} > 0$ and $\frac{\partial^2\pi(p,q_s)}{\partial p^2} = \frac{-2}{(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2} < 0$. Note that we assume that $4(1-\theta)u_L^2 - \alpha^2\theta(1-u_H)^2 > 0$ because otherwise, the optimal selling price will be at the boundary 0 or 1, leading to non-profitable situations. Note that one necessary condition for $n_o^S = q^S = \frac{(1+k)(1-\theta)u_L[2u_L-\alpha(1-u_H)]}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} > 0$ is $2u_L - \alpha(1-u_H) > 0$. The optimal $q_s^S = \frac{(1-\theta)(k+1)u_L(2u_L^2-\theta(-(1-\alpha)\alpha(1-u_H)^2+(2-\alpha)(1-u_H)u_L+2u_L^2))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}$. We have $\frac{\partial^2 q_s^S}{\partial\theta^2} = -\frac{4(2-\alpha)\alpha^2(k+1)(1-u_H)^3u_L^3(2u_L-\alpha(1-u_H))}{(4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2)^3} < 0$, which means that $\frac{\partial q_s^S}{\partial\theta}$ decreases in θ. We also have $\frac{\partial q_s^S}{\partial\theta}|_{\theta=0} = \frac{(k+1)(-(2-\alpha)(1-u_H)(2u_L-\alpha(1-u_H))-4u_L^2)}{8u_L} < 0$. Thus, $\frac{\partial q_s^S}{\partial\theta} < 0$ for all θ and q_s^S decreases in θ. It is obvious that when $\theta_1 = \frac{2u_L^2}{-\alpha(1-\alpha)(1-u_H)^2+(2-\alpha)(1-u_H)u_L+2u_L^2}$, $q_s^S = 0$. Thus, the solutions when $\theta \geq \theta_1$ equal to those when the firm does not join the sharing market. By substituting $p = p^S$ and $q_s = q_s^S$ into $q(p), p_s(p, q_s)$ and $\pi(p, q_s)$, we can easily obtain q^S, p_s^S, and π^S. The number of renters n_r^S can also be obtained given the product usage supply equals the demand $q^S(1-u_H) + q_s^S = n_r^S u_L$. □

Proof of Propositions 1 and 2. It is easy to observe that when $\theta < \theta_1$, we have $\Delta p = \frac{\alpha(k+1)(1-u_H)(2u_L^2-\theta((2-\alpha)(1-u_H)u_L-(1-\alpha)\alpha(1-u_H)^2+2u_L^2))}{2u_L(4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2)} < 0$. Also, it is also obvious that when $\theta < \theta_1$, we have $\frac{\Delta p}{\partial\theta} = \frac{-(2-\alpha)\alpha(1+k)(1-u_H)^2u_L(2u_L-\alpha(1-u_H))}{(4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2)^2} < 0$. In the same way, it is easy to obtain that when $\theta < \theta_1$, we have $\Delta q < 0$, $\frac{\partial q}{\partial\theta} > 0$; $\Delta p_s < 0$, $\frac{\partial p_s}{\partial\theta} > 0$; and $\Delta n_r > 0$, $\frac{\partial n_r}{\partial\theta} < 0$. □

Proof of Proposition 3. The difference between the firm's profits is $\Delta\pi = \tilde{c}_s - c_s$, where

$\tilde{c}_s = \frac{(1+k)^2(1-\theta)[-\alpha(1-\alpha)\theta(1-u_H)^2+(2-\alpha)\theta(1-u_H)u_L-2(1-\theta)u_L^2]^2}{-\alpha^2(1-\alpha)\theta^2(1-u_H)^4+(4-4\alpha-\alpha^2)\theta(1-\theta)(1-u_H)^2u_L^2+4(1-\theta)^2u_L^4}$, and $\tilde{c}_s > 0$. Thus, there exists $\tilde{c}_s$ such that $\Delta\pi > 0$ if $c_s < \tilde{c}_s$; and $\Delta\pi \leq 0$ if $c_s \geq \tilde{c}_s$. $\square$

Proof of Lemma 3. When the firm does not join the sharing market, its optimal profit with given quality k is $\pi^N = \frac{(1-\theta)\theta(k+1)^2((1-\alpha)(1-u_H)+u_L)^2}{4((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)} - ck^2$. Then, given the assumption of $c > \frac{(\theta-1)u_L(\alpha\theta(u_H-1)+u_L)}{\alpha^2\theta(u_H-1)^2+4(\theta-1)u_L^2}$, $\frac{\partial^2\pi^N}{\partial k^2} = \frac{(1-\theta)\theta((1-\alpha)(1-u_H)+u_L)^2}{4((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)} - c < 0$ because otherwise, the optimal quality will be as large or small as possible, which is unrealistic. So π^N is concave in k. Thus, the optimal quality decision k^N is obtained in $\frac{\partial\pi^N}{\partial k} = 0$. By substituting k^N into p^N, n_o^N, p_s^N, n_r^N, and π^N in Lemma 1, we can easily obtain $p_k^N, n_{ok}^N, p_{sk}^N, n_{rk}^N$, and π_k^N. $\square$

Proof of Lemma 4. When the firm joins the sharing market, its optimal profit with given quality k is $\pi^S = \frac{(1-\theta)(k+1)^2u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} - ck^2 - c_s$. Then, given the assumption of $c > \frac{(\theta-1)u_L(\alpha\theta(u_H-1)+u_L)}{\alpha^2\theta(u_H-1)^2+4(\theta-1)u_L^2}$, $\frac{\partial^2\pi^S}{\partial k^2} = \frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} - c < 0$, because otherwise the optimal quality will be as large or small as possible, which is unrealistic. So π^S is concave in k. Thus, the optimal quality decision k^S is obtained in $\frac{\partial\pi^N}{\partial k} = 0$. By substituting k^S into p^S, n_o^S, p_s^S, n_r^S, and π^S in Lemma 2, we can easily obtain $p_k^S, n_{ok}^S, p_{sk}^S, n_{rk}^S$, and π_k^S. $\square$

Proof of Proposition 4. Let $K_1 = (4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2)(\frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}-c) < 0$, $K_2 = (4((1-\alpha)\theta(1-u_H)^2 + (1-\theta)u_L^2))(\frac{(1-\theta)\theta((1-\alpha)(1-u_H)+u_L)^2}{4((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)} - c) < 0$, and $K_3 = (-(1-\alpha)\alpha\theta(1-u_H)^2 + (2-\alpha)\theta(1-u_H)u_L - 2(1-\theta)u_L^2)^2 > 0$. Thus, $\Delta k = \frac{(1-\theta)K_3}{K_1K_2} > 0$. Note that $\frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} > \frac{(1-\theta)\theta((1-\alpha)(1-u_H)+u_L)^2}{4((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)}$. $\square$

Proof of Proposition 5. We first investigate the difference between the selling prices. Let $K_4 = 2u_L^2 - \theta((2-\alpha)(1-u_H)u_L - (1-\alpha)\alpha(1-u_H)^2 + 2u_L^2) > 0$ when $\theta < \theta_1$, $\Delta p_k = \frac{K_4c}{K_1K_2u_L}K_5$, where $K_5 = c(2\alpha(1-u_H)((1-\alpha)\theta(1-u_H)^2 + (1-\theta)u_L^2) + (1-\theta)u_L((1-\alpha)(1-u_H) + u_L)((1-\theta)u_L - \theta(1-u_H)))$. If $(1-\theta)u_L - \theta(1-u_H) > 0$, $\Delta p_k > 0$; and if $(1-\theta)u_L - \theta(1-u_H) \leq 0$, $\Delta p_k > 0$ when $c > -\frac{(1-\theta)u_L((1-\alpha)(1-u_H)+u_L)((1-\theta)u_L-\theta(1-u_H))}{2\alpha(1-u_H)((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)}$. We have $c > \frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} > -\frac{(1-\theta)u_L((1-\alpha)(1-u_H)+u_L)((1-\theta)u_L-\theta(1-u_H))}{2\alpha(1-u_H)((1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2)}$. Thus, $\Delta p_k > 0$. Next, the difference between the selling quantities $\Delta q_k = -\frac{c(1-\theta)u_LK_4}{K_1K_2}K_6$, where $K_6 = 2(2-\alpha)(1-u_H)c - (1-\theta)((\alpha+1)(1-u_H) + u_L)$. $K_6 \geq 0$ when $c \geq \frac{(1-\theta)((1-\alpha)(1-u_H)+u_L)}{2(2-\alpha)(1-u_H)} >$

$\frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}$; and $K_6 < 0$ when $\frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} < c < \frac{(1-\theta)((1-\alpha)(1-u_H)+u_L)}{2(2-\alpha)(1-u_H)}$. We denote $\frac{(1-\theta)((1-\alpha)(1-u_H)+u_L)}{2(2-\alpha)(1-u_H)}$ as c_q. Thus, $\Delta q_k > 0$ when $c < c_q$; and $\Delta q_k \leq 0$ when $c \geq c_q$. $\square$

Proof of Proposition 6. We first investigate the difference between the sharing prices. $\Delta p_{sk} = \frac{K_4 c}{K_1 K_2 u_L} K_7$, where $K_7 = (1-\theta)u_L((2-\theta)u_L - (\alpha+1)\theta(1-u_H)) - c(4(1-\theta)u_L^2 - 2\alpha\theta(u_H-1)^2)$. We denote $c_{ps} = \frac{(1-\theta)u_L\{2u_L-\theta[(1+\alpha)(1-u_H)+u_L]\}}{4(1-\theta)u_L^2-2\alpha\theta(1-u_H)^2}$, and $c_{ps} > \frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2}$. Thus, $\Delta p_{sk} < 0$ if $c > c_{ps}$; and $\Delta p_{sk} \geq 0$ if $c \leq c_{ps}$. Then, we investigate the difference between the numbers of renters. $\Delta n_{rk} = \frac{K_4 c}{K_1 K_2} K_8$, where $K_8 = c(4(1-\theta)u_L^2 - 2\alpha\theta(u_H-1)^2) - (1-\theta)\theta((1-\alpha)(1-u_H)+u_L)(u_L-\alpha(1-u_H))$. $K_8 > 0$ because $c > \frac{(1-\theta)u_L(u_L-\alpha\theta(1-u_H))}{4(1-\theta)u_L^2-\alpha^2\theta(1-u_H)^2} > \frac{\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L][u_L-\alpha(1-u_H)]}{4(1-\theta)u_L^2-2\alpha\theta(1-u_H)^2}$. Thus, $\Delta n_{rk} > 0$. $\square$

Proof of Proposition 7. The difference between the firm's profits is $\Delta\pi_k = \hat{c}_s - c_s$, where $\hat{c}_s = \frac{(1-\theta)[-\alpha(1-\alpha)\theta(1-u_H)^2+(2-\alpha)\theta(1-u_H)u_L-2(1-\theta)u_L^2]^2}{[(4c-1)(1-\theta)u_l^2-\alpha^2\theta c(1-u_H)^2+\alpha\theta(1-\theta)u_L(1-u_H)]\{4c[(1-\alpha)\theta(1-u_H)^2+(1-\theta)u_L^2]-\theta(1-\theta)[(1-\alpha)(1-u_H)+u_L]^2\}}$, and $\hat{c}_s > 0$. Thus, there exists a threshold $\hat{c}_s$ such that $\Delta\pi_k > 0$ if $c_s < \hat{c}_s$; and $\Delta\pi_k \leq 0$ if $c_s \geq \hat{c}_s$. $\square$

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